

# Introduction to Medical Decision Making and Decision Analysis

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CENTER FOR PRIMARY CARE AND OUTCOMES RESEARCH

# Agenda

- Decision analysis
- Cost-effectiveness analysis
- Decision trees
- Sensitivity analysis
- Markov models
- Microsimulations

**WHAT IS A DECISION ANALYSIS?**

# What is a decision analysis?

- A quantitative method for evaluating decisions between multiple alternatives in situations of uncertainty

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- A quantitative method for evaluating decisions between multiple alternatives in situations of uncertainty

Decisions between multiple alternatives:

- Allocate resources to one alternative (and not the others)
- There is no decision without alternatives => making a choice

# What is a decision analysis?

- A quantitative method for evaluating decisions between multiple alternatives in situations of uncertainty

Quantitative method for evaluating decisions:

- Gather information
- Assess the consequences of each alternative
- Clarify the dynamics and trade-offs involved in selecting each
- Select an action to take that gives us the best expected outcome

**We employ probabilistic models to do this**

# The steps of a decision analysis

1. Enumerate all relevant alternatives
2. Identify important outcomes
3. Determine relevant uncertain factors
4. Encode probabilities for uncertain factors
5. Specify the value of each outcome
6. Combine these elements to analyze the decision

**Decision trees and related models important for this**

# What is a decision analysis called when its important outcomes include costs?

1. Enumerate all relevant alternatives
2. Identify important outcomes
3. Determine relevant uncertain factors
4. Encode probabilities for uncertain factors
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**Cost-effectiveness analysis** a type of decision analysis that includes costs as one of its outcomes



**WHAT IS A COST-EFFECTIVENESS  
ANALYSIS?**

# What is a cost-effectiveness analysis?

- In the context of health and medicine, a cost-effectiveness analysis (CEA) is a method for evaluating tradeoffs between health benefits and costs resulting from alternative courses of action
- CEA supports decision makers; it is not a complete resource allocation procedure

# Cost-Effectiveness Ratio (CER):

## How to compare two strategies in CEA

- **Numerator:** Difference between costs of the intervention (strategy) and costs of the alternative under study
- **Denominator:** Difference between health outcomes (effectiveness) of the intervention and health outcomes of the alternative

Incremental resources required  
by the intervention

Incremental health effects  
gained with the intervention

$$CER = \frac{C_i - C_{alt}}{E_i - E_{alt}}$$

# Models for decision analysis and CEAs

- **Decision model:** a *schematic* representation of all of the clinically and policy relevant features of the decision problem
  - Includes the following in its structure:
    - Decision alternatives
    - Clinical and policy-relevant outcomes
    - Sequences of events
  - Enables us to integrate knowledge about the decision problem from many sources (i.e., probabilities, values)
  - Computes expected outcomes (i.e., averaging across uncertainties) for each decision alternative

# Building decision-analytic model

1. Define the model's structure
2. Assign probabilities to all chance events in the structure
3. Assign values (i.e., utilities) to all outcomes encoded in the structure
4. Evaluate the expected utility of each decision alternative
5. Perform sensitivity analyses

**Simple enough to be understood; complex enough to capture problem's elements convincingly (assumptions)**

**“All models are wrong;  
but some models are  
useful”**

-- George Box and Norman Draper, 1987

# Building decision-analytic model

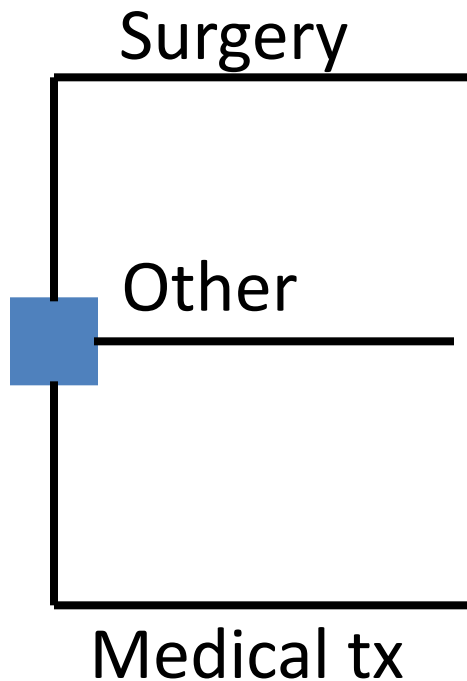
1. *Define the model's structure*
2. Assign probabilities to all chance events in the structure
3. Assign values (i.e., utilities) to all outcomes encoded in the structure
4. Evaluate the expected utility of each decision alternative
5. Perform sensitivity analyses

**WHAT ARE THE ELEMENTS OF A  
DECISION TREE'S STRUCTURE?**



# Decision node

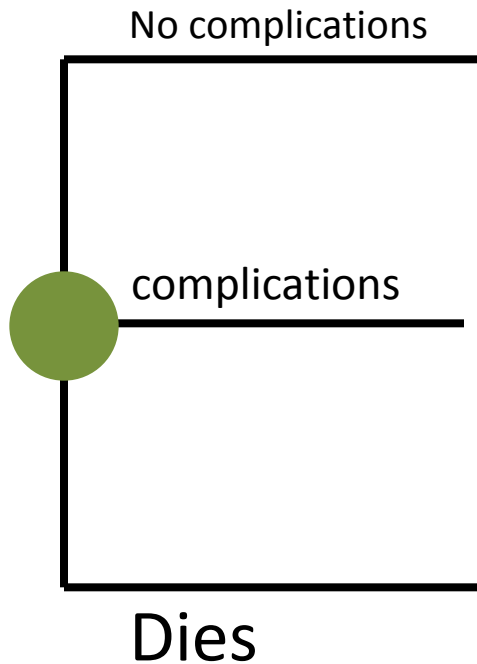
A place in the decision tree at which there is a choice between several alternatives



The example shows a choice between 2 alternatives, but a decision node can accommodate a choice between more alternatives ... provided alternatives are mutually exclusive.

# Chance node

A place in the decision tree at which chance determines the outcome based on probability



The example shows only 2 outcomes, but a chance node can accommodate more outcomes ... provided they are mutually exclusive AND collectively exhaustive.

# What do mutually exclusive and collectively exhaustive mean?

- Mutually exclusive

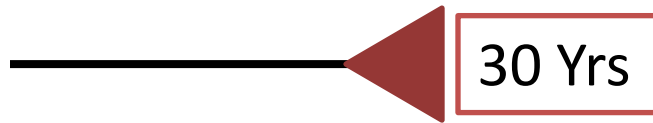
- Only one alternative can be chosen
- Only one event can occur

- Collectively exhaustive

- At least one event must occur
- One of the possibilities must happen
- Taken together, the possibilities make up the entire range of outcomes

# Terminal node

**Final outcome associated with each pathway of choices and chances**



Final outcomes must be valued in relevant terms (cases of disease, Life years, Quality-adjusted life years, costs) so that they can be used for comparisons

# Summary

- **Decision nodes:** enumerate a choice between alternatives for the decision maker
- **Chance nodes:** enumerate possible events determined by chance/probability
- **Terminal nodes:** describe outcomes associated with a given pathway (of choices and chances)

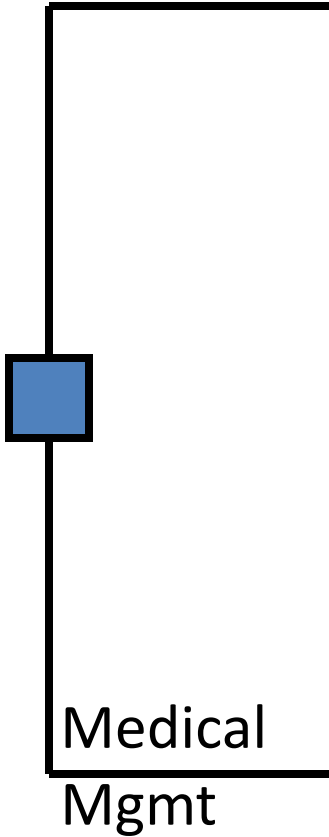
**The entire structure of the decision tree can be described with only these elements**

# Example: decision tree

- Patient presents with symptoms
- Likely serious disease; unknown w/o treatment
- Two treatment alternative:
  - Surgery, which is potentially risky
  - Medical management, which has a low success rate
- With surgery, one must assess the extent of disease and decide between curative and palliative surgery
- ***Goal: maximize life expectancy for the patient***

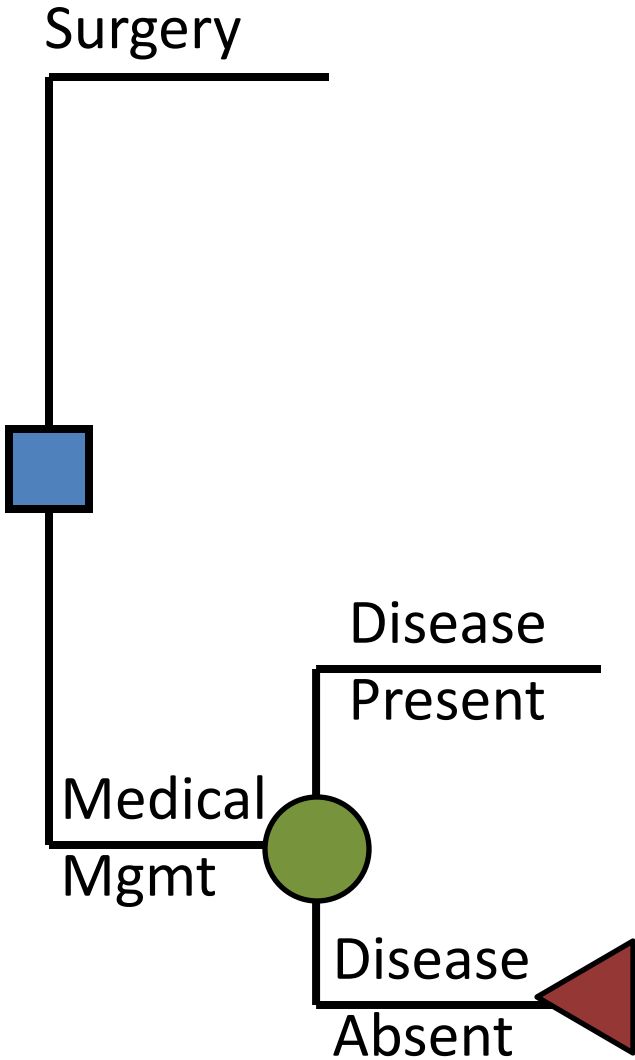
The initial decision is between surgery and medical management

Surgery



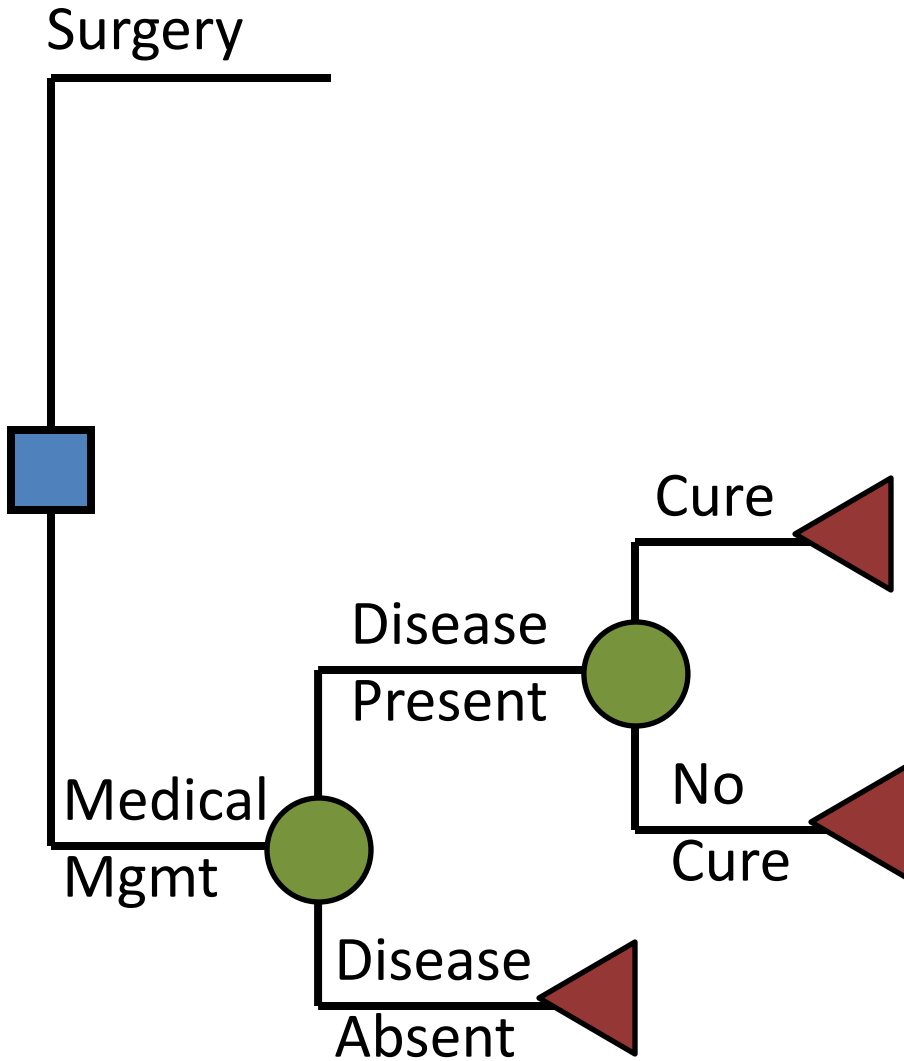
Medical  
Mgmt

Treatment is initiated on patients w/ symptoms; some w/o disease

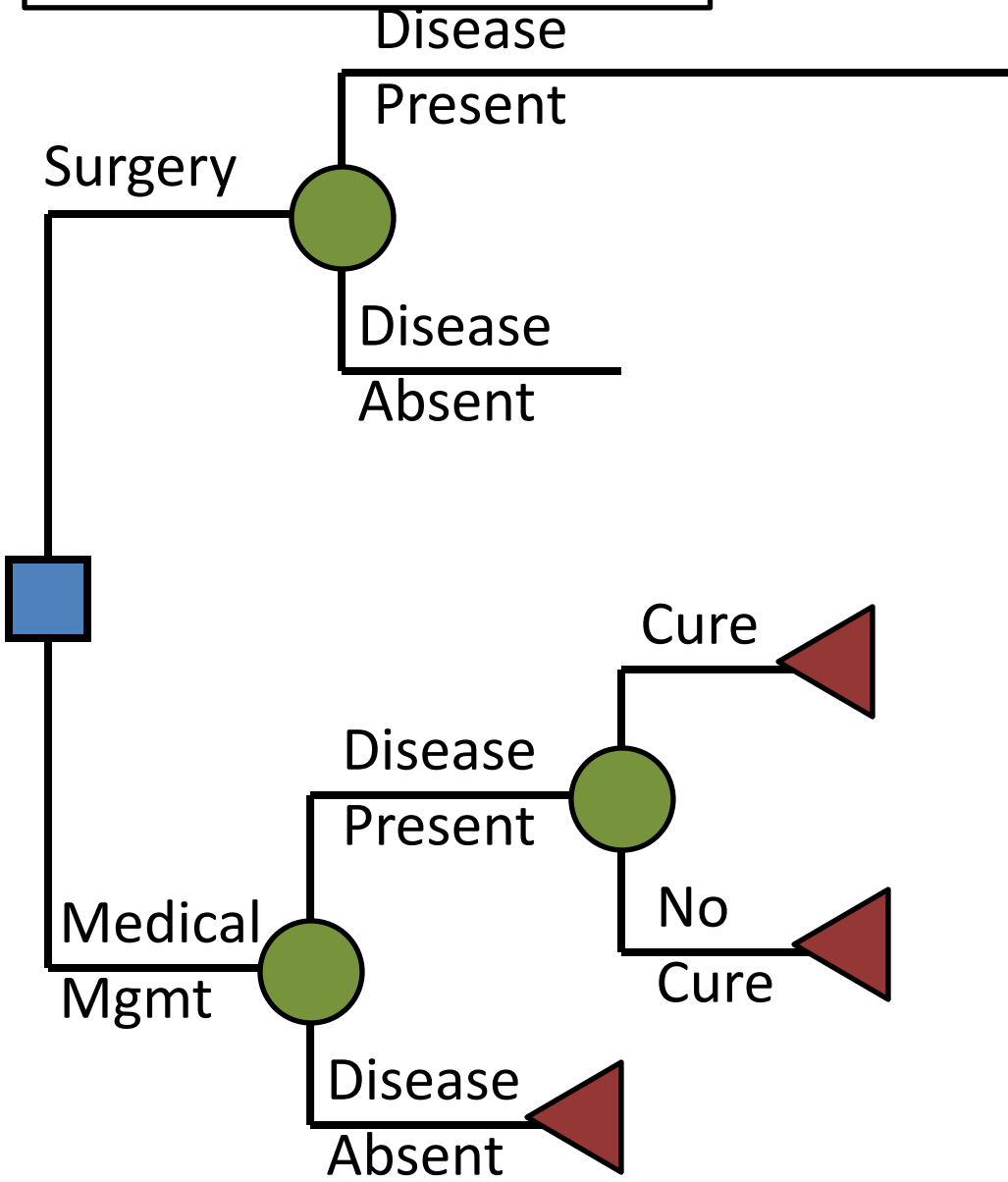




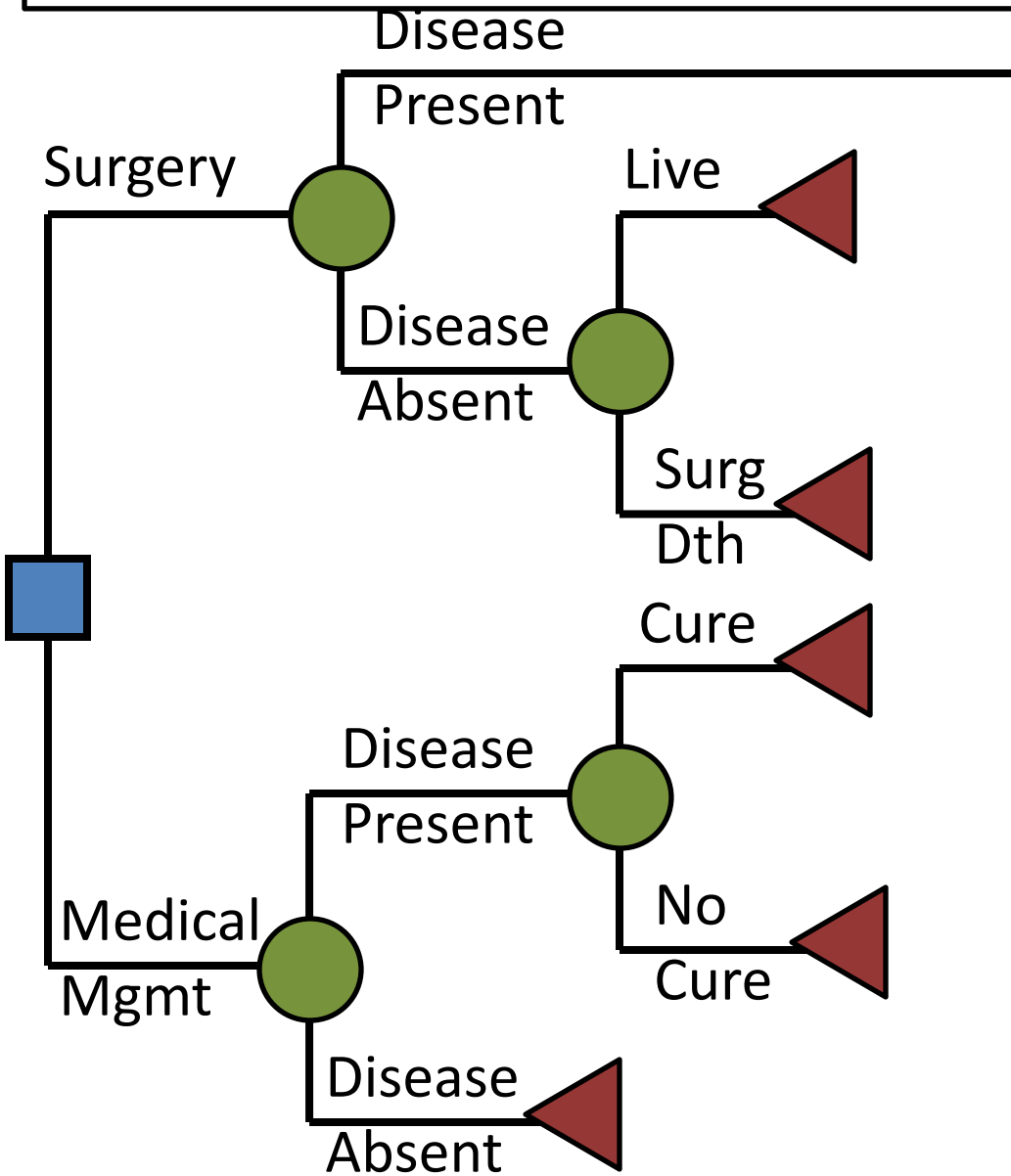
Those with disease have a chance to benefit from treatment

