

Report No. FAA-RD-74-34-II

**PRESTRESSED CONCRETE PAVEMENTS
VOLUME II
Design and Construction Procedures for Civil Airports**

Eugene C. Odom, Paul F. Carlton

**U. S. Army Engineer Waterways Experiment Station
Soils and Pavements Laboratory
Vicksburg, Miss. 39180**

PAVEMENT
BRANCH
FILE COPY



PAVEMENT
BRANCH
FILE COPY

**NOVEMBER 1974
FINAL REPORT**

Document is available to the public through the
National Technical Information Service,
Springfield, Virginia 22151

Prepared for

PAVEMENT
BRANCH
FILE COPY

**U.S. DEPARTMENT OF TRANSPORTATION
FEDERAL AVIATION ADMINISTRATION
Systems Research & Development Service
Washington, D.C. 20590**

NOTICE

This document is disseminated under the sponsorship of the Department of Transportation in the interest of information exchange. The United States Government assumes no liability for its contents or use thereof.

1. Report No. FAA-RD-74-34-II		2. Government Accession No.		3. Recipient's Catalog No.	
4. Title and Subtitle PRESTRESSED CONCRETE PAVEMENTS; VOLUME II: DESIGN AND CONSTRUCTION PROCEDURES FOR CIVIL AIRPORTS				5. Report Date November 1974	
				6. Performing Organization Code	
7. Author(s) Eugene C. Odom, Paul F. Carlton				8. Performing Organization Report No.	
9. Performing Organization Name and Address U. S. Army Engineer Waterways Experiment Station Soils and Pavements Laboratory Vicksburg, Miss. 39180				10. Work Unit No (TRAIS)	
				11. Contract or Grant No. FAY1WAI-218	
12. Sponsoring Agency Name and Address Federal Aviation Administration Systems Research & Development Service Washington, D. C. 20591				13. Type of Report and Period Covered Final report	
				14. Sponsoring Agency Code	
15. Supplementary Notes					
16. Abstract <p>This volume of this report recommends practices and procedures for design and construction of prestressed concrete pavements for civil airports. Volume I of the report describes and presents an analysis of the instrumentation and load tests of a prestressed concrete test road.</p> <p>For the design procedure, the basic load-stress relationships were developed from small-scale model tests employing static loadings. Stresses computed from Westergaard's theory for elastic behavior were adjusted by moment correction factors to reflect the redistribution of moments resulting from partial hinges that develop under the load. Effects of repetitive moving loads were examined both in small-scale models and in full-scale prototype test pavements. The design procedure permits interrelating magnitude of loading, load repetitions, flexural strength, subgrade conditions, pavement thicknesses, slab dimensions, and magnitude of prestress. Consideration also is given to the effects of elastic shortening, creep, and shrinkage of concrete, relaxation in steel tendons, anchorage systems, tendon friction, subgrade restraint, and temperature changes.</p> <p>Construction procedures and alternatives are examined based on a study of prototype test pavements and operational prestressed facilities constructed in this country and abroad. Recommendations are based on assessments of the relative merits of prestressing with and without tendons, pretensioning versus posttensioning, types of stressing tendons and conduits, bonded versus nonbonded tendons, expansion joints and joint seals, and subgrade friction-reducing layers.</p>					
17. Key Words Construction procedures Design procedures Load-stress relationships Prestressed concrete pavements			18. Distribution Statement Document is available to the public through the National Technical Information Service, Springfield, Va. 22151		
19. Security Classif. (of this report) Unclassified		20. Security Classif. (of this page) Unclassified		21. No. of Pages 77	22. Price

PREFACE

The preparation of this report was authorized under Inter-Agency Agreement FA71WAI-218, "Development of Airport Pavement Criteria," funded by the Federal Aviation Administration.

The study was conducted during August 1971-December 1973 under the general supervision of Mr. James P. Sale, Chief, Soils and Pavements Laboratory, of the U. S. Army Engineer Waterways Experiment Station (WES). This report was prepared by Messrs. Eugene C. Odom of WES and Paul F. Carlton of the Office, Chief of Engineers, U. S. Army.

Directors of WES during the conduct of the investigation and the preparation of this report were BG E. D. Peixotto, CE, and COL G. H. Hilt, CE. Technical Director was Mr. F. R. Brown.

TABLE OF CONTENTS

INTRODUCTION	7
BACKGROUND	7
PURPOSE AND SCOPE	9
DEVELOPMENT OF AN ANALYTICAL APPROACH TO DESIGN	11
DESIGN CONCEPT	11
LOAD-STRESS RELATIONSHIPS	12
SUBGRADE RESTRAINT	30
TEMPERATURE WARPING STRESS	31
DISCUSSION OF DESIGN METHOD	32
STRESS LOSSES IN PRESTRESSING TENDONS	36
ELASTIC SHORTENING OF CONCRETE	36
CONCRETE CREEP	37
CONCRETE SHRINKAGE	38
STEEL RELAXATION	39
ANCHORAGE LOSSES	39
TENDON FRICTION	40
APPLICATION OF DESIGN PROCEDURE	42
DESIGN EXAMPLE	42
DESIGN TABLE	43
STRESS LOSSES	45
DETERMINATION OF TENDON SIZE AND SPACING	51
DESIGN ALTERNATIVES	53
SYSTEMS FOR PRESTRESSING CONCRETE	53
AMOUNT OF PRESTRESSING	55
STRESSING TENDONS	56
CONDUITS	56
PLACEMENT OF STRESSING TENDONS	57
SUBGRADES AND SUBBASES	57
FRICTION-REDUCING LAYERS	58
SLAB DIMENSIONS	59
PROPERTIES OF CONCRETE	59
EXPANSION JOINTS	60
EXPANSION JOINT SEALS	62
CONSTRUCTION PROCEDURES	68
SUBBASE	68
FRICTION-REDUCING LAYER	68
PLACEMENT OF STRESSING TENDONS	68
PLACEMENT AND CURING OF CONCRETE	69
TENDON STRESSING	69
GROUTING	70
CONCLUSIONS	72

RECOMMENDATIONS	73
REFERENCES	74
BIBLIOGRAPHY	76

CONVERSION FACTORS, U. S. CUSTOMARY TO METRIC (SI)
UNITS OF MEASUREMENT

U. S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

<u>Multiply</u>	<u>By</u>	<u>To Obtain</u>
inches	2.54	centimeters
feet	0.3048	meters
square inches	6.4516	square centimeters
pounds	4.448222	newtons
pounds	0.45359237	kilograms
kips	0.45359237	metric tons
pounds per square inch	0.6894757	newtons per square centimeter
pounds per cubic inch	0.0276799	kilograms per cubic centimeter
pounds per cubic foot	16.01849	kilograms per cubic meter
Fahrenheit degrees	5/9	Celsius degrees or Kelvins*

* To obtain Celsius (C) temperature readings from Fahrenheit (F) readings, use the following formula: $C = (5/9)(F - 32)$. To obtain Kelvin (K) readings, use: $K = (5/9)(F - 32) + 273.15$.

INTRODUCTION

BACKGROUND

Prestressing to strengthen concrete has been used widely and successfully for bridges, buildings, storage tanks, and pressure pipes in the past 25 years; however, only a modest interest and limited investment of research funds have been directed at prestressed concrete as a pavement, particularly for airports. As a result, the current state-of-the-art in the design and construction of such pavements is not highly developed. Since the first prestressed concrete pavement on record (a bridge approach constructed at Luzancy, France, in 1946¹), only about 100 prestressed pavement test sections and test slabs have been constructed throughout the world. Historically, these sections have been about evenly divided between airports and highways. The most recent of these were prestressed pavement highway sections on an access road to Dulles International Airport and in Pennsylvania, which were designed and constructed by the Federal Highway Administration and the Pennsylvania Department of Transportation, respectively.

Gross weights of current and proposed commercial aircraft have reached such proportions and flight operations have reached such intensities that as much as 16 in.* or more of plain concrete may be required to provide an adequate pavement. In view of recent increased concern over a more effective use of the Nation's resources, there is a basis for renewed interest in the search for an improved method of constructing pavements. The desire to evaluate thoroughly the possible expanded role of prestressed pavements stems primarily from three basic advantages such pavements offer over conventional rigid pavements. First, it has been demonstrated, both analytically and by testing, that prestressed pavements permit a substantial reduction in pavement thickness (50 percent or more), with corresponding savings in construction materials and possibly costs. Second, prestressed pavements can be designed

* A table of factors for converting U. S. customary units of measurement to metric (SI) units is presented on page 5.

with fewer joints, a characteristic which results in quieter and smoother rides and eliminates the need for costly sealants and resealing programs. Third, the lesser number of joints and lower probability of crack formation (both load and nonload associated) can be projected into the likelihood of extended pavement life and reduced maintenance requirements.

There are, however, potential disadvantages associated with prestressed pavements which must also be considered. First, there is an increase in the complexity of construction which could lead to higher costs offsetting the savings of materials. There are strong indications that improved construction techniques which will result from increased usage of prestressed pavements will keep these costs minimal. Second, the joints that are required in a prestressed pavement are expensive to construct, and, due to the large horizontal movements, they are difficult to maintain. However, through the use of improved materials and construction techniques, more durable joint systems will probably be developed.

Data from only a relatively few full-scale test pavements, laboratory tests on small-scale models, and observations of the performance of a limited number of operational airport pavements are available to serve as a basis for extending and refining design and construction procedures. A review of the approaches to design employed in the construction of the various prestressed pavement test sections²⁻⁶ revealed that few attempts have been made to develop these designs by analytical techniques. Generally, pavement thicknesses and amounts of prestress have been selected on an arbitrary basis. Most highway pavement test sections have been 6 in. thick with only longitudinal prestressing, while airport pavement thicknesses have reached 9 in. with both longitudinal and transverse prestressing. Frequently, such empirical designs were subjected to static load tests following completion of the construction in order to evaluate the load-carrying capability of the pavement. Also, frequent attempts were noted at making quantitative assessments for such design parameters as: (a) the effects of hygrothermal stresses, (b) subgrade restraint, and (c) prestress losses associated with the stressing tendons and anchorage systems. Other than in studies

conducted by the Corps of Engineers, little evidence was found to indicate that the effect of frequency of load applications has been considered in design.

It is recognized that considerable research remains to be accomplished before completely validated design criteria can be established and the various construction techniques optimized. Based on the information that has been developed to date, it is considered that sufficient data are available to formulate interim criteria for the design and construction of prestressed pavements for airports. As with any criteria, however, refinements based upon performance and reanalyses of data will be needed for the validation process.

PURPOSE AND SCOPE

The purpose of this study was to develop suitable procedures based on available data for the design and construction of prestressed pavements at airports serving the civil aviation community.

The procedures presented herein are based upon state-of-the-art technology developed from a review of the literature. No field tests, other than some preliminary load-deflection measurements made on the Dulles International Airport access highway prestressed pavement constructed by the Federal Highway Administration, were conducted to develop or verify the procedures presented herein. The scope of the study included: (a) a review of literature included in the references and bibliography in this report, (b) selection of the design concepts that have been best validated by experimentation, (c) formulation of the design procedures, and (d) description of recommended construction procedures.

In reviewing the research that has been conducted to date, studies pertaining to prestressed highway pavements were included in addition to those for airport pavements. However, because of the differences in design requirements for highway and airport pavements, primary consideration was given to the research pertaining to airport pavements. In developing the procedures presented herein, the following items are discussed:

- a. Analytical concept for design.
- b. Selection of prestressing method.
- c. Types and properties of construction materials.
- d. Expansion joints.
- e. Effects of environmental parameters.

DEVELOPMENT OF AN ANALYTICAL APPROACH TO DESIGN

DESIGN CONCEPT

Before an analytical method of design for prestressed concrete pavements can be postulated, the mechanisms involved in determining load-stress-deflection interrelationships within the pavement structure must be understood, and the conditions leading to failure must be defined. To understand the behavior patterns of prestressed concrete pavements, it is helpful to first review those of conventional concrete pavements. Design procedures, such as Westergaard's, for conventional rigid pavements assume that load-induced stresses remain within the range of elastic behavior for the concrete. This characteristic in turn requires that the tensile stresses developed in the extreme fibers of the slab be limited to a value less than the flexural strength of the concrete.

The permanent compressive forces induced by prestressing can be used to increase the effective flexural strength of the concrete. For prestressed pavements designed based on elastic theory, this increase in strength permits some reduction in pavement thickness. Such thickness reductions, however, are relatively small with respect to the apparent increase in flexural strength.

If the structural benefits derived from prestressing were limited merely to increasing the stress range for elastic behavior of a rigid pavement, it is doubtful that prestressed pavements ever would have received serious consideration insofar as adapting this type of pavement to the needs of airports. Fortunately, prestressing permits the structural behavior of such pavements to be analyzed under concepts completely different from those employed in the familiar Westergaard analysis or other elastic theories applicable to the design of nonreinforced rigid pavements.

In addition to providing an increase in the stress range for purely elastic behavior, the presence of the prestress permits the pavement to be loaded beyond the elastic range. This characteristic in turn

results in the load-carrying capacity of the pavement being substantially increased over the limit of loading imposed by conventional linear elastic theory. Therefore, a revision is required in the criteria for defining pavement failure insofar as prestressed concrete pavements supporting operations of high-performance commercial aircraft are concerned.

For nonreinforced rigid airport pavements, failure usually is defined by the occurrence of structural cracking that is initiated by excessive tensile stresses induced in the bottom of the pavement and that extends through the full depth of the pavement. For prestressed concrete pavements, however, the initial tensile cracking that occurs in the bottom of the pavement serves as a momentary or partial plastic hinge under passage of the load imposed by the aircraft wheels. As the load passes, the force of the prestress closes the cracks, and the pavement regains most of its original rigidity.

In conjunction with the momentary hinging action which occurs under passage of the load induced by the aircraft wheels, there is a redistribution of the bending moments in the pavement such that the negative radial moments are increased substantially. After these negative moments have increased sufficiently to produce tensile cracking in the upper surface of the slab, a nominal increase in load or repetitions will cause severe cracking in the slab. Thus, for purpose of design, failure of a prestressed pavement has been defined as the occurrence of this secondary tensile cracking in the upper surface of the pavement.

Confirmation of this concept of prestressed pavement behavior has been established both theoretically and experimentally through independent research investigations of Levi,⁷ Cot and Becker,⁸ Carlton and Behrmann,⁹ and others.

LOAD-STRESS RELATIONSHIPS

The design procedure proposed in this report is based on determining the aircraft-induced load stresses by a two-step procedure. First, using the design load positioned in the interior portion of a pavement slab, the maximum stress is computed for elastic behavior using

the Westergaard solution. Second, the stress thus computed is then adjusted based on corrective moments reflecting the redistribution of moments due to the formation of the partial plastic hinge under the load. The moment correction factors used in this report are based on the results of small-scale model tests employing static loads. The design procedure does require an iterative approach in that the load-induced stress is determined on the basis of an assumed pavement thickness. Successive computations based on various pavement thicknesses may be required to arrive at a load stress that is acceptable.

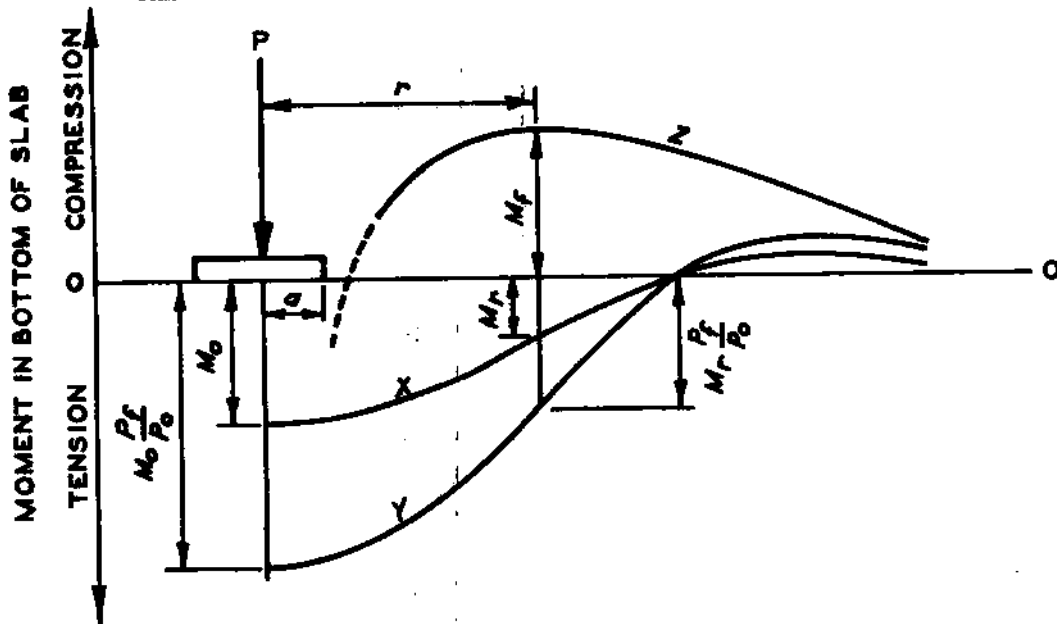
THE MODELS

Detailed descriptions of the small-scale model tests are contained in References 9-13. Essentially, the models consisted of 0.20-in.-thick, 16.6-in.-square neat gypsum cement prestressed slabs supported by a 12-in.-thick rubber "subgrade" with a modulus of subgrade reaction of 35 pci. The model slabs were equally prestressed longitudinally and transversely using tendons of piano wire placed at middepth of each slab. Static loads simulating single- and multiple-wheel gear loads were applied through rigid dies scaled to actual aircraft landing gear dimensions.

A total of 45 model slabs, 12 pretensioned and 33 posttensioned, were constructed and tested. Single-wheel loading tests were made on 24 of the model slabs, and multiple-wheel loading tests were made on the remaining 21 slabs. The three multiple-wheel configurations included in the tests were twin wheels, twin-tandem wheels, and twin-twin wheels.

Each slab was instrumented with SR-4 strain gages, and strain measurements were made at selected load intervals such that load-strain relationships were obtained through the elastic range and after the formation of the momentary hinge. Stresses and moments computed from the measured strains were used to analyze the effects of variations in loading as well as the redistribution of bending moments which occurred subsequent to the formation of initial cracking in the lower surface of the slab. Incremental increases in loading were continued until excessive negative radial moments induced cracking in the upper surface of the slab.

