NASA/TP—2006–214433

An Examination of Sunspot Number Rates of Growth and Decay in Relation to the Sunspot Cycle

Robert M. Wilson and David H. Hathaway
Marshall Space Flight Center, Marshall Space Flight Center, Alabama

June 2006

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ACRONYMS, SYMBOLS, AND ABBREVIATIONS

ASC	ascent duration in years
cl	confidence level
DES	descent duration in years
E(Rmin)	epoch of sunspot minimum occurrence
E(Rmax)	epoch of sunspot maximum occurrence
NSD	number of spotless days
NSD(E(Rmin))	number of spotless days during sunspot minimum year
n	sunspot cycle number
P	probability of obtaining the observed contingency table or one more suggestive of a departure from independence, computed using Fisher's exact test for 2×2 tables
PER	period or length of cycle in years
R	relative sunspot number
$<R>$	mean value of sunspot number
Rmax	sunspot number maximum amplitude
Rmin	sunspot number minimum amplitude
r	linear correlation coefficient
r^2	coefficient of determination (a measure of the amount of variance explained by the inferred regression)
$SLOPE_{ASC}$	average rate of growth in sunspot number during the ascent
$SLOPE_{DES}$	average rate of decay in sunspot number during the descent

ACRONYMS, SYMBOLS, AND ABBREVIATIONS (Continued)

$SLOPE_{DES}(GNV)$	greatest yearly negative change in sunspot number during the evolving descent duration
sd	standard deviation
se	standard error of estimate
T	elapsed time in years from sunspot maximum
$T(E(SLOPE_{DES}(GNV)))$	elapsed time in years from sunspot maximum to the occurrence of the greatest negative value of the evolving *SLOPE* during the decline of the sunspot cycle
t	elapsed time in years from sunspot minimum statistic for independent samples
$t(E(\Delta R_{GNV}))$	elapsed time in years from sunspot minimum to the occurrence of the greatest negative value of the change in R
$t(E(\Delta R_{GPV}))$	elapsed time in years from sunspot minimum to the occurrence of the greatest positive value of the change in sunspot number (R)
x	independent variable
$x1$, $x2$	variables
$Y.12$	refers to the bivariate fit, where 1 means parameter $x1$ and 2 means parameter $x2$
y	dependent variable
y_L	lower regression line
y_U	upper regression line
y_{x1x2}	the bivariate fit
ΔR_{GNV}	greatest negative value of the change in sunspot number
ΔR_{GPV}	greatest positive value of the change in sunspot number
χ^2	a statistical test statistic

TECHNICAL PUBLICATION

AN EXAMINATION OF SUNSPOT NUMBER RATES OF GROWTH AND DECAY IN RELATION TO THE SUNSPOT CYCLE

1. INTRODUCTION

Wolf's relative sunspot number (*R*) is one of the oldest and most enduring of the sunspot records.[1–5] Consequently, it is the one most often used to describe solar activity. Even so, recent studies have shown that its reliability is questionable for epochs earlier than the mid-1800s.[5–10] In particular, Hoyt and Schatten's group sunspot number has been shown to be virtually identical with Wolf's relative sunspot number, but only since about 1882.[9–11] Also, comparison of Wolf's relative sunspot number against Greenwich sunspot areas shows fairly good consistency from about 1874.[12,13] Hence, Wolf's relative sunspot number is generally recognized to be most reliable from the onset of cycle 12 in 1878 to the present.

In this study, the rates of growth and decay in annual sunspot number averages are examined relative to minimum and maximum amplitudes to ascertain their predictive behavior in providing early estimates of minimum and maximum amplitudes and the timing of their occurrences.[14–21]

2. RESULTS AND DISCUSSION

Figure 1 displays the cyclic behavior of several solar cycle parameters for cycles 12–23 including the following:

- Minimum sunspot number amplitude (*Rmin*).
- Maximum sunspot number amplitude (*Rmax*).
- Greatest positive value change in *R* from one year to the next (ΔR_{GPV}).
- Greatest negative value change in *R* from one year to the next (ΔR_{GNV}).
- Average sunspot number slope during ascent interval ($SLOPE_{ASC}$), computed as *Rmax*–*Rmin*/*ASC*, where *ASC* is ascent duration in years, or elapsed time between *Rmin* and *Rmax* occurrences; and average sunspot number slope during descent interval ($SLOPE_{DES}$), computed as *Rmin* (cycle *n*+1)–*Rmax* (cycle *n*)/*DES*, where *DES* is descent duration in years or elapsed time between *Rmax* occurrence cycle *n* and *Rmin* occurrence cycle *n*+1.

