NASA/TP—2008–215467

Using the Modified Precursor Method to Estimate the Size of Cycle 24

Robert M.Wilson and David H. Hathaway
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July 2008

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National Aeronautics and
Space Administration

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF ACRONYMS

DES	the descent duration
FLL	the cycle classification “Fast-Large-Long”
FLS	the cycle classification “Fast-Large-Short”
FSL	the cycle classification “Fast-Small-Long”
FSS	the cycle classification “Fast-Small-Short”
LP	long period
ME	modern era
NDD	the number of disturbed days
NDDM	the maximum value of NDD
NOAA	National Oceanic and Atmospheric Administration
SLL	the cycle classification “Slow-Large-Long”
SLS	the cycle classification “Slow-Large-Short”
SP	short period
SSL	the cycle classification “Slow-Small-Long”
SSS	the cycle classification “Slow-Small-Short”
TP	technical publication

NOMENCLATURE

aa	the aa geomagnetic index
aaI	the residual or following component of the aa index
$aaIM$	the maximum value of aaI
aaM	the maximum value of aa
Ap	the Ap geomagnetic index
ApM	the maximum value of Ap
ASC	the ascent duration
cl	the confidence level
$E(RM)$	the epoch of sunspot maximum
n	cycle number
nT	nanotesla (a measure of magnetic flux density)
P	probability
PER	period (cycle length)
r	coefficient of correlation
r^2	coefficient of determination
RM	maximum sunspot amplitude
$\sum PER$	the sum of consecutive sunspot cycle pair PER values
$\sum RM$	the sum of consecutive sunspot cycle pair RM values
se	the standard error of estimate
t	the elapsed time in months

NOMENCLATURE (Continued)

x	the independent variable in the regression equation
y	the dependent variable in the regression equation
y'	an alternate regression equation that excludes cycles 14 and 19

TECHNICAL PUBLICATION

USING THE MODIFIED PRECURSOR METHOD TO ESTIMATE THE SIZE OF CYCLE 24

1. INTRODUCTION

Recently, Dabas et al.[1] have examined the relationship between 12-month moving averages of the maximum amplitude of a sunspot cycle and the number of disturbed days (*Ap* index ≥ 25), which they call the "disturbance index," from cycle maximum of the preceding cycle. In particular, they find a strong correlation between the maximum amplitude of the following sunspot cycle and the value of the disturbance index measured about four years after the preceding cycle's maximum amplitude (corresponding to about three years or so before subsequent cycle minimum amplitude). Based upon their inferred correlation, they predicted cycle 24's maximum amplitude to be about 124 ± 23, occurring about 44 ± 5 months after its minimum amplitude occurrence (or about mid-to-late 2011, if the official start of cycle 24 is March 2008).

Over the years, many techniques have been proffered as providing a means whereby the size of a sunspot cycle might be estimated. Of particular interest are those techniques based on precursor geomagnetic information. For example, more than forty years ago Ohl[2] found a high correlation to exist between the minimum of geomagnetic activity near sunspot minimum and the later-occurring maximum sunspot amplitude. He also showed that the level of geomagnetic activity during the last few years of a sunspot cycle is well-correlated with the maximum amplitude of the following sunspot cycle.[3] More recently, Wilson[4] investigated a number of single variate and bivariate precursor techniques, as applied to cycle 22; Thompson[5] noted the importance of the number of disturbed days (*Ap* ≥ 25) in cycle prediction; and Hathaway, Wilson and Reichmann[6] described the so-called "Combined Precursor Method" and the "Combined Solar Cycle Activity Forecast Method," applying them to cycle 23. Several other precursor techniques have also appeared.[7–12]

The purpose of this Technical Publication (TP) is to re-examine the relationship as reported by Dabas et al. and determine if similar relationships exist using the *Ap*, *aa*, and *aaI* geomagnetic indices, where the *aaI* index is the residual or following recurrent component associated with high-speed streams, having removed the leading sporadic component due to the variation of the solar cycle.[13–16] This TP will also provide an estimation of the ascent duration for cycle 24 based on the Waldmeier effect and an examination of the variation of sunspot cycle lengths and Hale cycle effects.

2. RESULTS AND DISCUSSION

2.1 The Number of Disturbed Days (NDD)

Figure 1 plots individually the 12-month moving averages of the number of disturbed days (NDD) from maximum amplitude occurrence [E(*RM*)] for elapsed times in months *t* equal 0 to 84 months past *E*(*RM*) for cycles 17–23. Also shown are the maximum values of the NDD, their occurrence dates relative to *E*(*RM*), the occurrence dates for *E*(*RM*) and the descent durations (DES) of the cycles in months. As an example, for cycle 17, its NDD maximum (NDDM) equals 7.5, occurring 78 months past *E*(*RM*), or about October 1943, just four months prior to sunspot minimum for cycle 18. Clearly, the variation of NDD is best described as being episodic, with typically several peaks occurring during the descending portion of a sunspot cycle. Cycle 18 had the largest NDDM and is followed by cycle 19, the largest sunspot amplitude cycle during the modern era. NDDM for cycle 23 measures 9.8, occurring 40 months past *E*(*RM*), and this is the third largest value during cycles 17–23. Table 3 in the appendix gives a tabulation of NDD(*t*) values for elapsed time in months from *E*(*RM*) $t=0$–84 months for cycles 17–23.

Figure 2 shows the scatter plot of *RM* for cycle $n+1$ versus NDDM for cycle *n*. Shown are the inferred correlation *y*, the coefficient of correlation *r*, the coefficient of determination r^2 (a measure of the amount of variance explained by the inferred correlation), the standard error of estimate *se*, and the confidence level (*cl*) for the fit. While there appears to be a hint of positive correlation to exist between the size of the following sunspot cycle and the preceding maximum value of NDD, strictly speaking, the inferred correlation is not statistically important (obviously due to the brevity of the NDD record). The arrow marks the value of NDDM for cycle 23, which, according to the inferred correlation, suggests that cycle 24 could have an *RM* measuring about 156 ± 70 (the 90% prediction interval). It should be noted that, instead of comparing the preceding NDDM with the following *RM*, one could compare the maximum NDD of the "bump" during the latter half of the decline, $t > 42$ months, against the following *RM*. Doing so, one finds that the inferred correlation is statistically important and that cycle 24 should have $RM = 109 \pm 42$ (the 90% prediction interval).

Figure 3 displays the variation of the coefficients of determination r^2, resulting from a comparison of *RM* for cycle $n+1$ versus NDD(*t*) for cycle *n*, for elapsed time in months *t* equal 0 to 84 months past *E*(*RM*). This is essentially the same result as reported by Dabas et al.[1] Correlations prior to about three years after *E*(*RM*) are negative (inverse) relationships and those from about three years are positive relationships, with the ones around four years past *E*(*RM*) being the most statistically important. Unlike that found by Dabas et al., however, is that there appears to be a few months about two years after *E*(*RM*) where the inferred correlation seems to be statistically important ($cl \geq 95\%$), although the inferred correlation is not as statistically important as the one about four years past *E*(*RM*) of the preceding cycle. The two circled points 1 and 2 identify the two most important correlations for the two intervals ($t=25$ and 48 months). For circled point 1 ($t=25$ months), it has $r^2=0.719$, $r=-0.848$, $se=19.3$ and $cl > 95\%$. Since NDD($t=25$)$=3.0$ for cycle 23, one infers that *RM* for cycle 24 should measure about 190 ± 41 (the 90% prediction interval). For circled point 2, the strongest inferred correlation ($r^2=0.841$, $r=0.917$,

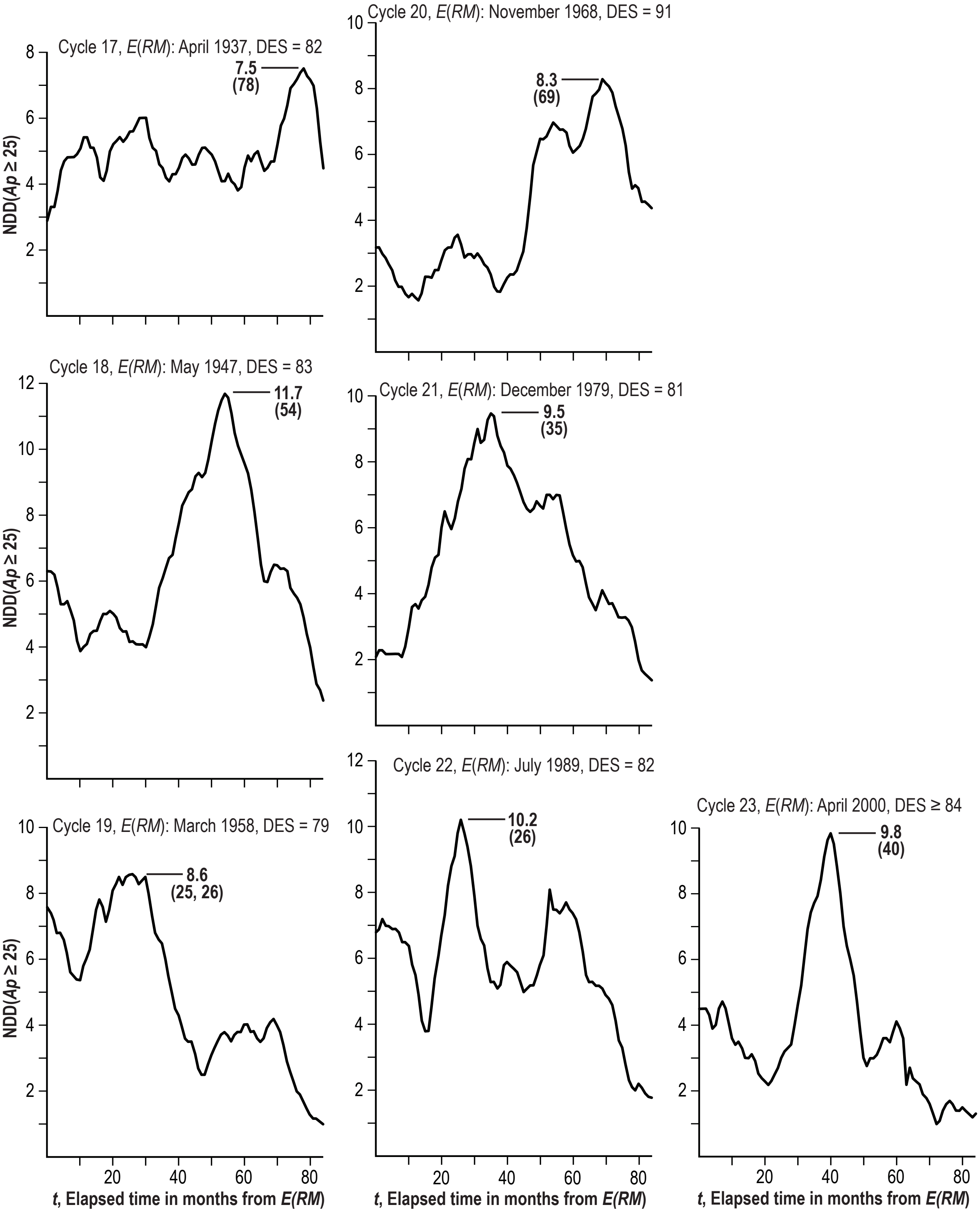


Figure 1. 12-month moving averages of NDD(t) for elapsed time in months from $t = 0$ to 84 months past $E(RM)$ for cycles 17–23.

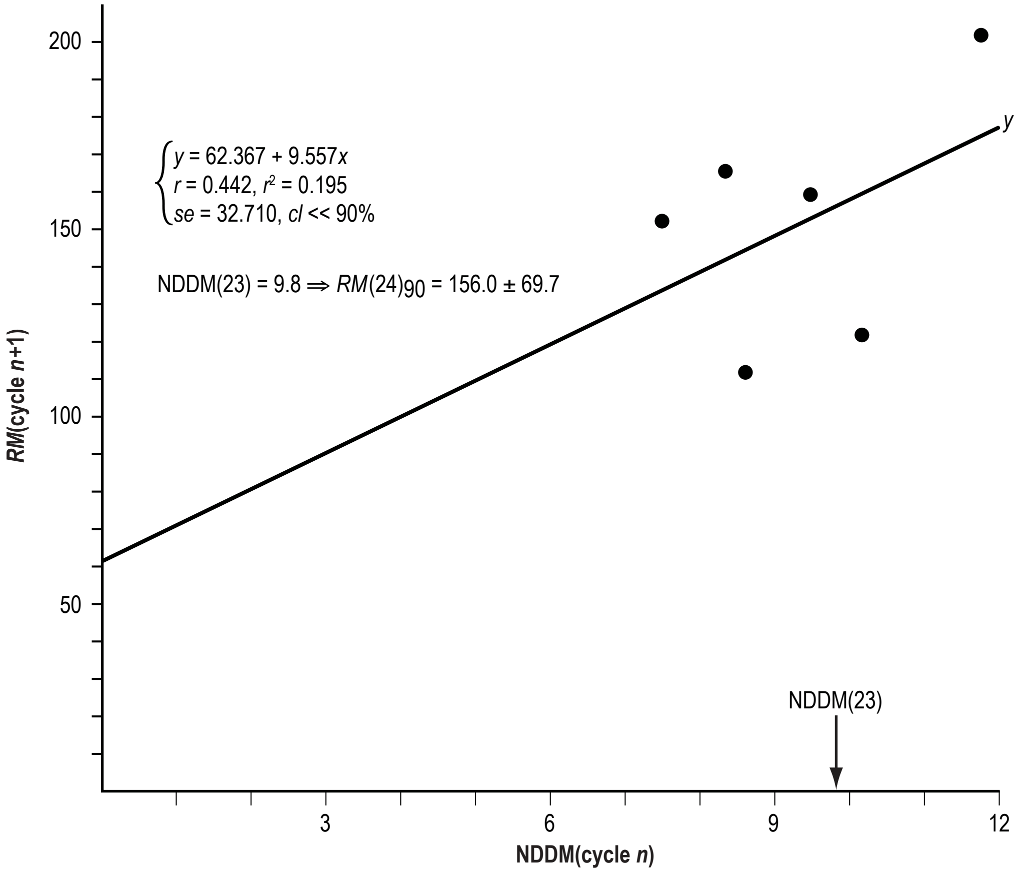


Figure 2. Scatter plot of *RM*(cycle $n+1$) versus NDDM.

$se = 13.5$ and $cl > 99\%$), it occurs at $t = 48$ months. For cycle 23, NDD($t = 48$) equals 4.7, which suggests a somewhat smaller *RM* for cycle 24, equal to 137 ± 31 (the 90% prediction interval).

2.2 The *Ap* Index

Figure 4 plots individually the 12-month moving averages of *Ap* from *E*(*RM*) for elapsed times in months *t* equal 0 to 84 months past *E*(*RM*) for cycles 17–23. Also shown are the maximum values of *Ap*, their occurrence dates relative to *E*(*RM*), the occurrence dates for *E*(*RM*) and the descending durations (DES) of the cycles in months. As an example, for cycle 17, its *Ap* maximum (*ApM*) equals 18.0 occurring 30 months past *E*(*RM*), or about October 1939, some 52 months prior to sunspot minimum for cycle 18. Clearly, the variation of *Ap* is best described as being episodic (quite similar to NDD, with the exception of cycle 17), with typically several peaks occurring during the descent duration of a sunspot cycle. Cycle's 18 and 22 had the largest *ApM* (= 25.0), and *ApM* for cycle 23 measures 22.3 occurring 40 months past *E*(*RM*), which is the third smallest value during cycles 17–23. Table 4 in the appendix gives a tabulation of *Ap*(t) values for elapsed time in months from *E*(*RM*) $t = 0$–84 months for cycles 17–23.

