Intercomparison of algorithms to estimate river depth from SWOT observations of slope and width

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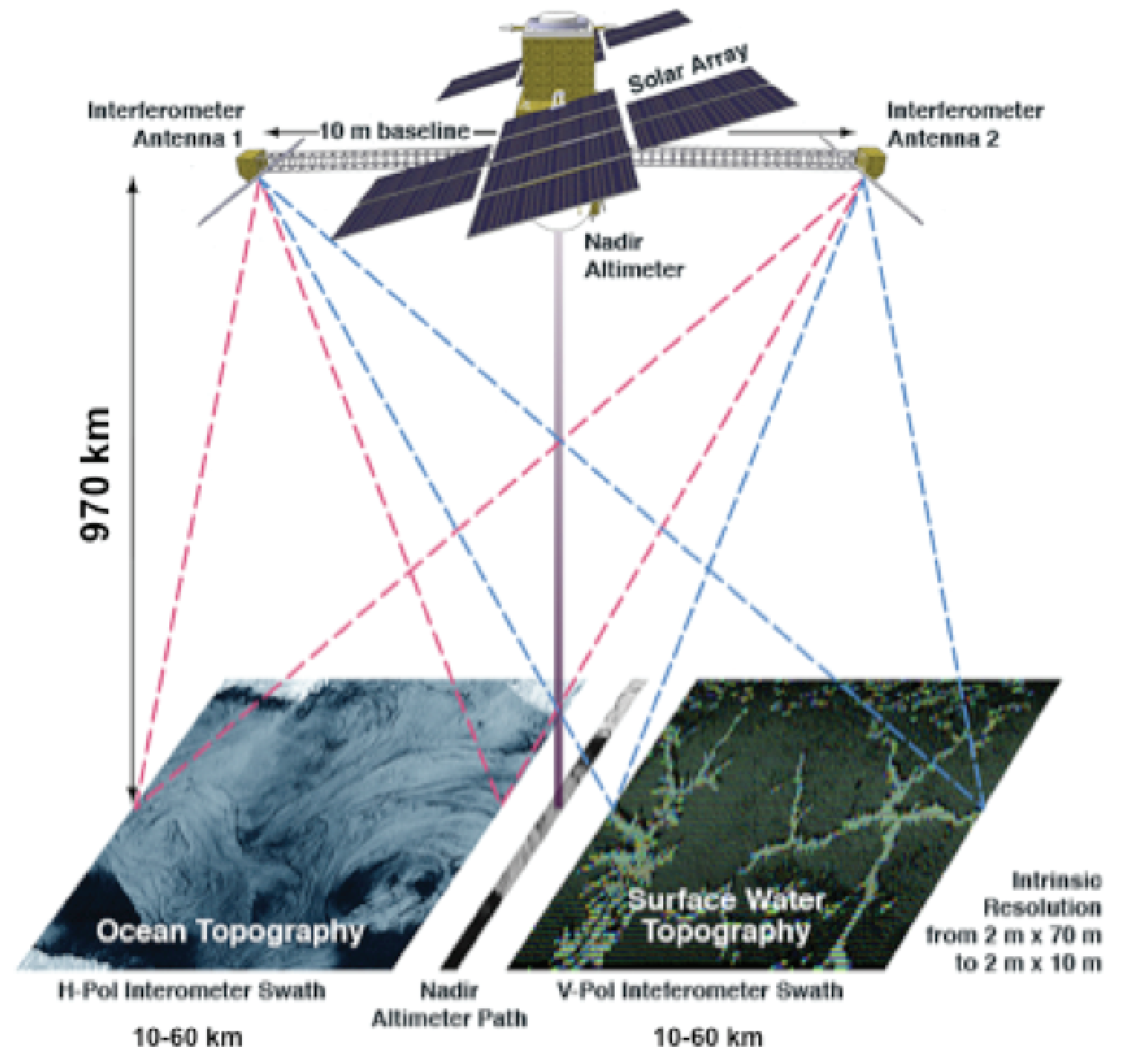
Summary and objectives

The proposed Surface Water and Ocean Topography mission would measure both ocean and inland water surface elevations , width, and the slope of the water surface. An estimate of river bathymetry is required in order to calculate cross-sectional flow area, and to derive an estimate of river discharge. Our goals are to estimate the expected accuracy of a depth estimate based on a two-tier approach where a “reach averaged” depth is calculated from an empirical model, and the gradually-varied flow equation is used to predict depth variations.

