Absolute Ionosphere Slant Delays From Ambiguous Carrier Phase Data

Dru A. Smith, Ph.D.
National Geodetic Survey
National Oceanic and Atmospheric Administration

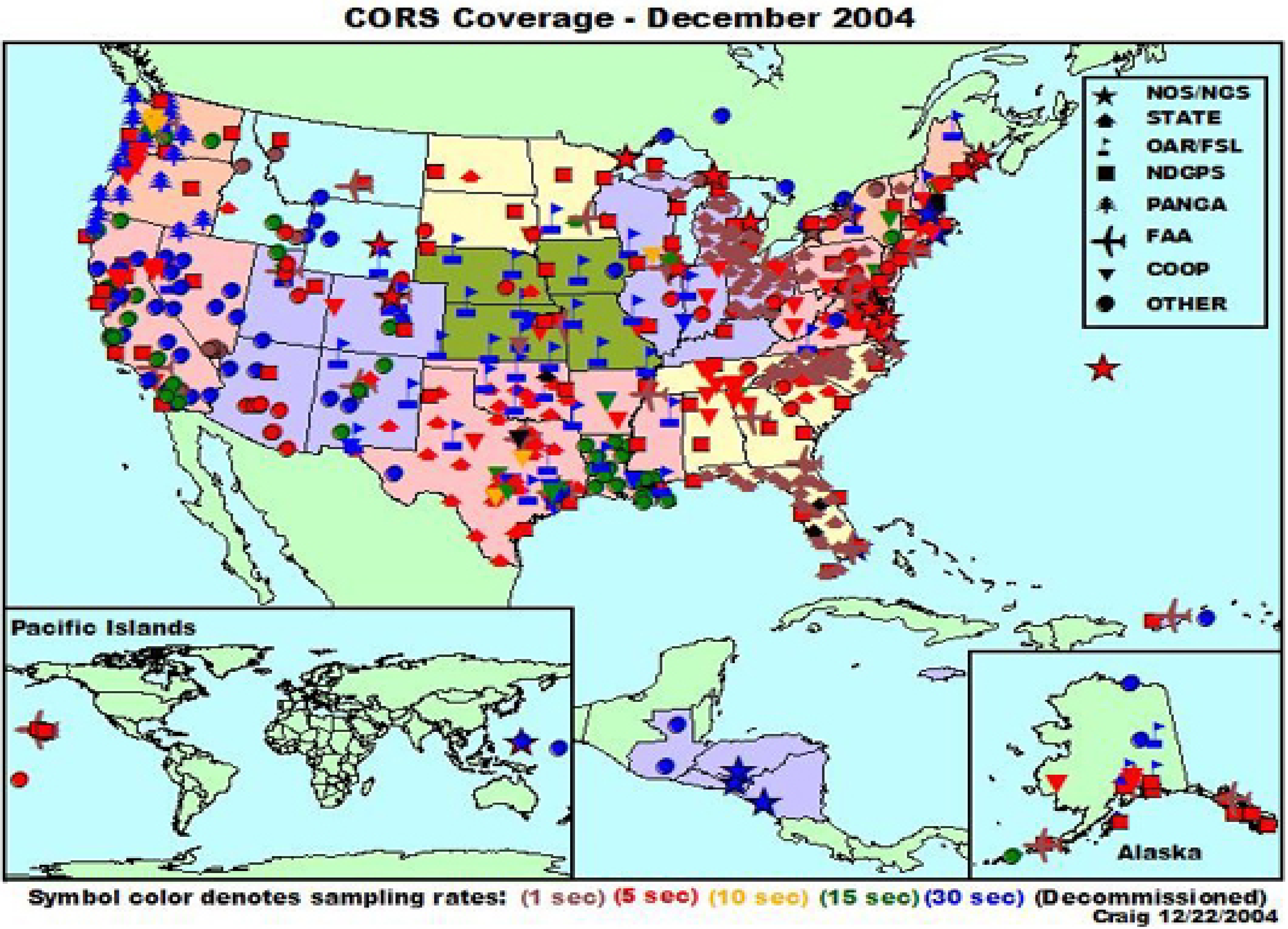
ION NTM 2005
January 25, 2005
San Diego, CA

Topics of Discussion

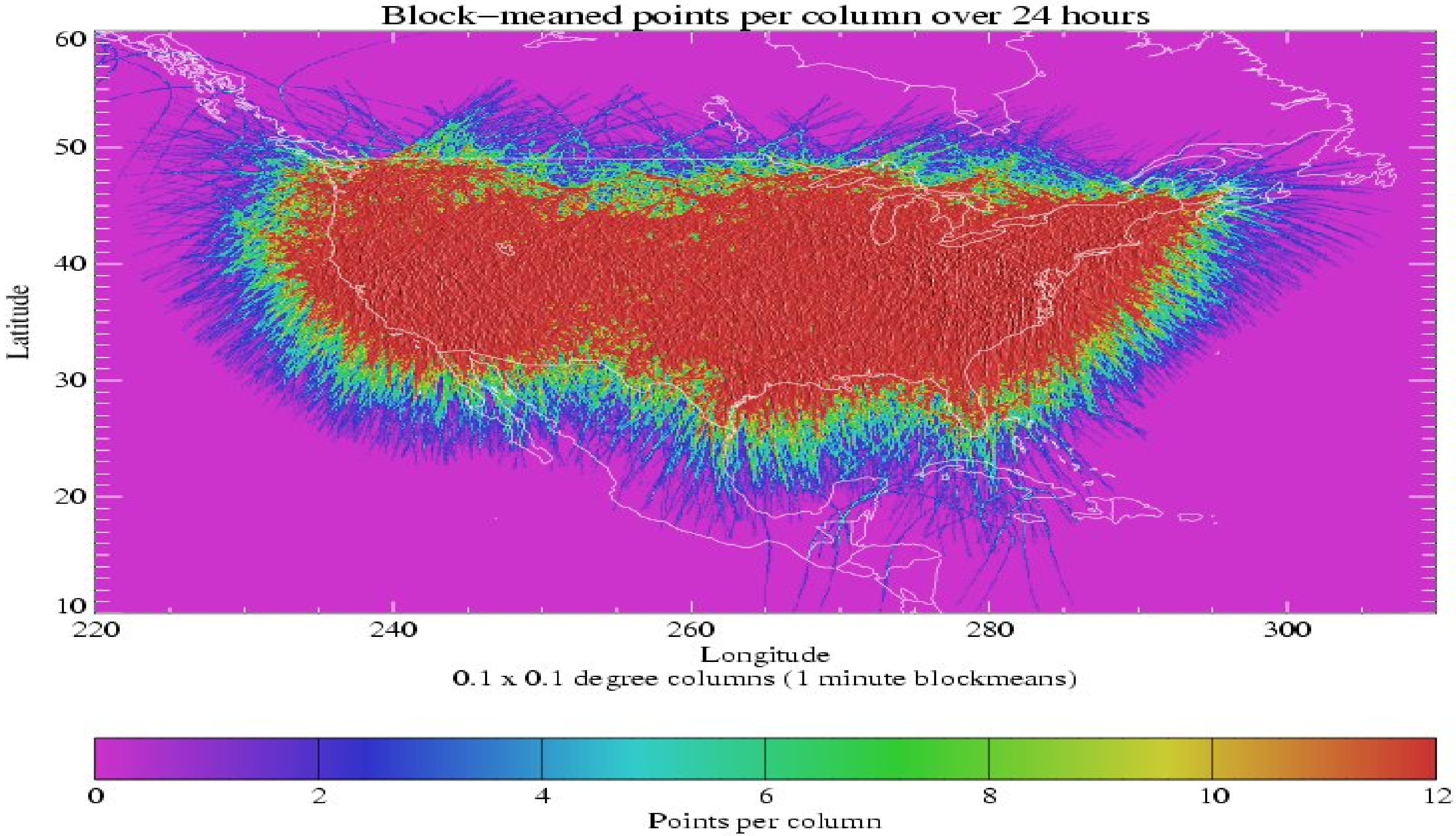
- Motivation for NGS ionosphere
- Model/Equations
- Comparisons/Analysis
- Conclusions

Geodetic need for ionosphere delays

- Dominant Frequency-dependent signals in GPS:
  - Ambiguities
  - Ionosphere
- Difficult to separate quickly
- NGS decision: model the ionosphere to get ambiguities faster
- Data wasn’t an issue: CORS



Smith, D.A., Absolute Ionosphere, ION NTM 2005



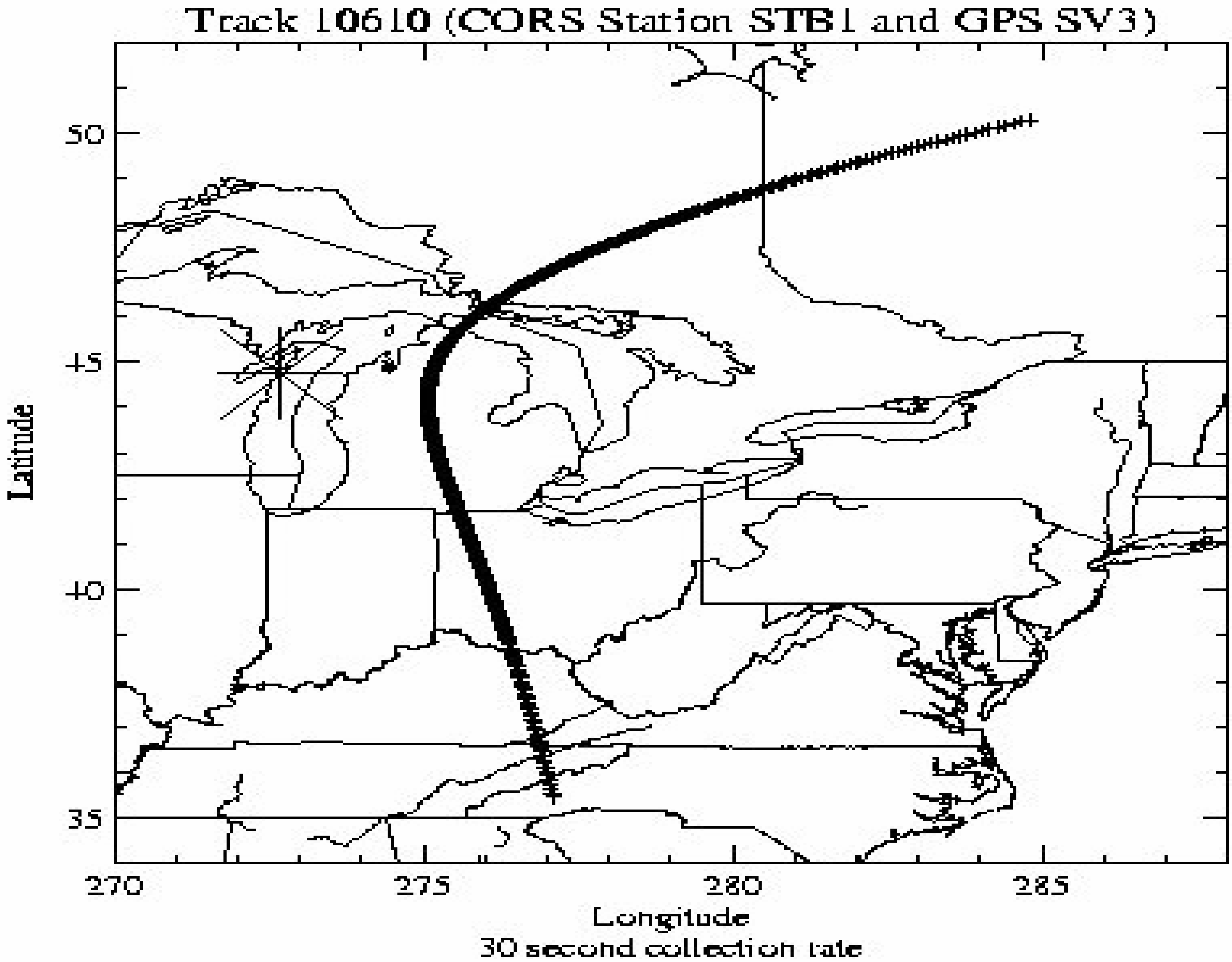
Nearly every part of the ionosphere above CONUS is viewed by CORS 12+ times daily (some >100 times a day)

Smith, D.A., Absolute Ionosphere, ION NTM 2005

Tools and Terms

- Terms:
  - Track = Sequential L1&L2 data for one CORS/SV combo without extended loss of lock
  - TECS=Total Electron Content along satellite/receiver vector

- CORS yields about 20-30k tracks every day



Smith, D.A., Absolute Ionosphere, ION NTM 2005

Primary Objective

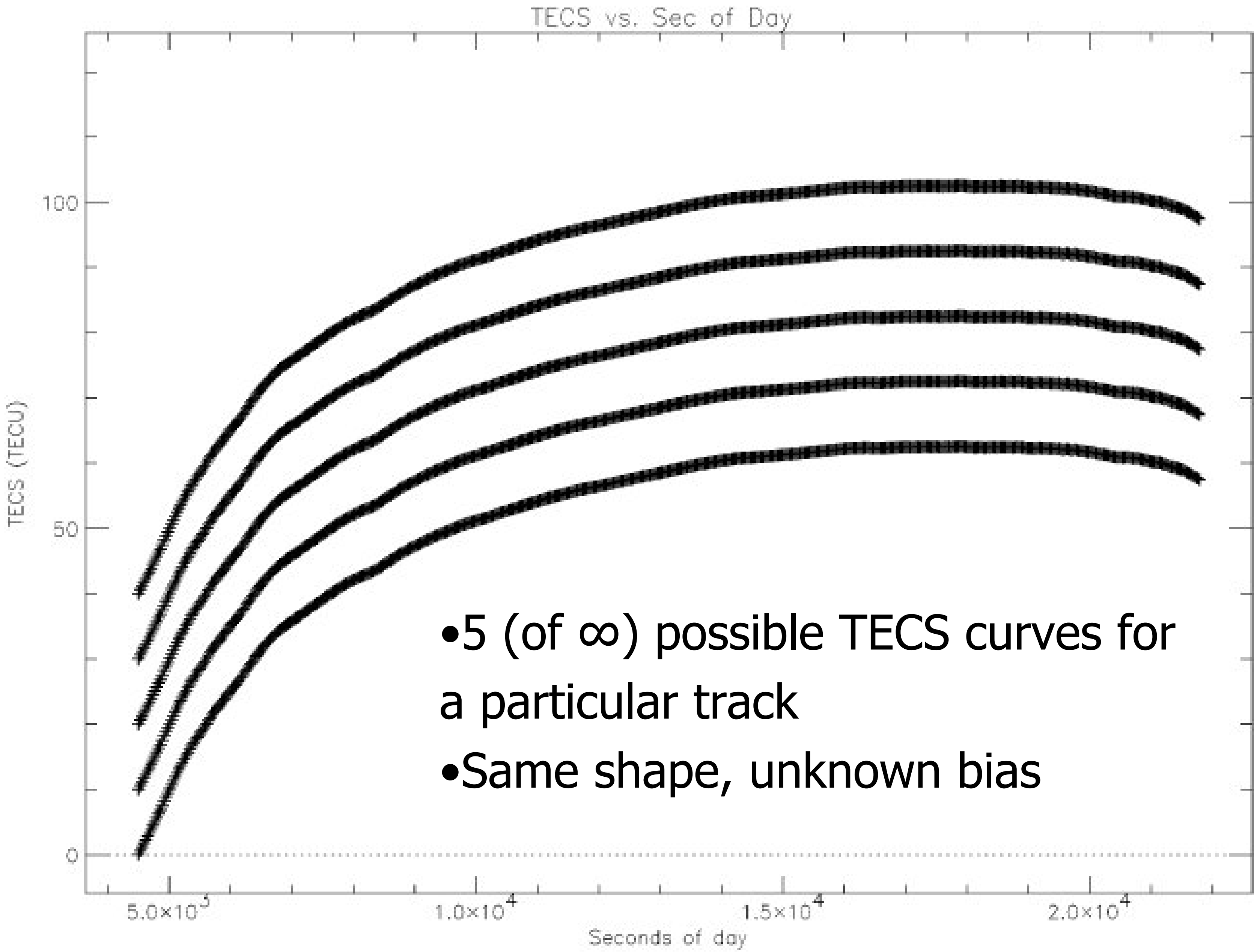
- Model absolute TECS data and maintain high resolution details of TECS for every track over CONUS
- Focus on *fast*, *accurate* ionosphere delays; not on modeling 4-D electron distribution