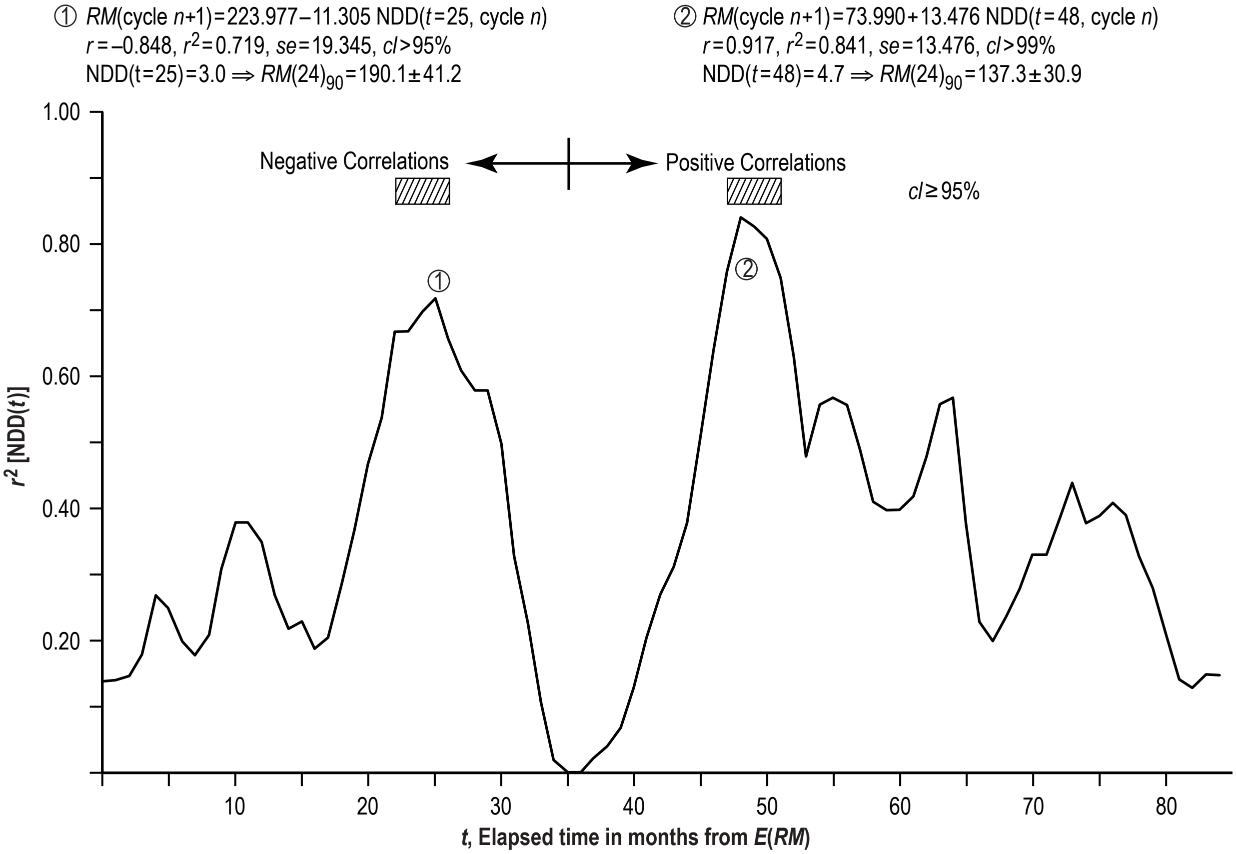


Figure 3. Variation of r^2 for $t=0$ to 84 months past $E(RM)$, resulting from a comparison of RM for cycle $n+1$ versus NDD(t) for cycle n.

Figure 5 shows the scatter plot of *RM* for cycle $n+1$ versus *ApM* for cycle *n*. Clearly, the inferred correlation between *RM* (cycle $n+1$) and *ApM* (cycle *n*) is not statistically important (again, due to the brevity of the *Ap* record). The arrow marks the value of *ApM* for cycle 23, which, according to the inferred correlation, suggests that cycle 24 could have an *RM* measuring about 151 ± 78 (the 90% prediction interval). As before for NDD, however, if instead of comparing *ApM* with the following *RM*, one compares the maximum *Ap* of the 'bump' during the latter half of the declining portion of the sunspot cycle, $t>42$ months, against the following *RM*, one finds that the inferred correlation is statistically important and that cycle 24 should have *RM* equal to about 121 ± 31, the 90% prediction interval.

Figure 6 displays the variation of the coefficients of determination r^2, resulting from a comparison of *RM* for cycle $n+1$ versus *Ap*(t) for cycle *n* for *t* equal 0 to 84 months past *E*(*RM*). This is essentially the same result as shown above for NDD(t). Namely, correlations prior to about three years after *E*(*RM*) are negative (inverse) relationships and those from about three years are positive relationships, with the ones around four years past *E*(*RM*) being the most statistically important. However, unlike that found for NDD(t), no statistically important relationship is found to occur prior to about four years past *E*(*RM*) of the preceding cycle. Circled point 1 identifies the most important correlation, and circled

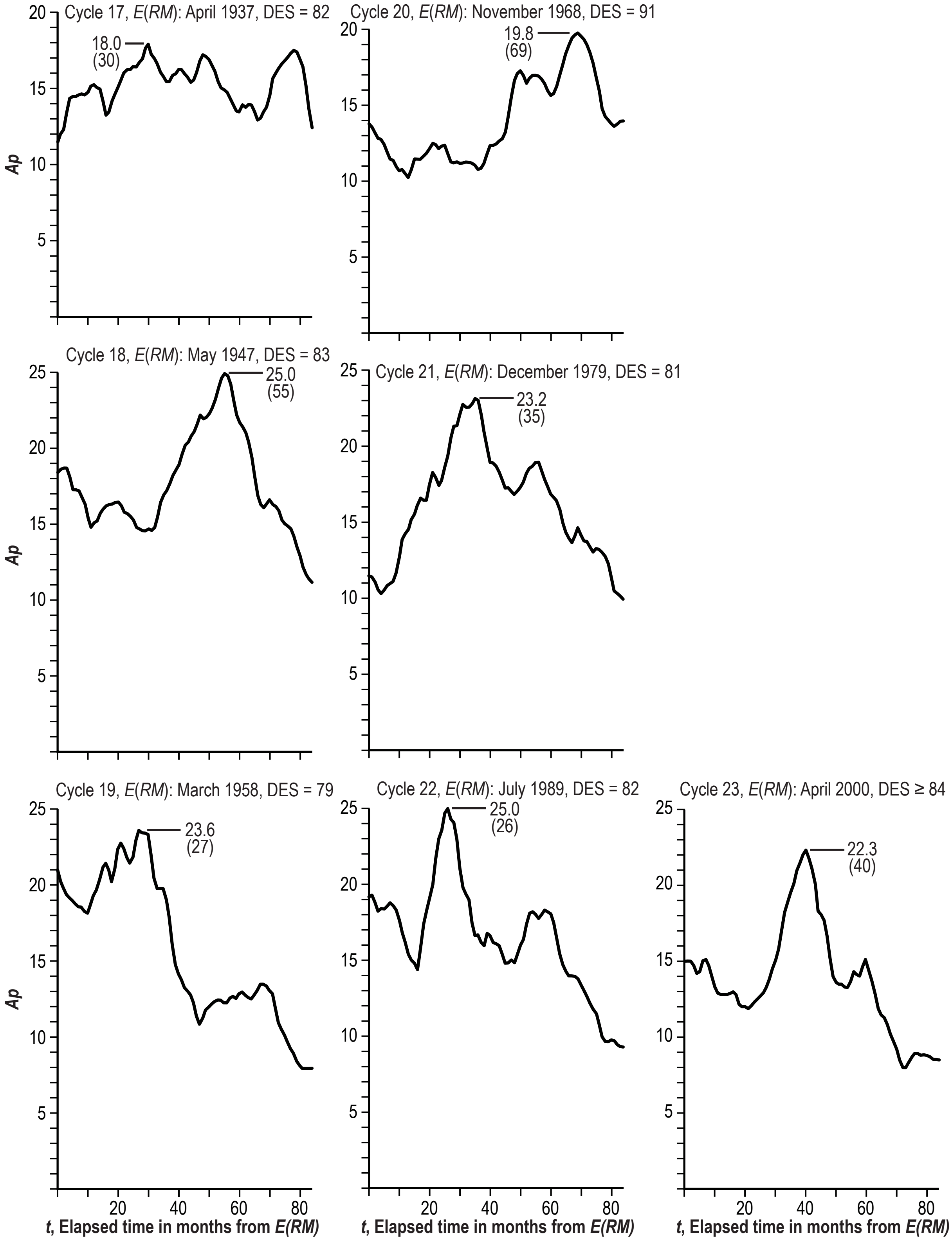


Figure 4. 12-month moving averages of $Ap(t)$ for elapsed time in months from $t = 0$ to 84 months past $E(RM)$ for cycles 17–23.

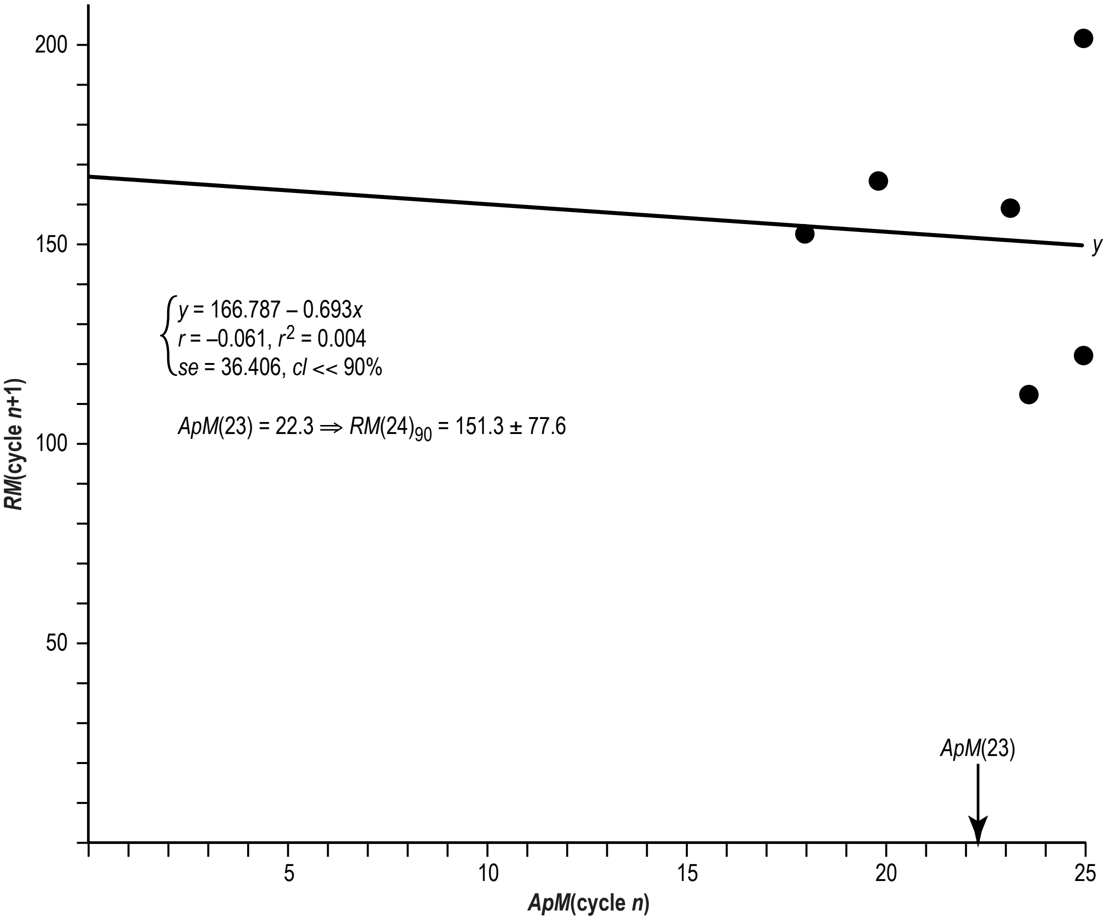


Figure 5. Scatter plot of *RM*(cycle $n+1$) versus *ApM*.

point 2 identifies another localized peak in r^2 that occurs slightly later in time, but one that also is statistically important. For circled point 1 ($t=49$ months), it has $r^2=0.910$, $r=0.954$, $se=11.0$ and $cl>99.5\%$. Since $Ap(t=49)=14.0$ for cycle 23, one infers that *RM* for cycle 24 should measure about 125 ± 23 (the 90% prediction interval). However, for circled point 2, it has $r^2=0.784$, $r=0.886$, $se=16.9$ and $cl>98\%$), occurring at $t=64$ months, and suggests *RM* for cycle 24 to be about 109 ± 36 (the 90% prediction interval). The overlap of the *RM* predictions for cycle 24 based on the two correlations [*RM* versus NDD($t=48$) and *RM* versus $Ap(t=49)$] is about 124 ± 21.

2.3 The *aa* and *aaI* Indices

Figure 7 shows individually the 12-month moving averages of the *aa* (upper curves) and *aaI* (lower curves) geomagnetic indices from *E*(*RM*) for elapsed times in months *t* equal 0 to 84 months past *E*(*RM*) for cycles 11–23, some six additional cycles as compared to the NDD or *Ap* data sets. As before, shown are the *E*(*RM*) occurrences dates, the descent durations (DES), and the peak values of *aa* and *aaI* and their occurrence dates relative to *E*(*RM*). Clearly, *aaI* is found to strongly mimic *aa*. Recall that *aaI* is the residual or following component of the *aa* index, having removed the leading sporadic component due to the sunspot cycle.[16] Additionally, it should be noted that the *aa* and *aaI* indices

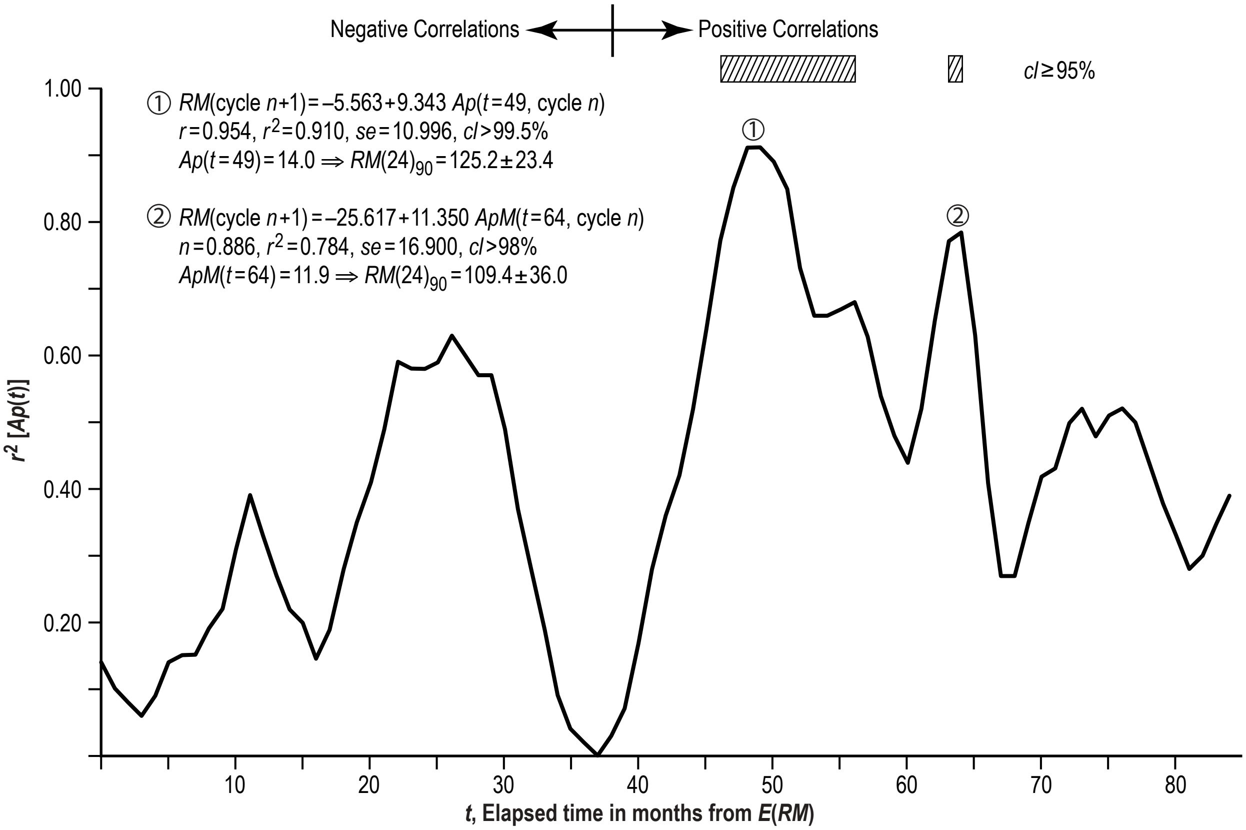


Figure 6. Variation of r^2 for $t=0$ to 84 months past $E(RM)$, resulting from a comparison of RM for cycle $n+1$ versus $Ap(t)$ for cycle n.

used here are those based on the adjusted values,[14,17] which compensate for changes in the repositioning of the magnetometers used for the computation of the *aa* geomagnetic index before 1957 (values prior to 1957 are slightly increased by 3 nT). Also, *aaI*, as employed here, is the 12-month moving average of the difference in monthly means of *aa* – aaR, where $\mathrm{aaR}=6.3+0.0462\ R$, determined from a straight-line fit of monthly means of *aa* and *R*, particularly, through the values for February 1880 and June 1999. (Using a different binning technique[18] would result in a somewhat stronger relationship between *aa* and *R*.)