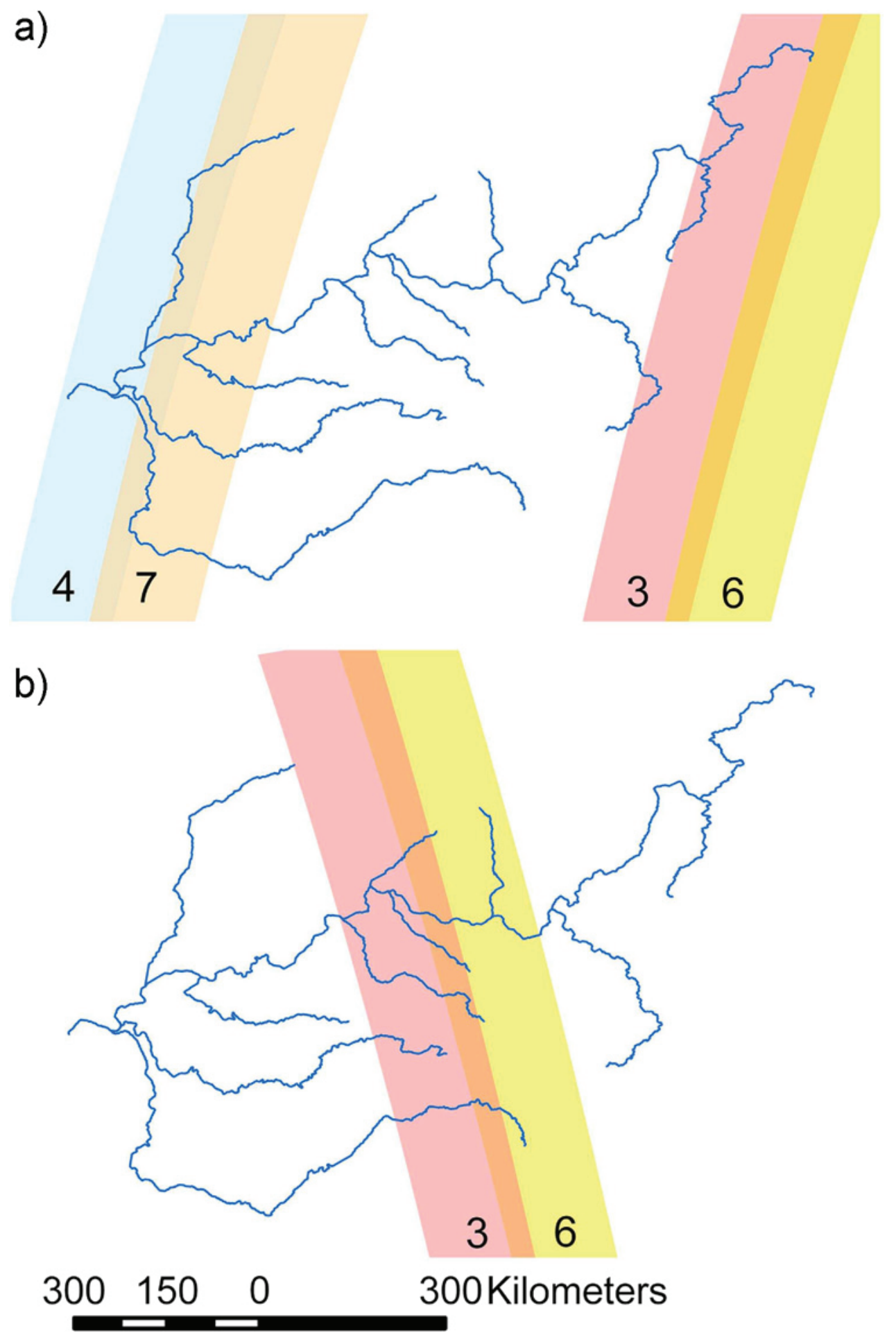
The SWOT mission and technology

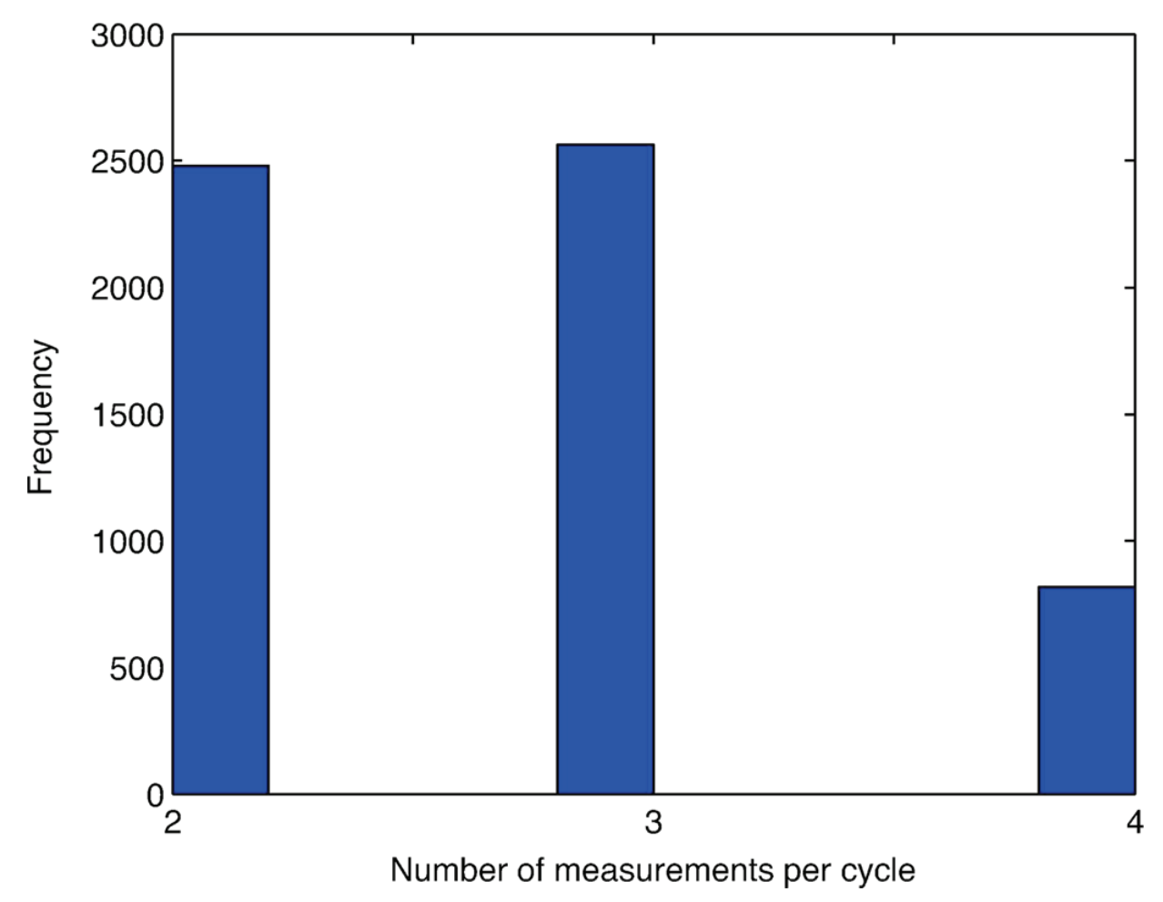


The proposed Surface Water and Ocean Topography (SWOT) mission is a radar altimeter that would provide new measurements of water surface elevation (WSE) for oceans, rivers, lakes, wetlands and reservoirs.

SWOT has been recommended by the National Research Council Decadal Survey for a launch date timeframe between 2013 – 2016. Technology for SWOT is a Ka-band radar inferometer (KaRIn), described in Alsdorf et al. (2007). The expected instrument spatial resolution in a best-case scenario results in rectangular pixels that are 2 m x 10 m in the far swath and 2m x 70 m in the near swath. Instrument