Estimating channel bathymetry from SWOT observations

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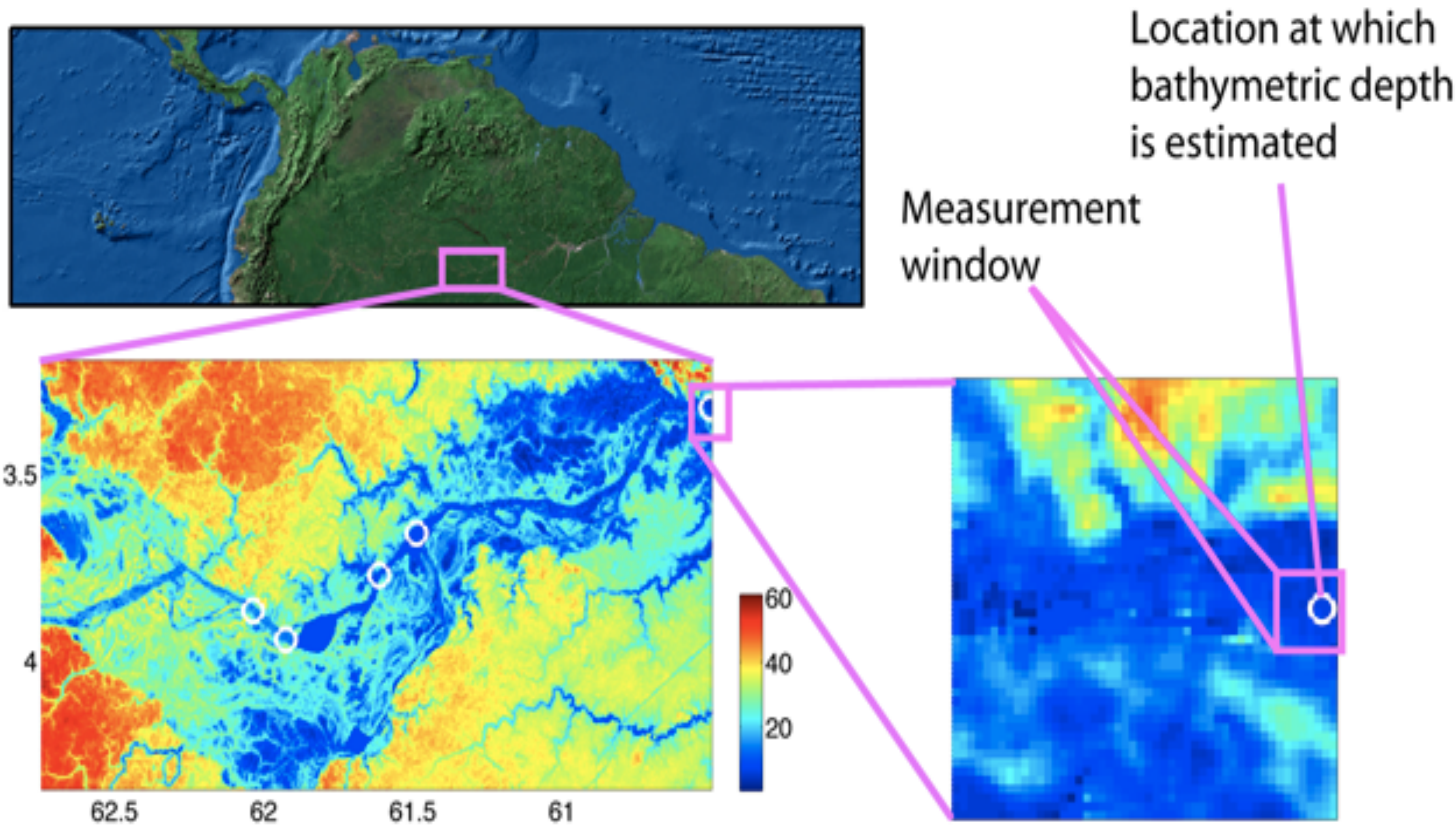
Bathymetry estimation for seasonally-flooded rivers

