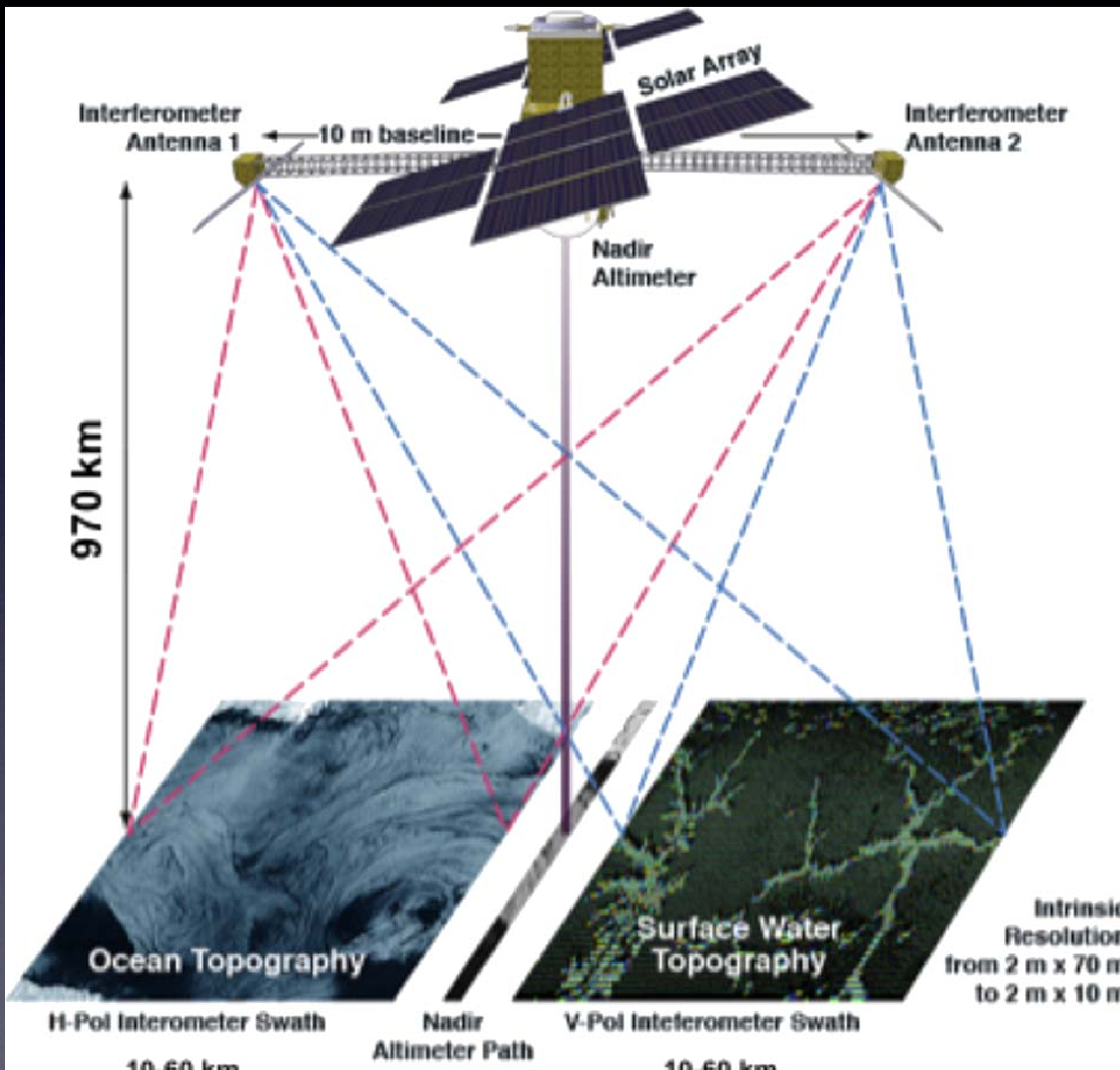
A Bayesian analysis scheme for estimating river depth and discharge using SWOT measurements



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American Meteorological Society
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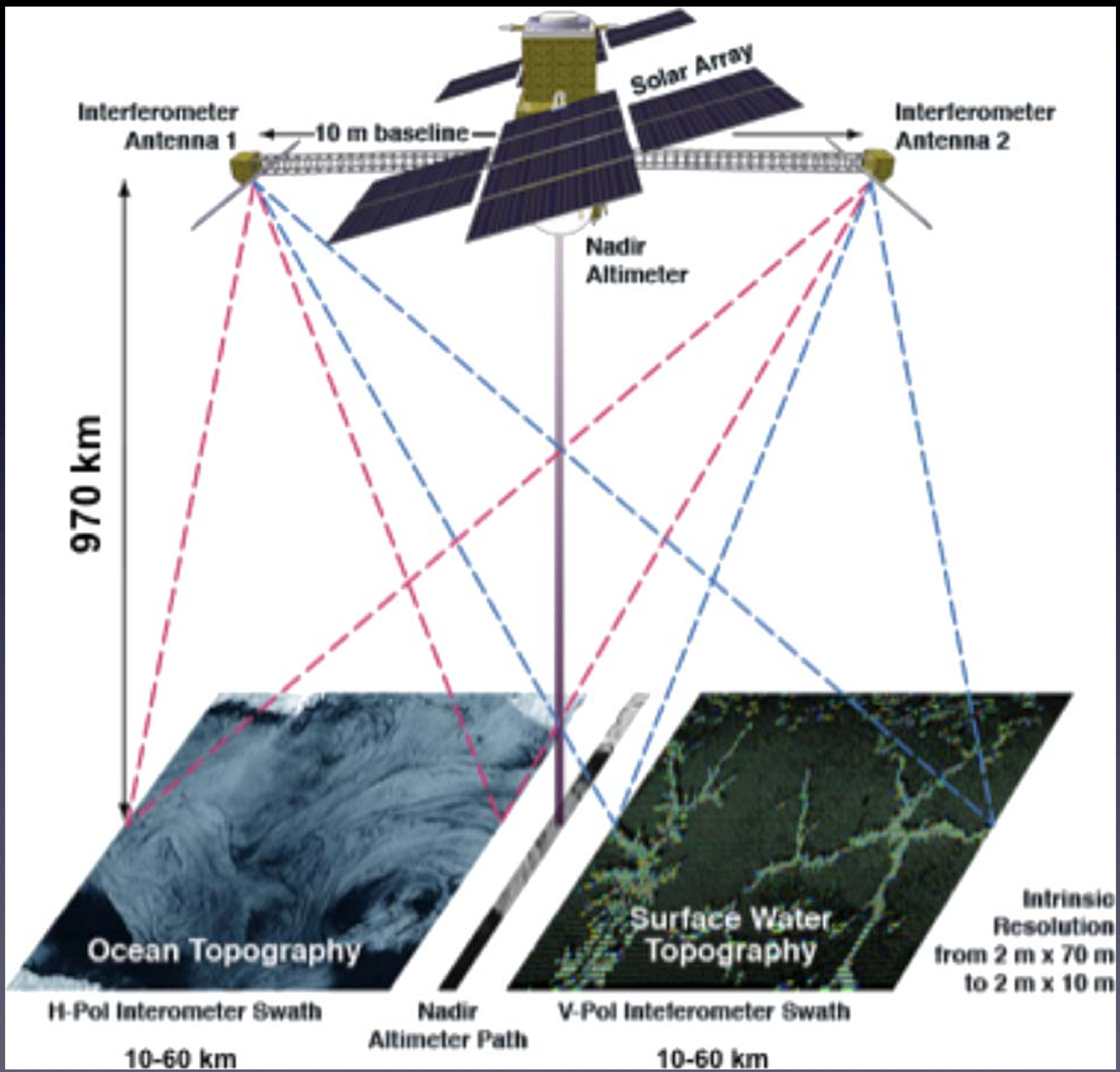
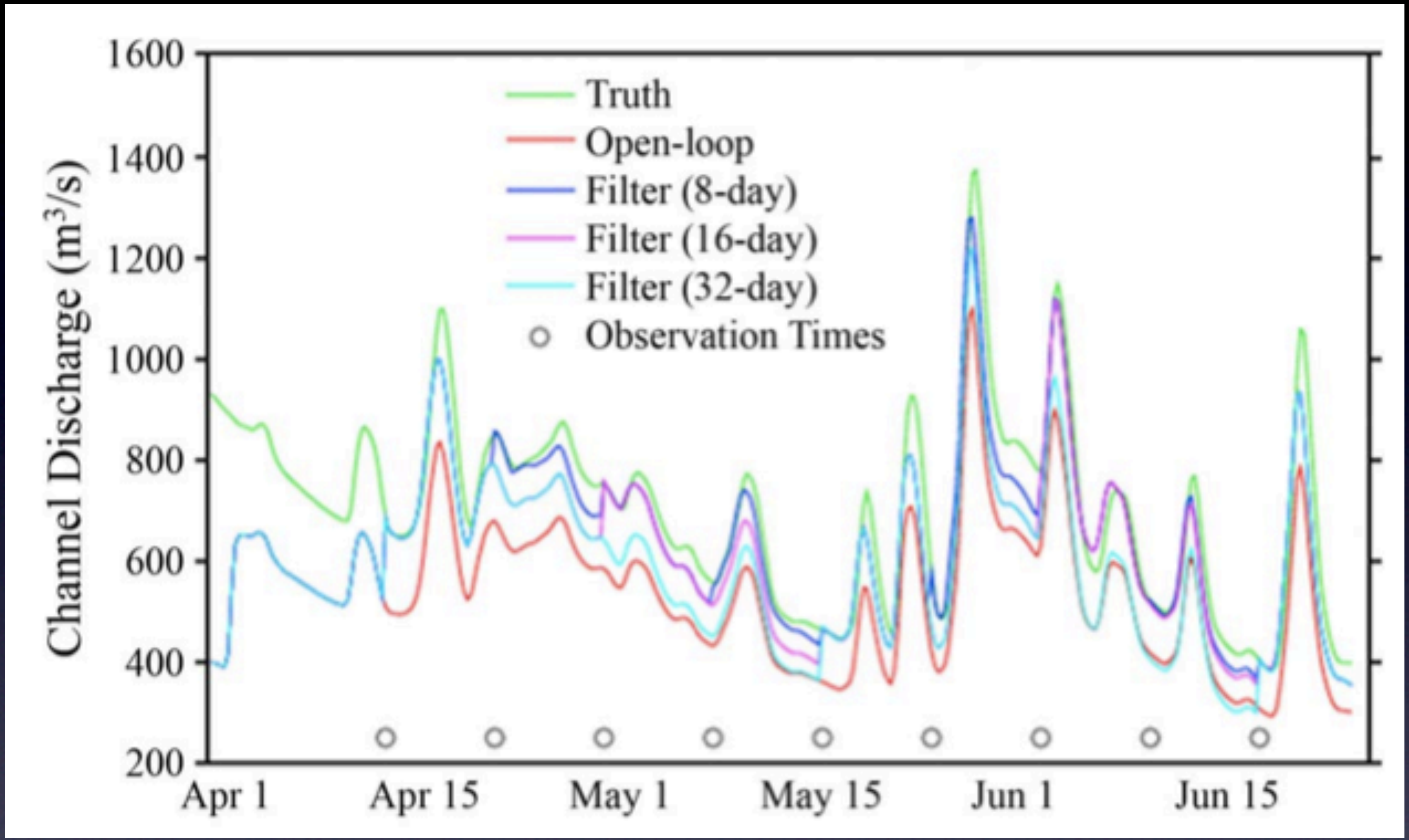
Funding: Physical Oceanography