In analyzing the data obtained from the model tests, it was assumed that tensile cracking occurred in the top surface of the slab when the maximum negative radial moment equaled the magnitude of the positive moment which produced the initial cracking in the bottom surface of the slab. A schematic diagram shown in Figure 1 illustrates



- h = SLAB THICKNESS
 - E = MODULUS OF ELASTICITY OF SLAB
 - μ = POISSON'S RATIO OF SLAB
 - K = MODULUS OF SUBGRADE REACTION
 - ξ = RADIUS OF RELATIVE STIFFNESS = $\left[\frac{Eh^3}{12(1-\mu^2)K} \right]^{1/4}$
 - a = RADIUS OF LOADED AREA
 - r = RADIUS OF NEGATIVE CRACK
 - P_o = LOAD PRODUCING CRACKING IN BOTTOM OF SLAB
 - P_f = LOAD PRODUCING CRACKING IN TOP OF SLAB
 - M_o = MAXIMUM POSITIVE MOMENT DUE TO P_o
- $$M_o = \frac{P_o(1+\mu)}{16\pi} \text{LOG} \frac{Eh^3}{Ka^4}$$
- M_f = MAXIMUM NEGATIVE RADIAL MOMENT AT DISTANCE r FROM CENTER OF LOAD AT LOAD P_f
 - M_r = RADIAL MOMENT AT DISTANCE r FROM CENTER OF LOAD AT LOAD P_o

Figure 1. Load-moment relationships for analysis of prestressed models

the relationships between radial moments for elastic and inelastic conditions in the slab.

In Figure 1, Curve X represents the radial moment in the bottom of the slab at the initial crack load P_o for the elastic condition. Had the slab continued to behave elastically after formation of the initial cracking, the radial moment for P_f (the load causing cracking in the top surface of the slab) would have been proportional to that for P_o . Moments increased by the ratio of P_f/P_o are indicated by Curve Y. Since the slab does not perform elastically after the initial cracking, the actual radial moment for load P_f is indicated by Curve Z. Values for Curve Z were determined experimentally and indicate the maximum negative radial moment occurring in the upper surface of the slab at a distance r from the center of the loaded area. Thus, r is the distance to the initial cracking in the upper surface that defines failure for prestressed pavements.

The difference in moment between Curves Y and Z at a distance r that is associated with increasing the load from P_o to P_f is expressed by the equation

$$\Delta M = M_r \frac{P_f}{P_o} - M_f \quad (1)$$

where

P_f = negative cracking load, lb

P_o = initial cracking load, lb

M_r = radial moment at a distance r from the center of load P_o

M_f = maximum negative radial moment at a distance r from the center of load P_f

This difference in radial moment at r can then be related to the maximum positive elastic moment per pound of applied load by a moment correction factor C as follows:

$$\Delta M = CM_o \frac{P_f}{P_o} \quad (2)$$

Since it is assumed that the negative cracking moment M_f is equal to

the positive cracking moment M_o , the moment correction factor can be obtained from Equations 1 and 2 by

$$C = \frac{\frac{M_r}{P_o} + \frac{M_o}{P_f}}{\frac{M_o}{P_o}} \quad (3)$$

Theoretical and experimental curves of radial moment per pound of wheel load for various offset distances from the center of the loaded area are shown in Figure 2. The data points defining the theoretical curve in Figure 2 were calculated using the influence charts presented by Pickett and Ray.¹⁴ The experimental curve in Figure 2 was obtained from SR-4 strain gage measurements of the tangential and radial strains at various offset distances. These strain data were used to calculate the radial moments needed to plot the experimental curve, which along with the theoretical curve shows the generally good agreement between the model and theory for loadings within the range of elastic behavior. This degree of agreement formed the basis for accepting the model as a valid representation of the performance of prestressed pavements under loadings producing elastoplastic behavior.

General curves of theoretical values of maximum radial moment per pound of wheel load under the wheel are shown in Figure 3 for both circular and elliptical tire contact areas. By plotting the moment coefficient M_o/P_o versus the nondimensional scale of loaded area divided by the radius of relative stiffness squared A/l^2 , the relationships depicted are applicable to all values of tire contact area, pavement thickness, and subgrade strength for the model slabs as well as for full-scale pavements.

On the basis of specific values for M_r/P_o , M_o/P_f , and M_o/P_o , it was possible to determine radial moment correction factors for the model slabs in accordance with Equation 3. The interrelationship of a/l , r/l , and the radial moment correction factor C is presented graphically in Figure 4. Similarly, the interrelationship of a/l , r/l ,

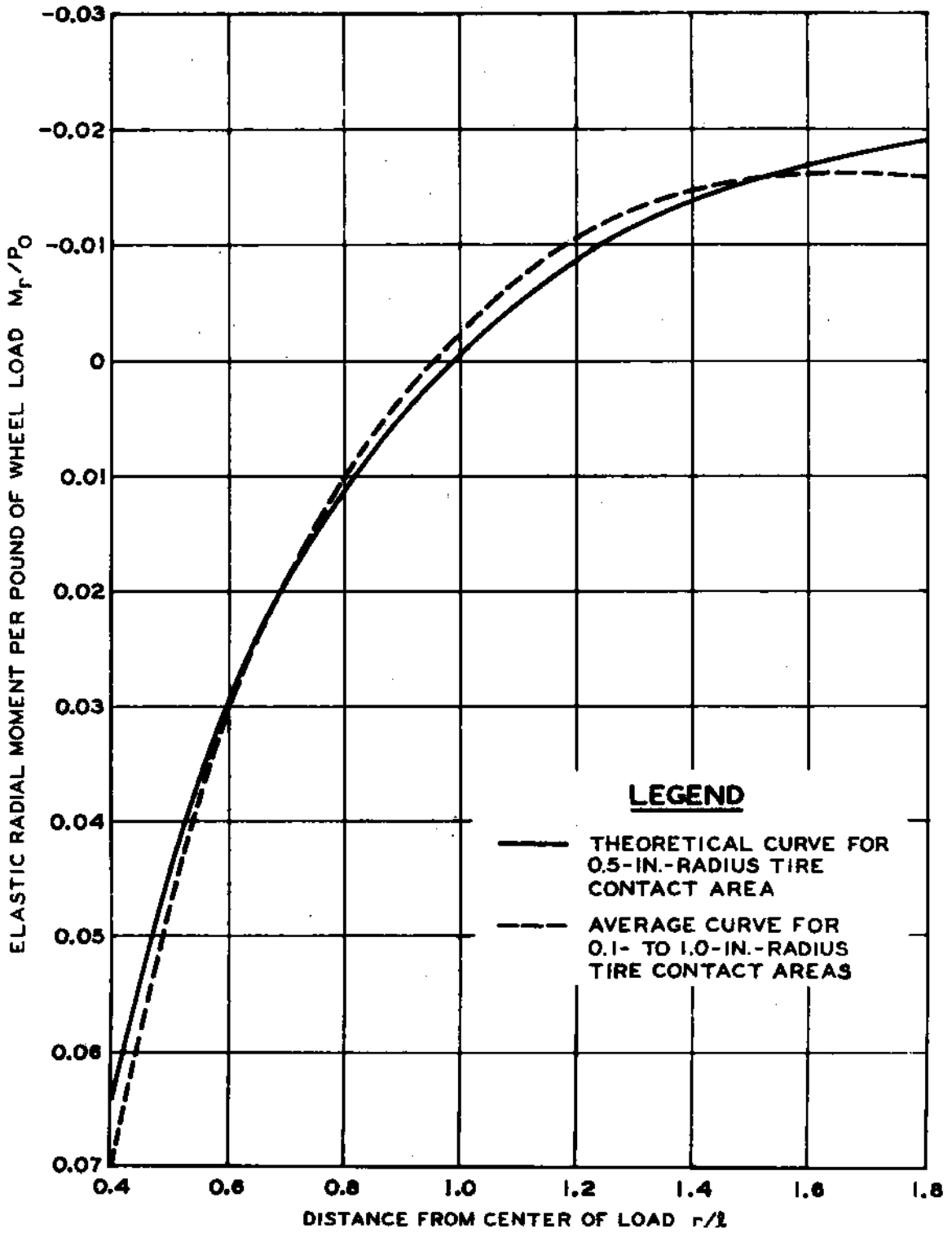


Figure 2. Radial moment coefficient M_r/P_0 for elastic behavior in the model (after Sale et al.¹⁰)

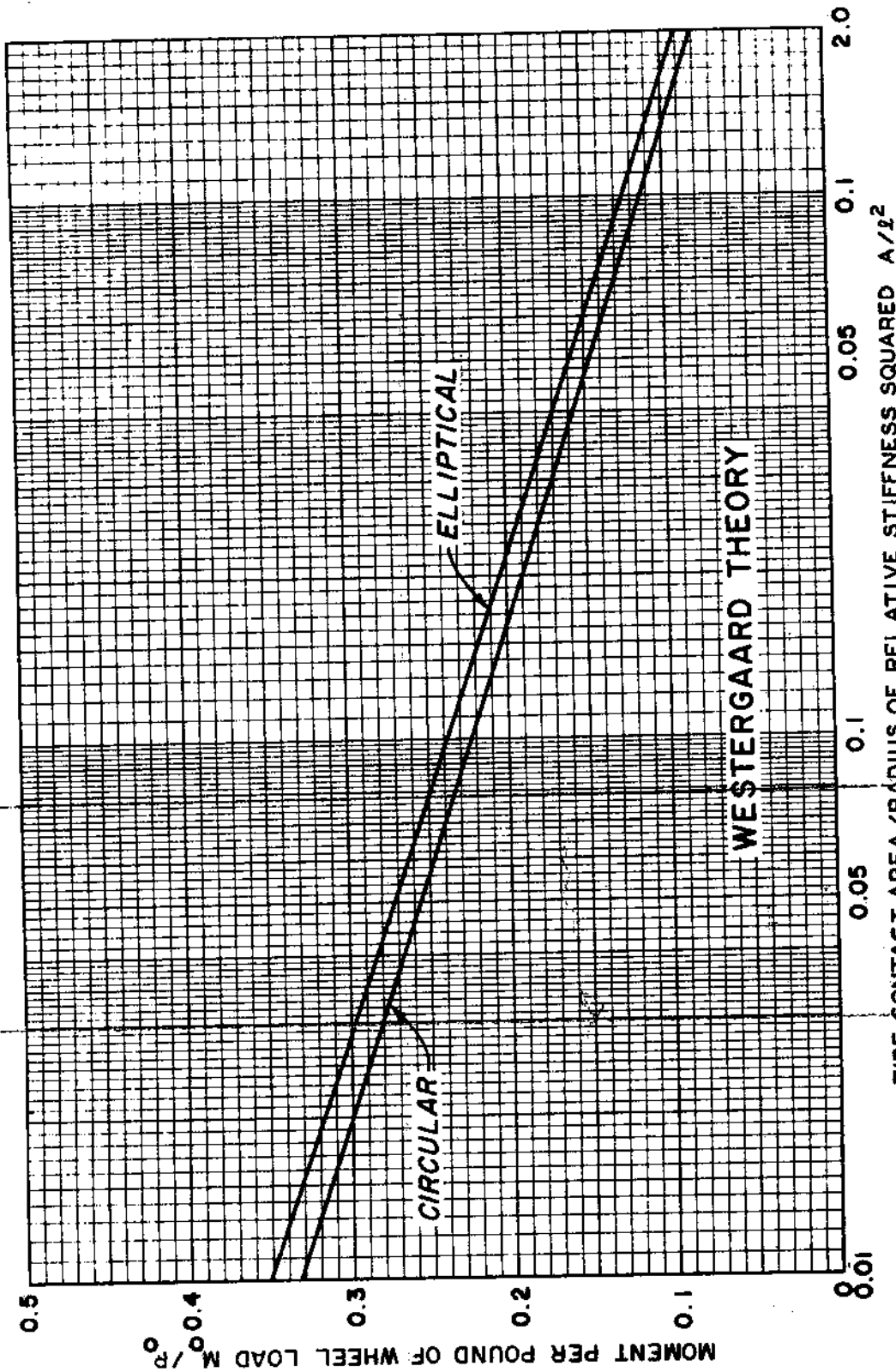


Figure 3. M_0/P_0 versus A/l^2 for interior loading of a single wheel (after Sale et al.¹⁰)

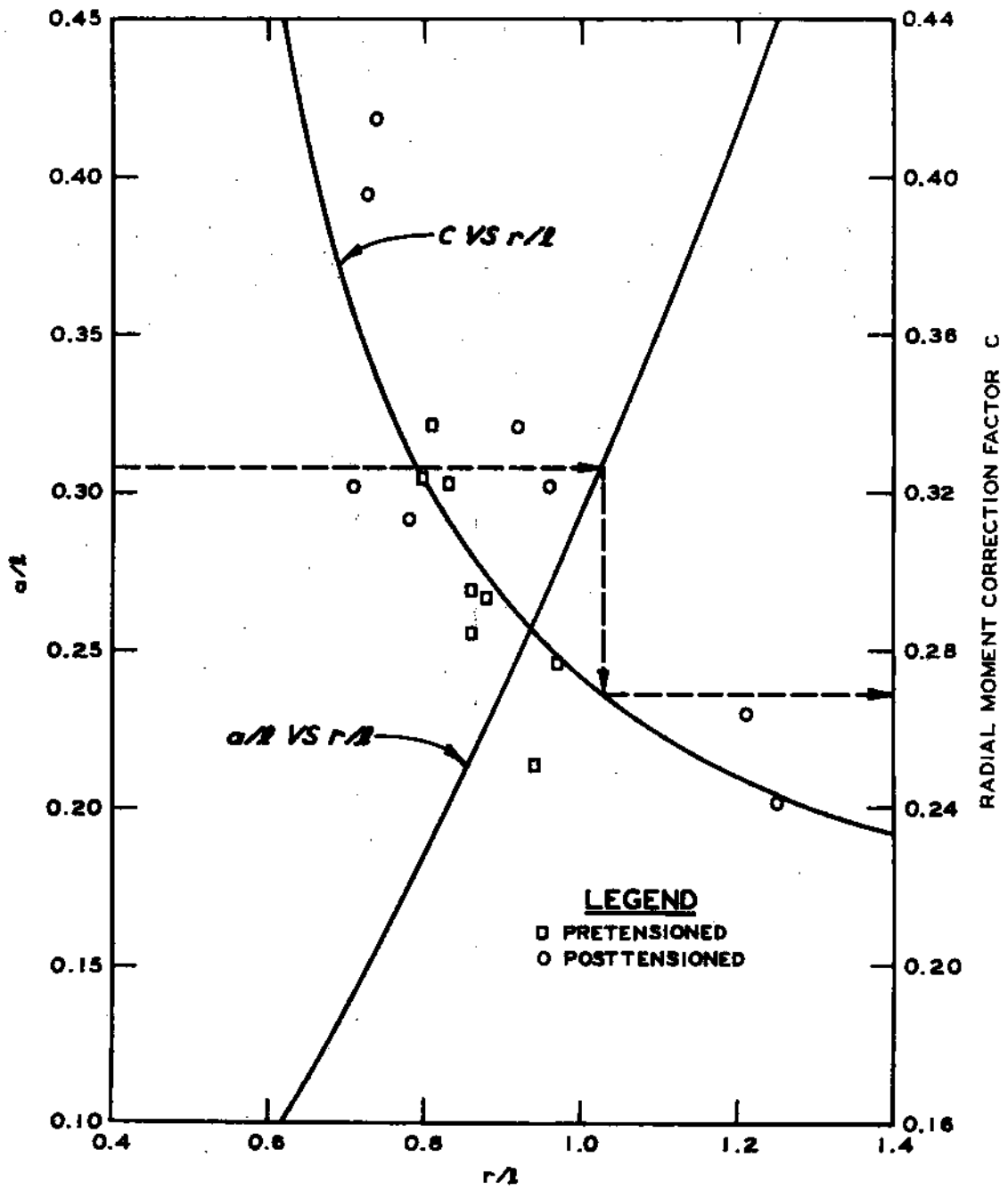


Figure 4. Interrelationship of a/l , r/l , and C (after Sale et al.¹⁰)

and M_r/P_o (from Figure 2) is presented graphically in Figure 5. As in Figure 3, the use of nondimensional relationships for a/l and r/l permits the curves in Figures 4 and 5 to apply to full-scale pavements as well as to the model slabs.

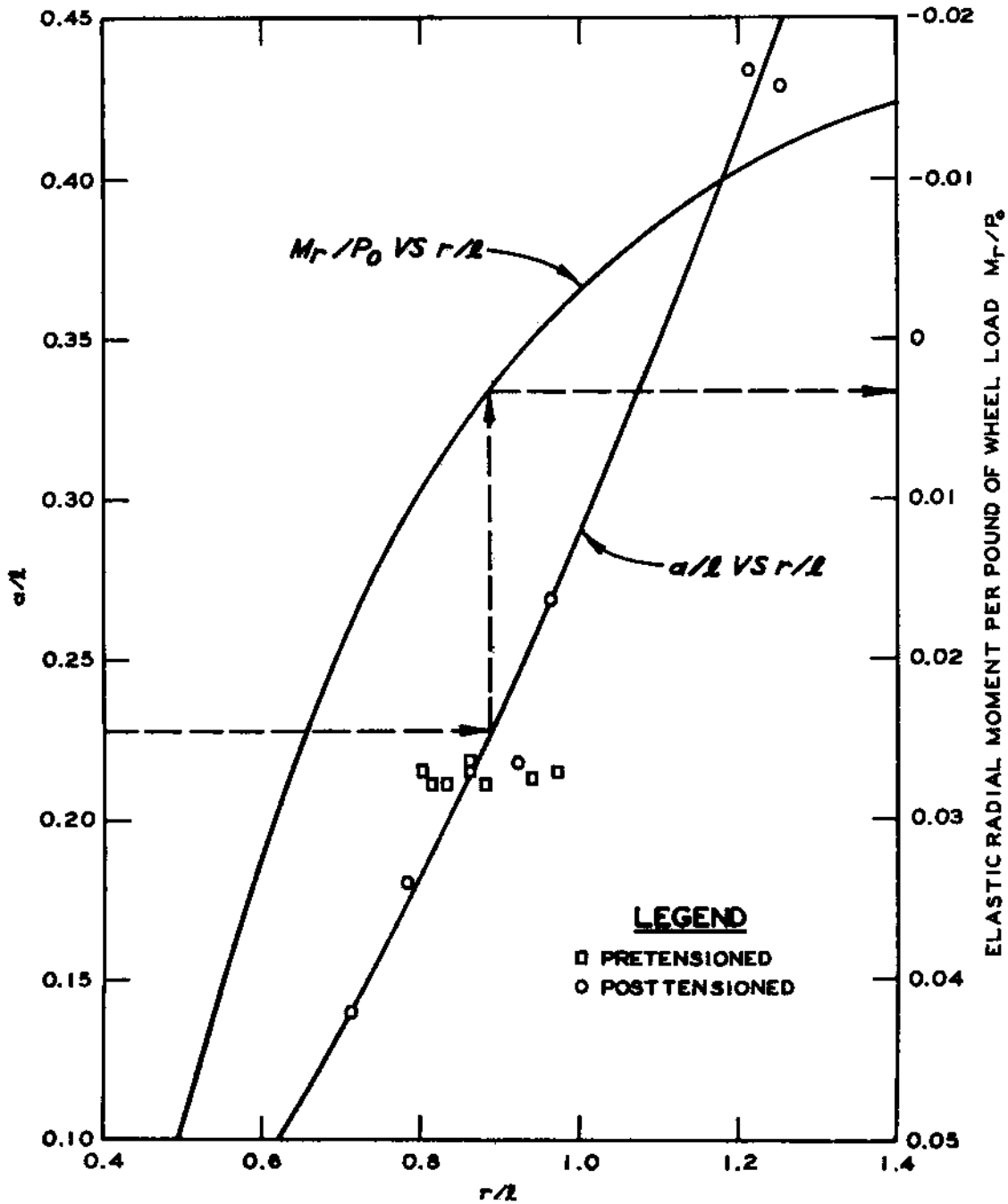


Figure 5. Interrelationship of a/l , r/l , and M_r/P_o (after Sale et al.10)

SINGLE-WHEEL LOADINGS

Once a methodology from the small-scale model tests had been established for determining the radial moment correction factors, it was possible to predict the failure load for a single static application of a single-wheel load on a particular slab by rearranging Equation 3 as follows:

$$P_f = \frac{M_o}{C \frac{M_o}{P_o} - \frac{M_r}{P_o}} \quad (4)$$

However, by definition, M_o is equal to the flexural cracking capacity of the slab so that

$$M_o = \frac{h^2}{6} (R + R_p) \quad (5)$$

where

h = thickness of the slab, in.