temporal resolution depends upon latitude and location within the swath. At tropics and mid-latitudes, most locations are sampled three times within the 22-day orbit. Coverage extends to 78 degrees latitude. For the Ohio River basin 34 N - 42 N latitude the three-day sub-cycle leads to the sampling pattern shown below for the first seven days of the 22-day orbit.

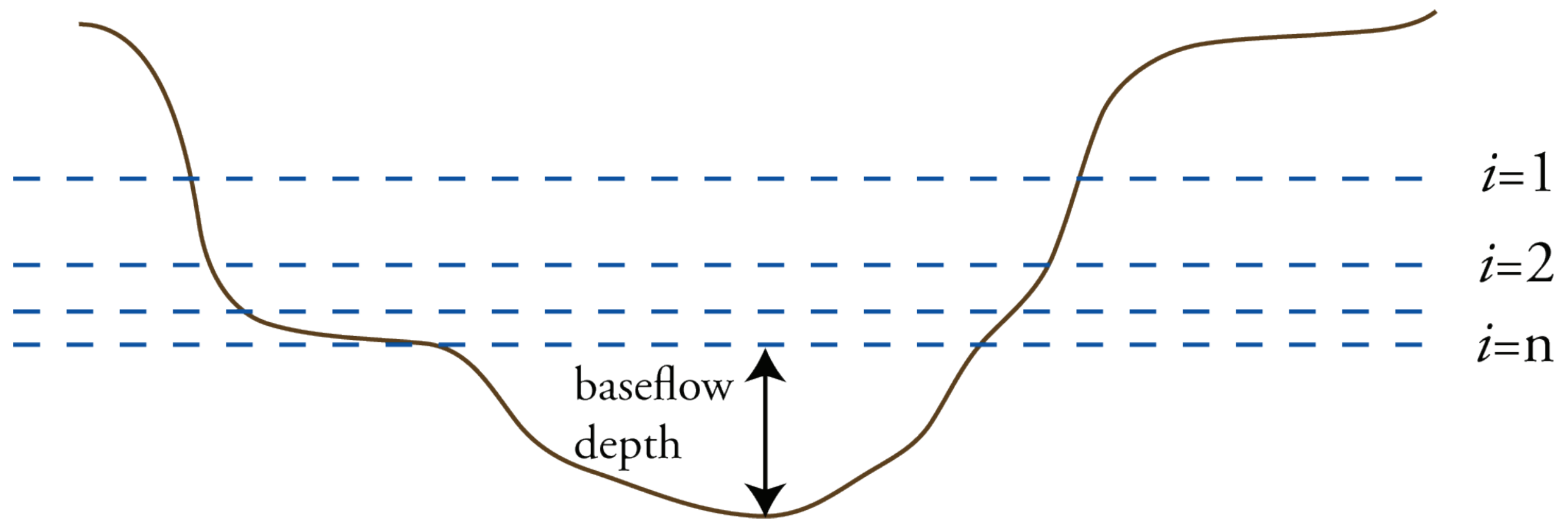
This sampling leads to between two and four measurements over the 22-day orbit (see histogram below). The expected random errors on height measurements is 50 cm for the small pixels shown above. The radar brightness can be used to distinguish top width, and along-stream changes in WSE give dh/dx.