SWOT measurements of rivers



- Launch date: 2019
- Will measure water elevation via Ka-band radar interferometer
- Spatial resolution: 2m-50m
- Temporal resolution: day - week
- Given water heights, find discharge

Discharge via data assimilation



Andreadis et al, GRL, 2007

- Hydrology model (e.g. VIC) gives prior discharge
- Hydraulics model (e.g. LISFLOOD-FP) relates discharge to height measurements: wildly expensive

Doing hydraulics backwards

- Remove hydraulic model (reduce computational expense)
- Simplified mass and momentum conservation: apply to a reach, in between SWOT overpasses

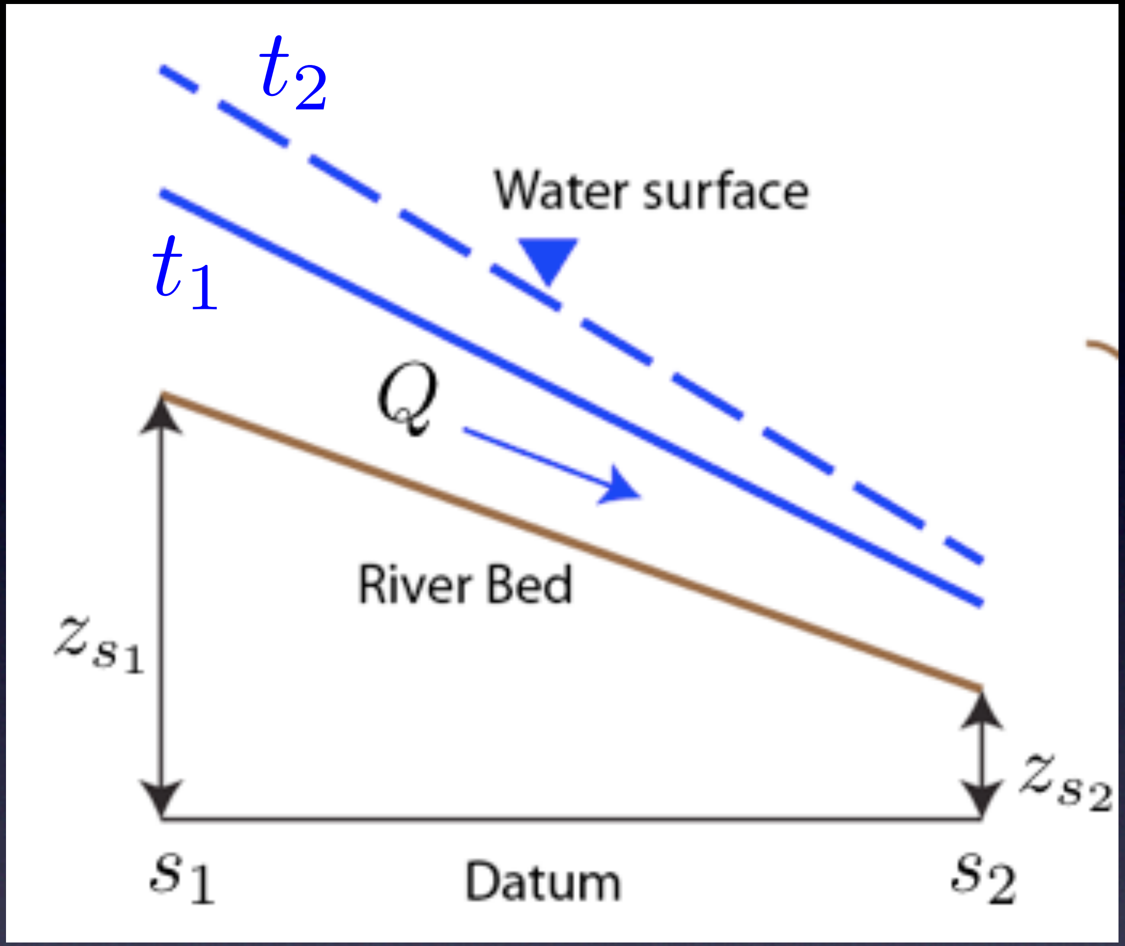
$$\frac{\partial Q}{\partial x} = -T\frac{\partial h}{\partial t} \qquad \frac{Q^2}{K^2} = -\frac{\partial h}{\partial x}$$

- Solve using Bayesian Metropolis MCMC algorithm

Conservation of mass

$$\frac{\partial Q}{\partial x} + T\frac{\partial h}{\partial t} = 0$$

Apply to a reach for period τ_1 between overpasses t_1, t_2



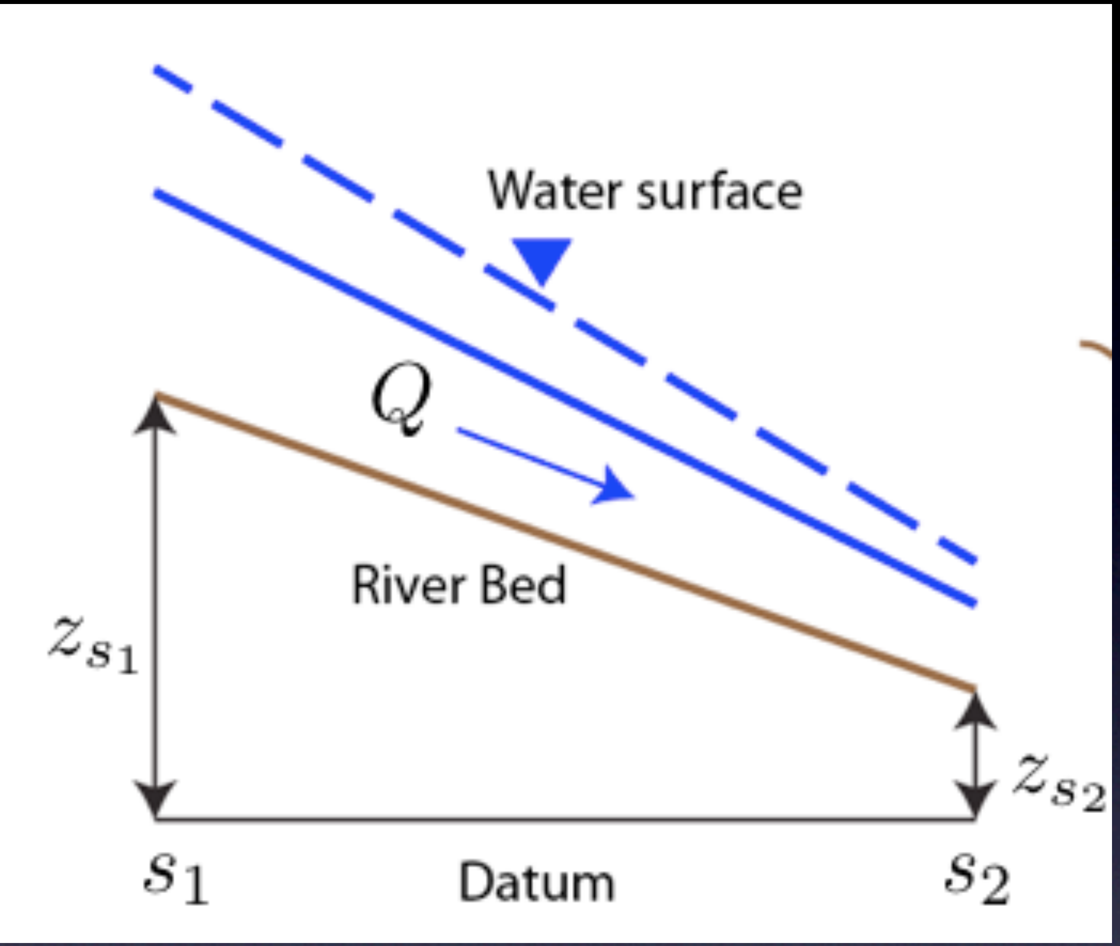
$$\overline{Q}_{s_1} - \overline{Q}_{s_2} = \frac{L}{2\tau_1}\left(\overline{T}_t + \overline{T}_{t+1}\right)\left(\overline{h}_{t_2} - \overline{h}_{t_1}\right)$$

