Excited states and hadron resonances from Lattice QCD

Daniel Mohler

Fermilab Theory Group
Batavia, IL, USA

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Fermilab

Collaborators: G. Engel, C. B. Lang, L. Lescovec, S. Prelovsek, R. M. Woloshyn

Outline

1. Introduction & Motivation

2. Extraction of excited energy levels
   - Why are excited states difficult?
   - Example results for light-quark mesons and baryons
   - Example results for the low-lying charmonium spectrum

3. From excited states to hadron resonances
   - The ρ resonance - a benchmark calculation
   - $K\pi$ scattering
   - $D\pi$ and $D^{\star}\pi$ scattering

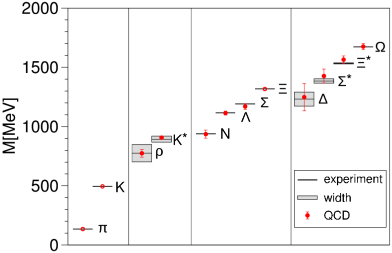
4. Summary & Outlook

Recent developments

- Light mesons and baryons

Ground states have been calculated with full control of systematic uncertainties

Example from Dürr et al. Science 322 (2008)



- Calculations at the physical pion mass

  `PACS-CS Phys.Rev. D81 (2010) 074503`

  `BMW Phys.Lett. B701 (2011) 265`

  `MILC arXiv:1212.4768`

  We will soon see more (RBC,. . .)

- First dynamical 2+1+1 flavor simulations (ETMC, MILC)

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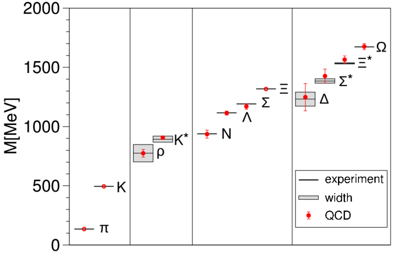

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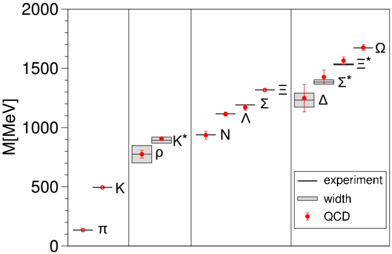

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Two kinds of progress . . .

(1) Much progress in controlling systematics for ground states in "gold-plated" channels
→ Studies with fully controlled systematics at the physical point.

(2) Qualitative studies provide/implement crucial insights/improvements and pave the way

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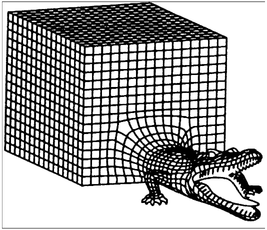

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Example of systematics: Extrapolations

- *Continuum limit*: $a(g, m) \to 0$
  - Need simulations at multiple different lattice spacings
- *Thermodynamic limit*: $L \to \infty \quad (L \cdot a = const.)$
  - Hadron physics in a small box → finite volume effects
    Typical volume $\approx 2.0 \ldots 3.5\mathrm{fm}$
- Calculation at physical quark masses or extrapolation to the *Chiral limit*: $m \to m_0 \quad (M_\pi \to M_{\pi,exp})$
  - Physical u, d quark masses small → Simulation very expensive!
  - Chiral Perturbation Theory (χPT) ↔ Lattice QCD

Many interesting issues

Renewed interest in hadron spectroscopy (in experiment and theory)

- X, Y and Z Charmonium-like states
- light scalar mesons
- D_s spectrum: $D^*_{s0}(2317)$ (0^+), $D_{s1}(2460)$
- Highly excited light-quark mesons and baryons

In addition puzzling lattice data for

- Roper resonance
- Λ baryons, especially $\Lambda(1405)$

Methods used for excited state spectroscopy interesting with regard to

- Radiative decays
- CP violation in charm
- Puzzles observed in semileptonic B decays

Example operators

Need: *Interpolating field operator* that creates states with correct quantum numbers.