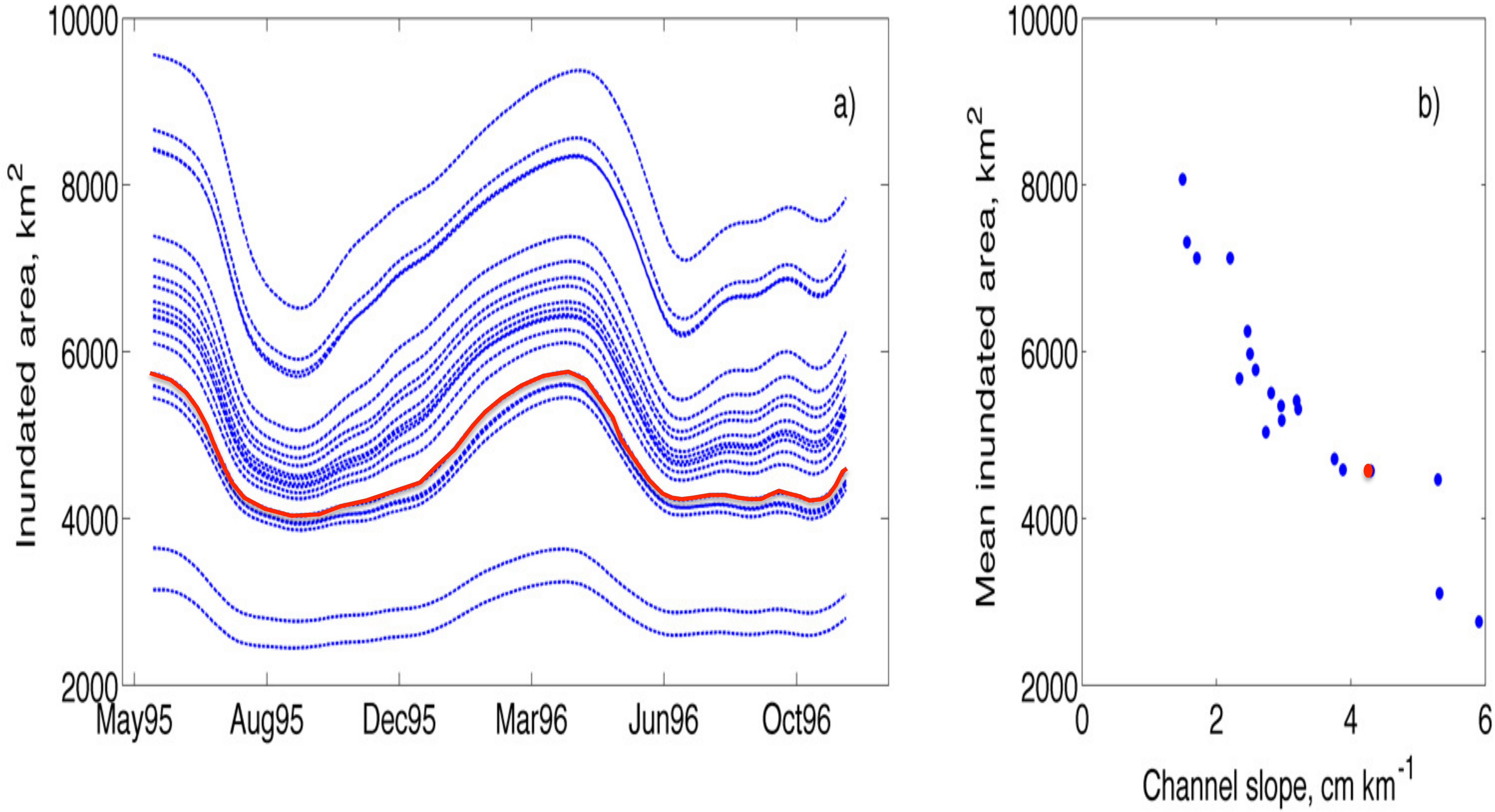
Photo: Bruce Forsberg

Study area: Amazon and Purus rivers



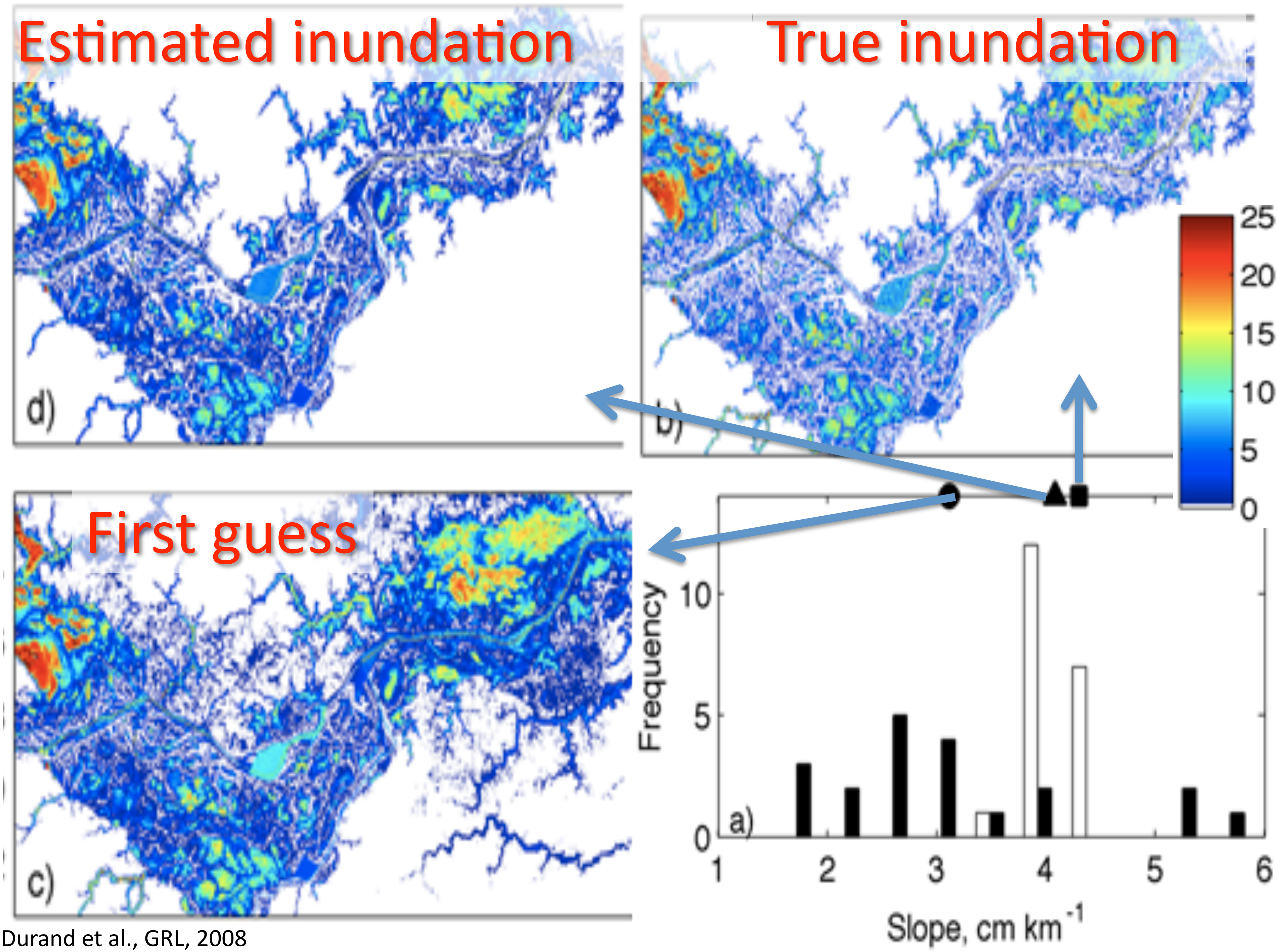
Durand et al., 2008

Bathymetric River Channel Slope



- SWOT can measure inundated area and total storage on floodplains.
- Knowing these through time, allows selection of channel bathymetric slope.