Depth or bathymetry estimate needed

A hydrodynamic model is used to relate WSE measurements and bathymetry. LISFLOOD-FP is a raster-based hydrodynamic model (Bates and DeRoo, 2000). Channel flow is represented by a 1-D finite difference solution to the kinematic wave approximation. Floodplain flow is represented by the decoupled 2-D diffusion wave. Model inputs include the Shuttle Radar Topography Mission (SRTM) DEM, at 270 m spatial resolution, channel bathymetry, and upstream flowrate boundary condition.



The blue dashed lines indicate conceptually the water elevation measurements that SWOT will provide, i=1...n. Above this elevation, cross-sectional flow area can be calculated explicitly from SWOT measurements. Below this level, the baseflow cross-sectional area will not be measured. This unobserved area or baseflow depth will need to be estimated using some methodology. An equivalent means to accomplish the same function is to estimate the river bathymetry directly. Two methods that have been explored in this regard is an inversion algorithm, and a data assimilation approach (Durand et al. 2008, Durand et al., 2009).

Problem statement

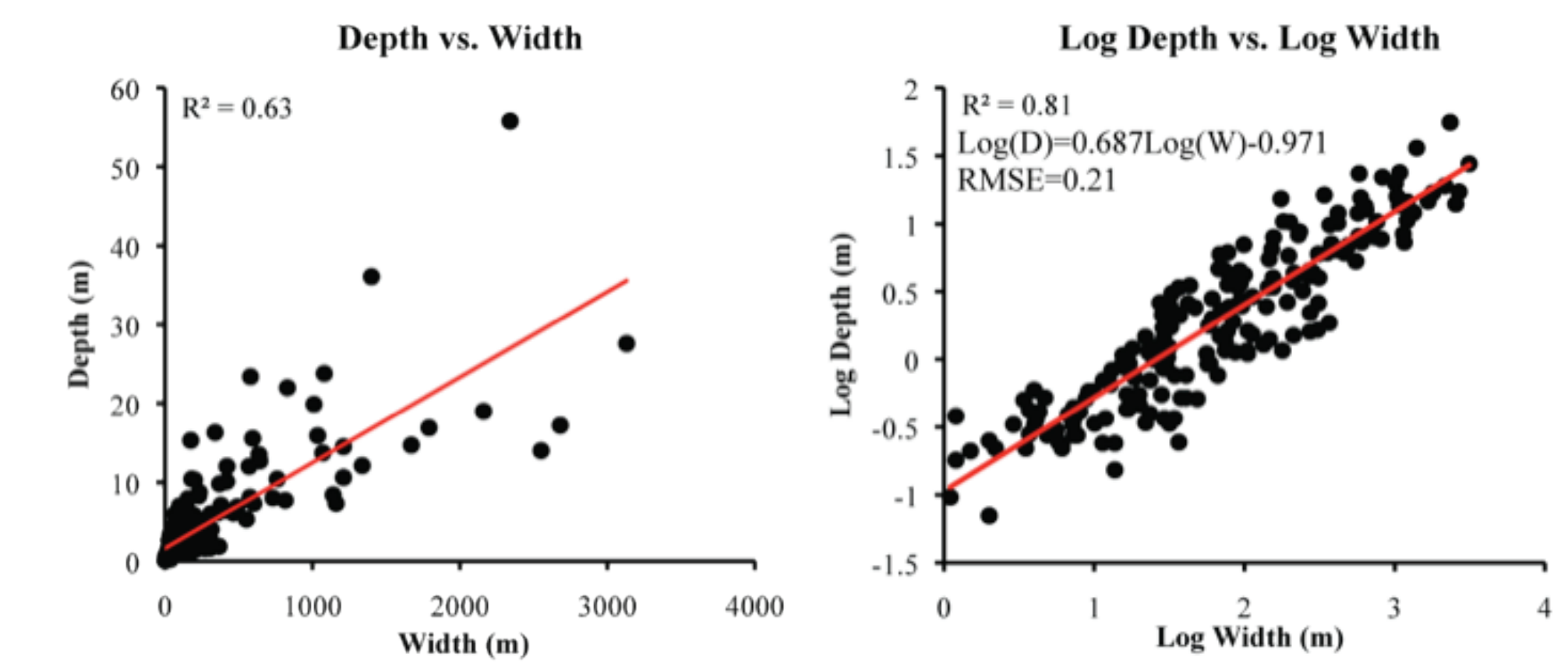
SWOT will measure river elevation, width and slope. In order to estimate river discharge, we seek an estimate of river depth. In that way, the depth problem becomes a classic inverse problem with regard to traditional comoputation of water surface elevation. In the discipline of Civil Engineering, inverse solutions are often hampered by lack of data, whereas in many traditional hydrologic models, inverse problems are defeated by the physical complexity of the system. In contrast, we seek an inverse solution to the shallow water equations, and will have a large number of measurments to work with.