- Example I: Pseudoscalar Mesons with $IJ^{PC} = 10^{-+}$

$$O_\pi^{(1)} = \bar{u}\gamma_5 d$$
$$O_\pi^{(2)} = \bar{u}\overleftrightarrow{D}\gamma_i\gamma_t\gamma_5 d$$

- Example II: Nucleon

$$O_N = \epsilon_{abc}\,\Gamma_1\,u_a\left(u_b^T\,\Gamma_2\,d_c - d_b^T\,\Gamma_2\,u_c\right)$$

- In practice: Many (slightly different) constructions possible!
- In a QFT they should all be OK; Overlap?

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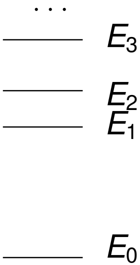
The problem with excited states

From the analysis of Euclidean correlators:

$$\left\langle \hat{O}_2(t)\hat{O}_1(0)\right\rangle_T \propto \sum_n e^{-tE_n} < 0|\hat{O}_2|n >< n|\hat{O}_1|0 >$$

- The whole tower of states contributes
- Ground state is dominant at large t
- Exited states appear as sub-leading exponentials
- Noisy background from limited statistics



- For a single correlator, fit to several exponentials leads to poor results
  → Advanced methods needed for excited states!

(My) Method of choice: The variational method

Matrix of correlators projected to fixed momentum (will assume 0)

$$C(t)_{ij} = \sum_n \mathrm{e}^{-tE_n} \langle 0|O_i|n\rangle \left\langle n|O_j^\dagger|0\right\rangle$$

Solve the generalized eigenvalue problem:

$$C(t)\vec{\psi}^{(k)} = \lambda^{(k)}(t)C(t_0)\vec{\psi}^{(k)}$$

$$\lambda^{(k)}(t) \propto \mathrm{e}^{-tE_k}\left(1+\mathcal{O}\left(\mathrm{e}^{-t\Delta E_k}\right)\right)$$

At large time separation: only a single state in each eigenvalue.
Eigenvectors can serve as a fingerprint.

`Michael Nucl. Phys. B259, 58 (1985)`

`Lüscher and Wolff Nucl. Phys. B339, 222 (1990)`

`Blossier et al. JHEP 04, 094 (2009)`

Angular momentum (mesons)

- *Reminder:* No unique spin assignment on the lattice.
  Five irreducible representations:

Irrep of O	J	Spinors in irrep
A_1	0,4,...	$1, \gamma_t, \gamma_5, \gamma_t\gamma_5$
A_2	3,6,...	
E	2,4,5,...	
T_1	1,3,4,5,...	$\gamma_i, \gamma_t\gamma_i, \gamma_5\gamma_i, \gamma_t\gamma_5\gamma_i$
T_2	2,3,4,5,...	

- Classification of interpolator basis by representations
- Unique identification of spin nontrivial

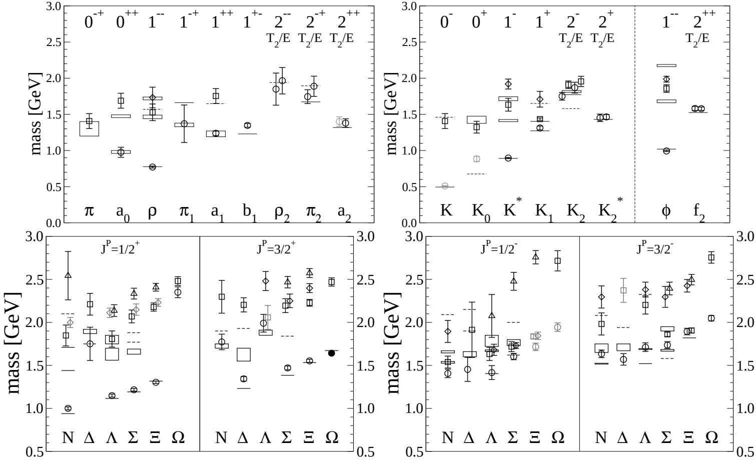
`Dudek et al., PRL 103 262001 (2009)`

Light mesons and baryons with CI fermions

- Simulation with 2 mass degenerate dynamic quarks and a heavier (strange) valence quark
- Pion masses ranging from 588MeV to 255MeV
- Scale set by the static quark potential and strange quark mass set by Ω baryon
- Errors are statistical only and systematic errors are non-negligible

