Analysis of Cirrus Cloud Particle Size Distributions Measured During Sparticus



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Data provided by Paul Lawson of SPEC, Inc.

Motivation

- Remote sensing of cirrus particle size distributions (PSD's)
- Maximize the posterior distribution of possible PSD params given remote observations, as given by Bayes' Rule:

$$p(\vec{x} \mid \vec{y}) \propto L(\vec{y} \mid \vec{x}) p(\vec{x})$$

Measurement and pram vectors

$$\vec{x} = \begin{bmatrix} p_1 \\ \dots \\ p_n \end{bmatrix} \qquad \qquad \vec{y} = \begin{bmatrix} Z \\ \dots \\ Tb \end{bmatrix}$$

Focus on the Prior Distribution: $p(\bar{x})$

• Often assumed to be Gaussian (e.g. Rogers, 2000) $p(\vec{x}) = \frac{1}{1 - exp} \left[-\frac{1}{2} (\vec{x} - \vec{x}_{-})^T S^{-1} (\vec{x} - \vec{x}_{-}) \right]$

$$p(x) = \frac{1}{(2\pi)^{n/2}} |S_a|^{1/2} \exp\left[-\frac{1}{2}(x - x_a) S_a^{-1}(x - x_a)\right]$$

Covariance generally assumed to be diagonal

$$S_a = \begin{bmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \sigma_2^2 & 0 \\ 0 & 0 & \sigma_3^2 \end{bmatrix}$$

First Two Questions:

- Can the prior distribution for cirrus PSD parameters be assumed Gaussian?
- Can the covariance matrix for cirrus PSD parameters be assumed diagonal?

 Investigate using PSD's measured by the 2-DS probe during Sparticus

PSD Fits

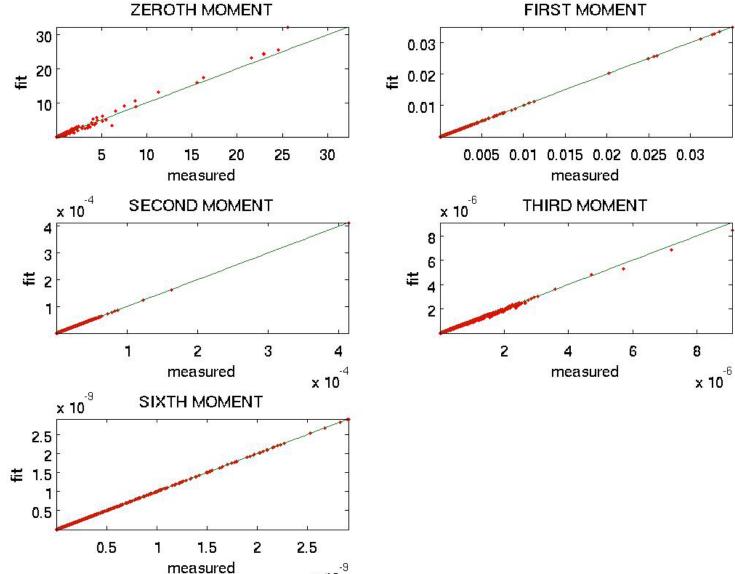


- Data from 58 of Sparticus' flight legs
- 2-DS distributions fit with

 $n(D) = N_0 (D/D_0)^{\alpha} \exp(-D/D_0); \quad \bar{x} = \begin{bmatrix} N_0 & D_0 & \alpha \end{bmatrix}$