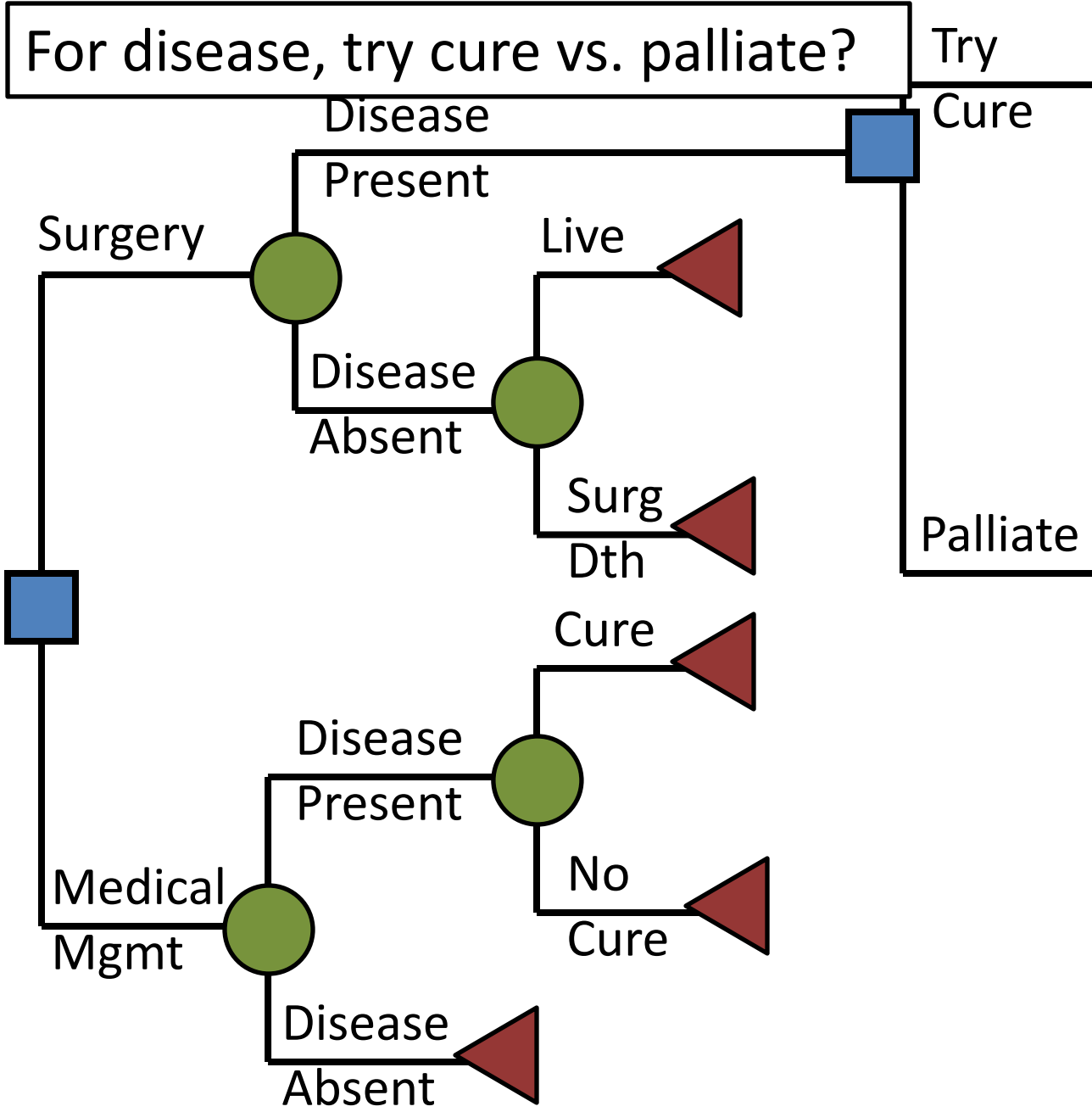


Likewise with surgery

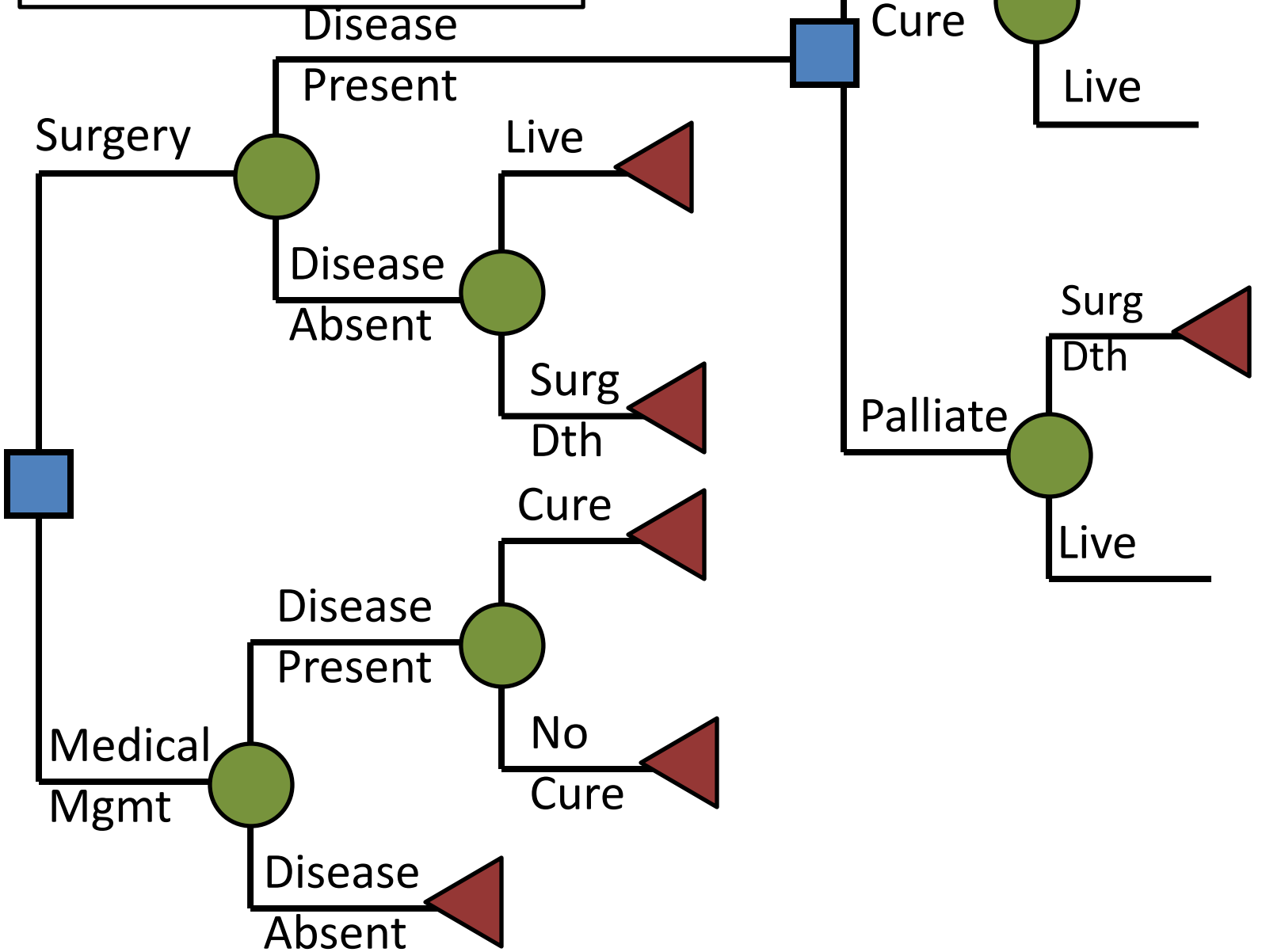


# Surgery is risky even for those with no disease

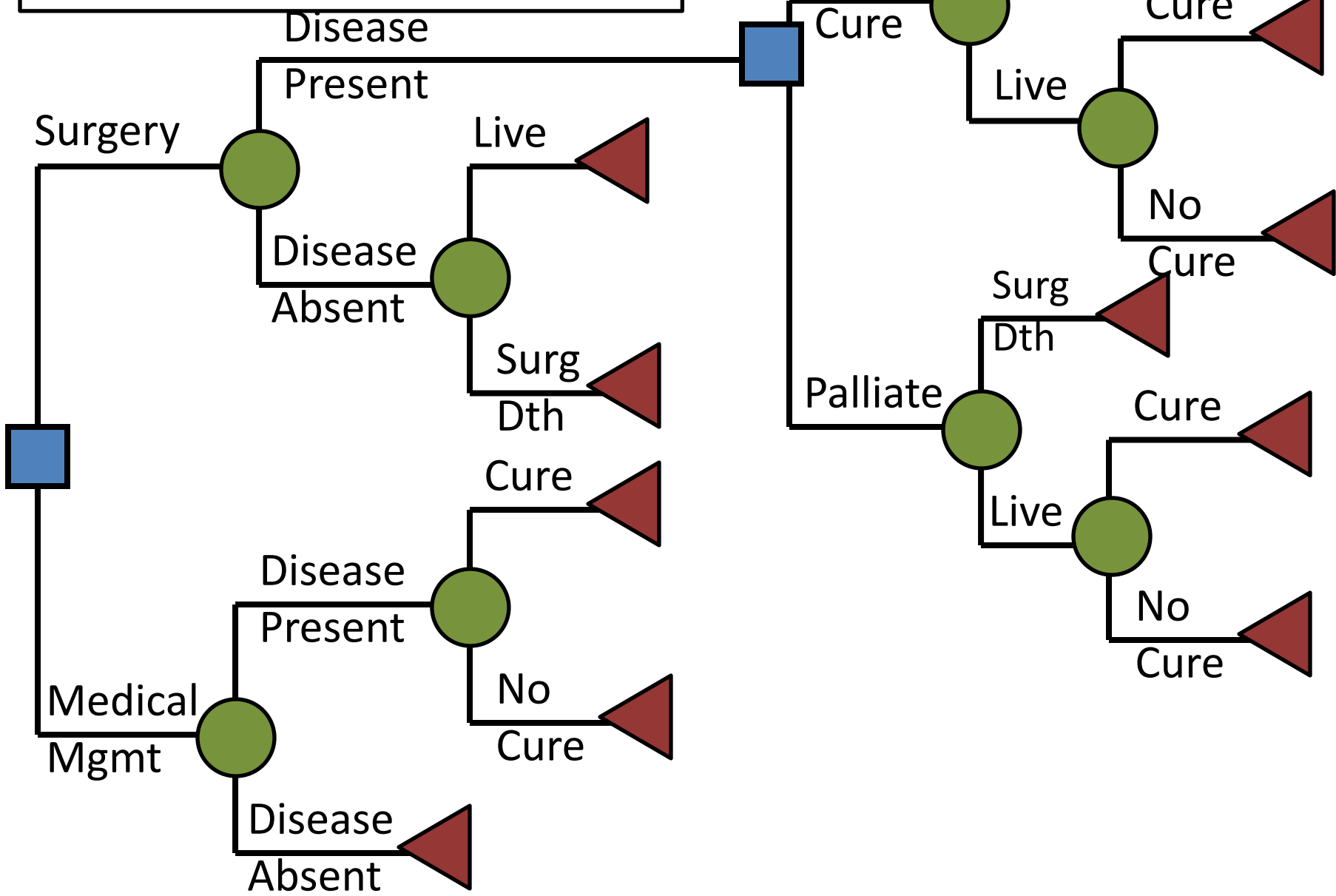




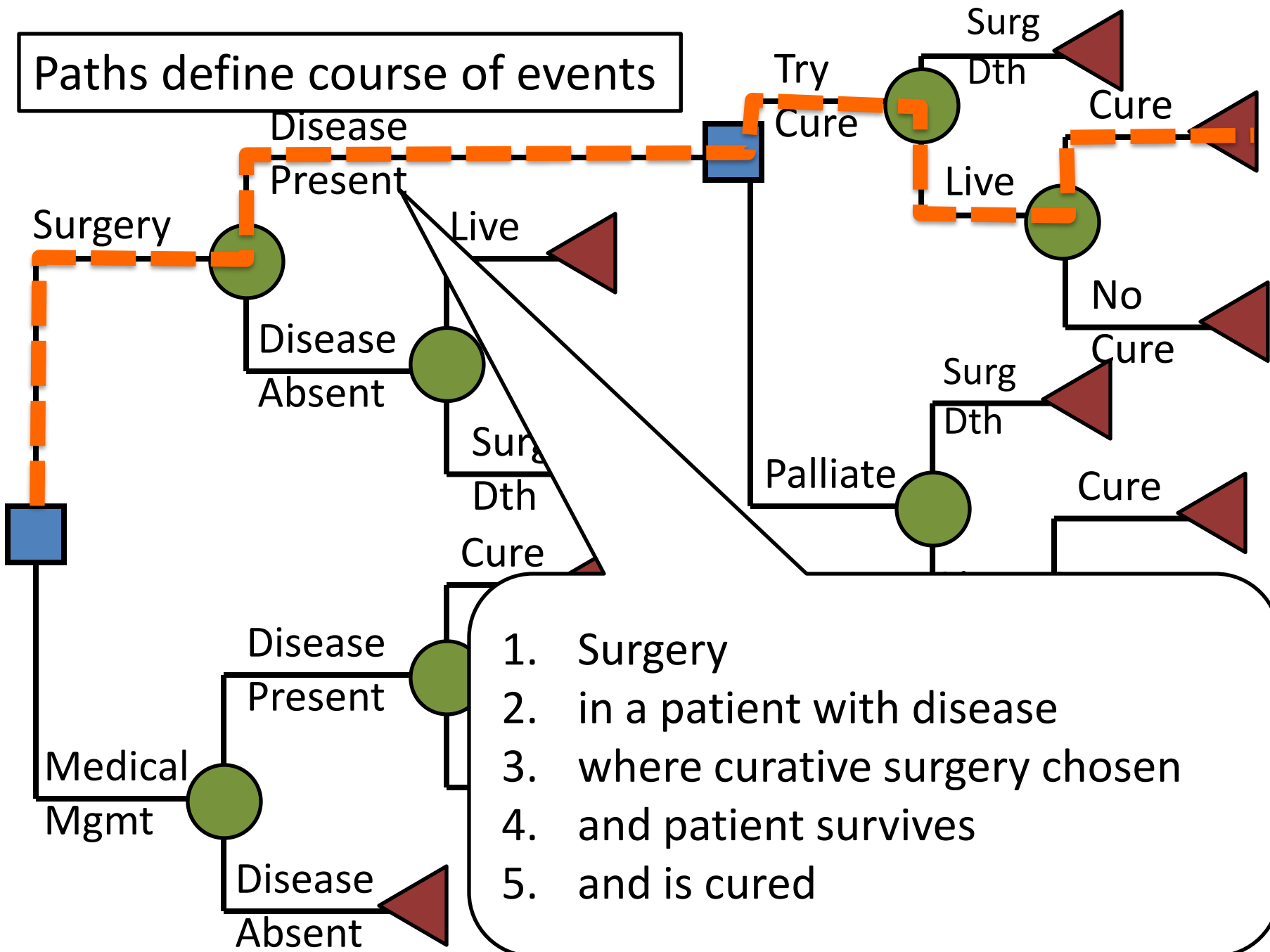
Surgical risks here too



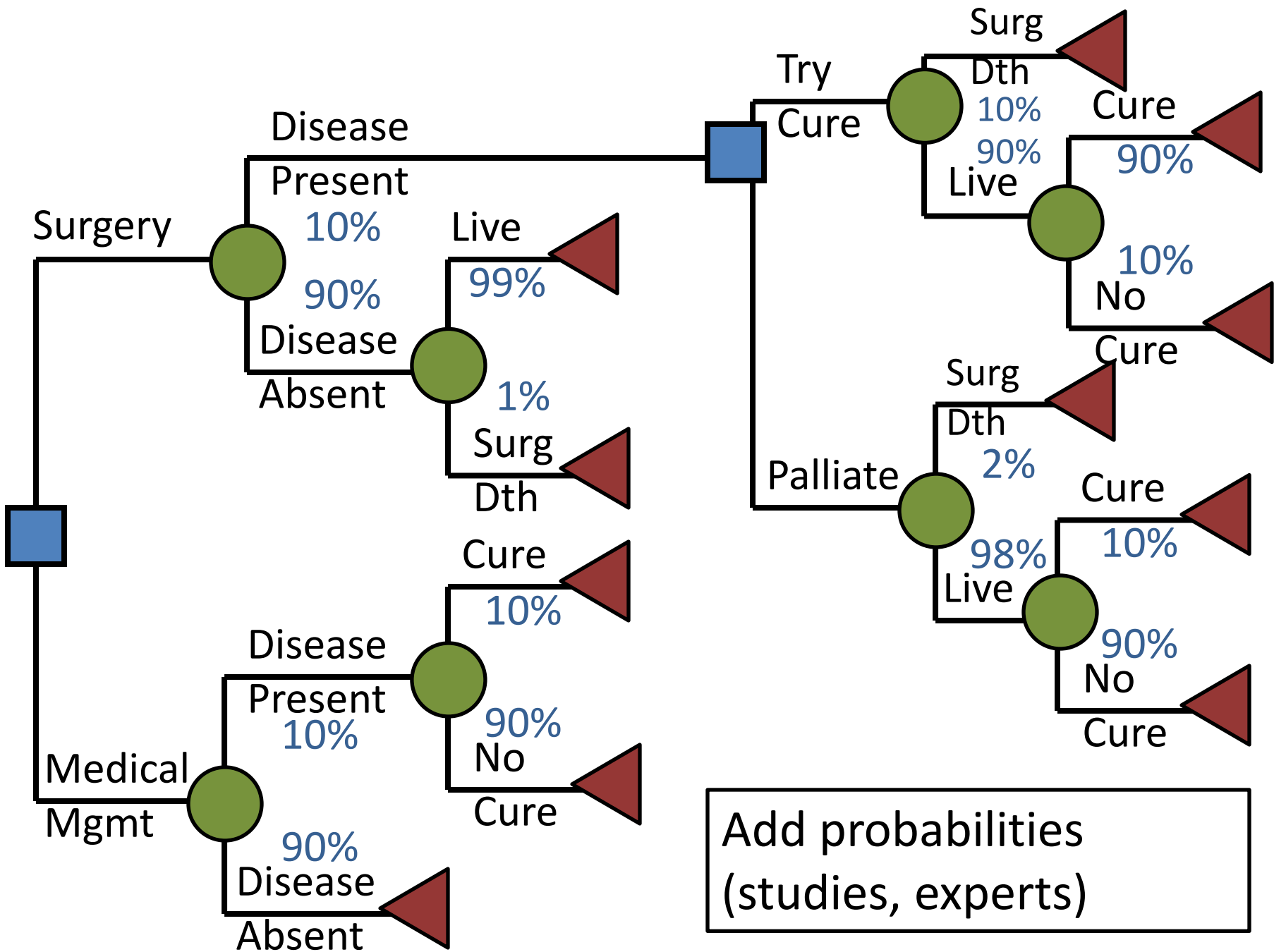
# Chance of cure for survivors



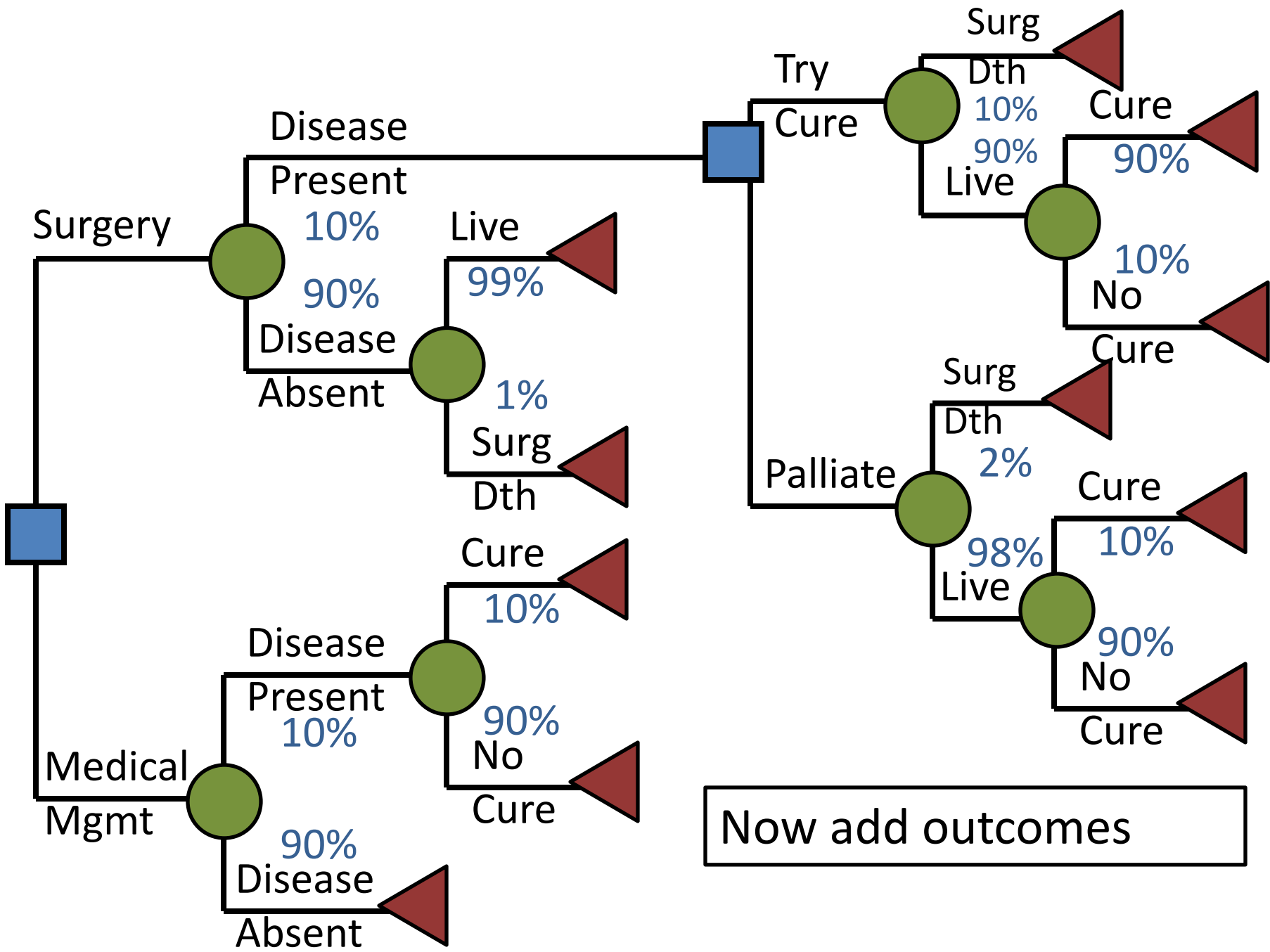
Paths define course of events



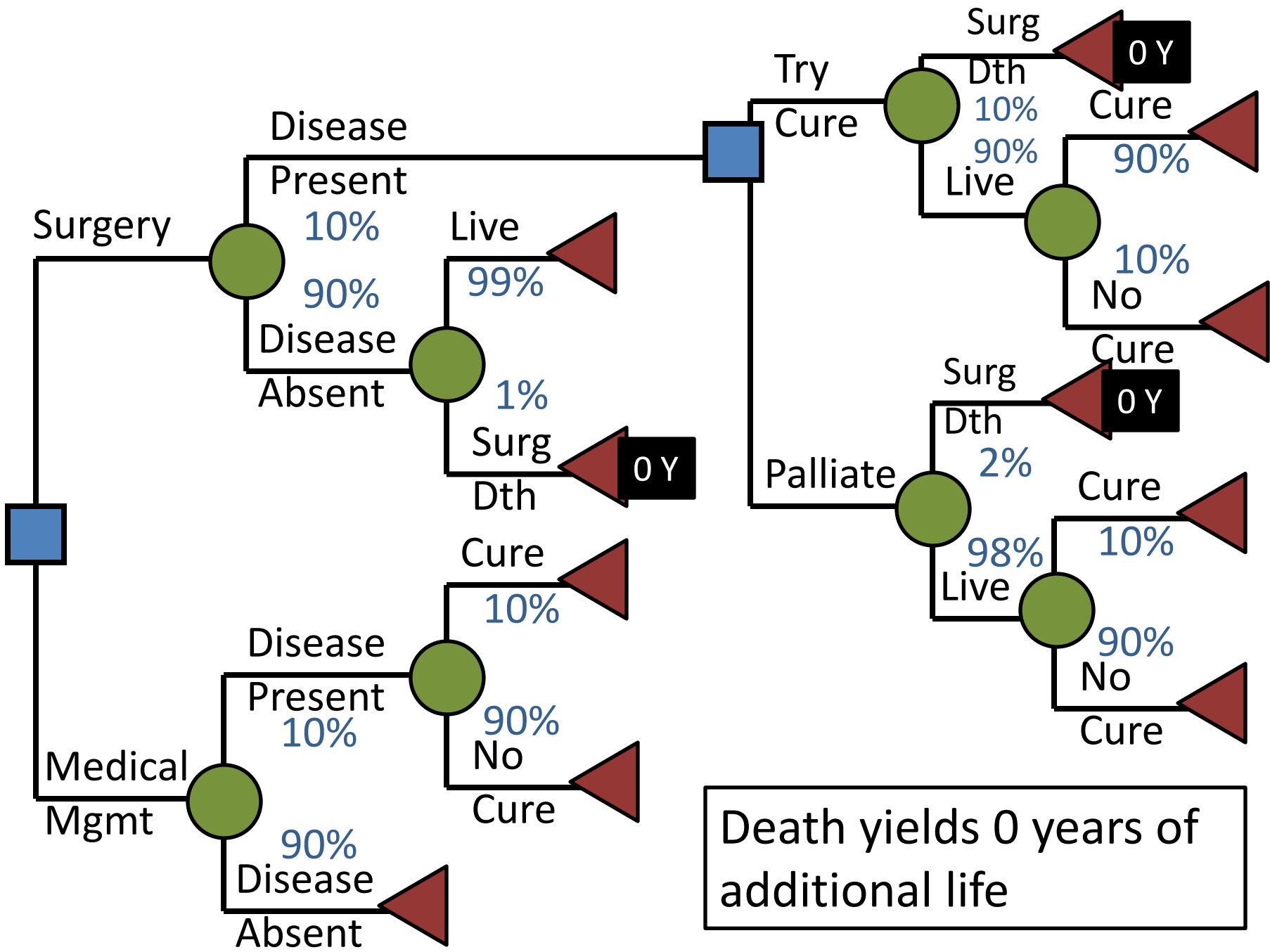
1. Surgery
2. in a patient with disease
3. where curative surgery chosen
4. and patient survives
5. and is cured