set	β_{LW}	m_0	configs.	m_π [MeV]	$L^3 \times T$ [a^4]	$m_\pi L$	a [fm]
A50	4.70	-0.050	200	596(5)	$16^3 \times 32$	6.40	0.1324(11)
A66	4.70	-0.066	200	255(7)	$16^3 \times 32$	2.72	0.1324(11)
B60	4.65	-0.060	300	516(6)	$16^3 \times 32$	5.72	0.1366(15)
B70	4.65	-0.070	200	305(6)	$16^3 \times 32$	3.38	0.1366(15)
C64	4.58	-0.064	200	588(6)	$16^3 \times 32$	6.67	0.1398(14)
C72	4.58	-0.072	200	451(5)	$16^3 \times 32$	5.11	0.1398(14)
C77	4.58	-0.077	300	330(5)	$16^3 \times 32$	3.74	0.1398(14)
LA66	4.70	-0.066	-0.012	97	$24^3 \times 48$	4.08	0.1324(11)
SC77	4.58	-0.077	-0.022	600	$12^3 \times 24$	2.81	0.1398(14)
LC77	4.58	-0.077	-0.022	153	$24^3 \times 48$	5.61	0.1398(14)

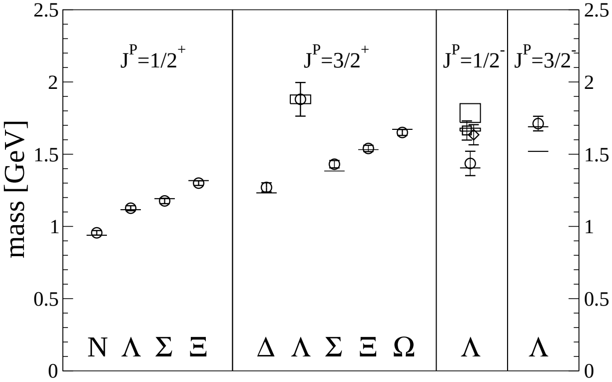
Light mesons and baryons from two flavor QCD



`Engel et al. PRD 85 034508 (2012); Engel et al. arXiv:1301.4318;`

- Errors are statistical only after chiral extrapolation

Low lying positive parity: infinite volume extrapolated



- Remaining shortcomings: 2 flavor, badly determined strange quark mass, no continuum extrapolation, ...

Charmonium spectrum: Charm quark treatment

- We use the *Fermilab method* for the heavy (charm) quark

`El-Khadra et al., PRD 55, 3933`

- We tune κ for the spin averaged kinetic mass $(M_{\eta_c} + 3M_{J/\Psi})/4$ to assume its physical value
- General form for the dispersion relation

`Bernard et al.  PRD83:034503,2011`

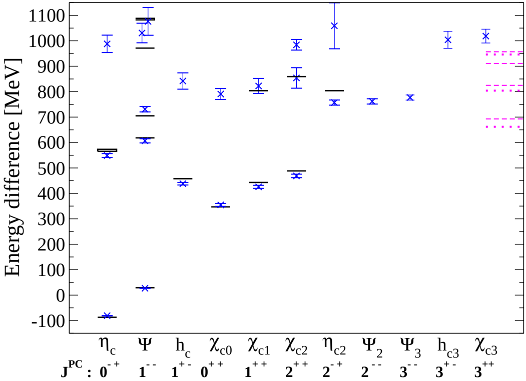
$$E(p) = M_1 + \frac{p^2}{2M_2} - \frac{a^3 W_4}{6} \sum_i p_i^4 - \frac{(p^2)^2}{8M_4^3} + \dots$$

