

Real World

General Relativity

Randall Sundrum Model

CFT AdS

4

5

6



The cosmological constant problem in scalar gravity

From: arXiv:1404.0002

Fornal: Theory Seminar February 2, 2015

D. Fornal: Quantum Gravity

CFT

RS3

Scalar gravity

Solution

The cosmological constant problem in scalar gravity

- Why is the cosmological constant so small?
- Why is the cosmological constant so small?
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CFT/AdS

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Analogy

The cosmological constant problem in scalar gravity

Prateek Agrawal
Fermilab

Fermilab Theory Seminar
February 7, 2013

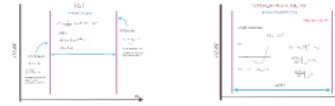
PA, Raman Sundrum
To appear

Real World

General Relativity



Randall Sundrum Model



4

	CFT	AdS
M^2	M^2	$M^2 = \frac{1}{2\pi\alpha'} \sqrt{(\alpha'k^2 + \alpha'^2)}$
Type of theory	Type II string theory or Super Yang Mills	Type IIB string theory or $SO(2,3)$ <i>Maldaena's</i>
$\mathcal{O}(x)$	$\mathcal{O}(x)$	$\psi(x)$
$\Delta_{\mathcal{O}}$	$\Delta_{\mathcal{O}}$	$m_{\mathcal{O}}^2$
$\phi(x)$	$\phi(x)$	$\varphi(x)$
$J(x)$	$J(x)$	$\int_{\mathcal{M}} \sqrt{g} \frac{\delta S}{\delta \varphi}$

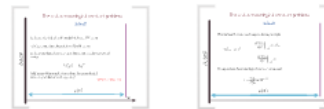
5

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CFT



RS3



Scalar gravity



Solution

Fields in the bulk

- $\psi(x)$ - scalar field
- $\varphi(x)$ - vector field
- $\chi(x)$ - spinor field

Boundary conditions

- Dirichlet
- Neumann
- Mixed

Asymptotic behavior

- AdS
- Schwarzschild

Conservation laws

- Energy
- Momentum
- Angular momentum

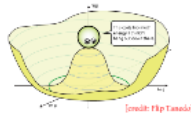
Analogy

General Relativity

The cosmological constant problem

For internal symmetries vacuum solutions preserving a subgroup generically exist

$$SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$$



In presence of cosmological constant, Einstein's equations do not admit a flat space solution

$$g_{\mu\nu} \rightarrow \eta_{\mu\nu}$$

The Poincaré solution requires fine-tuning

For constant fields, the dependence on g fixed $\propto \sqrt{g}$

Weinberg '89

A no-go theorem

Constant fields + matter equations of motion

$$\mathcal{L} = \sqrt{-g}V(\phi, \psi, \dots) = \rho c \sqrt{-g}$$

$$\frac{\partial \mathcal{L}}{\partial g^{\mu\nu}} = 0 \quad \text{no solution for trace of Einstein's equations}$$

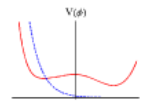
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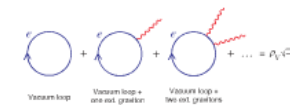
Can dynamics of matter fields choose $V=0$?

$$\frac{\partial V}{\partial \phi} = 0 \quad \text{implies} \quad V = 0$$

Only true for $V \sim e^{\phi}$



The cosmological constant problem is hard

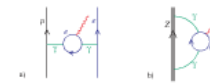


• Compositeness at 10 microns \times

• Long distance modification of GR \times

• Dynamical adjustment \times

...