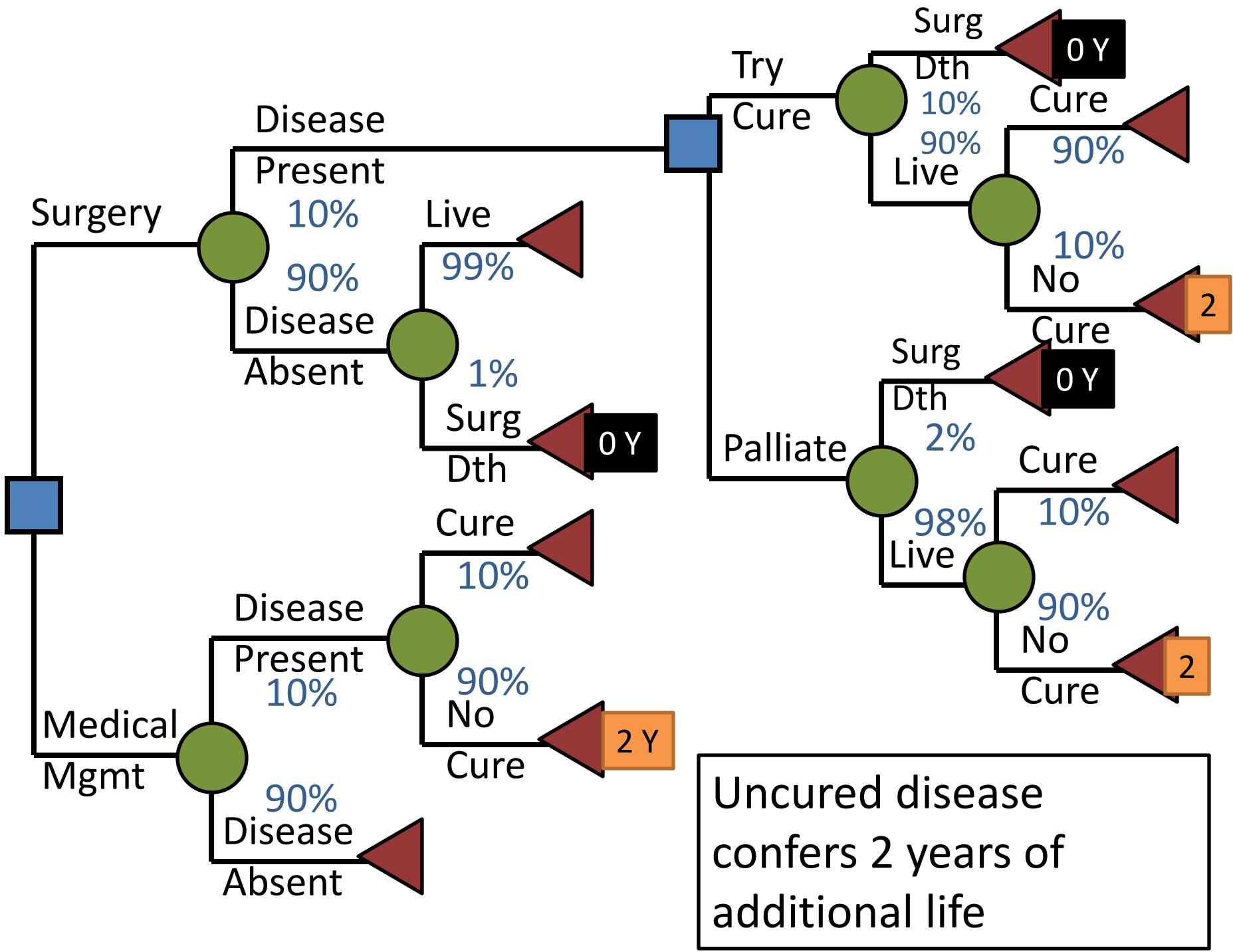


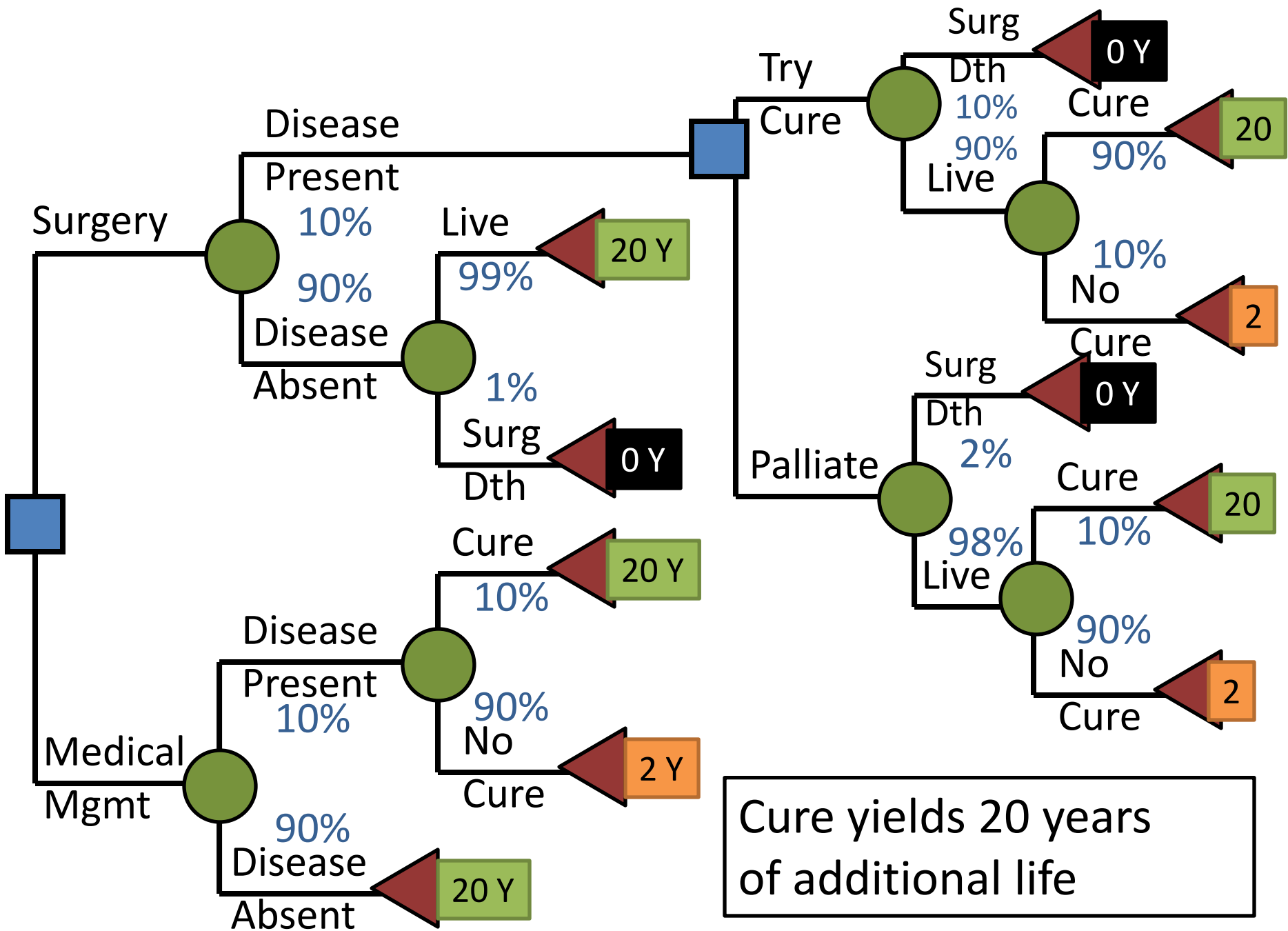


Now add outcomes



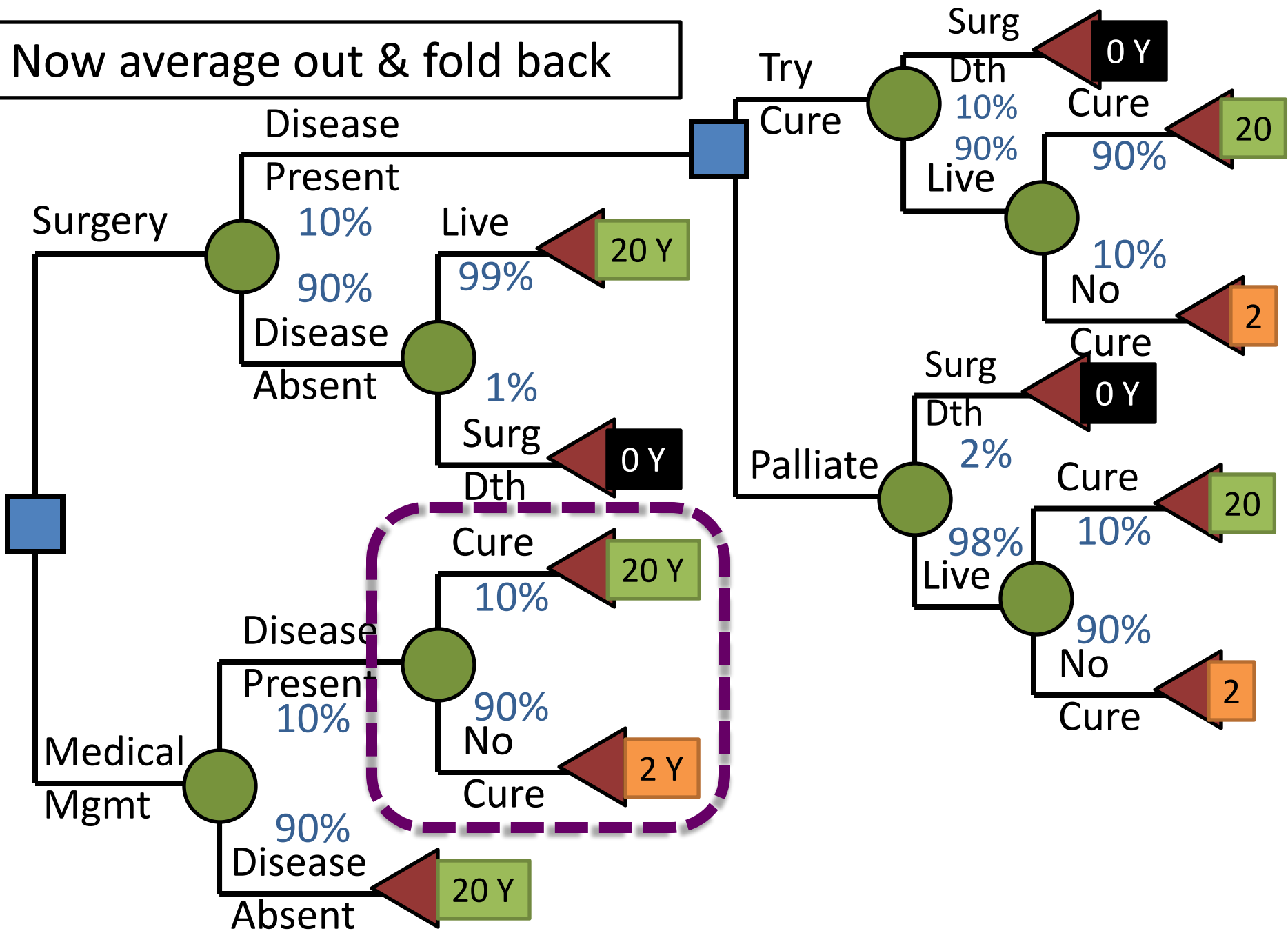
Death yields 0 years of additional life



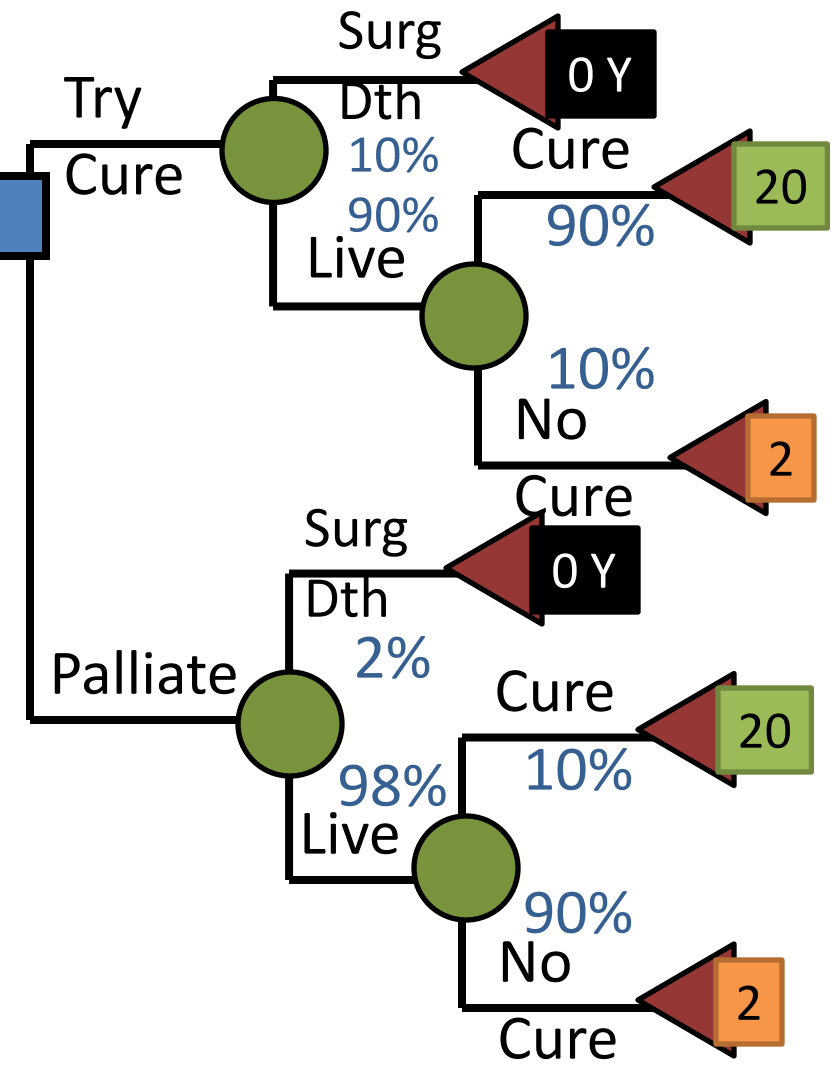
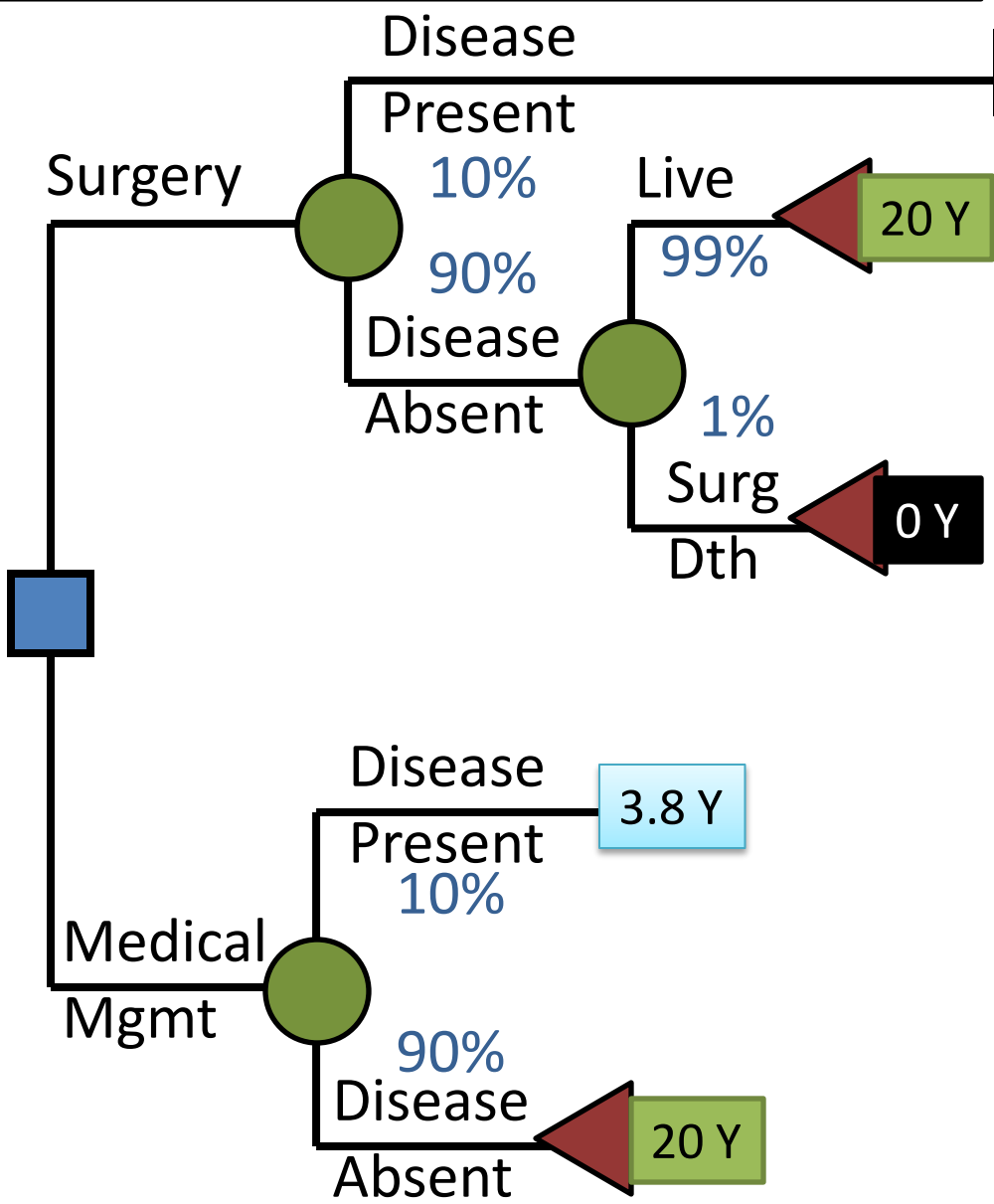


Cure yields 20 years of additional life

Now average out & fold back

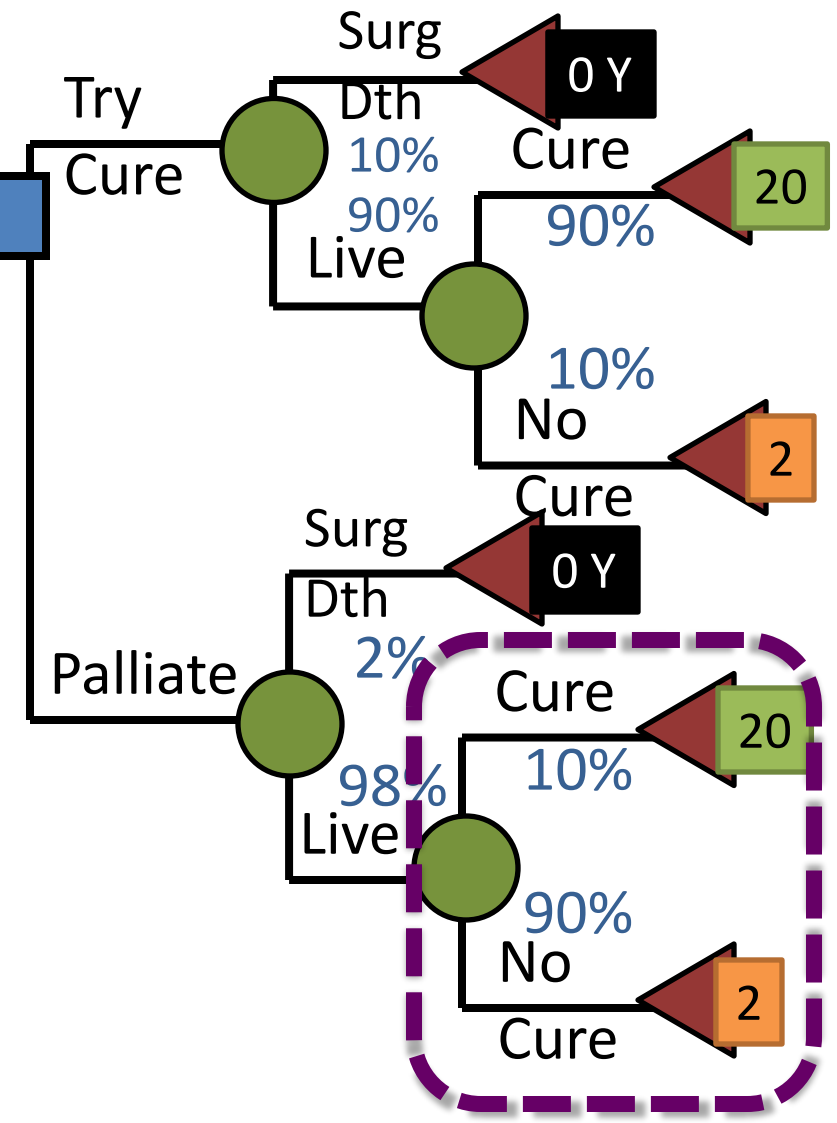
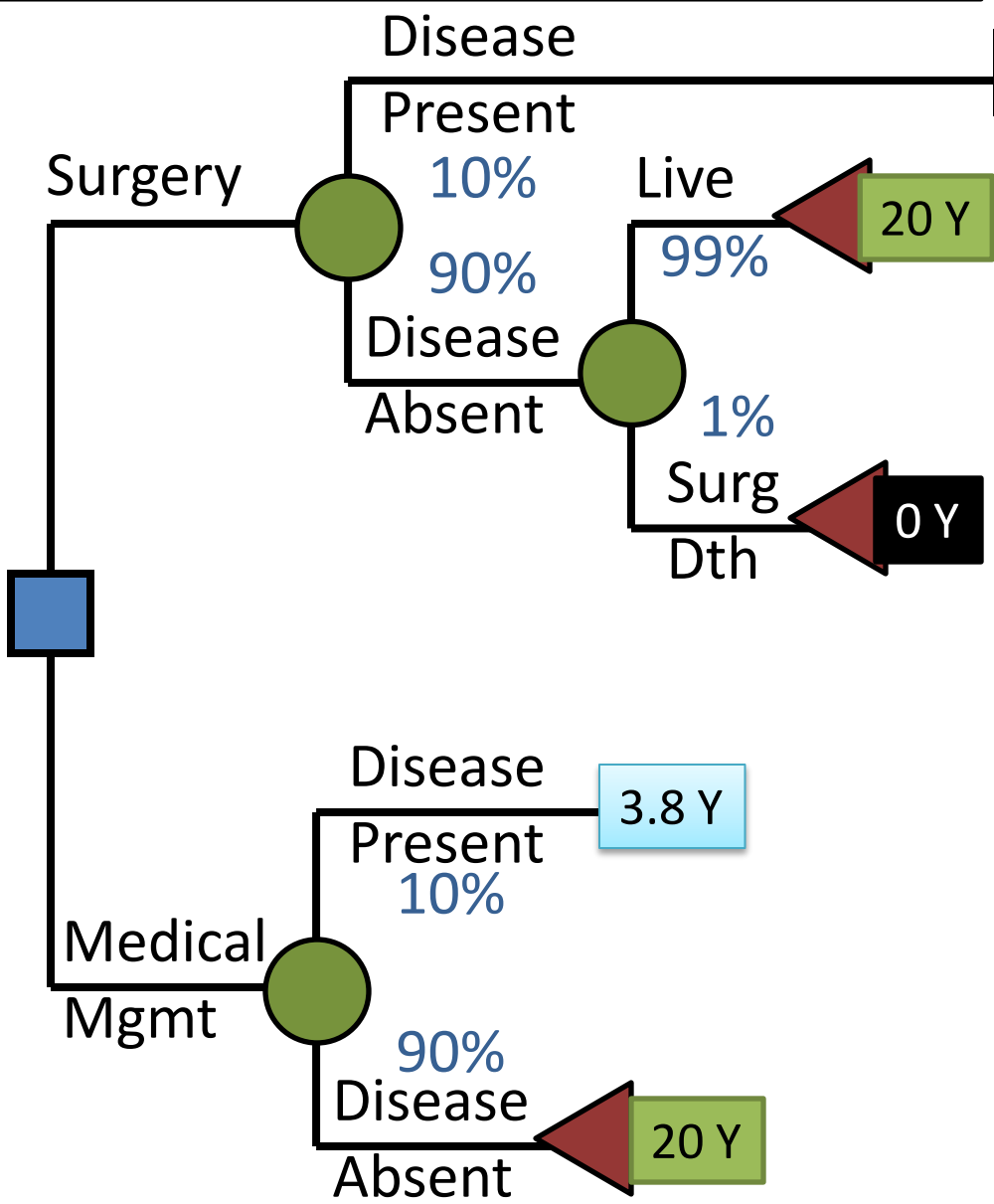


Now average out & fold back



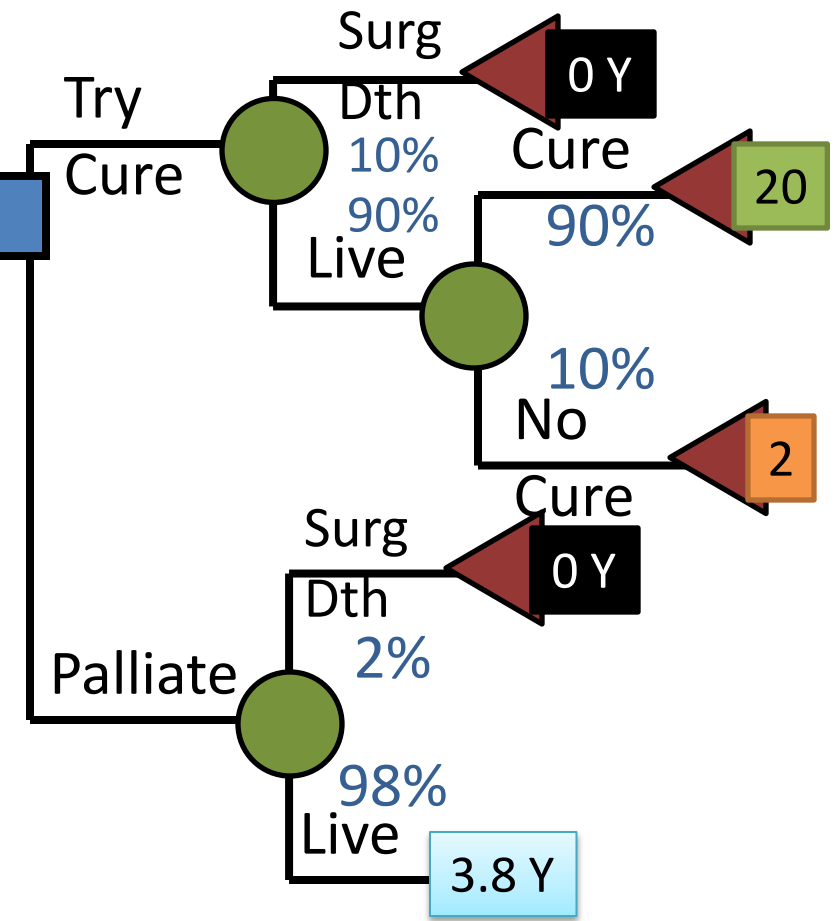
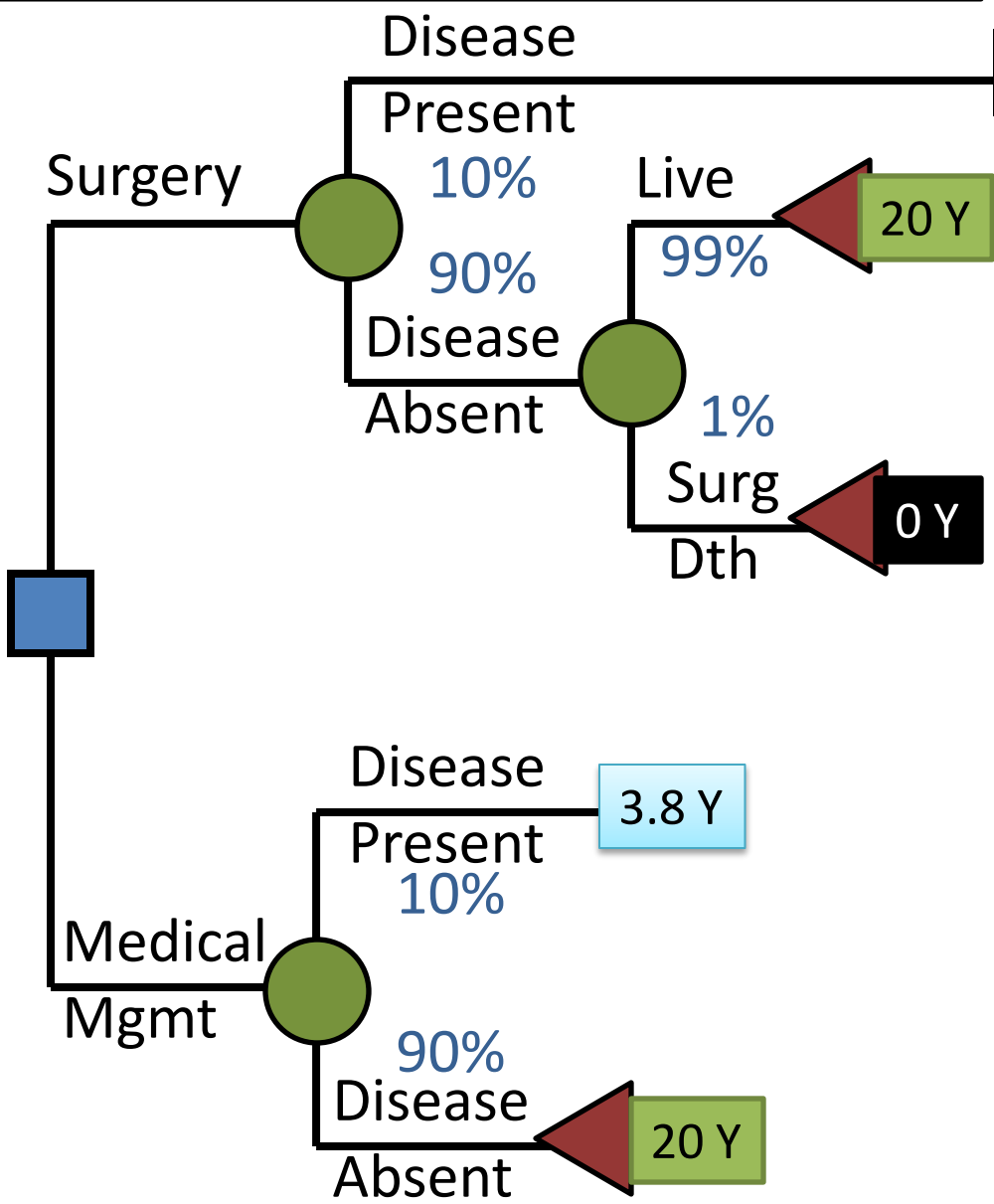
$$10\% * 20 + 90\% * 2 = 3.8 \text{ years (expected)}$$

Now average out & fold back



Same calculation here

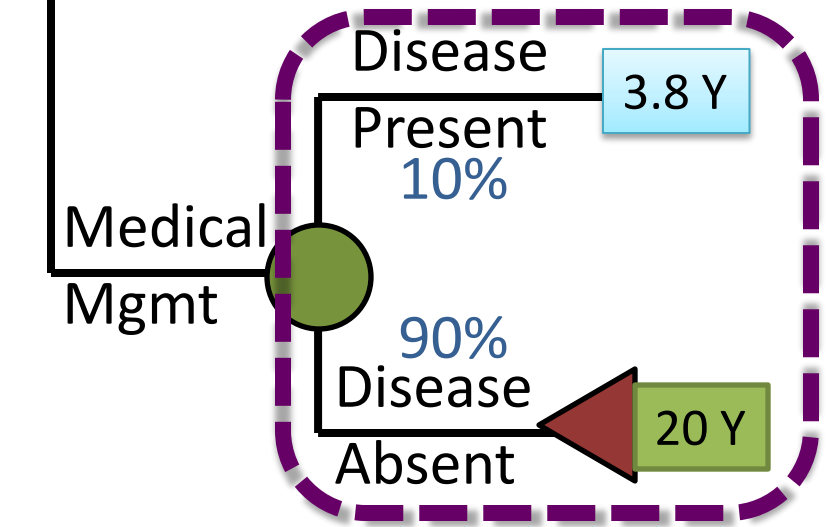
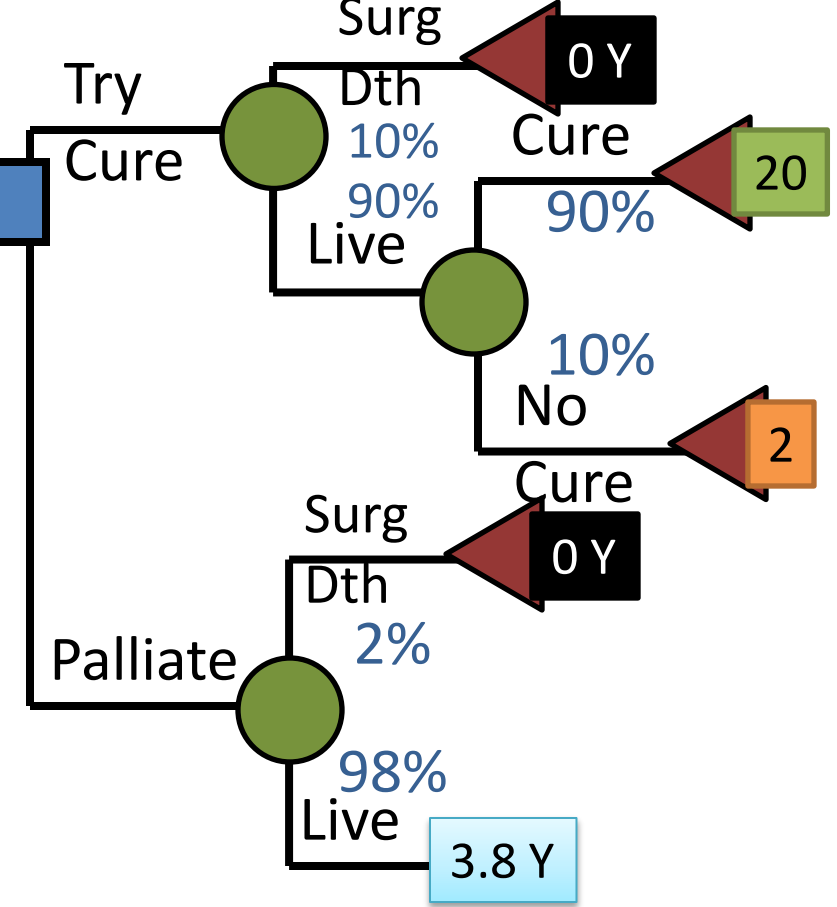
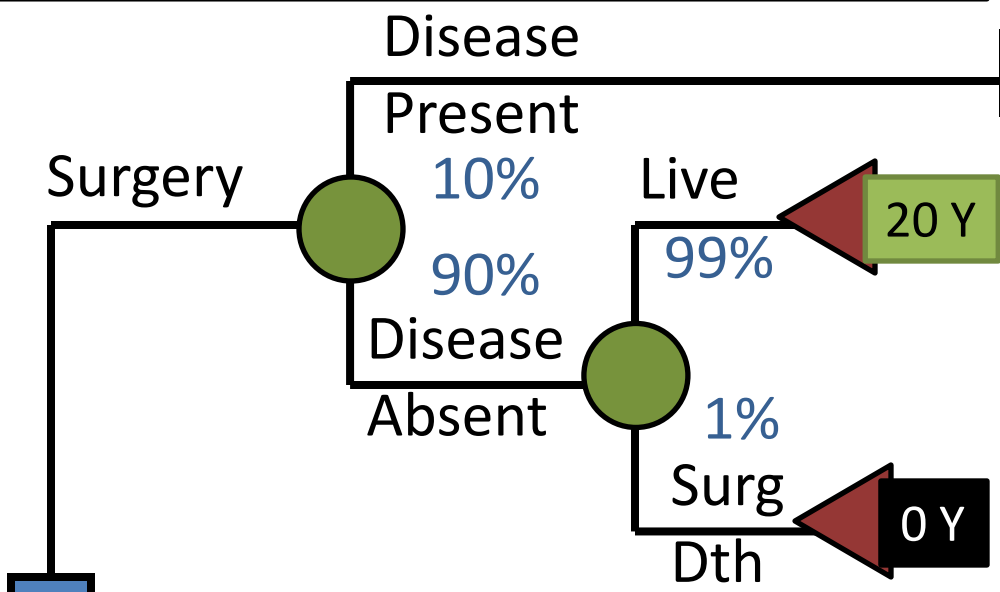
Now average out & fold back



$$10\% * 20 + 90\% * 2 = 3.8 \text{ years (expected)}$$

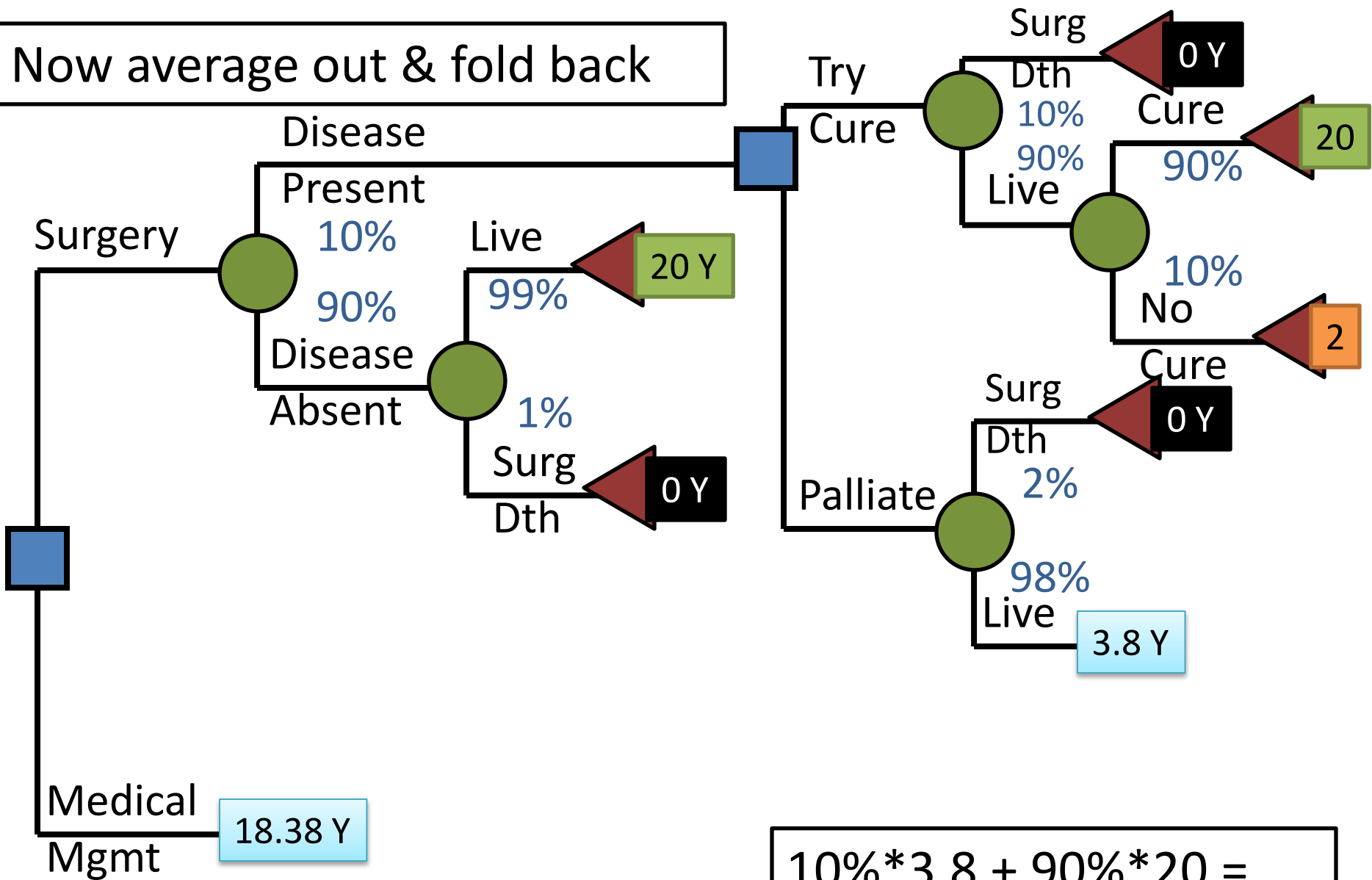


Now average out & fold back



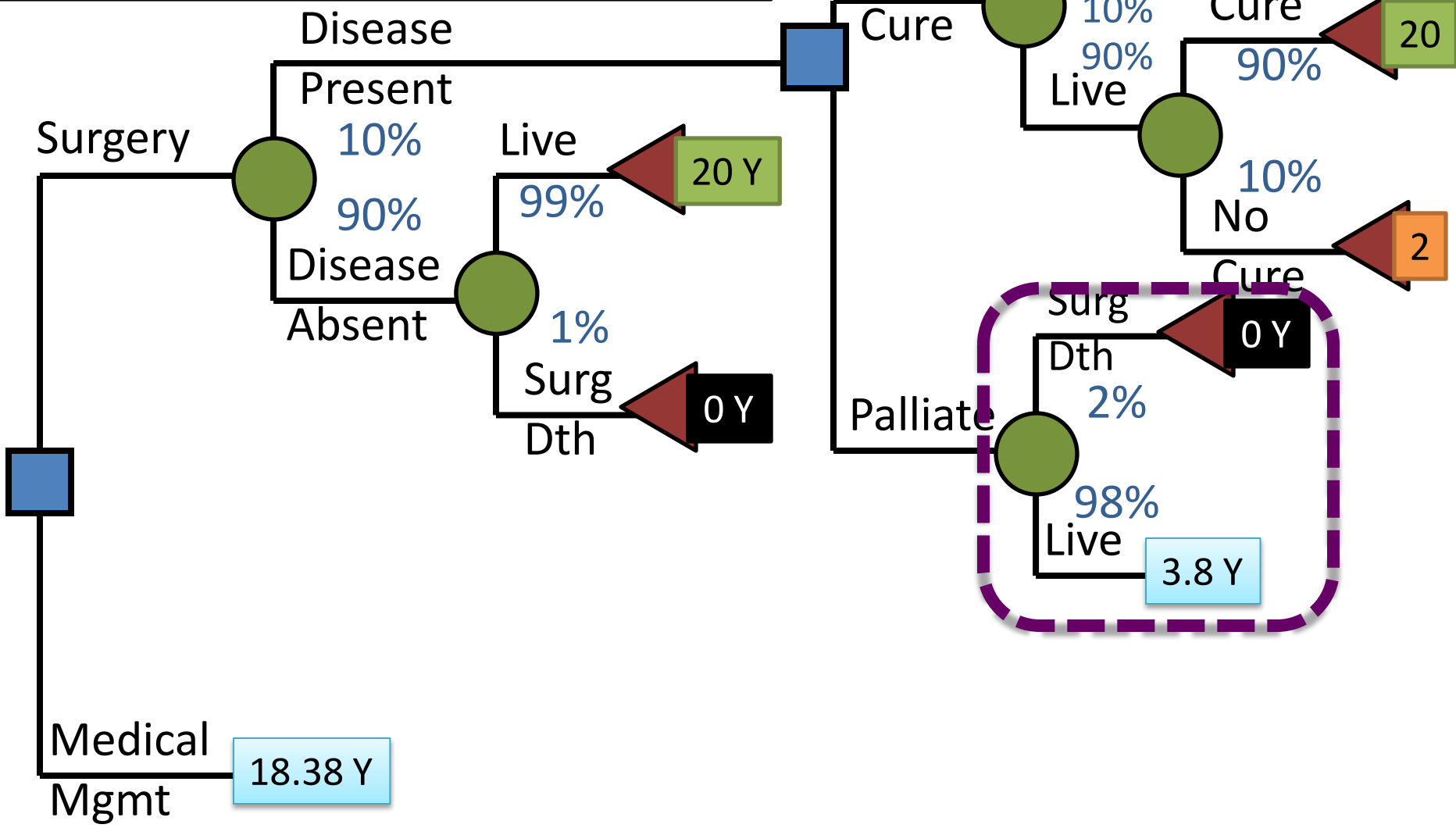
Since disease presence unknown, we do this again