- We compare results from three different fit strategies:
  1. Neglect the term with coefficient W_4
  2. Fit $E^2(p)$ and neglect $(p^2)^2$ term from mismatch of M_1, M_2 and M_4

$$E^2(p) \approx M_1^2 + \frac{M_1}{M_2} p^2 - \frac{M_1 a^3 W_4}{3} \sum_i (p_i)^4 \qquad (1)$$

  3. Set either $M_4 = M_1$ or $M_4 = M_2$

Charmonium spectrum using the distillation technique



`D. M., S. Prelovsek, R. M. Woloshyn, to appear in PRD;`

- For some technical detail see a later slide
- Data from 1 ensemble; Errors statistical + scale setting

Excited states - some key observations

- In practical calculations $\bar{q}q$ interpolators couple very weakly to multi-hadron states

  McNeile & Michael, Phys. Lett. B 556, 177 (2003); Engel et al. PRD 82, 034505 (2010); Bulava et al. PRD 82, 014507(2010); Dudek et al. PRD 82, 034508(2010);

- This is not unlike observations in string breaking studies

  Pennanen & Michael hep-lat/0001015;Bernard et al. PRD 64 074509 2001;

- This necessitates the inclusion of hadron-hadron interpolators

- We know: Energy levels $\neq$ resonance masses
  Naïve expectation: Correct up to $\mathcal{O}(\Gamma_R(m_\pi))$

Hadron resonances - experiment

- Most hadrons are resonances under the strong interaction

hadron	Γ [MeV]
$b_1(1235)$	142 ± 9
$a_1(1260)$	$250 - 600$

hadron	Γ [MeV]
$K^\star(1410)$	232 ± 31
$K_0^\star(1430)$	270 ± 80

hadron	Γ [MeV]
$D_0^\star(2400)$	267 ± 40
$D_1(2430)$	$384\pm^{130}_{110}$

- Widths and branching fractions often known poorly
- Experiment data is analyzed with a partial wave analysis
- Elastic scattering: Scattering amplitudes T_l and related phases δ_l:

$$T_l = \sin\delta_l e^{i\delta_l} = \frac{e^{2i\delta_l} - 1}{2i}$$

- Near a single relativistic Breit-Wigner shaped resonance

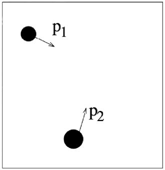
$$T_l = \frac{-\sqrt{s}\Gamma(s)}{s - s_R + i\sqrt{s}\Gamma(s)}$$

with resonance position $s_R = m_R^2$ and decay width Γ

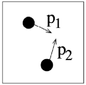
The Lüscher method for elastic scattering

M. Lüscher Commun. Math. Phys. 105 (1986) 153; Nucl. Phys. B 354 (1991) 531; Nucl. Phys. B 364 (1991) 237.

$$E = E(p_1) + E(p_2)$$



L >> few fm



L= few fm

$$E = E(p_1) + E(p_2) + \Delta_E$$

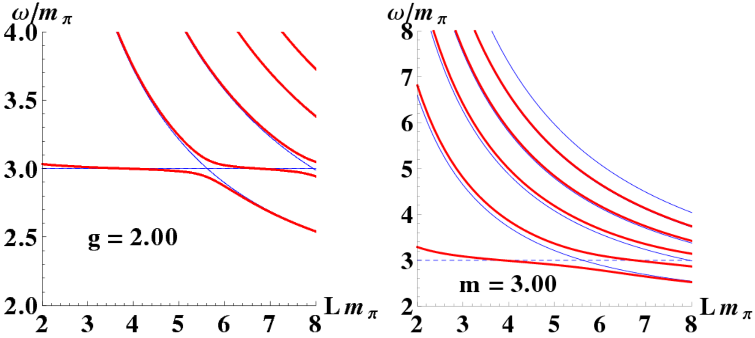
$$E_n(L) \quad \xrightarrow{(2)} \quad \delta_l \quad \xrightarrow{(3)} \quad m_R; \quad \Gamma_R \text{ or coupling } g$$

(1) Extract energy levels $E_n(L)$ in a finite box

(2) The Lüscher formula relates this spectrum to the phase shift of the continuum scattering amplitude