This equation is exact for the average quantities

Conservation of momentum

$$\frac{Q^2}{K^2} = -\frac{\partial h}{\partial x}$$

$$K = \frac{1}{n}(A_{base} + \delta A)^{5/3} T^{2/3}$$

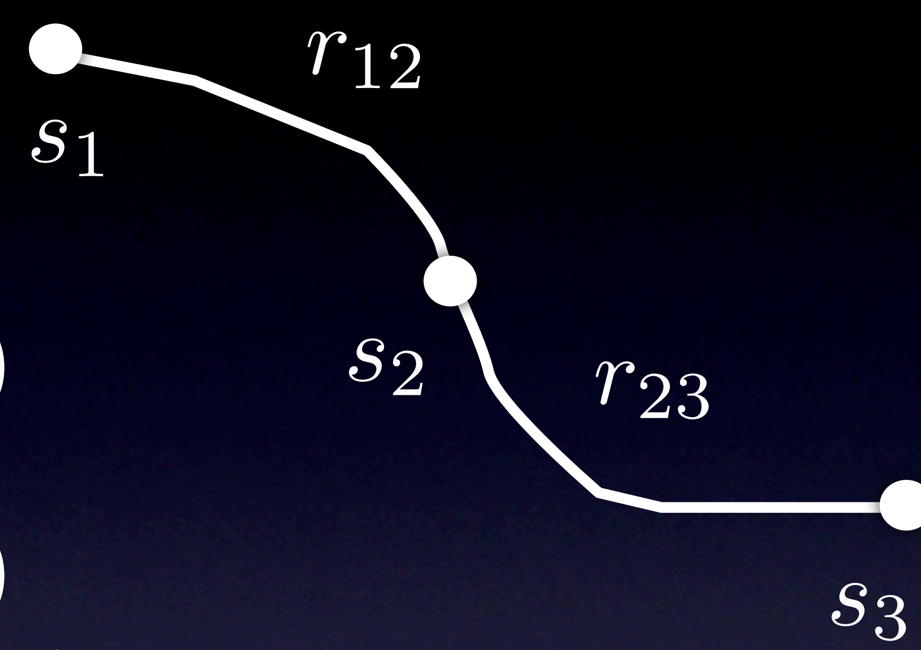


Two time-invariant unknowns per reach

Apply between two overpasses for one reach:

$$\overline{Q}_{s_1} + \overline{Q}_{s_2} = 2\overline{K}\left(-\overline{\frac{\partial h}{\partial x}}\right)^{1/2}$$

Application to two reaches



At t_2 :

$$\overline{Q}_{t_{12}s_1} - \overline{Q}_{t_{12}s_2} = L_1 {\tau_1}^{-1} \overline{T}_{r_1} \left(h_{t_2} - h_{t_1}\right)$$

$$\overline{Q}_{t_{12}s_2} - \overline{Q}_{t_{12}s_3} = L_2 {\tau_1}^{-1} \overline{T}_{r_2} \left(h_{t_2} - h_{t_1}\right)$$

$$\overline{Q}_{t_{12},s_1} + \overline{Q}_{t_{12},s_2} = 2\overline{K}_{r_{12}} \left(\overline{\left.\partial h / \partial x\right|}_{r_{12}} \right)^{1/2}$$

$$\overline{Q}_{t_{12},s_2} + \overline{Q}_{t_{12},s_3} = 2\overline{K}_{r_{23}} \left(\overline{\left.\partial h / \partial x\right|}_{r_{12}} \right)^{1/2}$$

Write two equations per reach, four total

After four overpasses, more equations than unknowns

A Bayesian approach

Posterior proportional to likelihood times prior:

$$p(y|z) \propto f(z|y)\pi(y)$$

y Unknowns: Discharge, bathymetry, roughness

z Observations: Water heights

1. Generate a new candidate y_{i+1}
2. Accept candidate with probability $r = \dfrac{f(z|y_{i+1})\pi(y_{i+1})}{f(z|y_i)\pi(y_i)}$

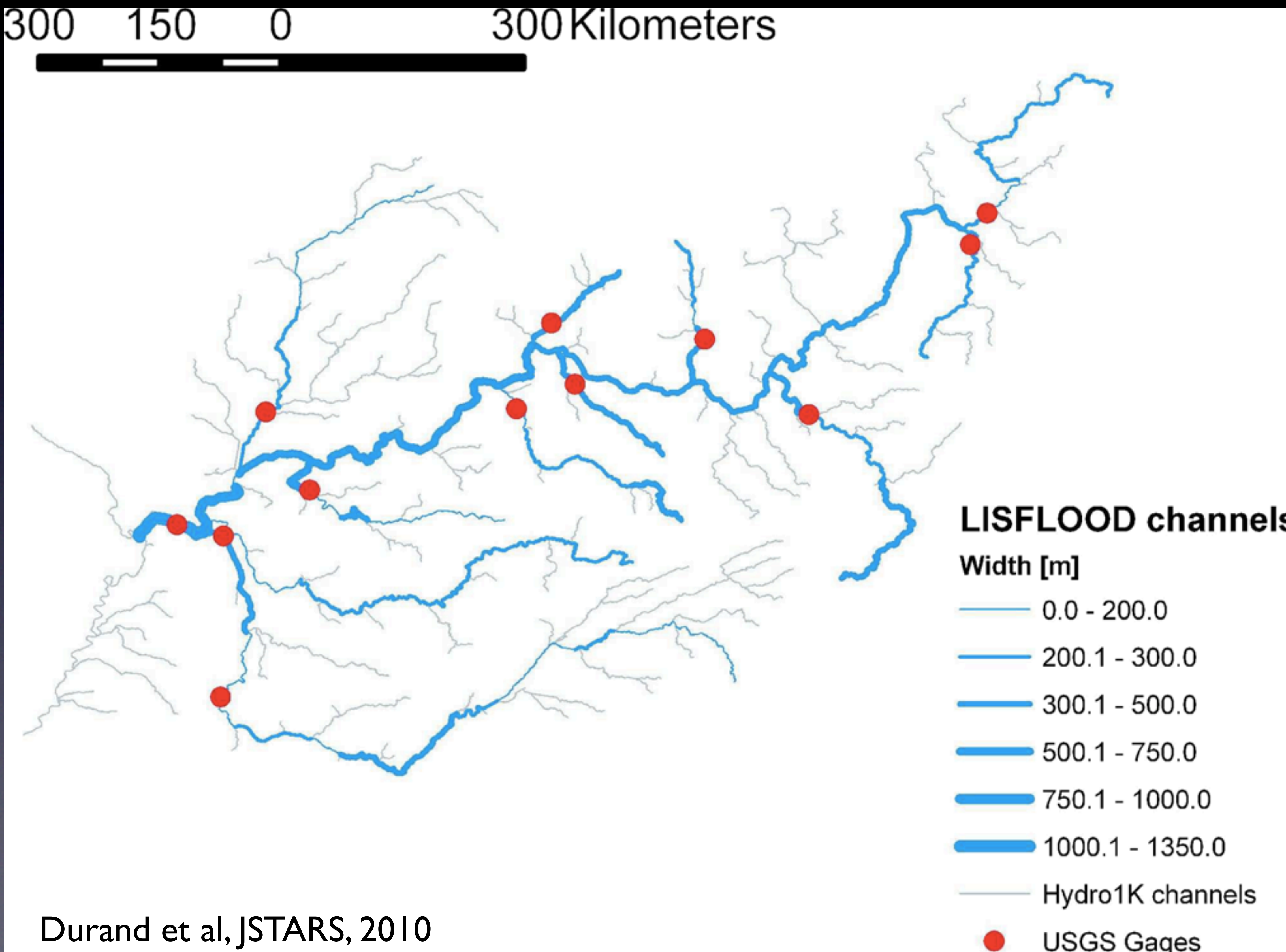
Using Gaussians for both:

$$\log f(z|y) \propto -(\frac{Q^2}{K^2} - \frac{\partial h}{\partial x})C^{-1}(\frac{Q^2}{K^2} - \frac{\partial h}{\partial x})^T \dots$$

$$\dots - (\frac{\delta Q}{LT} - \frac{\partial h}{\partial t})C_2^{-1}(\frac{\delta Q}{LT} - \frac{\partial h}{\partial t})^T$$

$$\pi(Q) \propto -(Q - \bar{Q})C_Q^{-1}(Q - \bar{Q})$$

Testing the algorithm

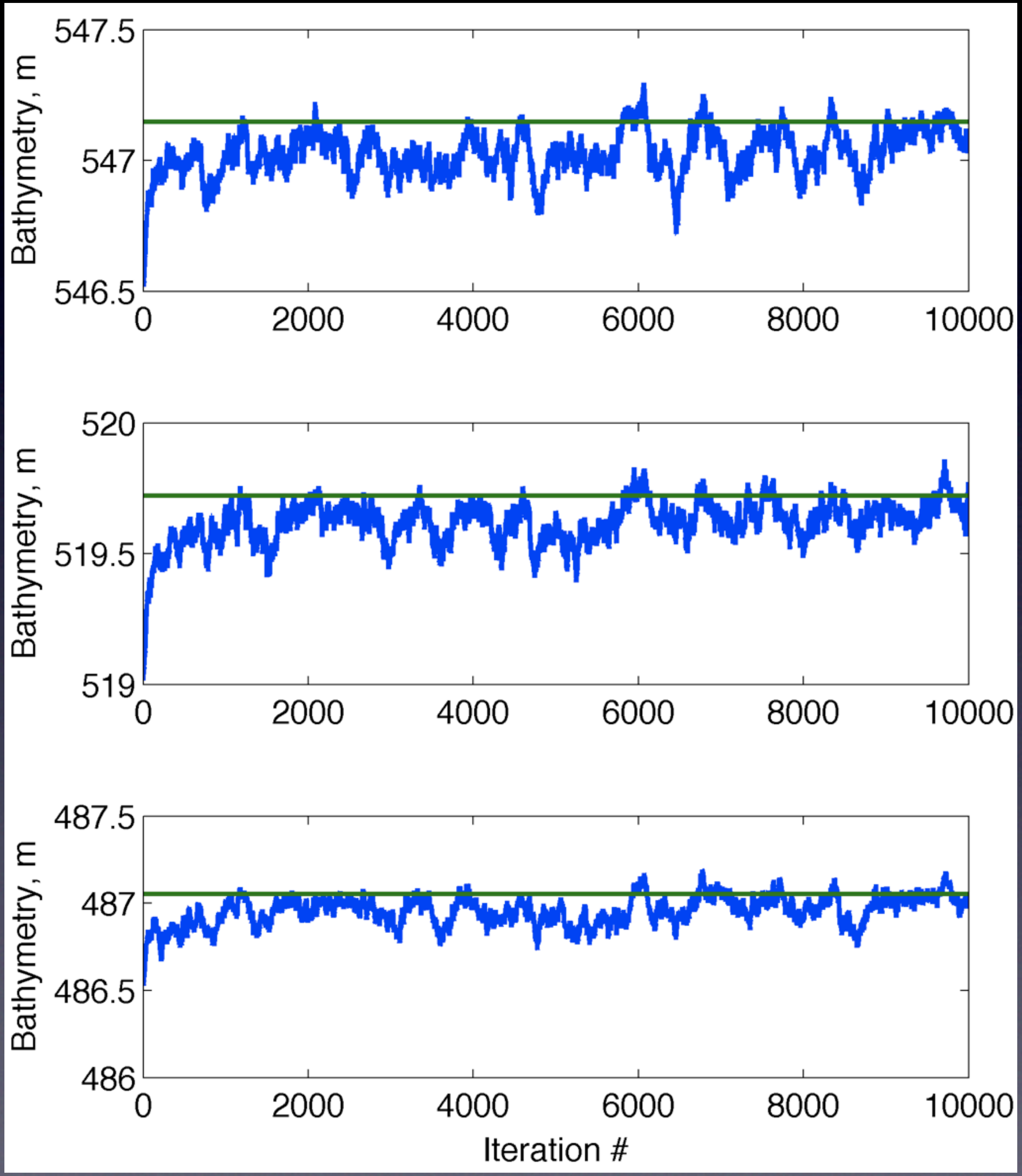


Durand et al, JSTARS, 2010

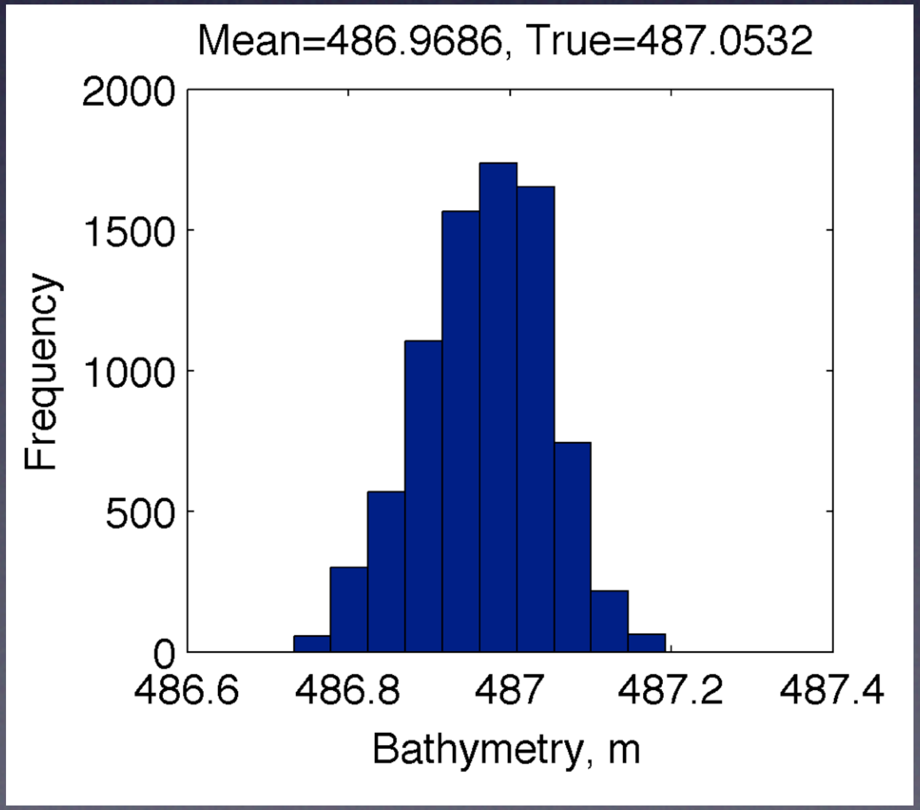
OSSE setup

- Generate “true” water surface heights using hydraulic model and true Q, z, n
- Corrupt true water surface heights using white noise (get synthetic measurements)
- Estimate Q, z, n for three reach system of Kanawha River using ten observations
- Reaches are 15 km in length
- Chains are 10,000 iterations, burned in after 2,000 iterations

