Econometrics Course: Cost at the Dependent Variable (II)

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Health Economics Resource Center

HSR&D HEALTH SERVICES RESEARCH & DEVELOPMENT

Review of Ordinarily Least Squares (OLS)

- Classic linear model
- Assume dependent variable can be expressed as a linear function of the chosen independent variables, e.g.:
- $Y_i = \alpha + \beta X_i + \varepsilon_i$

Review of OLS assumptions

- Expected value of error is zero $E(\varepsilon_i)=0$
- Errors are independent $E(\varepsilon_i\varepsilon_j)=0$
- Errors have identical variance $E(\varepsilon_i^2)=\sigma^2$
- Errors are normally distributed
- Errors are not correlated with independent variables $E(X_i\varepsilon_i)=0$

Cost is a difficult variable

- Skewed by rare but extremely high cost events
- Zero cost incurred by enrollees who don't use care
- No negative values

Review from last session

- Applying Ordinary Least Squares OLS to data that aren't normal can result in biased parameters
  - OLS can predict *negative* costs
- Log transformation can make cost more normally distributed
- Predicted cost is affected by re-transformation bias
  - Corrected using smearing estimator
  - Assumes constant error (homoscedasticity)

Topics for today's course

- What to do when there is heteroscedasticity?
- What to do when there are many zeros values?
- How to test differences in groups with no assumptions about distribution?
- How to determine which method is best?

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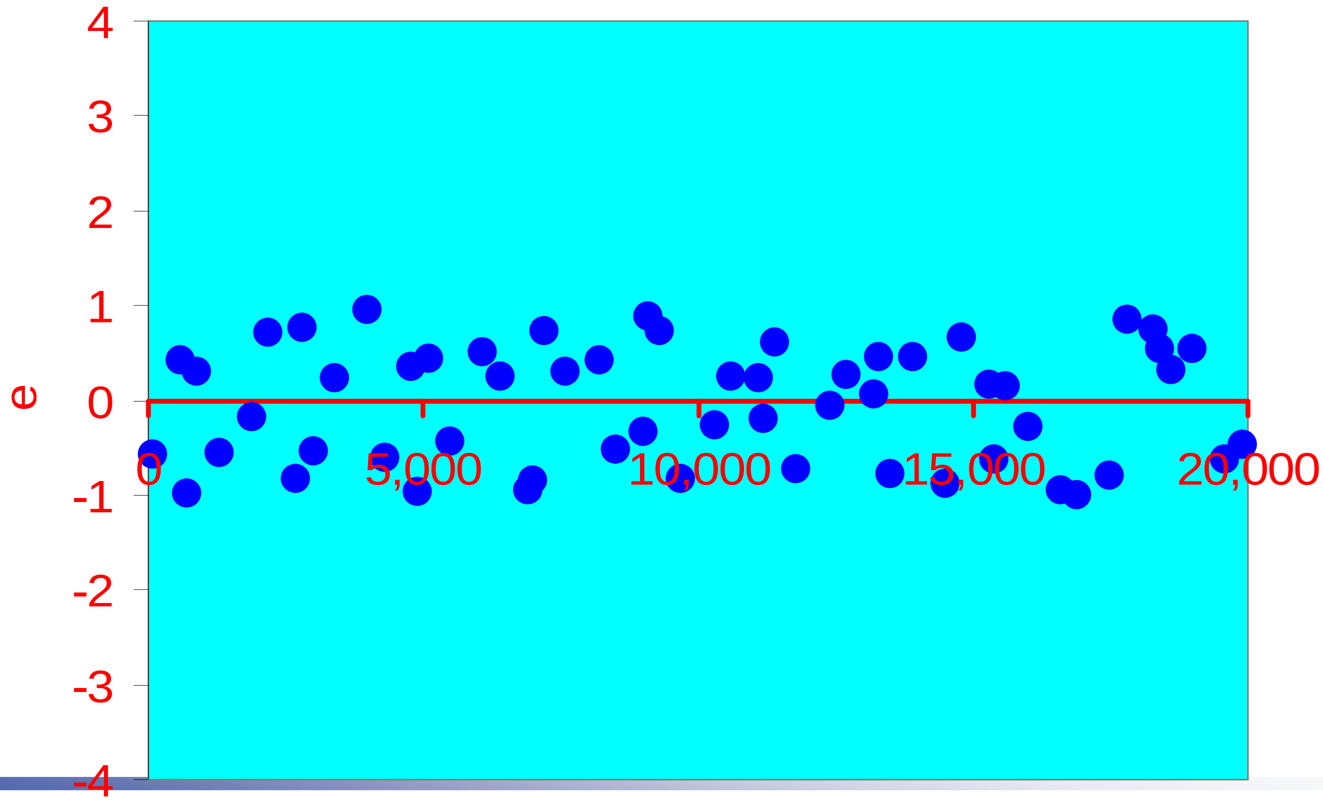
Properties of variance of the errors