The median, mean, and standard deviation (sd) for each parameter are shown.

Figure 1 reveals that cycles of late had values for these parameters that differ markedly from earlier cycles. Concerning *Rmin* for example, five of the last six cycles had an *Rmin* greater than both the median (6.1) and mean (7.0). Comparing *Rmin* for cycles 18–23 against *Rmin* for cycles 12–17, the difference in means is statistically important at the 2-percent level of significance. On the basis of hypothesis testing using the *t*-statistic for independent samples, note that a 5-percent level of significance means a confidence level (cl) of 95 percent, a level of significance of 1 percent means a cl of 99 percent, and so on.[22] Similarly, the other parameters show statistically significant differences in the means for the two groupings as follows:

- *Rmax* at the 0.2-percent level of significance.
- ΔR_{GPV} at the 2-percent level of significance.
- ΔR_{GNV} at the 0.1-percent level of significance.
- $SLOPE_{ASC}$ at the 0.5-percent level of significance.
- $SLOPE_{DES}$ at the 2-percent level of significance.

Thus, cycles 18–23 appear to be inherently more robust than cycles 12–17.

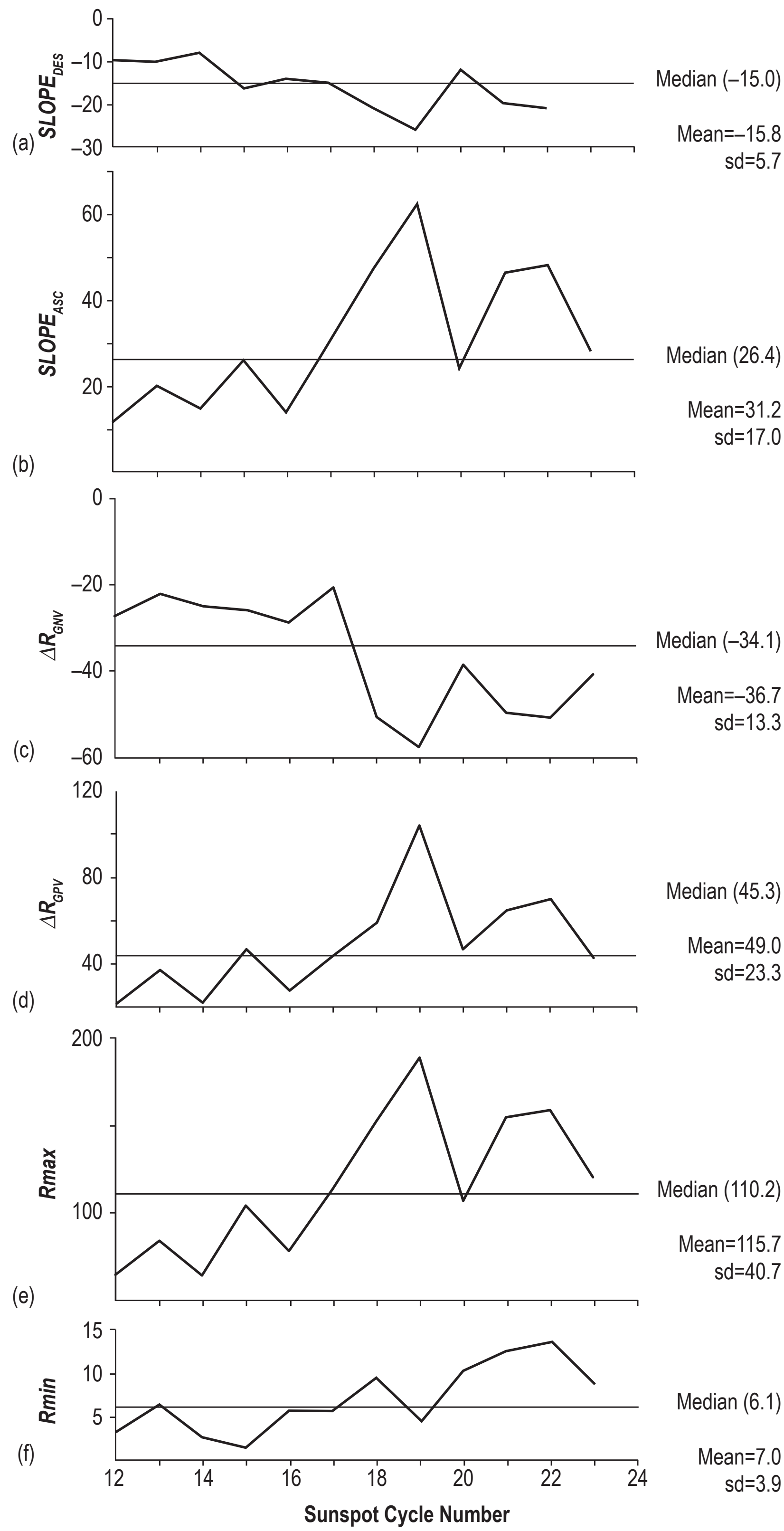

Figure 1. Cyclic variation of selected solar cycle parameters for cycles 12–23: (a) $SLOPE_{DES}$, (b) $SLOPE_{ASC}$, (c) ΔR_{GNV}, (d) ΔR_{GPV}, (e) *Rmax*, and (f) *Rmin*. The median value shown for each parameter is depicted as the thin horizontal line. Also identified are the mean and sd for each parameter.

Figure 2 compares yearly sunspot number averages for cycle 23 (filled circles) against the mean yearly averages (<*R*>) for cycles 12–22, relative to the epoch of *Rmin* occurrence (*E(Rmin)*). It also shows the relative sizes and times of *Rmax* occurrences, the ascent duration (*ASC*), and the relative length of the cycles (*PER*). On the basis of figure 2, it appears that cycle 23 is nearly to its end, with *Rmin* for cycle 24 expected either in 2006 or 2007, which corresponds respectively to year 10 or 11 of the sunspot cycle. For cycles 12–23, cycle 23 ranks fifth in relative size, having a maximum amplitude of 119.5 that occurred in year 4 of the sunspot cycle (counting the minimum year as year 0).