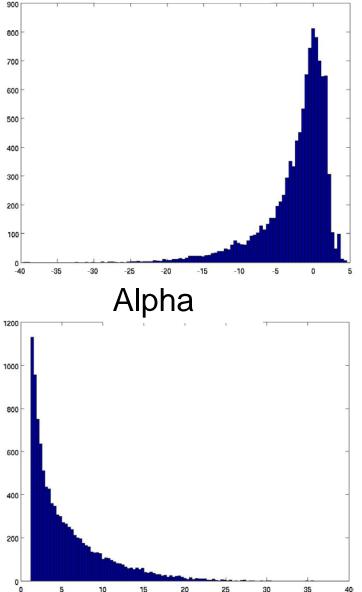
• Over 10,000 fits

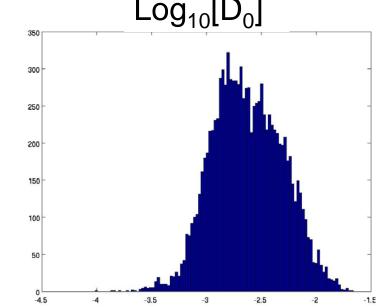
Fidelity of Computed Moments to Measured Moments



x 10

Distributions of Fitted Parameters Log₁₀[N₀]





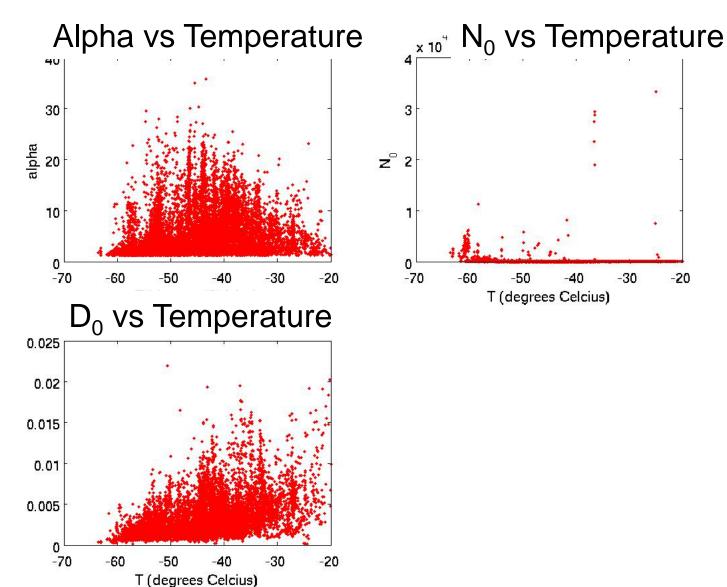
- Logarithms used to make orders of magnitude comparable
- PSD parameters can not be described with multivariate Gaussian

Covariance Matrix of Distribution Parameters

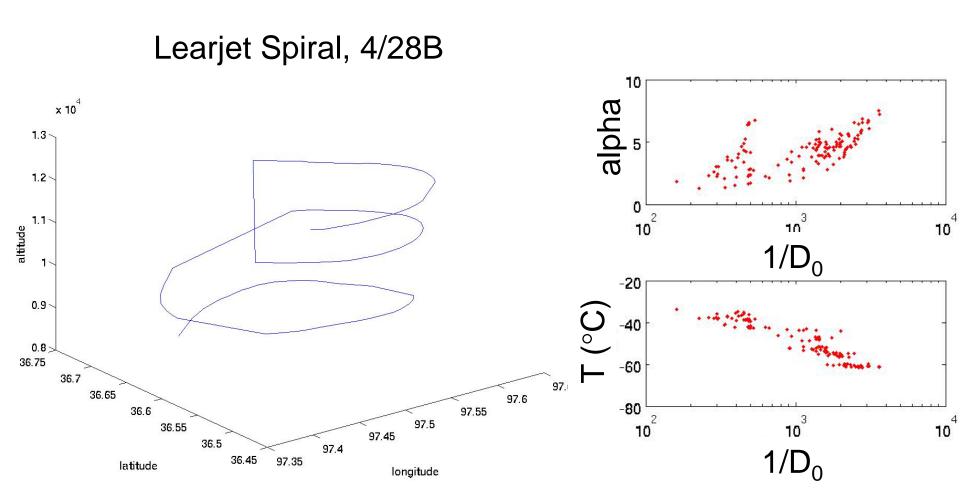
$$\operatorname{cov}\left(\begin{bmatrix} \log[N_0] \\ \log[D_0] \\ \alpha \end{bmatrix}\right) = \begin{bmatrix} 19.3851 & 0.4233 & -19.5493 \\ 0.4233 & 0.1016 & -0.6156 \\ -19.5493 & -0.6156 & 20.5703 \end{bmatrix}$$