- Homoscedasticity
  - Identical variance $E(\varepsilon_i^2)=\sigma^2$
- Heteroscedasticity
  - Variance depends on x (or on predicted y)

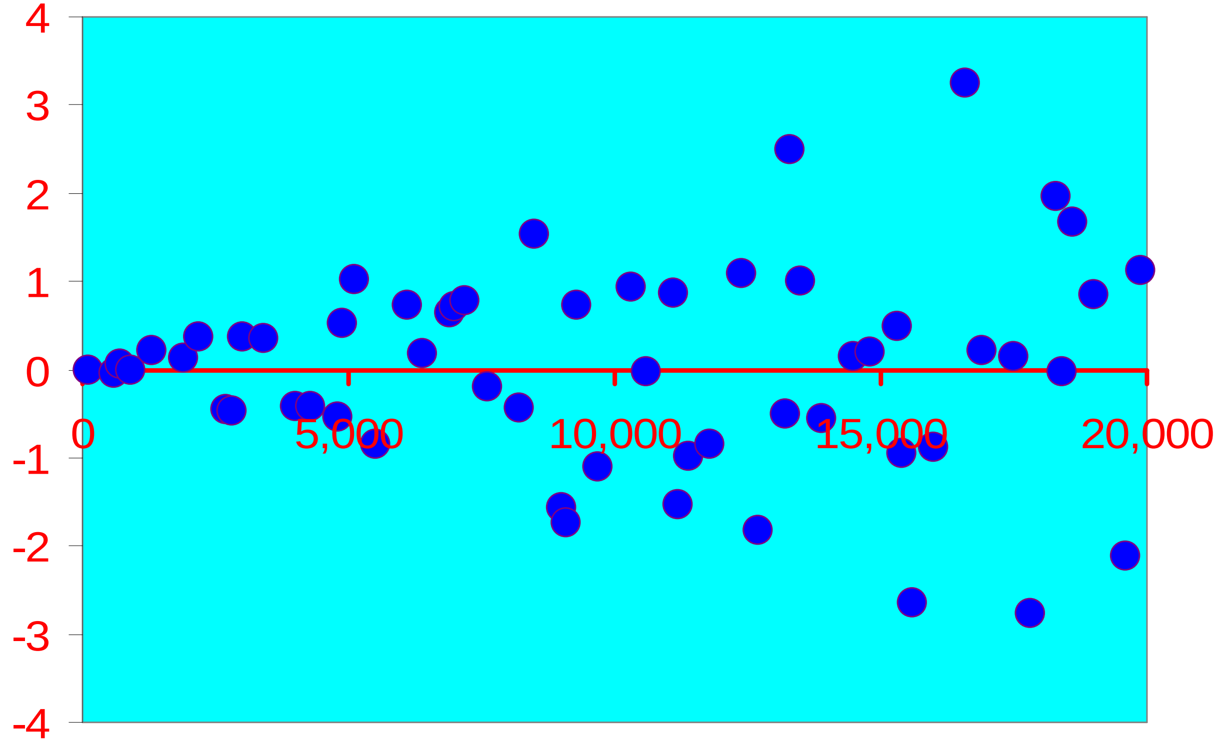
Homoscedasticity

– Errors have identical variance $E(\varepsilon_i^2)=\sigma^2$



Heteroscedasticity

– Errors depend on x (or on predicted y)



Why worry about heteroscedasticity?