R = flexural strength of the slab, psi

R_p = prestress of the slab, psi

Substituting the above value for M_o into the numerator in Equation 4, the expression for P_f becomes

$$P_f = \frac{h^2 (R + R_p)}{6 \left[C \left(\frac{M_o}{P_o} \right) - \frac{M_r}{P_o} \right]} \quad (6)$$

Equation 5 was not substituted for M_o in the denominator of Equation 4 because M_o/P_o can be easily determined from Figure 3. To simplify the solution of Equation 6, Figures 3-5 were combined in a curve of the load-moment factor B versus A/l^2 , where B is equal to $C (M_o/P_o) - M_r/P_o$. This curve is shown in Figure 6. If the single-wheel failure load for a particular prestressed concrete pavement is known, the

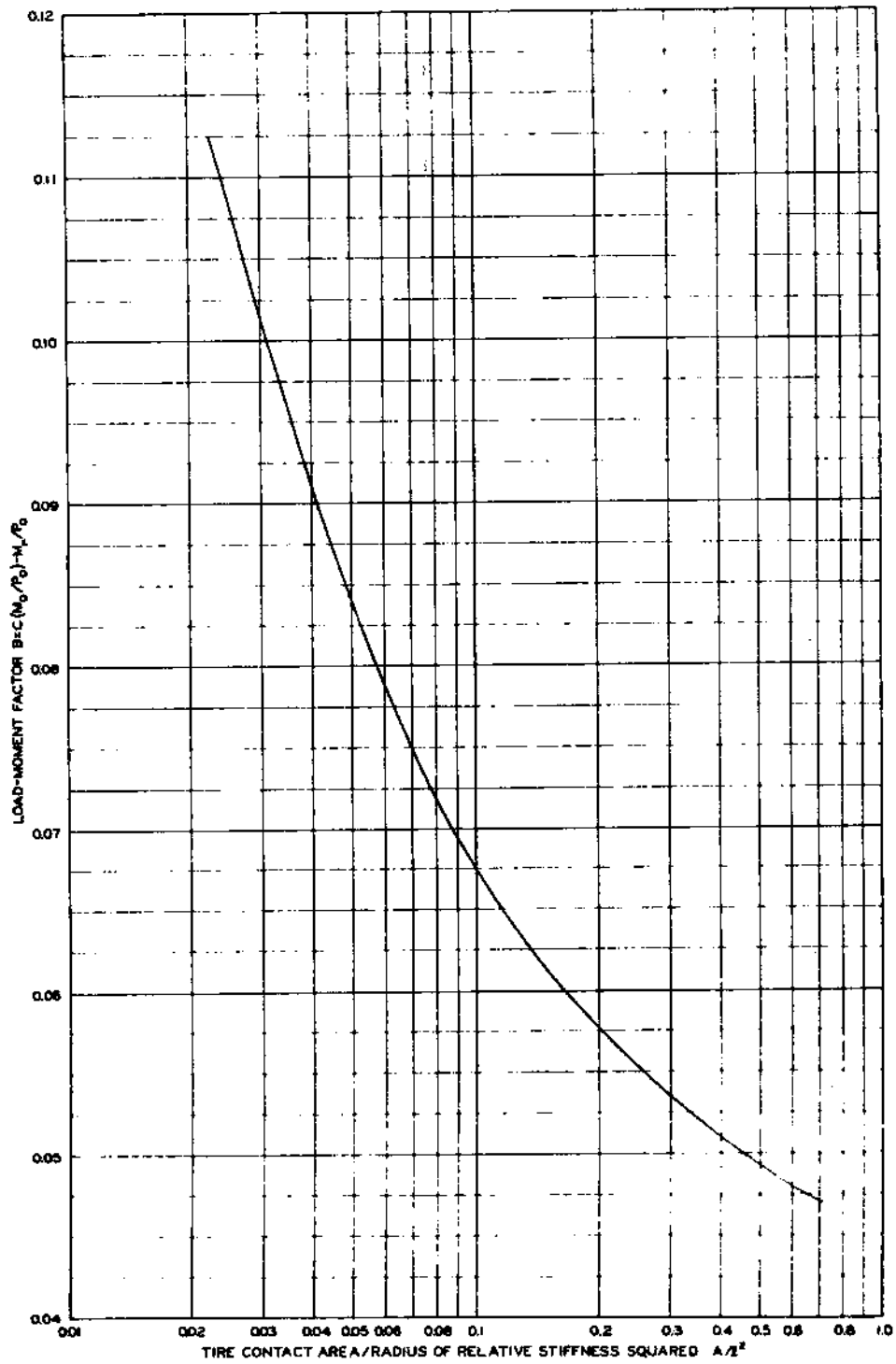


Figure 6. Load-moment factor versus A/l^2

effective prestress in the concrete can be determined by solving Equation 6 for R_p , i.e.,

$$R_p = \frac{6P_f B}{h^2} - R \quad (7)$$

Equation 7 can be used for design purposes in determining the effective prestress required in a prestressed concrete pavement by assuming that the load to be applied is the single-wheel failure load P_f . However, since Equation 7 only addresses the case of a single application of a single-wheel load, it is necessary to further expand the design procedure to include multiple-wheel loadings and repetitions of load (traffic).

MULTIPLE-WHEEL LOADINGS

The development of the relationships for M_o/P_o , C , and M_r/P_o for each multiple-wheel gear configuration found on operational aircraft would be a tedious and time-consuming effort either analytically or by means of the static load model. Earlier tests made use of the equivalent single-wheel load (ESWL) concept to simplify the analyses for multiple-wheel loadings, and this principle has been adopted and expanded in this study. Figure 7 shows the relationship between the loads producing negative cracking on three typical multiple-wheel gear configurations and the percent of single-wheel load determined by small-scale static load model tests on prestressed slabs. From this figure, an ESWL on a tire having a contact area of 267 sq in. can be determined which will produce the same negative moment causing failure in the prestressed concrete as would be produced by the multiple-wheel load. It is only necessary to divide the multiple-wheel gear loading by the percent of the single-wheel load to obtain the ESWL. By expressing the tire contact area in terms of the nondimensional A/t^2 , the relationships are made applicable to full-scale pavements as well as the model slabs. Likewise, for any given prestressed pavement, the multiple-wheel failure load can be determined using Equation 6 by multiplying the computed single-wheel failure load by the percent of single-wheel load

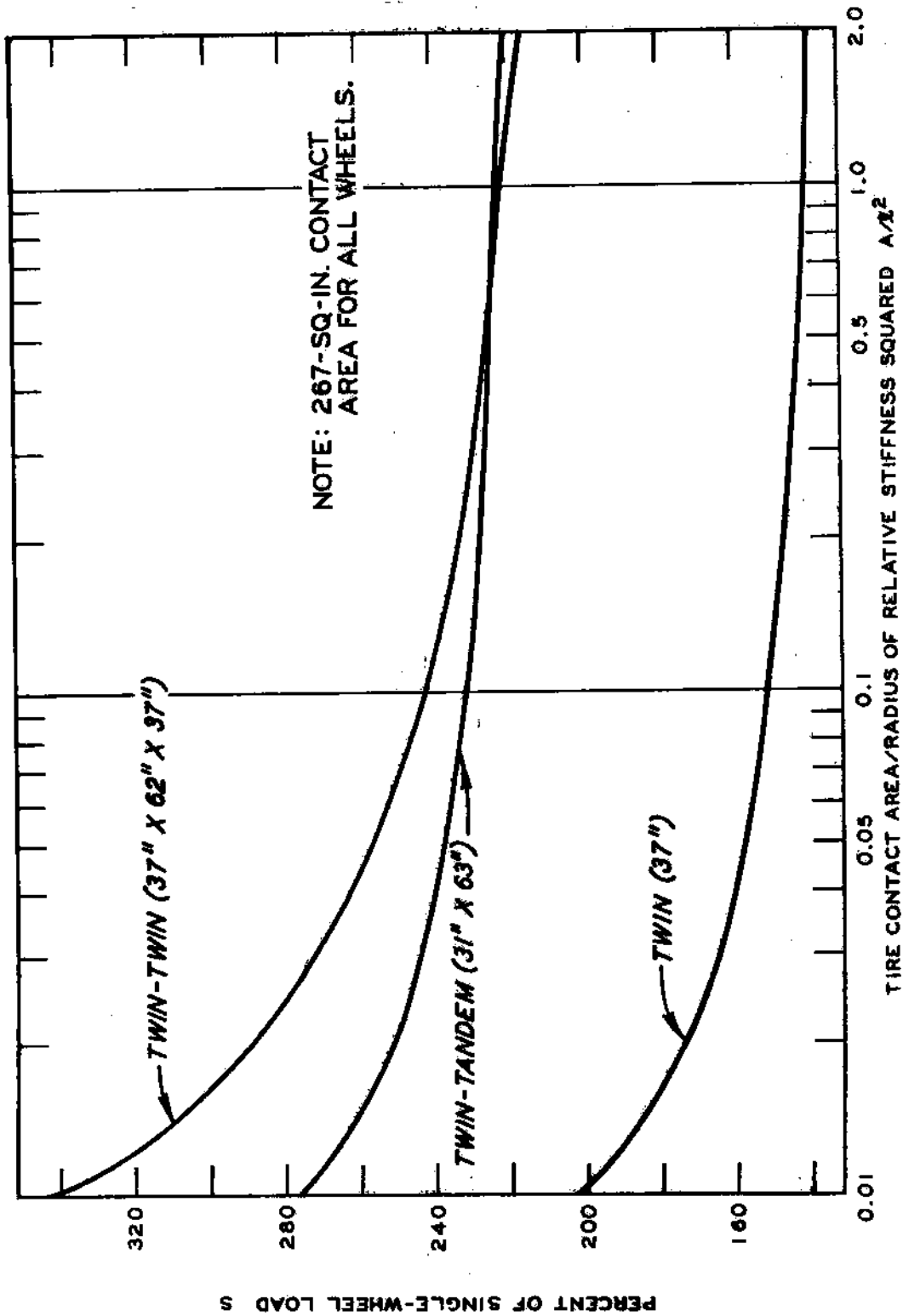


Figure 5. Multiple-wheel gear loads of military aircraft producing negative cracking (after Sale et al. 5)

determined from Figure 7. The failure load thus determined would be for a single application of the multiple-wheel load. An examination of Figure 7 shows that A/l^2 versus percent of single-wheel load is a function of the multiple-wheel gear configuration (number of wheels, contact area, wheel spacings, etc.), and thus separate relationships must be determined for each distinctly different gear configuration. Present-day civil aircraft have gear configurations that are different from those used to develop the relationships shown by Figure 7. Since the static load model was not available to develop the A/l^2 versus percent single-wheel load relationship for civil aircraft, it was necessary to provide another method. Therefore, a computer program based on a discrete element procedure for plates and slabs described in References 15-17 was used. The data derived from the static load model (Reference 10) used to develop the relationships in Figure 7 were used to establish the necessary input parameters for the computer model. These input parameters included the physical properties of the slabs and foundations. The computer program was then used to develop A/l^2 versus percent single-wheel load relationships for present-day civil aircraft as shown in Figure 8. In Figure 7, the percent single-wheel load was calculated based on the contact area of the equivalent single wheel (ESW) being 267 sq in. since this was the contact area for all three multiple-wheel gear configurations considered. For Figure 8, the contact area of the ESW used to calculate the percent single-wheel load for each aircraft was the same as the contact area of one of the tires of the particular multiple-wheel gear. The percent single-wheel load S values for the aircraft in Figure 7 were computed using the load of one of the two main gears as the multiple-wheel failure load. For the civil aircraft in Figure 8 that also have only two main gears, the values of S were computed in a similar manner. To allow the values of S for all aircraft in Figure 8 to be plotted on the same scale, the multiple-wheel failure loads of the civil aircraft that have more than two main gears were considered as being one-half of the total load on all main gears. Including this percent single-wheel load concept in the design procedure to account for multiple-wheel loadings, Equation 7 becomes

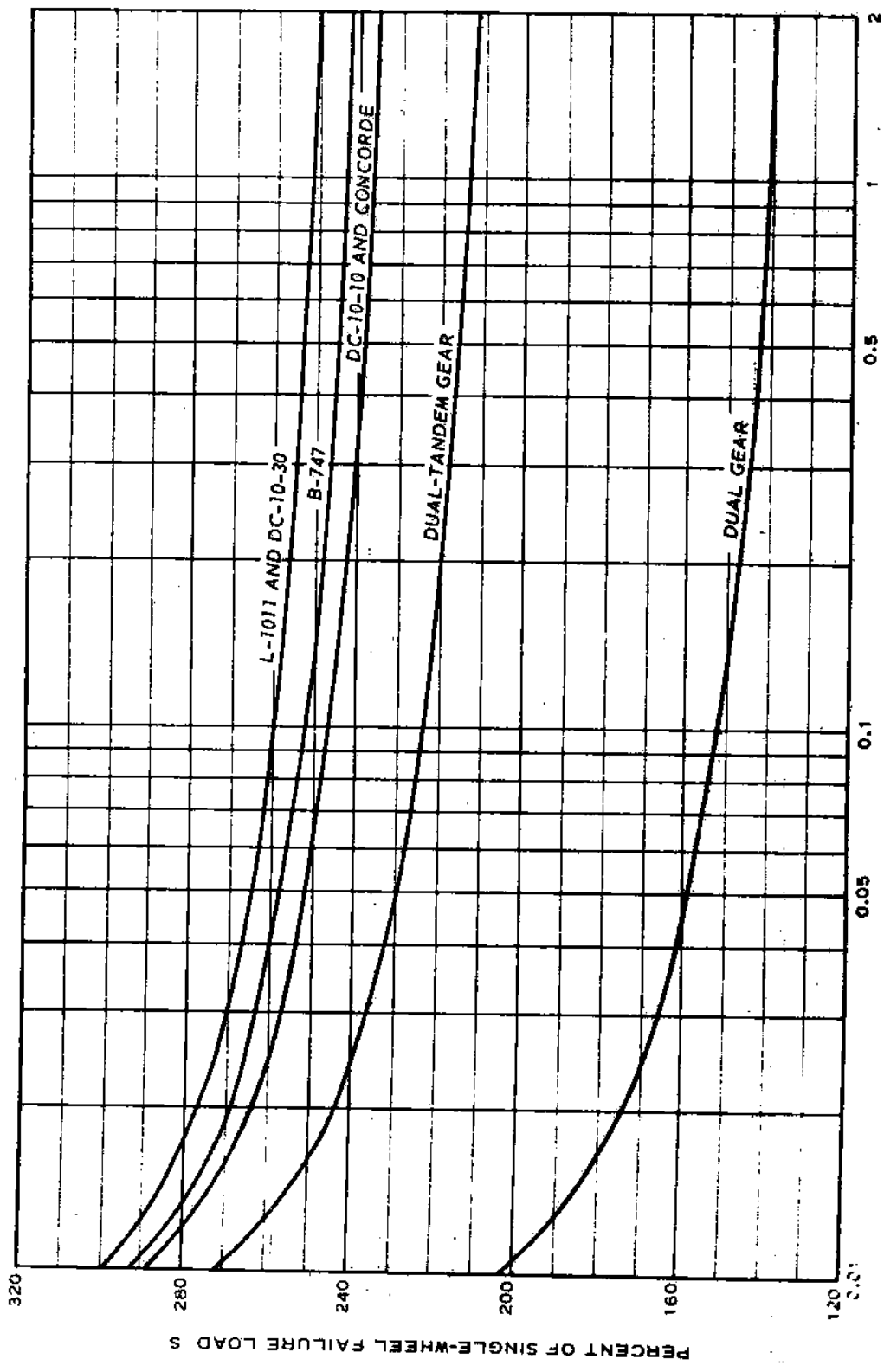


Figure 5. Multiple-wheel gear loads of civil aircraft producing negative cracking

$$R_p = \frac{6P_m B}{Sh^2} - R \quad (8)$$

where

S = percent single-wheel load for multiple-wheel gear loadings

P_m = multiple-wheel failure load, lb

LOAD REPETITIONS

The foregoing discussion outlines procedures for the determination of the level of prestress needed in a pavement of a selected thickness to withstand the single application of a given load. As with most materials, tests have indicated that prestressed pavement has a fatigue endurance level which is a function of the applied stress level and number of load applications. In the design procedure, a load repetition design factor is used which for a given load either increases the required level of prestress for a given pavement thickness or increases the required thickness for a given level of prestress, all other pertinent parameters being equal. Only a minimal amount of performance data (traffic to failure) are available for the development of the load repetition design factors for prestressed pavements. These data come from full-scale accelerated test sections reported in References 10 and 18 and from small-scale repetitive loading model tests reported in Reference 13. The plot of load repetition design factors versus traffic index (repetitions of load) is shown in Figure 9. The design factor N represents the ratio of the computed failure load for a single load application divided by the applied loading used for the traffic testing. As can be seen in Figure 9, two distinct sets of data have been obtained: one for full-scale test sections and one for small-scale repetitive loading model tests. To date, no satisfactory explanation for the variation of the data is available. It can also be seen that the small amount of performance data available are concentrated at a traffic index between 500 and 2000. The slope of the load repetition design factor versus traffic index curve is defined by these data and the value of 1.0, an assigned value which is necessary when the failure load and the applied load are equal and failure will occur at 1.0 load repetition. Considering