As with NDD and *Ap*, the *aa* and *aaI* indices display episodic variation, multiple peaks throughout the descending portion of the sunspot cycle, these peaks being associated with high-speed solar wind flows from the Sun.[16] Cycles 13 and 14 have the lowest *aaM* and *aaIM* values, while cycle 23 has the highest *aaM* and *aaIM* values. Tables 5 and 6 in the appendix give tabulations of *aa*(*t*) and *aaI*(*t*) for elapsed time in months from *E*(*RM*) *t* equal 0 to 84 months for cycles 11–23. It should be noted that the actual *aaM* and *aaIM* for cycles 12 and 13 occurred prior to their respective *E*(*RM*) dates. For cycle 12, its actual *aaM* and *aaIM* occurred simultaneously in September 1882, 15 months prior to *E*(*RM*), and measured, respectively, 26.8 and 17.8. For cycle 13, its actual *aaM* and *aaIM* occurred simultaneously in July 1892, 18 months prior to *E*(*RM*), and measured, respectively, 27.1 and 17.4. In tables 5 and 6, the occurrence dates of *aaM* and *aaIM* for cycles 12 and 13 are marked with apostrophes to indicate that these values are the maximum values during the declining portion of the sunspot cycle, being slightly smaller values as compared to their actual maximum values.

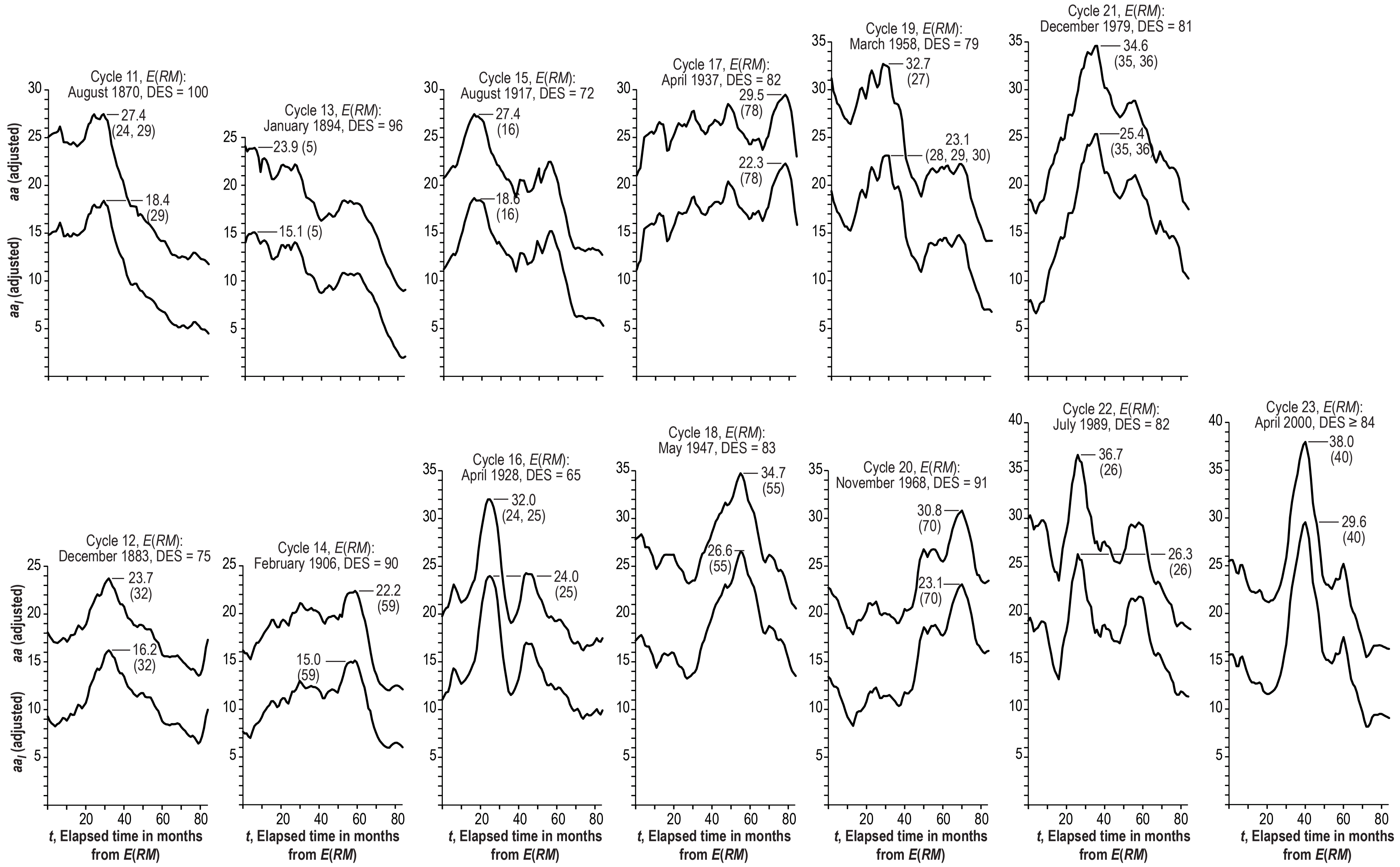


Figure 7. 12-month moving averages of $aa(t)$ (upper curves) and $aaI(t)$ (lower curves) for elapsed time in months from $t = 0$ to 84 months past $E(RM)$ for cycles 11–23.

Figure 8 shows the scatter plots of *RM* for cycle $n+1$ versus *aaM* for cycle *n* (left panel) and *aaIM* for cycle *n* (right panel). Both plots are statistically important, in contrast to that found before for NDDM (fig. 2) and *ApM* (fig. 5). Based on the *aaM* for cycle 23, one infers *RM* for cycle 24 could be about 168 ± 60 (the 90% prediction interval). Based on the *aaIM* for cycle 23, one infers *RM* for cycle 24 could be about 194 ± 41. The overlap in these predictions is about 190 ± 38, suggesting that *RM* for cycle 24 should be ≥152. Such a finding suggests that cycle 24's maximum amplitude will be greater than average size, possibly much greater than average size.[14,19–20] Plainly, it is the inclusion of cycles 11–16 that makes the correlation statistically important. The *aaM* and *aaIM* values plotted in fig. 8 for cycles 12 and 13 are those maximum values that occurred in their declines and not the actual maximum values. If instead, one used the actual maximum values, then, based on *aaM*, *RM*(24) would be predicted to be about 167 ± 64 and, based on *aaIM*, about 199 ± 44, yielding an overlap of 193 ± 38, essentially the same as above using the maximum values during the declines. Also, it should be noted that, as before for NDD and *Ap*, instead of comparing *aaM* or *aaIM* with the following *RM*, one compares their maximum values of the "bump" during the latter half of the decline, $t > 42$ months, against the following *RM*, one finds that the inferred correlations are statistically important and that cycle 24 should have *RM* equal to about 122 ± 27 and 118 ± 27, respectively, these being the 90% prediction intervals, thereby yielding an overlap of about 120 ± 25.

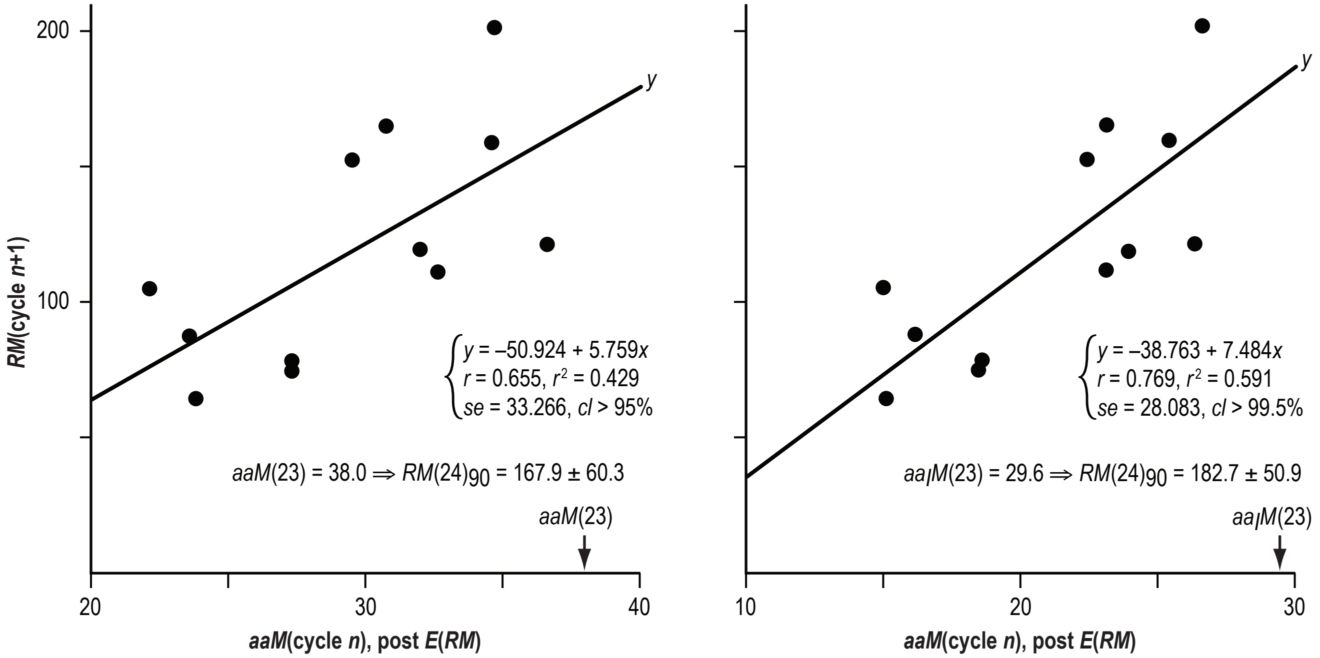


Figure 8. Scatter plot of *RM*(cycle $n+1$) versus *aaM* (left panel) and *aaIM* (right panel).

Figure 9 displays the variation of the coefficients of determination r^2 based on *RM* for cycle $n+1$ versus $aa(t)$ for cycle *n* (lower panel) and versus $aaI(t)$ for cycle *n* (upper panel) for *t* equal 0 to 84 months past $E(RM)$. Both panels display statistically important correlations beginning about $t=38$ months, with the greatest correlations occurring at $t=49$ months (also a secondary localized peak at $t=64$ months). Based on $aa(t=49$ months) for cycle 23, one infers *RM* for cycle 24 to be about 129 ± 27

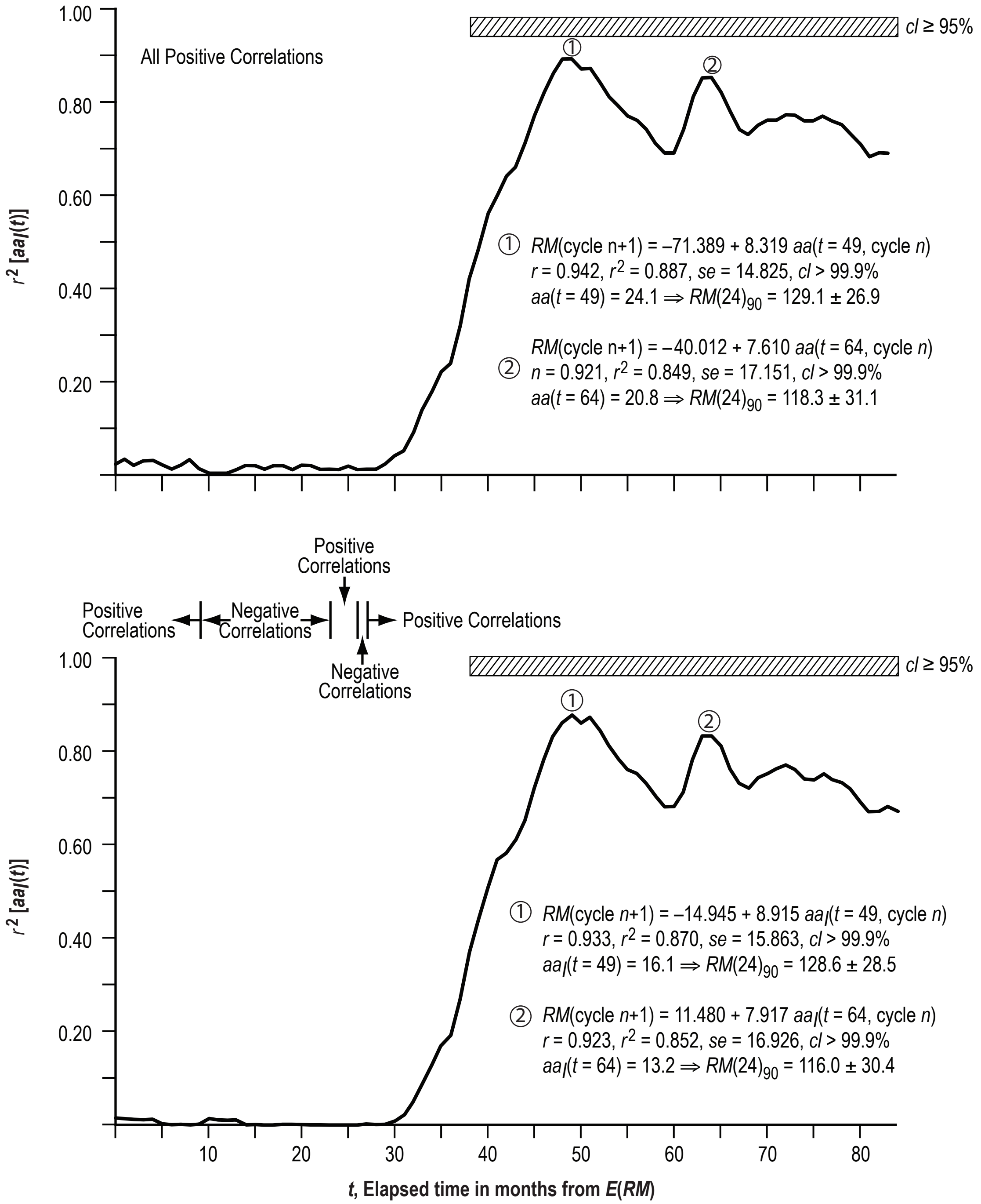


Figure 9. Variation of r^2 for $t=0$ to 84 months past $E(RM)$, resulting from a comparison of RM for cycle $n+1$ versus $aa(t)$ for cycle n (lower panel) and $aaI(t)$ for cycle n (upper panel).

(the 90% prediction interval) and, based on $aaI(t=49$ months) for cycle 23, one infers RM for cycle 24 to be about 127 ± 30, yielding an overlap of about 129 ± 27. Instead, based on the later-occurring correlation for $aa(t=64$ months), one infers RM for cycle 24 to be about 118 ± 31 (the 90% prediction interval) and, based on $aaI(t=64 \text{ months})$, one infers RM for cycle 24 to be about 114 ± 30 (the 90% prediction interval, yielding an overlap of about 116 ± 28. Together, the overlap of the combined estimates (based on aa and aaI) is about 123 ± 21. Based on the $t=49$ month estimates, there is only a 5% chance that cycle 24's RM will fall below about 100, suggesting that cycle 24 likely will be an above average size cycle[21–23], while, based on the $t=64$ month estimates, there is only a 5% chance that cycle 24's RM will fall above about 145. Plainly, a dilemma exists regarding the expected size of cycle 24's RM, using either the maximum values of the geomagnetic precursors (>152), or the values as measured about 4 or 5 yr past $E(RM)$ (<145).