- OLS with homoscedastic retransformation
  - “If error term ε is heteroscedastic, estimates can be appreciably biased”
  - Reminding Manning and Mullahy of Longfellow’s nursery rhyme:

    “When she was good, she was very, very good, but when she was bad, she was horrid”

JHE 20:461, 2001

Generalized Linear Models (GLM)

- Analyst specifies a link function g( )
- Analyst specifies a variance function
  – Key reading: "Estimating log models: to transform or not to transform," Mullahy and Manning JHE 20:461, 2001

Link function g( ) in GLM

- $g\,(E\,(y \mid x)\,) = \alpha + \beta x$
- Link function can be natural log, square root, or other function
  - E.g. $\ln\,(\,E\,(\,y \mid x)) = \alpha + \beta x$
  - When link function is natural log, then β represents percent change in y

GLM vs. OLS

- OLS of log estimate: E (ln (y) | x))
- GLM estimate: ln (E (y | x))
  - Log of expectation of y is not the same as expectation of log Y!
- With GLM to find predicted Y
  - No retransformation bias with GLM
  - Smearing estimator not used

Variance function

- GLM does not assume constant variance
- GLM assumes there is function that explains the relationship between the variance and mean
  - $v(y \mid x)$

Variance assumptions for GLM cost models

- Gamma Distribution (most common)
  – Variance is proportional to the square of the mean
- Poisson Distribution
  – Variance is proportional to the mean

Estimation methods

- How to specify log link and gamma distribution with dependent variable Y and independent variables X1, X2, X3
- Stata

```
GLM  Y X1 X2 X3, FAM(GAM) LINK(LOG)
```

- SAS (warning: SAS drops zero cost observations!!!!!!!)

```
PROC GENMOD MODEL Y=X1 X2 X3 /
  DIST=GAMMA LINK=LOG;
```

Choice between GLM and OLS of log transform

- GLM advantages:
  - GLM can correct for heteroscedasticity
  - GLM does not lead to retransformation error
- OLS of log transform advantages
  - OLS is more efficient (standard errors are smaller than with GLM)

Which link function?

- **Box-Cox regression**
  - **Stata command:**

boxcox cost {indep. vars} if y > 0

$$\frac{COST^{\theta}-1}{\theta}=\alpha+\beta x+\varepsilon$$

Which link function?

- **Box-Cox parameter**

Link function	Theta
Inverse (1/cost)	-1
Log(cost)	0
Square root (cost)	.5
Cost	1
Cost Squared	2

Which variance structure with GLM?

Modified Park test

- GLM regression & find residual
- Square the residuals
- Second regression by OLS
  - Dependent variable squared residuals
  - Independent variable predicted y

$$(Y_i - \hat{Y}_i)^2 = \gamma_0 + \gamma_1 \hat{Y}_i + \upsilon_i$$

Which variance structure with GLM?

- Parameter from GLM family test (modified Park test)

$$(Y_i - \hat{Y}_i)^2 = \gamma_0 + \gamma_1 \hat{Y}_i + \upsilon_i$$

γ_1	**Variance**
0	Gaussian (Norma)
1	Poisson
2	Gamma
3	Wald (Inverse Normal)

Other models for skewed data

- Generalized gamma models
  - Estimate link function, distribution, and parameters in single model
  - See: Basu & Rathouz (2005)

Questions?