Durand et al., GRL 2008

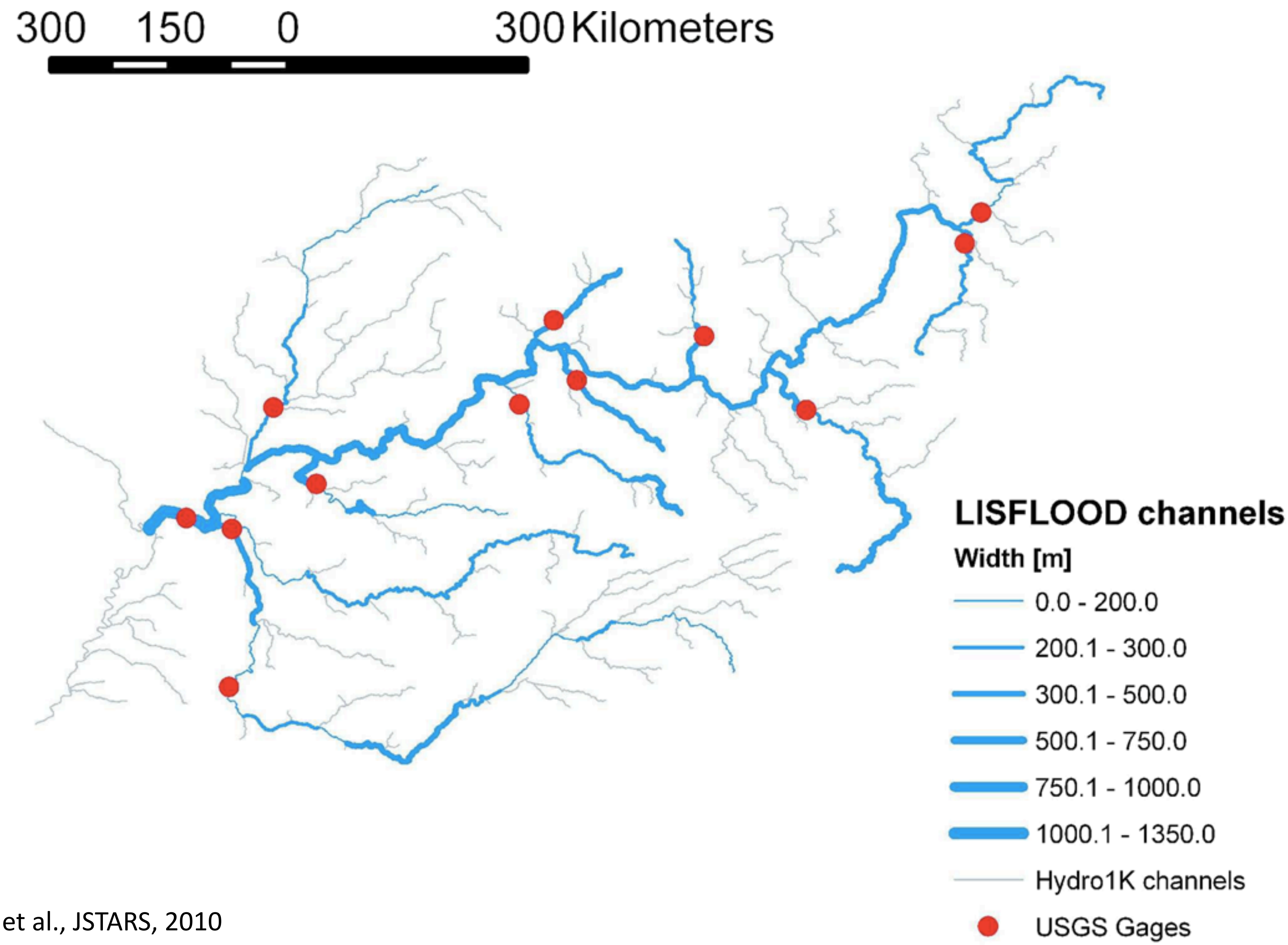


Durand et al., GRL, 2008

Bathymetry estimation for channelized rivers

Photo: US Army Corps of Engineers

Study area: The Ohio River basin



Durand et al., JSTARS, 2010

Algorithm to estimate depth

$$w_{s,t} \quad S_{s,t} \quad \delta z_{s,t}$$

$$Q_t = \frac{1}{n} w_t S_t^{\frac{1}{2}} \left(z_1 + \delta z_t \right)^{\frac{5}{3}}$$

$$Q_{s_1,t} = Q_{s_2,t}$$

$$\frac{1}{n_{s_1}} w_{s_1,t} S_{s_1,t}^{\frac{1}{2}} \left(z_{s_1,1} + \delta