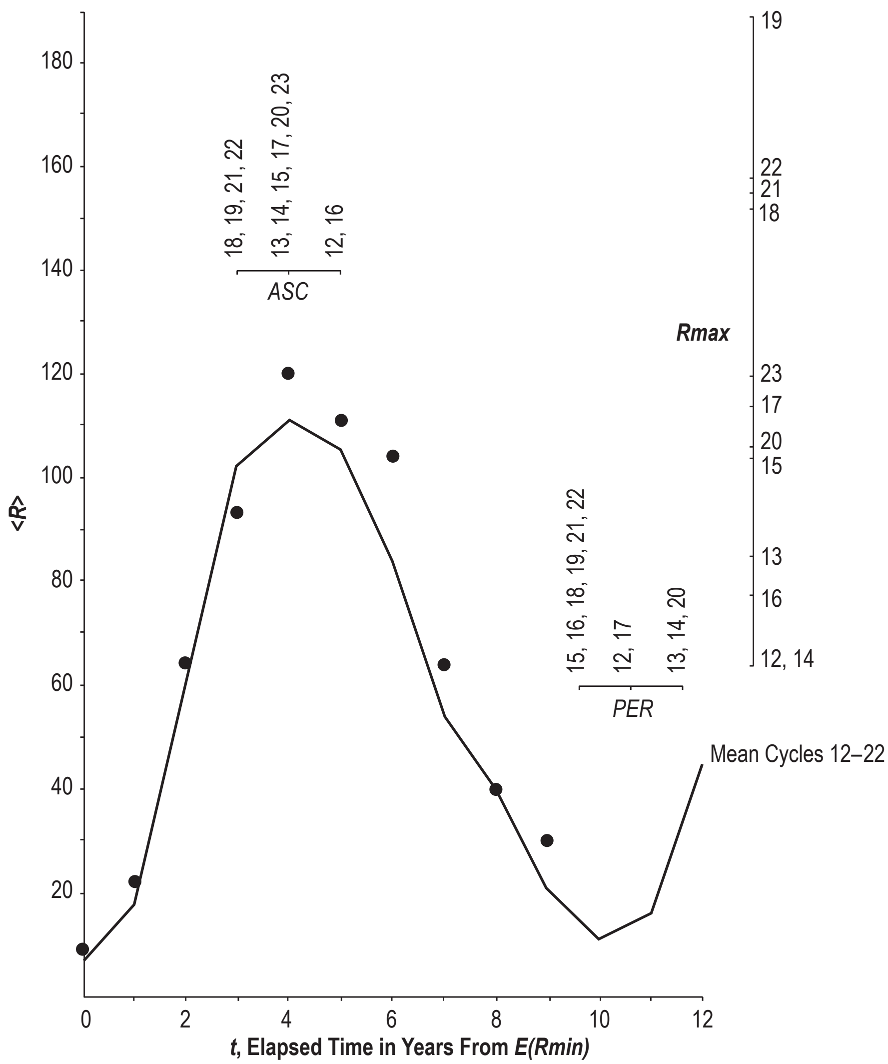

Figure 2. Comparison of cycle 23 *R*-values against the mean sunspot number (<*R*>) for cycles 12–22, relative to the elapsed time *(t)* in years from the epoch of sunspot minimum (*E*(*Rmin*)). Also shown are the relative sizes and times of occurrences of *Rmax*, the ascent duration (*ASC*), and the relative length of the cycle, period (*PER*). Cycle 23 is identified by the filled circles and the mean by the line. On the basis of this figure, there appears a strong indication that *Rmin* occurrence for cycle 24, the next sunspot cycle, will be in year *t*=10, corresponding to the year 2006.

Figure 3 compares yearly sunspot number averages for cycle 23 (filled circles) against the mean yearly averages (<*R*>) for cycles 12–22, relative to the elapsed time (*T*) in years from the epoch of sunspot maximum (*E(Rmax)*). It also shows the relative times of occurrences for the succeeding cycle *Rmin* descent duration (*DES*), thereby marking the conventional onset of the following cycle. On the basis of figure 3, it is suggested that cycle 23 will probably end in year seven from *E(Rmax)*, corresponding to 2007, inferring an 11-yr period for cycle 23 (*ASC+DES=PER*, or 4 +7 yr=11 yr). It should be noted, however, that minimum could come earlier in 2006, especially if cycle 24 proves to be above average in size, since robust cycles tend to start early, be fast risers, and often are of a shorter period.