Topics for today's course

- What to do when there is heteroscedasticity? (GLM models)
- What to do when there are many zeros values?
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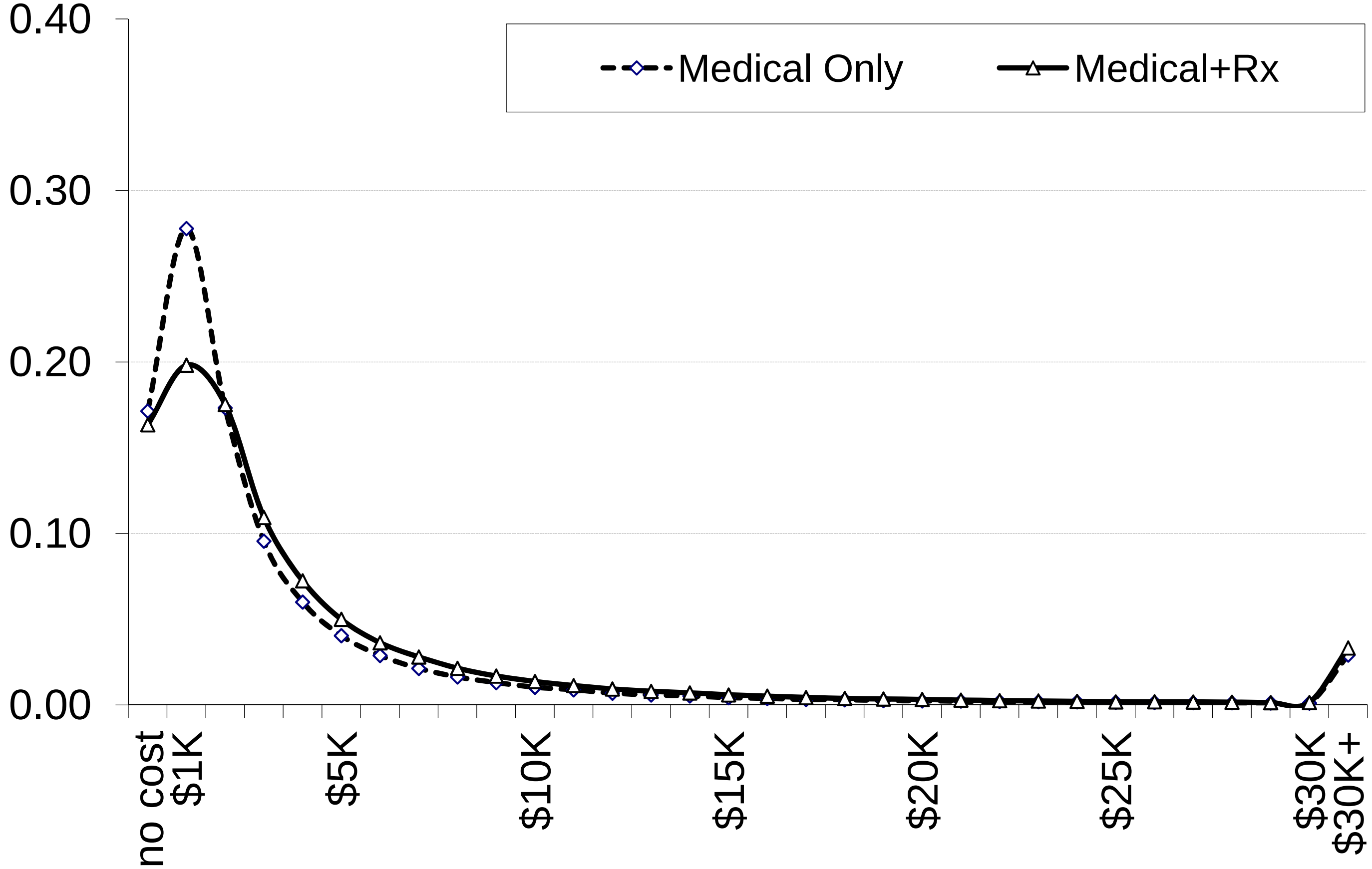
What to do when there are many zeros values?

- Example of participants enrolled in a health plan who have no utilization

Annual per person VHA costs FY09 *among those who used VHA in FY10*



The two-part model

- Part 1: Dependent variable is indicator any cost is incurred
  - 1 if cost is incurred (Y > 0)
  - 0 if no cost is incurred (Y=0)
- Part 2: Regression of how much cost, among those who incurred any cost

The two-part model

- Expected value of Y conditional on X

$$E(Y \mid X) = P(Y > 0) \mid X)E(Y \mid Y > 0, X)$$

Is the product of:

Part 1.
The probability that Y is greater than zero, conditional on X

Part 2.
Expected value of Y, conditional on Y being greater than zero, conditional on X