z_{s_1,t} \right)^{\frac{5}{3}} = \frac{1}{n_{s_2,t}} w_{s_2,t} S_{s_2,t}^{\frac{1}{2}} \left(z_{s_2,1} + \delta z_{s_2,t} \right)^{\frac{5}{3}}$$

$$\beta_{s_1,t} \left(z_{s_1,1} + \delta z_{s_1,t} \right) = \beta_{s_2,t} \left(z_{s_2,1} + \delta z_{s_2,t} \right)$$

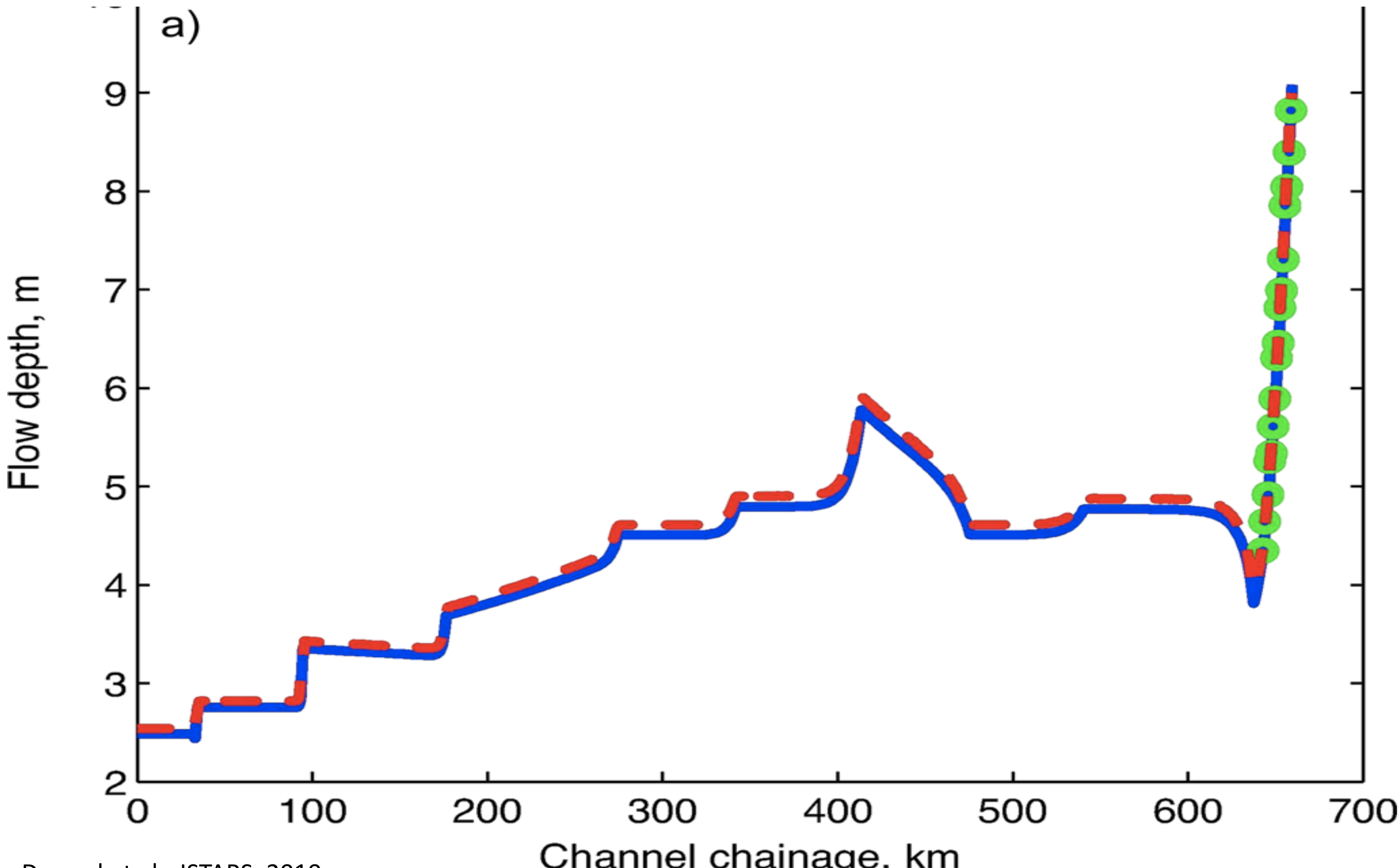
$$\overline{\mathrm{B}} x = c$$

1. Given: SWOT observables
2. Find: Estimate depth at initial time: z_1
3. Solution:
   a) Assume continuity between two pixels s_1 and s_2
   b) Rewrite for unknowns
   c) Solve over-constrained problem for unknown depth