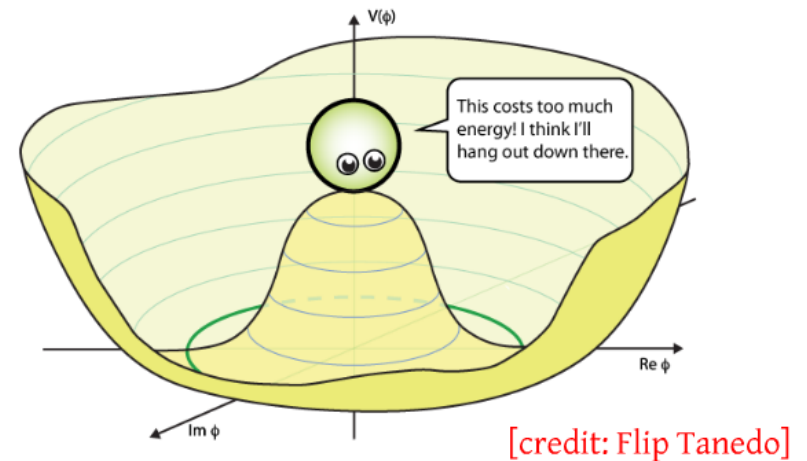


Polchinski '06

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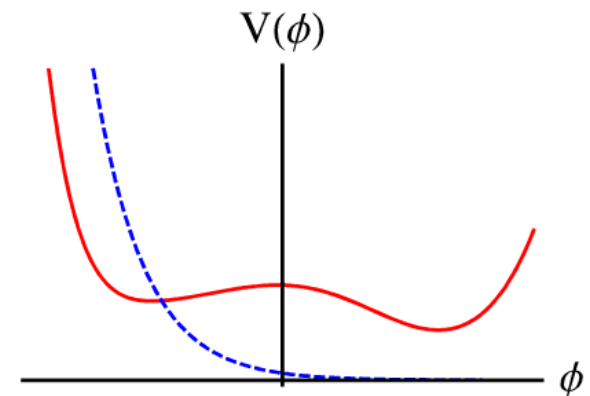
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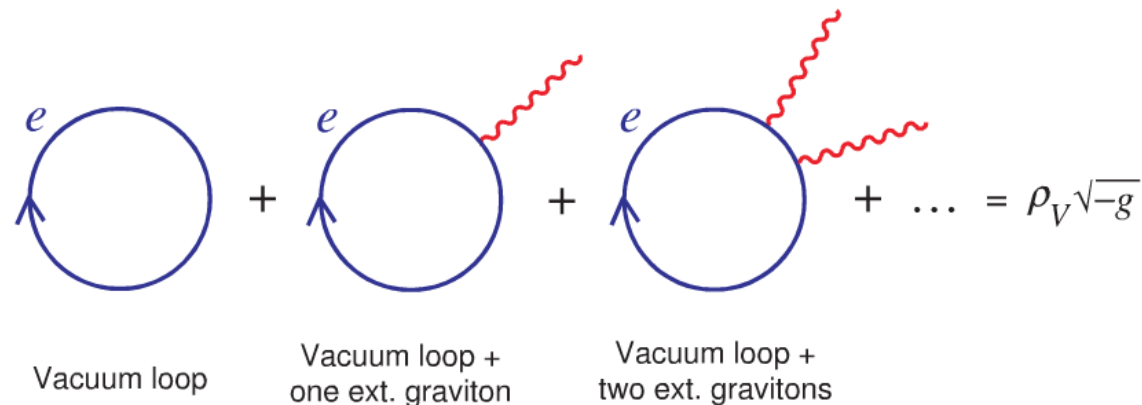
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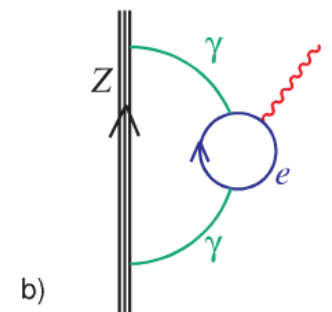
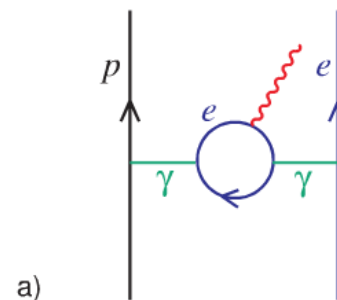
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- Compositeness at 10 microns ✘
- Long distance modification of GR ✘
- Dynamical adjustment ✘
- ...



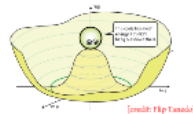
Polchinski '06

General Relativity

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[Smith-Hij-Tanaka]

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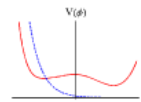
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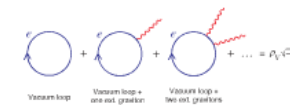
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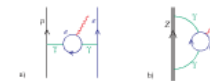
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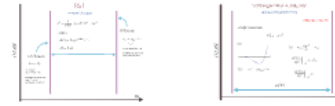
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γ	Type IIB string theory	Type IIB string theory on $S^1 \times S^3$
$\mathcal{O}(x)$	$\mathcal{O}(x)$	$\psi(x, r)$
$\Delta_{\mathcal{O}}$	$\Delta_{\mathcal{O}}$	$m_{\mathcal{O}}^2$
$\phi(x)$	$\phi(x)$	$\varphi(x)$
$J(x)$	$J(x)$	$\int_{\mathcal{M}} \frac{d^4x}{\sqrt{g}} \mathcal{L}(\varphi)$

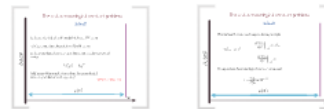
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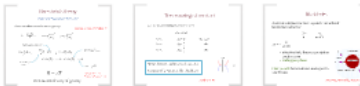
CFT



RS3



Scalar gravity



Solution

Fields in the bulk

- $\psi(x, r)$
- $\varphi(x, r)$
- $\mathcal{L}(\varphi)$

AdS/CFT

$\mathcal{O}(x)$

$\Delta_{\mathcal{O}}$

Boundary conditions

$\psi(x, r=0)$

$\partial_r \psi(x, r=0)$

Correlation functions

$\langle \mathcal{O}(x) \mathcal{O}(y) \rangle$

$\sim \frac{1}{|x-y|^{2\Delta_{\mathcal{O}}}}$

Analogy

CFT

Spontaneously broken conformal field theory

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu = f^2(x^\mu) \eta_{\mu\nu} dx'^{\mu} dx'^{\nu}$$