Getting TECS from carrier data

- For 1 track, between any two epochs (i, j):
  - $^{i,j}\Delta TECS = k(\text{"40.3"}, f_1, f_2) \times (^{i,j}\Delta L1 - {}^{i,j}\Delta L2)$

- Thus, every track has:
  - Very accurately known shape of ∂TECS/∂time (from carrier phase data)
  - One unknown TECS bias

- As per the Primary Objective:
  - Solve 1 TECS bias per track



- 5 (of ∞) possible TECS curves for a particular track
- Same shape, unknown bias

Solving for biases

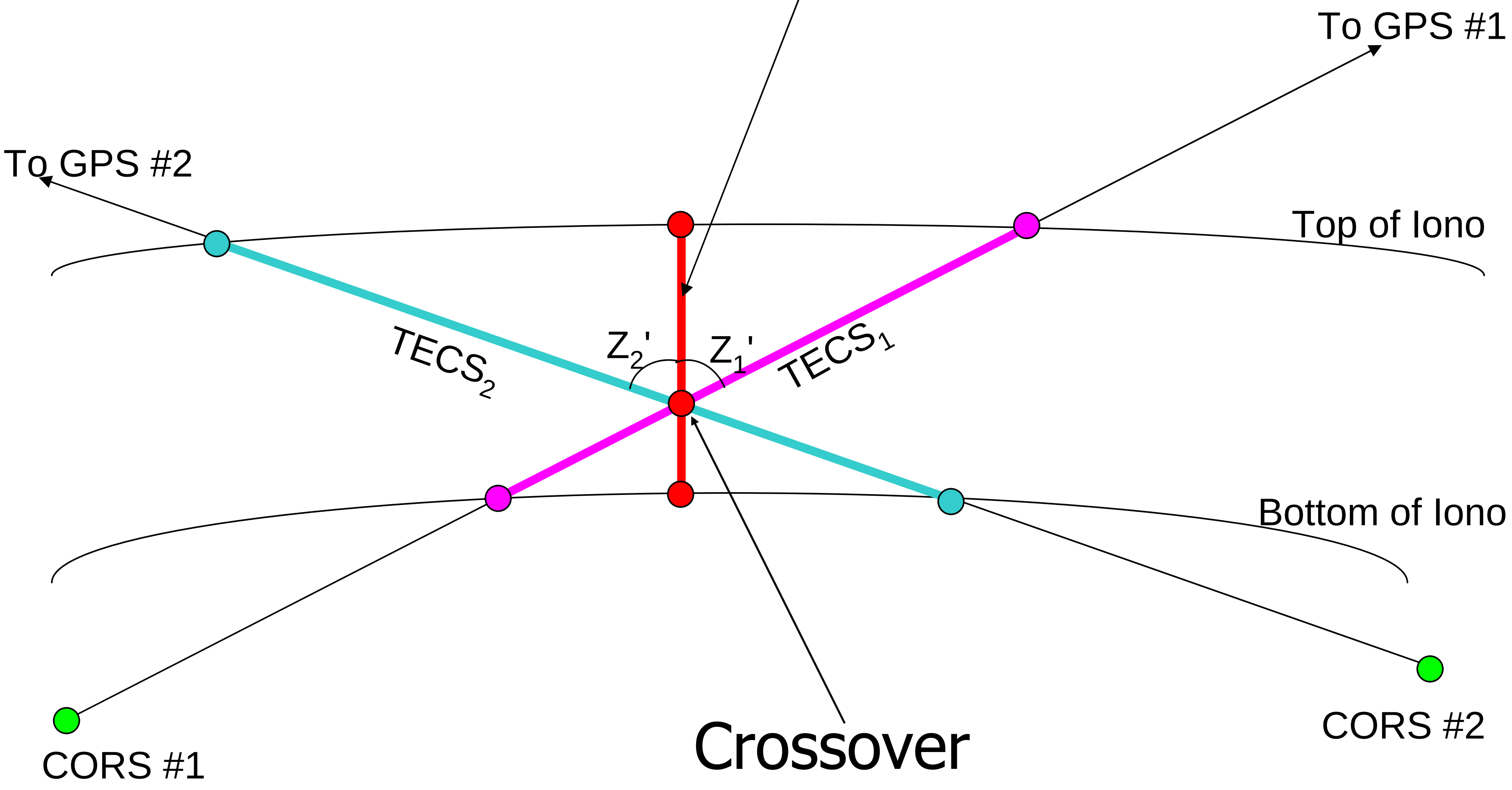
- Consider: Two receiver-satellite vectors of two different tracks "sufficiently close" to each other in time & space.
  – Call this a crossover
- Assumption at a crossover:
  – TECS(t, track a) = f [TECS(t±dt, track b)]
  – "sufficiently close" must be defined
  – Find an acceptable mapping function "f"

Mapping Functions

- Any mapping function can be used
  - Linear or non-linear
  - But, how good is your mapping function?

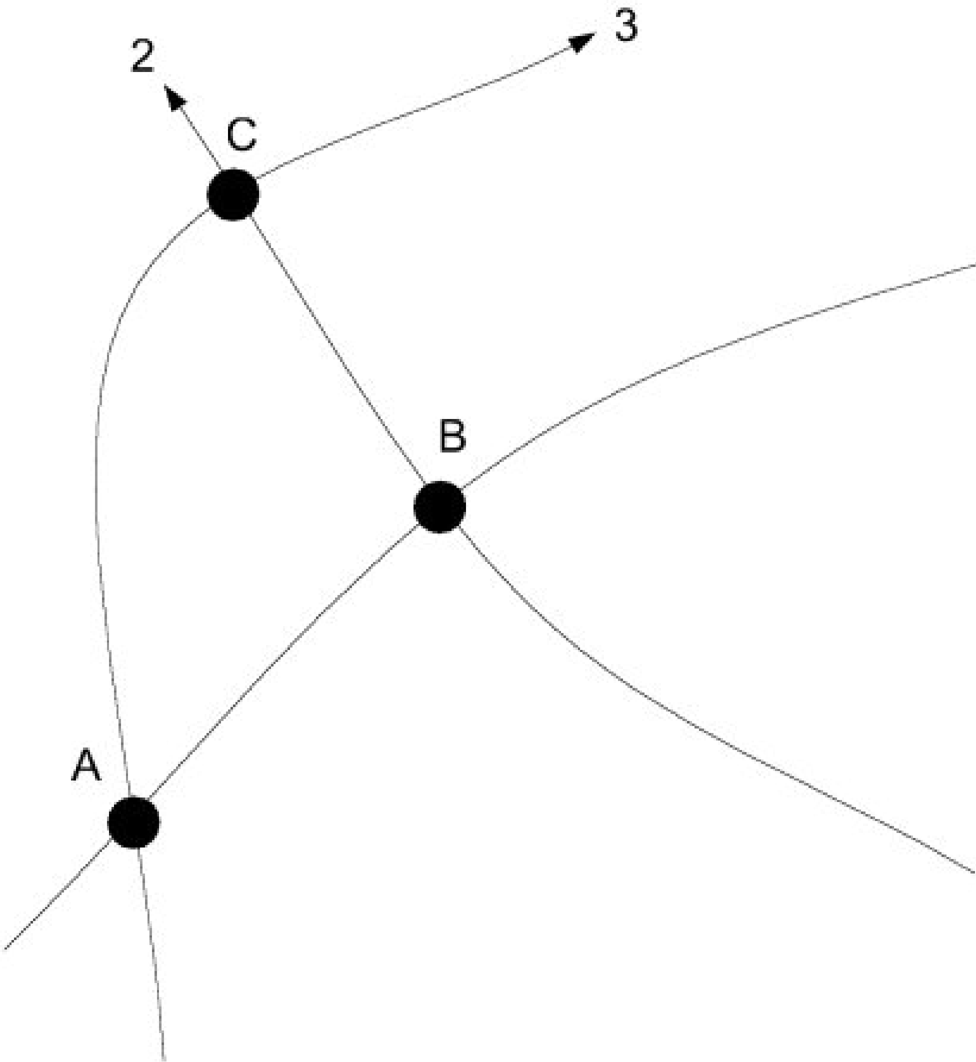
- NGS currently using the "vertical column equality" assumption
  - Crossovers defined by nearness of the two vectors at their 300 km altitude points
  - "Sufficiently close" generally at 0.1º x 0.1º x 60 sec

$$TECS_1 \times \cos(z_1') = \underline{\mathbf{TECR_1 = TECR_2}} = TECS_2 \times \cos(z_2')$$



Using Crossovers

- By itself, one crossover has:
  - 1 condition ($TECS_1 = f[TECS_2]$)
  - 2 unknowns (TECS biases for 2 tracks)
  - Thus, unsolvable as is
- Need conditions $\geq$ unknowns
- Closed polygons is the solution

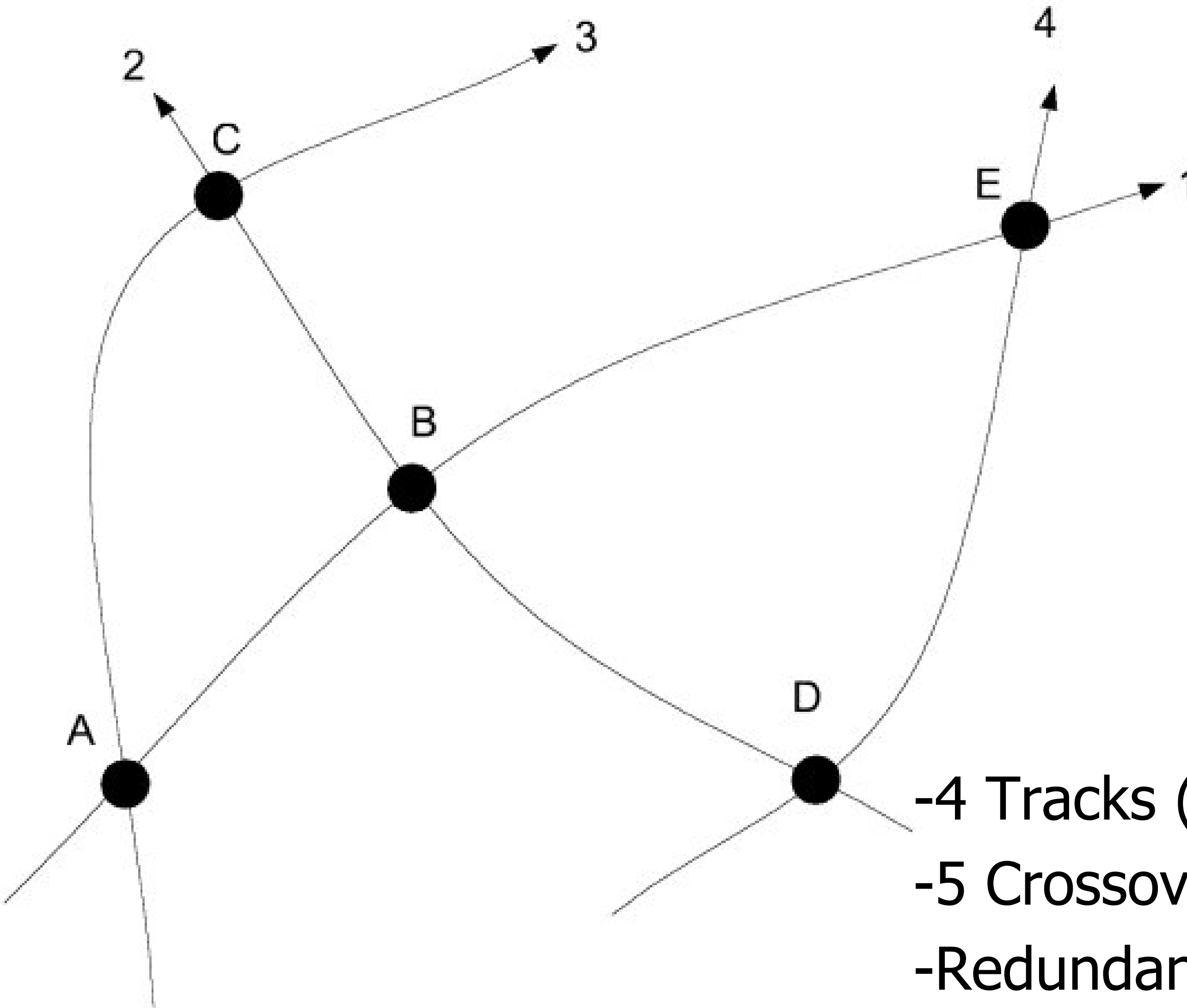


- 3 Tracks
- Crossovers A,B,C occur in sequential order
- Not as rare as it looks
- Forms a “closed polygon” of tracks
- *Uniquely* solvable in *absolute* TECS space