Note: $\beta_{s,t} = \left(\frac{1}{n_s} w_{s,t} S_{s,t}^{\frac{1}{2}} \right)^{\frac{3}{5}}$

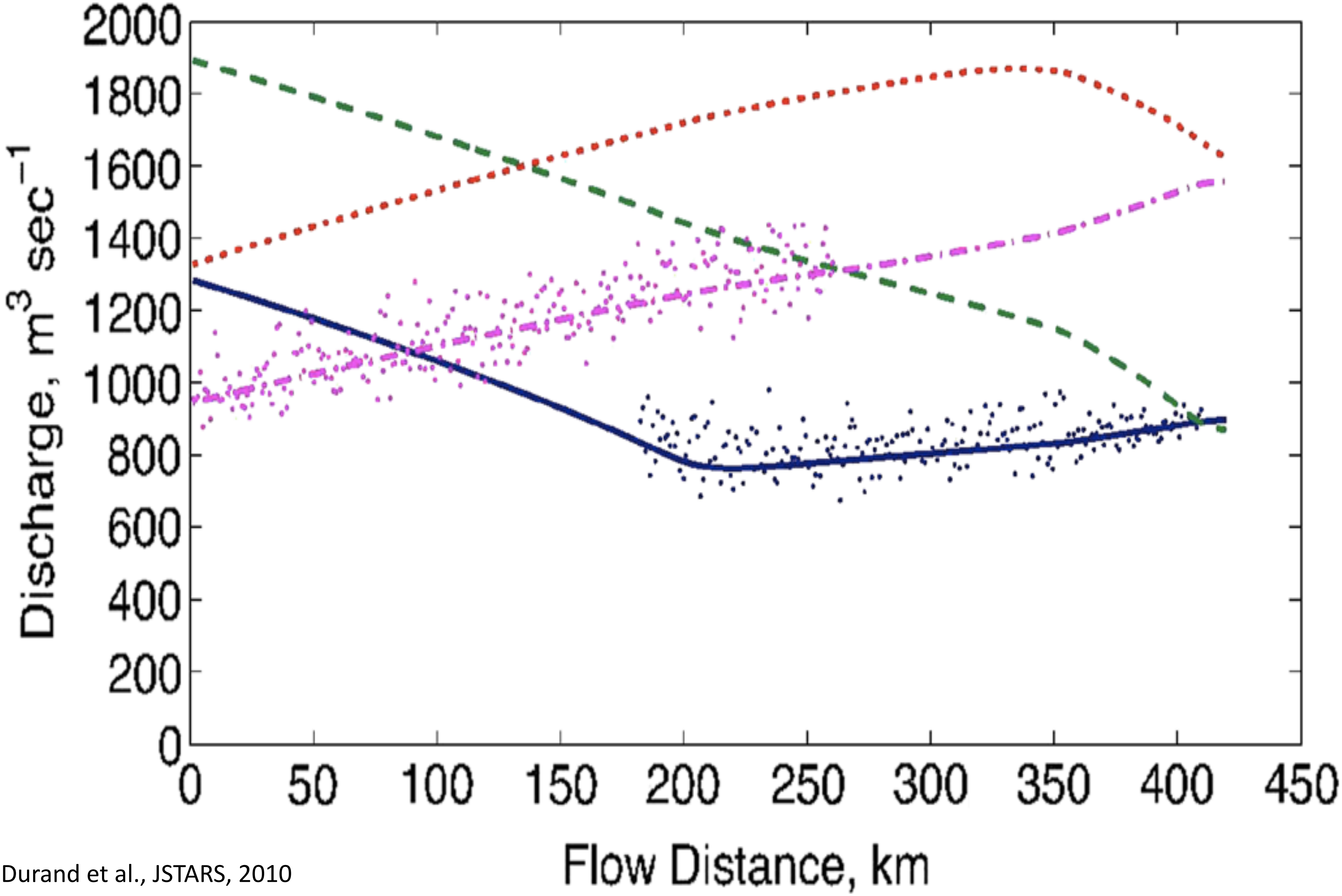
Durand et al., JSTARS, 2010

Depth results: Cumberland River



Durand et al., JSTARS, 2010

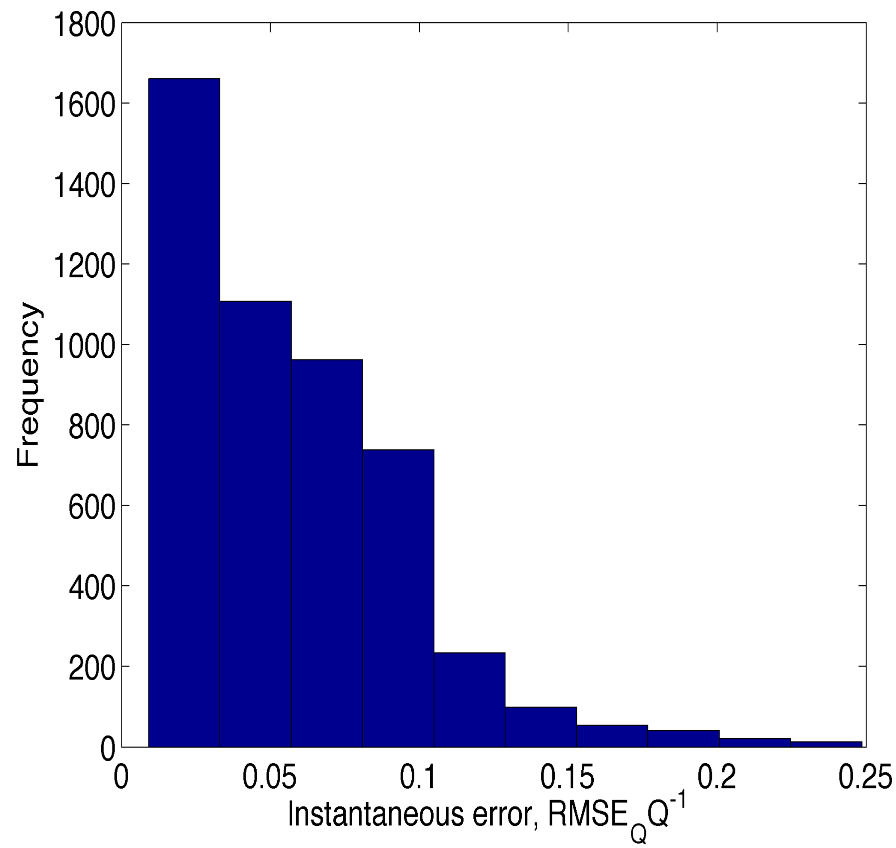
Discharge results: Kanawha River



Durand et al., JSTARS, 2010

Summary of Absolute Discharge Errors

- Error metric:
  - Pixelwise RMSE of discharge timeseries, normalized by mean Q
- Median: 11%
- 86 % of pixels have error less than 25 %
- Outliers should be easily identified