- A single scalar propagating Goldstone boson
- A dilaton
- $\phi(x) = M e^{-\int dx}$
- Other modes may be thought of as $\partial_\mu \phi$

- Under scale transformations

$$\phi'(D, x) = \frac{1}{3} \phi(x)$$

$$\pi'(D, x) = \pi(x) - M \log(3)$$

Chiral Lagrangian of CFT

Low energy effective theory of the dilaton

$$\mathcal{L}_{\text{dilaton}} = \int d^4x \left[\frac{1}{2} (\partial_\mu \phi)^2 + \lambda \phi^4 + \mathcal{L}_m + \mathcal{O}(\phi^6) + \dots \right]$$

Contains light particles protected by symmetry or by accident

$$\mathcal{L}_m > \tau(B + \eta_\mu \phi^\mu) + f(B + \eta_\mu \phi^\mu) - \frac{1}{2} F_{\mu\nu} F^{\mu\nu}$$

The ϕ^4 term is special to a dilaton goldstone

Quantum corrections

Dilaton or Higgs?

Running couplings break conformal invariance explicitly

$$\frac{1}{\Lambda^2} \frac{d}{d \ln(M)} \Lambda^2 = \frac{1}{\Lambda^2} \frac{d}{d \ln(M)} \Lambda^2 = \frac{1}{\Lambda^2} \frac{d}{d \ln(M)} \Lambda^2 = \frac{1}{\Lambda^2} \frac{d}{d \ln(M)} \Lambda^2$$

Conformal compensator

Higgs	Dilaton	
Λ^4	$\Lambda^4 \frac{\phi^4}{M^4}$	$\Lambda \rightarrow \Lambda \frac{\phi}{M}$
$\Lambda^2 \phi^2$	$\Lambda^2 \phi^2 \frac{\phi^2}{M^2}$	
$\phi^4 \log(\phi/\Lambda)$	$\phi^4 \log(\phi M/\Lambda \phi)$	

The equivalence principle

Equivalence of inertial and gravitational mass

No (tree-level) coupling to gluons

$$\text{Tr}(T_{\mu\nu}) = 0$$

Coupling to protons?

$$m_p \sim \mu \exp \left[\frac{-16\pi^2}{g^2(\mu)} \right]$$

$$\xrightarrow{\text{in a background}} \mu \exp \left[\frac{-16\pi^2}{g^2(\mu)} + \log(\phi/\Lambda f) \right] = \frac{\phi}{\Lambda} m_p$$

(massless quarks)

Spontaneously broken conformal field theory

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Contains light particles protected by symmetry or by accident

$$\mathcal{L}_m \supset \bar{e}(\not{D} + y_e \phi)e + \bar{p}(\not{D} + y_p \phi)p - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

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Quantum corrections



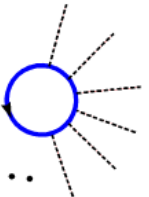
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$$\frac{1}{4g^2(\mu)} F_{\mu\nu} F^{\mu\nu} \rightarrow \frac{1}{4g^2(\mu\phi/M)} F_{\mu\nu} F^{\mu\nu}$$

$$\frac{1}{g^2(\mu\phi/M)} = \frac{1}{g^2(\mu)} - \beta_g \log(\phi/M)$$

Conformal compensator

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 \longrightarrow

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(massless quarks)

Scalar gravity

Nordström's theory

Understand the gravity of the situation

Covariant description for scalar gravity

Isham, Salam, Strathdee '71

$$g_{\mu\nu} = \frac{\phi^2(x)}{M^2} \eta_{\mu\nu} \quad \epsilon \rightarrow \epsilon \left(\frac{\phi}{M}\right)^2 \quad p \rightarrow p \left(\frac{\phi}{M}\right)^2$$

Cosmological constant

$$S = \int d^4x \sqrt{-g} \left[\frac{M^2}{12} R + \Lambda M^4 - \frac{1}{4} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - F_{\mu\nu} F^{\mu\nu} + \beta (e^{\gamma^\alpha} e'_\alpha D_\mu \phi - m_\phi)_\mu + \beta (e^{\gamma^\alpha} e'_\alpha D_\mu \phi - m_\phi)_\mu \right]$$

Opposite sign from GR

Gauge-gravity covariant derivative

$$m_\phi = \eta_\mu M$$

$$R = \kappa T$$

First metric theory of gravity

Nordström '13
Einstein, Fokker '14

The cosmological constant

$\Lambda \phi^4$ is the cosmological constant term

$$\partial^2 \phi = 4\Lambda \phi^3$$

$$\begin{aligned} \Lambda = 0 & \quad \langle \phi \rangle = M \\ \Lambda > 0 & \quad \langle \phi \rangle \propto \frac{1}{\sqrt{\Lambda}} \\ \Lambda < 0 & \quad \langle \phi \rangle \propto \frac{1}{\sqrt{|\Lambda|}} \end{aligned}$$