2.4 Hindcasts of RM(cycle $n+1$)

For this subsection and the next, only the fits between RM and the values of the geomagnetic precursors about four years past $E(RM)$ will be considered. Table 1 compares the observed RM with the predicted RM based on the previously mentioned techniques using $\mathrm{NDD}(t=48 \text{ months})$, $Ap(t=49 \text{ months})$, $aa(t=49 \text{ months})$, and $aaI(t=49 \text{ months})$. Of these particular techniques, the one based on $Ap(t=49 \text{ months})$ has the highest r (= 0.954) and r^2 (= 0.910), and the smallest se (= 11.0); hence, one expects it to be the best predictor. For cycle 24, it predicts RM for cycle 24 to be about 125 ± 23 (the 90% prediction interval).

Table 1. Hindcasts of RM(cycle $n+1$).

Cycle	RM(Obs.)	$RM[\mathrm{NDD}(t=48)]$	$RM[Ap(t=49)]$	$RM[aa(t=49)]$	$RM[aaI(t=49)]$
		$r=0.917$, $se=13.5$	$r=0.954$, $se=11.0$	$r=0.942$, $se=14.8$	$r=0.925$, $se=16.7$
12	74.6	–	–	68.4	62.8
13	87.9	–	–	85.8	94.1
14	64.2	–	–	72.5	72.0
15	105.4	–	–	90.0	90.4
16	78.1	–	–	103.3	105.2
17	119.2	–	–	119.1	129.1
18	151.8	142.7	155.1	164.0	162.3
19	201.3	199.3	200.9	190.7	184.4
20	110.6	107.7	104.7	98.3	95.0
21	164.5	150.8	154.2	146.6	141.1
22	158.5	162.9	154.2	155.7	162.3
23	120.8	144.1	138.3	142.4	138.3
24	–	137.3	125.2	129.1	127.3

For NDD ($t=48$ months), the average absolute error expressed as a percent of the predicted value is 6.4%. Five of the six cycles were predicted within ±10%, with only the RM for cycle 23 lying outside the ±10% range (–16.2%). For $Ap(t=49 \text{ months})$, the average absolute error expressed as a percent of the predicted value is 5.0%. Five of the six cycles were predicted within ±10%, again with only the RM for cycle 23 lying outside the ±10% range (–12.7%). For $aa(t=49 \text{ months})$, the average absolute error expressed as a percent of the predicted value is 9.9%. Half of the 12 cycles were predicted within ±10% and nine of 12 cycles were predicted within ±15%, with only cycles 15 (17.1%), 16 (–24.4%), and 23 (–15.2%) having RM values outside the ±15% range. For $aaI(t=49 \text{ months})$, the average absolute error expressed as a percent of the predicted value is 12.5%. Five of 12 cycles were predicted within ±10% and seven of 12 were predicted within ±15%. Only cycles 12 (18.8%), 15 (16.6%), 16 (–25.8%), 20 (16.4%), and 21 (16.6%) had *RM* values outside their ±15% ranges (cycle 23's observed *RM* was 12.7% below its predicted value).

Using ±15% as the expected uncertainty surrounding the predictions of *RM*, one computes that cycle 24 should have an *RM* of about 137.3 ± 20.6, based on the NDD($t = 48$ months) technique; 125.2 ± 18.8, based on the *Ap*($t = 49$ months) technique; 129.1 ± 19.4, based on the *aa*($t = 49$ months) technique; and 127.3 ± 19.1, based on the *aaI*($t = 49$ months) technique. The overlap of the predictions is 130.4 ± 13.7, indicating that cycle 24's *RM* likely should be expected to be greater than about 117, but no larger than about 144. Presuming the continued success of the modified precursor techniques, it appears highly unlikely, then, that the low prediction of 90 ± 10 as given by the NOAA Solar Cycle 24 Prediction Panel[21] is valid. It also appears that, while the high prediction of 140 ± 20 seems more likely, cycle 24's *RM* probably will fall either within the lower portion of the high prediction interval or, perhaps, just below it, at least, based on the modified precursor techniques described in this TP.

2.5 Cycle 24's Ascent Duration

Having what seems to be a reliable prediction for cycle 24's *RM*, it is now desirable to estimate its ascent duration. More than seventy years ago, Waldmeier[24] showed that the shape of the sunspot cycle curve for a given cycle is primarily determined by the height of its maximum. In particular, he found that larger amplitude cycles attained maximum amplitude more quickly than smaller amplitude cycles. This inverse relationship between the size of a sunspot cycle (*RM*) and its ascent duration (*ASC*) is often called the Waldmeier effect.[25–29]

Figure 10 displays the scatter plot of *ASC* versus *RM* for cycles 12–23, where *ASC* is simply the elapsed time in months from minimum to maximum sunspot amplitude, as measured using 12-month moving averages of monthly mean sunspot number. Each cycle is identified by its number beside the filled circles. The median values of *RM* (114.9) and *ASC* (47 months) are identified, respectively, by the vertical and horizontal lines. Thus, cycles 12–16 and 20 can be characterized as being smaller amplitude cycles, each having $ASC \geq 46$ months (the range is 46–60 months). Similarly, cycles 17–19 and 21–23 can be characterized as being larger amplitude cycles, each having $ASC \leq 47$ months (the range is 34–47 months). Based on the median values of *RM* and *ASC*, all of the first group except cycle 13 could be characterized as being slow rising–smaller amplitude cycles, and all of the second group except cycles 19 and 23 could be characterized as being fast rising–larger amplitude cycles (since the convention is to place values on the medians into the higher quadrant). If one were to invoke 48 months as marking the division between fast-rising and slow-rising cycles, then there would be no change in the first grouping, and all of the second grouping would be identified as fast risers.

Using all data points, linear regression analysis results in the inferred regression line, shown as the heavy line (y). It has $r = -0.640$, $r^2 = 0.410$ (meaning that the inferred regression can explain about 41% of the variance), $se = 5.5$ months, and $cl > 95\%$ (meaning that the inferred regression is considered statistically important at the 5% level of significance or the 95% confidence level). The result of Fisher's exact test[30] for 2×2 contingency tables is shown in the upper right portion of the figure. Thus, the probability (P)

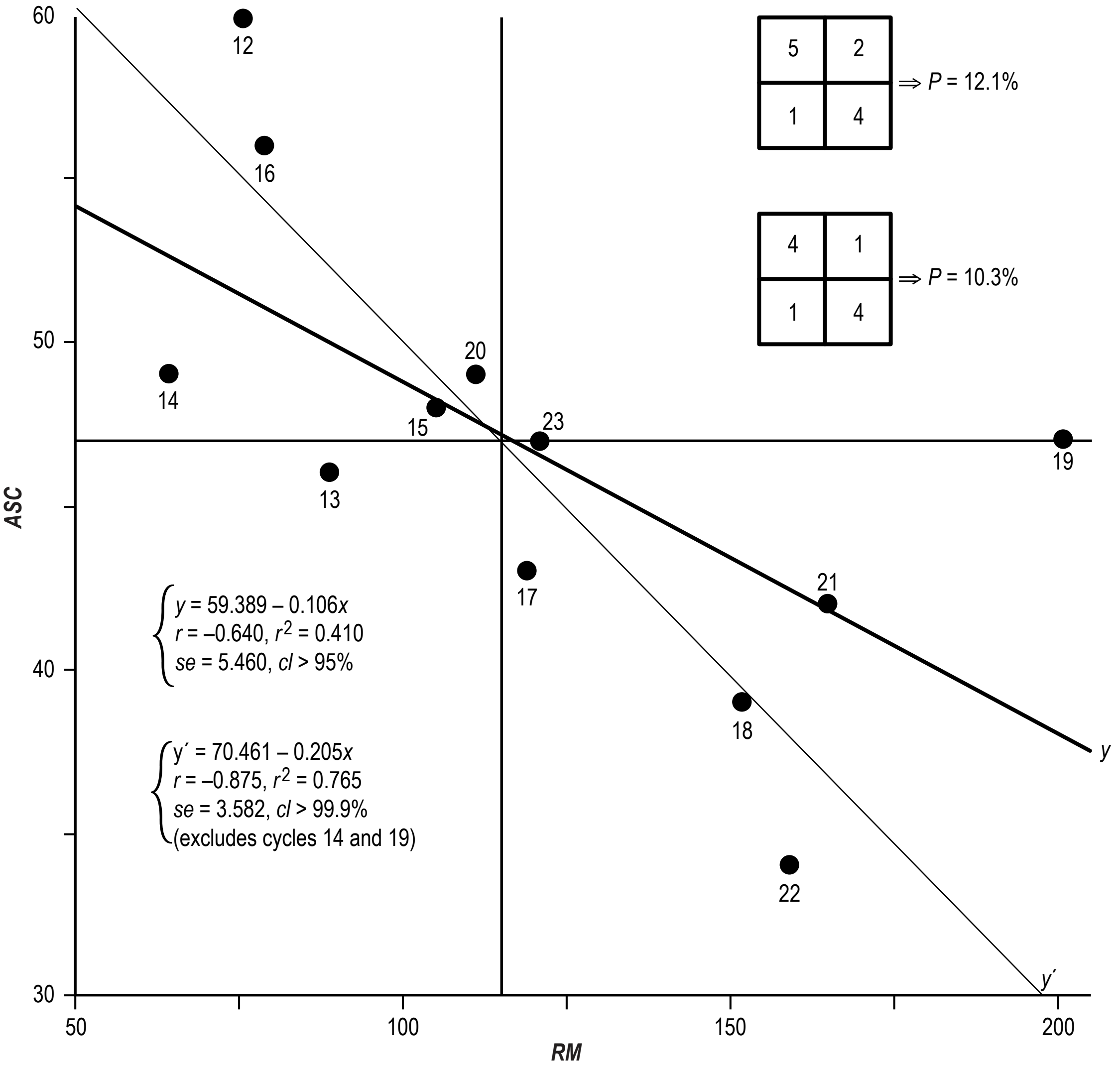


Figure 10. *ASC* versus *RM* (the Waldmeier effect).

of obtaining the observed contingency table, or one more suggestive of a departure from independence (chance), is 12.1%. Instead, using the 48 month division line, the observed table would be 5:0:6:1 rather than 5:2:4:1 and the probability is reduced to a mere 0.8%. Hence, cycles having larger amplitudes always have *ASC* <48 months, while smaller amplitude cycles almost always have *ASC* ≥48 months.

If one removes the extreme cycles in *RM* (cycles 14 and 19), the resultant inferred regression is highly statistically important identified as the thin line (y'). It has $r=-0.875$, $r^2=0.765$ (meaning that more than three-fourths of the variance can be explained by the inferred regression), $se=3.6$ months and $cl>99.9\%$. The result of Fisher's exact test for this 2×2 contingency table is also shown in the upper right portion of the figure, being $P=10.3\%$. Again, using the 48 month division line, the observed table would be 4:0:5:1 rather than 4:1:4:1 and the probability would be reduced to only 0.4%. It should be noted that cycle 19 is the main point of contention, being the largest amplitude cycle, but only of average rise time, so it truly is a statistical outlier. Removal of only cycle 19 yields the regression $y=66.009-0.172x$, $r=-0.822$, $r^2=0.675$, $se=4.0$ months, and $cl>99.8\%$.

Table 2 compares observed *ASC* with predicted *ASC* using the estimates of *RM* given in table 1 for each of the techniques, where the values in parentheses refer to the alternate prediction values that disregard cycles 14 and 19 (*y*′). Based on the predicted *RM* values using the NDD method and including all cycles (*y*), the average absolute error measures 10.4% and the range is –11.4 to +23.7%, with five of six cycles having errors within ±15% of the predicted *ASC* (only cycle 19 is out of bounds). Based on the predicted *RM* values (*y*′) using the NDD method and disregarding cycle 19, the average absolute error measures 7.6% and the range is –19.0 to +6.8%, with only cycle 22 having an error in excess of ±7%. Based on the predicted *RM* values using the *Ap* method and including all cycles, the average absolute error measures 10.5% and the range is –20.9 to +23.7%, with four of six cycles having errors within ±15% of the predicted *ASC* (cycles 19 and 22 are out of bounds). Based on the predicted *RM* values using the *Ap* method and disregarding cycle 19, the average absolute error is 6.5% and the range is –12.8 to +11.9%. Based on the predicted *RM* values using the *aa* method and including all cycles, the average absolute error is 9.9% and the range is –20.9 to +20.5%, with only cycles 12 (15.4%), 16 (16.7%), 19 (20.5%), and 22 (–20.9%) having errors larger than ±15%. Based on the predicted *RM* values using the *aa* method and disregarding cycles 14 and 19, the average absolute error is 8.9%, and the range is –13.2 to +14.6% (really, only cycle 19's *ASC* failed to fall within the ±15% boundary). Based on the predicted *RM* values using the *aaI* method and including all cycles, the average absolute error measures 8.7%, and the range is –19.0 to +17.5%, with only cycles 12, 16, 19, and 22 having errors larger than ±15%. Based on predicted *RM* values using the *aaI* method and disregarding cycles 14 and 19, the average absolute error is 5.5%, and the range is –9.8 to +14.3%. (Cycle 14's error is only –12.5%, so really only cycle 19's *ASC* fails to fall within the ±15% boundary.)

Table 2. Hindcasts of *ASC*(cycle *n*+1).

Cycle	***ASC*(Obs)**	***ASC*(Pred.)**	***ASC*(Pred.)**	***ASC*(Pred.)**	***ASC*(Pred.)**
		(NDD)	**(*Ap*)**	**(*aa*)**	**(*aaI*)**
12	60	–	–	52(56)	53(58)
13	46	–	–	50(53)	49(51)
14	49	–	–	52(56)	52(56)
15	48	–	–	50(52)	50(52)
16	56	–	–	48(49)	48(49)
17	43	–	–	47(46)	46(44)
18	39	44(41)	43(39)	42(37)	42(37)
19	47	38(30)	38(29)	39(31)	40(33)
20	49	48(48)	48(49)	49(50)	49(51)
21	42	43(40)	43(39)	44(40)	44(42)
22	34	37(42)	43(39)	43(39)	42(37)
23	47	41(44)	45(42)	44(41)	45(42)
24	–	45(42)	46(45)	46(44)	46(44)

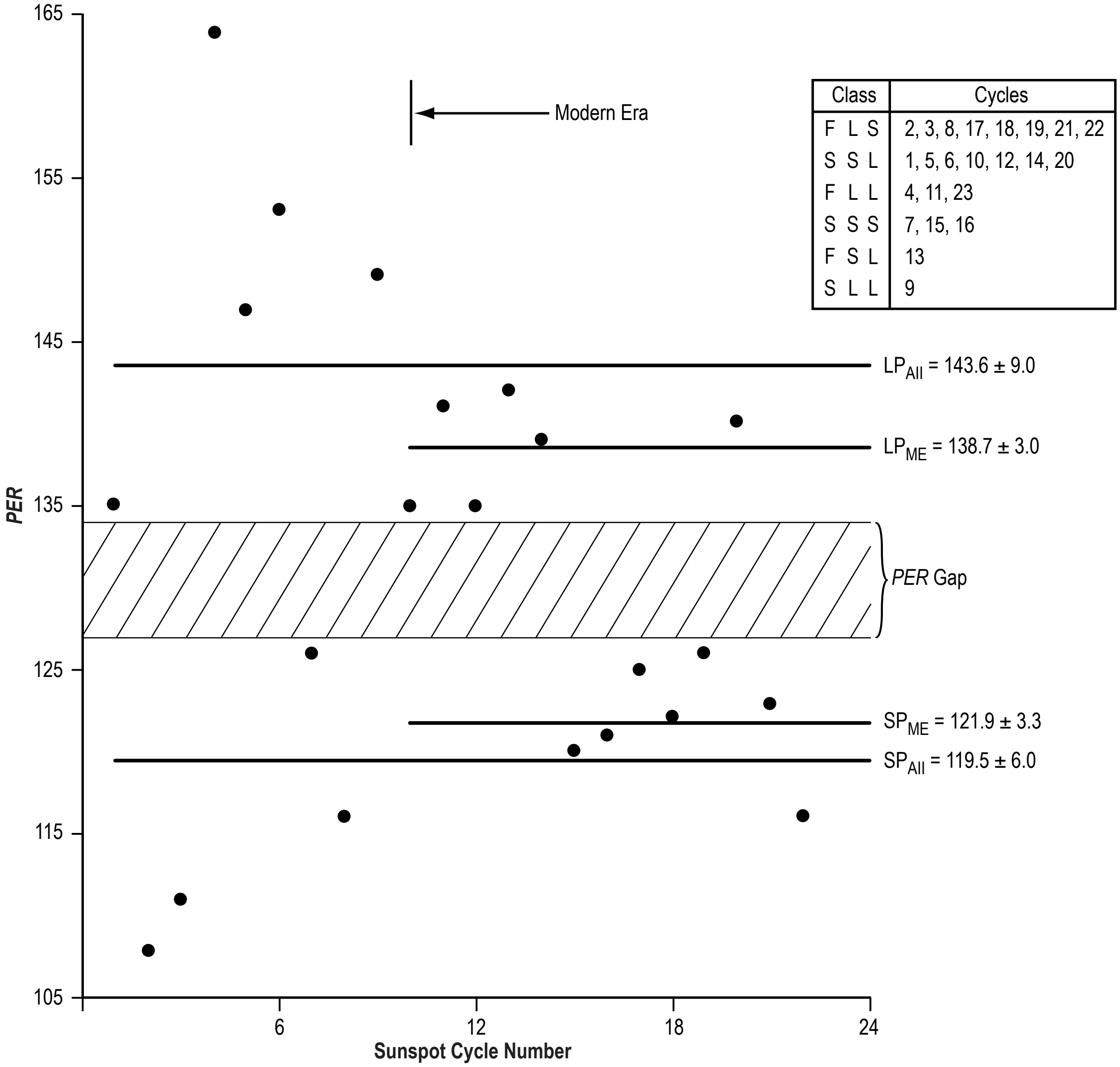
Using ±15% as the expected uncertainty surrounding the predictions of *ASC*, for the predicted *ASC* based on the NDD method (*RM*=137.3±20.6), one predicts cycle 24's *ASC* to be about 45±7 months (from *y*) or 42±6 months (from *y*’). Based on the *Ap* method (*RM*=125.2±18.8), one predicts cycle 24's *ASC* to be about 46 ± 7 months (from *y*) or 45 ± 7 months (from *y*'). Based on the *aa* and *aaI* methods (*RM* = 129.1 ± 19.4 and *RM* = 127.3 ± 19.1, respectively), one predicts cycle 24's *ASC* to be about 46 ± 7 months (from *y*) or 44 ± 7 months (from *y*'). The overlap of the predictions is 43.5 ± 4.5 months. Hence, cycle 24 should probably be considered a fast rising-large amplitude cycle, peaking fewer than 48 months after sunspot minimum. If cycle 24 has sunspot minimum in March 2008, as predicted by the NOAA Solar Cycle 24 Prediction Panel,[21] then clearly one should expect sunspot maximum for cycle 24 before March 2012.

