RUC CAPE vs. RAP CAPE

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The motivation for this is an email from Geoff Manikin (NCEP/EMC) concerning this subject resulting from a conversation he had with Bill Bua (COMET).

Summary:

- RUC CAPE is higher than RAP CAPE for large CAPE situations (with same temperature/moisture profile).
- RUC CAPE calculation used moist static energy formulation, different from the now-NCEP-consistent Unipost calculation with a more traditional CAPE calculation.
- RUC CAPE did not use virtual temperature for its CAPE calculation, but RAP CAPE (via Unipost) does.

When high values of moisture exist, the CAPE calculations become very sensitive to the mixing ratio of the lifted parcel (nonlinearity of the Clausius-Clapeyron equation) and the assumptions made in the calculations (particularly, the form of the first law of thermodynamics, and, less importantly, the form of the Clausius-Clapeyron equation for saturation vapor pressure of water as function of the temperature). The NMM (and prior, Eta) and RUC differ in these assumptions, with the RUC's assumptions tending to favor larger CAPE when dew points are high.

Detailed explanation

The RUC CAPE is based on conservation of moist static energy in parcels during ascent. This implies a more approximate form of the first law of thermodynamics than is used in the Unipost calculation, which I believe is based on conservation of θ_e during parcel ascent, where the formulation of θ_e follows Bolton (1980, *Mon. Wea. Rev.*).

Starting with a simplified form of the first law (already simplified from Bolton), in which the air is assumed dry except for latent-heat release,

$$c_{pd}\frac{DT}{Dt} - (1/\rho)\frac{Dp}{Dt} = -L_v\frac{Dr_s(T,p)}{Dt}, \quad r_v \geq r_s \qquad (1)$$

$$= 0, \text{ otherwise,}$$

where c_{pd} is specific heat of dry air at constant pressure and assumed constant, L_v, the latent heat of condensation, also assumed constant (i.e., not a function of temperature), ρ is air density (ignoring condensate and virtual temperature effects), r_v is mixing ratio of water vapor, r_s is the saturation (wrt liquid water) mixing ratio, and D/Dt represents the total derivative following an air parcel. With these assumptions,

$$\frac{D(c_{pd}T + L_v r_v)}{Dt} = \frac{1}{\rho}\frac{Dp}{Dt} \quad . \qquad (2)$$

Next comes the approximation I believe to be the most problematic for the RUC CAPE calculation: that the pressure change on the RHS of (2) following an air parcel can be described using the hydrostatic relation:

$$\frac{Dp}{Dt} = -\rho g\left(\frac{Dz}{Dt}\right), \qquad (3)$$

whence

$\frac{D}{Dt}\left(c_{pd}T + gz + L_v r_v\right) = 0 = \frac{Dh}{Dt}$, where $h = c_{pd}T + gz + L_v r_v$, the moist static energy as it is most commonly defined. Of course, $r_v = r_s(T, p)$ in a water-saturated parcel lifted above it's lifted condensation level.

The assumption (3) is that the pressure following the air parcel varies as if the parcel is in hydrostatic balance at it's particular density. But, this assumption conflicts with the fundamental assumption made in parcel ascent, that the pressure on the parcel is the same as the pressure in the environment, assumed to be in hydrostatic balance at (in general) a different density. So, (3) must be considered an *approximation*, and the error resulting from this approximation is proportional to the density difference. In particular, one can see that if ρ of the parcel is less (greater) than ρ of the environment, then a given decrease in pressure will correspond to a larger (smaller) increase in z than is actually the case if the pressure in environment and parcel are the same.

The approximation (3) does not appear explicitly in the calculations, so that the actual buoyancy calculation doesn't know anything about this explicitly, being based strictly on the starting *h* of the parcel and the comparison of this *h* to the *h* of the environment, assuming water saturation for both. In the buoyancy calculation, the geopotential height is assumed the same in both parcel and environment. This means that in a situation with large positive buoyancy, the larger *h* resulting from (3) is pushed into the $c_{pd}T$ and $L_v r_s(T, p)$ terms, resulting in inflation to the positive buoyancy.

The Unipost calculation of CAPE, which is used by the RAP, does not require (3), and also uses virtual temperature instead of non-virtual temperature. On both counts it should be more accurate. Frequently, particularly when buoyancy is modest and there is a rapid falloff of mixing ratio with height in the environment, the RAP CAPE for the same parcel and environment will be larger than the RUC CAPE. However, I doubt we will see as many extreme CAPE values (say > 6000 J/kg) in RAP as we have seen in RUC in the past.