(3) Extract resonance parameters with some degree of modeling/approximation

Energy levels in a box - an illustration



animations by C. B. Lang and DM

- **Left:** Expectations for ρ-like resonance at varying coupling $g_{\rho\pi\pi}$
- **Right:** Expectations for ρ-like resonance with physical $g_{\rho\pi\pi}$ and varying mass

Multiple possibilities

- Lüscher method
  - Rest-frame calculation in multiple spatial volumes L^3

    M. Lüscher Commun. Math. Phys. 105 (1986) 153; Nucl. Phys. B 354 (1991) 531; Nucl. Phys. B 364 (1991) 237.
  - Moving frames for equal mass hadrons $m_{h1} = m_{h2}$

    Rummukainen, Gottlieb, Nucl. Phys. B 450, 397 (1995);
    Kim, Sachrajda, Sharpe, Nucl. Phys. B 727, 218 (2005);
    Feng, Jansen, Renner, PoS LAT2010 104 (2010);
    Dudek, Edwards, Thomas, arXiv:1203.6041.
  - Moving frames for $m_{h1} \neq m_{h2}$: Even and odd l mix

    Fu, PRD 85 014506 (2012); Döring et al. arXiv:1205.4838;
    Göckeler et al. arXiv:1206.4141; Leskovec, Prelovsek, PRD 85 114507 (2012);
  - Calculations in multiple asymmetric boxes i.e. $L^2 \times L_z$
- Alternative approaches
  - Histogram method

    Bernard, Lage, Meißner, Rusetsky, JHEP 0808 (2008) 024
  - Correlator method

    Meißner, Polejaeva, Rusetsky, Nucl. Phys. B 846,1 (2011)

Meson - meson scattering and hadron resonances

- $N_f = 2$ flavors of nHYP smeared Wilson-clover quarks

$N_L^3 \times N_T$	κ_l	β	a[fm]	L[fm]	#configs	m_π[MeV]	m_K[MeV]
$16^3 \times 32$	0.1283	7.1	0.1239(13)	1.98	280/279	266(3)(3)	552(2)(6)

`Gauge ensemble from Hasenfratz et al. PRD 78 054511 (2008)`
`Hasenfratz et al. PRD 78 014515 (2008)`

- Basis of several $\bar{q}q$ and meson-meson interpolators
- Three separate studies
  - Coupled channel analysis of the ρ meson decay

    `Lang, DM, Prelovsek, Vidmar, PRD 84 054503 (2011)`
  - $K\pi$ scattering for isospin $\frac{1}{2}$ and $\frac{3}{2}$ in s-wave and p-wave

    `Lang, Leskovec, DM, Prelovsek, PRD 86 054508 (2012)`
  - D mesons including $D\pi$ and $D^\star\pi$ with relativistic charm quarks

    `DM, Prelovsek, Woloshyn - accepted by PRD`

The “Distillation” method

`Peardon et al. PRD 80, 054506 (2009)`
`Morningstar et al. PRD 83, 114505 (2011)`

- Idea: Construct separable quark smearing operator using low modes of the 3D lattice Laplacian
  Spectral decomposition for an $N \times N$ matrix:

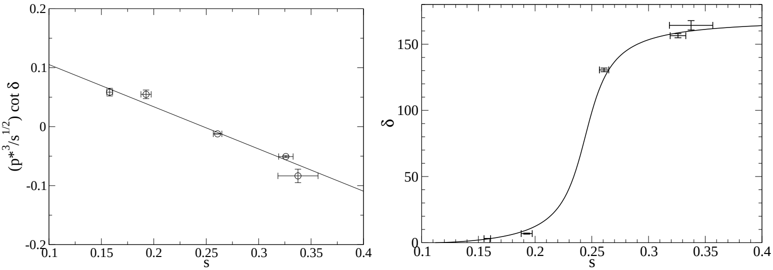
$$f(A) = \sum_{k=1}^{N} f(\lambda^{(k)})\; v^{(k)} v^{(k)\dagger}. \tag{2}$$