Now average out & fold back

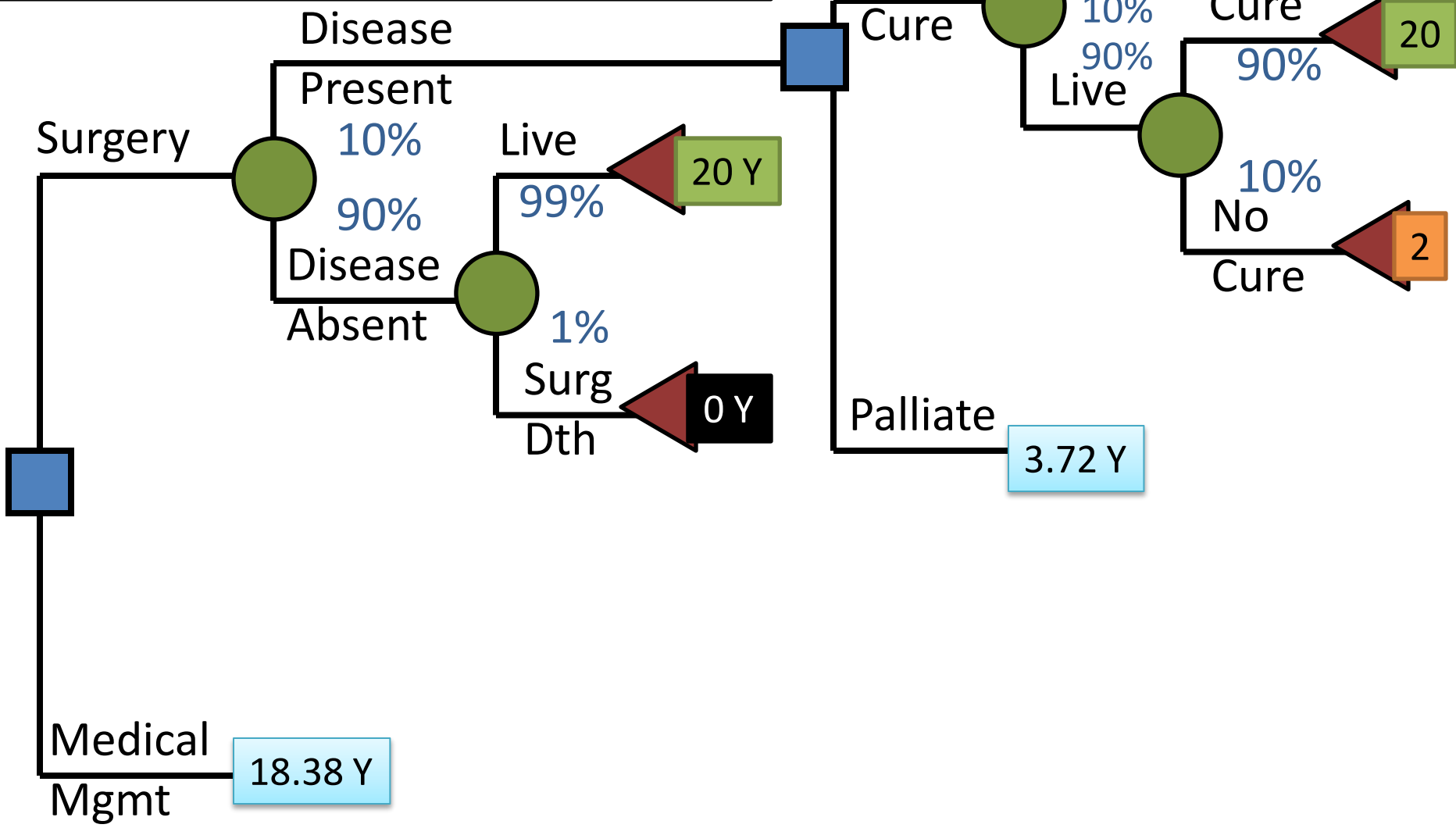


$$10\% * 3.8 + 90\% * 20 = 18.38 \text{ years}$$

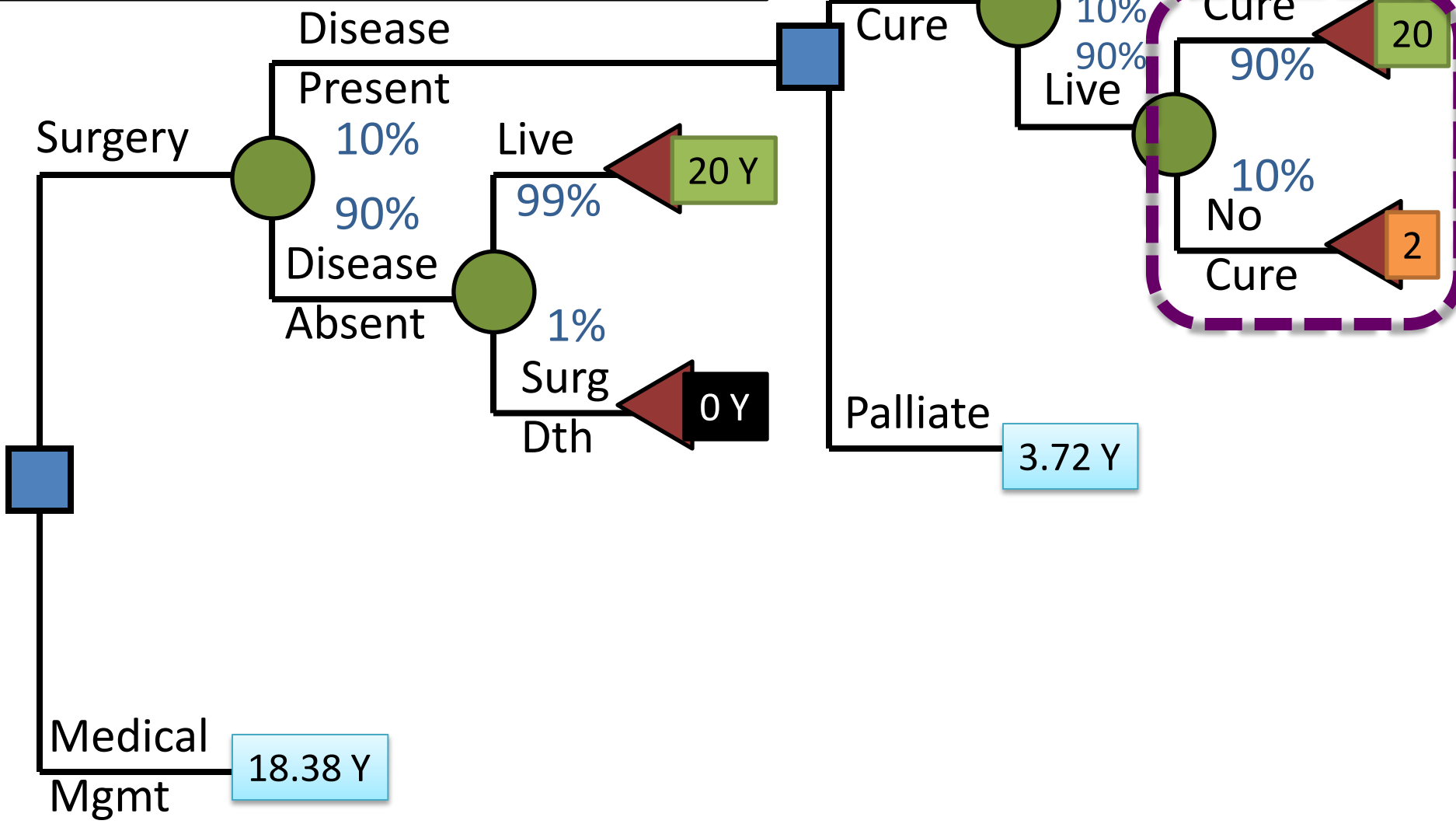
Now average out & fold back



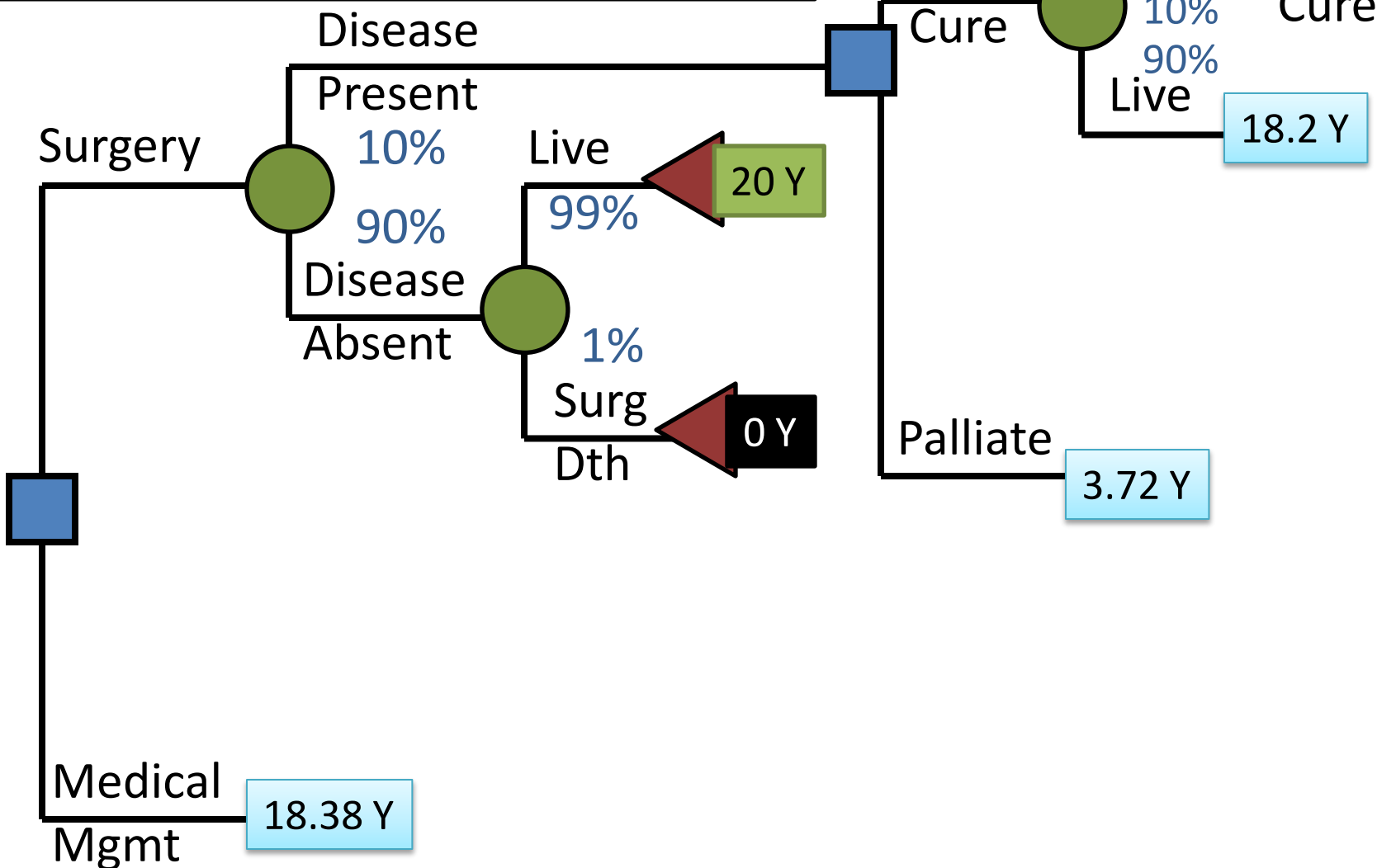
Now average out & fold back



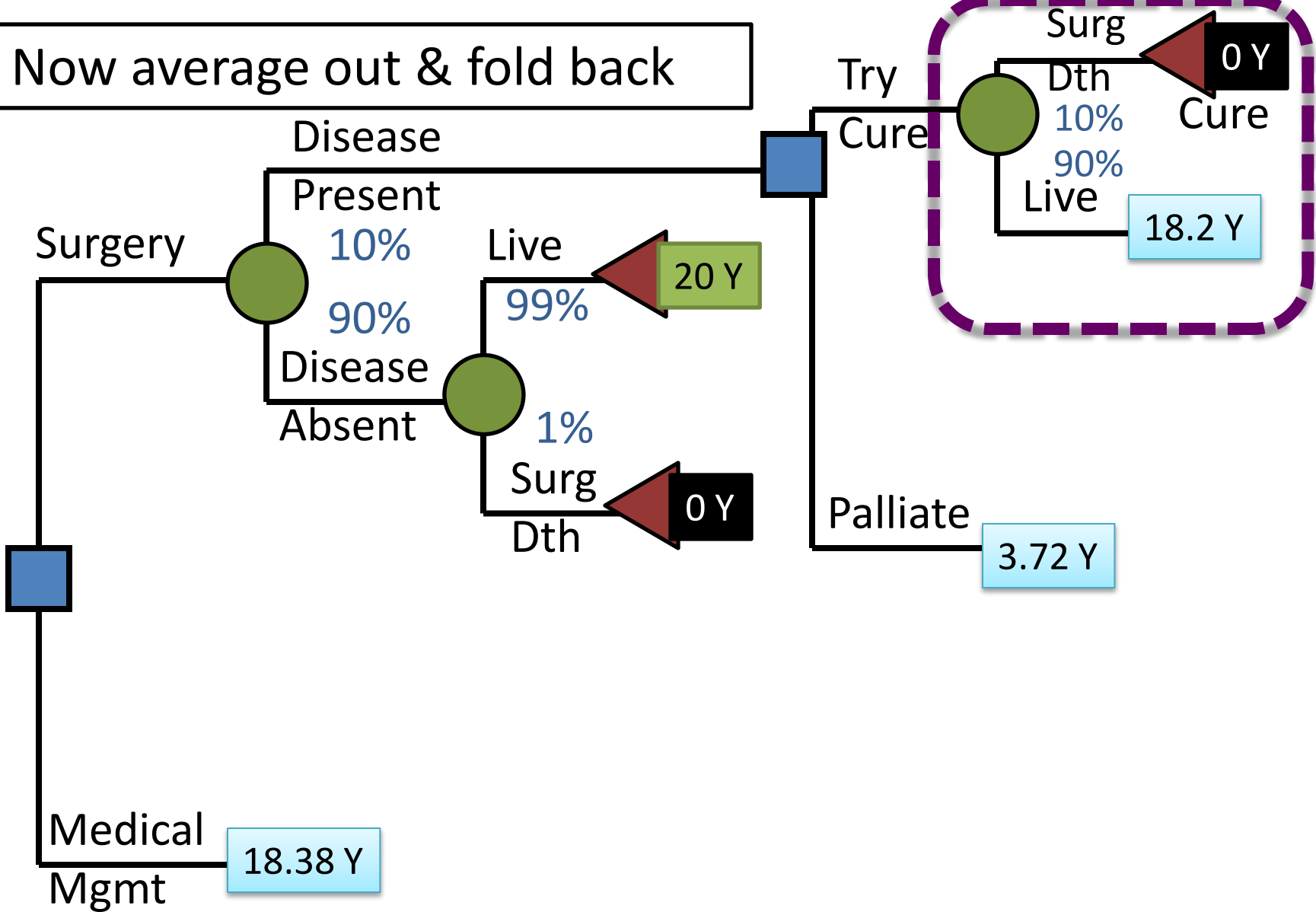
Now average out & fold back



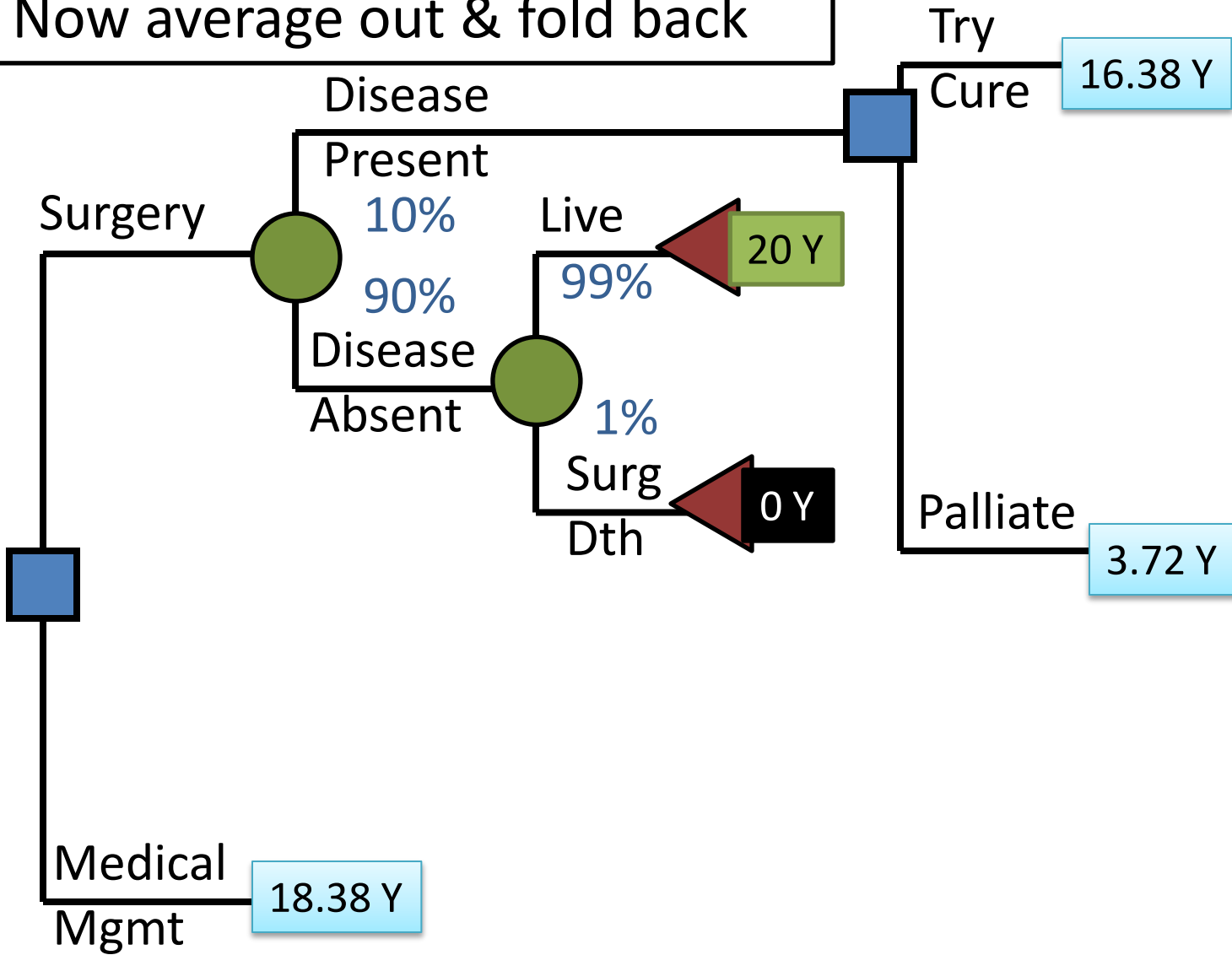
Now average out & fold back



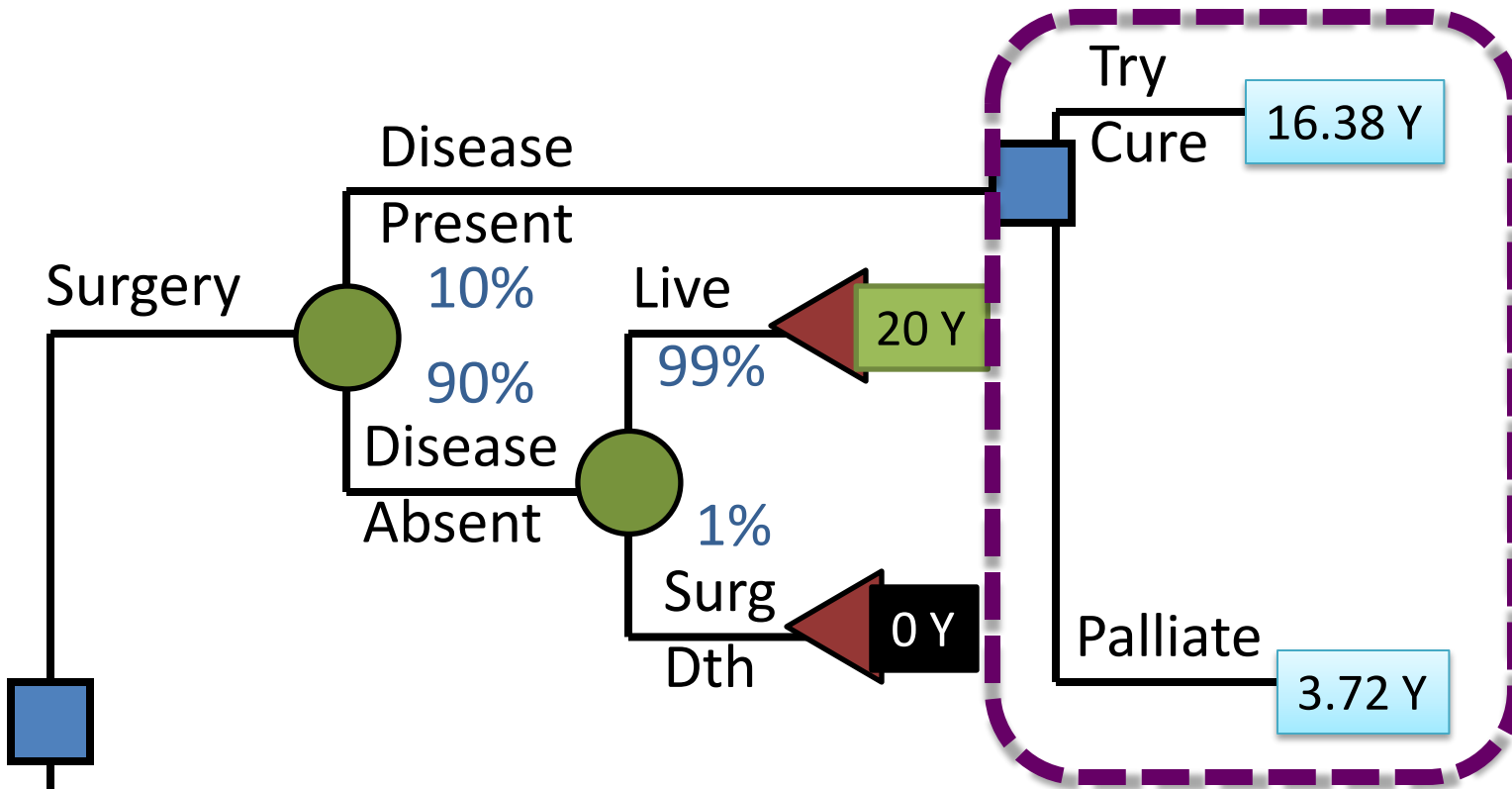
Now average out & fold back



Now average out & fold back

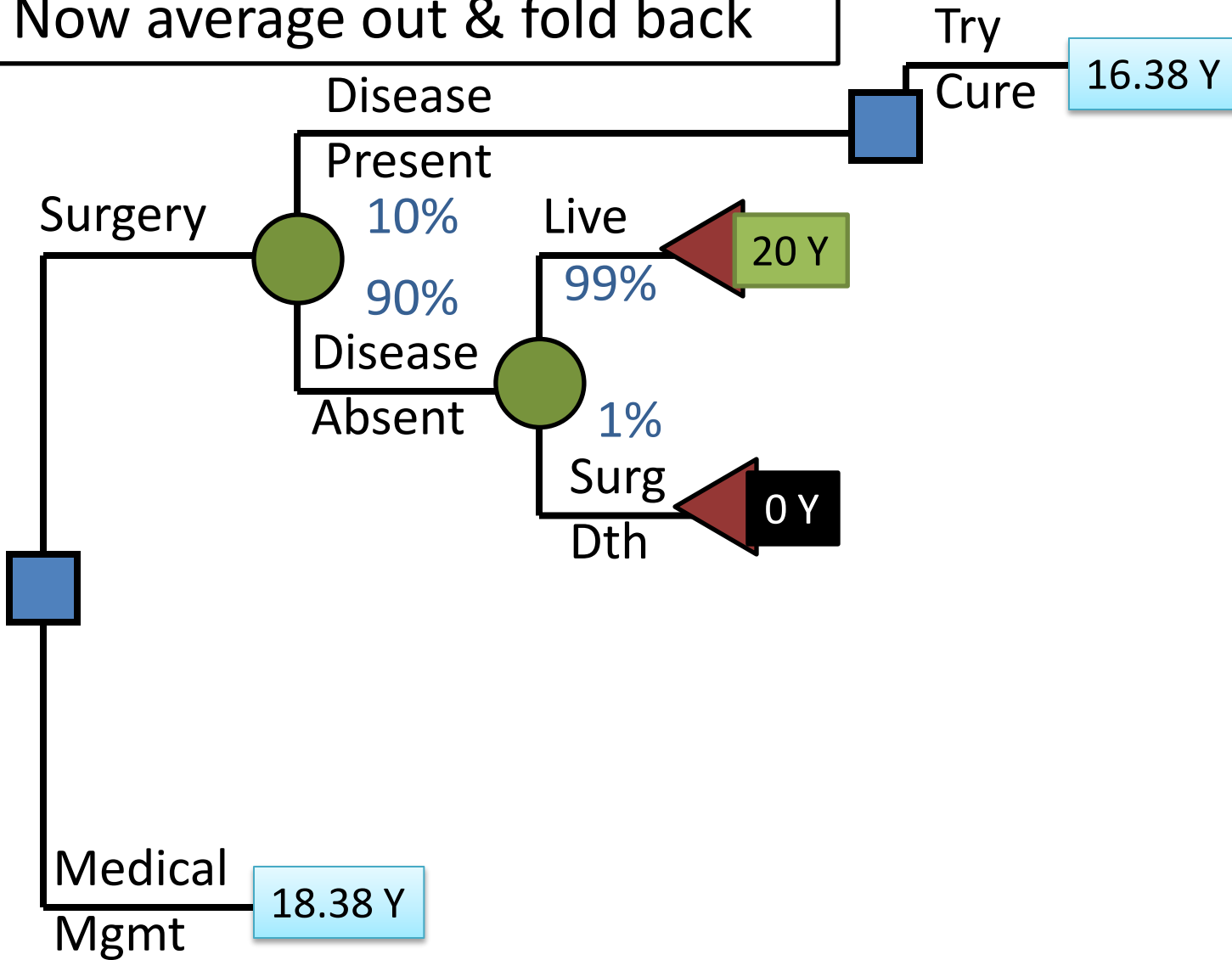




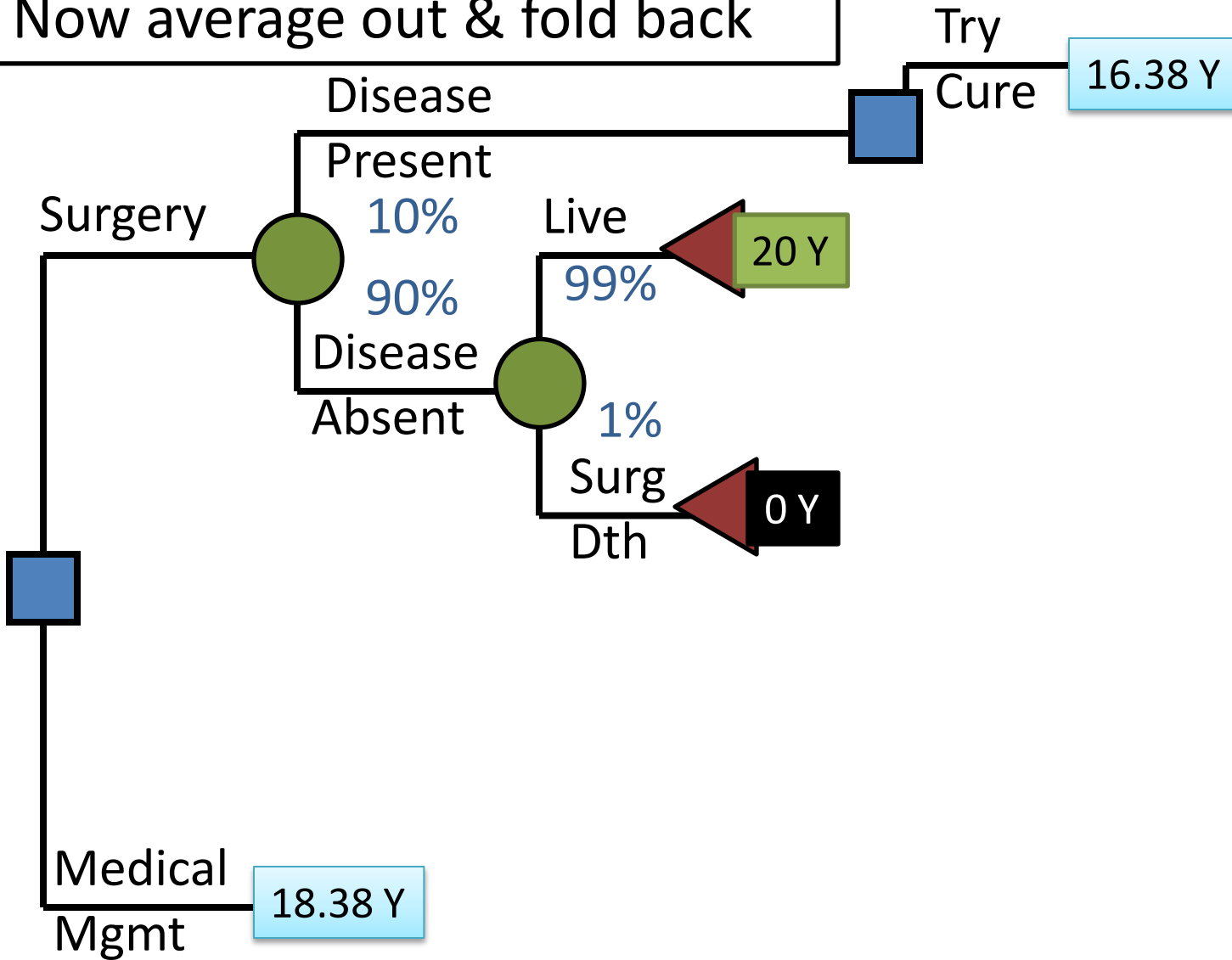


This one is different:  
 Decision node:  
 Surgeon picks option with  
 greatest expected benefit:  
 Try Cure (16.38 years) preferred  
*(called "folding back")*

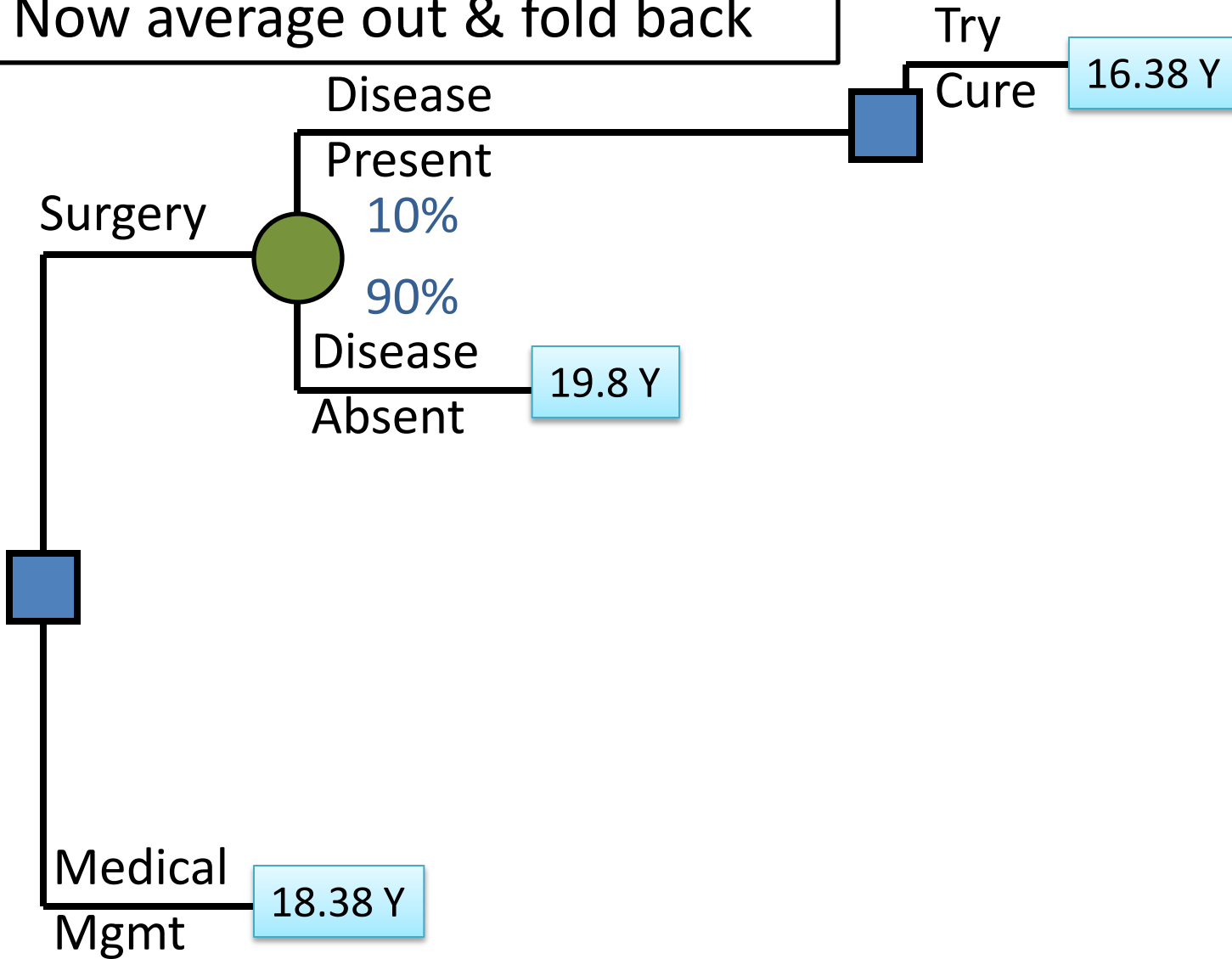
Now average out & fold back



Now average out & fold back



Now average out & fold back



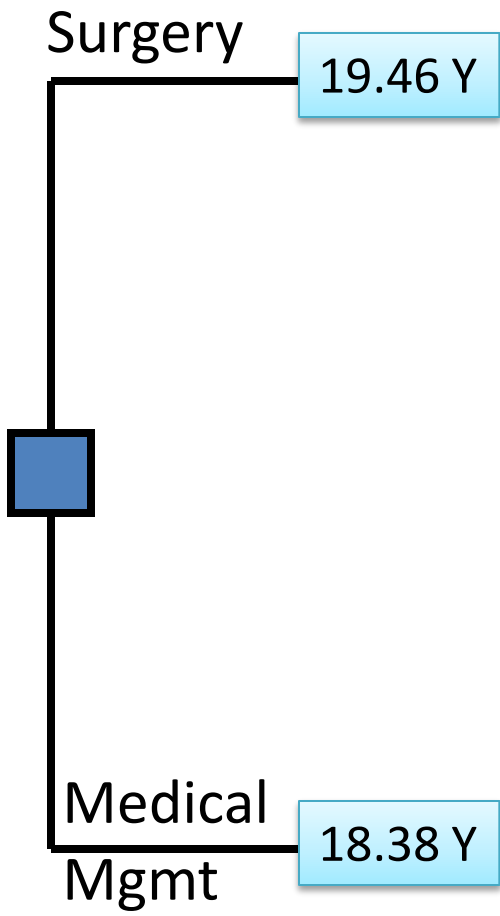
Surgery

19.46 Y



Medical  
Mgmt

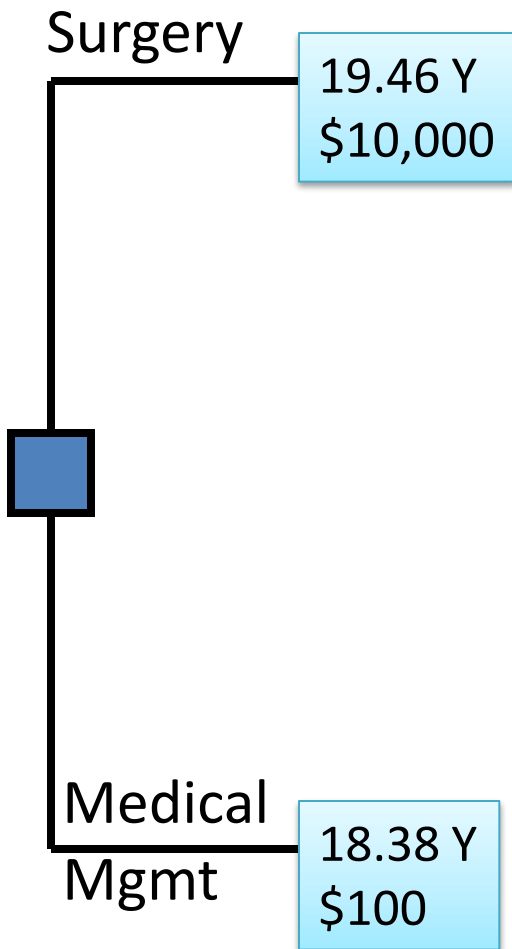
18.38 Y



Decision node again (overall)  