Poincaré

$$AdS_4$$

$$dS_4$$

No-go theorem applies to scalar gravity

Presence of ϕ^4 potential for the dilaton



Sundrum '03

Black holes

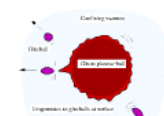
Classical solutions far from a point mass exhibit Newtonian behavior

$$\frac{\phi(r)}{M} = 1 - \frac{m}{4\pi M^2 r}$$

$$\text{At } r \sim \frac{m}{4\pi M^2}$$

- effective field theory description breaks down
- Unhiggsed phase

Plasma-balls have features analogous to black holes



Aharony, Minwalla, Wiseman '07

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$$\Lambda = 0$$

$$\langle\phi\rangle = M$$

Poincaré

$$\Lambda > 0$$

$$\langle\phi\rangle \propto \frac{1}{z}$$

AdS_4

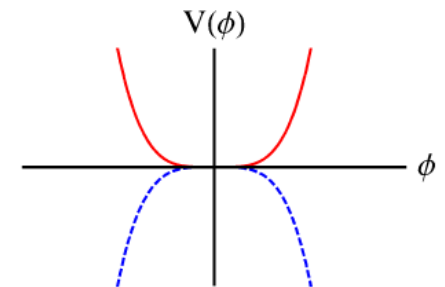
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dS_4

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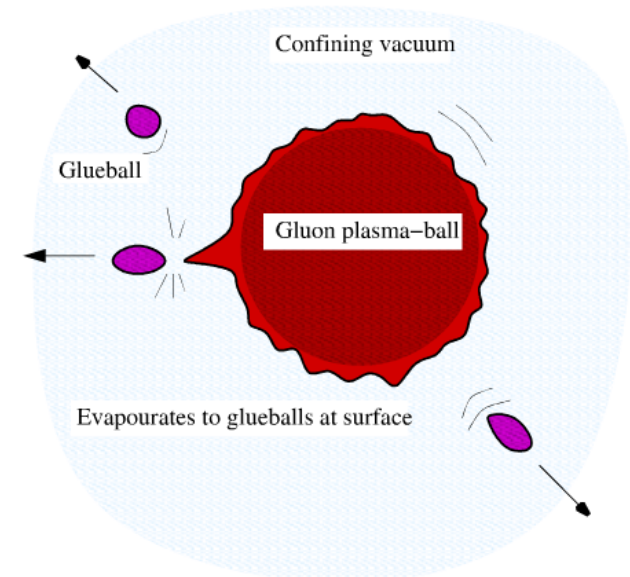
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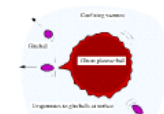
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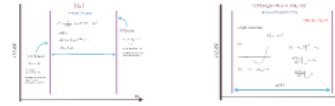
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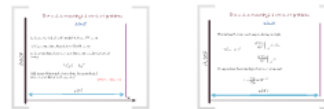
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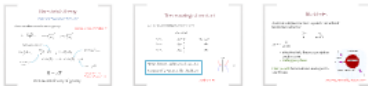
CFT



RS3



Scalar gravity



Solution

Fields in the bulk

- $\psi(x)$ - fermion
- $\phi(x)$ - scalar
- $A_\mu(x)$ - gauge field

Boundary conditions

- $\psi(x)$ - Dirichlet
- $\phi(x)$ - Neumann
- $A_\mu(x)$ - Dirichlet

Correlation functions

- $\langle \mathcal{O}(x) \mathcal{O}(y) \rangle$
- $\langle \mathcal{O}(x) \mathcal{O}(y) \mathcal{O}(z) \rangle$

Analogy

CFT

$Mink_4$

't Hooft limit of
 $\mathcal{N} = 4$ Super Yang-Mills

$\mathcal{O}(x)$

$\Delta_{\mathcal{O}}$

$\phi(x)$

$j(x)$

AdS

$$ds^2 = \frac{1}{k^2 w^2} [\eta_{\mu\nu} dx^\mu dx^\nu - dw^2]$$

Type IIB string theory
on $AdS_5 \times S^5$

Maldacena '98

$\chi(x, w)$

m_χ^2

$\varphi(x)$

$$\lim_{w \rightarrow 0} \frac{\chi(x, w)}{w^\Delta}$$

∂AdS

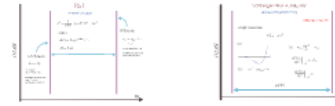
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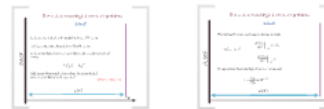
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CFT



RS3



Scalar gravity



Solution

Fields in the bulk

- ψ (fermion)
- φ (scalar)
- A_μ (gauge field)