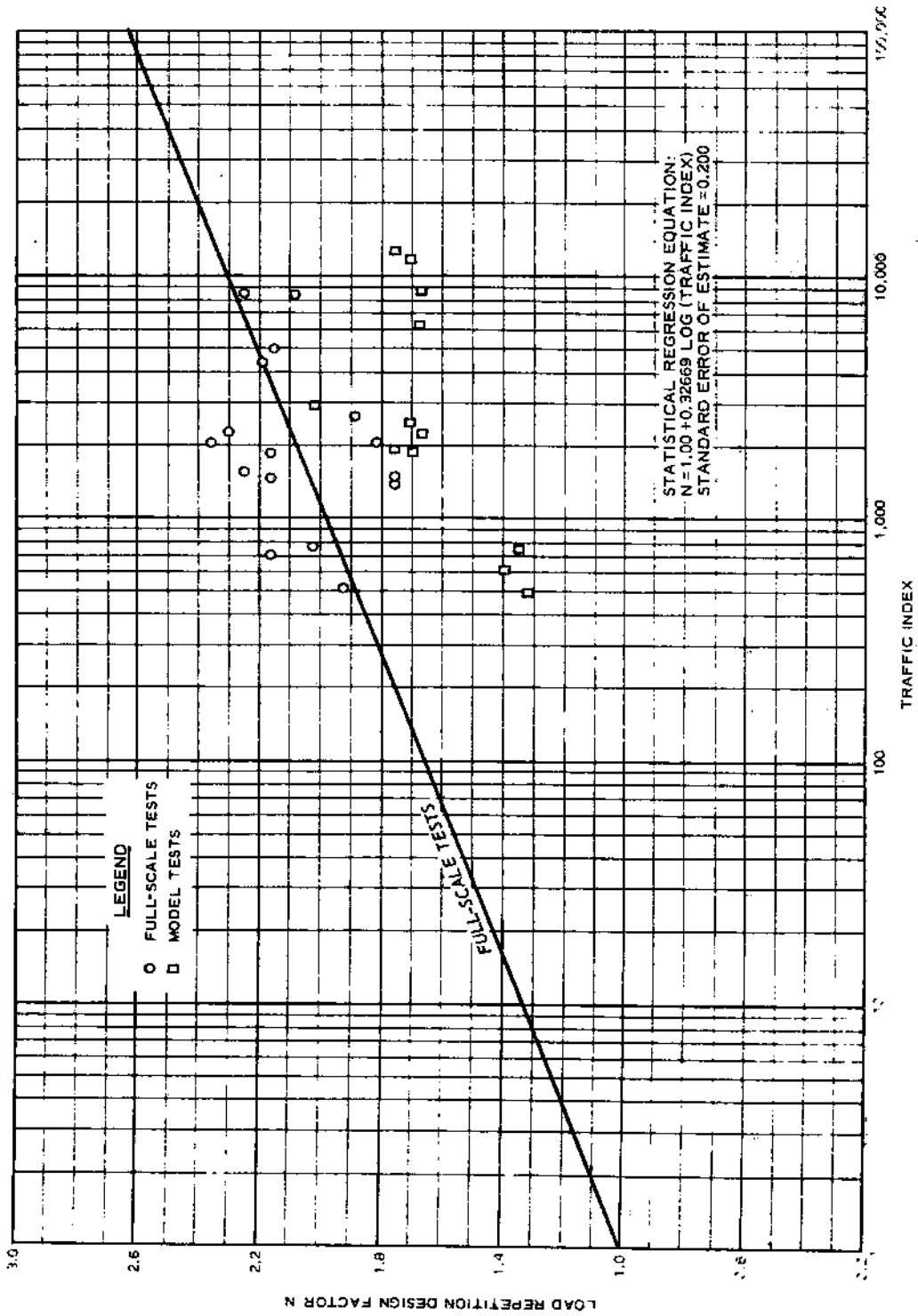


Figure 9. Load repetition design factor versus traffic index

the small amount of performance data available for prestressed pavements, it seems prudent to use conservative values for the load repetition design factor until more definitive data are available; thus, the relationships produced by the full-scale test section data have been selected for the design procedure. In the design procedure, the traffic index, which is a measure of repetitions of load, is computed by dividing the design level of aircraft passes by the appropriate conversion factor from Table 1. The conversion factors for each aircraft in Table 1 were

Table 1

Factors for Conversion of Aircraft Passes to Traffic Index

Type of Aircraft	Main Gear Spacing, in.	Tire Contact Area, sq in.	Conversion Factor	
			Taxiway	Runway
B-707	34.6 by 56	218.1	3.43	6.29
B-727	34	237.7	3.25	5.98
B-737	30.5	174.0	3.72	6.94
B-747F	44 by 58	204.0	3.82	5.34
B-747STR	44 by 58	245.6	3.48	4.87
DC-8-55-61F	30 by 55	209.0	3.35	6.33
DC-8-63F	32 by 55	220.0	3.33	6.17
DC-9	26	165.1	3.68	7.01
DC-10-10	54 by 64	294.1	3.64	5.80
DC-10-30	54 by 64	331.1	3.44	5.47
L-500	53 by 72	240.5	3.99	6.37
L-1011	52 by 70	282.0	3.64	5.88
Concorde	26.72 by 65.7	247.3	3.03	5.73

Note: Aircraft Passes ÷ Conversion Factor = Traffic Index.

calculated using the procedure from Brown and Thompson.¹⁹ The traffic index is then used to obtain a load repetition design factor N from Figure 9. Equation 8 then becomes

$$R_p = \frac{6P_m NB}{Sh^2} - R \quad (9)$$

If it is assumed that the total aircraft load is distributed 5 percent on the nose gear and 95 percent on the main gears, Equation 9 can then be rewritten as

$$R_P = \frac{2.85PNB}{Sh^2} - R \quad (10)$$

where P is the total aircraft load.

SUBGRADE RESTRAINT

Because of the large distances between joints, prestressed pavements undergo significant length changes as a result of changes in temperature within the concrete. This change in length results in stress within the concrete because of the frictional resistance between the pavement and the underlying material. The resulting stress is compressive and thus additive to the design prestress during slab elongation and is tensile and thus subtractive from the design prestress during slab shortening. The magnitude of the stress is a function of the coefficient of friction between the slab and the underlying material and the slab weight and dimensions. Assuming this stress is maximum at midlength of the slab, it can be expressed mathematically as

$$F = \frac{C_f L W h \gamma_c}{2hW(144)} = \frac{150C_f L}{288} \quad (11)$$

where

- F = subgrade restraint stress, psi
- C_f = coefficient of subgrade restraint
- L = slab length (or width), ft
- W = slab width (or length), ft
- h = slab thickness, ft
- γ_c = unit weight of concrete (assumed to be 150 pcf)

Coefficients of subgrade restraint C_f of up to 1.5 have been reported for slabs placed directly on granular material. However, it is an accepted practice to construct prestressed pavements on a friction-reducing layer such as sand, polyethylene, etc., which generally will result in C_f values ranging from 0.5 to 0.75.²⁰ For the prestressed concrete pavement at Dulles International Airport, Friberg and Pasko² reported an average C_f of 0.54. A value of 0.6 is generally accepted and recommended

for use when friction-reducing layers are used.¹⁰ Using this value of C_f , Equation 11 reduces to

$$F = 0.3125L \quad (12)$$

For convenience, the subgrade restraint values calculated using Equation 12 for various lengths and widths are listed in Table 2.

Table 2
Subgrade Restraint Stress F Values at Slab Midlength

Length ft	Longitudinal F psi	Width ft	Transverse F psi
300	94	25	8
400	125	50	16
500	156	75	23
600	188	100	31
700	219	125	39
800	250	150	47
		200	62
		250	78
		300	94

In the design of prestressed pavements, the subgrade restraint is considered as producing a tensile stress in the slab; thus, additional prestress must be added to that required for the aircraft loading and traffic. The design prestress determined by Equations 10 and 12 then becomes

$$R_p = \frac{2.85PNB}{Sh^2} - R + F \quad (13)$$

The design prestress is critical at the midpoint of the slab because F is at its maximum at the slab midpoint.

TEMPERATURE WARPING STRESS

Warping occurs in a concrete slab due to a variation in the temperature gradient through the slab thickness. When the temperature at the top of the pavement is lower than that at the bottom or vice versa, the pavement tends to warp toward the side with the lower temperature.

This warping is restricted by the weight of the slab. Thus, the side which has the lower temperature will be put into tension, while the opposite side will be put into compression. The maximum temperature warping stress may be computed by the following equation taken from Reference 21:

$$F_t = \frac{E_c t \epsilon_t}{2(1 - \mu)} = 15t \quad (14)$$

where

- F_t = warping stress, psi
- E_c = modulus of elasticity of concrete (assumed to be 4×10^6 psi)
- t = difference in temperatures between top and bottom of concrete, °F
- ϵ_t = coefficient of thermal expansion of concrete (assumed to be 6×10^{-6} in./in.)
- μ = Poisson's ratio (assumed to be 0.20)

Since failure in a prestressed concrete pavement occurs due to tensile cracking in the upper surface of the pavement, the critical condition for the temperature warping is the case in which there is a tensile stress at the upper surface due to the temperature of the top of the pavement being lower than the temperature of the bottom of the pavement. This condition occurs generally during a period of decreasing air temperature. For this condition, the temperature difference between the top and the bottom of the pavement can be assumed to be 1° F per inch of pavement thickness.²¹

Since warping can result in tensile stresses in the concrete, additional prestress must be added to offset the tensile stress. The tensile stress added to Equation 13 results in the total prestress required determined from the equation

$$R_p = \frac{2.85PNB}{Sh^2} - R + F + F_t \quad (15)$$

DISCUSSION OF DESIGN METHOD

Although the design method presented herein for prestressed concrete pavements was developed from analytical studies, small-scale model

tests, and full-scale accelerated traffic tests conducted several years ago, it has been modified and updated to be responsive to current aircraft usage on civil airports. Of the various design concepts reviewed, the methodology used to develop the design method is believed to be the most versatile of those available since it considers all of the significant parameters that affect the long-term performance of the pavement, namely: (a) load, (b) aircraft gear configurations, (c) traffic volume, (d) physical properties of the construction materials, and (e) most of the influencing environmental factors. In addition, the method has been used to design a taxiway pavement at a military airfield which has been subjected to a limited volume of traffic of the heavier of the military aircraft. The excellent structural performance of the taxiway pavement over a period of 15 years lends confidence to the design procedure.

At present, the greatest problem facing the use of prestressed concrete pavements is the design and construction of required transverse jointing systems which will exhibit long-term performance and accommodate the relatively large movements inherent at the joints. To date, this has been the only problem with the 15-year-old taxiway pavement at Biggs Air Force Base, Texas, and it appears that it may be a problem with similar recently constructed highway sections. It is believed that satisfactory jointing systems can be designed and constructed; however, at present, these joints will be expensive and tedious to construct. Several suggested jointing schemes are presented herein which should exhibit a range of construction costs and anticipated life expectancies that are in line with the design life of prestressed pavements. There are undoubtedly other systems and materials which will perform as well or better than those presented. Unfortunately, the evaluation of a jointing system requires either long-term testing or accelerated full-scale testing, both of which were beyond the scope of this study. The final selection of a jointing system requires a life-cycle cost analysis to compare the high initial cost of sophisticated systems with the maintenance costs of less costly and less sophisticated systems during the life of the pavement system.

During the original development of the design procedure, the relationship of tire contact area divided by the radius of relative

stiffness squared A/l^2 to the percent single-wheel load for military aircraft (Figure 7) was developed using a small-scale static load model. This relationship is used to simplify the computation of moments in the prestressed concrete due to complex multiple-wheel gear configurations. The procedure is based upon the concept of the formation of a plastic hinge (crack) in the bottom of the slab due to a positive moment under the applied load. This hinge causes a redistribution of moments within the slab and increases the negative moments at the surface of the pavement some distance from the applied load. Failure is when a crack occurs at the pavement surface because of excessive stresses resulting from the negative moments. The A/l^2 versus percent single-wheel load relationships were developed for the conventional gears then in operation: twin, twin-twin, and twin-tandem wheel gears (Figure 7). For the development of the criteria presented herein, consideration had to be given to significantly different gear configurations such as the B-747 and DC-10-30 and to different wheel spacings in the basic dual-tandem gear. Since the small-scale static load model used in the initial development was no longer available for use, the A/l^2 versus percent allowable single-wheel load relationships for these more modern gear configurations had to be developed analytically. This was accomplished by modifying an existing computer program to force it to yield the same relationships for the dual, dual-dual, and dual-tandem wheel gears as did the small-scale static load model. When this modification was completed, the program was then used to develop the relationships shown in Figure 8. These relationships appear to be reasonable, and there is no reason to suspect their validity; however, validation of the relationships by either small-scale models or full-scale tests was beyond the scope of this study. When using the design procedure, it should be noted that the stresses in the slab from negative moments become additive and sometimes controlling when the main gears or components of a main gear are closely spaced. This phenomenon is particularly true of the four dual-tandem configurations making up the main landing gears of the B-747 aircraft, where it is found that the maximum negative moments occur between the leading dual-tandem arrangements of the right and left main gears. It is also true, but less severe, for

the DC-10-30 for which the maximum negative moments occur between the dual-tandem main gear and the dual fuselage gear. This phenomenon means that these gear configurations, which were designed to distribute the load and be less severe for plain concrete pavements, become more severe for prestressed concrete pavements due to the additive nature of the negative moments than would higher concentrated loads that are more widely spaced. While, as mentioned previously, these concepts appear to be correct, they do need validation before widespread use of the criteria is made. Validation of the concept by small-scale model testing is feasible; however, the model would again have to be developed. Validation by means of a full-scale test section is also feasible but would be significantly more expensive.

In an effort to further validate and refine part of the design procedures herein, static and some moving load tests were conducted on an instrumented prestressed concrete highway test section constructed by the Federal Highway Administration near Dulles International Airport, Washington, D. C. A detailed analysis of these load tests is reported in Volume I of this report.²² The load tests conducted on the Dulles prestressed test road (see Figures 17-19 of Volume I) showed good correlation between measured subgrade behavior directly beneath the loads and that determined using linear elastic layer theory. For the subgrade conditions, pavement slab conditions, and load conditions at Dulles, this correlation indicates that the subgrade could be modeled by elastic layer theory. The load tests were all within the initial prefailure behavior of the pavements, and no cracks or failures were induced. The results of these tests imply that the initial maximum elastic deflections and stresses of the pavement slab may be closely approximated by the layer theory. Another implication is that the maximum subgrade deflections or deformations determined using the layer theory may be used for calculating the slab bending moments and stresses by the various slab behavior models. The layer theory model results can also be used with Westergaard's analysis and his correction factors based on measured deflections. Layer theory deflections could be specifically incorporated in Westergaard's subgrade reaction corrections. The above discussion applies specifically to the Dulles test pavements; future work should further investigate modeling the subgrade by linear elastic layer theory.

STRESS LOSSES IN PRESTRESSING TENDONS

In addition to the stress that must be applied to the prestressing tendons to produce the design effective prestress in the concrete, determined by Equation 15, additional stress must be added to the tendons to overcome stress losses which are produced by such things as elastic shortening of the concrete due to the prestress, concrete creep due to prestress, shrinkage of the concrete due to volume change, relaxation of the tendons under tension, slippage or deformation within the anchorage system, and frictional resistance between the tendon and its enclosure. The amount of stress loss in the stressing tendons due to each of the above must be determined and added to the applied stress of each tendon during the prestressing operations.

ELASTIC SHORTENING OF CONCRETE

PRETENSIONING METHOD

When prestressed pavements are constructed by the pretensioning method, a tensile force is applied to the steel prior to the placement of the concrete. After the concrete has gained sufficient strength, the steel is released from the anchors and the prestress is transferred to the concrete by the action of the bond between the steel and the concrete. As the concrete shortens due to the direct load of the prestressing, the tension in the steel is released. This shortening of the concrete is resisted by the friction between the concrete and the subgrade. The average frictional resistance to the shortening of the concrete will be equal to $F/2$, if it is assumed that the frictional resistance varies linearly from zero at the edge of the pavement to a maximum value at the center. Therefore, the average unit shortening of the concrete will be equal to $(R_p - F/2)/E_c$, and the loss of stress Δf_s in pounds per square inch for the steel due to this shortening is given by

$$\Delta f_s = \left(R_p - \frac{F}{2} \right) \frac{E_s}{E_c} = n \left(R_p - \frac{F}{2} \right) \quad (16)$$

where

R_p = required effective prestress, psi
 F = maximum subgrade restraint stress, psi
 E_s = modulus of elasticity of steel, psi
 E_c = modulus of elasticity of concrete, psi
 $n = E_s/E_c$

POSTTENSIONING METHOD

For pavements constructed by the posttensioning method, the steel is placed in tubes located in the pavement. After the concrete has gained sufficient strength, a tensile force is applied to the steel by jacking and anchoring against the pavement. If the stress is applied to all of the tendons simultaneously, the concrete shortens as the tendons are jacked against the slab, and there will be no stress loss in the tendons due to elastic shortening of the concrete.

In posttensioned systems where no stressing tendons are used, the prestress loss in the concrete due to elastic shortening is also eliminated by the simultaneous application of the stressing force along the joint.

For most posttensioned systems, there are a number of tendons, and these tendons are stressed in succession. The shortening of the concrete therefore increases as each succeeding tendon is jacked against the edges of the pavement. Thus, the stress loss due to elastic shortening in the first tendon stressed would be the same as the stress loss for a pretensioned tendon, while there would be no stress loss in the last tendon stressed. Therefore, when the tendons are stressed individually, the average stress loss due to elastic shortening is one-half that which occurs in a pretensioned system and is given by

$$\Delta f_s = \frac{n}{2} \left(R_p - \frac{F}{2} \right) \quad (17)$$

CONCRETE CREEP

The loss in steel stress due to creep in the concrete is similar to the loss due to elastic shortening of concrete since both losses are due to a strain in the concrete under a direct load. However, creep

occurs over a period of time rather than at the instant the load is applied. Since the strain in the concrete is inversely proportional to the modulus of elasticity, which increases with the concrete strength, concrete which is stressed within a few days after placement will have higher strains due to creep than concrete stressed later. A reasonable value from Reference 21 for the stress loss due to creep is

$$\Delta f_s = 5 \times 10^{-7} (R_p E_s) / l \text{ psi} \quad (18)$$

for pretensioned steel and

$$\Delta f_s = 4 \times 10^{-7} (R_p E_s) / l \text{ psi} \quad (19)$$

for posttensioned steel.