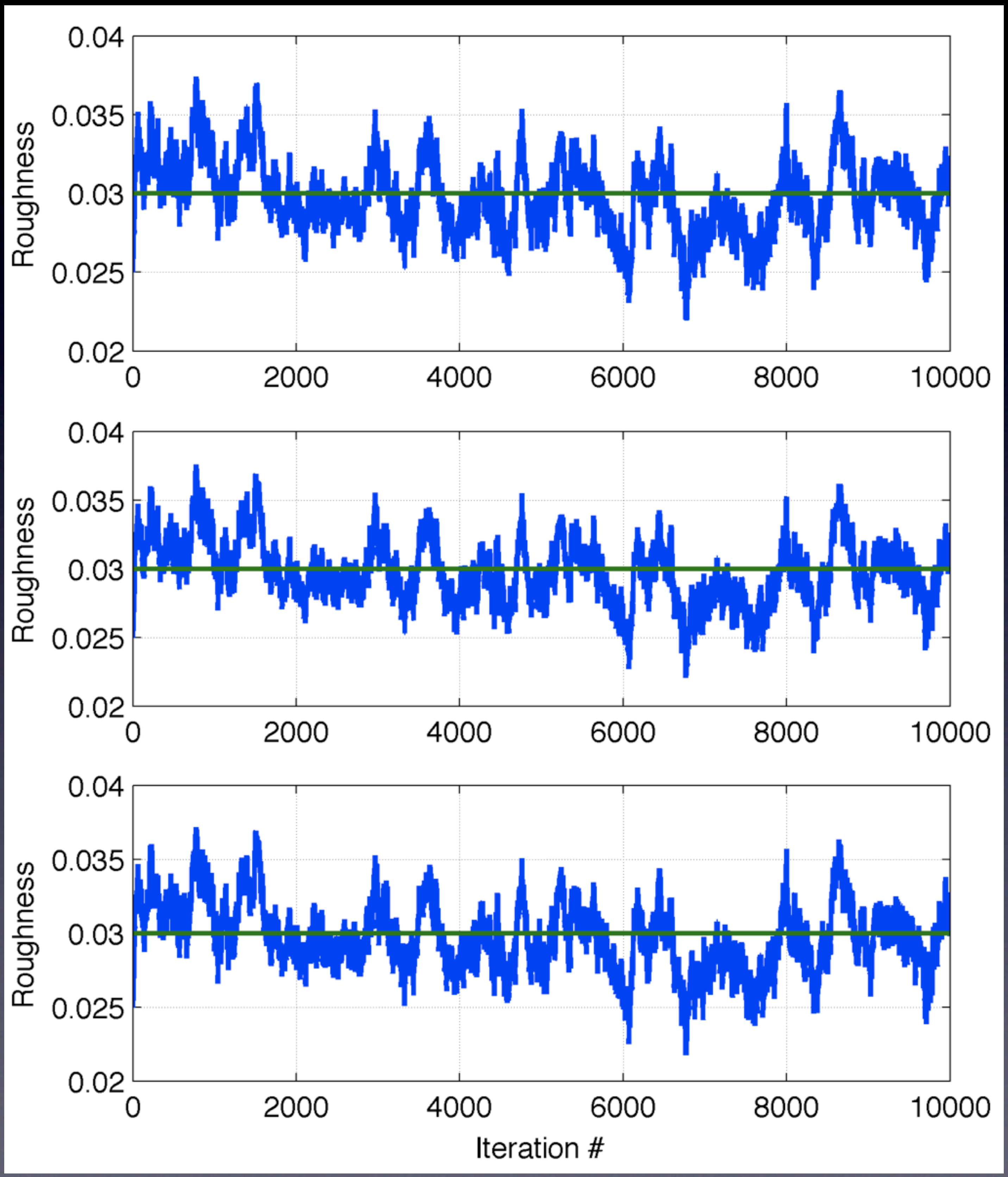
Results: Bathymetry



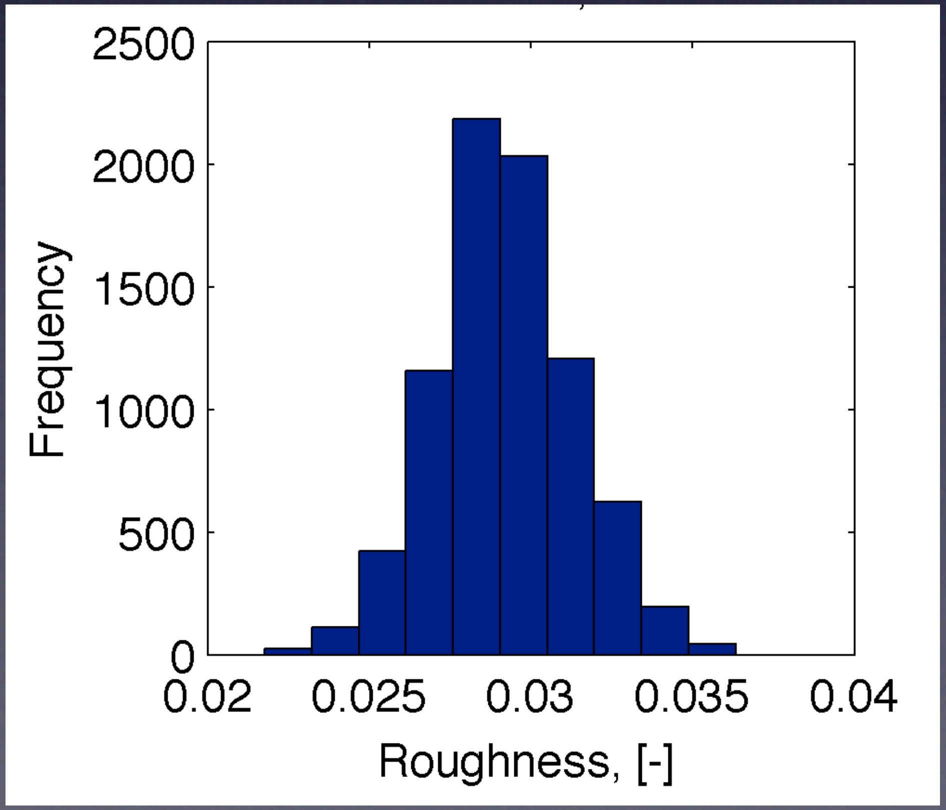
- Prior and first guess: ~50 cm too low
- RMS error: 9.7 cm
- Posterior standard deviation: 7.7 cm



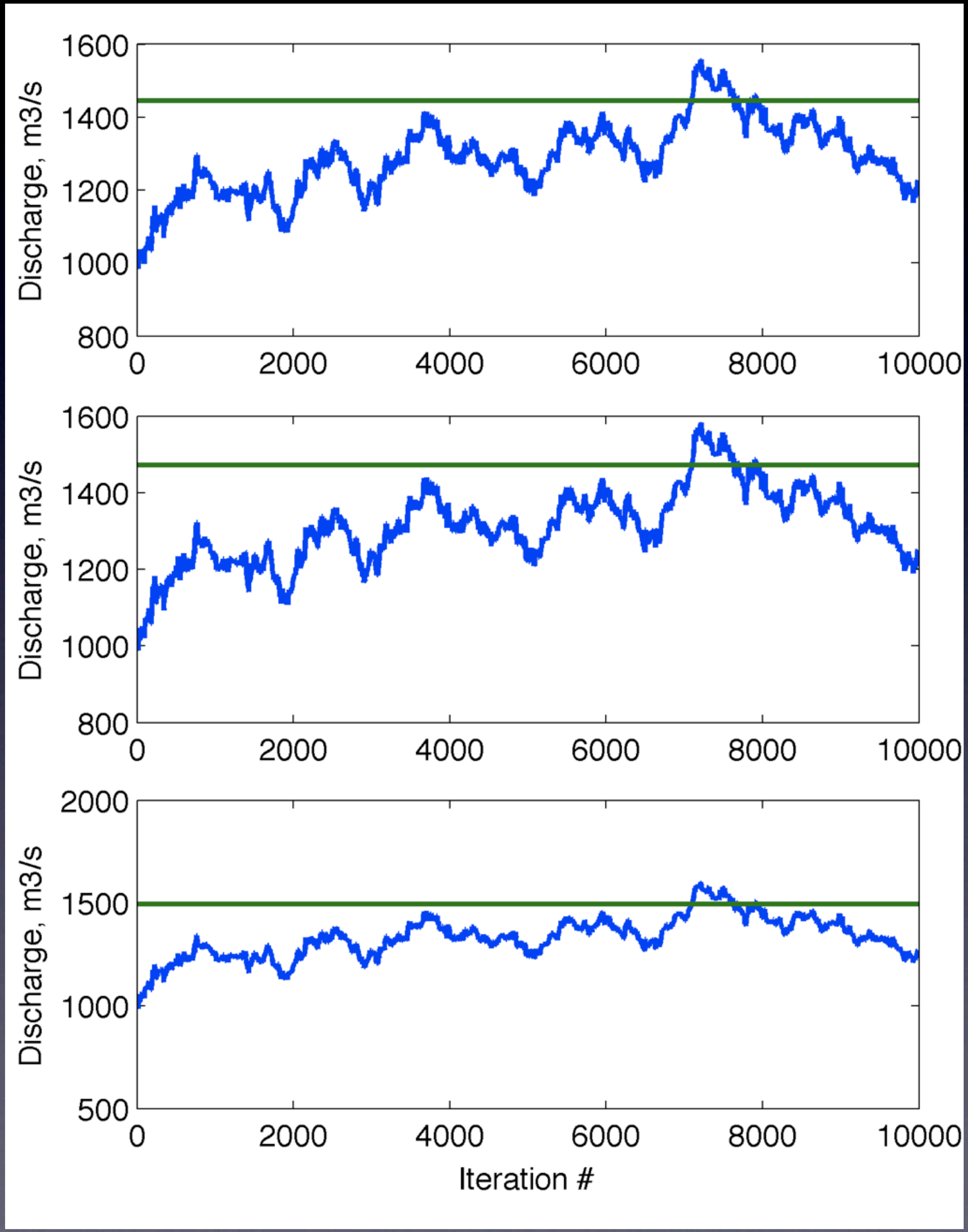
Results: Roughness



- Prior and first guess: 0.025, 17% low
- RMS: 0.00077 (2.5%)
- Posterior standard deviation: 0.0021 (7%)