Surgery is preferred to Medical Management because the incremental benefit of surgery is:

$$19.46 - 18.38 = 1.08 \text{ years}$$

**Recommendation: Choose surgery (with “try cure” surgical option)**



Use same approach for CEA but now with second set of outcomes

$$19.46 - 18.38 = \mathbf{1.08 \text{ years}}$$

$$\$10,000 - \$100 = \mathbf{\$9,900}$$

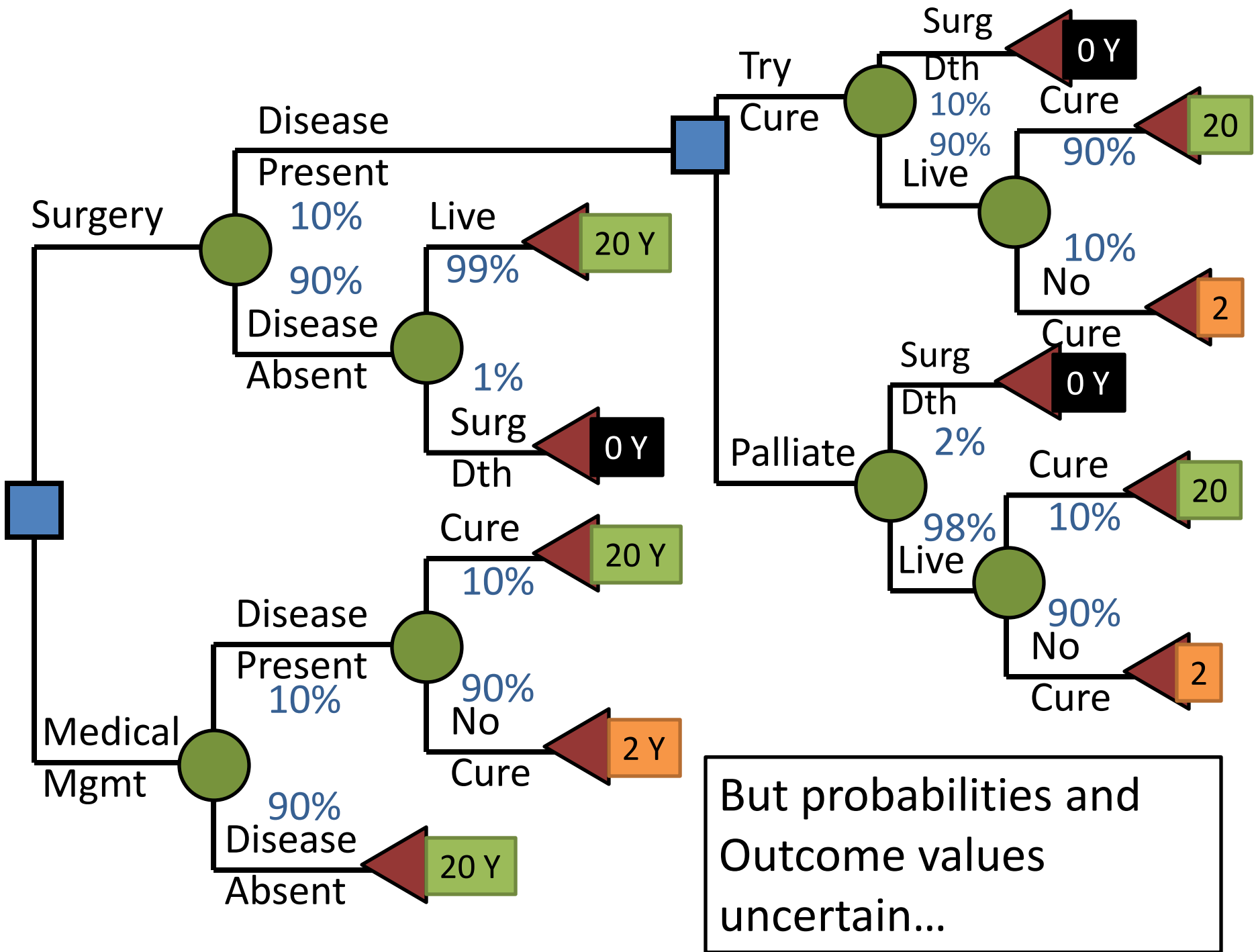
$$\mathbf{\$9,900 / 1.08 =}$$

$$\mathbf{\$9,167 \text{ per life year gained}}$$

Surgery if willing to pay at least \$9,167 per life year gained, otherwise medical management

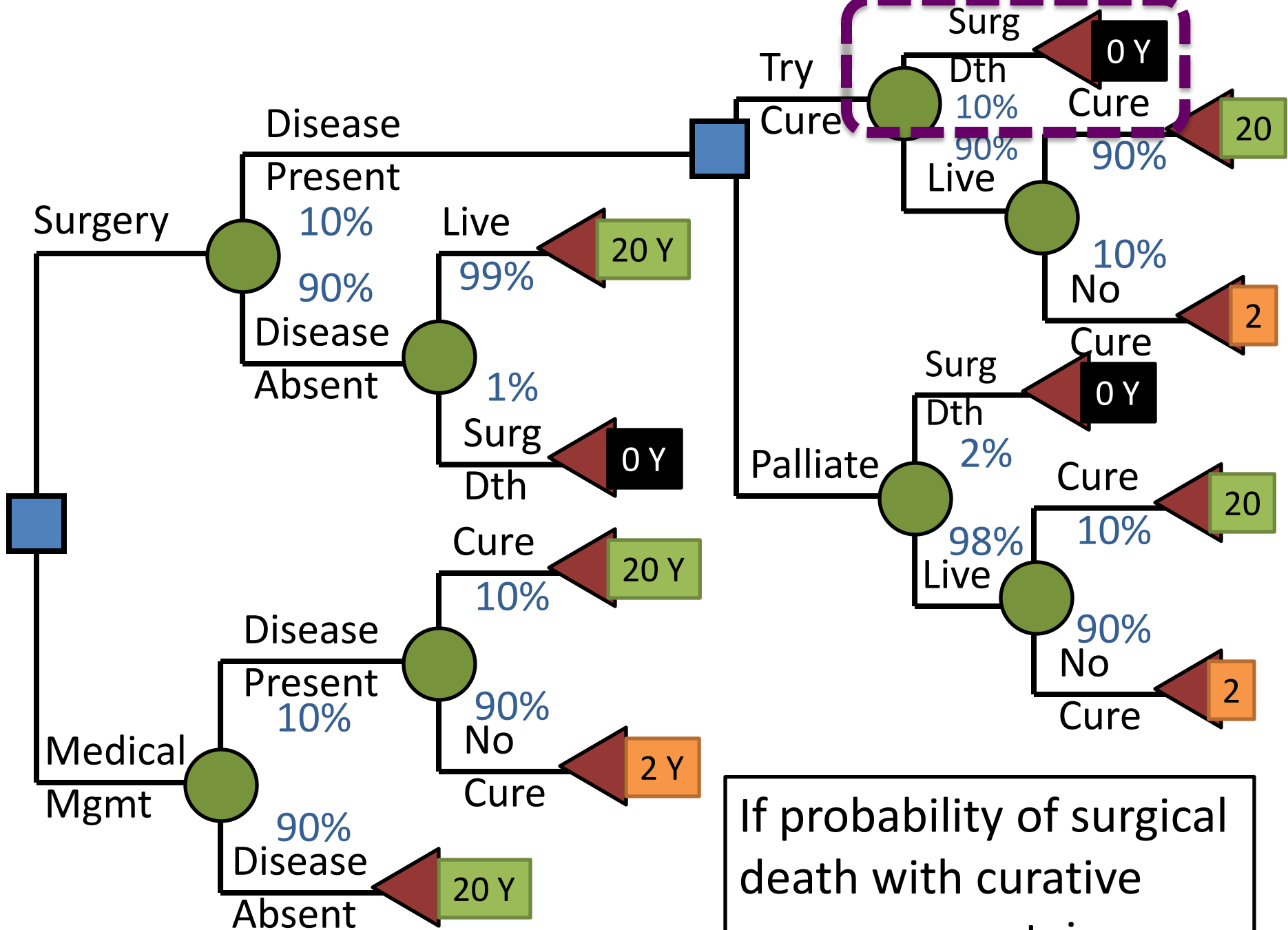
# **SENSITIVITY ANALYSIS**



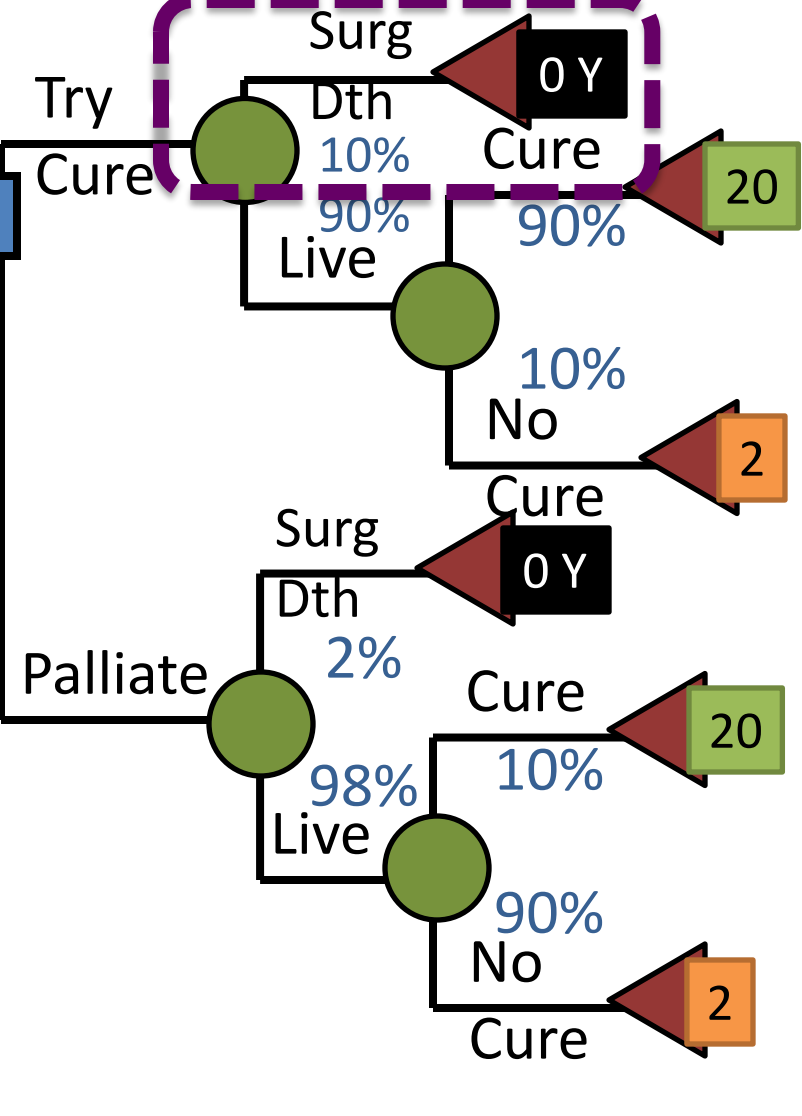


# Sensitivity Analysis

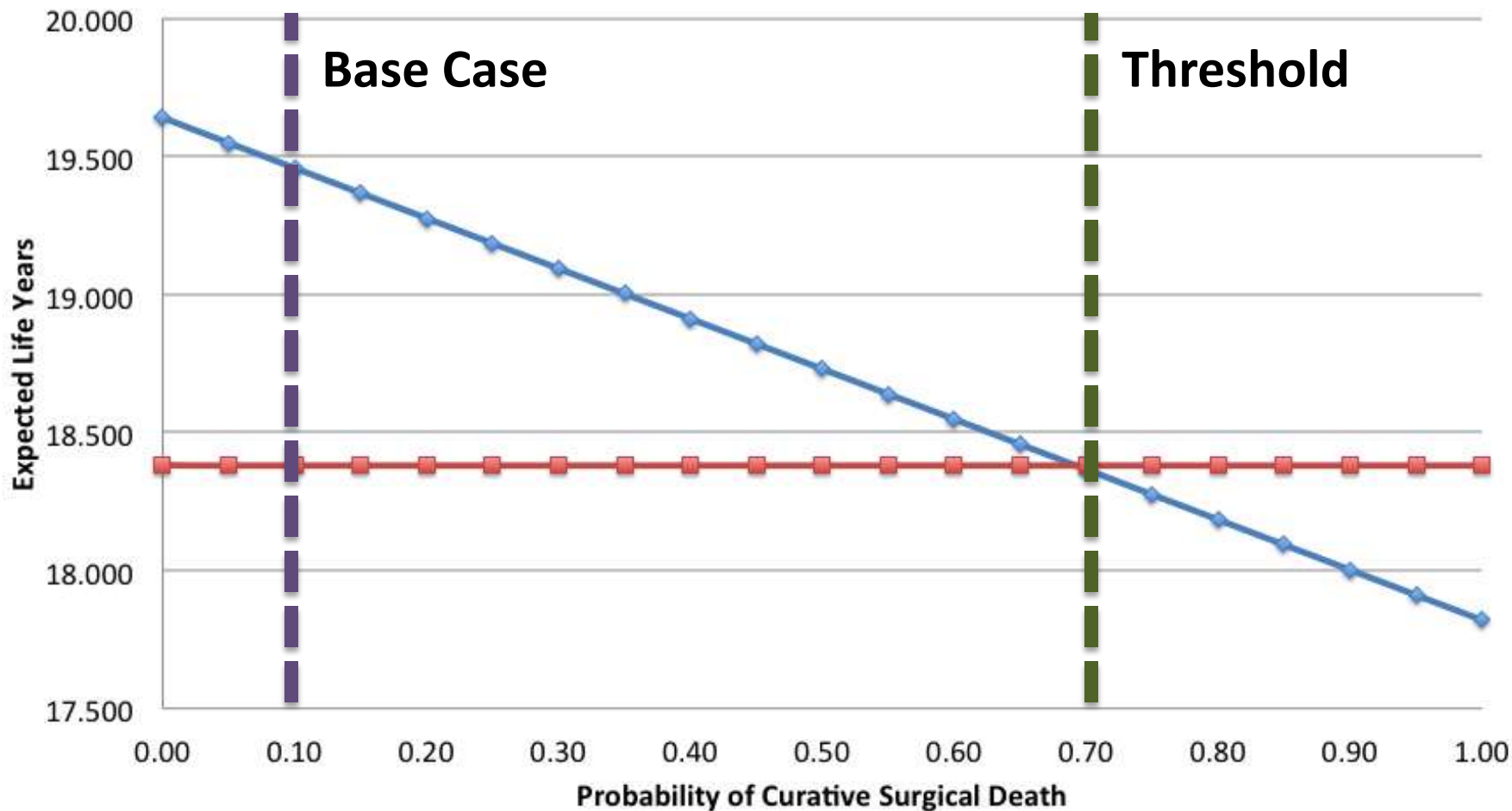
- Systematically asking “what if” questions to see how the decision result changes
- Determines how “robust” the decision is
  - **Threshold analysis:** one parameter varied
  - **Multi-way analysis:** multiple parameters systematically varied



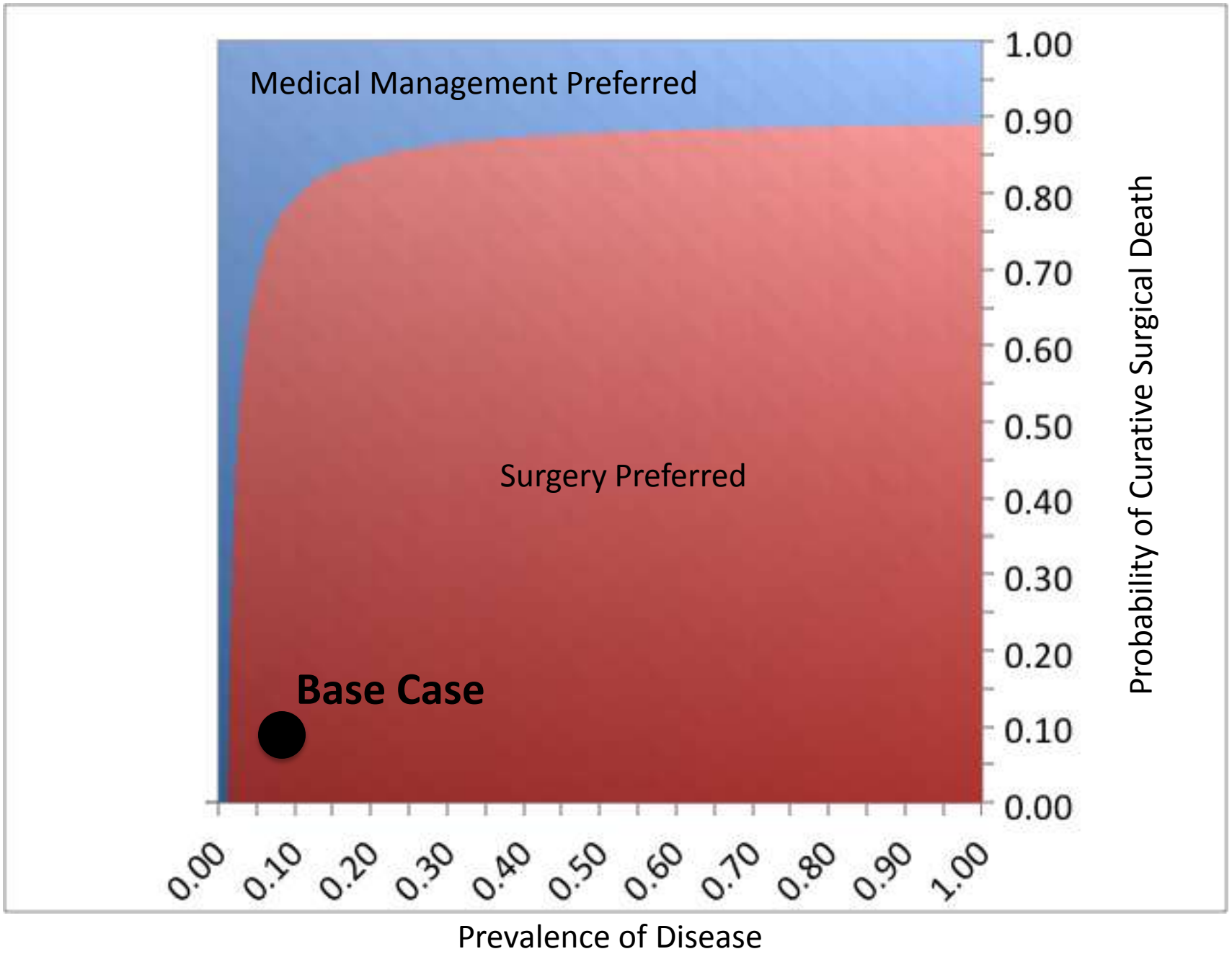
If probability of surgical death with curative surgery uncertain