Predicted cost in two-part model

- Predicted value of Y

$$E(Y \mid X) = P(Y > 0) \mid X)E(Y \mid Y > 0, X)$$

Is the product of:

Part 1.
Probability of any cost being incurred

Part 2.
Predicted cost conditional on incurring any cost

Question for class

$$P(Y > 0) \mid X)$$

- Part one estimates probability Y > 0
  - Y > 0 is dichotomous indicator
  - 1 if cost is incurred (Y > 0)
  - 0 if no cost is incurred (Y=0)
- What type of regression should be used when the dependent variable is dichotomous (takes a value of either zero or one)?

First part of model
Regression with dichotomous variable

- Logistic regression or probit
- Logistic regression uses maximum likelihood function to estimate log odds ratio:

$$\log \frac{P_i}{1-P_i} = \alpha + \beta_1 X$$

Logistic regression syntax in SAS

```
Proc Logistic;
Model Y = X1 X2 X3  / Descending;
Output out={dataset} prob={variable name};
```

- Output statement saves the predicted probability that the dependent variable equals one (cost was incurred)
- Descending option in model statement is required, otherwise SAS estimates the probability that the dependent variable equals zero

Logistic regression syntax in Stata

Logit Y = X1 X2 X3

Predict {variable name}, pr

- Predict statement generates the predicted probability that the dependent variable equals one (cost was incurred)

Second part of model
Conditional quantity

- Regression involves only observations with non-zero cost (conditional cost regression)
- Use GLM or OLS with log cost

Two-part models

- Separate parameters for participation and conditional quantity
  - How independent variables predict
    - participation in care
    - quantity of cost conditional on participation
  - each parameter may have its policy relevance
- Disadvantage: hard to predict confidence interval around predicted Y given X

Alternate to two-part model

- OLS with untransformed cost
- OLS with log cost, using small positive values in place of zero
- Certain GLM models

Topics for today's course

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Non-parametric statistical tests

- Make no assumptions about distribution, variance
- Wilcoxon rank-sum test
- Assigns rank to every observation
- Compares ranks of groups
- Calculates the probability that the rank order occurred by chance alone

Extension to more than two groups

- Group variable with more than two mutually exclusive values
- Kruskall Wallis test
  - is there any difference between any pairs of the mutually exclusive groups?
- If KW is significant, then a series of Wilcoxon tests allows comparison of pairs of groups

Limits of non-parametric test

- It is too conservative
  - Compares ranks, not means
  - Ignores influence of outliers
  - E.g. all other ranks being equal, Wilcoxon will give same result regardless of whether
    - Top ranked observation is $1 million more costly than second observation, or
    - Top ranked observation just $1 more costly
- Doesn’t allow for additional explanatory variables

Topics for today's course

- What to do when there is heteroscedasticity? (GLM models)
- What to do when there are many zeros values? (Two-part models)
- How to test differences in groups with no assumptions about distribution? (Non-parametric statistical tests)
- How to determine which method is best?

Which method is best?

- Find predictive accuracy of models
- Estimate regressions with half the data, test their predictive accuracy on the other half of the data
- Find
  - Mean Absolute Error (MAE)
  - Root Mean Square Error (RMSE)

Mean Absolute Error

- For each observation
  - find difference between observed and predicted cost
  - take absolute value
  - find the mean
- Model with smallest value is best

Root Mean Square Error

- Square the differences between predicted and observed, find their mean, find its square root
- Best model has smallest value

Evaluations of residuals

- Mean residual (predicted less observed)

  or

- Mean predicted ratio (ratio of predicted to observed)
  - calculate separately for each decile of observed Y
  - A good model should have equal residuals (or equal mean ratio) for all deciles

Formal tests of residuals

- Variant of Hosmer-Lemeshow Test
  - F test of whether residuals in raw scale in each decile are significantly different
- Pregibon's Link Test
  - Tests if linearity assumption was violated
    - See Manning, Basu, & Mullahy, 2005

Questions?