Polygon Crossover Equations

$$\begin{bmatrix} {}_1^A\Delta TECS \cos {}_1^A z' - {}_3^A\Delta TECS \cos {}_3^A z' \\ {}_1^B\Delta TECS \cos {}_1^B z' - {}_2^B\Delta TECS \cos {}_2^B z' \\ {}_2^C\Delta TECS \cos {}_2^C z' - {}_3^C\Delta TECS \cos {}_3^C z' \end{bmatrix} = \begin{bmatrix} -\cos {}_1^A z' & 0 & +\cos {}_3^A z' \\ -\cos {}_1^B z' & +\cos {}_2^B z' & 0 \\ 0 & -\cos {}_2^C z' & +\cos {}_3^C z' \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Smith, D.A., Absolute Ionosphere, ION NTM 2005



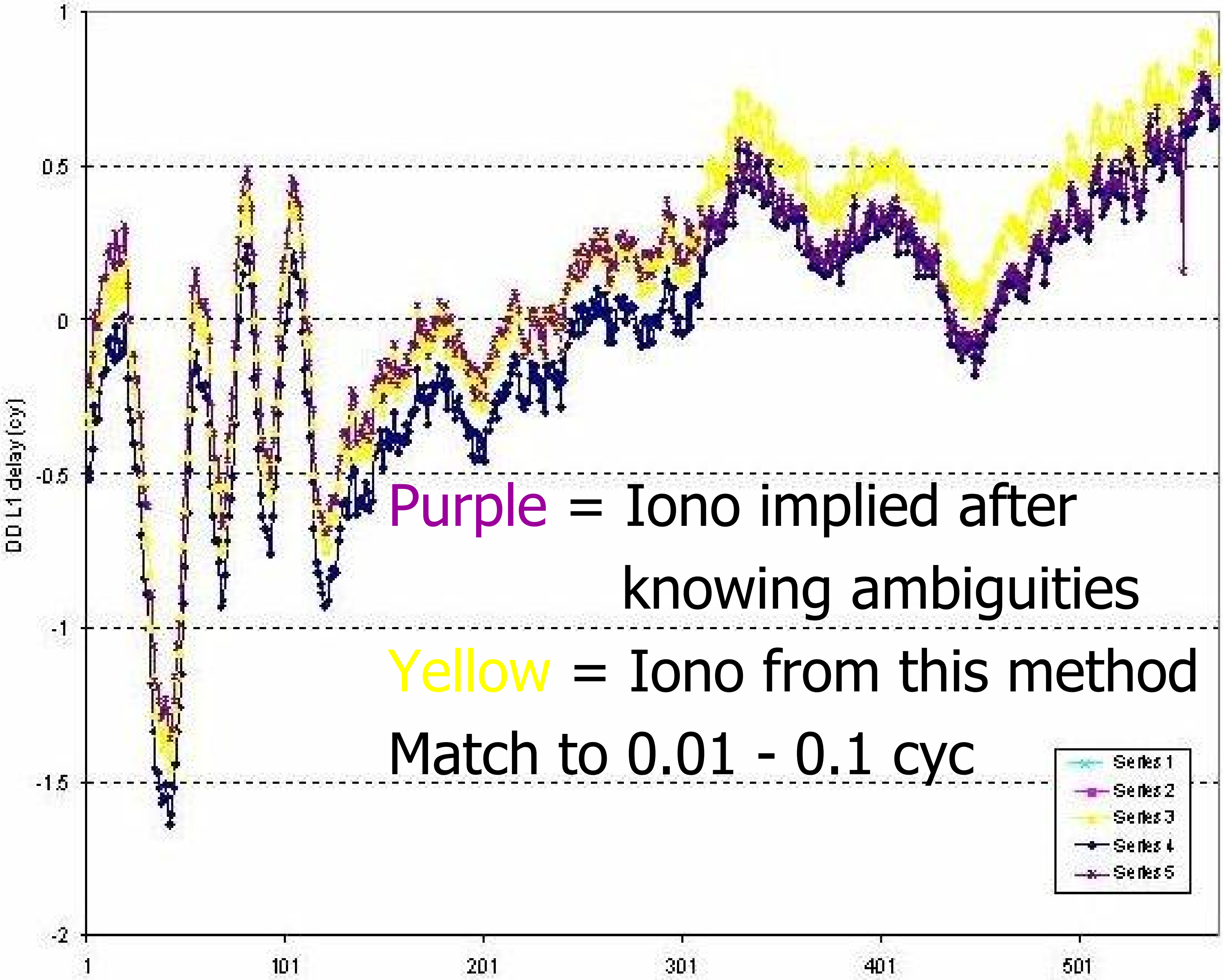
-4 Tracks (unknowns)

-5 Crossovers (conditions)

-Redundancy = Least Squares
Adjustment in *absolute* TECS space

Initial Tests(NGS)

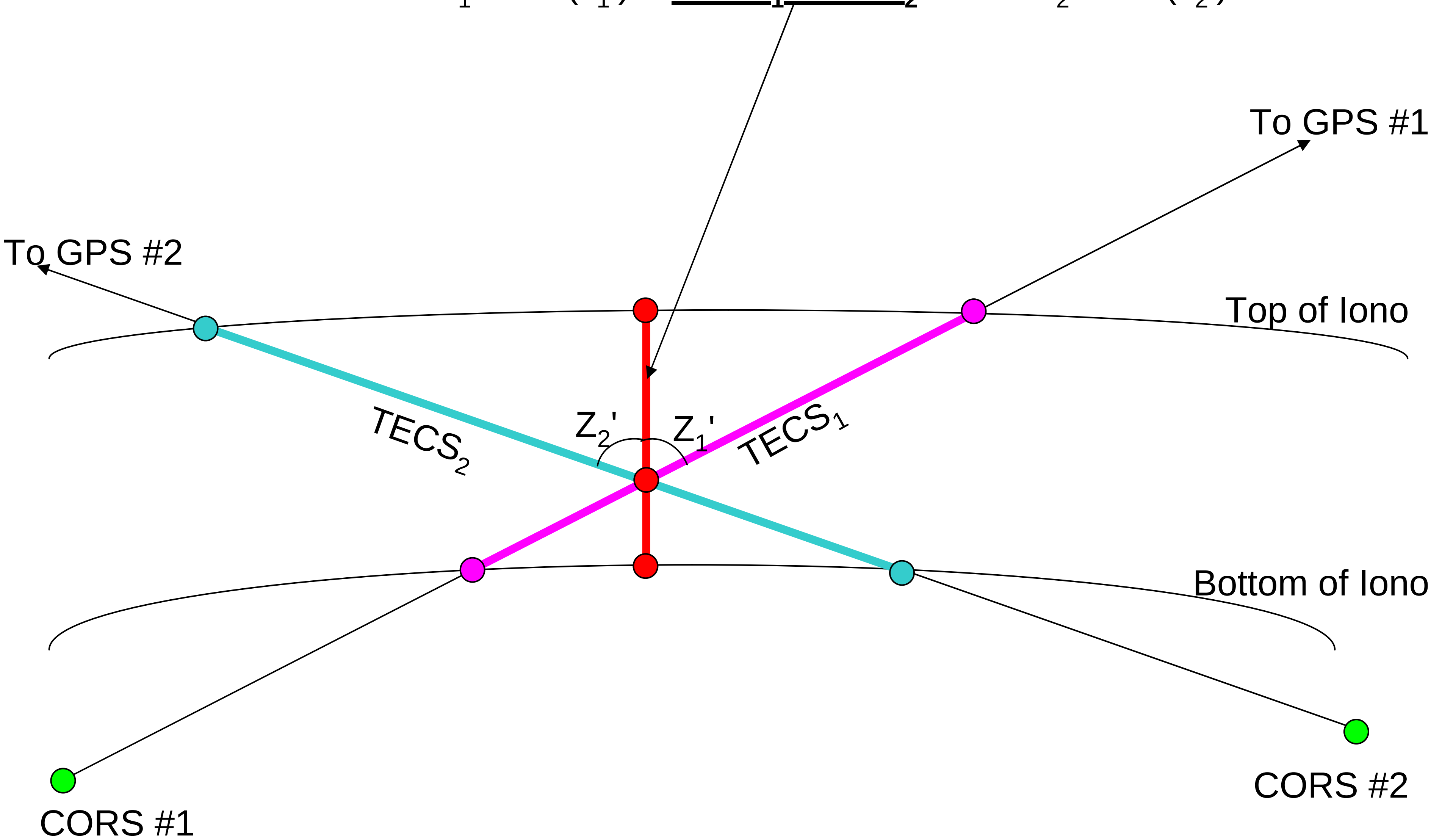
- Small "tracknets" of 10-12 tracks formed
- Proof-of-concept
- Absolute delays converted to double difference delays
- DD delays good to 0.1± 0.01 TECUs against "truth" (Ambiguity resolving software)


Purple = Iono implied after knowing ambiguities
Yellow = Iono from this method
Match to 0.01 - 0.1 cyc
DD L1 delay (cy)
1
0.5
0
-0.5
-1
-1.5
-2
1
101
201
301
401
501
Series 1
Series 2
Series 3
Series 4
Series 5

Initial Tests(OSU)

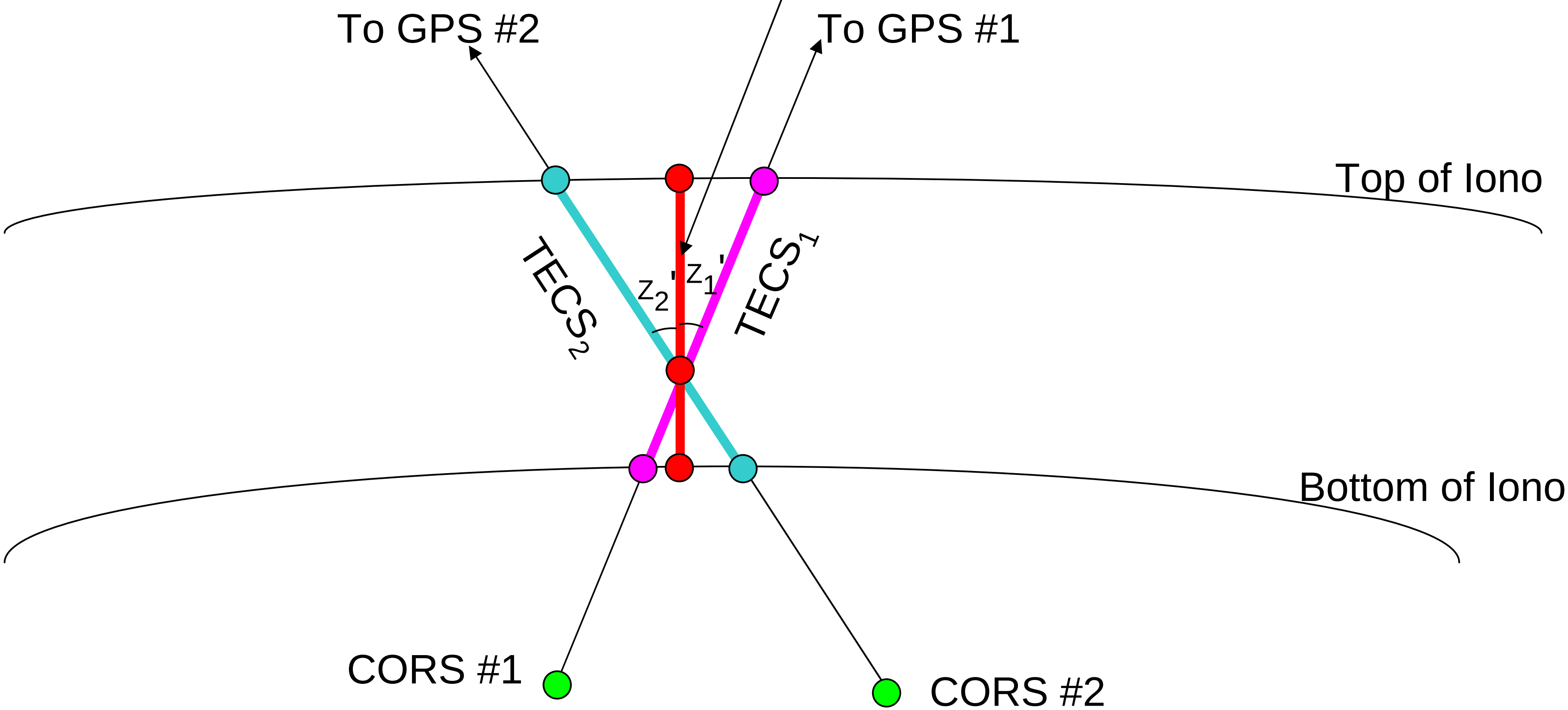
- The Ohio State University compared various Ionosphere estimates at Ohio CORS stations
- Double-difference mode
- Crossovers restricted to 40 degrees above the horizon
  - Avoids erroneous biases from low-elevation crossovers
  - Reduces number of tracks immediately solvable from tracknets (unsolved tracks need interpolation from nearby solved tracks)

$TECS_1 \times \cos(z_1') = \underline{\mathbf{TECR_1 = TECR_2}} = TECS_2 \times \cos(z_2')$



“Large” z’ makes the mapping of $TECS_1$ into $TECS_2$ questionable

$TECS_1 \times \cos(z_1') = \underline{\mathbf{TECR_1 = TECR_2}} = TECS_2 \times \cos(z_2')$



"Small" z' makes the mapping of $TECS_1$ into $TECS_2$ more reliable

Report for NOAA/NGS

On:

Accuracy analysis of various NGS ionosphere estimation models

Dorota Grejner-Brzezinska, Pawel Wielgosz, Israel Kashani

Department Of Civil & Environmental Engineering & Geodetic Science
The Ohio State University
470 Hitchcock Hall, 2070 Neil Avenue, Columbus, OH 43210
Tel: 614-292-0169; Fax: 614-292-2957
Email: dbrzezinska@osu.edu
Email: kashani.1@osu.edu
Email: wielgosz.1@osu.edu