### 1-way sensitivity analysis: Curative Surgical Death



—◆— Surgery - Curative    —■— Med Mgmt



# Advanced: Probabilistic Sensitivity Analysis (2nd order Monte Carlo)

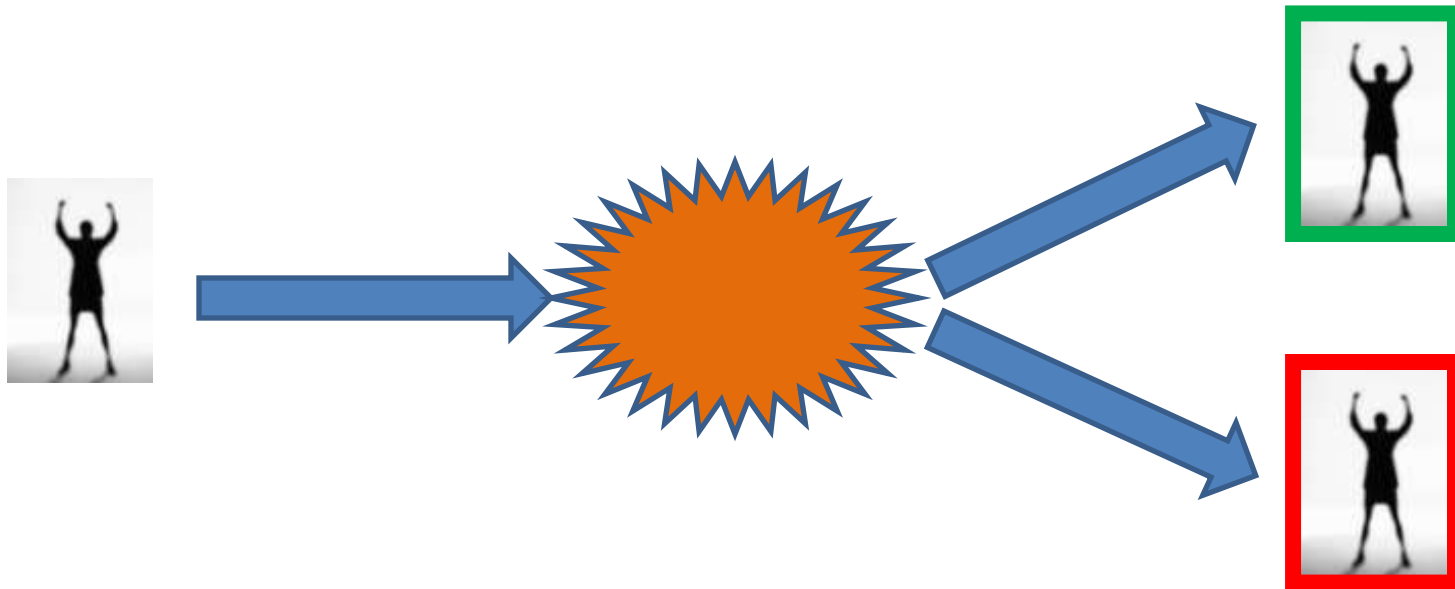
- Decision tree estimates of probabilities and utilities are replaced with probability distributions (e.g. logistic-normal)
- The tree is evaluated *many* times with random values selected from each distribution
- Results include means and standard deviations of the expected values of each strategy

# **MARKOV MODELS VS. DECISION TREES**

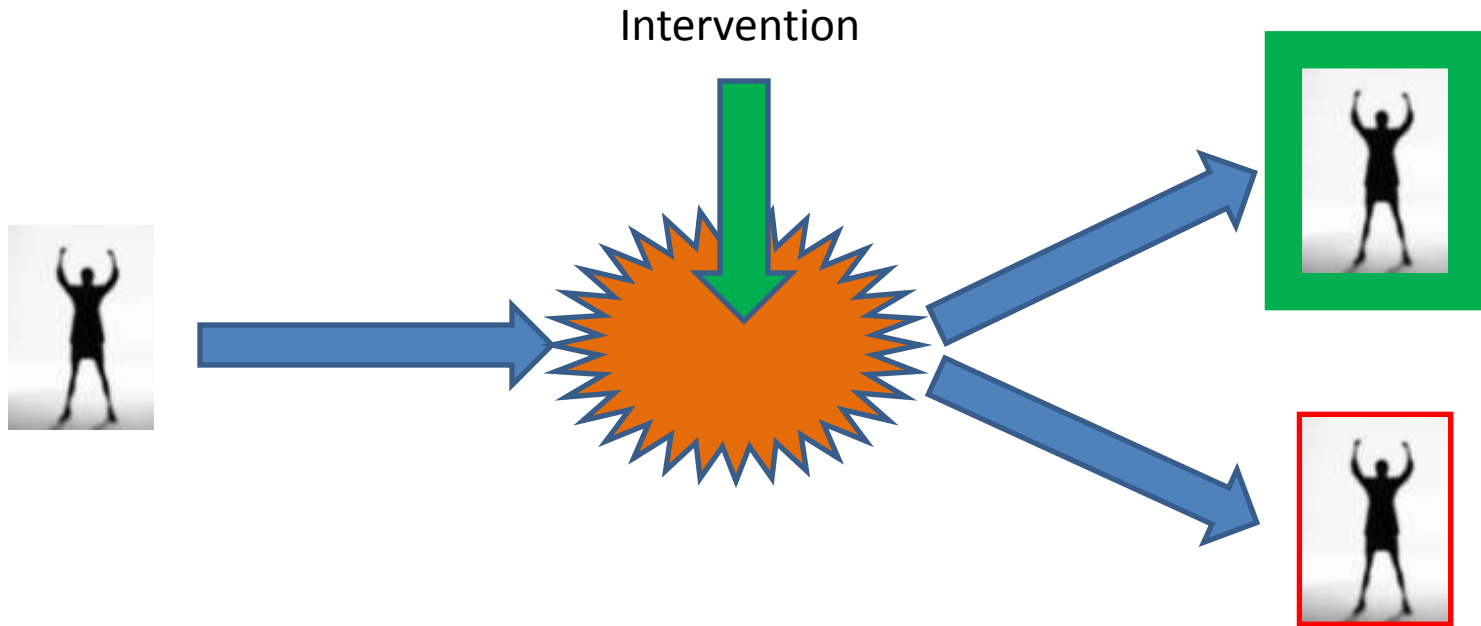
**WHAT TO DO WHEN THERE IS A  
POSSIBILITY OF REPEATED EVENTS  
AND/OR DECISIONS?**



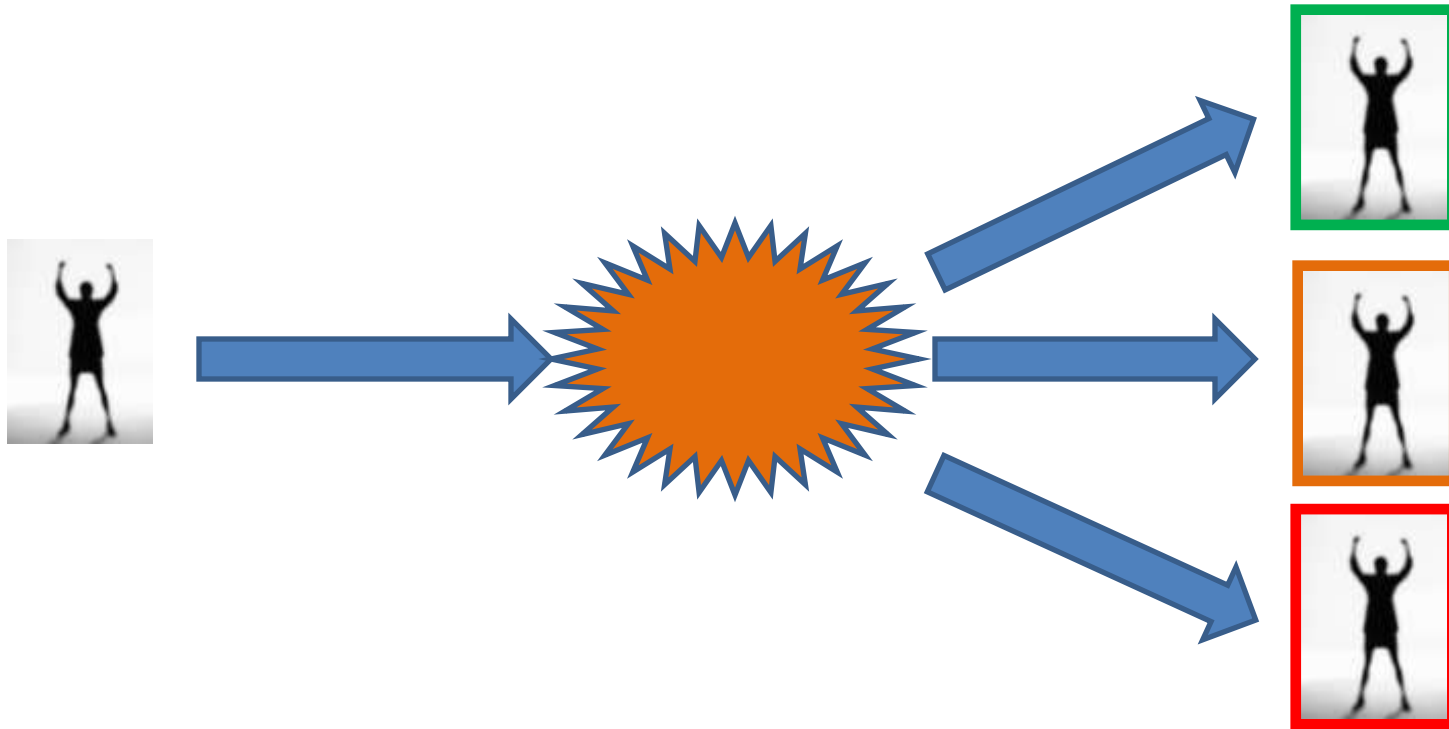
# Decision about one-time, immediate action



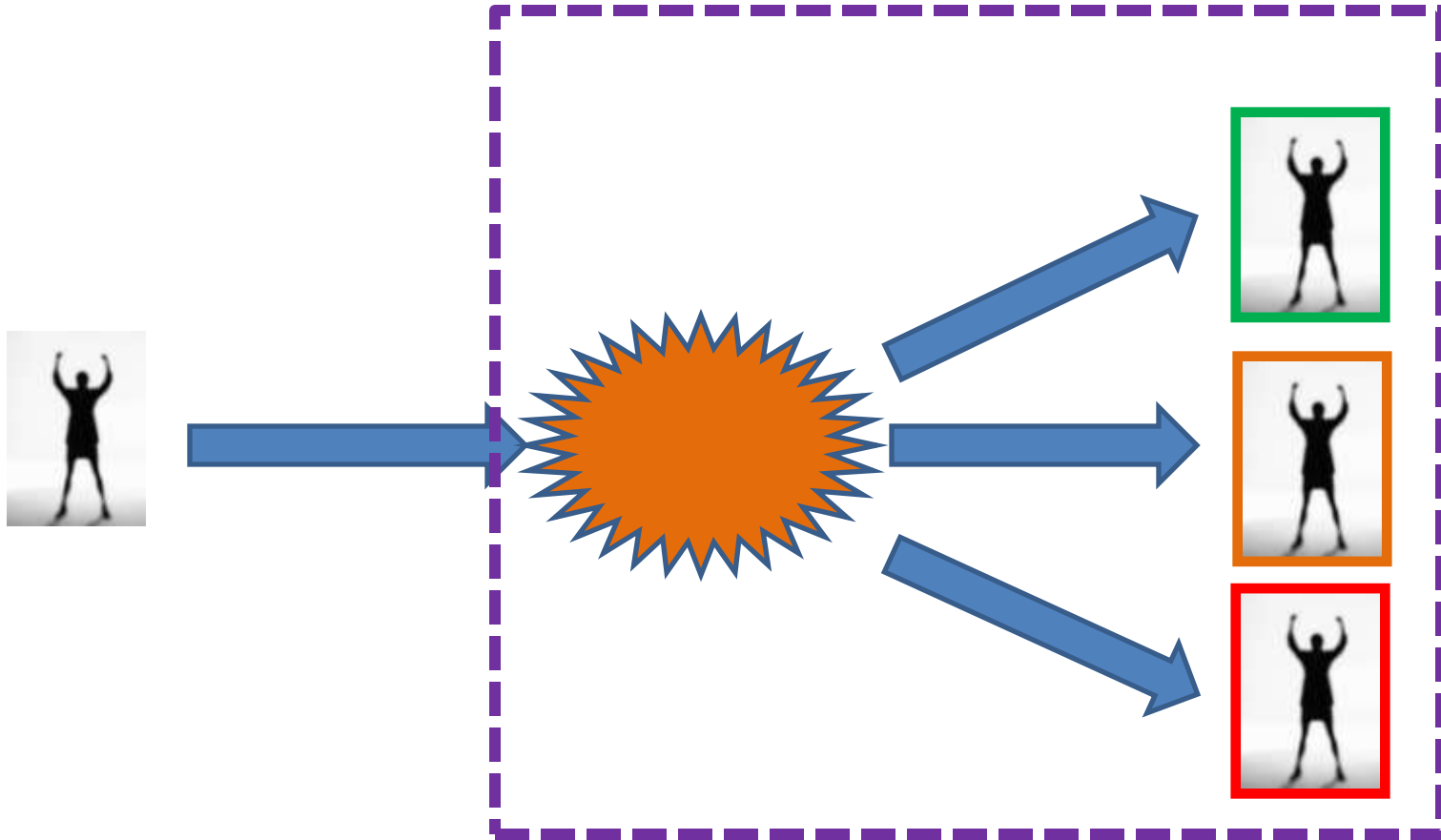
# Decision about one-time, immediate action



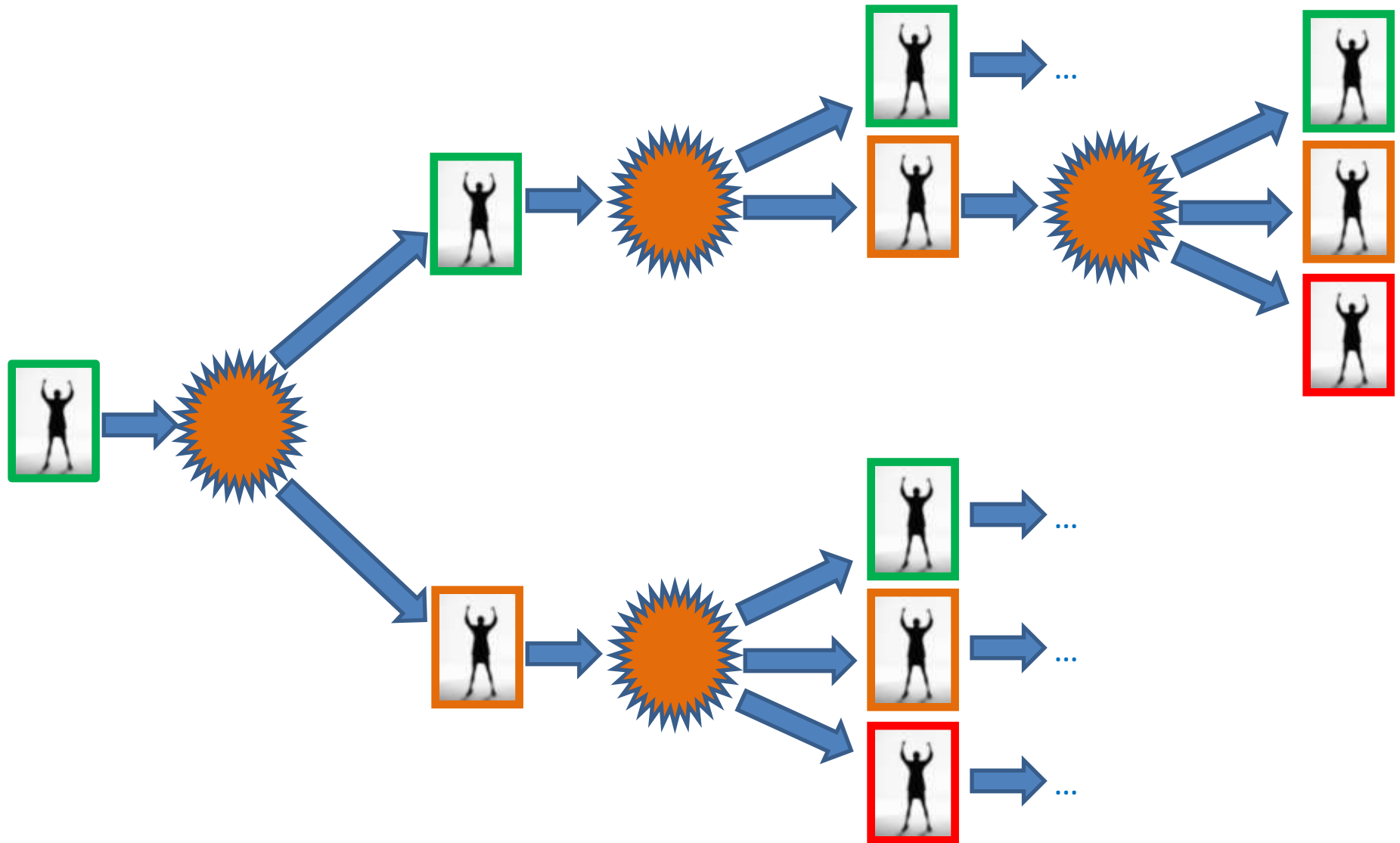
# Decisions: repeated actions and/or with time-dependent events



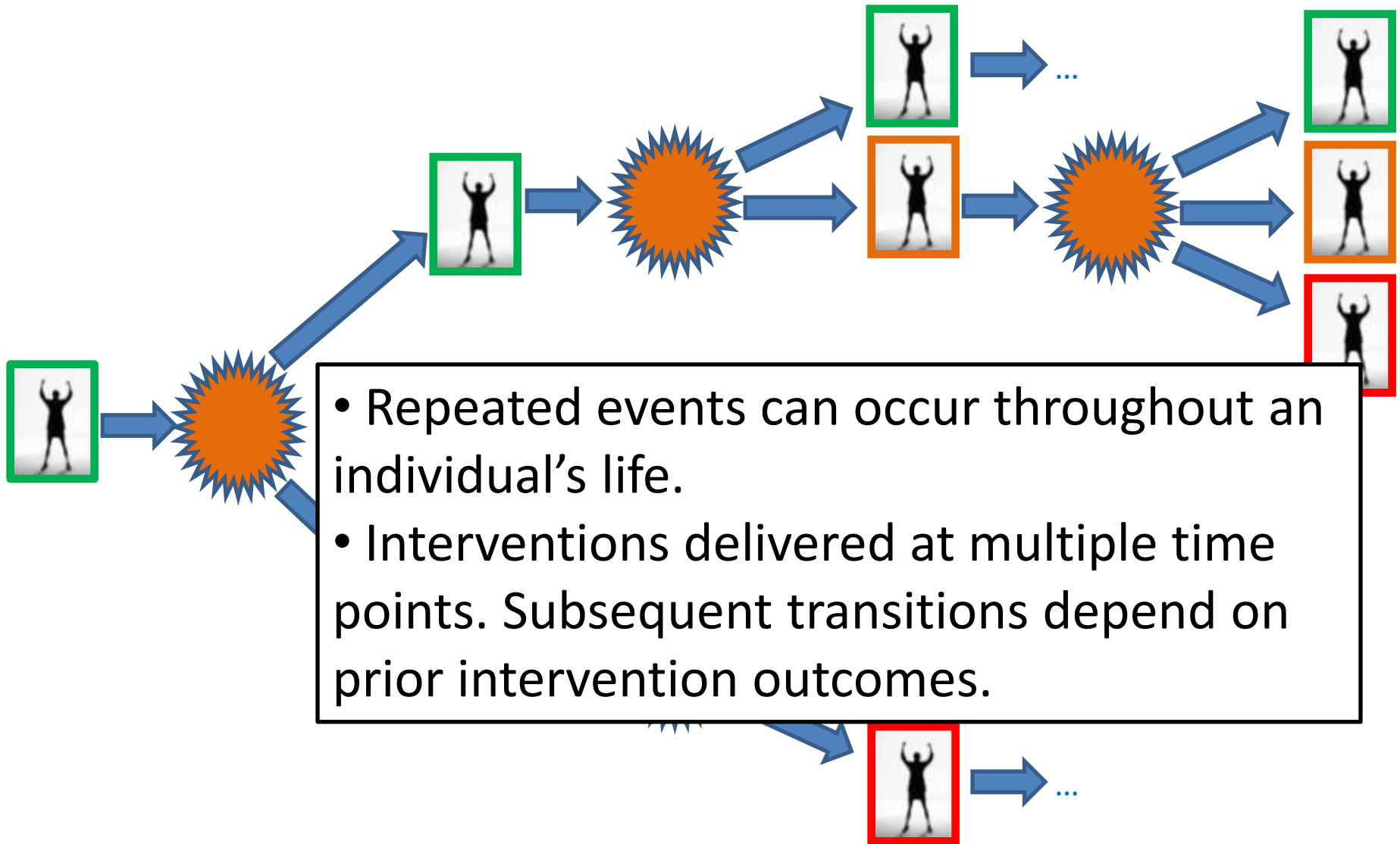
# Repeated in what sense?



# Disease process involves events occurring at multiple time points



# Intervention (can) be delivered repeatedly too



# What is a Markov Model?

- **Markov Model:** Mathematical modeling technique, derived from matrix algebra, that describes the transitions a cohort of patients make among a number of mutually exclusive and exhaustive health states during a series of short intervals or cycles

# Properties of a Markov Model

- Individuals are always in one of a finite number of health states
- Events are modeled as transitions from one state to another
- Time spent in each health state determines overall expected outcome
  - Living longer without disease yields higher life expectancy and quality adjusted life expectancy
- During each cycle of the model, individuals may make a transition from one state to another



# Constructing a Markov Model

- Define mutually exclusive health states
- Determine possible transitions between these health states
  - State transitions
  - Transition probabilities
- Determine clinically valid cycle length

# Cycle Length

- Short enough that for a given disease being modeled the chance of two events/transitions occurring in one cycle is essentially 0
  - Many applications: weekly or monthly
  - Some (e.g., ICU) may hourly or daily

# Natural history disease model: health states

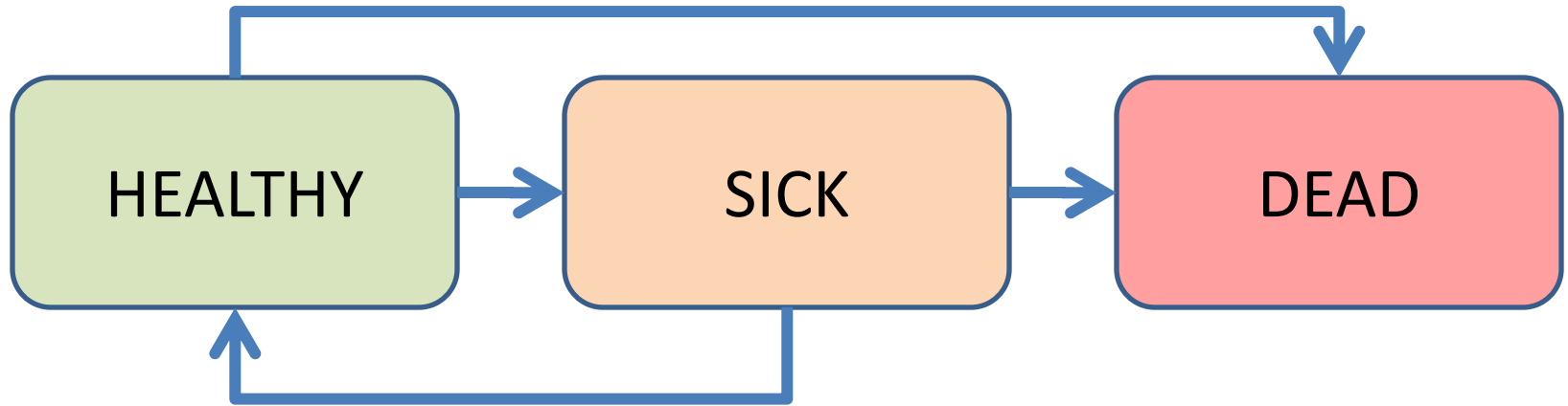
HEALTHY

SICK

DEAD

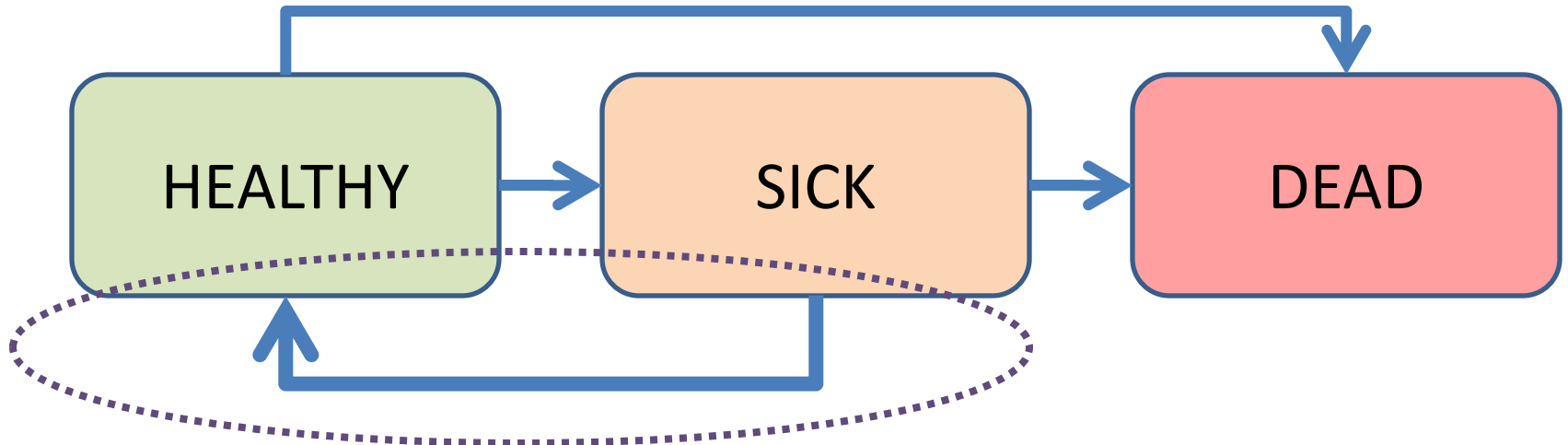
- Mutually exclusive and collectively exhaustive health states
- Best defined by actual biology/pathophysiology
- Markovian assumptions:
  - Homogeneity: All individuals in the same state have the same costs, quality of life, risks of transition
  - Memorilessness: The current state determines future risks
  - **Note: Stratification and tunnel states used to ensure Markov assumptions hold (advanced topic)**

# Natural history disease model: transitions



- Transitions between health states (arrows)
- The proportion that do not transition stay in current state
- Risk of death at all times and from all states!
- If no transition out of a state = absorbing state (i.e., death)

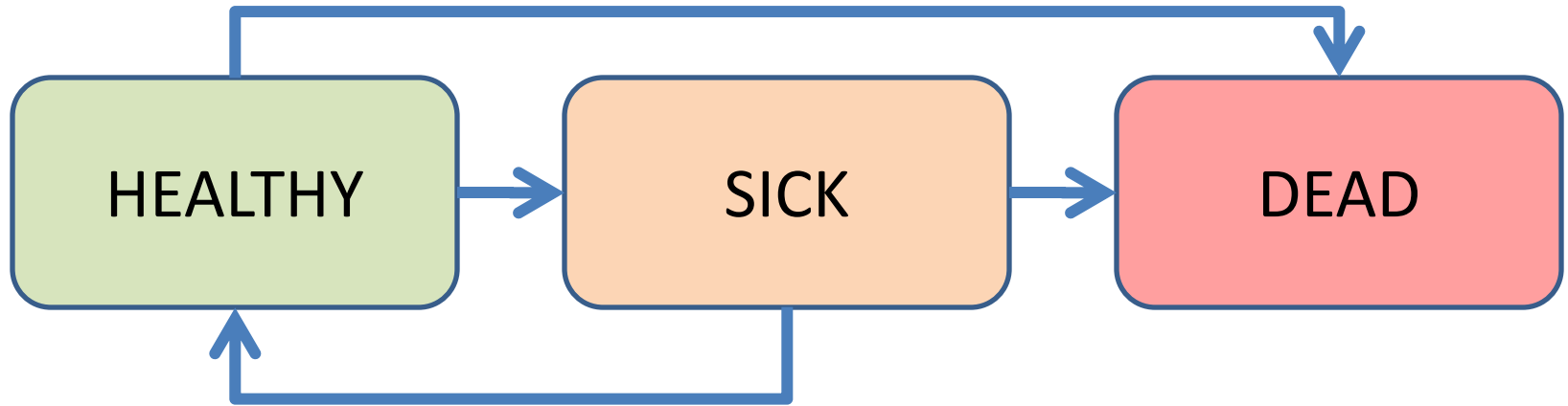
# Natural history disease model: time and matrix representation



$p_{HH}$	$p_{SH}$	0
$p_{HS}$	$p_{SS}$	0
$p_{HD}$	$p_{SD}$	1

For example  $p_{SH}$  is the Probability of going from Sick to Healthy

# Natural history disease model: time and matrix representation

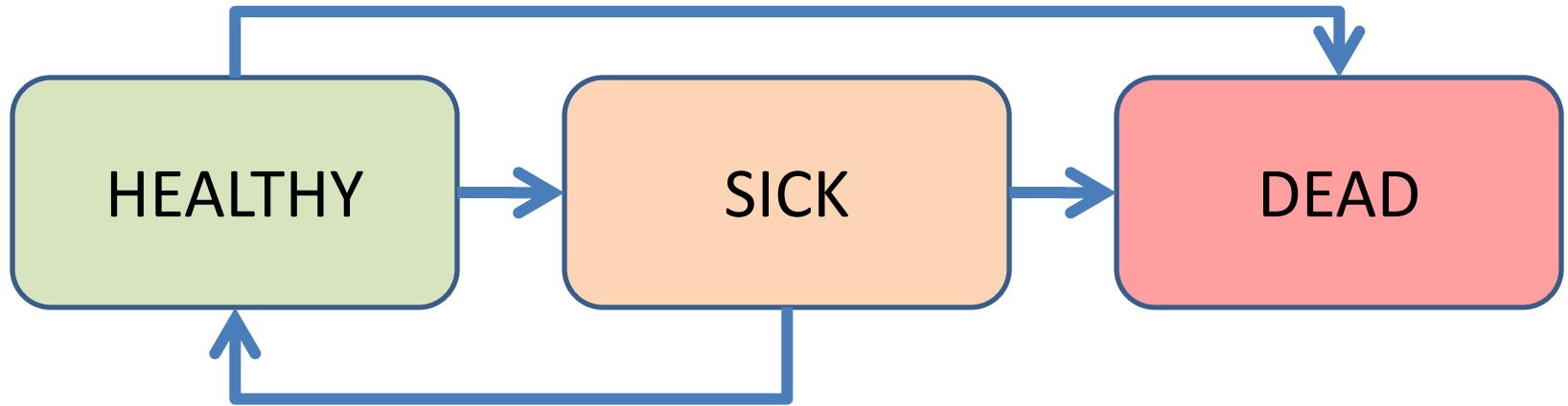


$$\begin{pmatrix} p_{HH} & p_{SH} & 0 \\ p_{HS} & p_{SS} & 0 \\ p_{HD} & p_{SD} & 1 \end{pmatrix} \begin{pmatrix} \text{propH} \\ \text{propS} \\ \text{propD} \end{pmatrix}$$

time=t

At time t, cohort has proportions in various states (Sum to 1!)