CONCRETE SHRINKAGE

The shrinkage of the concrete during the curing period will also result in the shortening of the tensioned steel, thus releasing part of the steel stress. When steel is pretensioned, all of the shrinkage takes place after the steel is tensioned; however, when steel is posttensioned, part of the shrinkage takes place before the steel is stressed. Therefore, the loss due to shrinkage would be greater for pretensioned steel than for posttensioned steel. While the amount of shrinkage will vary depending on the material physical properties and the placement environment, an average coefficient of length change is 0.0002 in./in. for pretensioned slabs and 0.00015 in./in. for posttensioned slabs.²¹ The stress loss would then be

$$\Delta f_s = 2 \times 10^{-4} E_s \quad (20)$$

for pretensioning and

$$\Delta f_s = 1.5 \times 10^{-4} E_s \quad (21)$$

for posttensioning.

STEEL RELAXATION

When steel is anchored at a high stress, there will be a relaxation of that stress due to an action similar to creep in the concrete. If the load were constant, the steel would continue to elongate under that load over a long period of time. However, since the distance between anchors remains relatively constant, there is a decrease of the tension in the steel. The rate of stress loss in the steel is greater at higher stresses than at lower stresses. From Reference 21, the loss of prestress due to the relaxation of steel is assumed to be

$$\Delta f_s = 0.06f_{si} \quad (22)$$

for steel anchored at six-tenths of the ultimate strength and

$$\Delta f_s = 0.10f_{si} \quad (23)$$

for steel tensioned at seven-tenths of the ultimate strength. (f_{si} is the stress in the steel at anchorage, which is equal to the steel stress required to obtain the effective prestress R_p plus the stress losses in the steel due to the elastic shortening of the concrete, the creep and shrinkage in the concrete, and the relaxation of the steel.)

ANCHORAGE LOSSES

After a prestressing tendon has been tensioned and anchored, it is expected that various amounts of slippage and/or deformation will take place in the various components of the anchorage system as the load is released from the jacks. Friction wedges which are used to hold tendons will slip for a short distance before the tendons are firmly gripped. For direct bearing anchorage, the head and nut are subject to a slight amount of deformation. Bearing plates and shims are subject to varying degrees of deformation. The high force in the end fittings for large strands may cause some slippage in the wires. Each of the above cases represents a shortening of the tensioned tendon and therefore a loss of prestress. The total shortening of the tendon in an anchorage system may be as much as 0.2 in. To compensate for the losses

in the anchorage system, an additional force should be applied to the prestressing tendons to provide additional elongation of the tendons at the time of anchorage. The amount of this extra stress should be

$$\Delta f_s = \frac{\Delta L E_s}{L} \quad (24)$$

where

ΔL = amount of slippage or deformation, in.

E_s = modulus of elasticity of steel, psi

L = length or width of the slab, in.

TENDON FRICTION

In posttensioned concrete, there is an additional loss of prestress due to friction between the posttensioned prestressing tendons and the enclosures. This friction may come from two sources: wobble of the enclosure due to displacement during construction or intentional bending of the tendon and enclosure due to changes in the grade of the pavement. The resistance to elongation of the tendons due to the wobble of the enclosures depends upon the number and intensity of variations of the enclosures from a straight line, the amount of space between the prestressing tendons and the enclosures, and the type of material used for the tendons and enclosures. The loss due to intentional bending of the tendons and enclosures depends upon the degree of curvature and the coefficient of friction between the tendons and enclosures.

The loss of prestress due to tendon friction is given by²¹

$$T_x = T_o e^{-(K_w X + K_e \phi)} \quad (25)$$

where

T_x = tension desired at a point x , lb

T_o = tension at the jack, lb

K_w = wobble coefficient per foot

X = distance from the jack, ft

K_e = coefficient of friction between the tendons and enclosures per radian

ϕ = change in the angle between the tendon at the jack and the tendon at a point x , radians

The actual values of the coefficients K_w and K_e will vary and should be determined in the field for the existing conditions. The generally expected ranges of values for the two coefficients are:²¹

<u>Type of Tendon</u>	<u>Type of Enclosure</u>	<u>K_w</u>	<u>K_e</u>
High-strength bars	Bright metal	0.0001 to 0.0005	0.08 to 0.30
Galvanized strands	Bright metal	0.0004 to 0.0020	0.15 to 0.30
Wire cables	Bright metal	0.0005 to 0.0030	0.15 to 0.35

For galvanized strands enclosed in steel conduits used in the prestressed concrete pavement at Dulles International Airport, Friberg and Pasko² reported values for K_w of 0.0004 and 0.0018 and a value for K_e of 0.26. For steel strands enclosed in plastic, they reported K_w values of 0.0014 and 0.0019 and a K_e value of 1.27.

To compensate for this friction loss, it is necessary to temporarily overstress the prestressing tendon and then release the tension to the design stress at anchorage f_{si} . The maximum value of this temporary stress should be eight-tenths of the ultimate strength of the steel, while the maximum design stress at anchorage should be seven-tenths of the ultimate strength of the steel.

APPLICATION OF DESIGN PROCEDURE

DESIGN EXAMPLE

As an application of the effective prestress design equation (Equation 15), assume that a prestressed pavement is to be designed to sustain 100,000 passes of a DC-8-63F aircraft. The gross aircraft load is 350,000 lb. For this example, the following conditions are used:

- a. Wheel spacings = 32 by 55 in.
- b. Tire contact area = 220 sq in. per wheel.
- c. Slab size = 500 ft long by 75 ft wide.
- d. Physical properties of the concrete:
 - (1) Modulus of elasticity $E_c = 4.0 \times 10^6$ psi
 - (2) Poisson's ratio = 0.20
 - (3) Design flexural strength $R = 700$ psi
 - (4) Unit weight = 150 pcf
- e. Modulus of subgrade reaction $k = 200$ pci.

Since both pavement thickness and magnitude of prestress must be determined, the design procedure requires that one of these two variables be assumed. For this example, the pavement thickness will be assumed, and the necessary design prestress will be computed.

For an assumed pavement thickness of 9 in., the radius of relative stiffness ℓ is 33.54 in. and A/ℓ^2 is 0.196. From Figure 6, the value of B is 0.058. From Figure 8, the ratio of the DC-8-63F dual-tandem gear failure load to the equivalent single-wheel failure load is 2.20, and, from Figure 9, the load repetition design factor N for 100,000 passes (traffic index = 30,000) is 2.46. For slab dimensions of 500 by 75 ft and a coefficient of subgrade restraint of 0.60, the maximum subgrade restraint stress F (from Table 2) is 156 psi longitudinally and 23 psi transversely. Assuming a maximum temperature gradient of 1° F per inch, the difference in temperature between the top and the bottom of the pavement is 9° F. From Equation 14, the maximum temperature warping stress is 135 psi.

Substituting the above values into the required effective

prestress equation (Equation 15) yields 390 psi as the required longitudinal design prestress and 257 psi as the transverse design prestress. Similarly, for an assumed pavement thickness of 10 in., the required longitudinal and transverse design prestresses are 272 and 139 psi, respectively. Since the two designs are comparable insofar as their load-carrying capabilities are concerned, the selection of the thickness to be used becomes one of economic and other considerations, which will be discussed later in this report.

In the design example above, the longitudinal and transverse design prestresses were set so that the limiting stress $(R + R_p - F - F_t)$ in the transverse direction was equal to the limiting stress in the longitudinal direction. For a single circular tire, failure of a prestressed slab occurs due to the maximum negative moment acting along a circle some distance away from the center of the load. The radial tensile stress causing cracking in the upper surface would be the same in all directions from the center of the load. Similarly, for a multiple-wheel gear load, the maximum negative moment occurs along an approximately circular pattern around the multiple-wheel gear, with the resulting load-induced radial tensile stress being approximately the same in all directions from the center of the load. Therefore, to best counteract this radial tensile stress, the limiting stress $(R + R_p - F - F_t)$ should be the same in the longitudinal and transverse directions. This need for transverse prestressing was shown in the full-scale traffic tests reported in References 10 and 18. In some of these test items, there was prestressing in both the longitudinal and transverse directions, while in other test items steel reinforcing was used in lieu of transverse prestressing. The results showed that, while the pavement items with only longitudinal prestressing performed better than plain concrete pavement, the pavement items with both longitudinal and transverse prestressing withstood many more load repetitions than the items with longitudinal prestressing only.

DESIGN TABLE

In designing a pavement for a particular aircraft using the

required effective prestress equation, a design table should be prepared. An example of a design table for the B-747STR aircraft is shown in Table 3.

Table 3
Design Table for Prestressed Pavement for 100,000 Passes
of a B-747STR Aircraft* in the Critical Area

k pci	h in.	l in.	A/l ²	B	S	2.85FN	Limiting
						Sn ² psi	Stress R _p + R - F - F _t psi
50	11	55.14	0.081	0.0714	2.53	17,800.3	1308
	12	58.85	0.071	0.0743	2.54	14,898.3	1139
	13	62.50	0.063	0.0773	2.55	12,644.6	1006
	14	66.07	0.056	0.0806	2.56	10,860.2	901
	15	69.58	0.051	0.0832	2.57	9,423.6	807
	16	73.03	0.046	0.0867	2.58	8,250.4	736
	17	76.43	0.042	0.0897	2.59	7,280.1	672
100	10	43.16	0.132	0.0631	2.51	21,710.0	1410
	11	46.37	0.114	0.0654	2.51	17,942.1	1208
	12	49.49	0.100	0.0676	2.52	15,016.5	1045
	13	52.55	0.089	0.0697	2.53	12,744.6	914
	14	55.56	0.080	0.0716	2.54	10,945.7	807
	15	58.51	0.072	0.0740	2.54	9,534.9	726
	16	61.41	0.065	0.0765	2.55	8,347.4	657
200	10	36.30	0.186	0.0586	2.49	21,884.3	1320
	11	38.99	0.162	0.0603	2.50	18,013.9	1118
	12	41.62	0.142	0.0620	2.50	15,136.7	966
	13	44.19	0.126	0.0638	2.51	12,846.1	844
	14	46.72	0.113	0.0656	2.51	11,076.5	748
	15	49.20	0.101	0.0674	2.52	9,610.6	667
	16	51.64	0.092	0.0691	2.53	8,413.4	598
300	10	32.80	0.228	0.0563	2.48	21,972.6	1273
	11	35.23	0.198	0.0578	2.48	18,159.2	1080
	12	37.61	0.174	0.0594	2.49	15,197.5	929
	13	39.93	0.154	0.0610	2.50	12,897.5	810
	14	42.21	0.138	0.0624	2.50	11,120.8	714
	15	44.46	0.124	0.0641	2.51	9,648.9	637
	16	46.66	0.113	0.0656	2.51	8,480.5	573
500	9	26.67	0.345	0.0525	2.46	28,148.1	1478
	10	28.87	0.295	0.0539	2.47	22,707.7	1224
	11	31.01	0.255	0.0552	2.47	18,766.7	1036
	12	33.10	0.224	0.0564	2.48	15,705.6	886
	13	35.15	0.199	0.0578	2.48	13,382.3	773
	14	37.16	0.178	0.0589	2.49	11,492.5	677
	15	39.13	0.160	0.0605	2.50	9,971.2	603
	16	41.06	0.146	0.0617	2.50	8,763.8	541

* Aircraft gross weight = 800 kips; tire contact area = 245.6 sq in.

The procedure for preparing the design table was as follows: For each modulus of subgrade reaction k value, the radius of relative stiffness l was calculated using several thickness values. Using the computed value of A/l^2 , B was obtained from Figure 6. The values of the percent single-wheel failure load were obtained from Figure 8, and the load repetition design factor N was read from Figure 9. (The design traffic level of 100,000 passes was used to obtain the load repetition design factor.) Then, the value of $2.85\text{BPN}/\text{Sh}^2$ was computed to obtain the limiting stress value $R_p + R - F - F_t$.

To formulate a design using a design table, the procedure is to select the limiting stress values of $R_p + R - F - F_t$ based on the subgrade k and several assumed thicknesses. From Table 2, the subgrade restraint stress F is then selected according to the design length and width. The maximum temperature warping stress in pounds per square inch is calculated using Equation 14. Using the 90-day design flexural strength R , the amount of the required effective prestress R_p in both the longitudinal and transverse directions can be calculated for each assumed thickness.

The final design thickness chosen should be the one with the most appropriate required prestress level based on other factors included in this report.

STRESS LOSSES

For an example of calculating the stress losses in a prestressed pavement, the same example for the application of the effective prestress design equation will be used. Assume that the 10-in. thickness is chosen with a required effective prestress of 272 psi in the longitudinal direction and 139 psi in the transverse direction. In this example, posttensioning is the prestressing method selected, and a steel strand with an ultimate strength of 240,000 psi is used to achieve the prestress. The other assumed conditions are:

- a. Slab size = 500 ft long by 75 ft wide.
- b. Final anchoring stress in the tendon = 168,000 psi (70 percent of ultimate).

c. Modulus of elasticity of steel $E_s = 30 \times 10^6$ psi.

d. Modulus of elasticity of concrete $E_c = 5 \times 10^6$ psi.

The calculations showing stress loss values for the example case are given below:

a. Elastic shortening of concrete:

$$\Delta f_s = \frac{n}{2} \left(R_p - \frac{F}{2} \right) \quad (17 \text{ bis})$$

(1) Longitudinal:
$$\Delta f_s = \frac{30 \times 10^6}{2(5 \times 10^6)} \left(272 - \frac{156}{2} \right)$$

= 580 psi

(2) Transverse:
$$\Delta f_s = \frac{30 \times 10^6}{2(5 \times 10^6)} \left(139 - \frac{23}{2} \right)$$

= 380 psi

b. Concrete creep:
$$\Delta f_s = 4 \times 10^{-7} (R_p E_s) / 1 \text{ psi} \quad (19 \text{ bis})$$

(1) Longitudinal:
$$\Delta f_s = 4 \times 10^{-7} (272) (30 \times 10^6) / 1 \text{ psi}$$

= 3260 psi

(2) Transverse:
$$\Delta f_s = 4 \times 10^{-7} (139) (30 \times 10^6) / 1 \text{ psi}$$

= 1670 psi

c. Concrete shrinkage:
$$\Delta f_s = 1.5 \times 10^{-4} E_s \quad (21 \text{ bis})$$

$$= (1.5 \times 10^{-4}) (30 \times 10^6)$$

= 4500 psi

d. Steel relaxation:
$$\Delta f_s = 0.10 f_{si} \quad (23 \text{ bis})$$

$$= 0.10 (168,000)$$

= 16,800 psi

e. Total:

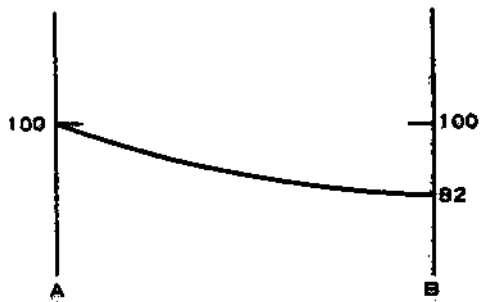
$$(1) \text{ Longitudinal: } \Delta f_s = 580 + 3260 + 4500 + 16,800 \\ = 25,140 \text{ psi}$$

$$(2) \text{ Transverse: } \Delta f_s = 380 + 1670 + 4500 + 16,800 \\ = 23,350 \text{ psi}$$

The above total is the stress loss to be expected in a tendon stressed at 168,000 psi. To insure that the tendon is stressed at this amount throughout its entire length, two other factors must be considered: anchorage loss and tendon friction. The additional stress needed to compensate for the anchorage loss can be determined by substituting in Equation 24. Assuming a tendon shortening of 0.2 in. with the anchoring system used, the extra stress needed would be 1000 psi in the longitudinal direction and 6670 psi in the transverse direction.

The calculation of the temporary additional stress required to overcome tendon friction is more complex. The amount of stress loss due to tendon friction can be determined by jacking the tendon to the desired stress at one end of the slab and measuring the stress applied to the tendon at the other end. A procedure for overcoming this stress loss is shown in Figure 10. Assume that for the 500-ft slab the measured stress at the end away from the jacking is 82 percent of the stress applied by the jack. The applied temporary stress needed to overcome the tendon friction would be the stress desired at final anchorage (168,000 psi) divided by the percent effectiveness (0.82), or 204,900 psi. However, this is greater than the allowed maximum value of 80 percent ultimate (192,000 psi). Therefore, the maximum final anchorage stress would have to be 82 percent of 192,000 psi, or 157,400 psi. This stress would not allow the most effective use of the steel.

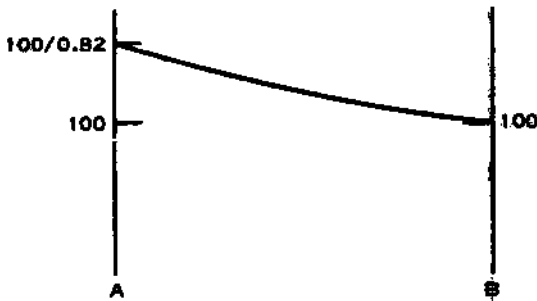
If 157,400 psi were chosen as the final design prestress, then the stress loss due to steel relaxation would have to be recalculated. For the final stressing operation, the tendon would be stressed at one end to 192,000 psi. The tendon would then be released to the final design stress (157,400 psi) plus the stress necessary to compensate



APPLY DESIGN PRESTRESS AT END A AND MEASURE THE PERCENT OF STRESS AT END B.

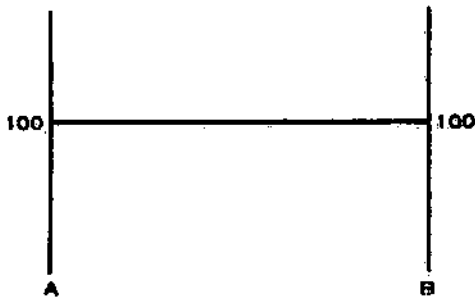
STEP 1

PERCENT OF DESIGN PRESTRESS



APPLY AT END A THE DESIGN PRESTRESS DIVIDED BY THE PERCENT OF STRESS AT END B TO ACHIEVE THE DESIGN PRESTRESS AT END B.