Boundary conditions

- $\psi = 0$ (Dirichlet)
- $\varphi = 0$ (Dirichlet)
- $A_\mu = 0$ (Dirichlet)

Boundary conditions

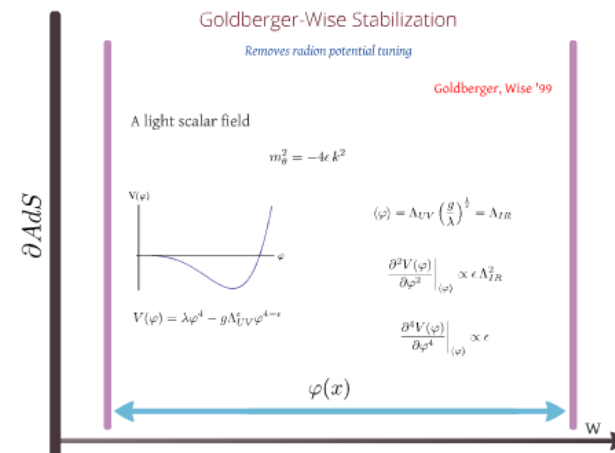
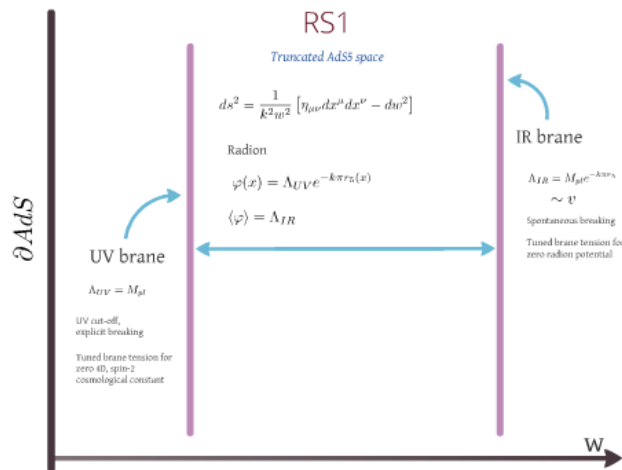
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Boundary conditions

- $\psi = 0$ (Dirichlet)
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Analogy

Randall Sundrum Model



∂AdS

RS1

Truncated AdS5 space

$$ds^2 = \frac{1}{k^2 w^2} [\eta_{\mu\nu} dx^\mu dx^\nu - dw^2]$$

Radion

$$\varphi(x) = \Lambda_{UV} e^{-k\pi r_5(x)}$$

$$\langle \varphi \rangle = \Lambda_{IR}$$

UV brane

$$\Lambda_{UV} = M_{pl}$$

UV cut-off,
explicit breaking

Tuned brane tension for
zero 4D, spin-2
cosmological constant

IR brane

$$\Lambda_{IR} = M_{pl} e^{-k\pi r_5} \\ \sim v$$

Spontaneous breaking

Tuned brane tension for
zero radion potential

W

UV brane

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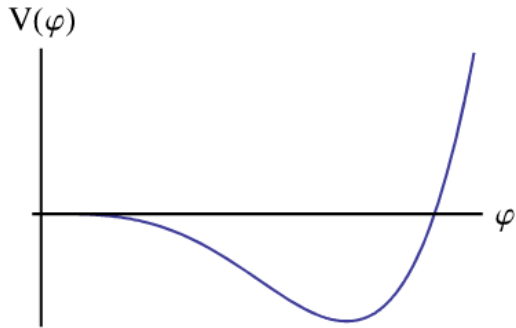
Goldberger-Wise Stabilization

Removes radion potential tuning

Goldberger, Wise '99

A light scalar field

$$m_\theta^2 = -4\epsilon k^2$$



$$V(\varphi) = \lambda\varphi^4 - g\Lambda_{UV}^\epsilon\varphi^{4-\epsilon}$$

$$\langle\varphi\rangle = \Lambda_{UV} \left(\frac{g}{\lambda}\right)^{\frac{1}{\epsilon}} = \Lambda_{IR}$$

$$\left.\frac{\partial^2 V(\varphi)}{\partial\varphi^2}\right|_{\langle\varphi\rangle} \propto \epsilon \Lambda_{IR}^2$$