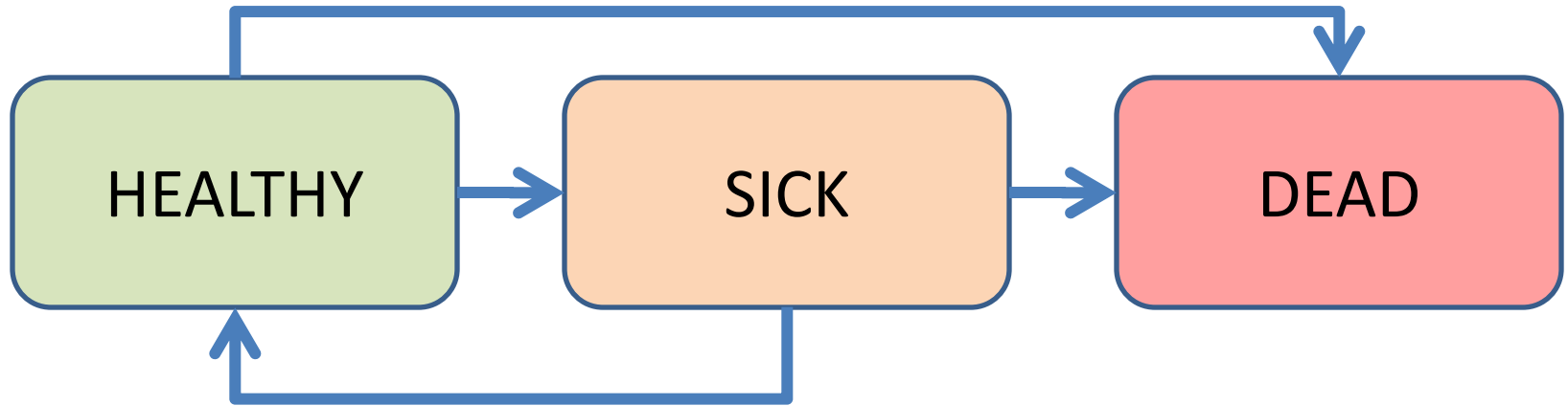
# Natural history disease model: time and matrix representation



$$\begin{pmatrix} p_{HH} & p_{SH} & 0 \\ p_{HS} & p_{SS} & 0 \\ p_{HD} & p_{SD} & 1 \end{pmatrix} \begin{pmatrix} \text{propH} \\ \text{propS} \\ \text{propD} \end{pmatrix}_{\text{time=t}} = \begin{pmatrix} \text{propH} \\ \text{propS} \\ \text{propD} \end{pmatrix}_{\text{time=t+1}}$$

***NOTE: transition probabilities can be time dependent as well***

# Natural history disease model: time and matrix representation

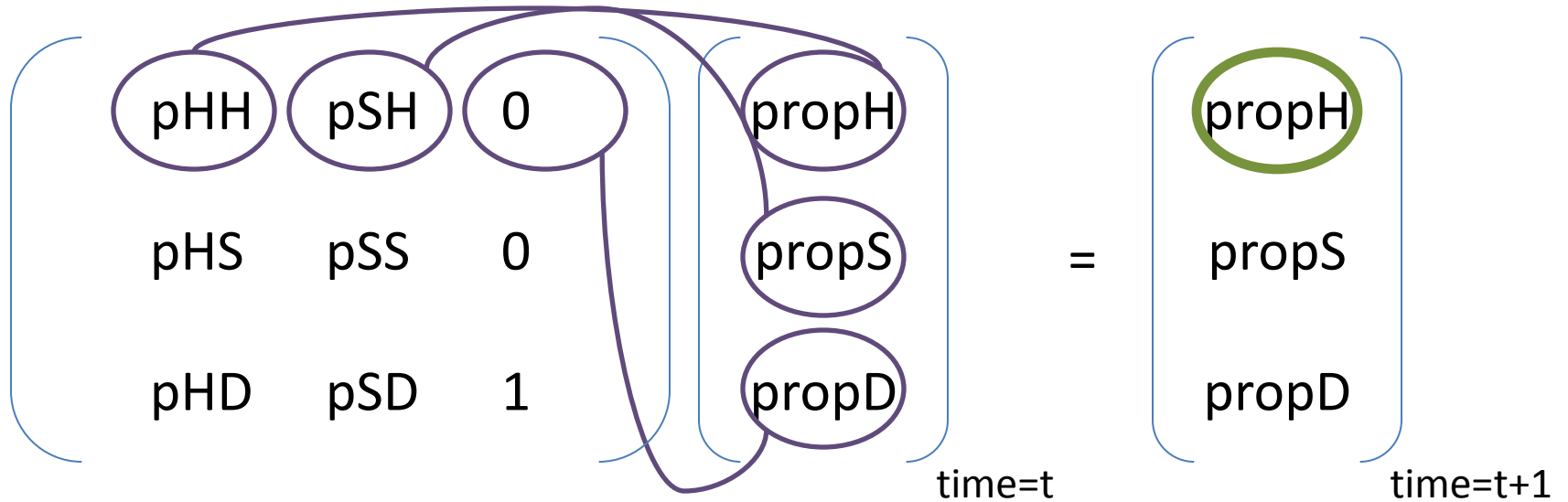
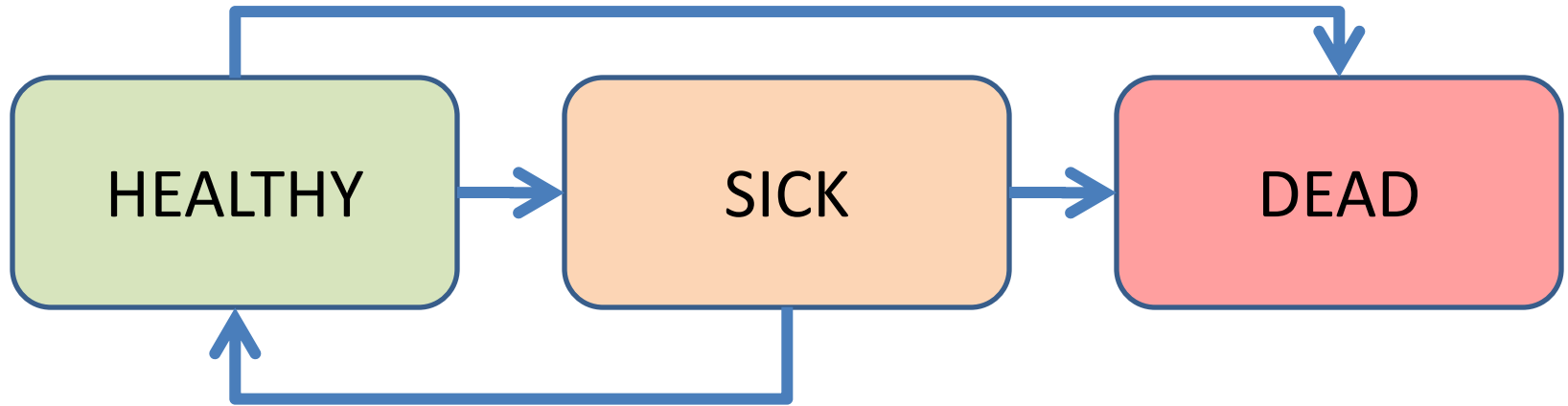


$$\begin{pmatrix} p_{HH} & p_{SH} & 0 \\ p_{HS} & p_{SS} & 0 \\ p_{HD} & p_{SD} & 1 \end{pmatrix} \begin{pmatrix} \text{propH} \\ \text{propS} \\ \text{propD} \end{pmatrix}_{\text{time}=t} = \begin{pmatrix} \text{propH} \\ \text{propS} \\ \text{propD} \end{pmatrix}_{\text{time}=t+1}$$

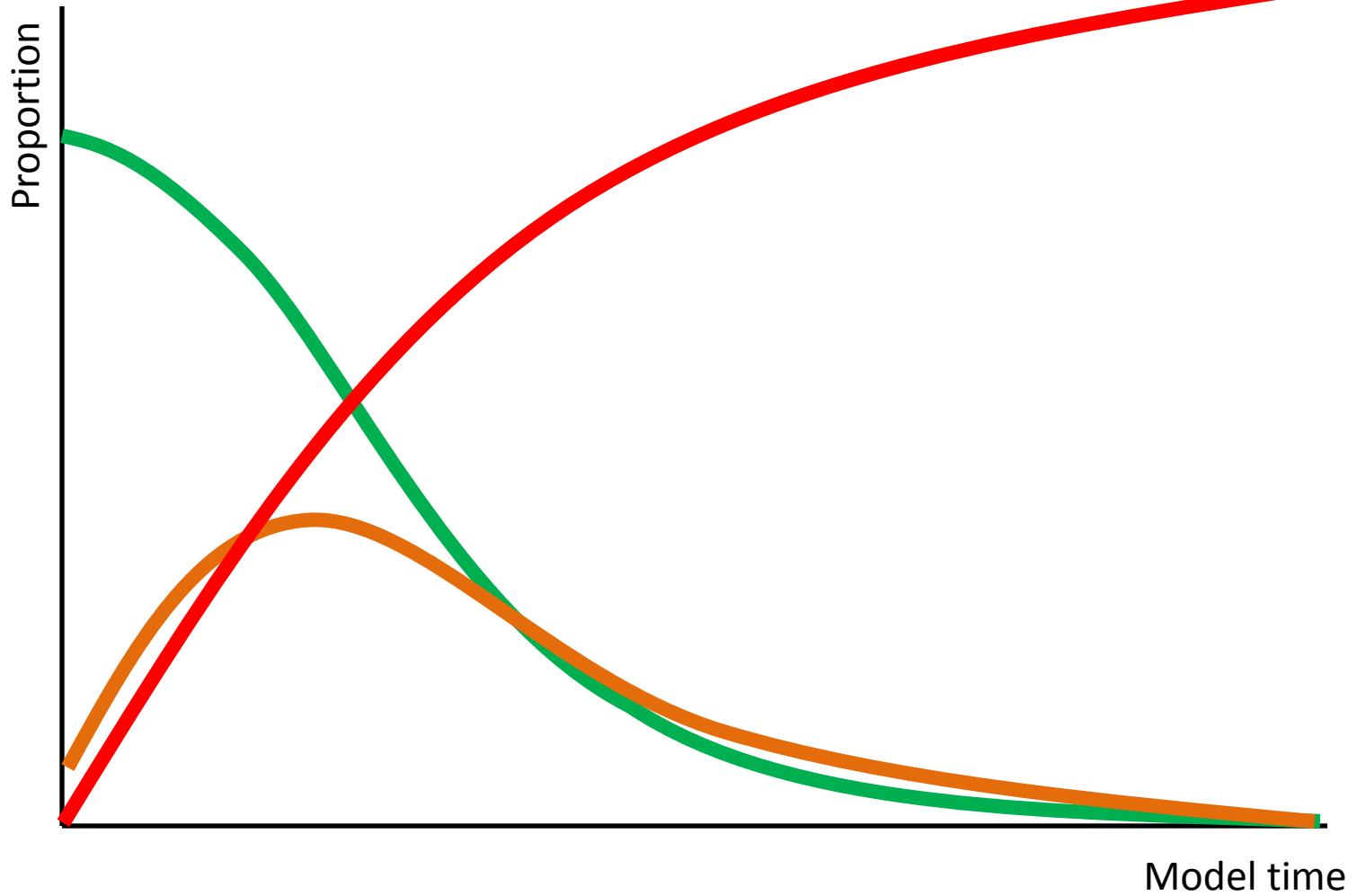
The diagram illustrates the matrix representation of the natural history disease model. It shows a transition matrix on the left, a vector of probabilities at time  $t$  in the middle, and a vector of probabilities at time  $t+1$  on the right. The transition matrix is a 3x3 matrix with elements  $p_{HH}$ ,  $p_{SH}$ ,  $0$ ,  $p_{HS}$ ,  $p_{SS}$ ,  $0$ ,  $p_{HD}$ ,  $p_{SD}$ , and  $1$ . The vector at time  $t$  contains  $\text{propH}$ ,  $\text{propS}$ , and  $\text{propD}$ . The vector at time  $t+1$  contains  $\text{propH}$ ,  $\text{propS}$ , and  $\text{propD}$ . The  $\text{propH}$  element in the time  $t+1$  vector is highlighted with a green circle.



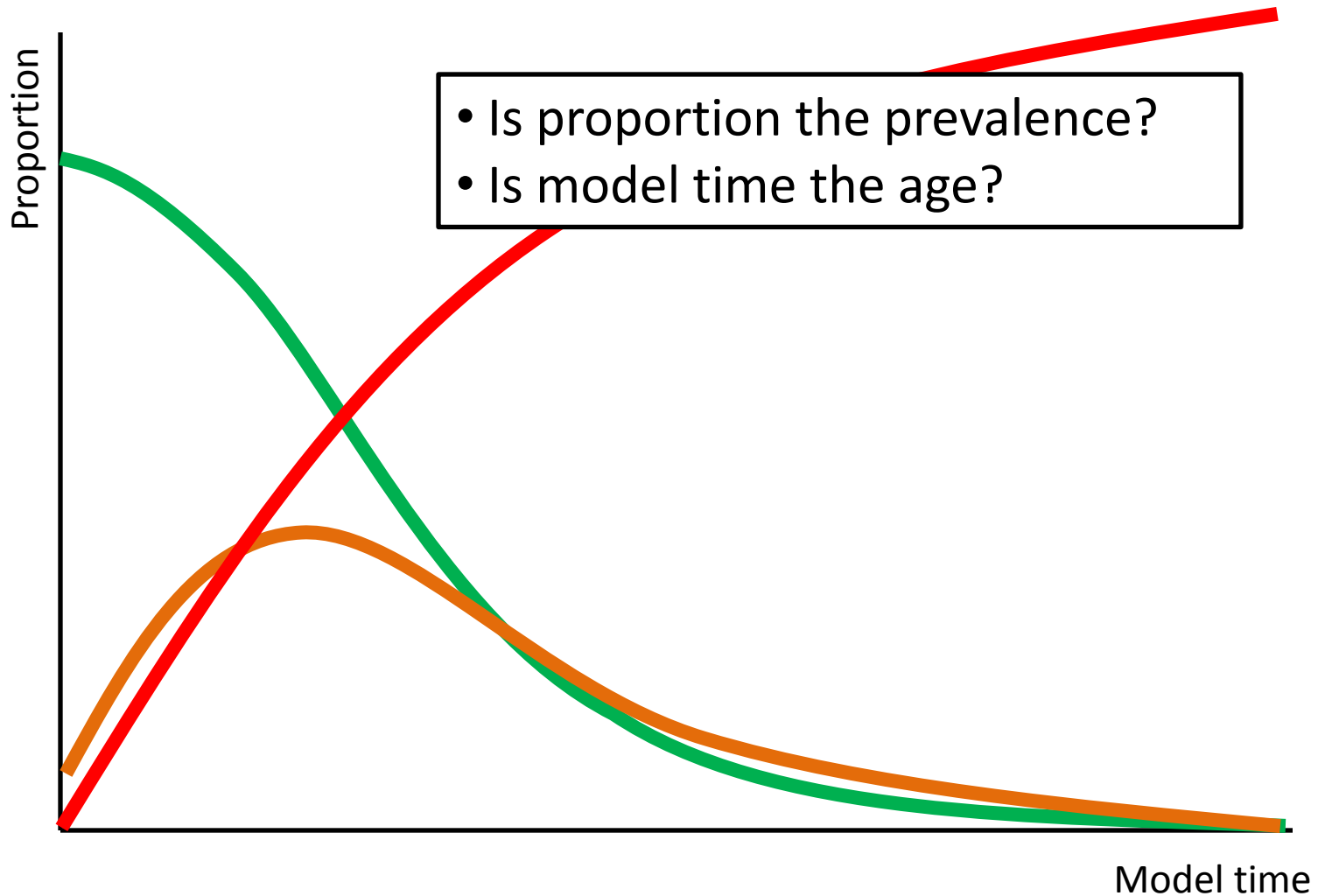
# Natural history disease model: time and matrix representation



# Model trace



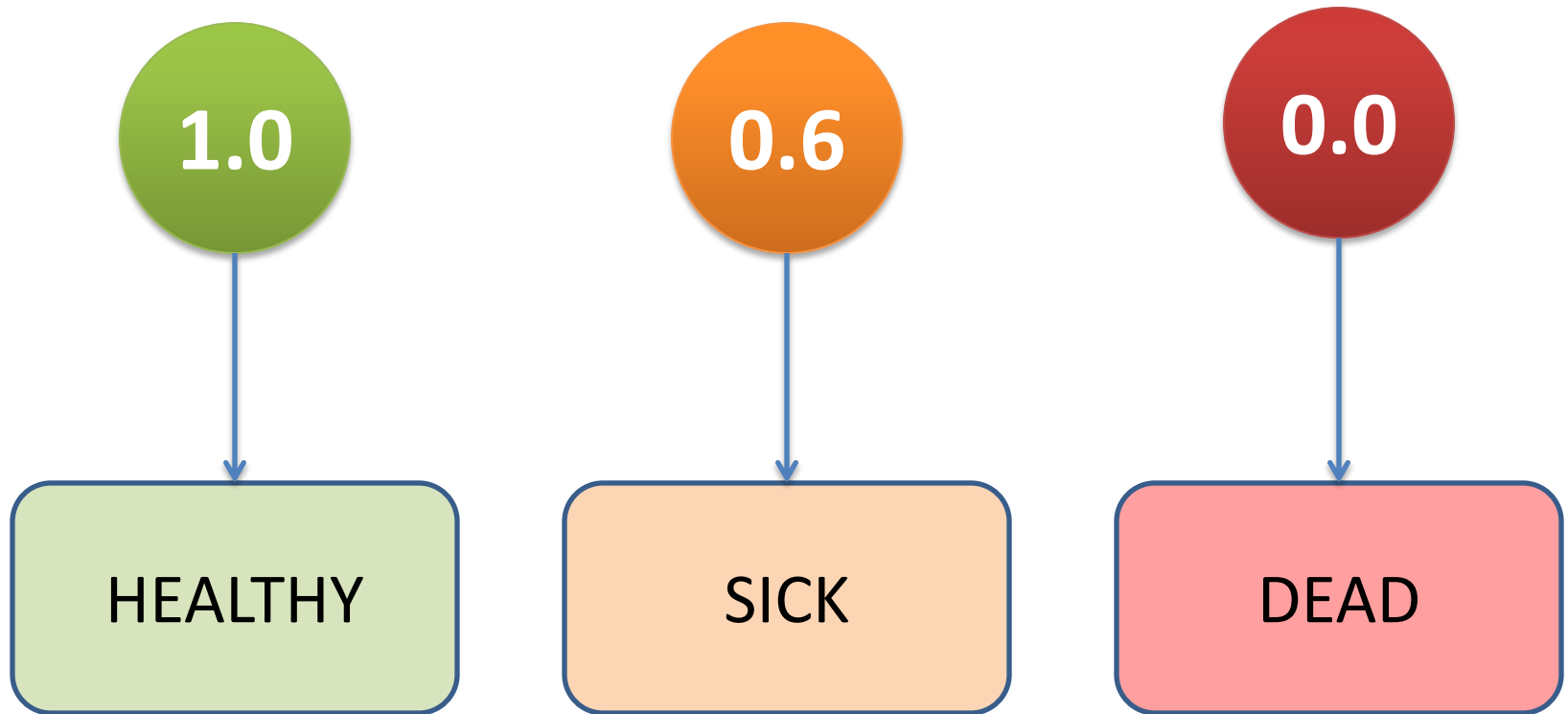
# Model trace



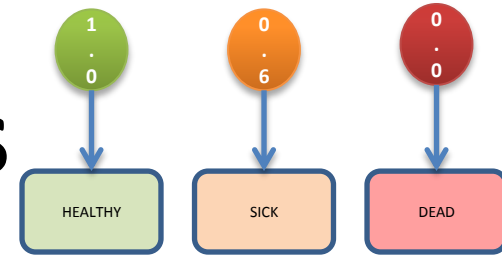
# Underlying the trace

Stage	propH_t	propS_t	propD_t	NotD
0	1.00	0.00	0.00	1.00
1	0.90	0.09	0.01	0.99
2	0.75	0.10	0.15	0.85
3	0.50	0.25	0.25	0.75
4	0.20	0.40	0.40	0.60
5	0.10	0.30	0.60	0.40
6	0.05	0.15	0.80	0.20
7	0.00	0.00	1.00	0.00

# Quality Adjusted Life Years (QALYS) & quality-of-life weights



# Valuing outcomes

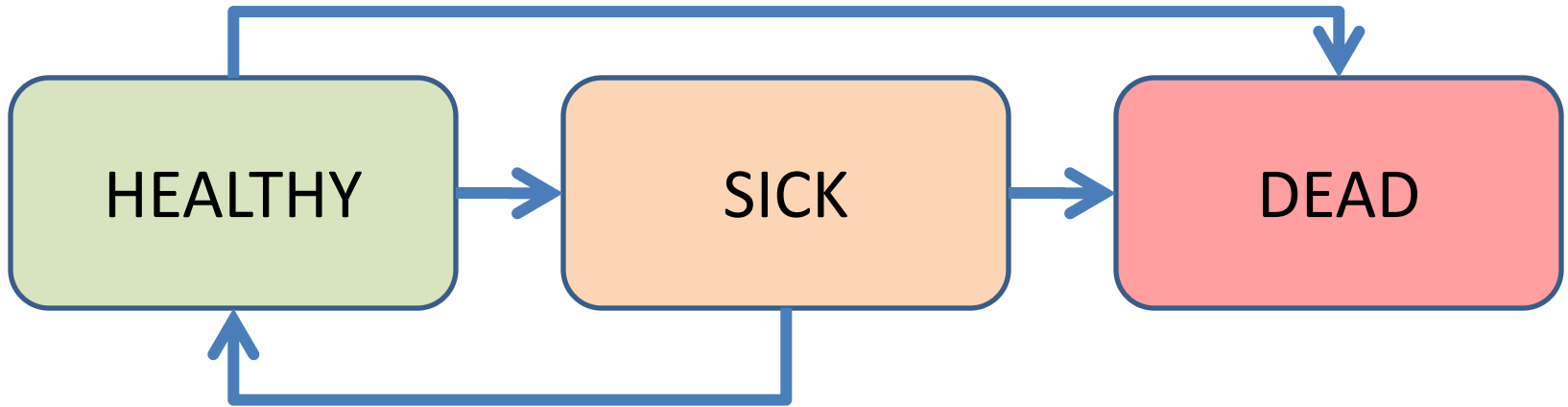


Stage	propH_t	propS_t	propD_t	NotD
0	1.00	0.00	0.00	1.00
1	0.90	0.09	0.01	0.99
2	0.75	0.10	0.15	0.85
3	0.50	0.25	0.25	0.75
4	0.20	0.40	0.40	0.60
5	0.10	0.30	0.60	0.40
6	0.05	0.15	0.80	0.20
7	0.00	0.00	1.00	0.00

$$QALYs = \sum_{t=0}^T \left( \text{prop}H_t * qH + \text{prop}S_t * qS + \text{prop}D_t * 0 \right)$$

$$COSTs = \sum_{t=0}^T \left( \text{prop}H_t * cH + \text{prop}S_t * cS + \text{prop}D_t * 0 \right)$$

# Interventions?



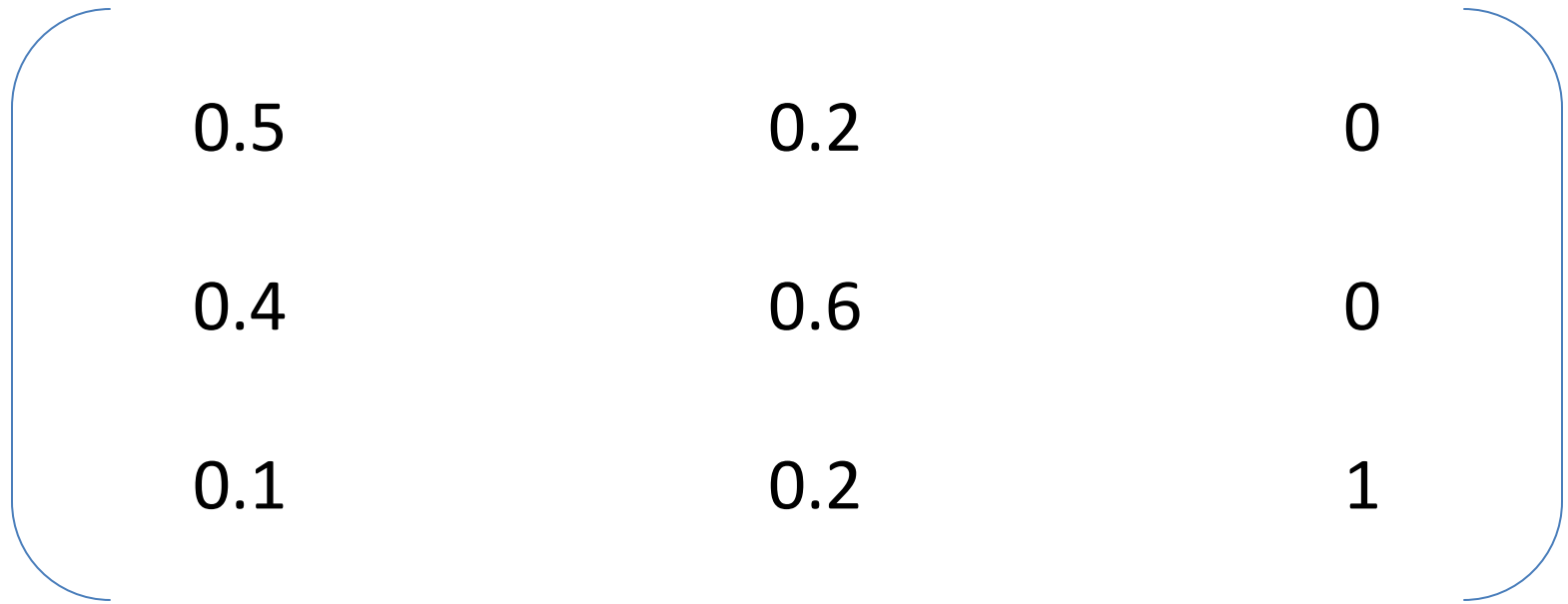
$$\begin{pmatrix} p_{HH} & p_{SH} & 0 \\ p_{HS} & p_{SS} & 0 \\ p_{HD} & p_{SD} & 1 \end{pmatrix} \begin{pmatrix} \text{propH} \\ \text{propS} \\ \text{propD} \end{pmatrix}_{\text{time=t}} = \begin{pmatrix} \text{propH} \\ \text{propS} \\ \text{propD} \end{pmatrix}_{\text{time=t+1}}$$