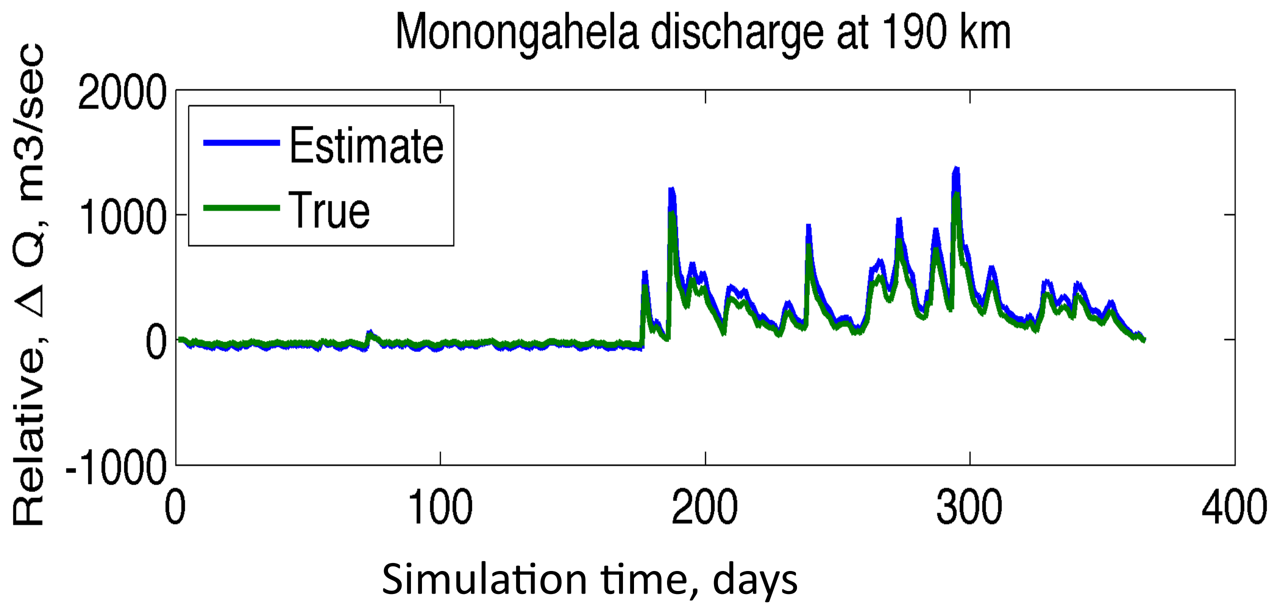
Durand et al., JSTARS, 2010

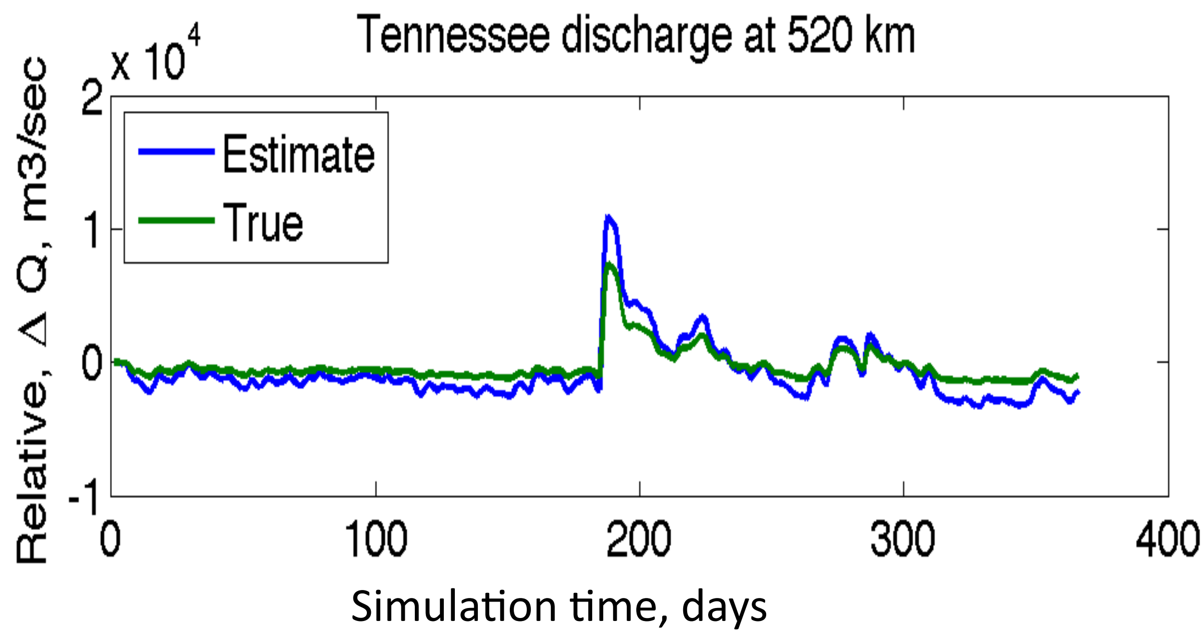
Summary of Relative Discharge Errors

- The WSE signal provides excellent information about discharge variations despite poor depth estimates
- Depth error is:

  0.5m or 100% error (top)

  5.0m or 200% error (bottom)





Durand et al., JSTARS, 2010

Ongoing and In-progress

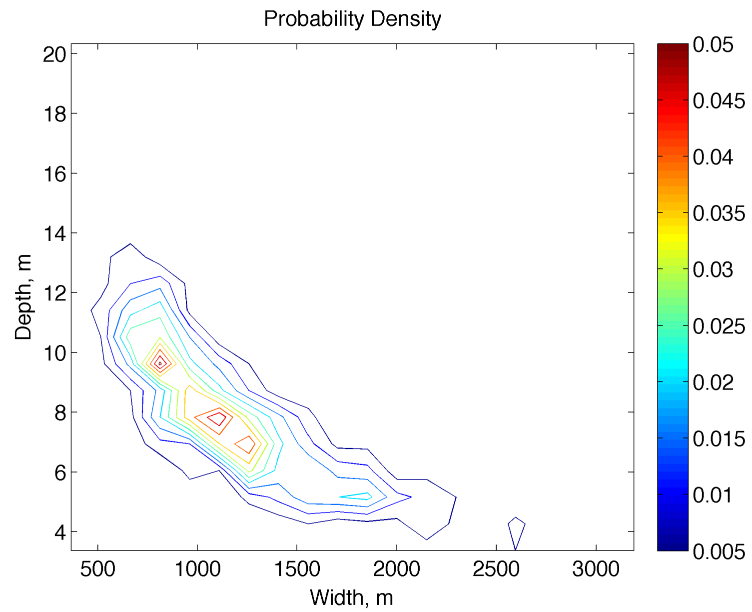
- Assimilation and inverse solution to depth estimation using hydraulic equations