  With $f(\nabla^2) = \Theta(\sigma_s^2 + \nabla^2)$ (Laplacian-Heavyside (LapH) smearing):

$$q_s \equiv \sum_{k=1}^{N} \Theta(\sigma_s^2 + \lambda^{(k)}) v^{(k)} v^{(k)\dagger}\, q \;=\; \sum_{k=1}^{N_v} v^{(k)} v^{(k)\dagger}\, q\,. \tag{3}$$

- Advantages: momentum projection at source; large basis possible
- Disadvantages: expensive; unfavorable volume scaling
- Stochastic approach (mostly) eliminates bad scaling

The ρ resonance - a benchmark calculation



From Lang, DM, Prelovsek, Vidmar, PRD 84 054503 (2011)

- We extract $g_{\rho\pi\pi}$ rather than Γ

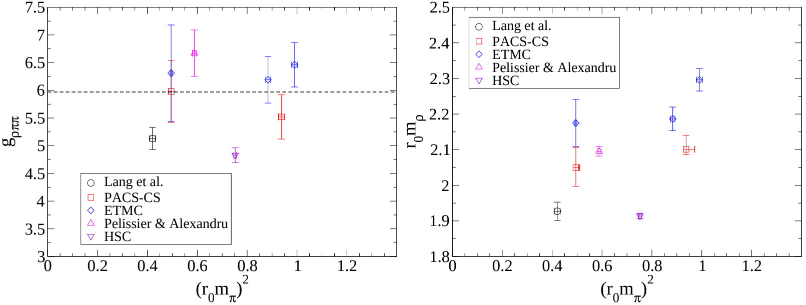
$$\Gamma(s) = \frac{p^{\star 3}}{s} \frac{g^2_{\rho\pi\pi}}{6\pi}$$

- Results for $m_\pi = 266(3)(3)\mathrm{MeV}$

$$g_{\rho\pi\pi} = 5.13(20) \qquad m_\rho = 792(7)(8)\ \mathrm{MeV}$$

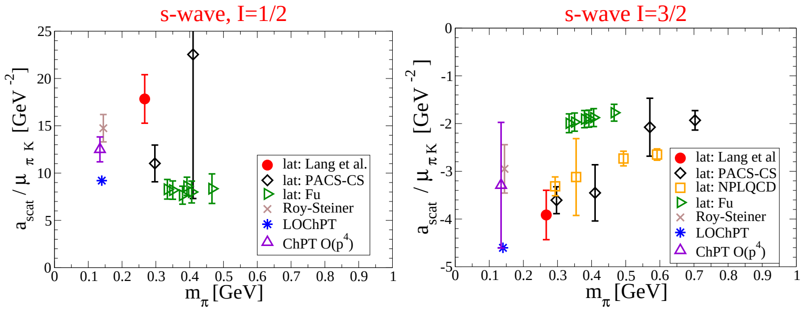
The ρ resonance - comparing results

- To compare the masses I use the values for $\frac{r0}{a}$ for each ensemble



- Simulations differ in several respects - time will tell

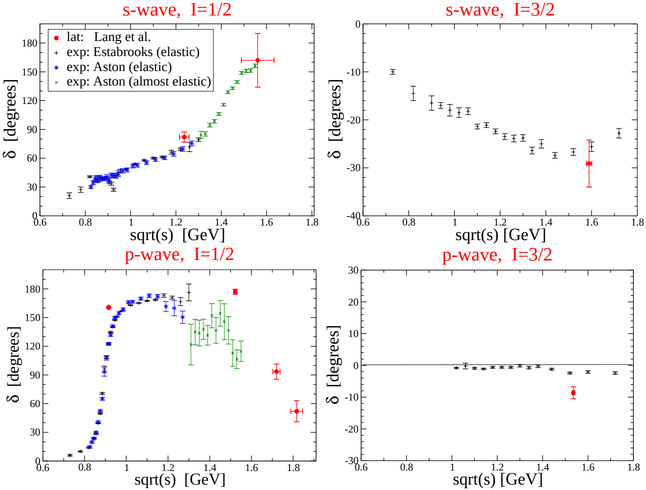
$K\pi$ scattering for isospin $\frac{1}{2}$ and $\frac{3}{2}$ - scattering length