# Screening before treatment

- Screening 70% sensitivity, 100% specific
- Treatment 90% effective
- Intervention occurs after natural hx transitions every cycle
  
- Calculations
  - $pHS_i = pHS*(0.3) + pHS*(0.7*0.1)$
  - $pSS_i = pSS*(0.3) + pSS*(0.7*0.1)$
  - $pSH_i = pSH + pSS*(0.7*0.9)$
  - $pHH_i = pHH + pHS*(0.7*0.9)$



# Natural History



0.5	0.2	0
0.4	0.6	0
0.1	0.2	1

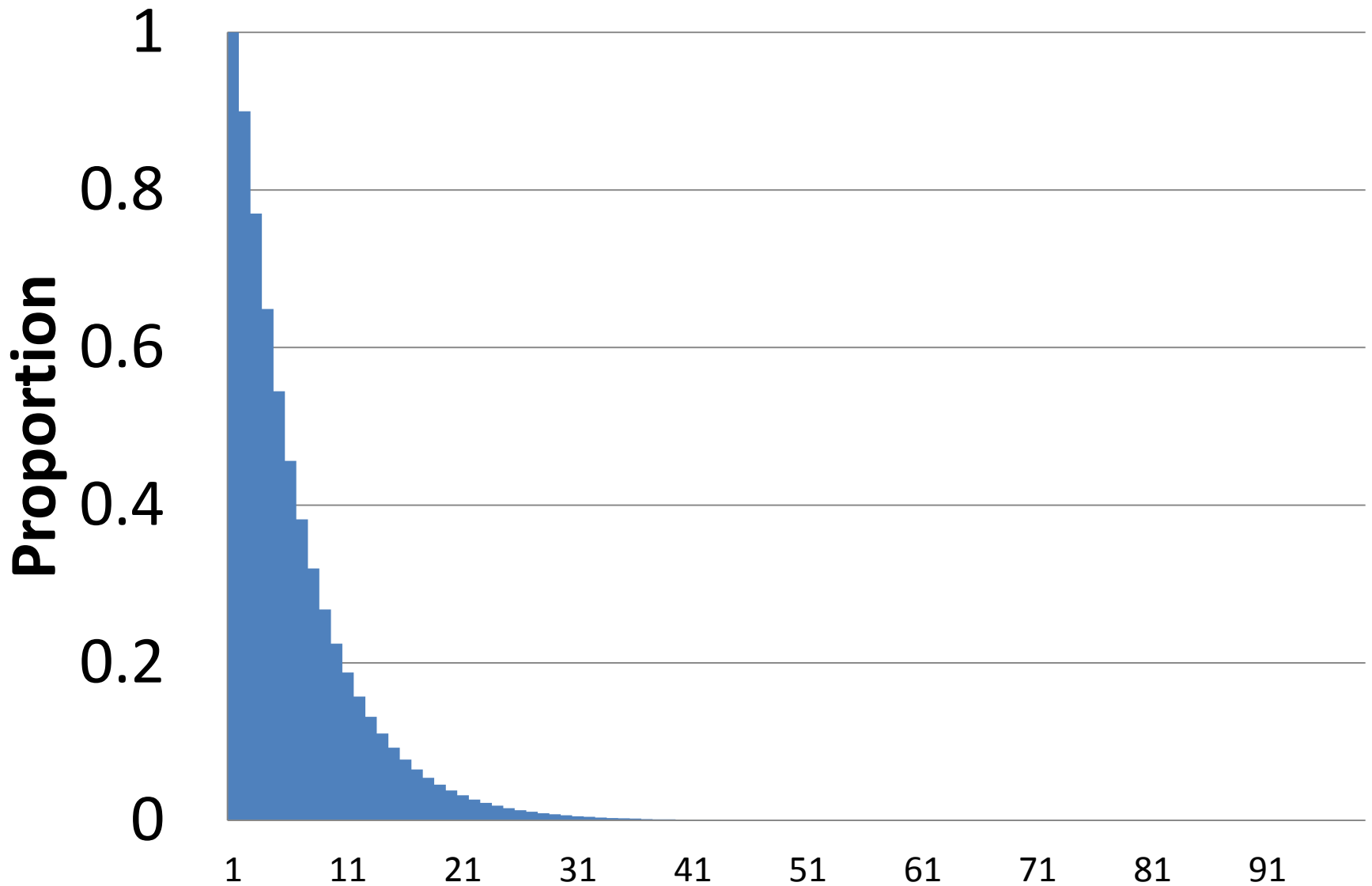
# Screening before treatment

pHH_i	pSH_i	0
pHS_i	pSS_i	0
pHD	pSD	1

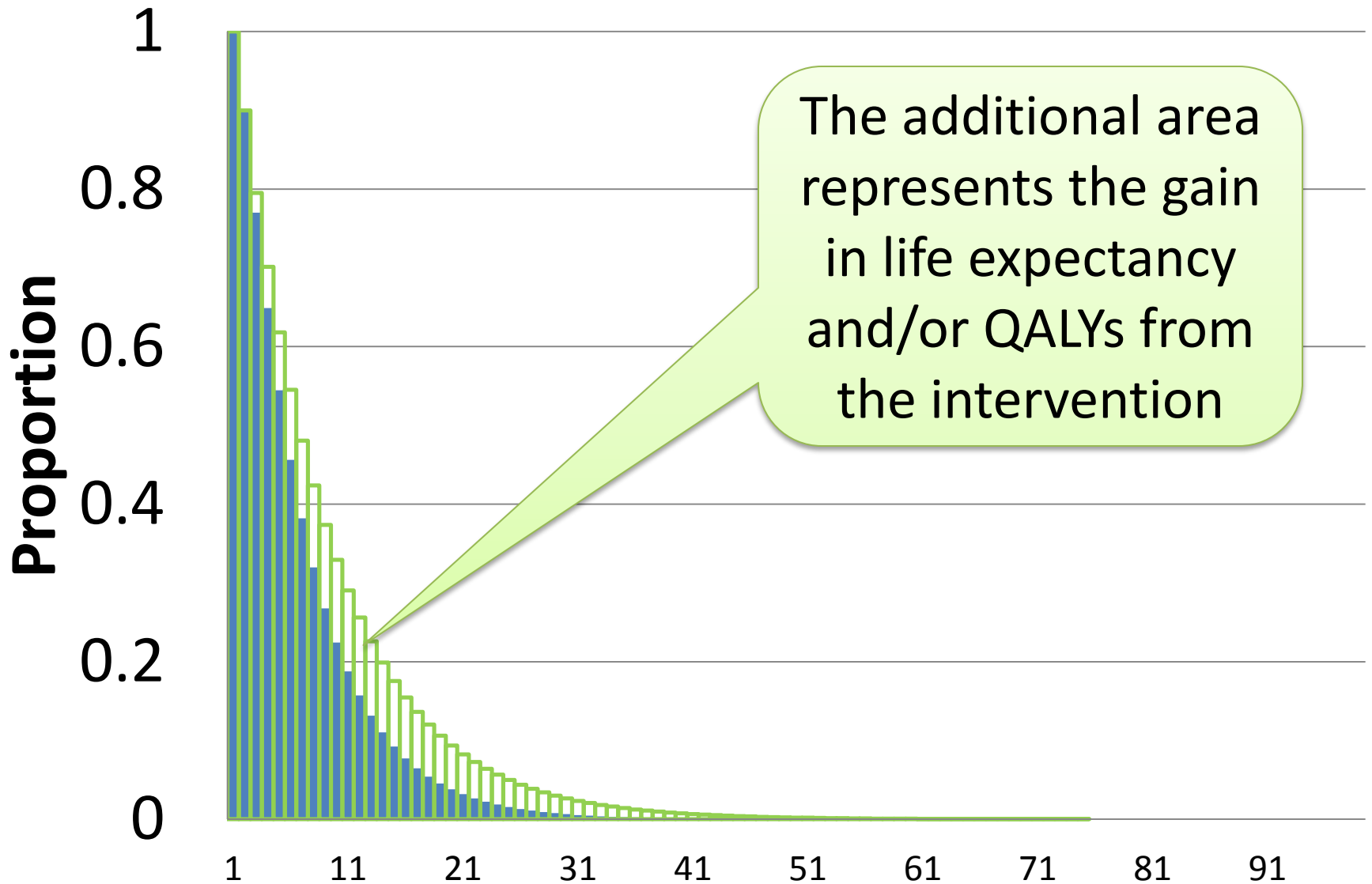
# Screening before treatment

0.752	0.222	0
0.148	0.578	0
0.100	0.200	1

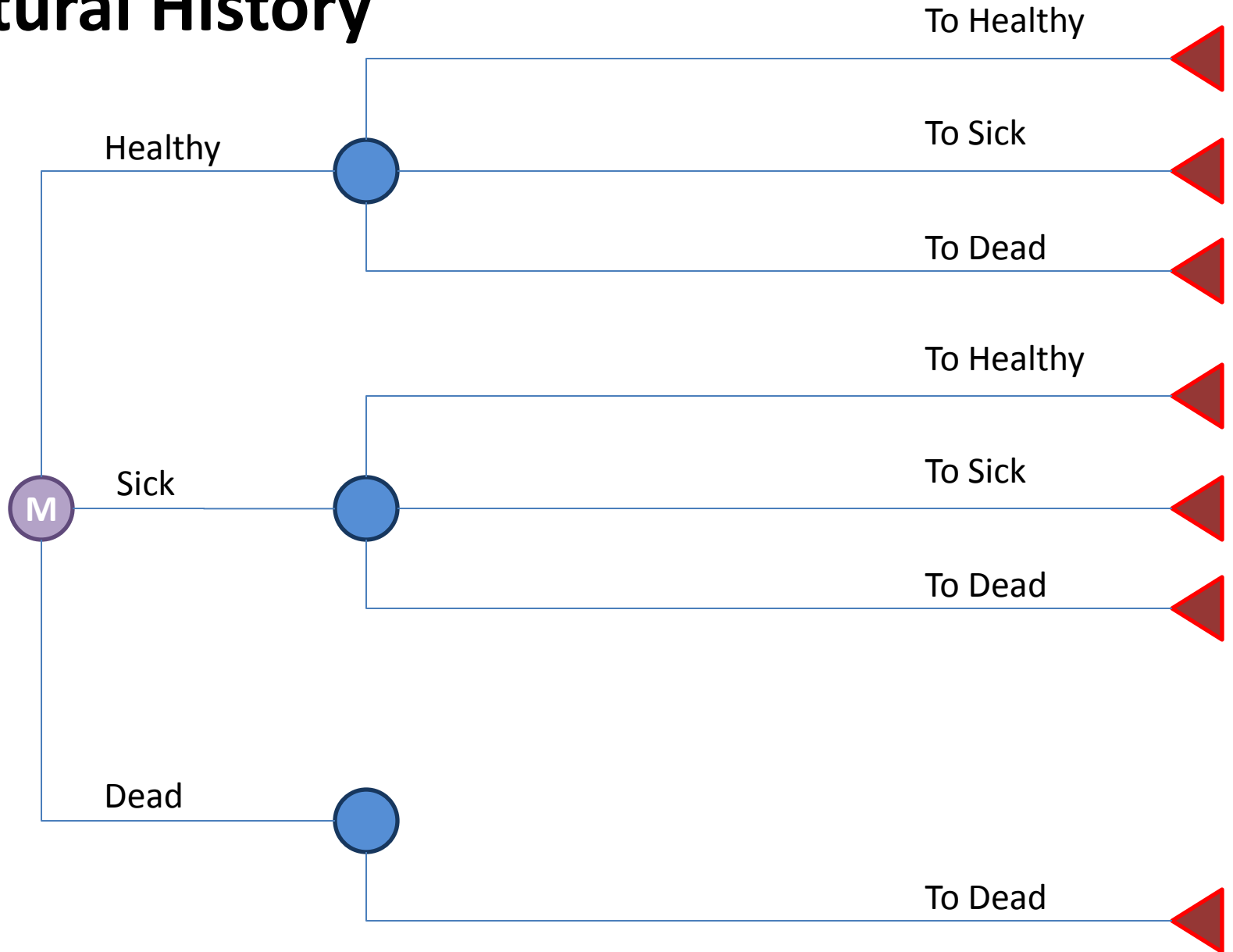
# With and w/o intervention



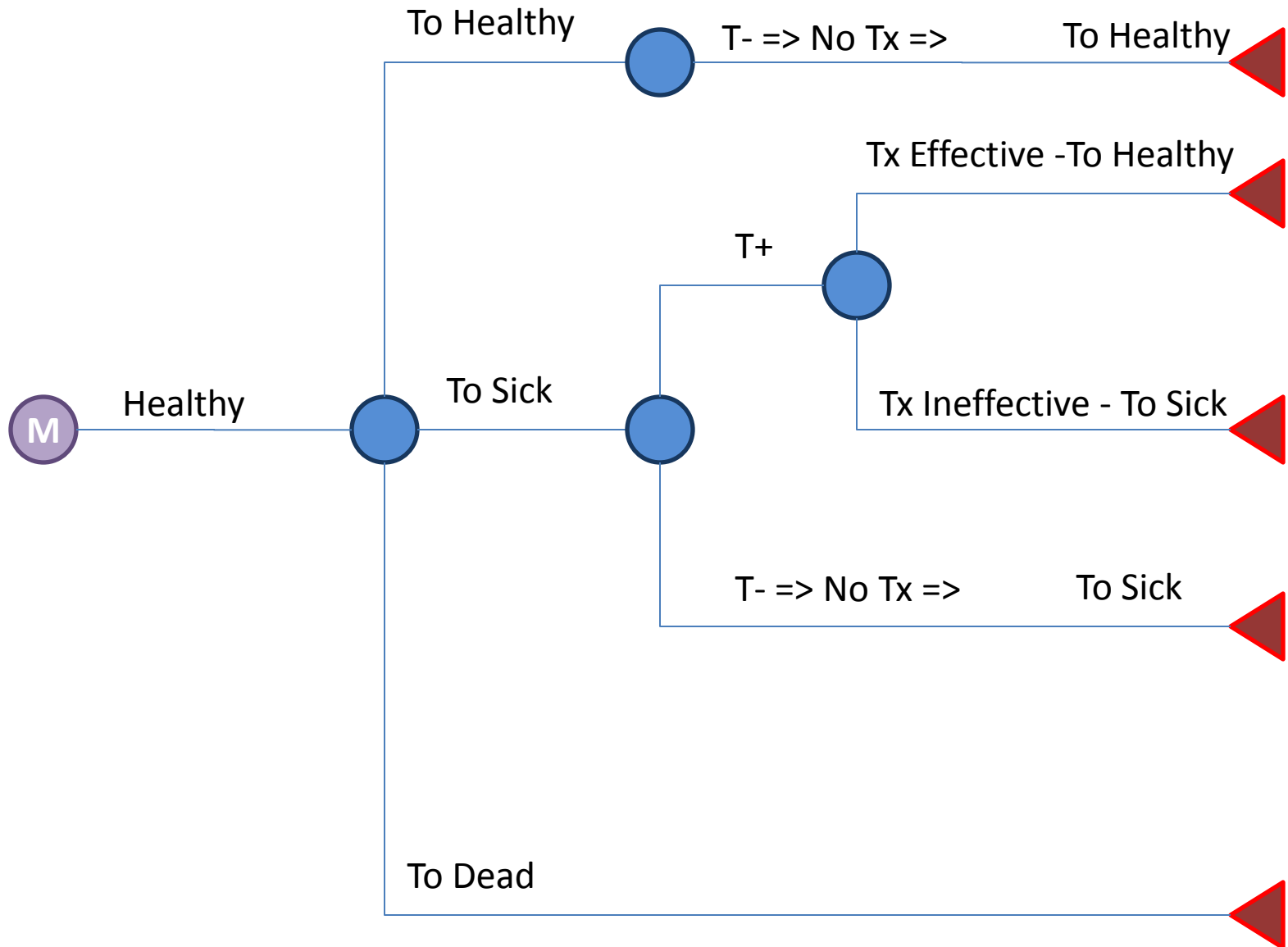
# With and w/o intervention



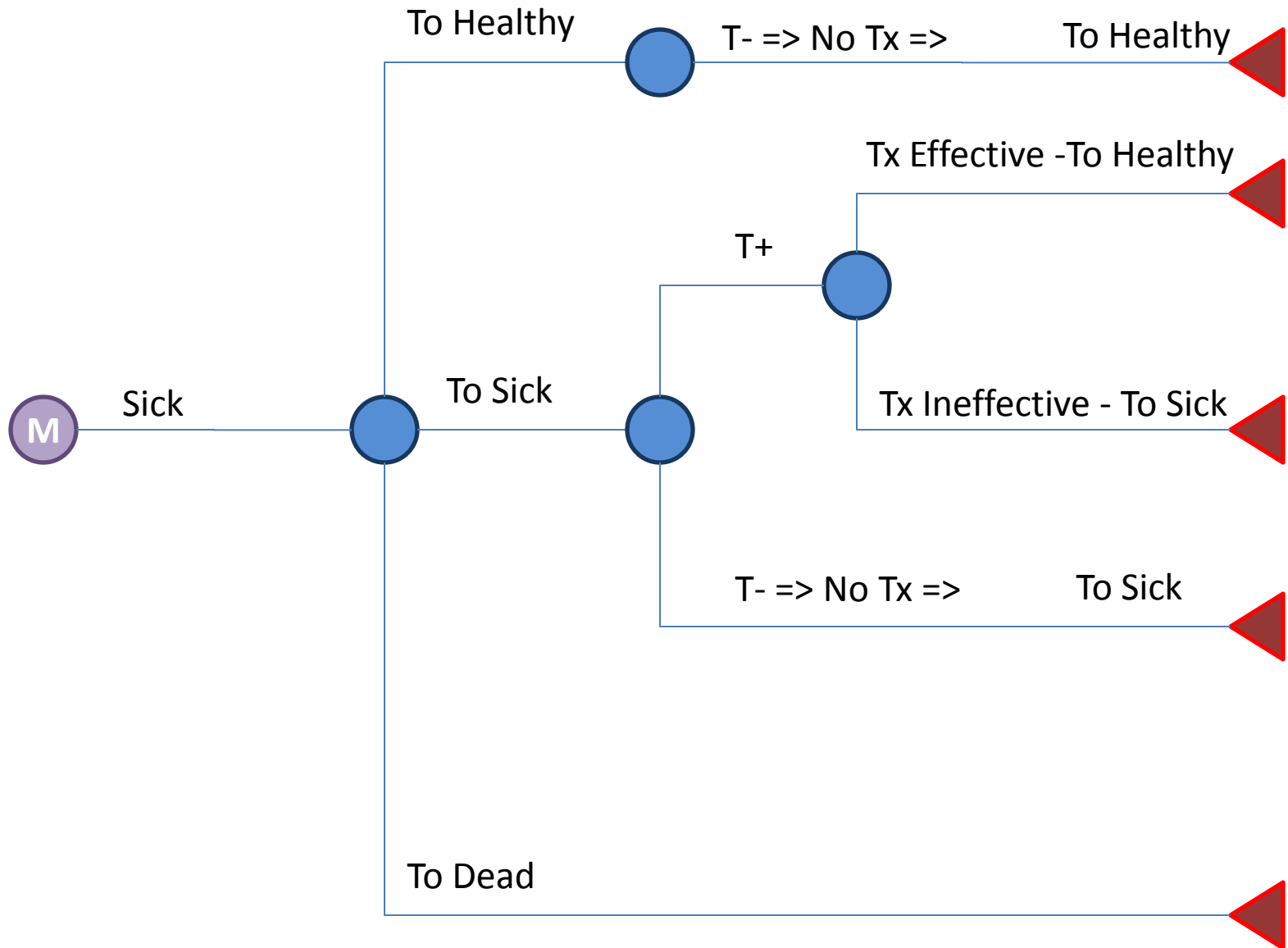
# Natural History



# Intervention



# Intervention





# Cohorts vs. individuals

## Deterministic vs. stochastic

- Markov cohort model (i.e., the matrix version) is smooth model (infinite population size) of the proportion of a cohort in each state at each time
- Can use same structure to simulate many individuals (first-order Monte Carlo) (simple microsimulation)
- The matrix becomes the probability of an individual transition from one state to another instead of the % of those in a given state who deterministically flow into another state

# Microsimulation

Healthy

Sick

Dead

0



1



2



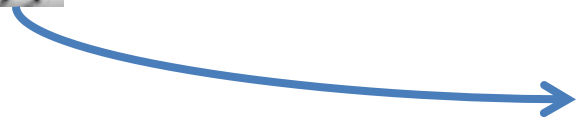
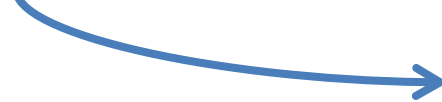
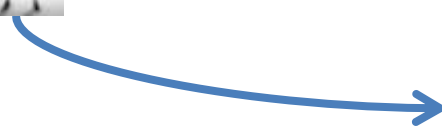
3



4



5



# Microsimulation

Healthy Sick Dead

0



pHS

1



pSS

2



pSH

3



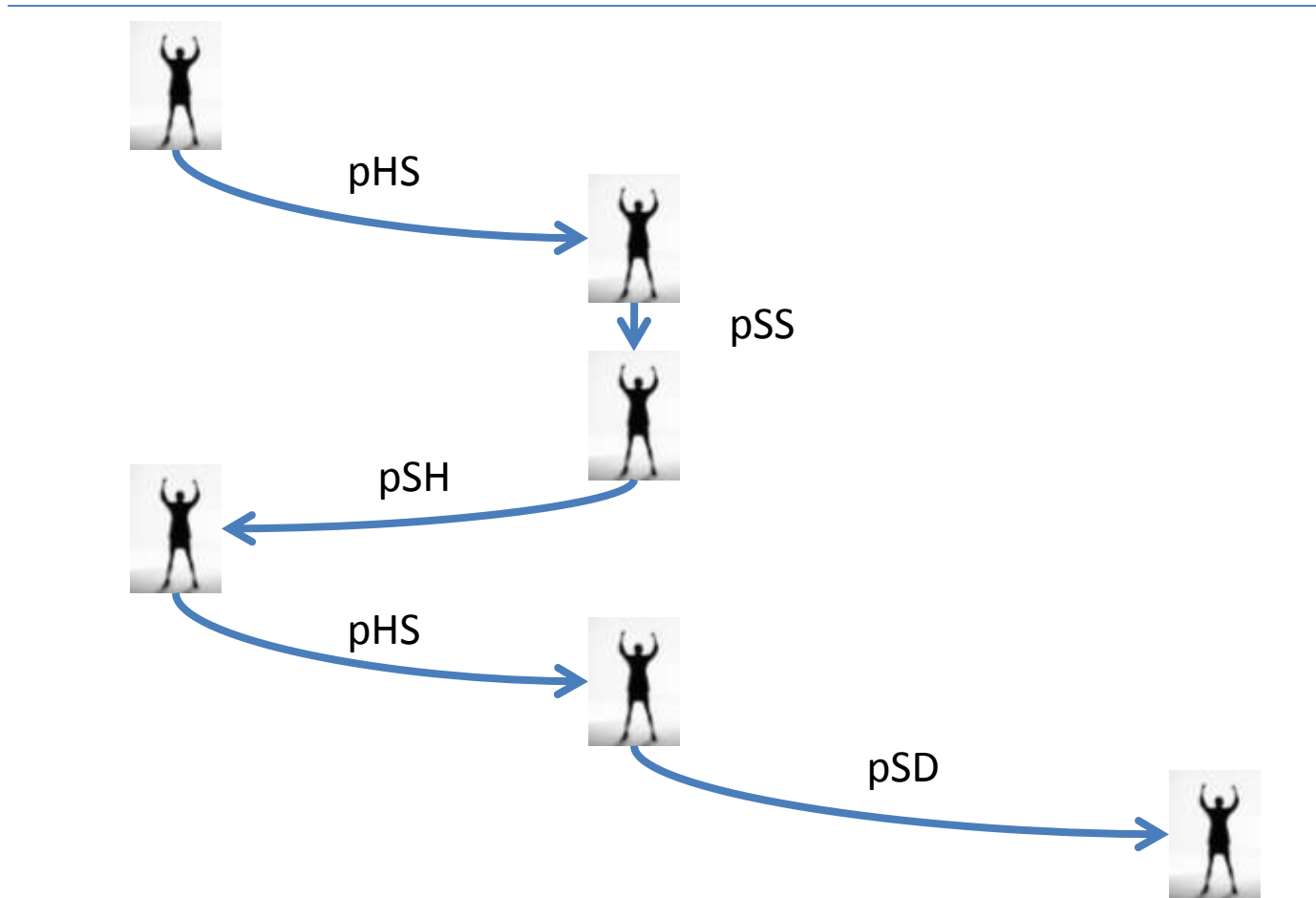
pHS

4



pSD

5



# Microsimulation

Healthy

Sick

Dead

0



1



2



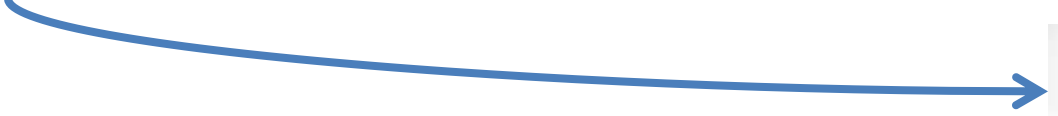
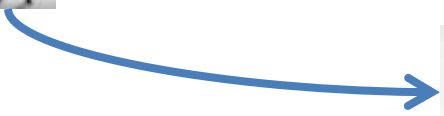
3



4



5



# Microsimulation

Healthy

Sick

Dead

0



1



2



3



4



5



# Recall the trace and calculation of outcomes from it

Stage	propH_t	propS_t	propD_t	NotD
0	1.00	0.00	0.00	1.00
1	0.90	0.09	0.01	0.99
2	0.75	0.10	0.15	0.85
3	0.50	0.25	0.25	0.75
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7	0.00	0.00	1.00	0.00

$$QALYs = \sum_{t=0}^T \left( \text{propH}_t * qH + \text{propS}_t * qS + \text{propD}_t * 0 \right)$$

$$COSTs = \sum_{t=0}^T \left( \text{propH}_t * cH + \text{propS}_t * cS + \text{propD}_t * 0 \right)$$

# Microsimulation

- Run with many individuals
- Calculate proportions in each state at each time (just like in our Markov cohort table)
  - Stage 2: 5100 sick / 100,000 people = 5.1%
- Approximates the “smooth” cohort version
  - 5.1% [CI] is  $\approx$  5.0% in “smooth” cohort
  - **Advanced**
    - Larger the number of individuals the closer to the smooth cohort (tighter the CI)
    - See Kuntz/Weinstein chapter of Michael Drummond’s book on Economic Evaluation for more on this for more on this

# Why consider microsimulation?

- It requires longer simulation times
- It is more complex
- Fewer people are familiar with it
- There is “Monte Carlo” noise (random error) even with simulating fairly large groups of individuals (at least for rare events)



# State explosion!

- Suppose you want to use a Markov model of a disease with 2 states and death (H,S,D)
- Suppose you need it stratified by sex and smoking status (3 levels), BMI (4 levels), hypertension (4 levels)
- Now you need  $2 \times 3 \times 4 \times 4 \times 2$  states (death is not stratified) = 192 states
- What if you need to stratify states by past history? (previous high hypertension, used to be obese) or Tx history (has a stent)?

# Microsimulation as alternative

- Simulate 1 individual at a time
- Assign a set of attributes to the individual
  - Sex=M, Smoking=Y, BMI=Overweight, HT=Y
- Define a function for the probability of transitioning from H to S
  - $P(H \text{ to } S \mid \text{Sex, Smoking, BMI, HT})$
- Have functions for changing attributes
  - $P(\text{BMI=Obese} \mid \text{Sex, BMI})$
- Track previous health states
  - $P(H \text{ to } S \mid \text{Sex, Smoking, BMI, HT, } S \text{ in the past})$
- **Note: Could estimate these functions from logistic regressions**

# Sage advice I have heard

- Know what information your consumers need
- Pick a model that is as simple as possible ... but no simpler
- Know the limits of what your model does and make statements within those limits – All research studies have limitations

# Summary:

## Medical Decision Analysis

- Clearly defines alternatives, events, and outcomes
- Formal method to combine evidence
- Can prioritize information acquisition
- Can help healthcare providers to make medical decisions under uncertainty

# Classic sources on about decision analysis and modeling

- Sox HC, Blatt MA, Higgins MC, Marton KI (1988) Medical Decision Making. Boston MA: Butterworth-Heinemann Publisher.
- Detsky AS, Naglie G, Krahn MD, Naimark D, Redelmeier DA. Primer on medical decision analysis: Parts 1-5. Med Decis Making. 1997;17(2):123-159.
- Sonnenberg FA, Beck JR. Markov models in medical decision making: a practical guide. Med Decis Making. 1993;13(4):322-38.
- Beck JR, Pauker SG. The Markov process in medical prognosis. Med Decis Making. 1983;3(4):419-458.
- Society for Medical Decision Making (<http://www.smdm.org>)

A wide-angle photograph of the Stanford University Main Quad at dusk. The central building, the Sather Gate, is illuminated from within, casting a warm glow. The sky is a mix of soft pinks, oranges, and blues. Palm trees and other trees are visible in the background. The foreground is a large, well-maintained green lawn with a central path leading towards the building.

# THANK YOU

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