- Width-to-depth algorithms
- Comparison of hydraulic geometry and assimilation solutions

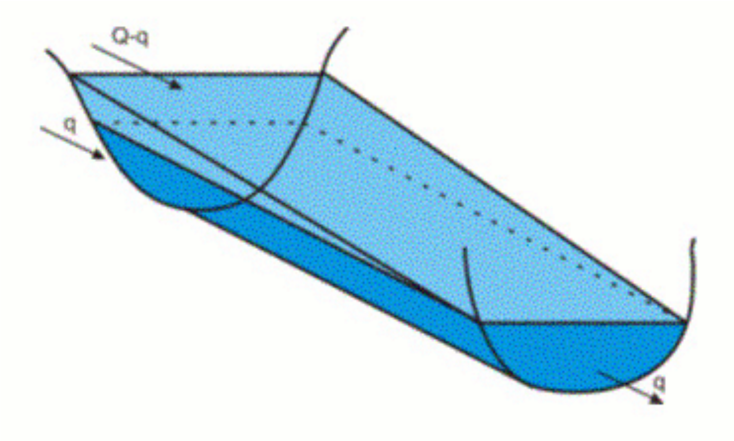
A Kalman-based width to depth algorithm

Basis: River depth adjusts spatially to changes in width, slope, and velocity, given continuity:

$$Q = A_1 V_1 = A_2 V_2$$

$$Q = w_1 d_1 V_1 = w_2 d_2 V_2$$





Some papers have looked at joint width (w) + depth (d) distribution as a **spatial** AR-1 process

$$x \sim \begin{bmatrix} \log w \\ \log d \end{bmatrix} \qquad x_n = \phi x_{n-1} + A_n$$