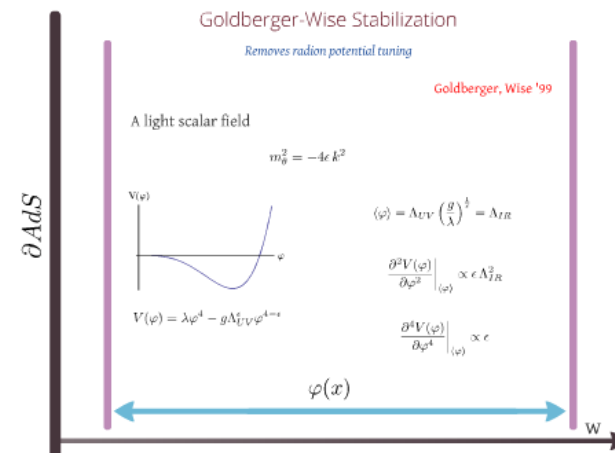
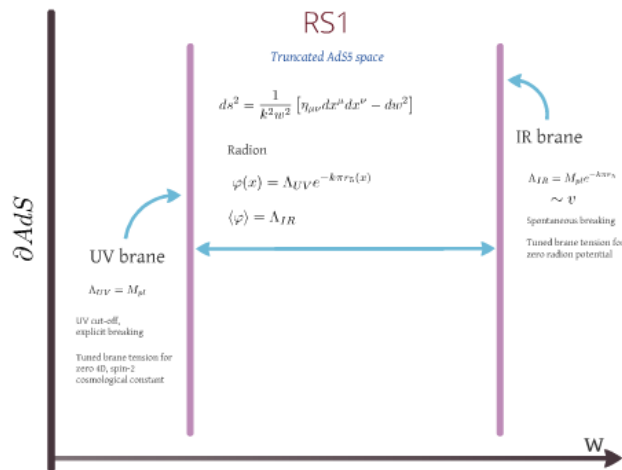
$$\left.\frac{\partial^4 V(\varphi)}{\partial\varphi^4}\right|_{\langle\varphi\rangle} \propto \epsilon$$

$\varphi(x)$

∂AdS

W

Randall Sundrum Model



RS3

The scalar cosmological constant problem
Solved!

Scalar gravity is dual to RS model without a UV brane

$\varphi(x)$ parametrizes the position of the IR brane

Scalar cosmological constant problem same as radion potential tuning

$$V(\varphi) = \lambda\varphi^4$$

Goldberger-Wise mechanism solves the cosmological constant problem in scalar gravity!
Rattazzi (Planck '10)

The scalar cosmological constant problem
Solved?

The GW mechanism breaks equivalence principle

$$m_0^2 = -4\epsilon k^2 \quad \frac{\partial^2 V(\varphi)}{\partial \varphi^2} \Big|_{\varphi} \propto \epsilon \Lambda_{IR}^2$$

$$\frac{\partial^4 V(\varphi)}{\partial \varphi^4} \Big|_{\varphi} \propto \epsilon$$

To reproduce the cosmological constant observed

$$\epsilon \sim \frac{\Lambda_{\text{obs}}}{M_{\text{pl}}^2} \simeq 10^{-120}$$

The scalar cosmological constant problem

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∂AdS

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∂AdS

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To reproduce the cosmological constant observed

$$\epsilon \sim \frac{\Lambda_{\text{obs}}}{M_{\text{pl}}^2} \simeq 10^{-120}$$

The scalar cosmological constant problem

Solved?

Scalar gravity is dual to 4D world without a cut-off scale

$\varphi(x)$ parameterizes the position of the Higgs

Scalar cosmological constant problem can be reformulated as finding

$$V(\varphi) = \lambda \varphi^4$$

Goldberger-Wise mechanism solves the cosmological constant problem in scalar gravity?

$\varphi(x)$

The scalar cosmological constant problem

Solved?

The GW mechanism breaks equivalence principle

$$\partial_\mu^2 \varphi = -\lambda \varphi^3$$

$$\frac{\partial^2 \varphi}{\partial x^2} = \lambda \varphi + A_4$$

$$\frac{\partial^2 \varphi}{\partial x^2} = \lambda \varphi$$

to reproduce the cosmological constant observed

$$r = \frac{\lambda}{M^2} = M^{-2} \lambda$$

$\varphi(x)$

Solution

Pseudo-Nambu Goldstone boson

- Goldberger - Wise field θ as a pseudo-Nambu Goldstone boson
- Inspiration: Higgs as pNG boson
 - implemented as A_2 of a gauge field in 5D
- θ as A_4 in a 6D model Georgi, Kaplan '94

Need a higher dimension!

A construction in six dimensions

Warped compactification

$$\frac{ds^2}{dx^2} = \frac{1}{w^2} \left(dt^2 + dx^2 + \sum_{i=1}^3 dx_i^2 \right)$$

$w = e^{kx}$

Goldberger-Wise

Wilson fermion operators

Weyl fermion solutions

$$\psi_L = e^{-kx} \psi_L(x)$$

$$\psi_R = e^{kx} \psi_R(x)$$

$$\psi(x) = \psi_L + \psi_R$$

$MS_2 \times S_1 / Z_2$

5D action: $M_5^3 \int d^5x \sqrt{g} \left[-\frac{1}{2} R_{5D} + \frac{1}{2} (\partial_\mu \theta)^2 + V(\theta) \right]$

4D effective theory

no tuning!