Lang, Leskovec, DM, Prelovsek, PRD 86 054508 (2012)

- $\frac{a_0^I}{\mu_{K\pi}}$ independent of $m_{K,\pi}$ in LOChPT

$K\pi$ scattering for isospin $\frac{1}{2}$ and $\frac{3}{2}$ - phase shift data

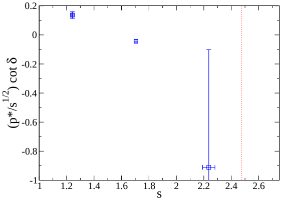


`Lang, Leskovec, DM, Prelovsek, PRD 86 054508 (2012)`

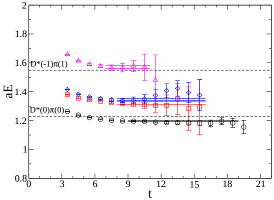
$D\pi$ and $D^\star\pi$ scattering

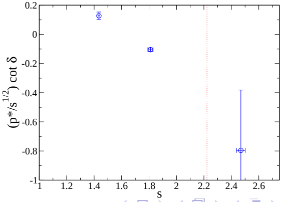
DM, Prelovsek, Woloshyn - accepted by PRD

- In the $J^P = 0^+$ $D_0^\star$ channel we extract three levels



- For the $J^P = 1^+$ channel there are two resonances $D_1(2420)$ and $D_1(2430)$

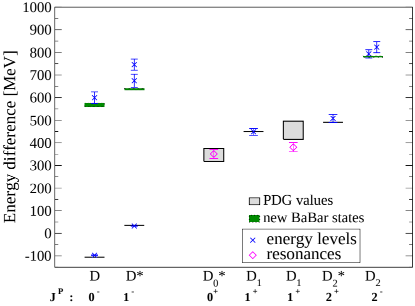




$D\pi$ and $D^\star\pi$ scattering

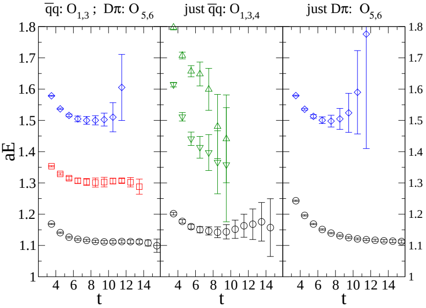
`DM, Prelovsek, Woloshyn - accepted by PRD`

- Motivated by the heavy quark limit, We assume one state is given by the naive energy level and fit the remaining data to obtain



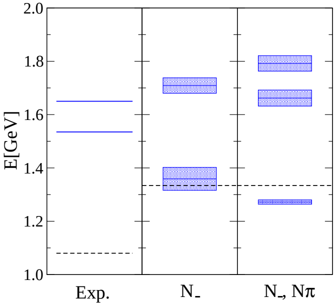
	$D_0^\star(2400)$	$D_1(2430)$
g^{lat} [GeV]	2.55 ± 0.21	2.01 ± 0.15
g^{exp} [GeV]	$\leq 1.92 \pm 0.14$	$\leq 2.50 \pm 0.40$

Comparing results with and without explicit scattering states



DM, Prelovsek, Woloshyn – accepted by PRD

- D meson spectrum in the D_0^* channel



C. B. Lang and V. Verduci, arXiv:1212.5055

- Negative parity Nucleon spectrum

- Similar observation also by Dudek, Edwards, Thomas, arXiv:1212.0830

Conclusions

- Studying (some) QCD resonances from first principles in fully dynamic simulations is becoming possible
- It seems necessary to include explicit multi-particle states to get good overlap and to disentangle these contributions
- Overall, studies of QCD resonances are still in their infancy
  → So far: Exploratory studies in small volumes
- The standard Lüscher method is restricted to the elastic case
- Extensions are not well developed or require modeling, although some progress has been made
- Looking at experiment, some interesting states can be studied but even low lying states can be quite difficult
  → There is much work ahead!