Data: lower Mississippi River, courtesy J. Moody

Fixed-lag Kalman smoother: width to depth

Kalman smoothing equations. Empirical literature values used for transition matrix

$$\begin{vmatrix} x_{n+1|n+1} \\ x_{n|n+1} \\ \cdots \\ x_{n-N+1|n+1} \end{vmatrix} = \Phi \begin{vmatrix} x_{n|n} \\ x_{n-1|n} \\ \cdots \\ x_{n-N|n} \end{vmatrix} + \begin{vmatrix} K_{n+1} \\ K_{n+1}^{(1)} \\ \cdots \\ K_{n+1}^{(N)} \end{vmatrix} \tilde{z}_{n+1}$$

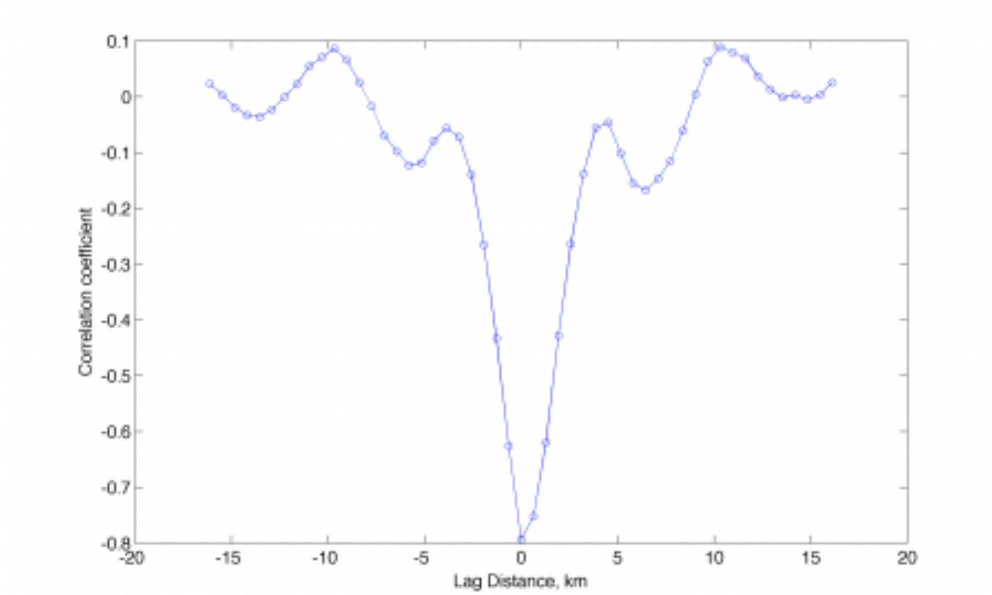
$$x \sim \begin{bmatrix} \log w \\ \log d \end{bmatrix}$$

State variables: width and depth

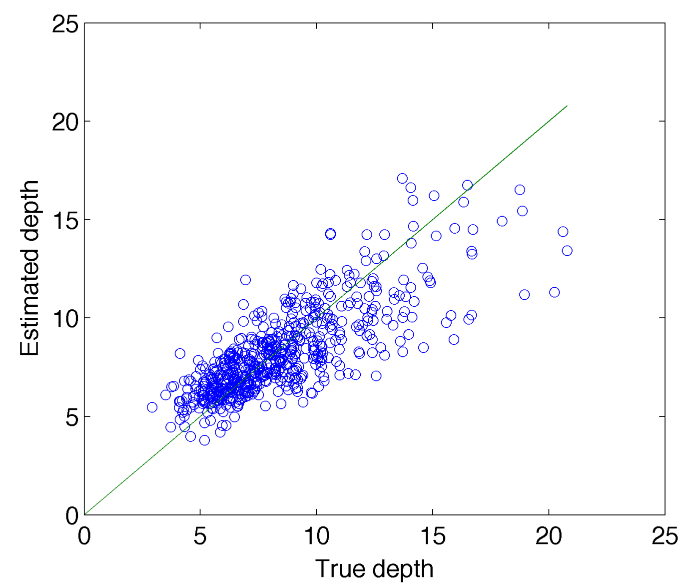
$$z \sim \left[\log w_{obs} \right]$$

SWOT width observations

Spatial correlations indicate smoothing:



Preliminary OSSE Results



Data: lower Missisippi River, courtesy J. Moody

Future work

- NASA OSTST project for river bathymetry estimation
  - Obtain detailed ground-truth bathymetry
  - Further develop two algorithms
    - UCLA: hydraulic geometry
    - OSU: ensemble data assimilation
  - Intercompare and hybridize the algorithms