2.6 Sunspot Cycle Lengths

Conventionally, the length of a sunspot cycle is reckoned from sunspot minimum occurrence to sunspot minimum occurrence of the following cycle using 12-month moving averages. Figure 11 displays the temporal variation of sunspot cycle periods for cycles 1–22. For all cycles the mean cycle period is 132.3 ± 14.4 months (one standard deviation). However, because the record is only reliable from about cycle 10, the beginning of the modern era[31–33] (perhaps only from about cycle 12), the mean cycle period might more reliably be determined to be about 130.8 ± 8.7 months (one standard deviation). Noticeable is that cycle periods do not cluster near the mean cycle period, but rather seem to be distributed both longer and shorter than the mean cycle period, whether one uses all cycle periods or just those of the modern era.[34] Hence, there is the perception that there are two distinct groupings of sunspot cycles: short-period (SP) cycles, having *PER* ≤126 months (the 90% distribution interval is 121.9 ± 6.4 months, based on the cycle lengths of the modern era sunspot cycles), and long-period (LP) cycles, having *PER* ≥135 months (the 90% distribution interval is 138.7 ± 6.0 months, based on the cycle lengths of the modern era sunspot cycles), with an eight-month gap separating them.

The NOAA Solar Cycle 24 Prediction Panel[21] has predicted that cycle 23 will be a cycle of longer duration, specifically 11.75 years (or 141 months). Through September 2007, the 12-month moving average of sunspot number equals 5.9 and cycle 23 has already persisted for 136 months; clearly, it is an LP cycle. An official start for cycle 24 in March 2008 corresponds to a cycle length of 141 months for cycle 23. Based on the 90% distribution interval of LP cycle lengths for modern era sunspot cycles, there is only a 5% probability that cycle 23's duration will persist longer than 145 months (June 2008).

In the upper right of fig. 11 is a table identifying (in descending order of frequency of occurrence) specific cycles according to a simple classification scheme, where the first letter refers to the ascent class (F: fast riser or S: slow riser, where the division is assumed to be 48 months), the second letter refers to the maximum amplitude class (L: large maximum amplitude or S: small maximum amplitude, where the division is assumed to be 114.9) and the third letter refers to the period class (L: long period or S: short period, where the division is assumed to be 132 months). The two largest groupings of cycle classes are FLS: eight entries and SSL: seven entries, using all cycles, or FLS: five entries and SSL: four entries, using only the modern era cycles. So, using all cycles, 8 of 11 fast-rising-large maximum-amplitude cycles have been cycles of shorter duration, the exceptions being cycles 4, 11, and 23. Also, using all cycles, 7 of 10 slow-rising-small maximum-amplitude cycles have been cycles of longer duration, the exceptions being cycles 7, 15, and 16. Cycles 9 (SLL) and 13 (FSL) fit none of the above primary classes, and, as yet, there have been no cycles classified as FSS or SLS.



Class	Cycles
F L S	2, 3, 8, 17, 18, 19, 21, 22
S S L	1, 5, 6, 10, 12, 14, 20
F L L	4, 11, 23
S S S	7, 15, 16
F S L	13
S L L	9

Figure 11. Variation of cycle lengths in months for cycles 1–22.

Since cycle 24 is predicted to be a fast-rising-large maximum-amplitude cycle (on the basis of the previously described modified geomagnetic precursor techniques), statistically speaking, it should also be a cycle of shorter duration. Recall that 8 of 11 (72.7%) previous fast-rising-large maximum-amplitude cycles have been cycles of shorter duration. If true, presuming that March 2008 marks the official start of cycle 24, then cycle 25 should not be expected to begin before March 2019. On the other hand, if cycle 24 turns out to be an odd-ball, like cycle 23 (FLL), then the onset of cycle 25 might be delayed until after March 2019.

2.7 Hale Cycle Effects

Another way to examine the behaviors of *RM* and *PER* is to determine the variation of their individual sums for consecutive cycle pairs (Hale cycle pairs). Figure 12 shows the variation of $\sum RM$ for sunspot cycle pairs 1/2, 2/3, …, 22/23, where $\sum RM$ is the sum of the maximum amplitudes for two consecutive sunspot cycles. Over the span of the sunspot record there has been an unmistakable rise in $\sum RM$ such that six of the past six sunspot cycle pairs have all had $\sum RM$ in the upper-right quadrant, as determined from the medians (the thin vertical and horizontal lines). Simple runs testing[35] suggests that the variation of $\sum RM$ is non-randomly distributed at the 5% level of significance (six runs with samples numbering 11 apiece) and linear regression analysis suggests a positive correlation that is marginally statistically important ($cl \geq 90\%$). Extension of the regression line to cycle pair 23/24 suggests that the $\sum RM$ for cycle pair 23/24 should measure about 278, inferring that cycle 24's *RM* should measure about 157 if it lies on the regression line (since cycle 23's *RM* measured about 121). It will measure below 157 if the sum falls below the regression line and it will measure above 103 if the sum is above the median value of 224. (From the previously described modified geomagnetic precursor techniques, recall that the overlap of the predictions is 130.4 ± 13.7 for cycle 24's *RM*, inferring that $\sum RM$ for cycle pair 23/24 should measure about 251 ± 14.)

Figure 13 shows the variation of $\sum PER$ for sunspot cycle pairs 1/2, 2/3, …, 22/23, where $\sum PER$ is the sum of the periods for two consecutive sunspot cycles. While there is no obvious trend in $\sum PER$ over the span of the sunspot record, it is apparent that since sunspot cycle pair 13/14, all sunspot cycle pairs have had $\sum PER \leq 266$ months, averaging about 251 months (the range is 239–266 months). Cycle pair 22/23 already has persisted 252 months and, presuming that cycle 24's official start will be March 2008, its sum will measure 258 months. Fisher's exact test for the 2×2 contingency table reveals that the probability of obtaining the observed result, or one more suggestive of a departure from independence, is $P = 6.3\%$ (it will actually improve to $P = 4.3\%$ if cycle pair 22/23 falls in the lower-right quadrant, as expected). Hence, cycles prior to cycle pair 12/13 have usually been of longer $\sum PER$ than cycles from cycle pair 12/13 (the exceptions are cycle pairs 1/2, 2/3 and 7/8). For cycle pair 22/23 to exceed the median (265) implies that *PER* for cycle 23 would exceed 149 months, which further implies a very late onset for cycle 24 (after October 2008), this not being expected (the longest *PER* in the modern era sunspot record has been 142 months, cycle 13).

Figure 14 displays the percentage change in $\sum RM$ (lower panel) and $\sum PER$ (upper panel) for cycle pairs 1/2–21/22. As an example, the $\sum PER$ for cycle pair 2/3 measures 219 and the $\sum PER$ for cycle pair 1/2 measures 243, yielding a difference of –24 months, which represents a –9.9% decrease (–24/243) in value from the value for cycle pair 1/2. For modern era sunspot cycle pairs, the average absolute percentage change in $\sum RM$ measures ±15.8% (the range is –24.5 to +37.4%) and the average absolute percentage change in $\sum PER$ measures ±3.4% (the range is –9.1 to +7.3%). Hence, one expects cycle pair 23/24 to have $\sum RM = 279.3 \pm 41.9$ (using an error of ±15%, a value that works for eight of 12 modern era cycle pairs, failing for cycle pairs 11/12, 16/17, 17/18 and 20/21). Also, one expects cycle pair 22/23 (presently ≥252 months) to have $\sum PER = 239 \pm 24$ months (using an error of ±10%, a value that works for 11 of 11 modern era sunspot cycle pairs), suggesting cycle 23's $PER \leq 147$ months; using an error of only ±5% (which works for seven of 11 cycle pairs) suggests cycle 23's $PER \leq 137$ months, which seems too short (implying cycle 24 onset October 2007), since the first confirmed high-latitude new cycle spot was not observed until January 2008 and high-latitude new cycle spots typically precede new cycle sunspot minimum by a few to several months.[29,36]

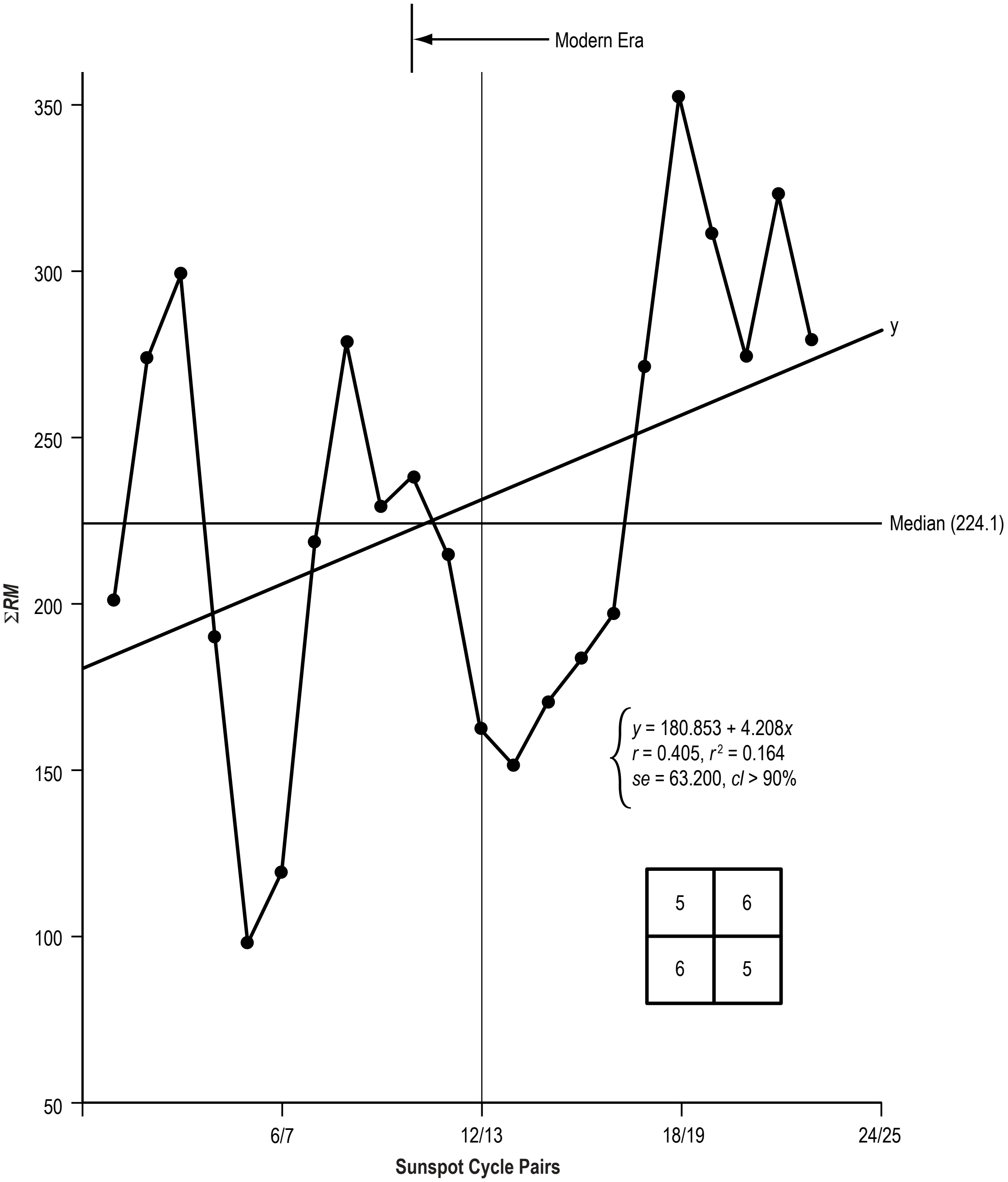


Figure 12. Scatter plot of $\sum RM$ versus consecutive sunspot cycle pairs 1/2 to 22/23.

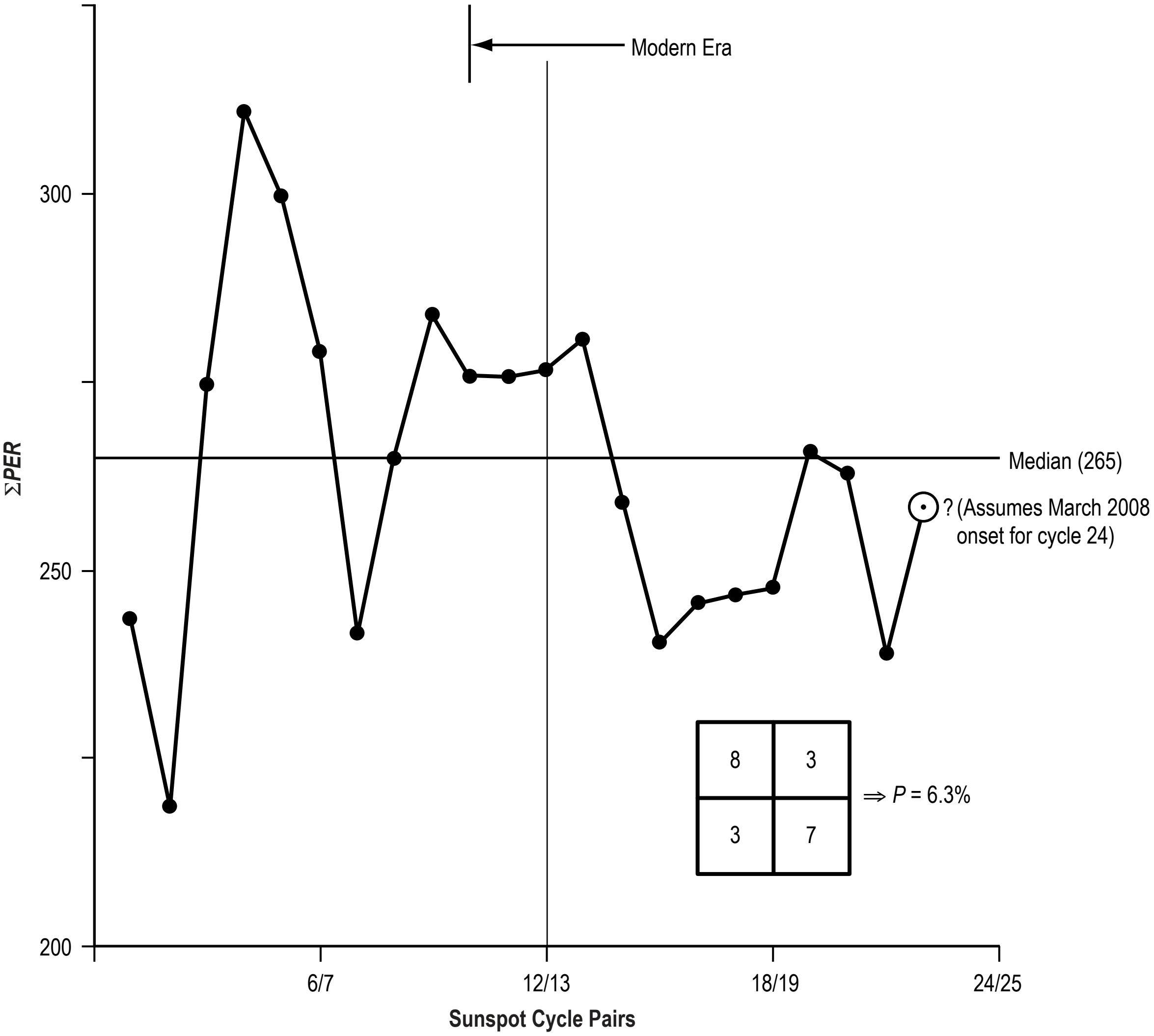


Figure 13. Scatter plot of $\sum PER$ versus consecutive sunspot cycle pairs 1/2 to 22/23.