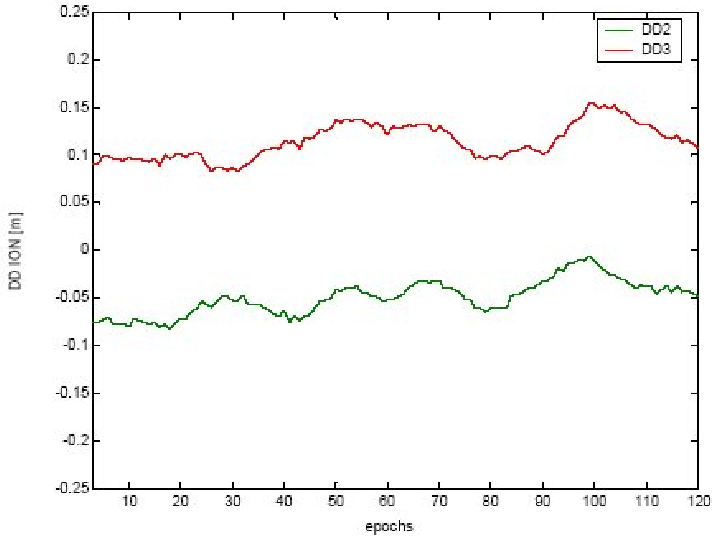


Fig. 22 NGSa DD iono (day-time)

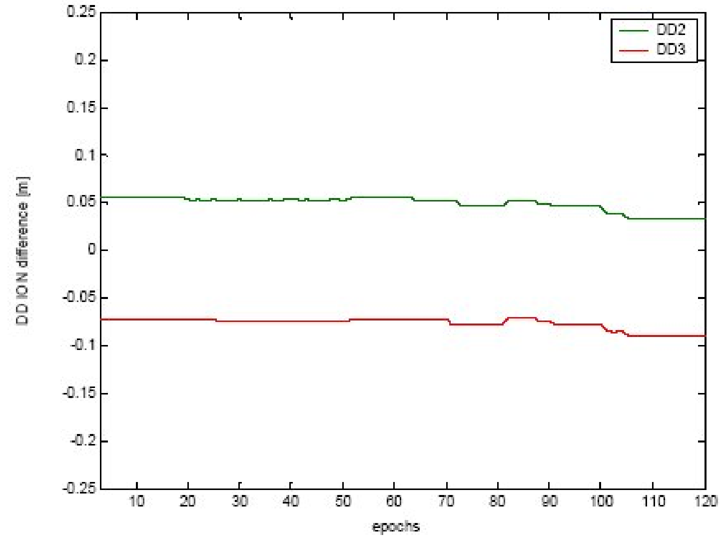


Fig. 23 NGSa DD iono differences from the "truth" (day-time)

This model

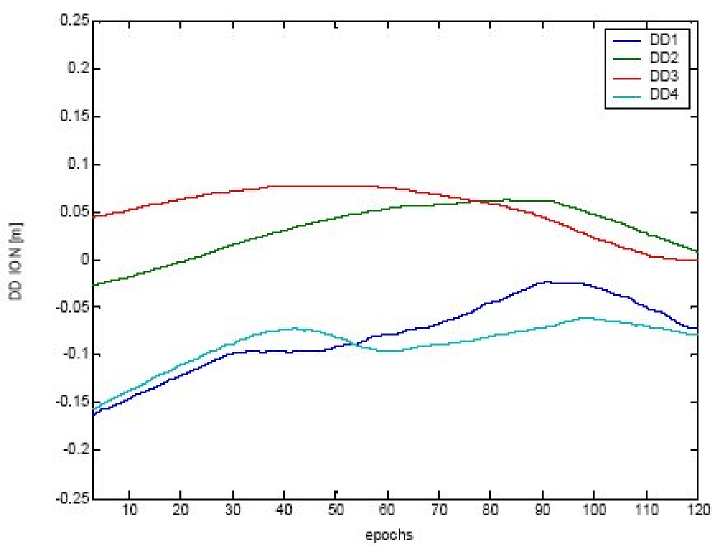


Fig. 24 NGSb DD iono (day-time)

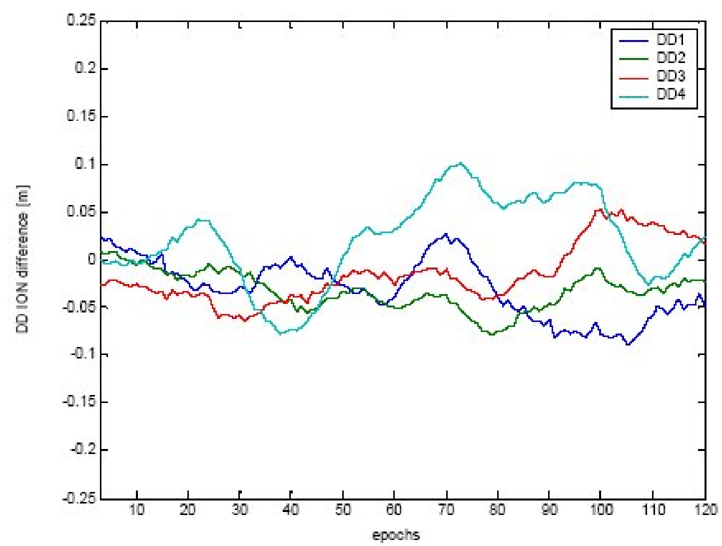


Fig. 25 NGSb DD iono differences from the "truth" (day-time)

NOAA's "Magic" model

Initial Tests(Results)

- In double-differenced mode, this method yields ~0.3 TECU agreement with independent estimates of the ionosphere
- Caveats:
  - One outlying bias can skew results of many tracks
  - Cycle slip detection/correction may be too strict
  - This method behaved worse in A.R. than MAGIC

Absolute TECS Sensitivity Analysis

- While mathematically consistent, this method is sensitive to choices:
  - What is a crossover?
    - "Sufficiently close" definition
  - How are the mapping functions applied?
    - Which one is used and where is it applied?

Absolute TECS Sensitivity Analysis (Crossover definition)

- Sensitivity to definitions of "sufficiently close"
  - Tested 5 different definitions for day 298 of 2004

0.20° x 0.20° x 300 s	14,657 tracks solvable
0.15° x 0.15° x 150 s	13,941 tracks solvable
0.10° x 0.10° x 60 s	12,698 tracks solvable
0.05° x 0.05° x 30 s	9,129 tracks solvable
0.01° x 0.01° x 10 s	0 tracks solvable

Sensitivity of TECS values: ±1.98 TECU

Smith, D.A., Absolute Ionosphere, ION NTM 2005

Absolute TECS Sensitivity Analysis (Mapping Function Location)

- Sensitivity to location of mapping function
  - Tested 5 different locations for day 298 of 2004

250 km	12,041 tracks solvable
300 km	12,698 tracks solvable
350 km	12,680 tracks solvable
400 km	12,905 tracks solvable
450 km	13,044 tracks solvable

Sensitivity of TECS values: ±1.26 TECU

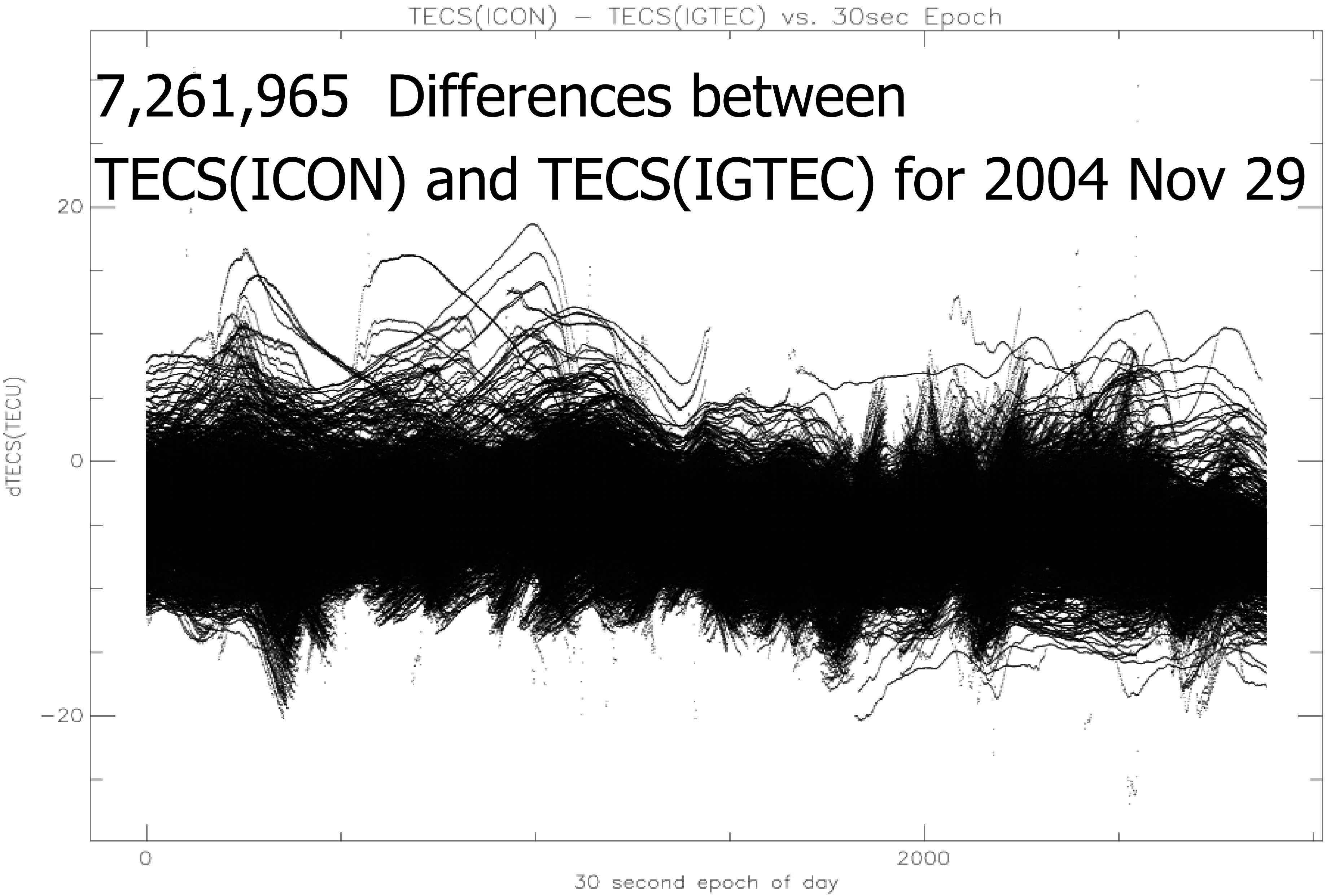
ICON: Prototype Model

- After internal testing, a prototype production was established at NGS (Nov 1, 2004) to encourage independent validations
- Daily solutions (~15k crossovers, ~30k tracks)
  - Sparse matrix solution = 2 minutes
  - Reading data/uncompressing/gridding/making pretty pictures = 3 hours
- "ICON" (Ionosphere over CONus)
- www.ngs.noaa.gov/ionosphere

Absolute Comparison with IGTEC

- ~ 1 month of data (Dec 2004)
- ICON – IGTEC
- Daily bias between models ~ -3 to -4 TECU
- Daily σ around bias ~ ±2 to 3 TECU
- Possible causes:
  - Resolution differences
  - Model errors

7,261,965 Differences between TECS(ICON) and TECS(IGTEC) for 2004 Nov 29

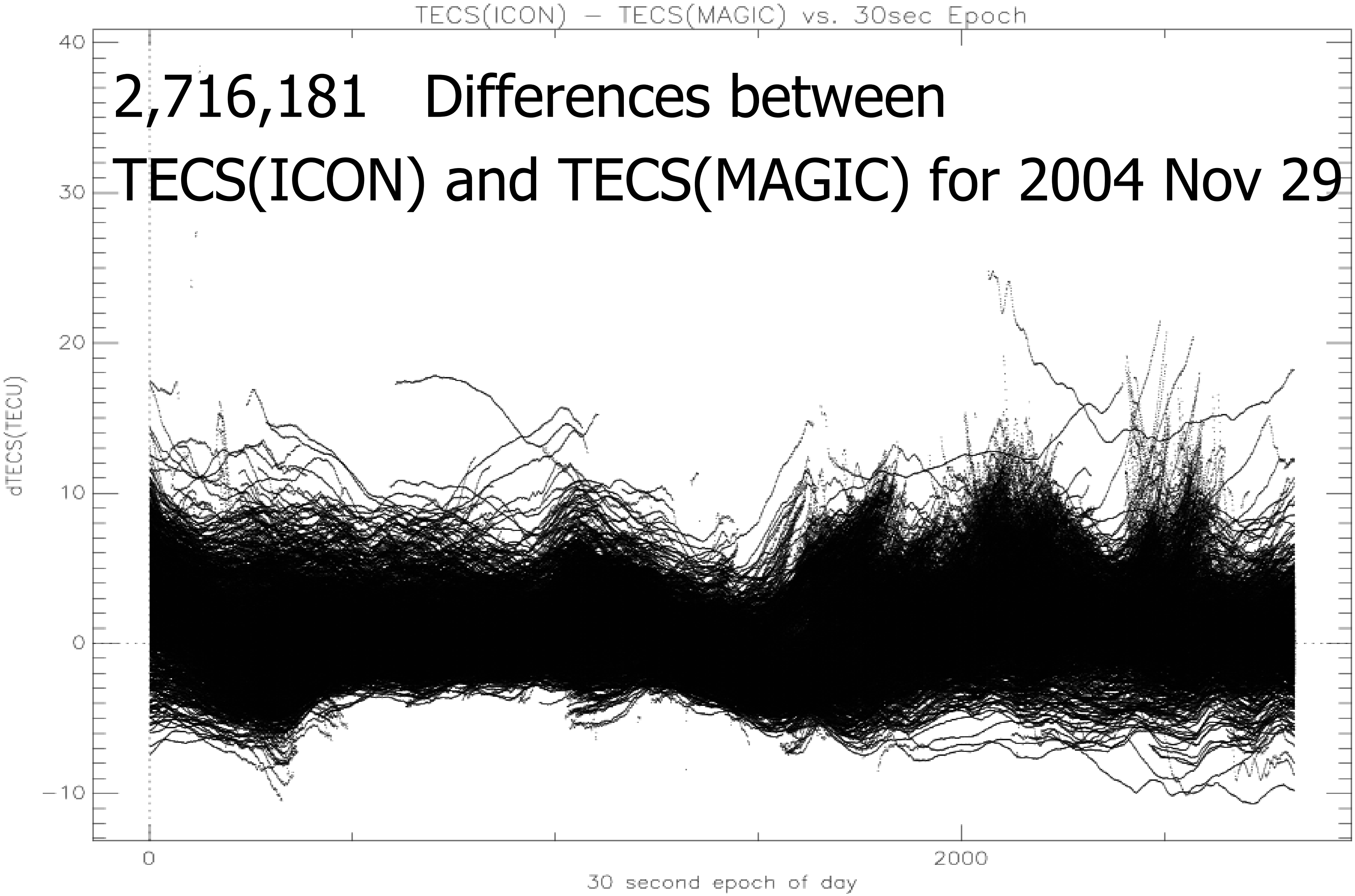


Smith, D.A., Absolute Ionosphere, ION NTM 2005

Absolute Comparison with MAGIC(NGS)