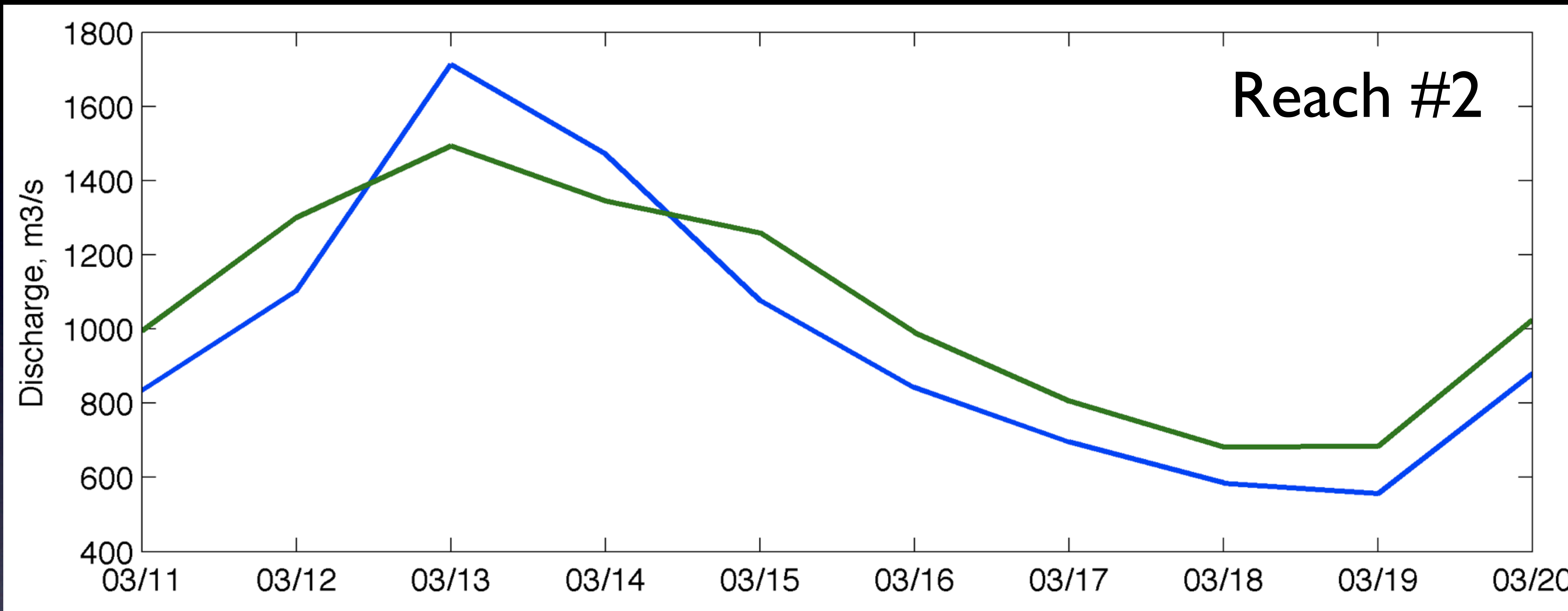
Results: Discharge



Between March 14-15:

- Prior and first guess: Q=1000 m3/s
- Bias = -127 m3/s (~8%)
- Standard deviation = 79 m3/s

Results: Discharge



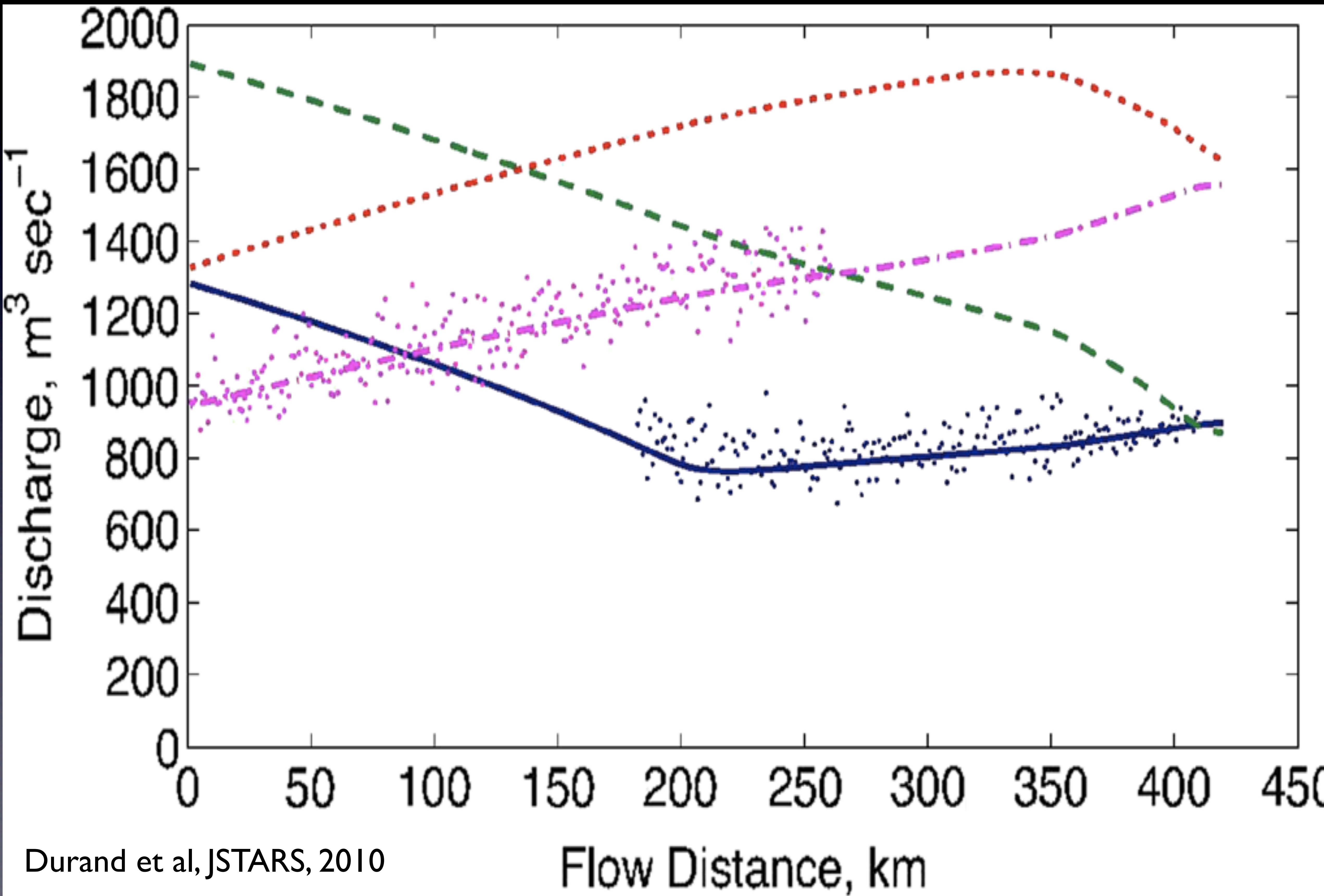
- Prior and first guess: Q=1000 m3/s
- RMS = 156 m3/s (16.9%)
- Standard deviation = 94 m3/s

Conclusions

- Estimation of discharge to within 17 % RMS
- Simultaneous estimation of roughness, bathymetry
- No hydraulic model used
- Caveat: SWOT spatio-temporal sampling and true error structure not considered
- Improvement: Include more prior information on discharge