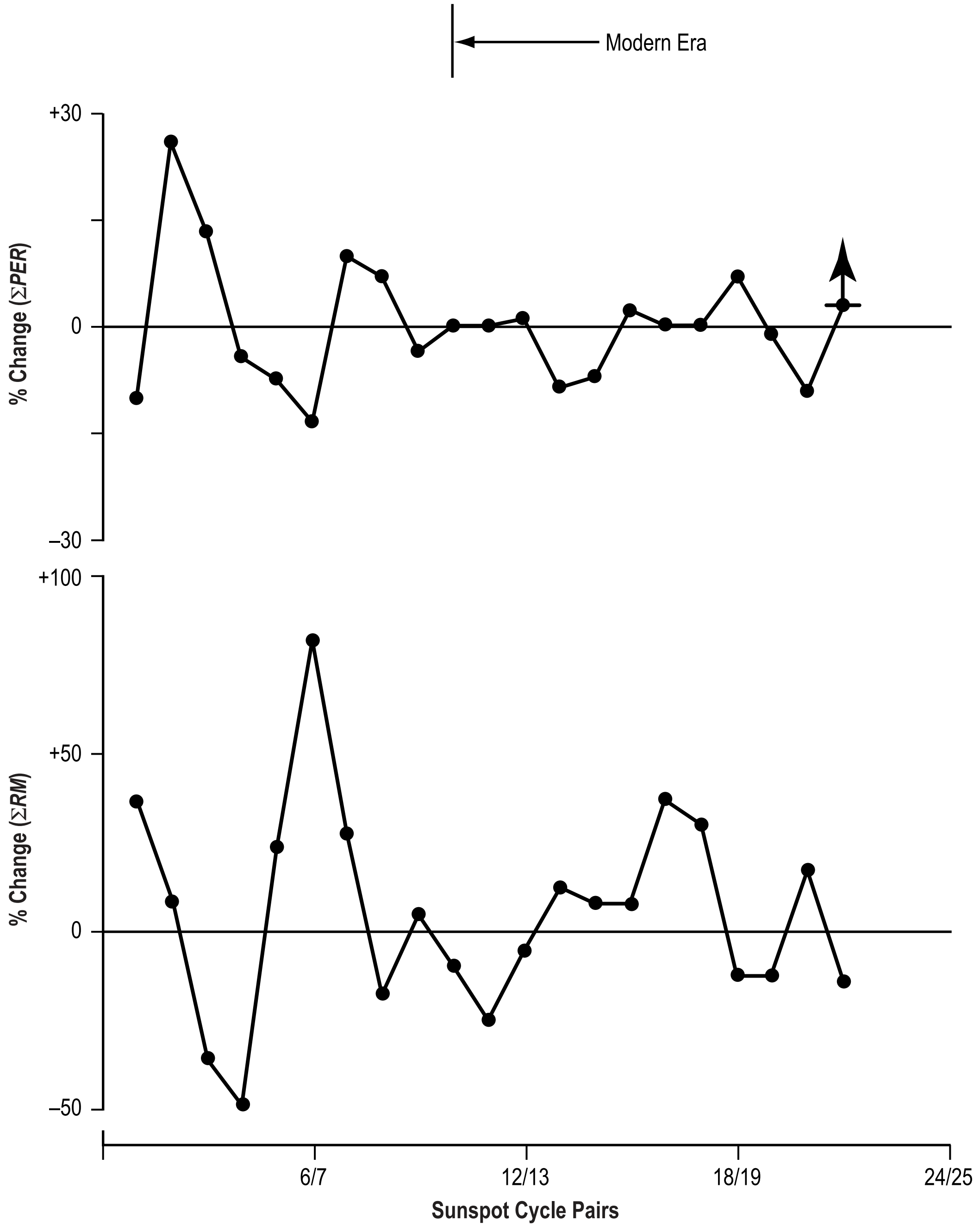


Figure 14. Variation of the percentage of change for $\sum RM$ (lower panel) and $\sum PER$ (upper panel) for consecutive sunspot cycle pairs 1/2 to 21/22.

3. SUMMARY

As first noted in the introduction, Dabas et al.[1] have proffered a modified geomagnetic precursor technique based on the number of disturbed days (*Ap* ≥25) about four years after cycle maximum that is statistically important and provides what appears to be a fairly reliable (within 10–15% uncertainty) means for predicting the following cycle's maximum amplitude. On the basis of their technique, they predict cycle 24's maximum amplitude to be of about 124 ± 23, peaking about 44 ± 5 months after sunspot minimum occurrence, or about mid-to-late 2011 if cycle 24's minimum occurs in March 2008. In this TP, we have reexamined the Dabas et al. method and extended it to other data sets, including *Ap*, *aa*, and *aaI*. We confirm the general conclusions of the Dabas et al. results that cycle 24 will be slightly larger than average size and have a faster than average rise time (presuming the continued success of the modified precursor technique based on values of the geomagnetic indices at about four years past *E*(*RM*) of the preceding cycle). In particular, we find that the *Ap* data seem to provide a more reliable prediction of *RM* (based on hindcasting) and that the combined (overlap) prediction for cycle 24's *RM* based on all four data sets is 130.4 ± 13.7, highly suggestive that cycle 24 will be larger than average size and have an *RM* that is outside the range of the consensus low prediction (90 ± 10) of the NOAA Solar Cycle 24 Prediction Panel.[21] Such a value also compares quite favorably with the secular trend based on group sunspot number described in Hathaway, Wilson, and Reichmann[27] (136.5 ± 41.3, the 90% prediction interval for cycle 24). The combined predicted *RM* suggests that cycle 24 will have *ASC* = 44 ± 5 months, in agreement with that predicted by Dabas et al., and implying that maximum amplitude for cycle 24 will be about November 2011 (±5 months), presuming an official start of March 2008. [Slightly smaller estimates for *RM* result when using geomagnetic indices at 64 months past *E*(*RM*).]

Also examined were sunspot cycle lengths and Hale cycle effects of *RM* and *PER*, based on the behavior of consecutive sunspot cycle pairs. Eight of 11 fast-rising large maximum-amplitude cycles have been cycles of shorter than average duration, and 7 of 10 slow-rising small maximum-amplitude cycles have been cycles of longer than average duration, a behavior that describes two-thirds of all sunspot cycles (three-fourths of all modern era sunspot cycles). Cycle 23, however, does not fit this paradigm. In contrast, it is a fast-rising large maximum-amplitude long-period cycle (like cycles four and 11). Because cycle 24 is predicted to be a fast-rising large maximum-amplitude cycle, statistically speaking, one expects it to also be a cycle of shorter than average duration, unless, of course, it too is another statistical outlier like cycle 23. The predicted $\sum RM$ for sunspot cycle pair 23/24 is about 278 ± 63 (one standard error accuracy), suggesting that cycle 24 should measure about 157 ± 63, based on the inferred statistically important upward secular trend. Based on the predicted value for cycle 24's *RM* (= 130 ± 14), one computes that sunspot cycle pair 23/24 should have $\sum RM = 251 \pm 14$, well within the bounds of the inferred upward secular trend prediction, which if true will mark the second straight decrease in $\sum RM$ (the peak occurred in sunspot cycle pair 18/19: 353.1). Beginning with sunspot cycle pair 14/15, $\sum PER$ has always been ≤266 months and it appears that sunspot cycle pair 22/23 will also have $\sum PER \leq 266$ months, although its value will exceed that observed for sunspot cycle pair 21/22 (= 239 months). Presently, sunspot cycle pair 22/23 has $\sum PER \geq 252$ months (through September 2007), and it very probably will be ≤263 months, implying that the official start for cycle 24 will occur before September 2008.

APPENDIX A—NDD(*t*), *Ap*(*t*), *aa*(*t*), AND *aaI*(*t*) FOR CYCLES 17–23

This appendix provides tabulations for NDD(*t*), *Ap*(*t*), *aa*(*t*), and *aaI*(*t*) values for elapsed time in months from *E*(*RM*) *t*=0–84 months for cycles 17–23.

Table 3. 12-month moving averages of NDD(*t*) for cycles 17–23 from *t*=0 to 84 months past *E*(*RM*).

t	**Cycles**							**Comments**
	17	**18**	**19**	**20**	**21**	**22**	**23**	
0	2.9	6.3	7.6	3.2	2.1	6.8	4.5	
1	3.3	6.3	7.4	3.2	2.3	6.9	4.5	
2	3.3	6.2	7.2	3.0	2.3	7.2	4.5	
3	3.8	5.8	6.8	2.9	2.2	7.0	4.3	
4	4.4	5.3	6.8	2.7	2.2	7.0	3.9	
5	4.7	5.3	6.6	2.5	2.2	6.9	4.0	
6	4.8	5.4	6.2	2.2	2.2	6.9	4.5	
7	4.8	5.1	5.6	2.0	2.2	6.8	4.7	
8	4.8	4.8	5.5	2.0	2.1	6.5	4.5	
9	4.9	4.2	5.4	1.8	2.4	6.5	4.1	
10	5.1	3.9	5.4	1.7	3.0	6.4	3.6	
11	5.4	4.0	5.8	1.8	3.6	5.8	3.4	
12	5.4	4.1	6.0	1.7	3.7	5.5	3.5	
13	5.1	4.4	6.3	1.6	3.6	4.9	3.3	
14	5.1	4.5	6.9	1.9	3.8	4.1	3.0	
15	4.8	4.5	7.5	2.3	3.9	3.8	3.0	
16	4.2	4.8	7.8	2.3	4.3	3.8	3.1	
17	4.1	5.0	7.6	2.3	4.8	4.5	2.9	
18	4.4	5.0	7.2	2.5	5.1	5.4	2.5	
19	5.0	5.1	7.5	2.5	5.2	6.1	2.4	
20	5.2	5.0	8.1	2.8	6.0	6.7	2.3	
21	5.3	4.9	8.3	3.1	6.5	7.3	2.2	
22	5.4	4.6	8.5	3.2	6.2	8.2	2.3	
23	5.3	4.5	8.3	3.2	6.0	8.8	2.5	
24	5.4	4.5	8.5	3.5	6.3	9.1	2.7	
25	5.6	4.2	8.6	3.6	6.8	9.8	3.0	
26	5.6	4.2	8.6	3.3	7.2	10.2	3.2	NDDM(19), NDDM(22)
27	5.8	4.1	8.5	2.9	7.8	9.8	3.3	
28	6.0	4.1	8.3	3.0	8.1	9.4	3.4	
29	6.0	4.1	8.4	3.0	8.1	8.8	4.0	
30	6.0	4.0	8.5	2.9	8.6	7.9	4.6	

Table 3. 12-month moving averages of NDD(t) for cycles 17–23 from $t = 0$ to 84 months past $E(RM)$ (Continued).

	Cycles							
t	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**Comments**
31	5.4	4.3	8.0	3.0	9.0	7.0	5.2	
32	5.1	4.7	7.3	2.9	8.6	6.6	6.1	
33	5.0	5.2	6.8	2.7	8.7	6.4	6.9	
34	4.6	5.8	6.6	2.6	9.3	5.7	7.4	
35	4.5	6.1	6.5	2.4	9.5	5.3	7.7	NDDM(21)
36	4.2	6.4	6.0	2.0	9.4	5.3	7.9	
37	4.1	6.7	5.5	1.9	8.8	5.1	8.4	
38	4.3	6.8	5.0	1.9	8.5	5.2	9.1	
39	4.3	7.3	4.5	2.1	8.3	5.8	9.6	
40	4.5	7.8	4.3	2.3	7.9	5.9	9.8	NDDM(23)
41	4.8	8.3	4.0	2.4	7.8	5.8	9.5	
42	4.9	8.5	3.6	2.4	7.6	5.7	8.8	
43	4.8	8.7	3.5	2.5	7.4	5.6	8.0	
44	4.6	8.8	3.5	2.8	7.1	5.3	7.0	
45	4.6	9.2	3.1	3.1	6.8	5.0	6.4	
46	4.9	9.3	2.7	3.8	6.6	5.1	6.0	
47	5.1	9.2	2.5	4.8	6.5	5.2	5.5	
48	5.1	9.3	2.5	5.7	6.6	5.2	4.7	
49	5.0	9.7	2.8	6.1	6.8	5.5	3.8	
50	4.9	10.3	3.1	6.5	6.7	5.8	3.0	
51	4.7	10.8	3.3	6.5	6.6	6.1	2.8	
52	4.4	11.2	3.5	6.6	7.0	7.0	3.0	
53	4.1	11.5	3.7	6.8	7.0	8.1	3.0	
54	4.1	11.7	3.8	7.0	6.9	7.5	3.1	NDDM(18)
55	4.3	11.6	3.7	6.9	7.0	7.5	3.3	
56	4.1	11.1	3.5	6.8	7.0	7.4	3.6	
57	4.0	10.5	3.7	6.8	6.5	7.5	3.6	
58	3.8	10.1	3.8	6.7	6.0	7.7	3.5	
59	3.9	9.9	3.8	6.3	5.5	7.5	3.8	
60	4.5	9.6	4.0	6.1	5.2	7.4	4.1	
61	4.8	9.3	4.0	6.2	5.0	7.2	3.9	
62	4.7	8.8	3.8	6.3	5.0	6.8	3.6	
63	4.9	8.1	3.8	6.5	4.8	6.2	2.2	
64	5.0	7.3	3.6	6.8	4.3	5.5	2.7	
65	4.7	6.5	3.5	7.3	3.9	5.3	2.4	
66	4.4	6.0	3.6	7.8	3.7	5.3	2.3	
67	4.5	6.0	3.9	7.9	3.5	5.2	2.2	
68	4.7	6.3	4.1	8.0	3.8	5.2	1.9	
69	4.7	6.5	4.2	8.3	4.1	5.1	1.8	NDDM(20)

Table 3. 12-month moving averages of NDD(t) for cycles 17–23 from $t=0$ to 84 months past $E(RM)$ (Continued).

t	**Cycles**							**Comments**
	17	**18**	**19**	**20**	**21**	**22**	**23**	
70	5.2	6.5	4.0	8.2	3.9	4.9	1.6	
71	5.8	6.4	3.8	8.1	3.7	4.8	1.3	
72	6.0	6.4	3.4	7.9	3.7	4.6	1.0	
73	6.4	6.3	2.9	7.5	3.5	4.1	1.1	
74	6.9	5.8	2.6	7.2	3.3	3.5	1.4	
75	7.0	5.6	2.3	6.8	3.3	3.3	1.6	
76	7.2	5.5	2.0	6.3	3.3	2.8	1.7	
77	7.4	5.3	1.9	5.5	3.2	2.3	1.6	
78	7.5	4.9	1.7	5.0	3.0	2.1	1.4	NDDM(17)
79	7.3	4.4	1.5	5.1	2.6	2.0	1.4	
80	7.2	4.0	1.3	5.0	2.0	2.2	1.5	
81	7.0	3.4	1.2	4.6	1.7	2.1	1.4	
82	6.3	2.9	1.2	4.6	1.6	1.9	1.3	
83	5.3	2.7	1.1	4.5	1.5	1.8	1.2	
84	4.5	2.4	1.0	4.4	1.4	1.8	1.3	

Table 4. 12-month moving averages of $Ap(t)$ for cycles 17–23 from $t=0$ to 84 months past $E(RM)$.

t	**Cycles**							**Comments**
	17	**18**	**19**	**20**	**21**	**22**	**23**	
0	11.5	18.5	21.4	13.8	11.5	19.2	15.0	
1	12.0	18.7	20.3	13.6	11.5	19.3	15.0	
2	12.3	18.8	19.9	13.2	11.1	18.8	15.0	
3	13.4	18.8	19.4	12.9	10.6	18.3	14.7	
4	14.4	18.2	19.2	12.8	10.4	18.4	14.2	
5	14.5	17.4	19.0	12.5	10.6	18.4	14.3	
6	14.5	17.4	18.8	12.0	10.9	18.6	15.0	
7	14.6	17.3	18.6	11.5	11.0	18.8	15.1	
8	14.7	16.8	18.6	11.4	11.1	18.6	14.7	
9	14.6	16.4	18.3	11.0	11.7	18.3	14.0	
10	14.8	15.5	18.2	10.7	12.8	17.6	13.3	
11	15.2	14.9	18.7	10.8	13.9	16.8	12.9	
12	15.3	15.2	19.3	10.5	14.3	16.2	12.8	
13	15.1	15.3	19.7	10.3	14.6	15.4	12.8	
14	15.0	15.8	20.4	10.8	15.2	15.0	12.8	
15	14.2	16.1	21.2	11.5	15.6	14.8	12.9	
16	13.3	16.3	21.4	11.5	16.1	14.4	13.0	

Table 4. 12-month moving averages of $Ap(t)$ for cycles 17–23 from $t=0$ to 84 months past $E(RM)$ (Continued).