Review of presentation

- Cost is a difficult dependent variable
  - Skewed to the right by high outliers
  - May have many observations with zero values
  - Cost is not-negative

When cost is skewed

- OLS of raw cost is prone to bias
  - Especially in small samples with influential outliers
  - "A single case can have tremendous influence"

When cost is skewed (cont.)

- Log transformed cost
  - Log cost is more normally distributed than raw cost
  - Log cost can be estimated with OLS

When cost is skewed (cont.)

- To find predicted cost, must correct for retransformation bias
  - Smearing estimator assumes errors are homoscedastic
  - Biased if errors are heteroscedasctic
- "When she was good, she was very, very good, but when she was bad, she was horrid"

When cost is skewed and errors are heteroscedastic

- GLM with log link and gamma variance
  - Considers heteroscedasctic errors
  - Not subject to retransformation bias
  - May not be very efficient
  - Alternative specification
    - Poisson instead of gamma variance function
    - Square root instead of log link function

When cost has many zero values

- Two part model
  - Logit or probit is the first part
  - Conditional cost regression is the second part

Comparison without distributional assumptions

- Non-parametric tests can be useful
- May be too conservative
- Don’t allow co-variates

Evaluating models

- Mean Absolute Error
- Root Mean Square Error
- Other evaluations and tests of residuals

Next lecture

Non-linear dependent variables

Ciaran Phibbs

May 30, 2012

Key sources on GLM

- MANNING, W. G. (1998) The logged dependent variable, heteroscedasticity, and the retransformation problem, *J Health Econ*, 17, 283-95.
- * MANNING, W. G. & MULLAHY, J. (2001) Estimating log models: to transform or not to transform?, *J Health Econ*, 20, 461-94.
- * MANNING, W. G., BASU, A. & MULLAHY, J. (2005) Generalized modeling approaches to risk adjustment of skewed outcomes data, *J Health Econ*, 24, 465-88.
- BASU, A. & Rathouz P.J. (2005) Estimating marginal and incremental effects on health outcomes using flexible link and variance function models, *Biostatistics* 6(1): 93-109, 2005.

Key sources on two-part models

- * MULLAHY, J. (1998) Much ado about two: reconsidering retransformation and the two-part model in health econometrics, *J Health Econ*, 17, 247-81
- JONES, A. (2000) Health econometrics, in: Culyer, A. & Newhouse, J. (Eds.) *Handbook of Health Economics*, pp. 265-344 (Amsterdam, Elsevier).

References to worked examples

- FLEISHMAN, J. A., COHEN, J. W., MANNING, W. G. & KOSINSKI, M. (2006) Using the SF-12 health status measure to improve predictions of medical expenditures, *Med Care*, 44, I54-63.
- MONTEZ-RATH, M., CHRISTIANSEN, C. L., ETTNER, S. L., LOVELAND, S. & ROSEN, A. K. (2006) Performance of statistical models to predict mental health and substance abuse cost, *BMC Med Res Methodol*, 6, 53.

References to work examples (cont).

- MORAN, J. L., SOLOMON, P. J., PEISACH, A. R. & MARTIN, J. (2007) New models for old questions: generalized linear models for cost prediction, *J Eval Clin Pract*, 13, 381-9.
- DIER, P., YANEZ D., ASH, A., HORNBROOK, M., LIN, D. Y. (1999). Methods for analyzing health care utilization and costs Ann Rev Public Health (1999) 20:125-144 (Also gives accessible overview of methods, but lacks information from more recent developments)

Link to HERC Cyberseminar HSR&D study of worked example

Performance of Statistical Models to Predict Mental Health and Substance Abuse Cost

Maria Montez-Rath, M.S. 11/8/2006

The audio:

- http://vaww.hsrd.research.va.gov/for_researchers/cyber_seminars/HERC110806.asx

The Power point slides:

- http://vaww.hsrd.research.va.gov/for_researchers/cyber_seminars/HERC110806.pdf

Book chapters

- MANNING, W. G. (2006) Dealing with skewed data on costs and expenditures, in: Jones, A. (Ed.) *The Elgar Companion to Health Economics*, pp. 439-446 (Cheltenham, UK, Edward Elgar).