Extra slides

Trading time for space with SWOT



Durand et al, JSTARS, 2010

Checking diffusive assumption

After solution is obtained, calculate $V = Q/A$

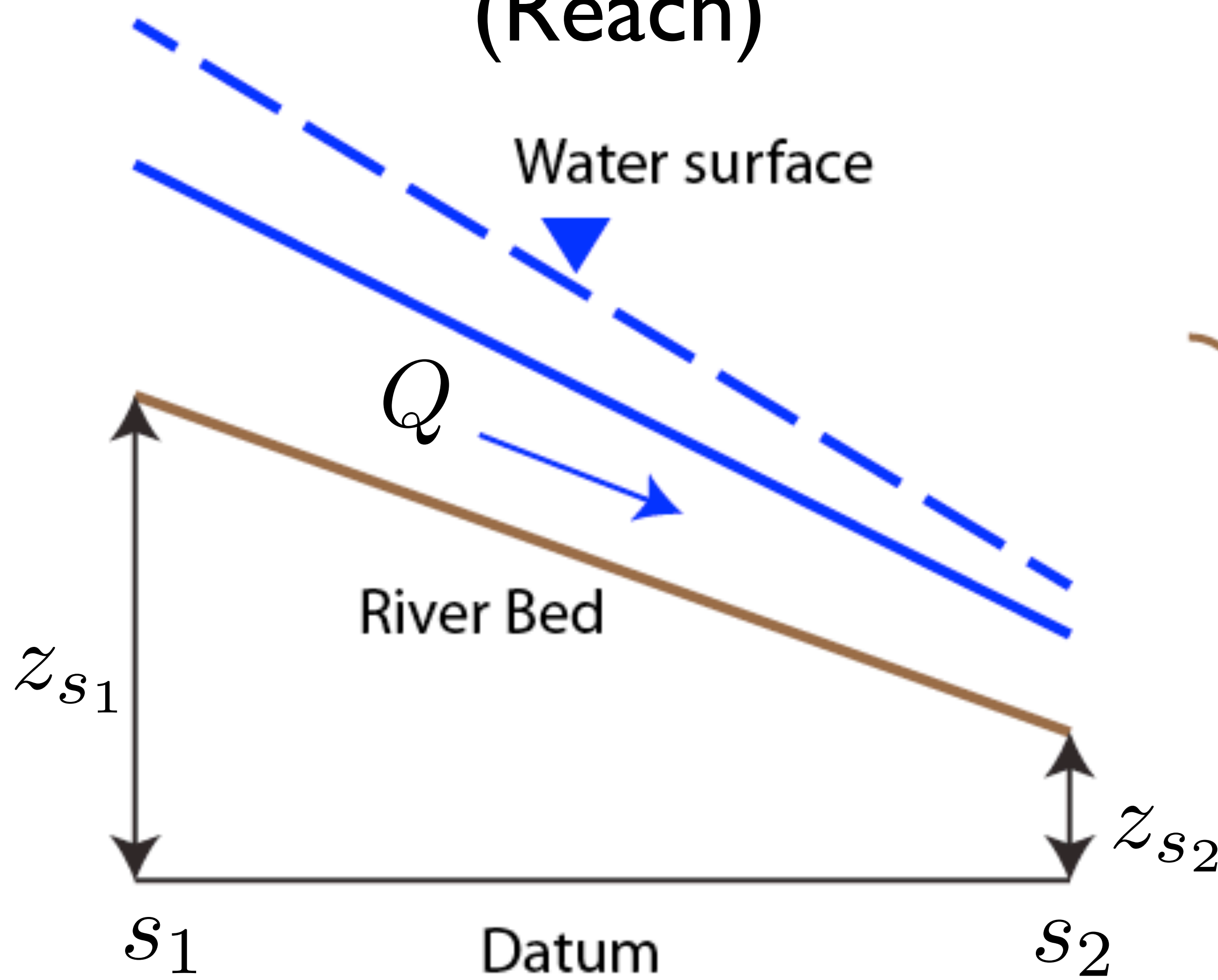
Linear momentum equation:

$$\frac{\partial y}{\partial x} + \frac{V}{g}\frac{\partial V}{\partial x} + \frac{1}{g}\frac{\partial V}{\partial t} = S_0 - S_f$$

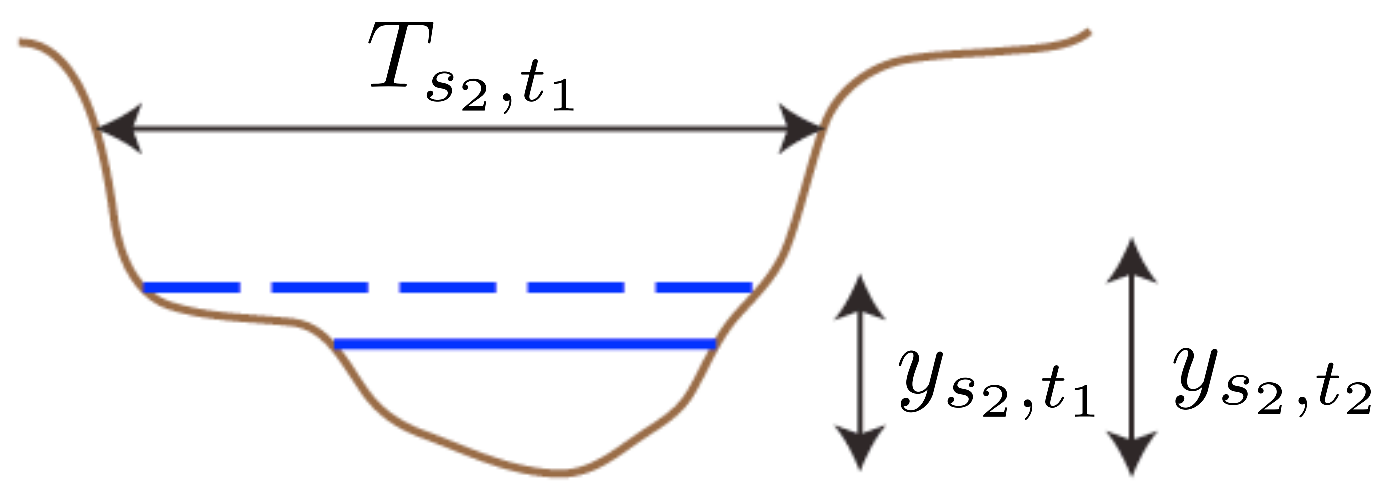
Calculate magnitude of momentum terms, verify assumption: $\frac{V}{g}\frac{\partial V}{\partial x}$ $\frac{1}{g}\frac{\partial V}{\partial t}$

Discharge and river cross sections

Along-stream (Reach)



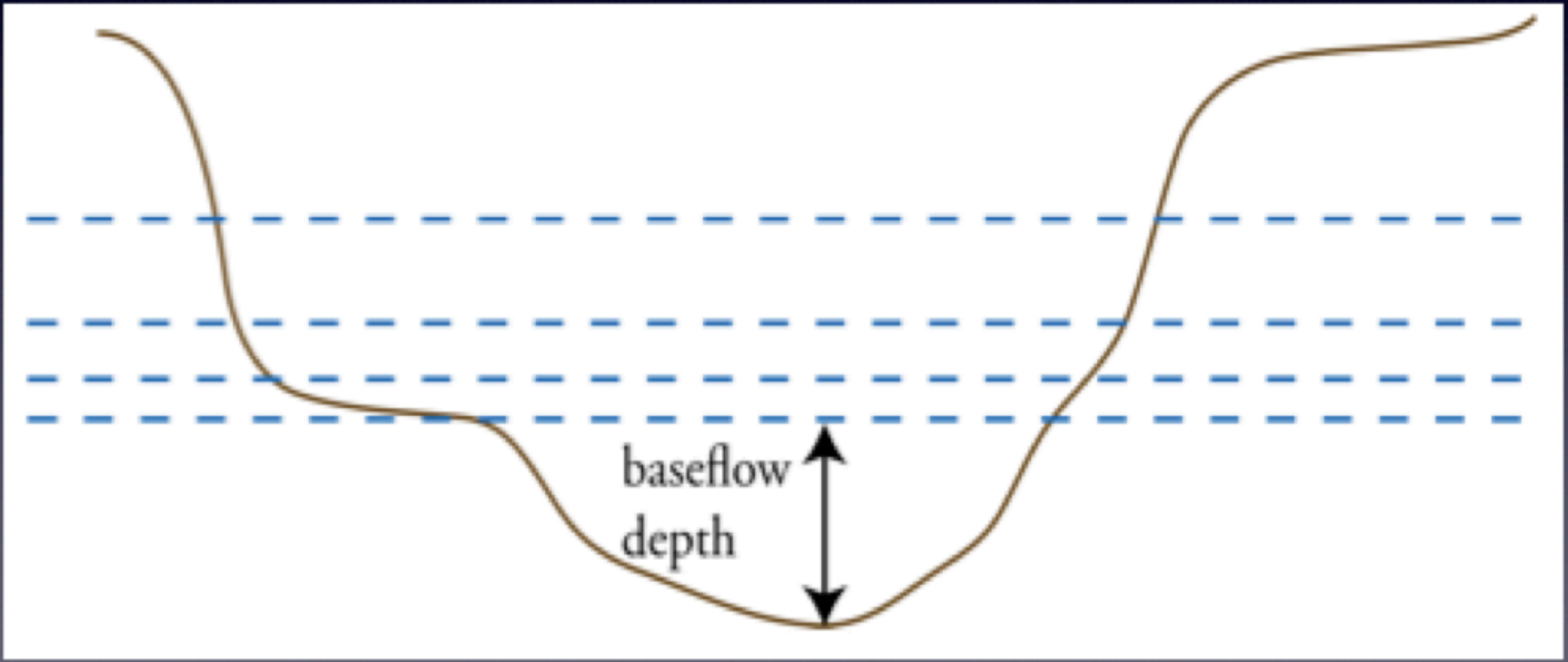
Across-stream (Cross-section s_2)



T	Top width
z_{s_1}, z_{s_2}	Bed elevation at cross-sections
Q	River discharge
$y_{s,t}$	Water depth
$h = z + y$	Water elevation
$A_{s,t}$	Flow area
$V = Q/A$	Flow velocity
$\left.\frac{\partial h}{\partial x}\right\rvert_t$	Water slope
$T\left.\frac{\partial h}{\partial t}\right\rvert_s$	River storage change

The depth problem

SWOT does not observe depth below the lowest height measurement



Assumptions for a new algorithm

Diffusion Wave Analogy

$$\frac{\partial y}{\partial x} + \frac{V}{g}\frac{\partial V}{\partial x} + \frac{1}{g}\frac{\partial V}{\partial t} = S_0 - S_f$$