Note that estimation of a mean depth in time over the period is equivalent to solving our problem. The mean annual depth could be used to approximate this. For a given river network, if even a few gages existed inside, the reach-averaged mean annual depth could be estimated using standard downstream hydraulic geometry relationships.

The spatial variability of river depth should be able to be predicted to some extent using the measurements of river width and slope, and solving an inverse problem.

Estimating reach-averaged depth

Data below from Moody and Troutman (2002) show a strong positive correlation between log of depth and width, though with significant scatter. The fit was better between depth and width than with width and drainage contributing area, pulled from geolocating each point on the Hydrosheds drainage accumulation maps.


Depth vs. Width

Log Depth vs. Log Width

Estimating depth variations

I wanted to work out a slope+ width-to-depth algorithm where we are just solving for the predicted spatial variations in depth given width and slope. Let’s say there is a river with steady flow (no changes in time), with gradual variations in space in the slope of the bed and the width. How do the variations in width and slope impact the river depth? The kinematic wave approximation predicts that any change in width will be accommodated by an immediate change in depth to keep discharge constant, as predicted by Manning’s equation. In reality, on a mild slope, changes downstream in width and bed slope will affect depth upstream. This is a class of problems under steady, gradually-varied flow within open channel hydraulics.

$$\frac{dz}{dx} = \frac{S_0 - S_e}{1 - F^2}$$

Following Dingman’s formulation, where S0 is bed slope, Se is the slope of the so-called energy grade line, and F is the Froude number. Substituting for F, we get:

$$\frac{dz}{dx} = \frac{S_0 - S_e}{1 - \left(\frac{Q^2}{gw^2z^3}\right)}$$

where Q is the discharge, g is acceleration due to gravity, and w is river width. Here, we'll assume that we can neglect head losses due to expansion and contraction of the channel and other such things; in that case, Se can be approximated directly from Manning's equation:

$$S_e = \left(\frac{nQ}{z^{5/3}w}\right)^2$$

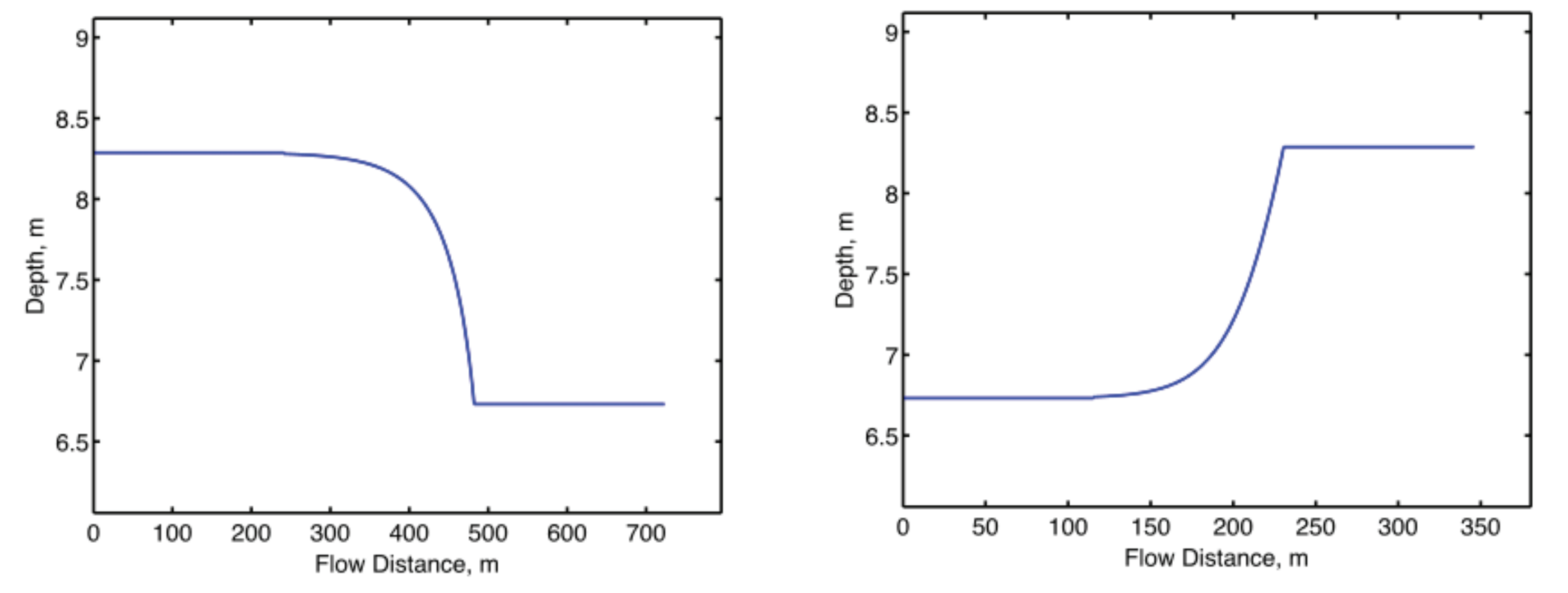
In *Fluvial Hydrology* Dingman reworks these as:

$$\frac{dz}{dx} = S_0\frac{N}{D}$$

where:

$$N = 1 - \left(\frac{z_n}{z}\right)^{\frac{10}{3}} \qquad D = 1 - \left(\frac{z_c}{z}\right)^3$$

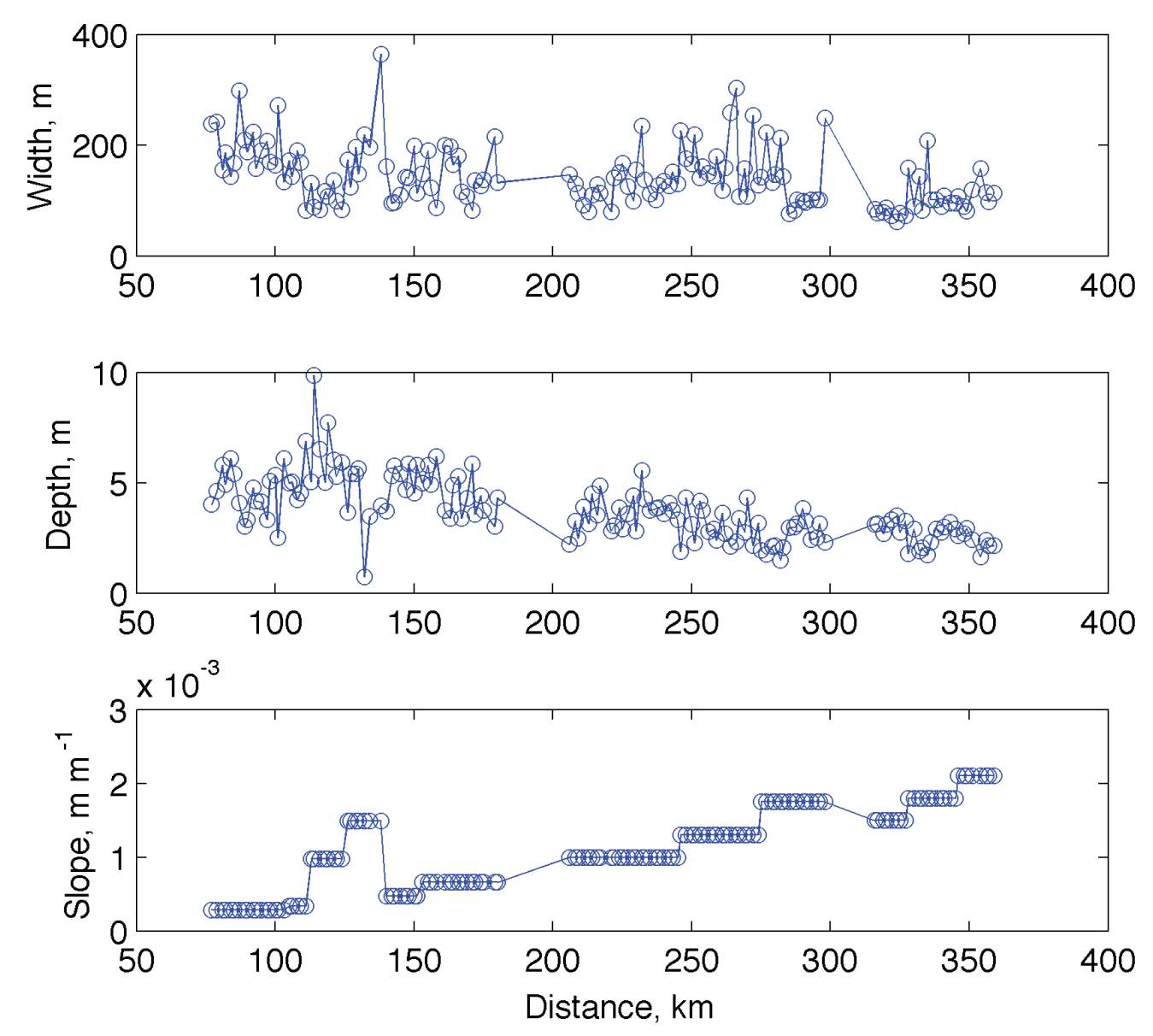
So the effect of a change in width or slope on the river depth is expressed as a gradual transition over some distance, from one so-called “normal” depth (where Manning’s equation is satisfied) to another.