- Data generally unavailable currently
- ICON – MAGIC
- Daily bias between models ~ +1 TECU
- Daily σ around bias ~ ±2 to 3 TECU
- Possible causes:
  - Resolution differences
  - Model errors

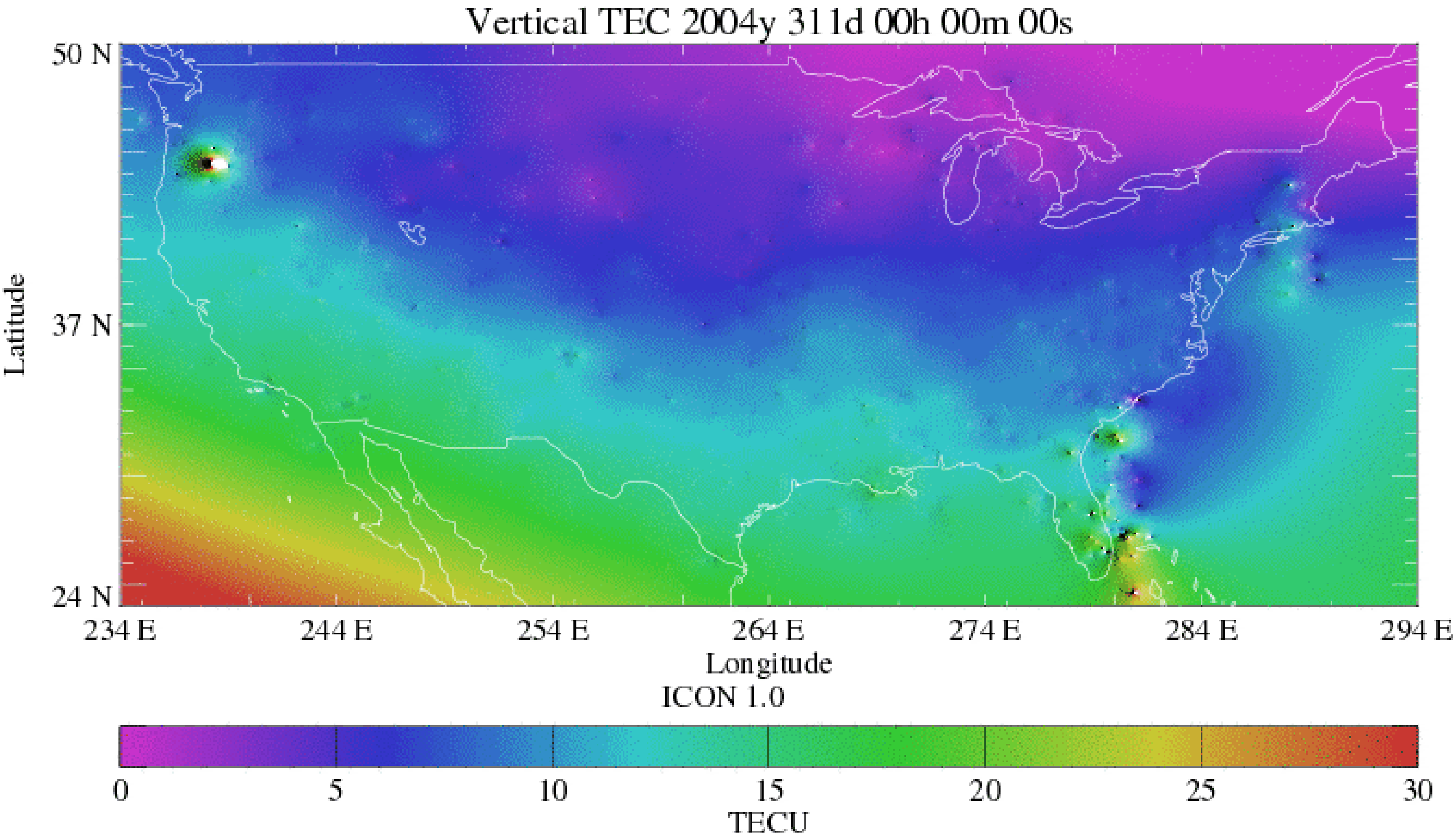
2,716,181 Differences between TECS(ICON) and TECS(MAGIC) for 2004 Nov 29



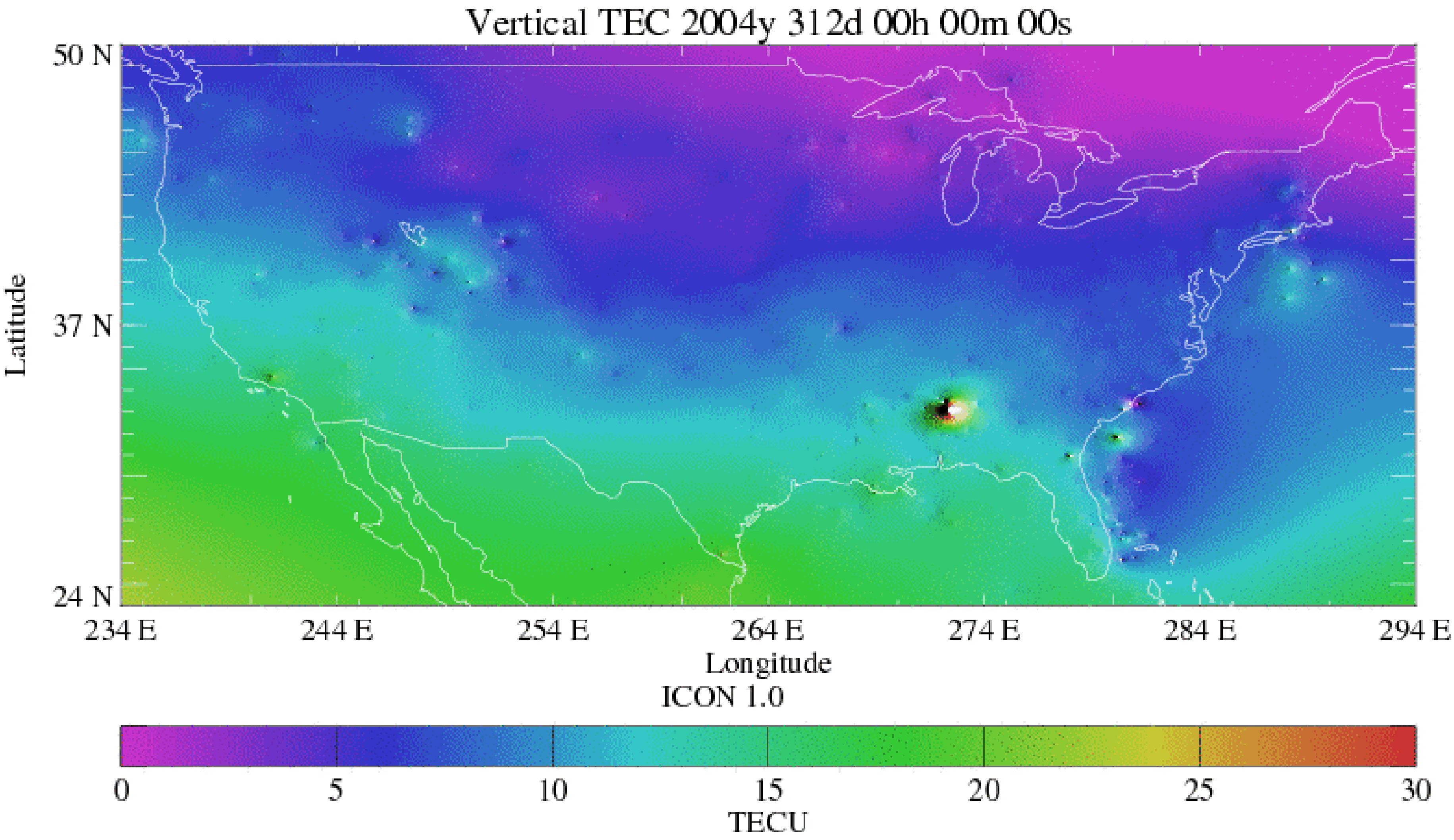
Smith, D.A., Absolute Ionosphere, ION NTM 2005

Grids

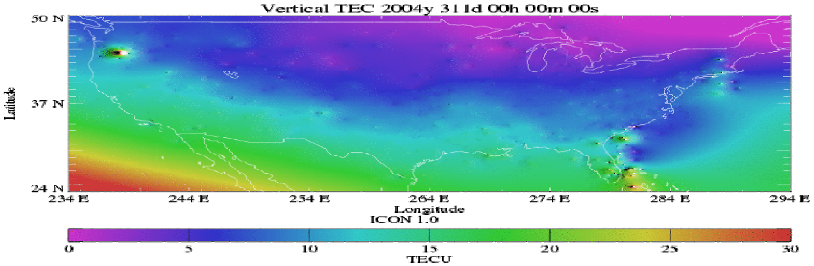
- As a secondary product, ICON produces radial TEC (TECR) on a grid in IONEX and GIF formats
- Mostly for analysis: Grid to slant delays introduce another error source
  - Useful for seeing outliers, storms and small ionosphere features



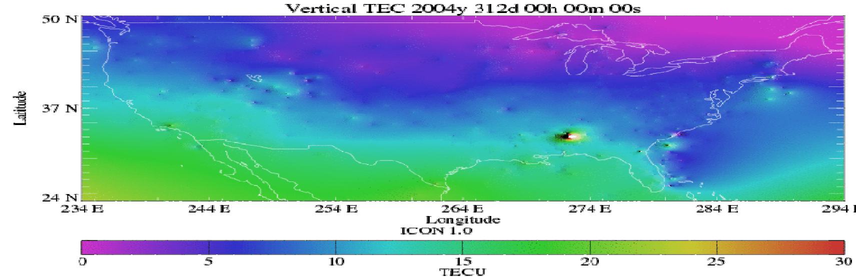
Smith, D.A., Absolute Ionosphere, ION NTM 2005

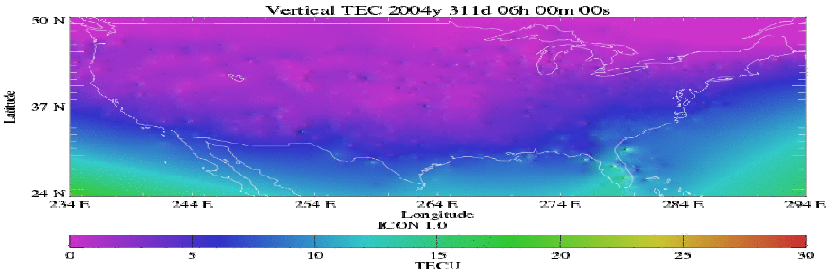


Smith, D.A., Absolute Ionosphere, ION NTM 2005

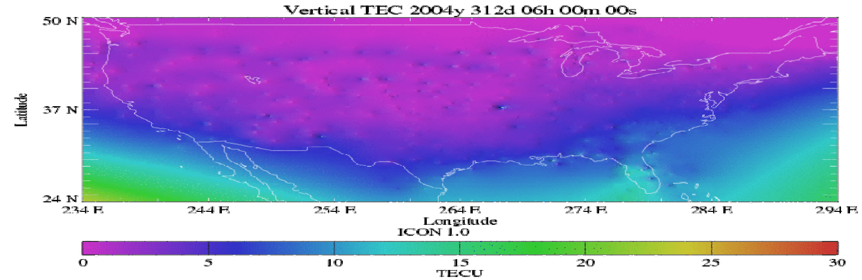


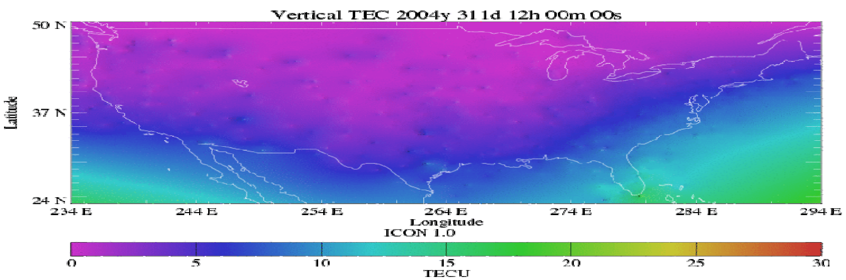
7 pm



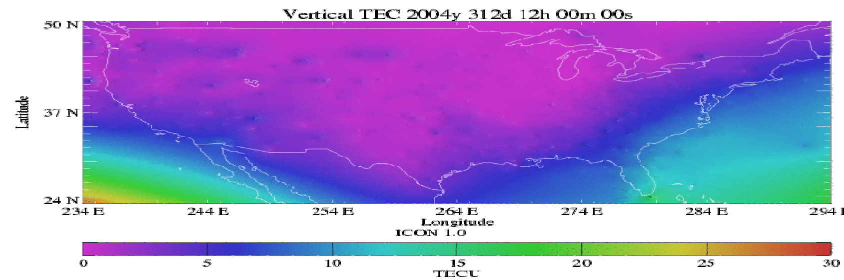


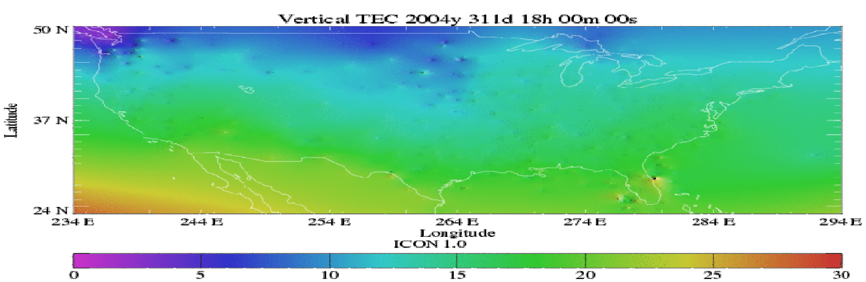
1 am



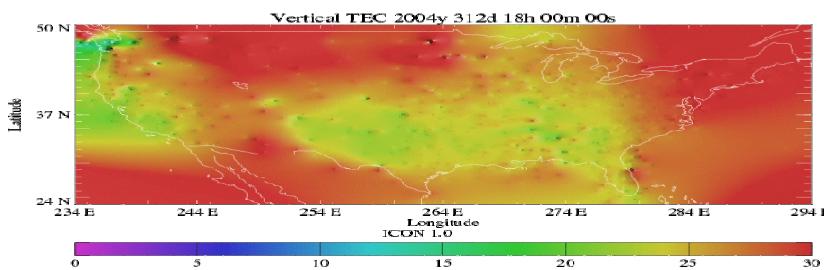


7 am





1 pm



Nov 6-7, 2004

Nov 7-8, 2004

Summary and Conclusions

- Absolute TECS is mathematically determinable from ambiguous carrier phase data under 4 assumptions:
  - Network of Ground Stations
  - Dual Frequency
  - ${}^{i,j}\Delta TECS = k(\text{"40.3"}, f_1, f_2) \times ({}^{i,j}\Delta L1 - {}^{i,j}\Delta L2)$
  - $TECS_1 = f[TECS_2]$ when "sufficiently close"

Summary and Conclusions

- Relying on simple cosine mapping functions, a model for the ionosphere can be computed as an entire network
  - to ~4 TECU RMS (absolute)
  - to ~0.3 TECU RMS (5 cm on L1) agreement with Double Difference estimates, subject to cycle-slip fixing and outlier biasing
- Interpolation can yield ± 5 cm (L1) biases from nearby tracks

Future Work

- Removal of outliers and general improvement in regional correlation
- Usefulness of method in A.R. must be improved
- Move from daily solutions to progressive epoch by epoch solutions

Contact Information

- Dr. Dru A. Smith
- 301-713-3202 x 149
- Fax: 301-713-4172
- Dru.Smith@noaa.gov
- www.ngs.noaa.gov/ionosphere

Questions?