STEP 2



RELEASE STRESS AT END A TO THE DESIGN PRESTRESS.

STEP 3

Figure 10. Procedure 1 for overcoming tendon friction stress loss

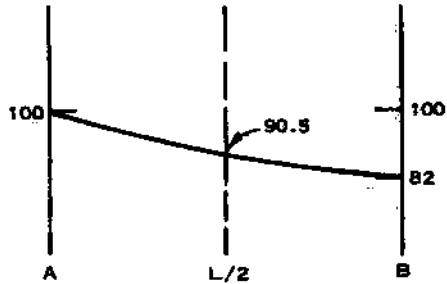
for the anchorage stress loss (1,000 psi).

An alternate method of overcoming the tendon friction, which results in a lower temporary stress, may be used. In this method, which is shown in Figure 11, the tendons are stressed at both ends. To determine the amount of temporary stress, it is necessary to solve the equation for tendon friction (Equation 25). If there are no horizontal or vertical curves, then the angular change ϕ is zero, and the equation reduces to $T_x/T_o = e^{-K_v X}$.

For the above example in which the stressing effectiveness (T_x/T_o) is 0.82 and the distance is 500 ft, the wobble coefficient is 0.0004. By stressing the tendons at both ends, the length along which the tendon friction loss occurs is reduced to one-half of the length of the slab. By substituting 250 ft and the wobble coefficient 0.0004 into the tendon friction equation, the stressing effectiveness at the mid-length of the slab can be calculated. In this case, the stressing effectiveness is 0.905. The temporary stress needed at each end would be 168,000 psi divided by 0.905, or 185,600 psi. Since this is less than 80 percent of ultimate, the tendons can be anchored at 70 percent of ultimate. For the final stressing operation, one end of the tendon would be jacked with a force to obtain a stress of 185,600 psi and anchored. The other end would then be stressed to 185,600 psi. This would result in the stress at the midlength being 168,000 psi. Then, both ends would be released to the design prestress (168,000 psi) plus the stress needed to overcome the anchorage stress loss (1,000 psi).

For most long slabs, it will be necessary, as in the above example, to stress the tendons from both ends to achieve a more effective use of the steel, i.e. the final design prestress being close or equal to seven-tenths of the ultimate strength of the steel. To determine whether to stress the tendons at one end or both ends, the cost of requiring more steel versus the cost of stressing at both ends should be considered. With shorter slabs or lower friction values, it may be possible to achieve the maximum anchoring stress in the tendons by stressing at only one end of the tendon.

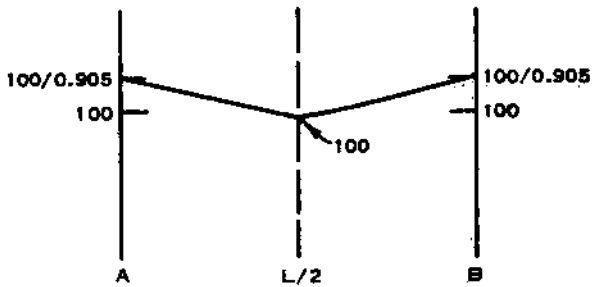
The temporary stress required in the transverse direction would



STEP 1

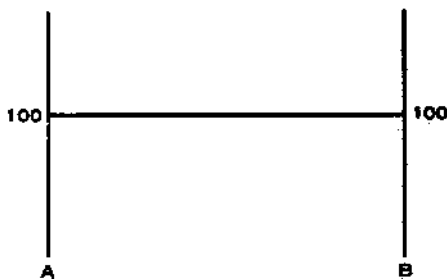
APPLY DESIGN PRESTRESS AT END A AND MEASURE THE PERCENT OF STRESS AT END B. CALCULATE STRESS AT MIDLENGTH.

PERCENT OF DESIGN PRESTRESS



STEP 2

APPLY AT ENDS A AND B THE DESIGN PRESTRESS DIVIDED BY THE PERCENT OF STRESS AT MIDLENGTH TO ACHIEVE THE DESIGN PRESTRESS AT MIDLENGTH.



STEP 3

RELEASE STRESS AT ENDS A AND B TO THE DESIGN PRESTRESS.

Figure 11. Procedure 2 for overcoming tendon friction stress loss

be calculated in a manner similar to that used for the longitudinal direction. Assuming that in the above example the pavement sloped downward at a rate of 1 percent from the taxiway center line, the angular change ϕ in the tendon friction equation (Equation 25) would be the change in slope from one end of the tendon to the other end, or 0.02 radians. If the wobble coefficient K_w was the same as that for the longitudinal tendons and the friction coefficient K_e was 0.20, then the stressing effectiveness would be

$$\begin{aligned} T_x/T_o &= e^{-(K_w X + K_e \phi)} && (25 \text{ bis}) \\ &= e^{-[0.0004(75) + 0.20(0.02)]} \\ &= 0.967 \end{aligned}$$

It can be seen that generally there will be a lower percentage of friction loss transversely than longitudinally due to the decreased length since the angular change has a minimal effect on the friction stress loss. As a result of the higher stressing effectiveness, most transverse prestressing can be done at one end of the tendon and still attain the maximum anchoring stress.

While the stressing effectiveness should be checked in the field by measuring the force transmitted from one end of the tendon to the other, the tendon friction equation is useful in determining an approximation of stress loss for design purposes. It is also necessary to obtain the value of the wobble coefficients using the tendon friction equation if the final prestressing is to be applied at both ends of the tendon.

DETERMINATION OF TENDON SIZE AND SPACING

To determine the size and spacing of the prestressing tendons, the following procedure can be used. First, the final anchoring stress f_{si} of the tendons should be selected, which is the maximum stress (70 percent of the ultimate) for most effective use of the prestressing steel. However, other factors may dictate the selection of a final anchoring stress less than the allowable maximum. Based on the final

anchoring stress, the stress losses Δf_s can then be calculated as in the above example. The stress in the tendons available for prestressing the concrete is the final anchoring stress minus the stress losses $(f_{si} - \Delta f_s)$. The total force needed to achieve the effective prestress required in the concrete is the design effective prestress R_p from Equation 15 times the cross-sectional area of the concrete slab. The cross-sectional area of the concrete slab is the width times the thickness for transverse prestressing or the length times the thickness for longitudinal prestressing. The total area of the steel A_s required for the prestressing tendons is the total force needed in the concrete divided by the stress in the tendons, or

$$A_s = \frac{R_p A_c}{f_{si} - \Delta f_s} \quad (26)$$

The total area of the steel A_s is equal to the cross-sectional area of one tendon times the number of tendons in the slab. Therefore, for a tendon of a given size, the number of tendons required can be determined. The number of tendons required and the dimensions of the slab govern the spacing of the tendons. A number of tendon sizes, with each size requiring a different spacing, could probably be used to satisfy the design requirements. The final selection of a tendon size and tendon spacing should be based on engineering judgement including design considerations discussed later in this report.

DESIGN ALTERNATIVES

It has been shown in past prestressed construction projects that several alternatives in construction methods and materials are available to the designer. A general discussion of some design alternatives is included below with recommendations based on a study of theoretical concepts, past construction experience, and past prestressed pavement performance.

SYSTEMS FOR PRESTRESSING CONCRETE

The amount of prestressing required may be achieved by the use of three different prestressing systems: pretensioning, posttensioning, and poststressing. In some cases, a combination of systems has been used.

PRETENSIONING

In the pretensioning system, the prestressing tendons are stressed and anchored before the concrete is placed. The tendons may consist of wires, strands, or bars of high-tensile-strength steel. After the concrete has been placed and has gained sufficient strength, the tendons are released from anchorage. The stress in the tendons is then transferred to the concrete through the bond between the concrete and the steel. The primary disadvantage of this system is the rather massive anchorage system required to pretension the steel. This system is used primarily for prefabricated prestressed units.

POSTTENSIONING

For the posttensioning system, the prestressing tendons are installed before the concrete is placed but are not fully stressed until after concrete placement. To prevent the tendons from bonding to the concrete before prestressing, they must be placed in a conduit or coated with a bond-breaking material. To maintain the correct alignment of the tendons in flexible conduits during concrete placement, the tendons are sometimes slightly stressed before the concrete placement. After the concrete compressive strength is high enough, the tendons are stressed until the desired prestress level is reached. For the conduit-enclosed

tendons, grout is often injected into the conduits after stressing to bond the tendons to the conduit. This bonding will prevent corrosion of the tendons and will prevent the complete loss of the prestress in the event of a future failure of a steel tendon at any one location.

There are two methods of posttensioning the concrete slabs. One method consists of using external jacks at the edge of the slab to stress the tendons. After the tendons have been properly stressed, they are securely anchored against the slab. Another method of posttensioning uses a system of internal jacks placed in a gap between two slabs. Each longitudinal tendon goes through the entire length of the slab, passes through the gap, and extends through the other slab. In both slabs, the tendons are anchored in the end of the slab away from the jacks in the gap. When the concrete has reached a certain strength, the jacks are used to force the two slabs apart, stressing the tendons and inducing a compressive stress in the concrete. The gaps are then filled with concrete. The internal jacks may or may not be removed from the gap before the concrete is placed.

POSTSTRESSING

Longitudinal prestressing in the concrete pavement can also be done without tendons. In the poststressing system, two abutments are anchored in the ground at the ends of the section to be stressed. The concrete slab is constructed between these two abutments and supported against one fixed abutment while it is jacked against the other to stress it. A series of slabs may be placed between two abutments with a system of jacks between each pair of slabs. When the jacking has been completed, the spaces occupied by the jacks are filled with concrete.

COMPARISON OF SYSTEMS

Posttensioning has several advantages over the pretensioning system. A principal advantage is the lack of fixed anchors needed in the pretensioning system. There is also a smaller loss of prestress in the posttensioning system, giving it a higher rate of efficiency for the steel used in the tendons. An advantage of pretensioning when compared

with posttensioning is the savings in the cost of conduits used in the posttensioning system. The use of the poststressing system does not require the tendons used in posttensioning or pretensioning. However, in poststressing, small orders of hygrothermal contraction and creep in the concrete can seriously reduce the compressive stresses in the pavement. For this reason, the jacking systems have to be designed to permit periodic reapplications of prestress to maintain the design prestress level. In the pretensioning and posttensioning systems, the losses due to contraction and creep do not seriously affect the compressive stresses because only a small percentage of the tension in the tendon is relieved. Another disadvantage of poststressing is the cost of the fixed abutments.

Based on a review of the aspects of the different systems, it is recommended that the posttensioning system using external jacks be used, because it has been proven that this method can achieve the amount of prestressing required with the highest degree of certainty.

AMOUNT OF PRESTRESSING

The amount of anchoring prestress applied to the prestressing tendons must be equal to the amount of effective prestress required in the pavement plus the amount of prestress required to overcome the stress losses. Other considerations must be taken into account in determining the final prestress level. For example, an excessive amount of prestressing may induce warping or buckling of the slab. Based on the effective required prestress equation, the higher the prestress level, the lesser the thickness required. While the load-carrying capacity of a higher prestress and lesser thickness pavement may be theoretically the same as that of a pavement with lower prestress and greater thickness, the larger deflections due to the decrease in thickness may be detrimental to pavement performance. For the above factors, a reasonable maximum value of prestress should be specified. The recommended maximum level of effective prestress in the pavement, based on past experience and engineering judgement, is 400 psi.

There should also be a minimum amount of prestressing. To insure that the pavement will be stressed throughout the entire length of the

slab under all conditions, the amount of prestress should be at least 100 psi more than the maximum subgrade restraint stress F . The value of the design anchoring prestress will influence such factors as the size and spacing of the stressing tendons, the force applied to the prestress tendons, the capacity of the jacks required for prestressing, etc. The costs of the above factors should be taken into account in determining the most economical design.

STRESSING TENDONS

The stressing tendons used may consist of high-strength wires, strands, or bars. All three types have been used effectively in the past. The selection of the type of tendon to be used for each design case should be based on the availability and total cost of each type. Some factors in determining the total cost of each tendon type would be the amount of steel required, the unit material cost, and the expense of handling with respect to the transportation, placing, stressing, and anchoring of the tendons. Since the tendons are to be bonded in conduits after stressing, there is the additional cost of placing the tendons into the conduits. This step may be done either in the field or by the manufacturer. The amount of steel required will vary from case to case depending upon the amount of prestress required and the ultimate tensile strength of the tendons.

CONDUITS

The conduits used for enclosing the tendons may be either rigid or flexible metal tubing, or the tendons may be plastic encased. The plastic-encased tendons have as an advantage the fact that they can be acquired from the manufacturer with a corrosion-resistant grease between the steel tendon and the plastic conduit, thus eliminating the necessity to inject grout into the conduit. With the plastic-encased tendons, however, there is a greater amount of friction between the tendon and the conduit and therefore a larger loss of stress in the tendon. This loss in stress could result in a need for larger sizes or quantities of tendons to achieve the design prestress. With the rigid or flexible

conduits, there must be enough clearance between the tendons and conduits to minimize the prestress loss due to friction between the tendons and the conduits and to allow for an efficient grouting operation. However, if the conduits are too large, there will be a tendency for the concrete to crack. The inside diameter of the metal conduits should be at least 0.25 in. larger than the diameter of the stressing tendons. The allowable minimum cover between the conduits and the upper surface of the concrete should be 3 in., while the minimum cover for the lower surface should be 2 in.

PLACEMENT OF STRESSING TENDONS

The purpose of the prestressing tendons is to provide a hinge effect after bottom tensile cracking and thus prevent upper surface tensile cracking caused by the negative moments. Therefore, the prestressing tendons should be placed at about the middepth of the slab to perform both functions. With long concrete slabs, there is a tendency for the slab ends to warp upward. In some prestressed concrete projects, the longitudinal tendons have been placed slightly below middepth to counteract this warping tendency. The standard practice, however, has been to place the longitudinal tendons at middepth and to place the transverse tendons either below the longitudinal tendons or alternately above and below the longitudinal tendons. The lower transverse tendons are usually placed first to help support the longitudinal tendons. It is recommended that the longitudinal tendons be set at middepth with the transverse tendons immediately below them.

The horizontal spacing of the prestressing tendons varies greatly from case to case. The spacing will depend on the amount of prestress needed, the size and ultimate strength of the tendons used, and the allowable bearing stress of the anchorage system. Based upon past experience, it is recommended that the spacing be from two to four times the slab thickness for longitudinal tendons and three to six times the slab thickness for transverse tendons.

SUBGRADES AND SUBBASES

Since the required thickness for prestressed pavements will be

less than the required thickness for conventional concrete pavements for the same loading conditions and foundation strengths, the deflections of prestressed pavements will be larger than those of conventional pavements. The increased deflections increase the possibility of pumping action under the slab and reduce the ability of the slab to sustain repetitive loadings unless adequate protection is provided.

Any increase in stress due to the increase in deflections is accounted for in the design procedure leading to the effective prestress required. However, the increased possibility of pumping action requires the use of adequately designed subbases under the prestressed slabs. The subbase should consist of an open-graded granular material to permit adequate moisture drainage to the shoulders.

Even though the small-scale model and full-scale traffic tests have indicated that a low-strength foundation is suitable for prestressed pavements with sufficient thickness, most prestressed pavements have been constructed on high-strength foundations with a modulus of subgrade reaction k of 200 pci or higher. This construction procedure has been used to avoid the larger deflections with the lower strength foundations. Therefore, a minimum k of 200 pci is recommended.

FRICION-REDUCING LAYERS

Several types of layers have been used to reduce the friction between the prestressed pavement slabs and the underlying subgrade. These layers reduce the subgrade restraint stress caused by contraction of the slab. To satisfy the design requirements, the coefficient of friction between the pavement and the underlying subgrade should be equal to or less than 0.6. Some friction-reducing layers which have been successful in the past include various thicknesses of sand layers, a polyethylene sheet on a sand layer, a sheet of waterproof paper over sand, and two sheets of polyethylene lying on a sand layer or directly on the subgrade. A problem using sand by itself is the difficulty in maintaining a uniform thickness of sand for the entire length of the slab during construction. When the sand layer becomes too thin or uneven in thickness, the resulting nonuniform condition can cause increased friction. Two polyethylene

sheets are effective when resting on a smooth surface, but the friction coefficient becomes larger as the subgrade surface becomes progressively rougher. It is recommended that two polyethylene sheets over a 1/4-in. sand layer of uniform size be used as the friction-reducing layer. The sand should be used primarily to smooth out the surface irregularities of the subgrade.

SLAB DIMENSIONS

Prestressed concrete pavement slabs of various lengths and widths have been constructed with success. The most economical slab length seems to be in the range of 400 to 600 ft. With slabs longer than this, there is the cost of providing more prestress to offset the larger subgrade restraint stress and the cost of providing joints to withstand the larger slab end movements. With shorter slabs, there is the need for more transverse joints per total length of pavement. It is recommended that the design length of the prestressed slabs be 500 ft.

The design width of the prestressed slabs will depend on the capability of the construction equipment. This will usually be either 25 or 50 ft.

PROPERTIES OF CONCRETE

In the selection of the concrete mix design used for prestressed concrete, certain properties of the concrete are desired. The flexural strength has a considerable effect on the design prestress, and, to minimize the thickness required, the concrete mix should yield a high flexural strength. A minimum 90-day flexural strength of 700 psi is recommended. The concrete strength should be as uniform as possible. Type II cement has sometimes been used because it was felt that better cement quality control could be gained. Also, this type cement generates less heat during curing. A low water content should be used to reduce concrete shrinkage and creep. This property will also lessen the amount of prestress loss.

For long slabs, the total length change is critical in the design of expansion joints. To keep this length change at a reasonable level,

the aggregate should be selected to give the concrete a thermal expansion coefficient less than or equal to 6×10^{-6} in./in. per degree Fahrenheit, if economically feasible. Selection of the aggregate should be determined by tests of several candidate aggregates.