	Cycles							
t	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**Comments**
17	13.5	16.4	21.0	11.5	16.6	15.7	12.8	
18	14.2	16.4	20.3	11.7	16.5	17.4	12.1	
19	14.7	16.5	21.1	11.8	16.5	18.3	12.0	
20	15.1	16.6	22.4	12.1	17.5	19.1	12.0	
21	15.6	16.3	22.8	12.5	18.3	20.0	11.9	
22	16.1	15.9	22.5	12.5	18.0	21,7	12.1	
23	16.3	15.8	21.9	12.2	17.5	23.0	12.3	
24	16.3	15.6	21.5	12.3	17.8	23.6	12.5	
25	16.5	15.4	21.9	12.4	18.7	24.7	12.7	
26	16.5	14.9	23.0	11.9	19.4	25.0	12.9	*ApM*(22)
27	16.8	14.8	23.6	11.3	20.6	24.3	13.3	*ApM*(19)
28	17.0	14.7	23.5	11.2	21.4	24.1	13.8	
29	17.7	14.7	23.5	11.3	21.4	23.0	14.5	
30	18.0	14.8	23.4	11.2	22.1	21.1	15.1	*ApM*(17)
31	17.3	14.7	22.1	11.2	22.8	19.8	15.8	
32	17.0	14.9	20.5	11.3	22.7	19.4	17.1	
33	16.7	15.6	19.8	11.3	22.6	19.0	18.2	
34	16.1	16.5	19.8	11.2	22.9	17.5	18.9	
35	15.8	17.0	19.8	11.1	23.2	16.7	19.5	*ApM*(21)
36	15.5	17.3	19.1	10.8	23.1	16.7	20.1	
37	15.5	17.8	17.9	10.9	22.1	16.2	21.0	
38	15.9	18.3	16.1	11.2	21.1	16.0	21.5	
39	16.0	18.6	14.8	11.8	20.0	16.8	22.0	
40	16.3	19.0	14.2	12.4	19.0	16.7	22.3	*ApM*(23)
41	16.3	19.7	13.8	12.4	19.0	16.2	21.8	
42	16.0	20.3	13.3	12.5	18.8	16.1	21.1	
43	15.8	20.5	13.0	12.7	18.4	16.0	20.0	
44	15.5	20.9	12.8	12.8	17.8	15.4	18.3	
45	15.6	21.0	12.3	13.3	17.3	14.9	18.1	
46	16.1	21.7	11.4	14.3	17.3	14.9	17.7	
47	16.9	22.3	10.9	15.6	17.1	15.0	16.7	
48	17.3	22.0	11.3	16.7	16.9	14.9	15.2	
49	17.2	22.1	11.8	17.1	17.1	15.4	14.0	
50	17.0	22.4	12.0	17.3	17.3	16.0	13.6	
51	16.6	22.7	12.2	17.0	17.7	16.4	13.5	
52	16.2	23.3	12.4	16.5	18.3	17.4	13.5	
53	15.5	24.1	12.5	16.8	18.6	18.1	13.3	
54	15.1	24.7	12.5	17.0	18.7	18.2	13.3	
55	15.0	25.0	12.3	17.0	19.0	18.1	13.7	*ApM*(18)

Table 4. 12-month moving averages of $Ap(t)$ for cycles 17–23 from $t=0$ to 84 months past $E(RM)$ (Continued).

t	**Cycles**							**Comments**
	17	**18**	**19**	**20**	**21**	**22**	**23**	
56	14.8	24.9	12.3	17.0	19.0	17.8	14.3	
57	14.4	24.3	12.6	16.8	18.5	18.0	14.1	
58	14.0	23.2	12.7	16.5	17.9	18.3	14.0	
59	13.5	22.3	12.6	16.0	17.4	18.2	14.6	
60	13.5	21.9	12.9	15.7	16.9	18.1	15.1	
61	14.0	21.5	13.0	15.8	16.7	17.5	14.4	
62	13.9	21.1	12.8	16.3	16.5	16.5	13.7	
63	14.0	20.5	12.7	17.0	15.9	15.5	12.8	
64	14.0	19.5	12.6	17.7	15.0	14.7	11.9	
65	13.5	18.3	12.8	18.4	14.4	14.3	11.5	
66	13.0	17.0	13.1	19.0	14.0	14.0	11.3	
67	13.1	16.4	13.5	19.5	13.7	14.0	10.8	
68	13.5	16.2	13.5	19.7	14.1	14.0	10.2	
69	13.8	16.5	13.4	19.8	14.7	13.8	9.7	*ApM*(20)
70	14.6	16.7	13.2	19.7	14.2	13.4	9.2	
71	15.7	16.4	12.9	19.4	13.8	13.0	8.5	
72	16.1	16.3	12.0	19.0	13.8	12.6	8.0	
73	16.4	16.0	11.0	18.5	13.4	12.2	8.0	
74	16.7	15.4	10.6	17.8	13.1	11.8	8.3	
75	16.9	15.2	10.2	16.9	13.3	11.5	8.7	
76	17.1	15.0	9.7	16.0	13.3	10.8	8.9	
77	17.4	14.8	9.3	14.8	13.1	10.0	8.9	
78	17.6	14.3	9.0	14.3	12.8	9.7	8.8	
79	17.5	13.6	8.5	14.1	12.3	9.7	8.8	
80	17.1	13.0	8.2	13.8	11.4	9.8	8.8	
81	16.5	12.3	8.0	13.7	10.5	9.7	8.7	
82	15.3	11.8	8.0	13.8	10.4	9.5	8.5	
83	13.7	11.5	8.0	14.0	10.2	9.4	8.5	
84	12.5	11.3	8.0	14.0	10.0	9.3	8.5	

Table 5. 12-month moving averages of $aa(t)$ for cycles 11–23 from $t=0$ to 84 months past $E(RM)$.

t	**Cycles**													**Comments**
	11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	
0	25.1	18.1	23.0	16.1	20.7	19.8	21.0	27.8	31.2	22.7	18.5	30.0	25.5	
1	25.2	17.6	23.6	15.8	20.9	20.2	21.5	28.0	29.9	22.6	18.5	30.3	25.6	
2	25.4	17.4	23.8	15.8	21.2	20.6	21.8	28.1	29.4	22.1	18.1	29.8	25.6	
3	25.5	17.1	23.8	15.4	21.5	20.7	23.4	28.3	28.6	21.8	17.4	28.9	25.1	

Table 5. 12-month moving averages of *aa*(*t*) for cycles 11–23 from *t* = 0 to 84 months past *E*(*RM*) (Continued).

	Cycles													
t	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**Comments**
4	25.4	17.0	23.8	15.2	21.7	21.3	25.0	27.9	28.4	21.7	17.2	29.2	24.3	
5	25.7	17.0	23.9	15.8	22.0	22.3	25.2	27.1	28.0	21.4	17.7	29.2	24.2	*aaM*’(13)
6	26.1	17.1	23.6	16.5	21.9	23.0	25.3	27.0	27.4	20.8	18.2	29.5	25.1	
7	25.5	17.3	22.9	16.9	22.1	22.7	25.5	27.0	27.0	20.2	18.4	29.8	25.1	
8	24.5	17.5	21.4	16.9	22.9	22.2	25.6	26.5	26.9	19.8	18.5	29.8	24.5	
9	24.5	17.4	22.7	17.2	23.5	21.6	25.4	26.1	26.6	19.1	19.3	29.5	23.7	
10	24.5	17.1	22.8	17.5	24.2	21.2	25.7	25.2	26.5	18.5	20.5	28.6	23.0	
11	24.3	17.4	22.6	17.9	24.9	21.3	26.3	24.7	27.1	18.5	21.6	27.2	22.5	
12	24.3	17.8	22.1	18.2	25.6	21.4	26.6	25.0	27.7	18.1	22.0	26.1	22.3	
13	24.5	17.8	21.3	18.5	26.2	21.7	26.4	25.3	28.3	17.8	22.3	25.0	22.3	
14	24.3	17.9	20.6	18.8	26.7	22.1	26.4	25.9	29.1	18.4	23.1	24.4	22.2	
15	24.1	18.4	20.6	19.3	27.1	22.7	25.2	26.2	30.0	19.2	23.7	24.0	22.3	
16	24.2	18.8	20.8	19.3	27.4	23.2	23.6	26.2	30.2	19.3	24.4	23.5	22.5	*aaM*(15)
17	24.4	18.6	21.0	18.9	27.2	23.5	23.7	26.2	29.7	19.3	25.0	24.9	22.3	
18	24.6	18.4	21.5	18.5	27.2	24.7	24.4	26.2	29.1	19.5	24.8	26.8	21.6	
19	24.8	18.6	22.0	18.5	27.0	26.6	25.0	26.2	30.1	19.8	24.9	27.9	21.4	
20	25.7	18.9	22.1	19.0	26.9	28.2	25.5	26.2	31.5	20.3	26.3	28.7	21.3	
21	26.4	19.5	21.9	19.3	26.6	29.4	26.1	25.6	32.0	21.0	27.4	29.7	21.2	
22	26.5	20.4	21.9	19.2	25.8	30.5	26.5	25.1	31.6	21.1	27.4	31.8	21.4	
23	27.2	20.8	21.8	18.9	25.0	31.5	26.4	24.8	30.9	20.9	27.3	33.7	21.4	
24	27.4	21.2	21.5	18.7	24.0	32.0	26.3	24.7	30.3	21.1	27.8	34.6	21.6	*aaM*(11)
25	27.1	21.7	21.7	19.5	23.4	32.0	26.4	24.4	30.6	21.3	29.0	36.1	22.1	*aaM*(16)
26	27.0	22.0	22.1	20.0	23.1	31.7	26.2	23.7	32.0	20.7	29.9	36.7	22.5	*aaM*(22)
27	26.9	21.9	22.0	20.0	22.6	31.2	26.6	23.4	32.7	20.0	31.3	36.0	23.0	*aaM*(19)
28	27.3	22.0	21.4	20.1	22.1	30.3	27.0	23.3	32.6	19.9	32.1	35.9	23.8	
29	27.4	22.6	21.0	20.5	22.0	29.1	27.7	23.4	32.5	20.0	32.3	34.7	24.9	*aaM*(11)
30	27.0	23.2	20.4	21.0	21.6	27.1	27.8	23.6	32.3	20.0	33.3	32.6	25.9	
31	26.7	23.5	19.4	20.9	21.3	24.9	26.9	23.6	30.9	19.9	34.0	31.2	27.6	
32	25.7	23.7	18.7	20.6	20.9	23.2	26.6	24.0	29.3	19.9	33.8	30.7	30.1	*aaM*’(12)
33	24.3	23.4	18.6	20.4	20.6	21.7	26.3	24.8	28.6	19.7	33.7	30.2	31.7	
34	23.5	22.9	18.5	20.3	20.6	20.5	25.8	25.7	28.6	19.6	34.2	28.5	32.8	
35	22.7	22.6	18.3	20.4	20.3	19.5	25.7	26.3	28.5	19.4	34.6	27.4	33.8	
36	22.0	22.0	18.1	20.5	19.9	19.1	25.4	26.5	27.9	19.0	34.6	27.5	34.7	*aaM*(21)
37	21.4	21.4	17.5	20.3	19.2	19.2	25.6	27.0	26.8	19.0	33.4	26.9	36.0	
38	20.8	21.0	16.9	20.3	18.7	19.6	26.2	27.5	24.6	19.1	32.3	26.6	36.9	
39	20.5	21.0	16.5	20.2	19.6	20.0	26.3	27.9	22.9	19.8	31.1	27.5	37.7	
40	19.7	20.7	16.3	20.0	20.6	20.6	26.5	28.5	22.0	20.5	30.2	27.6	38.0	*aaM*(23)
41	18.8	20.1	16.4	19.6	20.5	21.5	26.8	29.1	21.5	20.3	30.2	27.2	37.2	
42	18.3	19.6	16.6	19.0	20.5	22.9	26.7	29.7	20.9	20.4	29.8	27.1	36.2	

Table 5. 12-month moving averages of $aa(t)$ for cycles 11–23 from $t=0$ to 84 months past $E(RM)$ (Continued).