Apply for a reach without lateral inflows

Wetted Perimeter

$$P \approx T$$

Stationary Channel

$$\frac{\partial z}{\partial t} = 0 \qquad \frac{\partial n}{\partial t} = 0$$

Conservation laws

$$\frac{\partial Q}{\partial x} + T\frac{\partial h}{\partial t} = 0$$

Mass

$$\frac{\partial y}{\partial x} + \frac{V}{g}\frac{\partial V}{\partial x} + \frac{1}{g}\frac{\partial V}{\partial t} = S_0 - S_f$$

Momentum

$$S_f = Q^2/K^2$$

$$K = \frac{1}{n}A^{5/3}P^{2/3}$$

$$Q = AV$$

Unknowns:

$$Q, y, A, V, n, P, K, S_0$$

Unsteady algorithm: Conveyance

Apply to a reach r_{12} from s_1, s_2 averaged from t_1, t_2

$$\overline{Q}_{s_1} + \overline{Q}_{s_2} = 2\overline{K}_{r_{12}} \left(\overline{\frac{\partial h}{\partial x}} \right)^{1/2}$$

$$\overline{K_r} = \frac{K_{r,t_1} + K_{r,t_2}}{2}$$

$$K_{r,t} = \frac{1}{n_r} (A_{base,r} + \delta A_{r,t})^{5/3} T_{r,t}^{2/3}$$

Two time-invariant unknowns per reach

Application to a single reach

At t_2 two equations, four unknowns

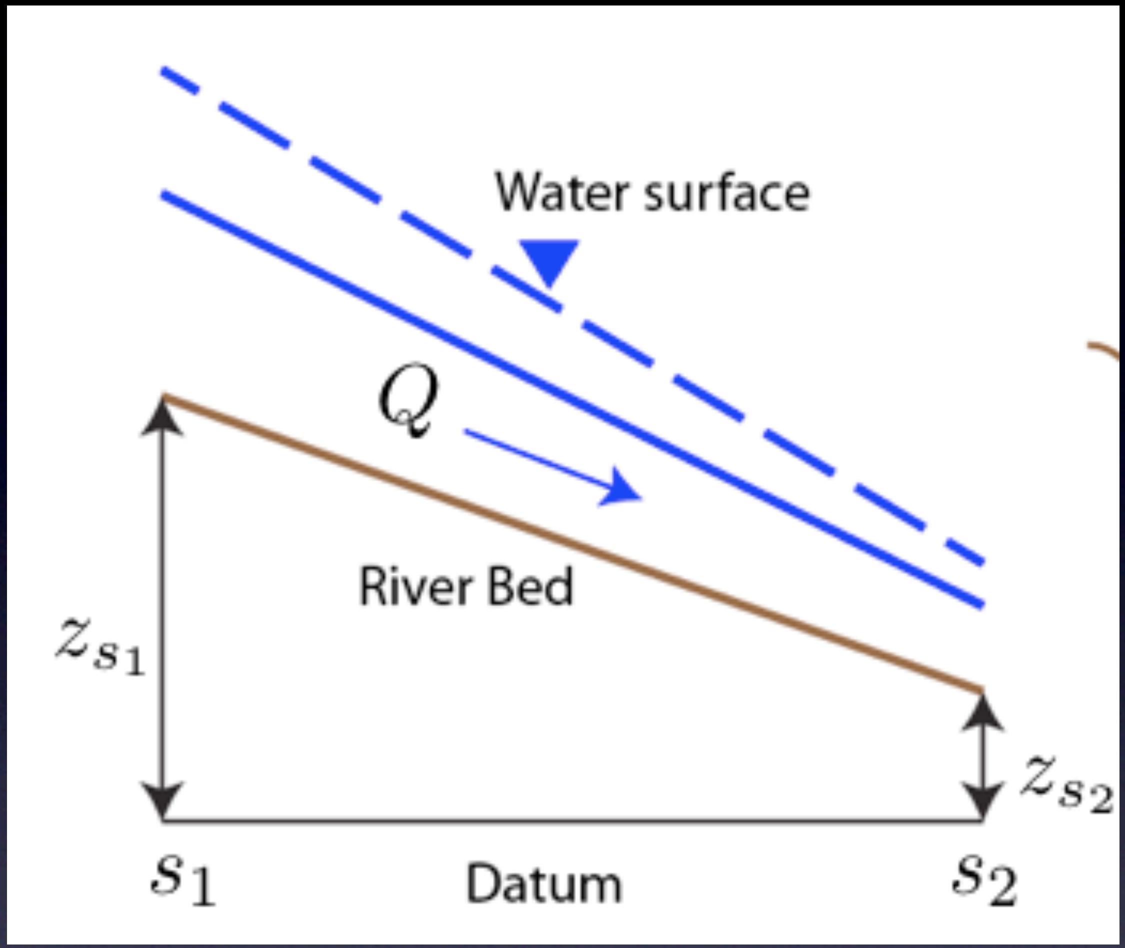
$$\overline{Q}_{t_{12}s_1} - \overline{Q}_{t_{12}s_2} = L\overline{T}\left(\frac{h_{t_2} - h_{t_1}}{\tau_1}\right)$$

$$\overline{Q}_{t_{12}s_1} + \overline{Q}_{t_{12}s_2} = 2\overline{K}_r\left(\overline{\frac{\partial h}{\partial x}}\right)^{1/2}$$

At t_3 four equations, six unknowns

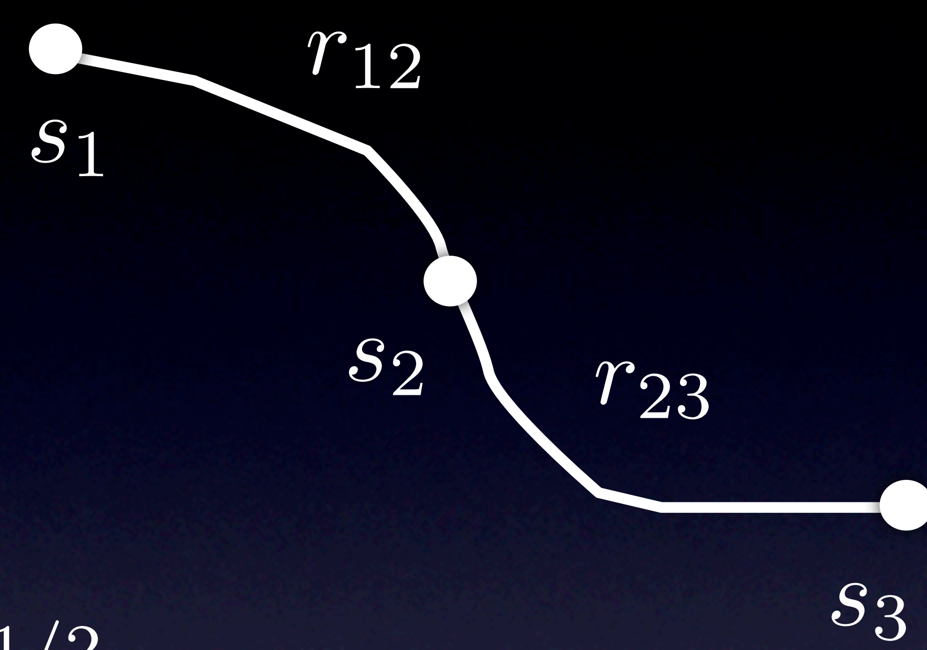
$$\overline{Q}_{t_{23}s_1} - \overline{Q}_{t_{23}s_2} = L\overline{T}\left(\frac{h_{t_3} - h_{t_2}}{\tau_2}\right)$$

$$\overline{Q}_{t_{23}s_1} + \overline{Q}_{t_{23}s_2} = 2\overline{K}_r\left(\overline{\frac{\partial h}{\partial x}}\right)^{1/2}$$



At t_3 six equations, eight unknowns...

Application to two reaches



At t_3 add these equations:

$$\overline{Q}_{t_{23}s_1} - \overline{Q}_{t_{23}s_2} = L_1{\tau_2}^{-1}\overline{T}_{r_1}\left(h_{t_3} - h_{t_2}\right)$$

$$\overline{Q}_{t_{23}s_2} - \overline{Q}_{t_{23}s_3} = L_2{\tau_1}^{-1}\overline{T}_{r_2}\left(h_{t_3} - h_{t_2}\right)$$

$$\overline{Q}_{t_{23},s_1} + \overline{Q}_{t_{23},s_2} = 2\overline{K}_{r_{12}}\left(\overline{\partial h/\partial x}\Big|_{r_{12}}\right)^{1/2}$$

$$\overline{Q}_{t_{23},s_2} + \overline{Q}_{t_{23},s_3} = 2\overline{K}_{r_{23}}\left(\overline{\partial h/\partial x}\Big|_{r_{12}}\right)^{1/2}$$

We now have eight equations

We have six discharge unknowns but the same two time-invariant conveyance unknowns per reach: ten total