EXPANSION JOINTS

Because of the length of prestressed slabs, it is necessary to have expansion joints which can withstand large end movements. The magnitude of the seasonal and daily movements will vary depending upon the temperature change and slab length. Another factor affecting the seasonal length change is the prevailing moisture conditions. If the changes due to temperature are not considered, the concrete pavements will be longer during the wetter months, usually winter, and will be shorter during the drier months, usually summer. Based on a thermal coefficient of 6×10^{-6} in./in. per degree Fahrenheit and a seasonal moisture length change coefficient of 1×10^{-4} in./in., the summer to winter length change of a 500-ft slab undergoing a 100° F temperature decrease would be as follows:

- a. Contraction due to temperature change:

$$\Delta L = (6 \times 10^{-6} \text{ in./in. per degree Fahrenheit}) (500 \text{ ft}) \\ (12 \text{ in./ft}) (100^\circ \text{ F}) = +3.60 \text{ in.}$$

- b. Expansion due to seasonal moisture change:

$$\Delta L = (1 \times 10^{-4} \text{ in./in.}) (500 \text{ ft}) (12 \text{ in./ft}) = -0.60 \text{ in.}$$

- c. Total contraction:

$$\Delta L = 3.60 - 0.60 = 3.00 \text{ in.}$$

The thermal coefficient used in the above calculation is a normal value for concrete, while the seasonal length change coefficient due to moisture is based on past tests of prestressed concrete as reported in Best et al.²³ The slab movement at the joints can be reduced by one-half if two transverse joints are provided per prestressed slab. Unpaved sections are left between the prestressed slabs to provide working space for the prestressing process. These transition areas are paved after the prestressing has been completed. An expansion joint is then placed at each side of the transition slab adjacent to the prestressed slab.

Typical sections of the two general methods of constructing the transition slabs are shown in Figure 12. Type A consists of having the

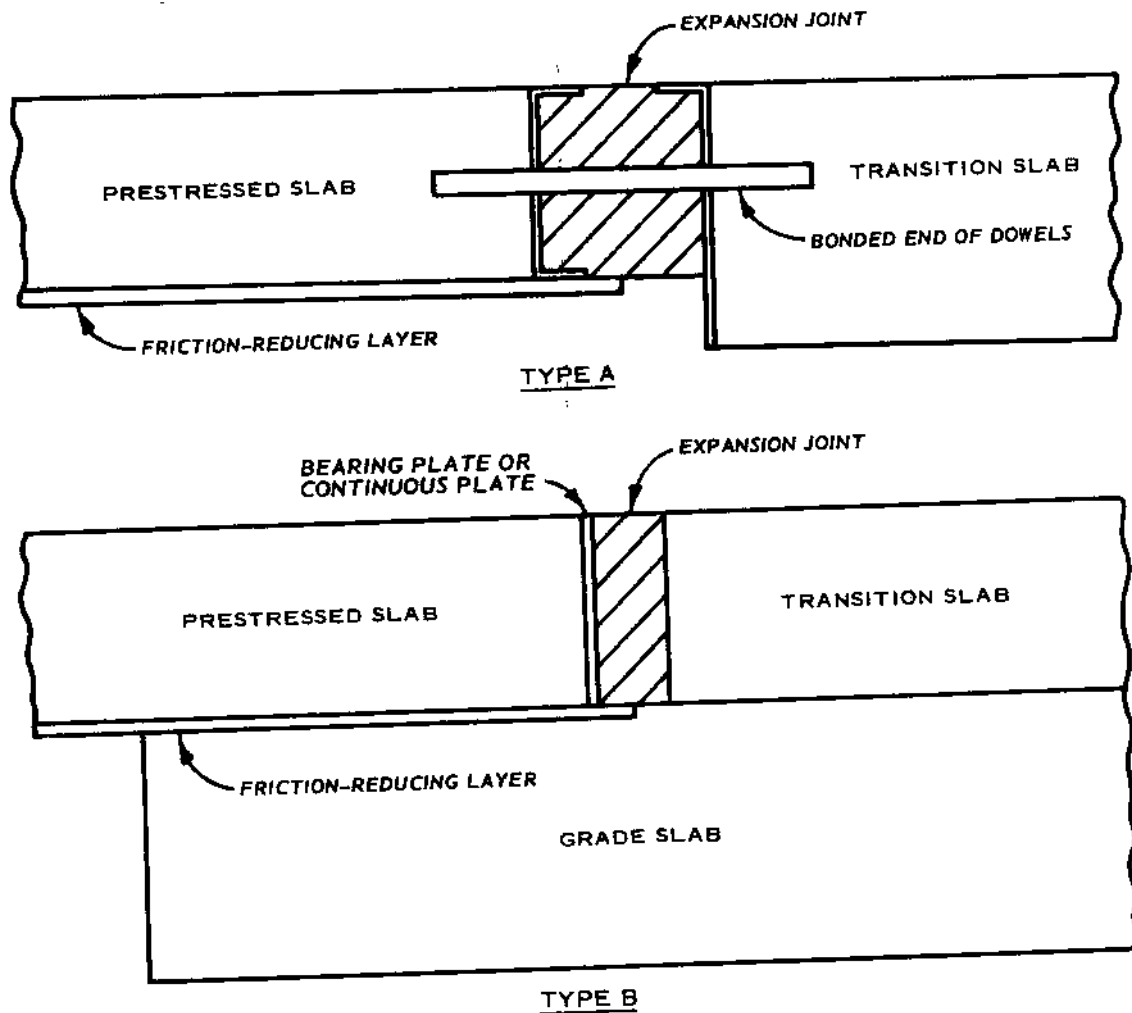


Figure 12. Types of expansion joints and transition slabs

transition slab resting on the subbase. It should be built with the thickness required for plain concrete pavements. This type of transition slab should be connected to the prestressed slabs with dowel bars to provide load transfer across the joint. The transition slabs of type B use concrete grade beams underlying the joint to provide load transfer. The thickness of the transition slab of this type should be the same as the thickness of the prestressed slabs. In determining the thickness of the underlying grade beam, the transition slab may be considered as an

overlay pavement. Steel beams, channels, or plates should be placed next to the prestressed slabs of both types to reinforce the slab edges, to provide an anchor for the prestressing tendons, and to act as a guide for the dowel bars in type A. In addition, the transition slabs of type A will probably require a left-in-place form due to the presence of the dowel bars.

For prestressed concrete pavements with widths that require two or more adjacent slabs, several types of longitudinal joints have been used in the past. The full-scale traffic tests reported in References 10 and 18 included a study of longitudinal joints. The four types of longitudinal joints incorporated in the test items having both longitudinal and transverse prestressing were (a) doweled, (b) keyed, (c) plain butt-type, and (d) grouted butt-type. All four types were compression joints in that the transverse prestressing was extended through the longitudinal joint. The face of the grouted butt-type joint was sandblasted and washed with a neat cement grout prior to placing the adjacent slab. Under accelerated traffic testing of the test items, no failures occurred at the longitudinal compression joint. Based on these and other tests, it appears that the most economical longitudinal joints would be the compression butt-type or keyed joints.

EXPANSION JOINT SEALS

Several types of expansion joint seals are available commercially. These range from poured-in-place seals to complex combinations of neoprene rubber glands and steel beams. There is a wide range in price and performance from the conventional poured joint seal to the complex combinations.

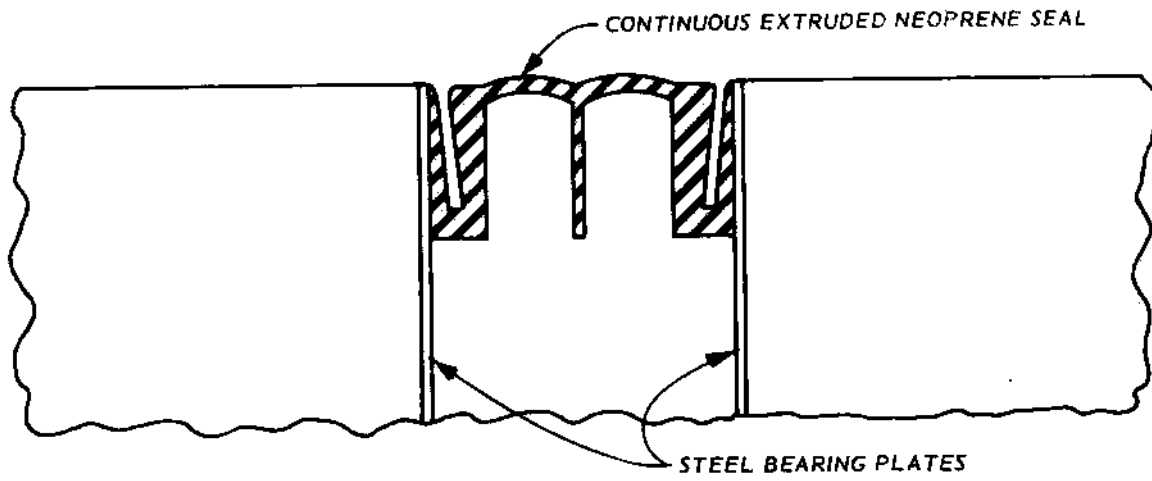
A poured- or foamed-in-place compressible material has been used in expansion joints similar to joint type A in Figure 12 constructed both with and without steel I-beams. Without the I-beams, experience has shown that the sealant material must be replaced within 2 or 3 years because of large movements of the joints. The flange of the I-beams permits the distance between the slab ends to increase, allowing a smaller percentage of length change for the compressible material and yet keeping the opening of the joint as small as possible. Steel

channels with the flanges extending over the joint can also be used in place of the I-beams. This type of expansion joint has been used only recently, and very little performance data is available. An expansion joint of this type, consisting of 6-in. I-beams and foamed-in-place polyurethane, was used in the 6-in. prestressed concrete test road at Dulles International Airport. Detailed information on this pavement is presented in References 2 and 24. While still performing adequately 2 years later, the polyurethane shows signs of weathering and cracking. In addition, the steel I-beams adjacent to one of the five transition slabs have warped, and cracking and severe spalling in the transition slab have occurred. Based on the performance of these joints at Dulles and other similar joints, this type of joint and joint seal combination must be considered to be of short life and will require periodical resealing. However, the lower cost of this joint type may offset the additional cost of frequent resealing.

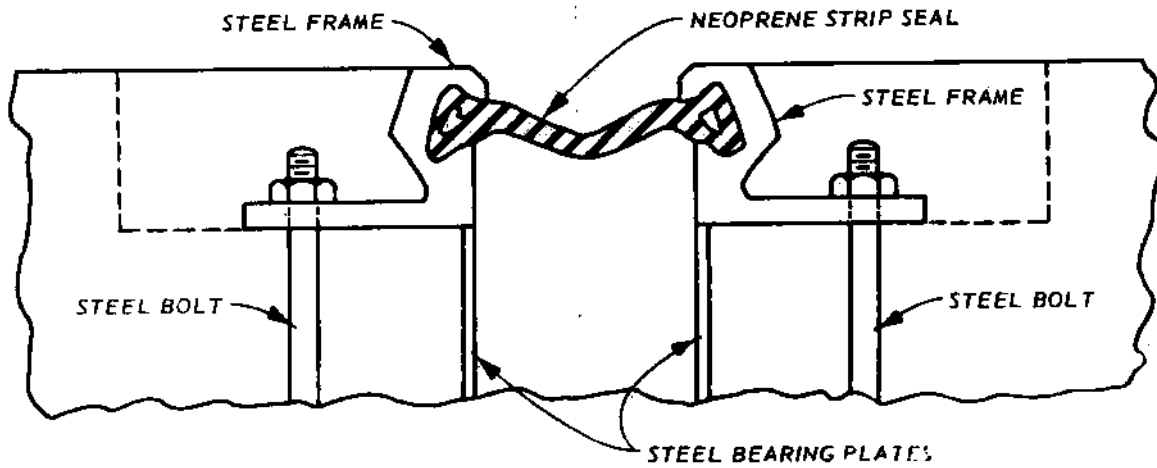
For the movement that is anticipated in the joints of long prestressed slabs, it is doubtful that any poured seal will perform satisfactorily without the extended flanges of I-beams or channels built into the joints. Similarly, compression seals will not keep the joint sealed during periods of contraction of the slabs. Long-term performance requires a joint seal that can be attached to the concrete slabs. Six types of joint seals that are available are shown in Figures 13-15. Although all of these six types have not been fully tested, it is expected that they will perform adequately.

The type A seal shown in Figure 13 is composed of a polyurethane core covered by a neoprene rubber shield. Installation of this seal requires priming of the surfaces, applying adhesive, and placing the seal by the use of clamps to compress the seal to a width less than the width of the opening to prevent loss of adhesive when the seal is placed in the opening. The clamps are then removed, and the seal comes in contact with the walls of the joint. The approximate cost of this type installation is \$18.50 per lineal foot. This price includes the seal, primer, and adhesive but does not include the installation clamps.

The type B seal (Figure 13) consists of an extruded neoprene

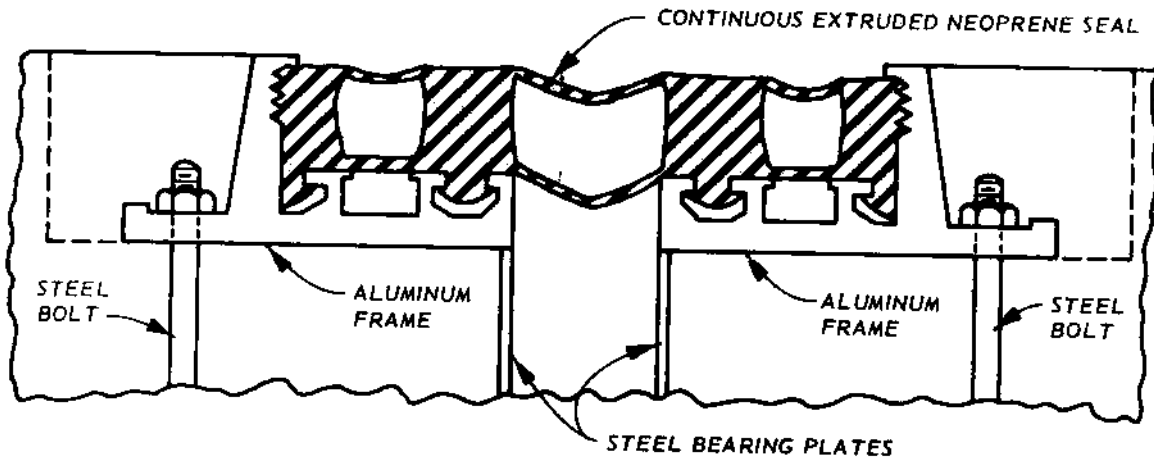


TYPE A

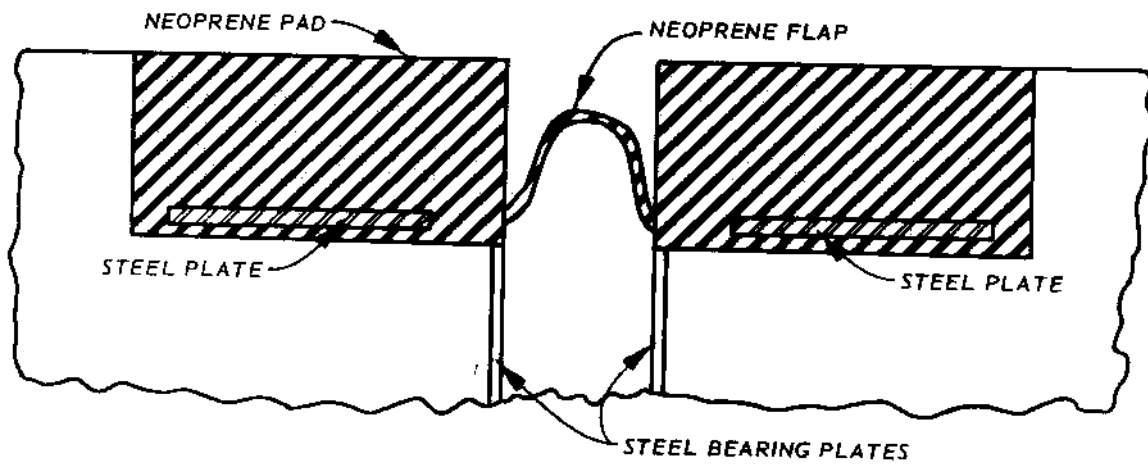


TYPE B

Figure 13. Types A and B joint seals

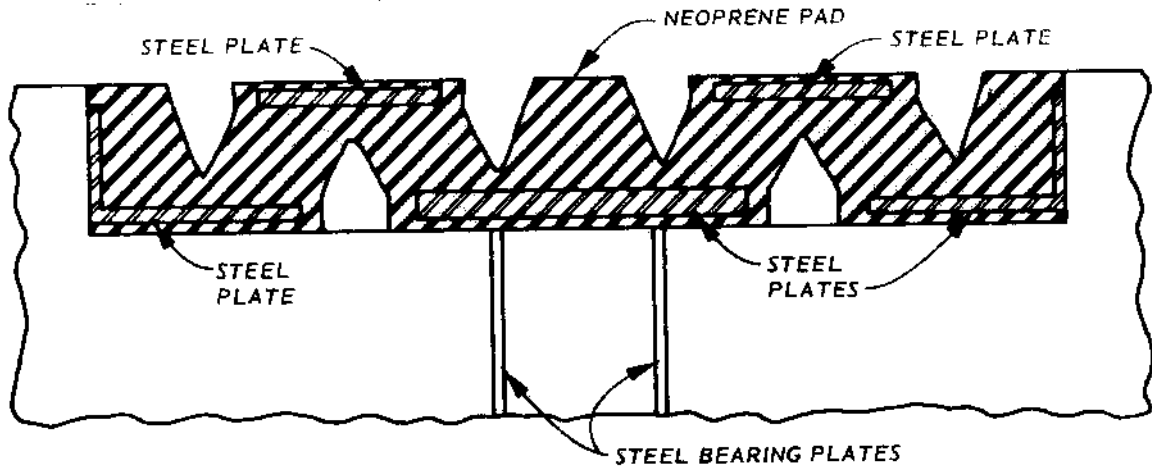


TYPE C

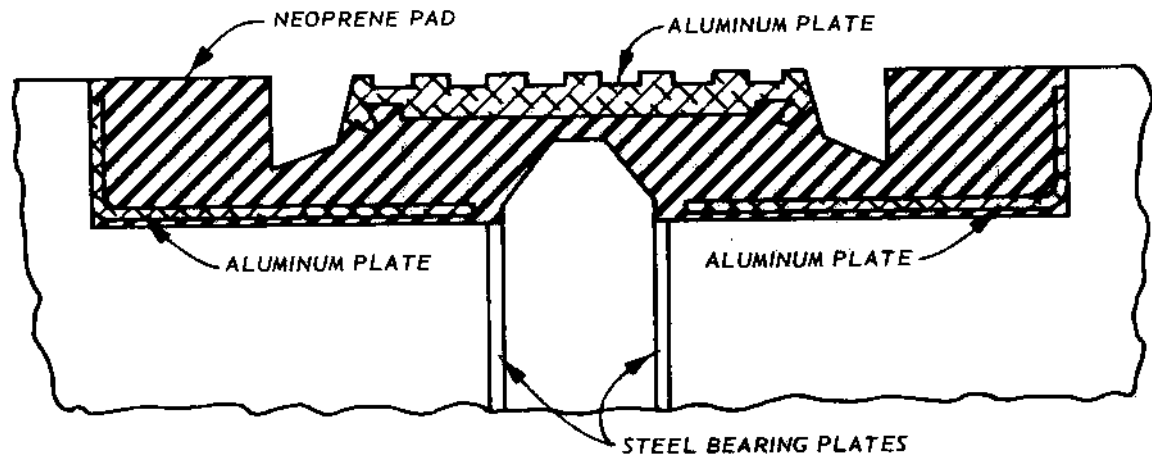


TYPE D

Figure 14. Types C and D joint seals



TYPE E



TYPE F

Figure 15. Types E and F joint seals

rubber gland installed in steel extrusions that are anchored in the concrete slabs. The cost of this joint is approximately \$25.00 per lineal foot. By welding a steel angle to the bottom of the steel extrusion, end anchorages for the stressing cables can be provided.