Cascade of effective theories

5D effective theory has a light scalar

$$S_{5D, \text{bulk}} = M_5^3 \int d^5x \sqrt{g} \left[-\frac{1}{2} R_{5D} + \frac{1}{2} (\partial_\mu \theta)^2 + V(\theta) \right]$$

$$S_{5D, \text{brane}} = M_5^3 \int d^5x \sqrt{g} [T + V(\theta) + \mathcal{L}_m]$$

4D effective theory

- small cosmological constant
- tiny violation of equivalence principle

$$S_{4D} = \int d^4x \left[\frac{1}{2} (\partial_\mu \pi)^2 - m_\pi^2 \pi^2 - \Lambda_{4D} + \epsilon (D + \gamma_M(M + \pi)) + \dots \right]$$

no tuning!

Pseudo-Nambu Goldstone boson

- Goldberger - Wise field θ as a pseudo-Nambu Goldstone boson
- Inspiration: Higgs as pNG boson
 - implemented as A_5 of a gauge field in 5D
- θ as A_6 in a 6D model

Georgi, Kaplan '84

Need a higher dimension!

A construction in six dimensions

Warped compactification

$$A_M : (-, -)$$

$$A_6 : (+, +)$$

6D Goldberger-Wise

$$S_{bulk} = M_6^4 \int d^5x \int_0^{\pi r_6} du \sqrt{-G} \left[-\frac{1}{4}R + \Lambda_{bulk} + \frac{1}{2}G^{ab}\partial_a\chi\partial_b\chi - \frac{1}{2}m_\chi^2\chi^2 \right. \\ \left. - \frac{1}{4}G^{ac}G^{bd}F_{ab}F_{cd} + \frac{1}{2}G^{ab}(D_a\psi)^\dagger D_b\psi - \frac{1}{2}m_\psi^2\psi^2 \right]$$

$$S_1 = k_6 M_6^4 \int d^5x \int_0^{\pi r_6} du \delta(u) \sqrt{g_1} [T_1 + \lambda_1 \chi + \frac{1}{2} \mu_2 \chi^2]$$

$$S_2 = k_6 M_6^4 \int d^5x \int_0^{\pi r_6} du \delta(u - \pi r_6) \sqrt{g_2} [T_2 + \lambda_2 \chi + \frac{1}{2} \mu_2 \chi^2]$$

Untuned brane tensions

Maximally symmetric solutions

DeWolfe, Freedman, Gubser, Karch 2000

$$ds^2 = e^{-2\sigma(u)} dx^M dx^N \bar{g}_{MN} - du^2$$

$$\bar{g} = e^{-2ky} dx^\mu dx^\nu \eta_{\mu\nu} - dy^2$$

$$\chi(x, u) = \chi(u)$$

$$AdS_5 \times S_1/Z_2$$

Potential for 6D gauge field

- Gauge invariance prohibits 6D mass term
- Other potential terms absent

$$\int du \partial_u A^6$$

- Potential arises from non-local Aharanov-Bohm effect

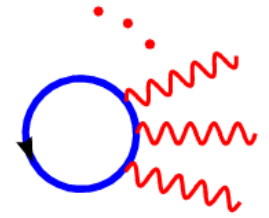
Wilson loops using Schwinger proper time

(flat spacetime case)

Strassler '92

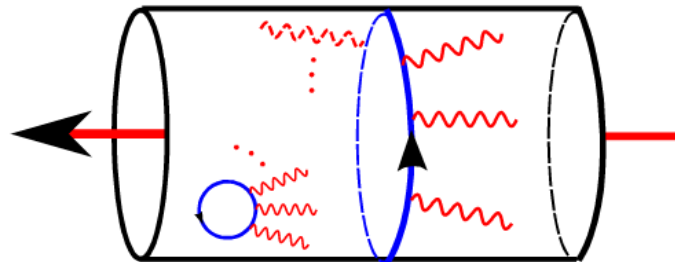
Effective action as a path integral over charged particle worldlines

$$\begin{aligned}\Gamma[A] &= -\log(\det(-D^2 - m_\psi^2)) \\ &= -\text{tr} \log(-D^2 - m_\psi^2) \\ &= \int_0^\infty \frac{dT}{T} \text{tr} \exp\left(\frac{1}{2}\mathcal{E}T(-D^2 - m_\psi^2)\right)\end{aligned}$$



$$\begin{aligned}\Gamma[A] &= \int_0^\infty \frac{dT}{T} \int \mathcal{D}x \mathcal{D}p \exp\left[\int_0^T d\tau ip \cdot \dot{x}\right] \exp\left[-\frac{1}{2}\mathcal{E} \int_0^T d\tau (p - eA)^2 + m_\psi^2\right] \\ &= \int_0^\infty \frac{dT}{T} \mathcal{N} \int \mathcal{D}x \exp\left[-\int_0^T d\tau \frac{1}{2\mathcal{E}} \dot{x}^2 + \frac{\mathcal{E}}{2} m_\psi^2 + ieA(x) \cdot \dot{x}\right] \\ &= \int_0^\infty \frac{dT}{T} \mathcal{N} \int \mathcal{D}x \exp\left[-m_\psi \int_0^T d\tau \sqrt{\eta_{\mu\nu} \dot{x}^\mu \dot{x}^\nu}\right] \exp\left[ig \oint dx \cdot A(x)\right]\end{aligned}$$