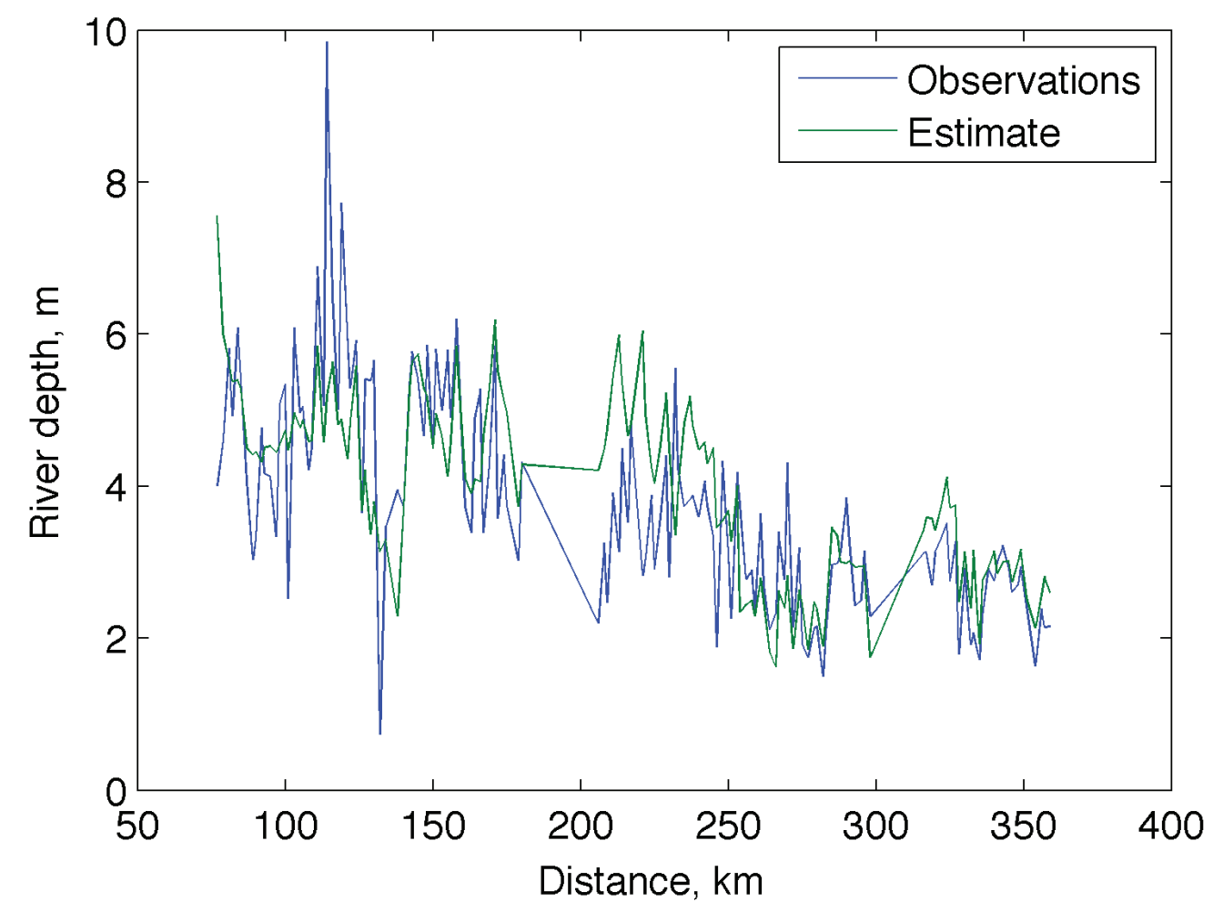


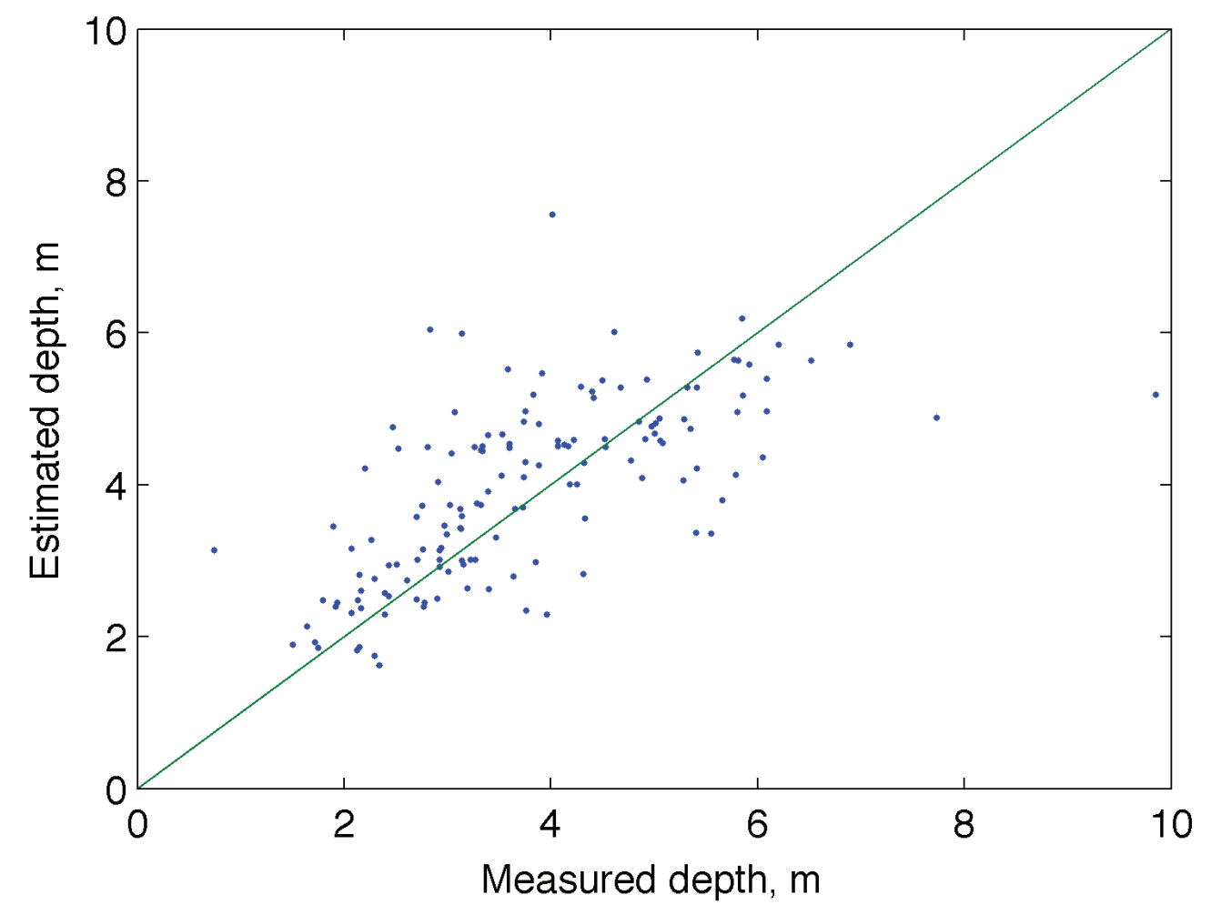
The exponential shape of the curves above and the simple differential equation for dz/dx above suggest an exponential function to calculate the transition between two normal depths. The equation below fits these curves very well, where xt marks the location where a transition has taken place. Note that the coefficient a below can be approximated quite well by the slope S0, which is a SWOT product.

$$z(x') = \left|\begin{array}{cc} z_0^{dn} & x' \le x'_t \\ z_n^{up} + (z_n^{dn} - z_n^{up})e^{-a(x'-x'_t)} & x' > x'_t \end{array}\right|$$

The depth, width, and slope data at the right were made by John Pitlick on the Colorado River, between Parachute, CO and Moab, UT. Measurements were made at a median interval of 2 km, with some exceptions. Slopes were made based on DGPS from a boat.







The algorithm described previously was used to estimate depth from slope and width. The algorithm requires an estimate of the starting depth, which could come from a known cross-section (where available). Discharge is also an input, so the equations would need to be solved iteratively.

An alternative to a brute-force iterative solution is to use a Bayesian estimator, solving the gradually-varied flow problem in reverse for bathymetry and discharge. In this approach, a prior guess is specified, and a Markov Chain Monte Carlo algorithm is used to find the optimal value of depth and discharge, similar to a data assimilation scheme.

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Acknowledgments

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