The type C seal (Figure 14) consists of aluminum extrusions fastened to the concrete slabs by means of anchor bolts with an extruded neoprene rubber seal installed in the extrusions. The cost of this joint is approximately \$40.00 per lineal foot.

More costly joint seals are manufactured by several companies, including types D, E, and F in Figures 14 and 15. These seals are similar in design, and they consist of integrally molded neoprene rubber and steel reinforcement. The wearing surfaces of two are rubber while that of the other is aluminum. The price of joint seals of these types ranges from about \$65.00 to about \$90.00 per lineal foot depending on the manufacturer. The anticipated life of these seals is approximately 20 years. Constructing leave-outs in the concrete slabs and placing of anchor bolts are necessary for installation of these seals.

While the seals of types A-F are considerably more costly than the poured-in-place sealants, their anticipated longer life spans may offset the additional cost. The selection of a seal type for joints will require a cost effective-type study based upon actual installed costs of the seals.

CONSTRUCTION PROCEDURES

While the number of prestressed pavements has been limited, there has been a large variance of construction procedures used. Like the recommended selection of design alternatives, the recommended construction procedures that follow are based on a study of past prestressed pavement construction and the performance of the pavements.

SUBBASE

To protect against pumping, an open-graded granular material should be used to construct the subbase to provide adequate moisture drainage to the shoulders. In addition, the subbase should be designed and constructed so that the modulus of subgrade reaction k will be a minimum of 200 pci.

FRICITION-REDUCING LAYER

In constructing the friction-reducing layer, the sand should be spread evenly over the subbase. Any convenient means can be used to place the sand. It should then be screeded to a minimum thickness of 0.25 in. immediately before the two polyethylene sheets are placed. The screeding will insure that the sand layer has a smooth even surface. If windy conditions are expected, a means of holding the polyethylene sheets in place must be provided. (In the past, high winds have displaced the polyethylene even with the prestressing tendons and conduits in place.) The polyethylene sheets can be held in place with edge anchors.

PLACEMENT OF STRESSING TENDONS

As stated previously, the longitudinal tendons should be placed at middepth with the transverse tendons immediately below them. The transverse conduits should be placed on metal chairs at the desired depth while the longitudinal conduits can rest on the transverse conduits. All of the conduits should be tied firmly in place to maintain the proper alignment during placement of the concrete.

PLACEMENT AND CURING OF CONCRETE

Care should be taken so that the placement of the concrete does not disturb the position of the prestressing conduits. The concrete vibrators should be lifted over the conduits to prevent displacement of them. The concrete for each prestressing slab should be placed in one continuous operation. Use of machine vibrators at the sides and ends of the prestressed slabs should be supplemented with use of hand vibrators and should be carefully controlled to insure proper placement of the concrete in the vicinity of the jacking force used for prestressing. The concrete should be properly cured to prevent early shrinkage and contractions in the concrete prior to prestressing. Wet-curing with either cotton or burlap mats covered by waterproof paper or polyethylene sheets is recommended. Waterproof paper, curing compound, or polyethylene sheets by themselves should not be used for curing the prestressing slabs during the initial curing period. The mats should be kept wet, and the wet-curing should continue until the prestress in the concrete has reached one-half of the design prestress but for not less than 72 hours. For cold weather conditions, the pavement should be protected from frost action by protective curing for at least 10 days.

TENDON STRESSING

Early tendon stressing is important in preventing cracking in the long prestressed slabs during curing. If the pavement is stressed too soon, however, it may fail due to an insufficient gain of concrete compressive strength. This is the reason that longitudinal prestressing is sometimes applied in stages rather than having the final design prestress applied as the concrete gains strength. The amount of prestressing that can be applied to the pavement at any one time should be determined based on the compressive strength of cylinders made from the concrete that is placed in the slab ends being stressed. This is the area in which compressive failure is most likely to occur. The maximum bearing stress on the concrete should be six-tenths of the concrete compressive strength. The bearing stress on the concrete will depend on

the amount of prestressing and the properties and dimensions of the bearing plate used to transmit the force from the jack to the concrete.

For the first step of the stressing procedure, any preliminary tension in the tendons should be released. The tendons should then be pulled back and forth in the conduits a short distance several times to reduce the tendon friction. The longitudinal tendon stressing should then be applied in three stages with the amount to stress of each successive stage being 25, 50, and 100 percent of the design prestress. The stress in the first two stages can be applied at just one end of the slab. To overcome the stress loss due to tendon friction, the stress in the final stage should be applied according to the procedure for stress losses.

In prestressing the transverse tendons of a pavement with a width requiring two or more adjacent prestressed slabs, the prestressing tendons must pass through the longitudinal joints, and the multiple-lane pavement should be prestressed as a unit. A number of paving sequences can be used. An interior slab can be placed first with the paving sequence then working outward paving each successive slab, or the paving can progress from the outer slabs to the center. Another method is to pave alternate slabs and fill the other slabs in later. Whatever the sequence used, the transverse prestressing should be done all in one stage after all of the adjacent slabs have been placed since there should be no danger of longitudinal shrinkage cracking due to the relatively narrow slab widths.

GROUTING

After the design prestress is achieved, the tendons should be grouted in place with a neat, cement grout. For this purpose, grouting connections should be provided at both ends of the conduits and along the conduits at intervals not to exceed 150 ft. The conduits should be flushed with water immediately before the grouting operation. The grouting for the longitudinal conduits should start at the lower end of the slab and proceed upward along the conduit in the following manner. The grout should be pumped into the end vent until grout is ejected from

the first interior grouting vent. The end vent should then be sealed, and the pumping should continue at the first vent until grout is ejected from the next interior vent. The grouting operation should proceed in a similar manner until grout is ejected from the vent at the other end of the conduit. The grouting pump should be able to inject the grout at a minimum pressure of 150 psi.

CONCLUSIONS

Prestressed concrete pavements can be designed for airport pavements with a reasonable degree of accuracy with respect to the load-carrying capability and the number of passes that can be sustained.

Construction of airport pavements with prestressed concrete rather than conventional concrete will result in a savings of concrete due to the smaller thicknesses required. In addition, the long prestressed slabs will result in fewer joints to be maintained and will provide a smoother operating surface.

The recommendations concerning the design criteria and construction procedures are considered to be conservative in some areas. This tendency is due to the uncertain state-of-the-art in these areas. All recommendations are subject to being refined pending further study.

~~SYNOPSIS~~

Based upon the review of the literature describing past experience with prestressed pavements and the results of analytical studies and instrumentation data collected as a part of this study, the following recommendations on design and construction of prestressed pavements for civil airports are made:

- a. To achieve the prestress in the concrete, the posttensioning system using external jacks should be used.
- b. The maximum level of effective prestress in the concrete should be 400 psi, while the minimum level should be 100 psi more than the maximum subgrade restraint stress F .
- c. The longitudinal prestressing tendons should be set at mid-depth with the transverse tendons immediately below them.
- d. An open-graded granular material should be used to construct the subbase.
- e. The subbase should be designed and constructed so that the modulus of subgrade reaction k is a minimum of 200 pci.
- f. The friction-reducing layer should consist of two polyethylene sheets over a 1/4-in. sand layer of uniform depth.
- g. The design length of the prestressed slabs should be 500 ft.
- h. The 90-day flexural strength of the concrete used in the prestressed slabs should be a minimum of 700 psi.
- i. The thermal expansion coefficient of the concrete should be a maximum of 6×10^{-6} in./in. per degree Fahrenheit.
- j. The prestressing tendons should be stressed in three stages as described herein.

In addition to the above recommendations, the following experimental work is recommended:

- a. The A/ℓ^2 versus percent single-wheel failure load S relationship depicted by Figure 8 plays an important role in the design of prestress level and pavement thickness and should be verified by means of model tests using the procedures described in Reference 9 before widespread use of the criteria is made.
- b. Selected sections of prestressed pavements which can be instrumented, evaluated, and observed should be constructed for a range of foundation types to gain verification of the design criteria presented herein and to modify the criteria as necessary.

REFERENCES

1. Armstrong, W. E. J., "Pre-Stressed Concrete Roads," Chartered Civil Engineer, May 1952, pp 11-15.
2. Friberg, B. F. and Pasko, T. J., Jr., "Prestressed Concrete Highway Pavement at Dulles International Airport: Research Progress Report to 100 Days," Report No. FHWA-RD-72-29, Aug 1973, Federal Highway Administration, Offices of Research and Development, Washington, D. C.
3. Friberg, B. F., "Municipal Airport, Rochester, Minnesota, Design Study for 60 Ft by 300 Ft Taxiway Concrete Pavement Prestressed in Longitudinal Direction by Pre-Tensioned Steel Strand," Contract Report, Jan 1969, prepared for the City of Rochester, Minn.
4. Sargious, M. and Wang, S. K., "Economical Design of Prestressed Concrete Pavements," Journal of the Prestressed Concrete Institute, Vol 16, No. 4, Jul-Aug 1971, pp 64-79.
5. _____, "Design of Prestressed Concrete Airfield Pavements Under Dual and Dual Tandem Wheel Loading," Journal of the Prestressed Concrete Institute, Vol 16, No. 6, Nov-Dec 1971, pp 19-30.
6. Huyghe, G. and Celis, R., "Joint-Free Experimental Prestressed Pavement," Journal of the Prestressed Concrete Institute, Vol 17, No. 1, Jan-Feb 1972, pp 58-72.
7. Levi, F., "Experimental Theoretical Study of a Prestressed Slab on an Elastic Support Beyond the Limits of Elasticity," Jun 1953, Annales de L'Institut du Batiment et des Travaux Publics.
8. Cot, P. D. and Becker, E., "Design of Prestressed Concrete Runways," No. 292, May 1956, Revue Generale des Routes et des Aerodromes.
9. Carlton, P. F. and Behrmann, R. M., "Model Studies of Prestressed Rigid Pavements for Airfields," Highway Research Board Bulletin 179, 1958, pp 32-50.
10. Sale, J. P., Hutchinson, R. L., and Carlton, P. F., "Development of a Procedure for Designing Prestressed Airfield Pavements," Proceedings, Highway Research Board, Vol 40, 1961, pp 205-234.
11. Carlton, P. F. and Behrmann, R. M., "Small-Scale Model Studies of Prestressed Rigid Pavements; Development of the Model and Results of Exploratory Tests," Technical Report No. 4-13, Part I, May 1962, U. S. Army Engineer Ohio River Division Laboratory, CE, Cincinnati, Ohio.
12. _____, "Small Scale Model Studies of Prestressed Rigid Pavements for Military Airfields; Single-Wheel Loadings on Pre-Tensioned and Post-Tensioned Slabs," Technical Report No. 4-25, Part II, May 1963, U. S. Army Engineer Ohio River Division Laboratory, CE, Cincinnati, Ohio.

13. Carlton, P. F. and Behrmann, R. M., "Repetitive Loading Model Tests of Prestressed Rigid Pavement on a Low-Strength Subgrade," Technical Report No. 4-28, May 1964, U. S. Army Engineer Ohio River Division Laboratory, CE, Cincinnati, Ohio.
14. Pickett, G. and Ray, G. K., "Influence Charts for Concrete Pavements," Transactions, American Society of Civil Engineers, Vol. 116, Paper No. 2425, 1951, pp 49-73.
15. Hudson, W. and Matlock, H., "Discontinuous Orthotropic Plates and Pavement Slabs," Research Report 56-6, May 1966, Center for Highway Research, The University of Texas, Austin, Tex.
16. Stelzer, C. F., Jr., and Hudson, W. R., "A Direct Computer Solution for Plates and Pavement Slabs," Research Report 56-9, Oct 1967, Center for Highway Research, The University of Texas, Austin, Tex.
17. Panak, J. J. and Matlock, H., "A Discrete-Element Method of Multiple-Loading Analysis for Two-Way Bridge Floor Slabs," Research Report 56-13, Jan 1970, Center for Highway Research, The University of Texas, Austin, Tex.
18. Renz, C. F. and Williams, F. H., "Design and Construction Report, Prestressed Concrete Test Tracks P1 and P2, Sharonville, Ohio," Technical Report No. 4-14, Nov 1960, U. S. Army Engineer Ohio River Division Laboratory, CE, Cincinnati, Ohio.
19. Brown, D. N. and Thompson, O. O., "Lateral Distribution of Aircraft Traffic," Miscellaneous Paper S-73-56, Jul 1973, U. S. Army Engineer Waterways Experiment Station, CE, Vicksburg, Miss.
20. Subcommittee on Prestressed Concrete Pavements, HRB Committee on Rigid Pavement Design, "Prestressed Concrete Pavements," Special Report 78, 1963, Highway Research Board, National Academy of Science--National Research Council, Washington, D. C.
21. U. S. Army Engineer Ohio River Division Laboratory, CE, "Tentative Concepts for the Design of Prestressed Concrete Airfield Pavements," Unpublished Report, Mar 1958, Cincinnati, Ohio.
22. Odom, E. C. and Ledbetter, R. H., "Prestressed Concrete Pavements; Dulles Test Road Instrumentation and Load Tests," Report No. FAA-RD-74-34-I, Vol I, November 1974, Department of Transportation, Federal Aviation Administration, Washington, D. C.
23. Best, J. L. et al., "Length Changes in Prestressed Concrete Slabs on Subgrades," Aug 1960, Missouri School of Mines and Metallurgy, University of Missouri, Rolla, Mo.
24. Federal Highway Administration, Offices of Research and Development, "Final Report, Prestressed Concrete Pavement Construction," Feb 1973, Washington, D. C.

BIBLIOGRAPHY

- Caludon, J. G., "Study of Stresses in Prestressed Concrete Pavements at Maison-Blanc Airport," Paper presented at 42d Annual Meeting of the Highway Research Board, Jan 1963.
- Cholnoky, T., "Prestressed Concrete Pavements for Airfields," Paper presented at American Concrete Institute 51st Annual Convention, Feb 1955, Milwaukee, Wis.
- _____, "Theoretical and Practical Aspects of Prestressed Concrete for Highway Pavements," Highway Research Board Bulletin 179, Jan 1958, pp 13-31.
- Christensen, A. P., "Load Tests on Post-Tensioned Pavement Slabs," Highway Research Board Highway Research Record No. 60, 1963, pp 95-115.
- Harris, A. J., "Prestressed Concrete Runways: History, Practice and Theory," Airport Paper No. 31, Oct 1956, The Institution of Civil Engineers Airport Division Meeting, London.
- McIntyre, J. P. and Mellinger, F. M., "Prestressed Concrete Taxiway, Biggs Air Force Base, Texas, U. S. A.," Paper prepared for Fourth International Congress of the "Federation Internationale Precontrainte," Nov 1961, Rome, Italy.
- Mellinger, F. M., "Prestressed Concrete Airfield Pavements," Proceedings, The World Conference on Prestressed Concrete, Aug 1957, San Francisco, Calif.
- _____, "Summary of Prestressed Concrete Pavement Practices," Journal of the Air Transport Division, American Society of Civil Engineers, Vol 87, No. AT2, Aug 1961, pp 163-177.
- Melville, P., "Review of French and British Procedures in the Design of Prestressed Pavements," Highway Research Board Bulletin 179, Jan 1958, pp 1-12.
- Meyerhof, G. G., "Load-Carrying Capacity of Concrete Pavements," Journal of the Soil Mechanics and Foundations Division, American Society of Civil Engineers, Vol 88, No. SM3, Jun 1962, pp 89-116.
- Moreell, B., Murray, J. J., and Heinzerling, J. E., "Experimental Prestressed Concrete Highway Project in Pittsburgh, Pennsylvania," Paper presented at 37th Annual Meeting of the Highway Research Board, Jan 1958.
- Pasko, T. J., Jr., "Prestressed Highway Pavement at Dulles Airport for TRANSPO 72," Journal of the Prestressed Concrete Institute, Vol 17, No. 2, Mar-Apr 1972, pp 46-54.
- Pickett, G. et al., "Deflections, Moments and Reactive Pressures for Concrete Pavements," Bulletin No. 65, Oct 1951, Kansas State College, Manhattan, Kans.

Preload Engineers, Inc., "Review and Study of Foreign and Domestic Test Data on Prestressed Concrete Pavements," Contract Report, May 1954, Arlington, Va. (prepared for U. S. Army Engineer Ohio River Division Laboratory).

Renz, C. F., "Prestressed Concrete Taxiway No. 1, Naval Air Station, Lemoore, California," Memorandum Report, Aug 1960, U. S. Army Engineer Ohio River Division Laboratory, CE, Cincinnati, Ohio.

Renz, C. F. and Williams, F. H., "Design and Construction Report, Prestressed Concrete Overlay and Buckling Slabs, Sharonville, Ohio," Technical Report No. 4-10, Nov 1958, U. S. Army Engineer Ohio River Division Laboratory, CE, Cincinnati, Ohio.

Renz, C. F. et al., "Experience with Prestressed Concrete Airfield Pavements in the United States," Journal of the Prestress Concrete Institute, Vol 6, No. 1, Mar-Apr 1961.

Smith, J. R. and Lightholder, R. K., "Moving Load Test on Experimental Prestressed Concrete Highway Slab," Paper presented at University of Pittsburgh, Jul 1963.

Subcommittee VI of ACI Committee 325, "Prestressed Pavement - A World View of Its Status," Proceedings, American Concrete Institute, Vol 55, 1959, pp 829-838.

Westergaard, H. M., "New Formulas for Stresses in Concrete Pavements of Airfields," Transactions, American Society of Civil Engineers, Vol 113, Paper No. 2340, 1948, pp 425-444.