- The covariance matrix is not diagonal
- This impacts both the estimation of remotely sensed PSD parameters as well as their uncertainty estimate

On the whole, no discernible relationship between fit prams and any single physical pram



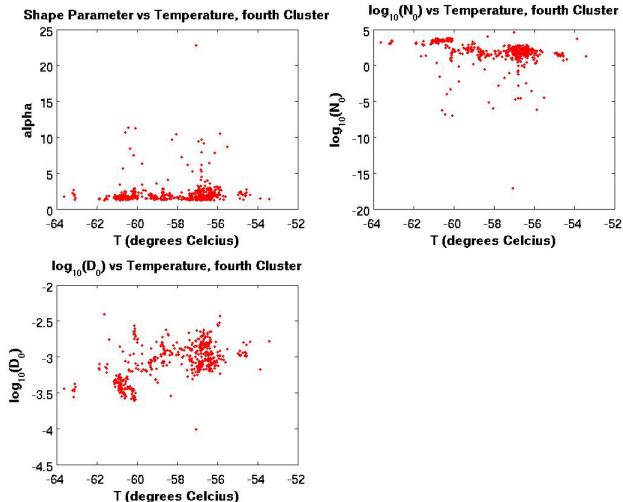
There are discernible relationships in spiral flights



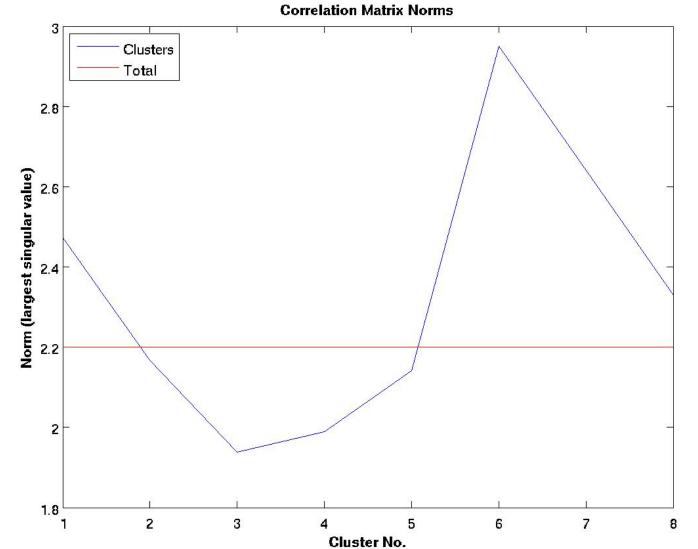
Next Question:

- Can the whole data set be partitioned so as to reveal various regions of cloud with similar PSD parameter relationships?
- Hierarchical clustering, stopped at 8 clusters
- Clustered by temperature, pressure, and relative humidity over ice

Not with clustering scheme chosen: only fourth cluster exhibits weak relationship with temperature



However, half of the clusters resulted in PSD pram covariance matrices with tighter correlations



Summary

- Within cirrus cloud PSD retrieval schemes, PSD prior distributions are often incorrectly assumed to be Gaussian
- PSD prior covariance matrices are often incorrectly assumed to be diagonal
- Retrieval schemes using the Bayesian approach must be re-formulated to account for these facts
- Although the first attempt bore sickly fruit, it is believed that proper cluster analysis can reveal PSD parameter patterns in certain physical regimes that will yield prior covariance matrices with tighter correlations