Extra Slides

Conclusions

- Average a-posteriori σ_{bias} of ±1.1 TECU reasonable, but larger than hoped for
- Sub-TECU crossover residuals show tight "locking" or consistency of tracknet
- Overall noise in grids needs improvement
- General conclusion:
  - "Promising" but not by any means "done"
  - Initial analysis indicates **near-horizon crossovers are the primary error source** (TECS=TECR/cos z' unreliable)

Summary and Conclusions (cont)

- Further sensitivity studies:
  - Removing near-horizon crossovers (nearly done)
  - Shell height
  - CORS thinning
- Independent tests forthcoming:
  - Against other ionosphere models
  - In ambiguity resolving software
- Production:
  - Daily solutions expected to begin in Fall 2004

```
     2.10           OBSERVATION DATA    G (GPS)             RINEX VERSION / TYPE
teqc  2000Jul20     CORS-ADM Account    20040108 05:18:03UTCPGM / RUN BY / DATE
dqua                                                        MARKER NAME
xxx                                                         MARKER NUMBER
CORS/NGS/NOAA       OAR/FSL                                 OBSERVER / AGENCY
3735A20424          TRIMBLE   4000SSI   7.19                REC # / TYPE / VERS
3328A68603          TRM14532.00                             ANT # / TYPE
  -395445.7142 -5271710.3365  3556709.0313                  APPROX POSITION XYZ
        0.0000        0.0000        0.0000                  ANTENNA: DELTA H/E/N
     1     1                                                WAVELENGTH FACT L1/2
     8    C1    L1    L2    P1    P2    D1    D2    I1      # / TYPES OF OBSERV
    30.0000                                                 INTERVAL
Forced Modulo Decimation to 30 seconds                      COMMENT
  2004     1     7     0     0    0.0000000     GPS         TIME OF FIRST OBS
This is an IINEX file, not strictly RINEX.                  COMMENT
The difference is that an I1 variable, representing         COMMENT
the computed Ionosphere delay on L1, in cycles of L1,       COMMENT
has been introduced.  This value should generally           COMMENT
always be positive.                                         COMMENT
I1 was computed by Dru Smith, NOAA/NGS with the following   COMMENT
parameters (see D. Smith for details):                      COMMENT
Year = 2004 Day of Year = 007                               COMMENT
Shell Height (km) = 0300                                    COMMENT
Track Cleaning Criteria Index      : 001                    COMMENT
Crossover Spacing Criteria Index   : 001                    COMMENT
Tracknet Formation Criteria Index  : 009                    COMMENT
LSA Weighting Scheme Index         : 004                    COMMENT
Flag for post-LSA interpolation    : 000                    COMMENT
                                                            END OF HEADER
 04  1  7  0  0  0.0000000  0  9G 5G13G24G 6G17G10G 4G30G29  0  0  0 0.000000000
  21696356.31340  10363943.65748   8142514.00747         0.00000  21696361.84440
     -4100.07840         0.00000      15.799
  25107871.38340   3708841.58542   2259556.02945         0.00000  25107875.26240
     -3307.67240         0.00000  -99999.000
  21512358.68840  18328193.84048  14366529.97347         0.00000  21512363.50440
     -3686.29740         0.00000  -99999.000
  23862591.76640  -1725935.55946  -1150692.59045         0.00000  23862597.87140
       706.65640         0.00000      24.278
  21126131.32040   3655026.98748   2892249.13247         0.00000  21126136.67640
     -1809.82840         0.00000       9.850
  20301784.44540  -5022796.20048  -3846505.25347         0.00000  20301789.52040
     -1548.39140         0.00000  -99999.000
  24034690.15640  41863677.08547  32692773.82046         0.00000  24034694.44540
     -4906.23440         0.00000  -99999.000
  21762273.85940   -282123.29347   -147989.82246         0.00000  21762278.34840
     -1713.00040         0.00000      16.644
  24560456.50840    -45620.85743         0.00000         0.00000         0.00000
       704.09440         0.00000  -99999.000
```

CORS Network

- ***Currently 400+ 24/7 receivers***
  - Dual frequency, carrier-phase
  - Multi-agency
  - Administered by NGS
  - All 50 states, Central America, others
  - *Ideally suited to serve as an ionosphere monitoring network for geodetic applications in the USA*

Equations

$${}^{i}R_k = b_k + {}^{i}r + c({}^{i}\delta t) + {}^{i}T + {}^{i}I_k (+{}^{i}m_k) = \lambda_k \, {}^{i}\Phi_k^{\mathrm{RINEX}} \qquad \text{(biased range, m, epoch "i", freq "k")}$$

$$I_k = -\frac{40.3}{f_k^2} TECS \qquad \text{(m)}$$

$$\therefore \quad \lambda_1 \, {}^{i}\Phi_1^{\mathrm{RINEX}} - \lambda_2 \, {}^{i}\Phi_2^{\mathrm{RINEX}} = (b_1 - b_2) + ({}^{i}I_1 - {}^{i}I_2)$$

$$\therefore \quad {}^{i}TECS = \left(\frac{1}{40.3}\right)\left(\frac{1}{f_1^2} - \frac{1}{f_2^2}\right)^{-1} \left[\lambda_1 \, {}^{i}\Phi_1^{\mathrm{RINEX}} - \lambda_2 \, {}^{i}\Phi_2^{\mathrm{RINEX}}\right]$$

$$- \left(\frac{1}{40.3}\right)\left(\frac{1}{f_1^2} - \frac{1}{f_2^2}\right)^{-1} (b_1 - b_2)$$

$$\therefore \quad {}^{i,j}\Delta TECS = {}^{j}TECS - {}^{i}TECS$$

$$= \left(\frac{1}{40.3}\right)\left(\frac{1}{f_1^2} - \frac{1}{f_2^2}\right)^{-1} \left(\lambda_1 \, {}^{i,j}\Delta\Phi_1^{\mathrm{RINEX}} - \lambda_2 \, {}^{i,j}\Delta\Phi_2^{\mathrm{RINEX}}\right)$$

Closed Polygons

- Altimetry or Leveling (ΔH & H-equality):
  - # conditions = # vertices – 1
- Ionosphere (ΔTECS & TECR-equality)
  - # conditions = # vertices
- Any time that a closed polygon is formed we have:
  - # Conditions = # Unknowns

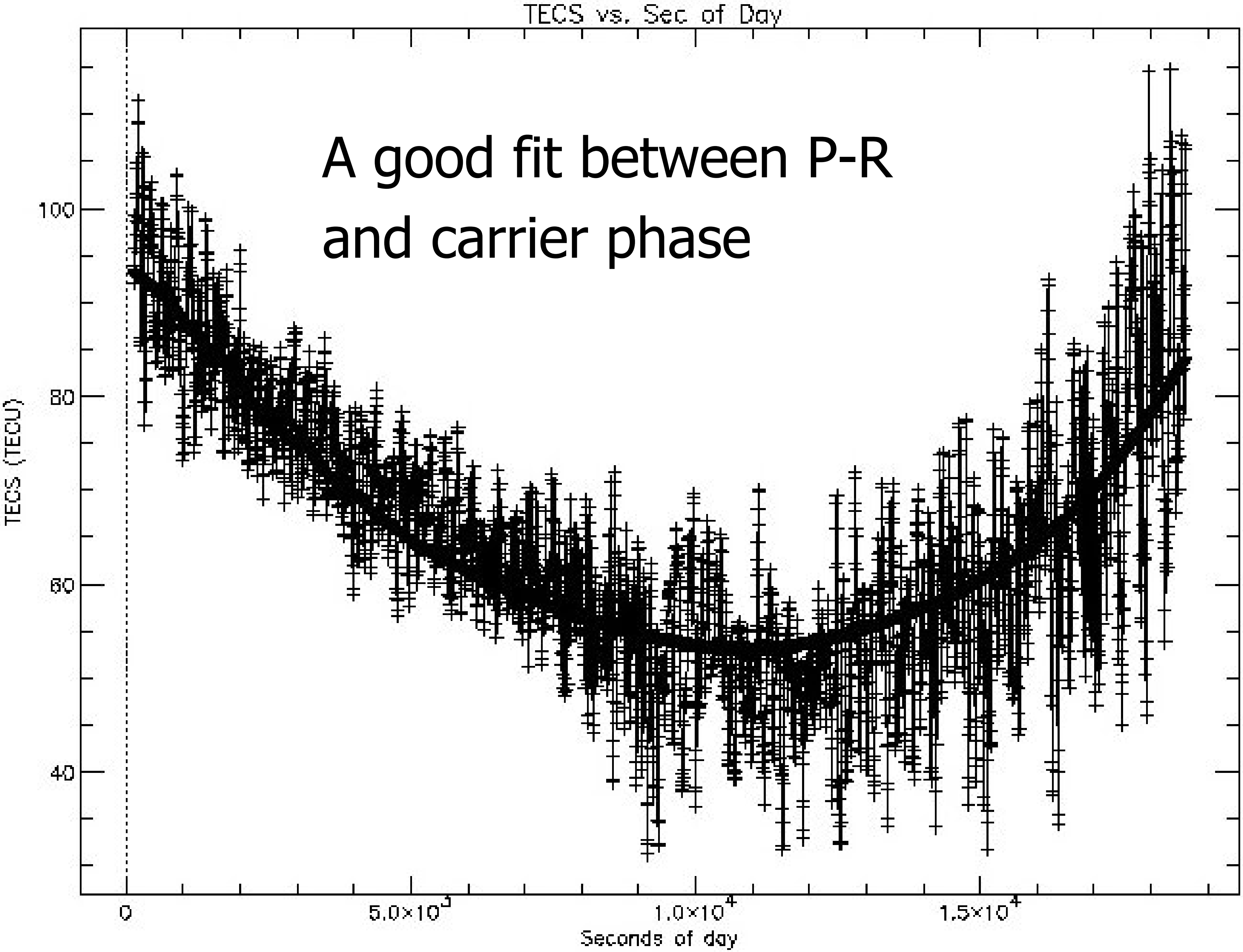
Polygon Crossover Equations

$$\begin{bmatrix} {}^{A}_{1}\Delta TECS \cos {}^{A}_{1}z' - {}^{A}_{3}\Delta TECS \cos {}^{A}_{3}z' \\ {}^{B}_{1}\Delta TECS \cos {}^{B}_{1}z' - {}^{B}_{2}\Delta TECS \cos {}^{B}_{2}z' \\ {}^{C}_{2}\Delta TECS \cos {}^{C}_{2}z' - {}^{C}_{3}\Delta TECS \cos {}^{C}_{3}z' \end{bmatrix} = \begin{bmatrix} -\cos {}^{A}_{1}z' & 0 & +\cos {}^{A}_{3}z' \\ -\cos {}^{B}_{1}z' & +\cos {}^{B}_{2}z' & 0 \\ 0 & -\cos {}^{C}_{2}z' & +\cos {}^{C}_{3}z' \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

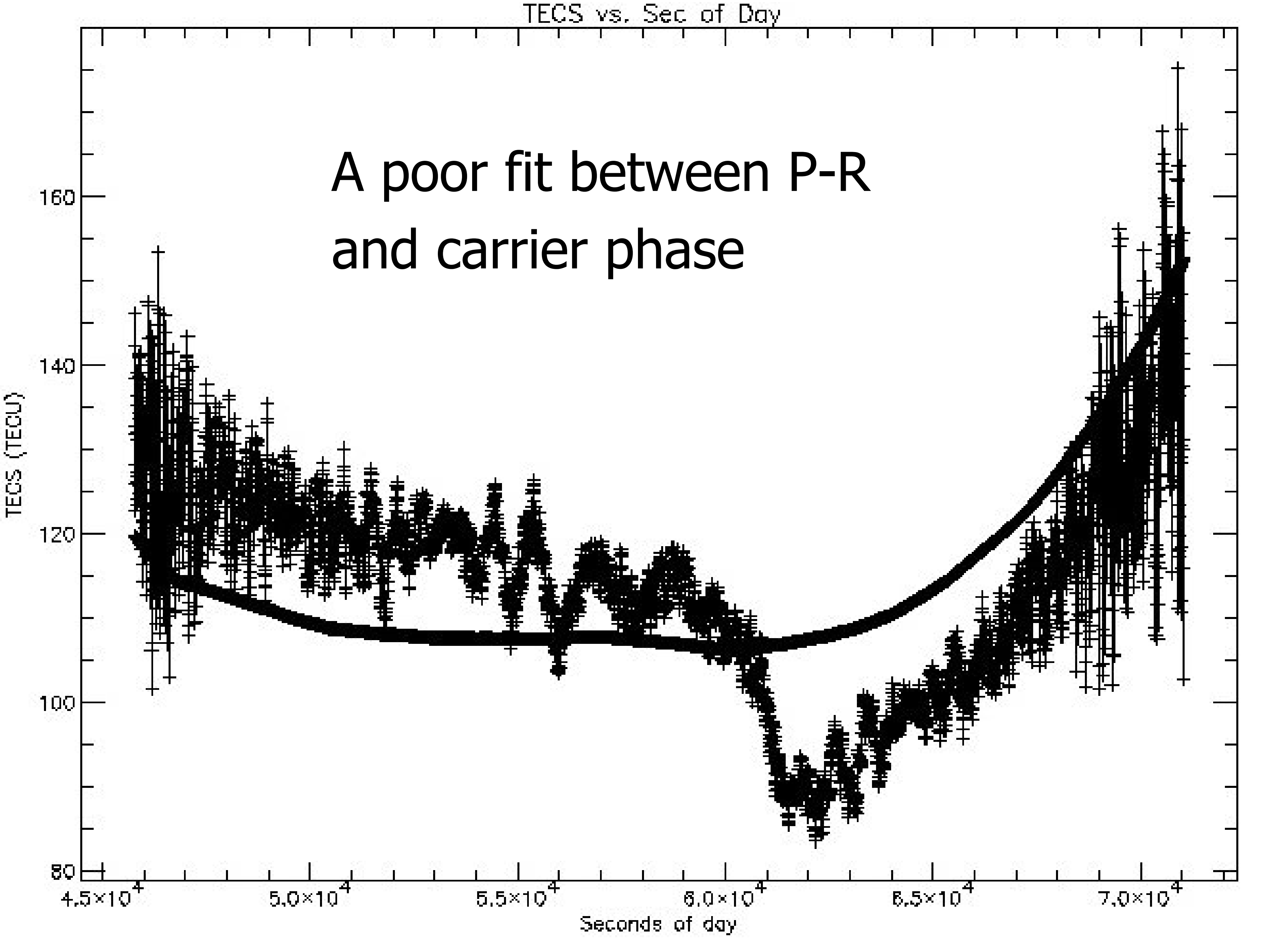
Smith, D.A., Absolute Ionosphere, ION NTM 2005