t	**Cycles**													**Comments**
	11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	
43	17.9	19.5	16.7	18.9	19.9	24.0	26.6	30.0	20.7	20.5	29.4	26.9	34.1	
44	17.8	19.3	17.0	19.3	19.3	24.3	26.3	30.4	20.4	20.7	28.7	26.2	31.6	
45	17.8	18.9	16.9	19.6	19.3	24.2	26.5	30.6	19.9	21.5	28.0	25.6	30.4	
46	17.7	18.8	16.6	19.6	19.4	24.2	27.0	31.2	19.2	22.7	27.7	25.5	29.5	
47	16.8	18.7	16.7	19.7	19.7	24.2	28.0	31.7	18.9	24.3	27.4	25.5	28.0	
48	16.9	18.8	17.0	19.7	20.3	23.7	28.5	31.4	19.6	25.6	27.1	25.3	25.9	
49	16.8	18.9	17.3	19.4	21.0	22.9	28.3	31.5	20.4	26.2	27.3	25.7	24.1	
50	16.5	18.8	17.9	19.3	21.6	22.2	28.0	31.9	21.0	26.7	27.6	26.4	23.4	
51	16.2	18.5	18.3	19.8	21.0	21.8	27.6	32.3	21.3	26.4	28.0	26.9	23.1	
52	16.0	18.4	18.4	19.6	20.3	21.5	26.9	33.0	21.7	26.0	28.6	28.1	23.1	
53	15.9	18.4	18.3	20.9	20.8	21.0	26.1	33.8	21.8	26.4	28.7	29.1	22.8	
54	15.6	18.4	18.1	21.6	21.4	20.3	25.8	34.3	21.6	26.7	28.7	29.4	22.6	
55	15.3	18.0	18.3	22.0	21.9	19.8	25.7	34.7	21.4	26.7	28.9	29.3	23.0	$aaM(18)$
56	15.0	17.5	18.2	22.1	22.4	19.7	25.4	34.5	21.5	26.7	28.9	29.1	23.7	
57	14.6	17.2	18.0	22.1	22.4	19.8	25.0	33.9	21.9	26.4	28.3	29.3	23.6	
58	14.3	17.0	18.1	22.1	22.0	19.6	24.8	32.8	22.0	26.0	27.7	29.6	23.7	
59	14.2	16.4	18.1	22.2	21.3	19.3	24.3	31.9	21.8	25.6	27.1	29.4	24.7	$aaM(14)$
60	14.2	15.8	18.0	22.1	20.8	19.3	24.3	31.5	22.1	25.5	26.4	29.3	25.3	
61	14.1	15.6	17.8	21.5	20.1	19.4	24.8	31.0	22.1	25.7	26.1	28.4	24.5	
62	13.7	15.6	17.3	20.7	19.0	19.4	24.7	30.6	21.6	26.2	25.9	27.0	23.4	
63	13.3	15.6	16.8	19.9	18.1	19.1	24.8	29.8	21.4	27.1	25.2	25.8	22.1	
64	12.9	15.5	16.4	19.2	17.5	18.8	24.9	28.7	21.2	27.9	24.2	24.7	20.8	
65	12.7	15.6	16.1	18.3	16.7	18.6	24.4	27.5	21.5	28.8	23.5	24.0	20.2	
66	12.7	15.6	16.0	17.1	15.6	18.3	23.8	26.4	21.9	29.4	22.8	23.4	20.0	
67	12.6	15.6	15.6	15.8	14.7	17.6	24.1	25.7	22.3	30.0	22.4	23.2	19.3	
68	12.4	15.7	15.1	14.8	14.0	17.2	24.8	25.5	22.3	30.4	22.9	23.4	18.5	
69	12.4	15.5	14.7	14.0	13.5	17.0	25.1	25.9	22.0	30.6	23.6	23.2	17.9	
70	12.5	15.3	14.2	13.5	13.3	17.0	26.0	26.2	21.6	30.8	23.1	22.7	17.1	$aaM(20)$
71	12.5	15.0	13.5	12.9	13.4	17.1	27.2	26.1	21.1	30.3	22.6	22.2	16.2	
72	12.4	14.9	12.9	12.6	13.4	16.9	27.7	25.9	19.9	29.7	22.5	21.8	15.5	
73	12.2	14.6	12.4	12.4	13.3	16.4	28.1	25.6	18.7	29.1	22.1	21.4	15.6	
74	12.3	14.4	11.8	12.2	13.2	16.2	28.6	24.8	18.1	28.3	21.8	21.2	15.9	
75	12.6	14.2	11.4	12.0	13.1	16.5	28.9	24.5	17.5	27.4	21.9	21.0	16.4	
76	12.9	14.3	11.2	11.9	13.2	16.8	29.0	24.6	16.8	26.2	21 9	20.1	16.7	
77	12.9	14.2	10.7	11.9	13.3	16.7	29.3	24.4	16.3	24.8	21.7	19.1	16.7	
78	12.7	13.8	10.2	12.1	13.3	16.7	29.5	23.9	15.8	24.1	21.3	18.7	16.7	$aaM(17)$
79	12.5	13.6	9.9	12.3	13.4	16.8	29.2	23.2	15.2	24.0	20.7	18.8	16.7	
80	12.2	13.7	9.5	12.4	13.3	17.1	28.6	22.4	14.5	23.5	19.4	19.0	16.7	
81	12.2	14.2	9.2	12.4	13.2	17.5	27.7	21.6	14.2	23.3	18.3	19.0	16.6	

Table 5. 12-month moving averages of *aa*(t) for cycles 11–23 from $t=0$ to 84 months past *E*(*RM*) (Continued).

t	**Cycles**													**Comments**
	11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	
82	12.1	15.2	9.0	12.3	13.2	17.2	26.2	21.1	14.2	23.2	18.1	18.8	16.5	
83	11.9	16.5	8.9	12.2	13.0	17.1	24.4	20.9	14.2	23.3	17.8	18.7	16.4	
84	11.7	17.3	9.0	12.0	12.7	17.5	23.0	20.6	14.2	23.4	17.5	18.5	16.3	

Table 6. 12-month moving averages of *aaI*(t) for cycles 11–23 from $t=0$ to 84 months past E(RM).

t	**Cycles**													**Comments**
	11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	
0	12.3	8.3	12.6	6.8	9.5	9.9	9.2	14.9	16.0	11.3	4.6	17.4	13.6	
1	12.4	7.9	13.3	6.6	9.8	10.3	9.7	15.1	15.2	11.2	4.7	17.5	13.7	
2	12.7	7.7	13.7	6.7	10.1	10.7	10.1	15.3	15.3	10.7	4.3	16.8	13.8	
3	12.8	7.5	13.7	6.5	10.5	10.9	11.9	15.6	15.0	10.4	3.6	15.9	13.3	
4	12.8	7.4	13.7	6.5	10.9	11.5	13.6	15.3	14.8	10.4	3.6	16.1	12.5	
5	13.3	7.6	13.9	7.0	11.3	12.6	13.8	14.5	14.5	10.2	4.1	16.3	12.5	aaIM’(13)
6	13.8	7.8	13.7	7.5	11.3	13.4	13.9	14.4	14.2	9.6	4.8	16.8	13.5	
7	13.4	8.1	13.1	7.7	11.7	13.2	14.1	14.4	14.1	9.0	5.0	16.8	13.6	
8	12.6	8.5	12.7	7.7	12.6	12.8	14.2	13.7	14.1	8.6	5.2	16.6	13.0	
9	12.8	8.5	13.0	8.0	13.2	12.2	14.1	13.2	13.8	7.9	6.0	16.4	12.4	
10	13.0	8.3	13.1	8.5	14.0	11.9	14.3	12.4	13.6	7.4	7.3	15.6	11.9	
11	12.8	8.6	13.0	8.9	15.0	12.2	15.0	12.0	14.2	7.4	8.5	14.6	11.4	
12	12.9	9.0	12.7	9.4	15.7	12.5	15.4	12.4	15.1	7.0	9.1	14.1	11.1	
13	13.2	8.8	12.0	9.6	16.4	12.7	15.1	12.7	15.7	6.7	9.6	13.1	11.0	
14	13.2	8.9	11.3	9.8	16.9	12.9	15.1	13.3	16.3	7.3	10.3	12.4	10.9	
15	13.2	9.5	11.3	10.1	17.3	13.4	13.8	13.4	17.1	8.0	10.8	12.0	10.9	
16	13.4	9.9	11.6	10.1	17.5	13.9	12.4	13.1	17.6	8.1	11.5	11.6	10.9	aaIM(15)
17	13.5	9.8	11.7	9.9	17.3	14.3	12.6	13.1	17.3	8.0	12.1	12.9	10.7	
18	13.8	9.7	12.3	9.6	17.4	15.6	13.3	13.2	16.7	8.3	12.0	14.6	10.0	
19	14.0	9.9	12.9	9.8	17.4	17.5	13.9	13.5	17.8	8.6	12.2	15.7	9.7	
20	14.8	10.3	12.8	10.4	17.4	19.3	14.4	13.6	19.3	9.1	13.5	16.5	9.7	
21	15.4	11.0	12.4	10.6	17.2	20.6	15.4	13.1	20.0	10.0	14.5	17.5	9.6	
22	15.4	11.9	12.6	10.6	16.5	21.9	16.1	12.6	19.8	10.3	14.6	19.7	9.7	
23	16.2	12.4	12.6	10.3	15.7	23.0	16.0	12.3	19.0	10.3	14.6	21.4	9.9	
24	16.4	13.0	12.3	10.0	14.8	23.5	15.9	12.2	18.4	10.7	15.2	21.8	10.2	
25	16.1	13.7	12.6	10.7	14.2	23.6	16.1	11.8	18.8	11.3	16.4	23.0	10.8	
26	16.1	14.1	13.1	11.3	14.0	23.6	16.1	11.2	20.3	11.1	17.5	23.7	11.3	aaIM(16), aaIM(22)
27	16.2	14.1	13.0	11.4	13.7	23.3	16.7	11.1	21.1	10.5	19.0	23.2	12.0	
28	16.7	14.3	12.5	11.5	13.4	22.6	17.2	11.3	21.3	10.6	20.1	23.2	12.9	

Table 6. 12-month moving averages of *aaI*(t) for cycles 11–23 from $t = 0$ to 84 months past E(RM). (Continued).

t	**Cycles**													**Comments**
	11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	
29	17.1	15.0	12.2	12.0	13.6	21.3	17.9	11.5	21.5	10.9	20.7	22.3	14.2	aaIM(11), aaIM(19)
30	16.9	15.7	11.6	12.4	13.4	19.4	18.0	11.8	21.5	11.0	22.0	20.6	15.4	
31	16.6	16.1	10.7	12.1	13.2	17.3	17.4	11.9	20.3	11.0	23.1	19.6	17.4	
32	15.9	16.3	10.3	11.8	12.8	15.5	17.2	12.4	18.9	11.1	23.3	19.4	20.0	aaIM'(12)
33	14.7	16.1	10.5	11.7	12.5	14.2	16.8	13.3	18.4	11.0	23.6	19.2	21.7	
34	14.1	15.7	10.5	11.7	12.5	12.9	16.2	14.5	18.6	10.8	24.3	17.6	22.9	
35	13.4	15.6	10.4	11.8	12.3	12.0	16.1	15.4	18.8	10.5	24.8	16.6	24.0	
36	12.9	15.0	10.2	12.0	12.0	11.7	16.0	15.9	18.5	10.1	24.8	17.0	25.2	aaIM(21)
37	12.4	14.4	9.7	12.0	11.3	11.9	16.3	16.7	17.5	9.8	23.7	16.7	26.5	
38	12.0	14.1	9.2	12.1	10.7	12.3	16.9	17.4	15.6	9.6	22.7	16.6	27.6	
39	11.7	14.1	8.8	11.9	11.6	12.8	16.9	17.9	14.0	10.2	21.7	17.7	28.5	
40	10.9	13.8	8.7	11.7	12.6	13.4	17.1	18.7	13.3	10.8	21.0	17.9	28.9	aaIM(23)
41	10.1	13.2	8.9	11.3	12.5	14.4	17.5	19.5	12.7	10.7	21.0	17.5	28.2	
42	9.6	12.7	9.1	10.8	12.4	15.8	17.6	20.0	12.2	10.7	20.7	17.5	27.2	
43	9.2	12.5	9.2	10.9	11.9	17.0	17.6	20.3	12.0	11.0	20.3	17.4	25.2	
44	9.2	12.3	9.5	11.4	11.4	17.4	17.4	20.8	11.7	11.3	19.6	16.9	22.7	
45	9.3	12.0	9.4	11.7	11.5	17.2	17.6	21.1	11.4	12.2	19.0	16.4	21.8	
46	9.3	11.9	9.1	11.8	11.6	17.3	18.2	21.7	10.8	13.6	18.9	16.5	20.9	
47	9.0	11.8	9.2	11.9	12.0	17.3	19.2	22.1	10.7	15.2	19.1	16.6	19.6	
48	8.8	12.0	9.5	12.0	12.6	16.8	19.8	21.9	11.5	16.5	19.2	16.5	17.5	
49	8.8	12.2	9.8	11.8	13.4	16.0	19.6	22.0	12.3	17.3	19.6	17.0	15.8	
50	8.5	12.1	10.4	11.7	14.1	15.4	19.5	22.4	12.9	18.1	20.0	17.9	15.2	
51	8.3	11.8	10.8	12.3	13.5	15.0	19.1	23.0	13.3	18.0	20.6	18.5	15.0	
52	8.2	11.7	10.8	13.0	13.0	14.6	18.4	23.8	13.7	17.6	21.3	19.9	14.9	
53	8.2	11.8	10.7	13.7	13.6	14.1	17.6	24.7	13.9	18.1	21.5	21.0	14.8	
54	8.1	12.2	10.6	14.6	14.2	13.5	17.2	25.6	13.8	18.5	21.6	21.4	14.6	
55	8.0	12.2	10.8	15.0	14.9	13.1	17.1	26.2	13.7	18.6	21.9	21.5	15.1	
56	7.7	11.7	10.8	15.1	15.4	13.0	16.8	26.2	13.8	18.7	21.9	21.2	15.8	
57	7.4	11.4	10.7	15.2	15.4	13.1	16.7	25.7	14.2	18.5	21.3	21.4	15.7	aaIM(18)
58	7.2	11.2	10.8	15.2	15.0	12.9	16.7	24.7	14.3	18.2	20.7	21.8	15.8	
59	7.1	10.6	10.8	15.4	14.4	12.7	16.3	24.0	14.1	17.8	19.9	21.8	16.8	aaIM(14)
60	7.1	10.0	10.7	15.3	13.9	12.7	16.5	23.7	14.4	17.7	19.1	21.7	17.6	
61	7.0	9.8	10.6	14.7	13.4	12.8	17.0	23.3	14.5	18.0	18.9	20.9	16.8	
62	6.7	9.8	10.2	14.0	12.4	12.8	17.0	22.8	14.0	18.4	18.6	19.5	15.8	
63	6.3	9.7	9.8	13.3	11.5	12.6	17.2	22.1	13.8	19.2	18.0	18.3	14.5	
64	6.0	9.6	9.5	12.6	10.9	12.3	17.3	21.1	13.6	20.1	17.1	17.2	13.2	
65	5.9	9.7	9.3	11.7	10.1	12.2	16.9	19.9	13.9	20.9	16.3	16.5	12.7	
66	5.8	9.4	9.1	10.5	9.1	11.8	16.4	18.8	14.4	21.5	15.7	16.0	12.5	

Table 6. 12-month moving averages of $aaI(t)$ for cycles 11–23 from $t = 0$ to 84 months past E(RM). (Continued).

t	Cycles													Comments
	11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	
67	5.8	9.0	8.6	9.3	8.1	11.1	16.8	18.2	14.8	22.1	15.3	15.9	11.9	
68	5.6	9.1	8.1	8.3	7.4	10.7	17.6	18.1	14.9	22.5	15.8	16.0	11.2	
69	5.5	9.0	7.7	7.5	6.9	10.4	17.9	18.6	14.7	22.8	16.6	16.0	10.6	
70	5.6	8.7	7.1	7.0	6.7	10.4	18.8	19.0	14.4	23.0	16.0	15.5	10.0	aaIM(20)
71	5.7	8.5	6.5	6.5	6.8	10.5	20.0	18.9	14.0	22.6	15.5	15.1	9.1	
72	5.5	8.3	5.9	6.1	6.8	10.3	20.5	18.8	12.9	22.1	15.5	14.7	8.4	
73	5.4	8.1	5.4	6.0	6.8	9.8	20.9	18.5	11.8	21.6	15.1	14.4	8.5	
74	5.5	7.8	4.8	5.7	6.7	9.6	21.5	17.9	11.3	20.9	14.9	14.3	8.8	
75	5.8	7.7	4.3	5.6	6.5	9.8	21.8	17.7	10.7	20.1	15.0	14.1	9.4	
76	6.0	7.7	4.1	5.4	6.5	10.0	22.1	17.8	10.0	18.9	15.0	13.3	9.7	
77	6.0	7.6	3.7	5.5	6.5	9.9	22.4	17.6	9.5	17.6	14.7	12.3	9.7	
78	5.8	7.2	3.4	5.6	6.5	9.8	22.7	17.2	9.1	17.1	14.4	12.0	9.8	aaIM(17)
79	5.6	7.0	3.2	5.8	6.5	9.9	22.4	16.5	8.4	17.0	13.8	12.0	9.9	
80	5.3	7.0	2.9	5.9	6.4	10.1	21.9	15.8	7.7	16.6	12.5	12.3	9.8	
81	5.3	7.5	2.6	5.9	6.2	10.3	21.1	15.1	7.4	16.3	11.4	12.3	9.8	
82	5.2	8.4	2.4	5.9	6.2	10.0	19.5	14.6	7.3	16.2	11.2	12.1	9.7	
83	5.0	9.6	2.4	5.8	5.9	9.7	17.7	14.4	7.3	15.3	10.9	12.0	9.6	
84	4.9	10.2	2.5	5.6	5.5	10.1	16.3	14.1	7.4	16.3	10.5	11.8	9.6	

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Modified geomagnetic precursor techniques for predicting the size of the following sunspot cycle are developed, where these techniques use the 12-month moving averages of the number of disturbed days (when $Ap \geq 25$), the Ap index, the aa index, and the aaI index at about 4 yr during the declining portion of the preceding sunspot cycle. For cycle 24, these techniques suggest that its RM will measure about 130 ± 14, a value outside the consensus prediction interval of the low prediction (90 ± 10) given by the NOAA Solar Cycle 24 Prediction Panel. Furthermore, cycle 24 is predicted to be a fast-rising cycle ($ASC = 44 \pm 5$ months), peaking before April 2012, presuming the official start of cycle 24 in March 2008. Also discussed are the variation of solar cycle lengths and Hale cycle effects, as related to cycles 23 and 24.

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