Wilson loops using Schwinger proper time



$$V_{eff}[A] \propto \int_0^\infty \frac{dT}{T} \int \mathcal{D}x \exp \left[-m_\psi \int_0^T d\tau \sqrt{\eta_{\mu\nu} \dot{x}^\mu \dot{x}^\nu} \right] \exp \left[ig \oint dx \cdot A(x) \right]$$
$$\propto e^{i\theta} e^{-2\pi m_\psi r_6}$$

Charged matter needs to be only modestly heavier than $\frac{1}{r_6}$

Cascade of effective theories

5D effective theory has a light scalar

$$S_{5D,bulk} = M_5^3 \int d^5x \sqrt{g} \left[-\frac{1}{4} R_{5D} + \hat{k}_5^2 + g^{MN} \partial_M \theta \partial_N \theta + \epsilon V(\theta) \right]$$

$$S_{5D,brane} = M_5^3 k \int d^5x \sqrt{g} [T + V(\theta) + \mathcal{L}_m]$$

4D effective theory

- scalar gravity
- small cosmological constant
- tiny violation of equivalence principle

$$S_{4D} = \int d^4x \left[\frac{1}{2} (\partial_\mu \pi)^2 - m_\pi^2 \epsilon \pi^2 - \Lambda \epsilon \phi^4 + \bar{e} (\not{D} + y_e (M + \pi)) e + \dots \right]$$

no tuning!

Real World

General Relativity

Randall Sundrum Model

	CFT	AdS
M	M_{Planck}	$M^2 = \frac{1}{2\pi\alpha'} \sqrt{g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu}$
λ	Type IIB string theory	Type IIB string theory on $S^1 \times S^5$
$\mathcal{O}(x)$	$\phi(x)$	$\psi(x)$
$\Delta_{\mathcal{O}}$	$\Delta_{\mathcal{O}}$	$m_{\mathcal{O}}^2$
$\langle \mathcal{O}(x) \rangle$	$\langle \phi(x) \rangle$	$\langle \psi(x) \rangle$
$J(x)$	$J(x)$	$J(x) = \frac{1}{2\pi\alpha'} \int dx^\mu \dot{x}^\mu$

W

4

5

6

CFT

RS3

Scalar gravity

Solution

Analogy

The Anthropic principle

The scalar grass is greener

- The 5D theory is a theory of gravity, presumably with string UV completion
- Scalar gravity plausibly exists in the landscape
- The cosmological constant experienced by SM matter is robustly small.
- In the non-relativistic limit, scalar gravity reduces to Newtonian gravity
- Why does the anthropic selection principle choose our universe over scalar gravity?

Summary

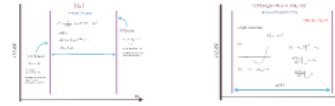
- Scalar gravity is a powerful analogy for spin-2 gravity
- Possesses an analog to the cosmological constant problem
- Cosmological constant in the scalar gravity need not be fine-tuned.
- Vacuum energy relaxes via small violation of equivalence principle
- Sharpens the question of anthropic selection of the spin-2 cosmological constant

Real World

General Relativity



Randall Sundrum Model



4

	CFT	AdS
M^2	M^2	$M^2 = \frac{1}{2\pi\alpha'} \sqrt{-(\dot{x}^\mu - A^\mu)^2}$
Type of theory	Type II string theory or Super Yang Mills	Type IIB string theory or $SO(2,3)$ <i>Maldecena's</i>
$\mathcal{O}(x)$	$\mathcal{O}(x)$	$\psi(x)$
$\Delta_{\mathcal{O}}$	$\Delta_{\mathcal{O}}$	$m_{\mathcal{O}}^2$
$\phi(x)$	$\phi(x)$	$\varphi(x)$
$J(x)$	$J(x)$	$\int_{\mathcal{M}} \sqrt{g} \frac{\delta S}{\delta \varphi}$

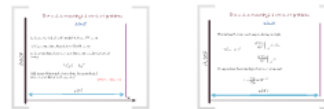
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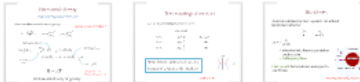
CFT



RS3



Scalar gravity



Solution

Fields in the bulk

- ψ (scalar)
- ϕ (vector)
- χ (spinor)

Asymptotic boundary conditions

- $\psi \sim \psi_0 + \dots$
- $\phi \sim \phi_0 + \dots$
- $\chi \sim \chi_0 + \dots$

Conservation of energy

- $\nabla_\mu T^{\mu\nu} = 0$
- $\nabla_\mu \psi = 0$
- $\nabla_\mu \phi = 0$
- $\nabla_\mu \chi = 0$

Analogy