Polygon Crossover Equations

- The existence of the cos z’ values on the RHS allows for matrix inversion
  - (as opposed to +1,0 and -1 for altimetry)
- Solvability
- Can we have redundancy?
  - YES



TECS vs. Sec of Day
A good fit between P-R
and carrier phase
TECS (TECU)
100
80
60
40
0
5.0×10³
1.0×10⁴
1.5×10⁴
Seconds of day


TECS vs. Sec of Day
A poor fit between P-R
and carrier phase
TECS (TECU)
80
100
120
140
160
4.5×10⁴
5.0×10⁴
5.5×10⁴
6.0×10⁴
6.5×10⁴
7.0×10⁴
Seconds of day

Initial Tests

- Parameters:
  - Crossover height = 300 km
  - Crossover definition: 0.1° x 0.1° x 1 min
  - Cut-off angle: 10° (for data and crossovers)

Initial Tests (all contain the 4 base tracks)

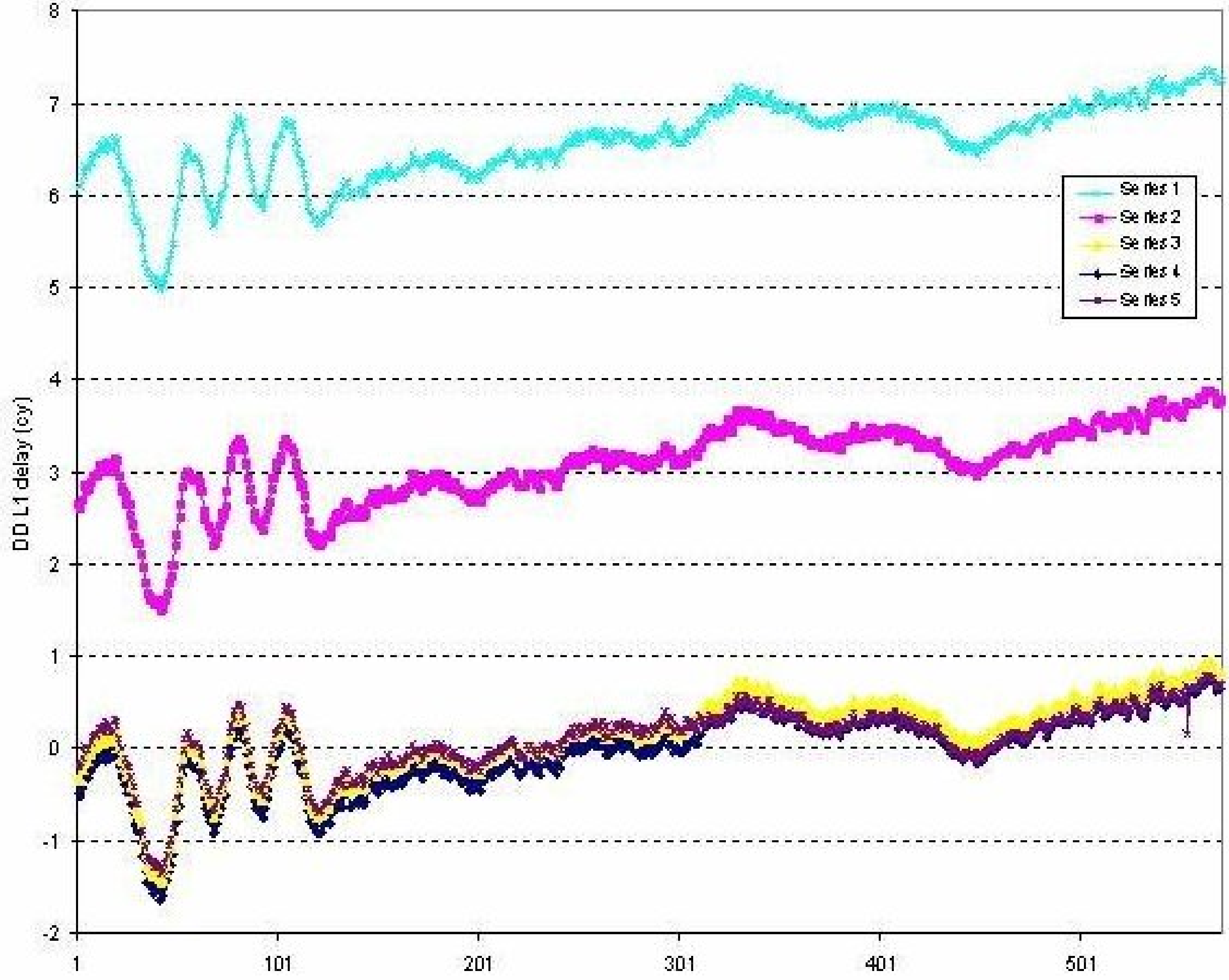
- Solution 1 (smallest tracknet possible containing the 4 base tracks)
  - 8 tracks, No polygons, PR-fit 6 of 8 tracks
- Solution 2
  - 10 tracks, 2 polygons, PR-fit 7 of 10 tracks
- Solution 3
  - 10 tracks, 2 polygons, no PR-fitting
- Solution 4
  - 10 tracks, 2 polygons, PR-fit 1 of 10 tracks

Formal σ_{bias} estimates for first tracknet tests (in TECU)

Track #	Soln 1 (PR fit to 6 of 8; no polygons)	Soln 2 (PR fit to 7 of 10; 2 polygons)	Soln 3 (No PR fit; 2 polygons)	Soln 4 (PR fit to 1 of 10; 2 polygons)
4300 (base)	3.5	2.9	0.1	1.2
4303 (base)	8.8	4.7	0.2	2.1
9484 (base)	9.3	4.6	0.2	2.0
9487 (base)	9.4	3.1	0.1	1.3
2253	13.6	5.9	0.3	2.5
10146	9.7	3.3	0.1	1.4
11416	6.5	4.9	0.2	2.0
12565	6.1	3.9	0.2	1.6
2224	-	4.3	0.2	1.7
11580	-	3.0	0.1	1.2

Initial Tests (cont)

- Individual ionosphere delays for each SV/CORS combo were estimated:
  - I_{4300}(SV1/GODE), I_{4303}(SV2/GODE), I_{9484}(SV1/RED1), I_{9487}(SV2/RED1) all estimated individually (as well as for all other tracks in the tracknet)
- Double Difference delays were then computed:
  - $I_{DD}=(I_{4300}-I_{9484})-(I_{4303}-I_{9487})$ computed and compared to independent estimates from NGS ambiguity resolving software



8
7
6
5
4
3
2
1
0
-1
-2
DD L1 delay (cy)
1
101
201
301
401
501
Series 1
Series 2
Series 3
Series 4
Series 5

First tracknet tests

- Pseudo-range fitting tends to bias the tracknet
- Better fit to Double Difference estimated ionosphere by using just polygons and no P-R fitting

Full day solution (cont)

- Interpolation from tracks to grids and/or other tracks:
  - Track-to-grid-to-Track
    - Useful for grid-distributed Ionosphere model and animations
    - 0.00 ± 0.38 TECU (±6 cm on L1)
  - Track-to-Track
    - Useful for RINEX-distributed Ionosphere model
    - 0.00 ± 0.25 TECU (±5 cm on L1)

- Full day solution was gridded and animated

Example 2. 17:00-18:00 UT (day-time)

The "truth" DD ionospheric delays are presented in figure 14 with the corresponding satellite elevation map in figure 15. Figures 16–25 represents the derived DD ionosphere form each method and the difference from the "truth" (in pairs). The mean and standard deviation of the ionospheric residuals from the "truth" are shown in Table 2.

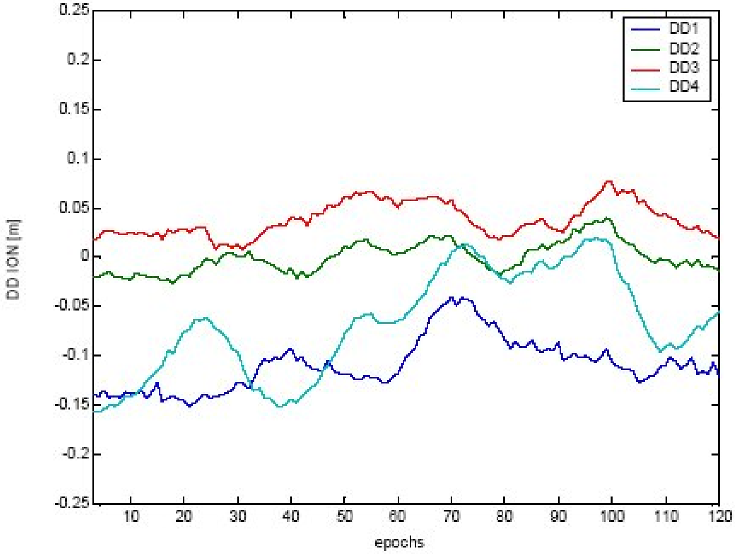


Fig. 14 "Truth" DD iono (day-time)

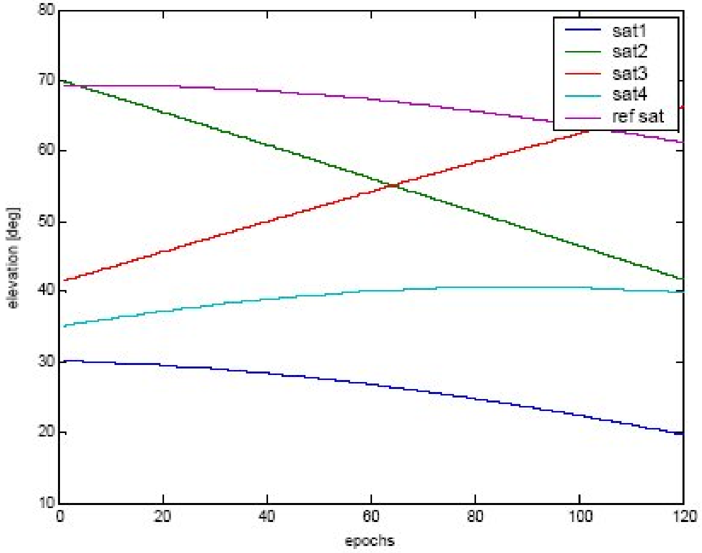


Fig. 15 Satellite elevations

"Truth" (Iono after ambiguity fixing)

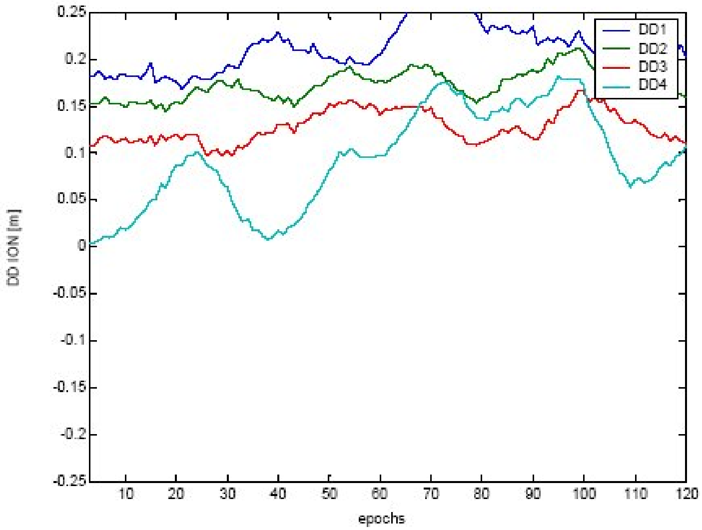


Fig. 16 P4 DD iono (day-time)

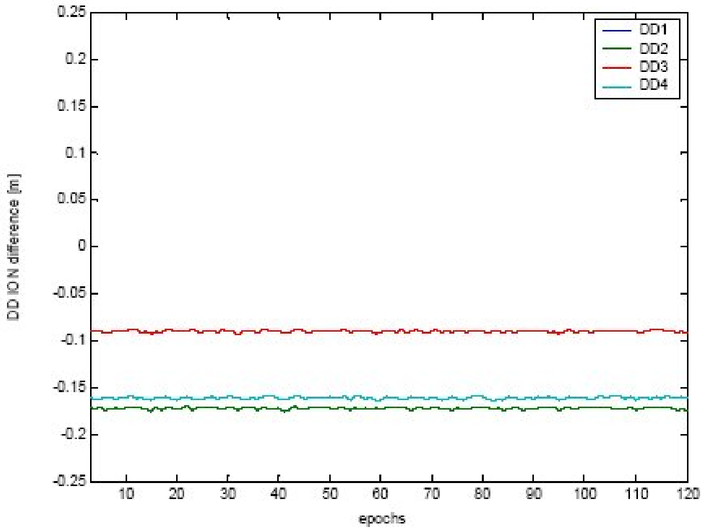


Fig. 17 P4 DD iono differences from the "truth" (day-time)

Smoothed Pseudorange Estimates

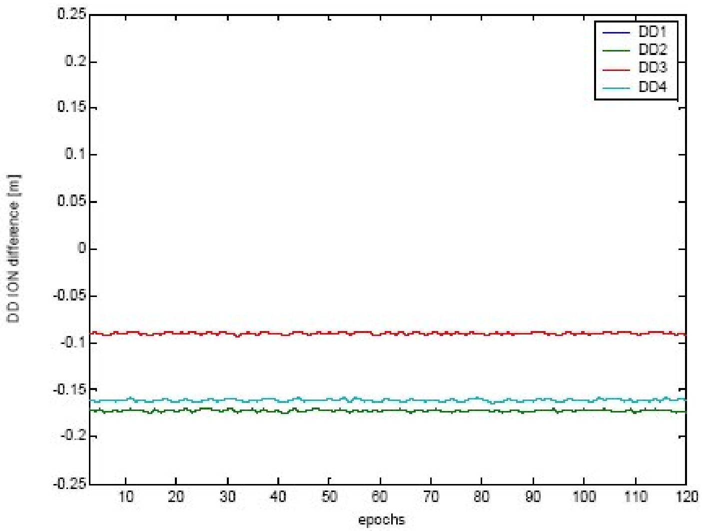


Fig. 18 GIM DD iono (day-time)

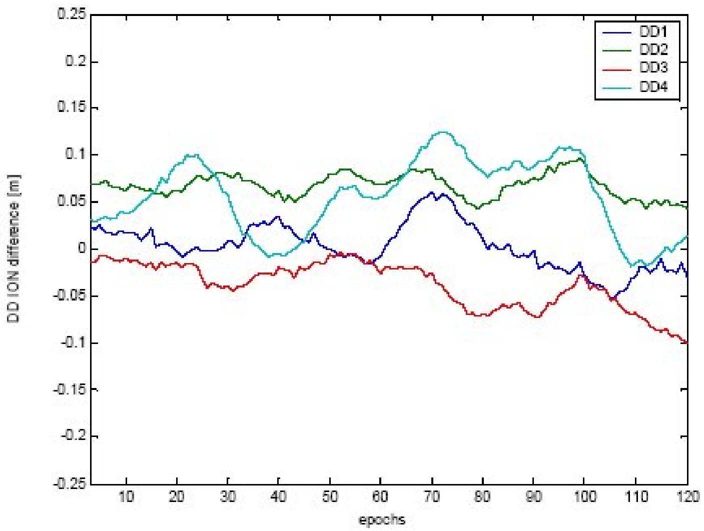
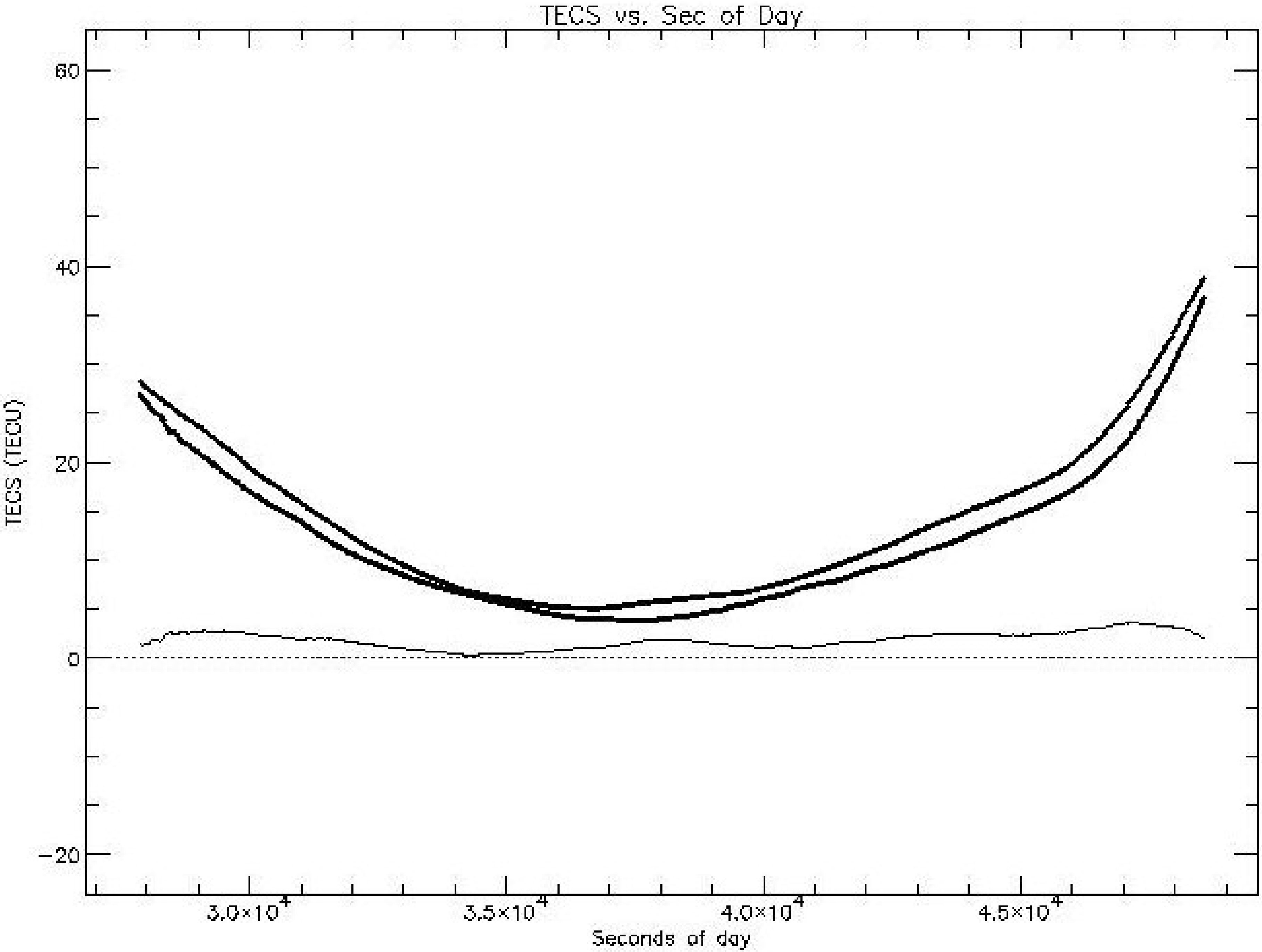
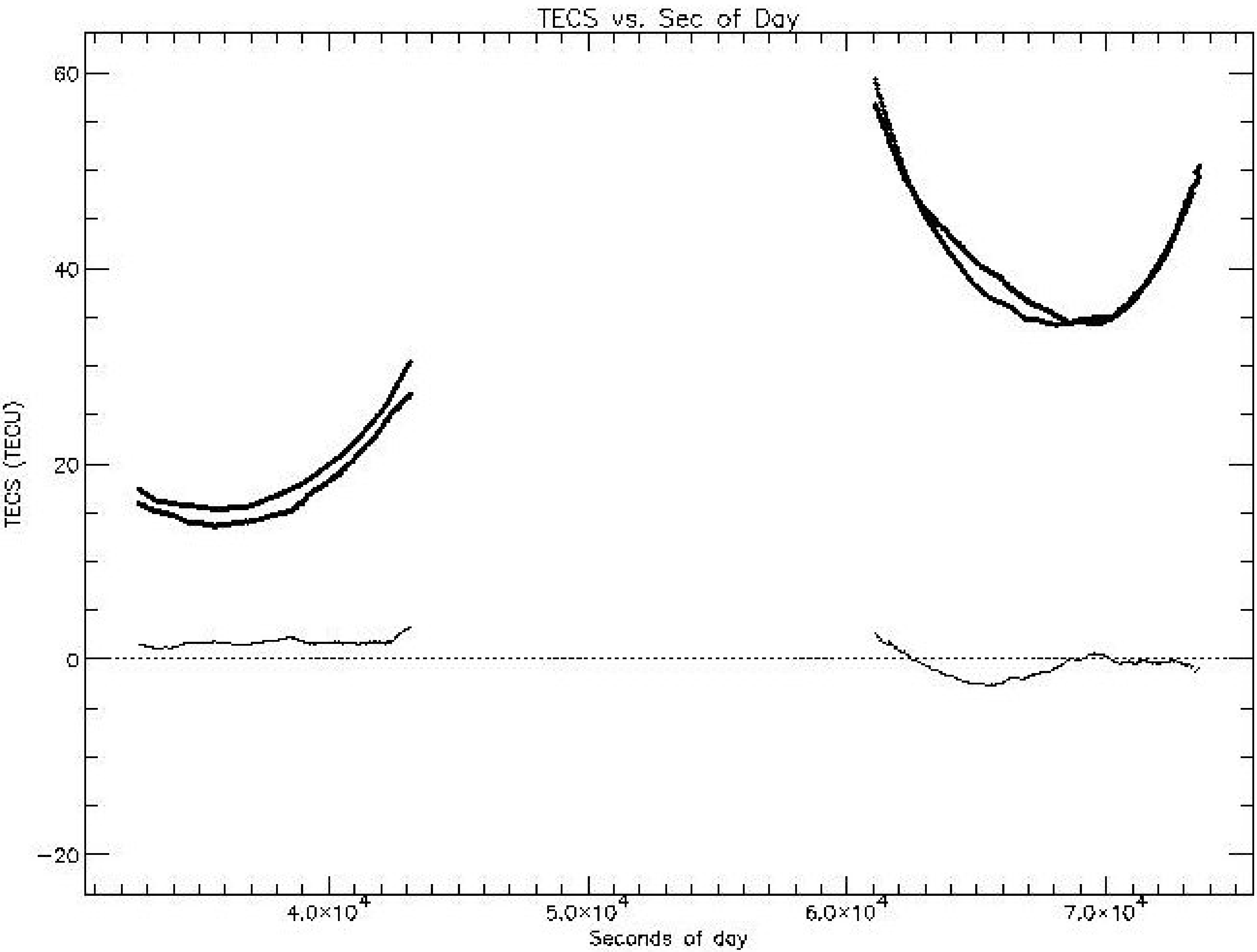


Fig. 19 GIM DD iono differences from the "truth" (day-time)

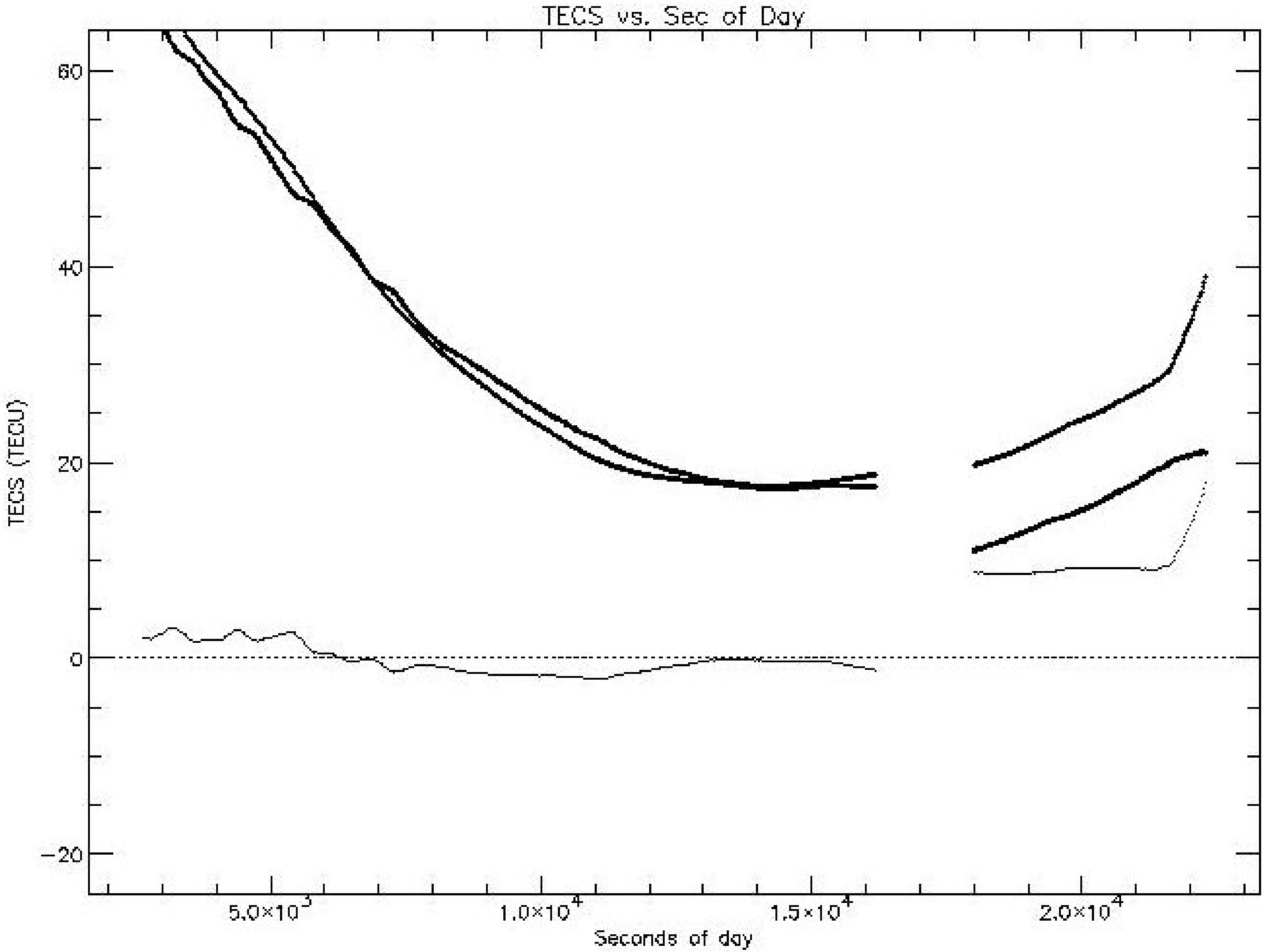
OSU's MPGPS method



TECS vs. Sec of Day
60
40
20
0
−20
TECS (TECU)
3.0×10⁴
3.5×10⁴
4.0×10⁴
4.5×10⁴
Seconds of day



TECS vs. Sec of Day
60
40
20
0
-20
TECS (TECU)
4.0×10^4
5.0×10^4
6.0×10^4
7.0×10^4
Seconds of day



TECS vs. Sec of Day
TECS (TECU)
60
40
20
0
−20
5.0×10^3
1.0×10^4
1.5×10^4
2.0×10^4
Seconds of day