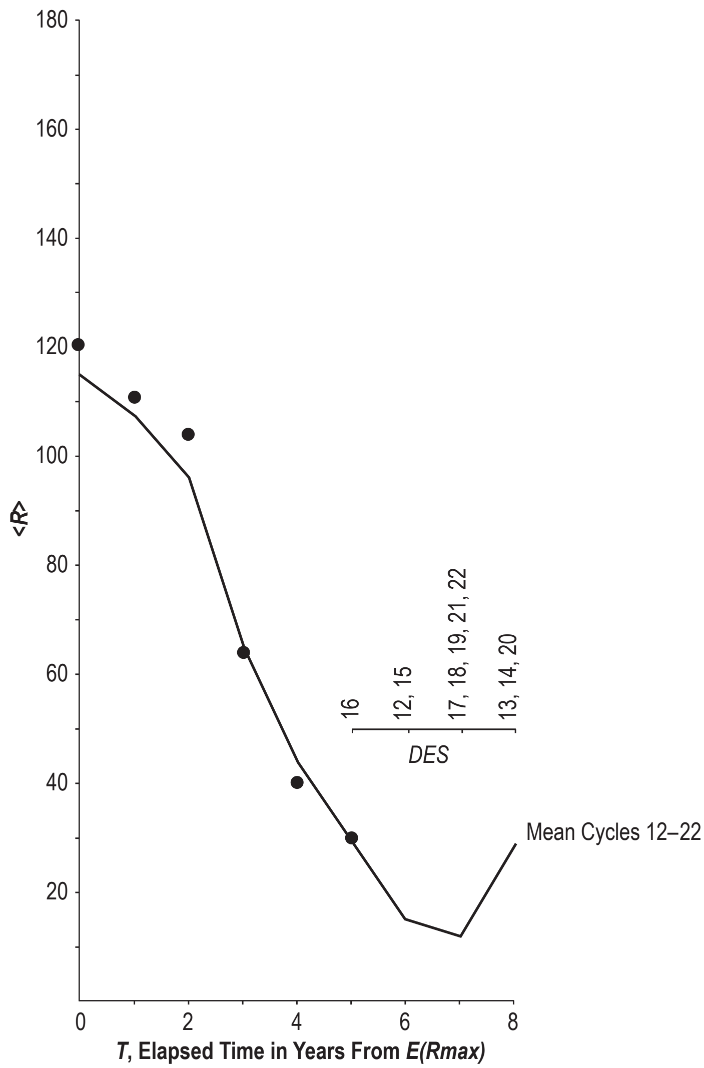

Figure 3. Comparison of cycle 23 *R*-values against the mean sunspot number (<*R*>) for cycles 12–22, relative to the elapsed time (*T*) in years from the epoch of sunspot maximum (*E*(*Rmax*)). Also shown are the relative occurrences of the descent duration (*DES*). Cycle 23 is identified by the filled circles and the mean by the line. On the basis of this figure, there appears a strong indication that *Rmin* for cycle 24, the next sunspot cycle, will be in year *T*=6 or 7, corresponding to the years 2006 or 2007.

Figure 4 shows the distribution of cycles based on cycle length (*PER*) and *ASC* for cycles 12–22. All cycles have *ASC* of 3, 4, or 5 yr and *PER* of 10, 11, or 12 yr. A χ^2 test of the observed 3×3 distribution yields χ^2=8.13, suggesting that *ASC* and *PER* might be weakly associated at the 10-percent level of significance.[23] At the 5-percent level of significance, the two parameters must be viewed as being independent of each other.

		PER		
		10	**11**	**12**
ASC	**3**	18, 19, 21, 22 (4)	(0)	(0)
	4	15 (1)	17 (1)	13, 14, 20 (3)
	5	16 (1)	12 (1)	(0)

χ^2=8.13

Figure 4. A 3×3 contingency table comparing ascent durations (*ASC*) and periods (*PER*) for cycles 12–22. Individual cycle numbers are identified in each bin. The numbers in parentheses give the frequency of occurrence. A χ^2 test yields χ^2=8.13, which is a marginally significant result (at the 10-percent level of significance).

Figure 5 compares yearly sunspot number averages for cycle 23 (filled circles) against the mean yearly averages ($<R>$) for cycles of *ASC*=3 yr (thick line), 4 yr (thin line), and 5 yr (dashed line). Throughout its rise, cycle 23 yearly sunspot numbers fell below the mean for *ASC*=3 yr and above the mean for *ASC*=4 yr, making it difficult to accurately determine whether it would have a 3 yr or 4 yr rise. The cycle 23 decline appears to be more like the decline found for *ASC*=3 yr rather than *ASC*=4 yr, except for the last year or two. The cycle 23 R value at t=9 yr (2005) equals 29.9, which is slightly smaller than was seen for the same t in cycles 17 (30.6) and 20 (38.2) and slightly larger than was seen for the same t in cycle 19 (27.9). The suggestion from figure 5 is that minimum for cycle 24, the next sunspot cycle, will probably occur either at t=10 yr (2006) or t=11 yr (2007).

Figure 6 compares yearly sunspot number averages for cycle 23 (filled circles) against the mean yearly averages ($<R>$) for cycles of *PER*=10 yr (thick line), 11 yr (thin line), and 12 yr (dashed line). Throughout its rise and fall, cycle 23's behavior has closely mimicked the mean behavior of cycles having 10-yr lengths. Hence, it may be that cycle 23 is also a cycle of *PER*=10 yr. If true, then 2006 should mark the onset year for cycle 24, on the basis of annual averages.

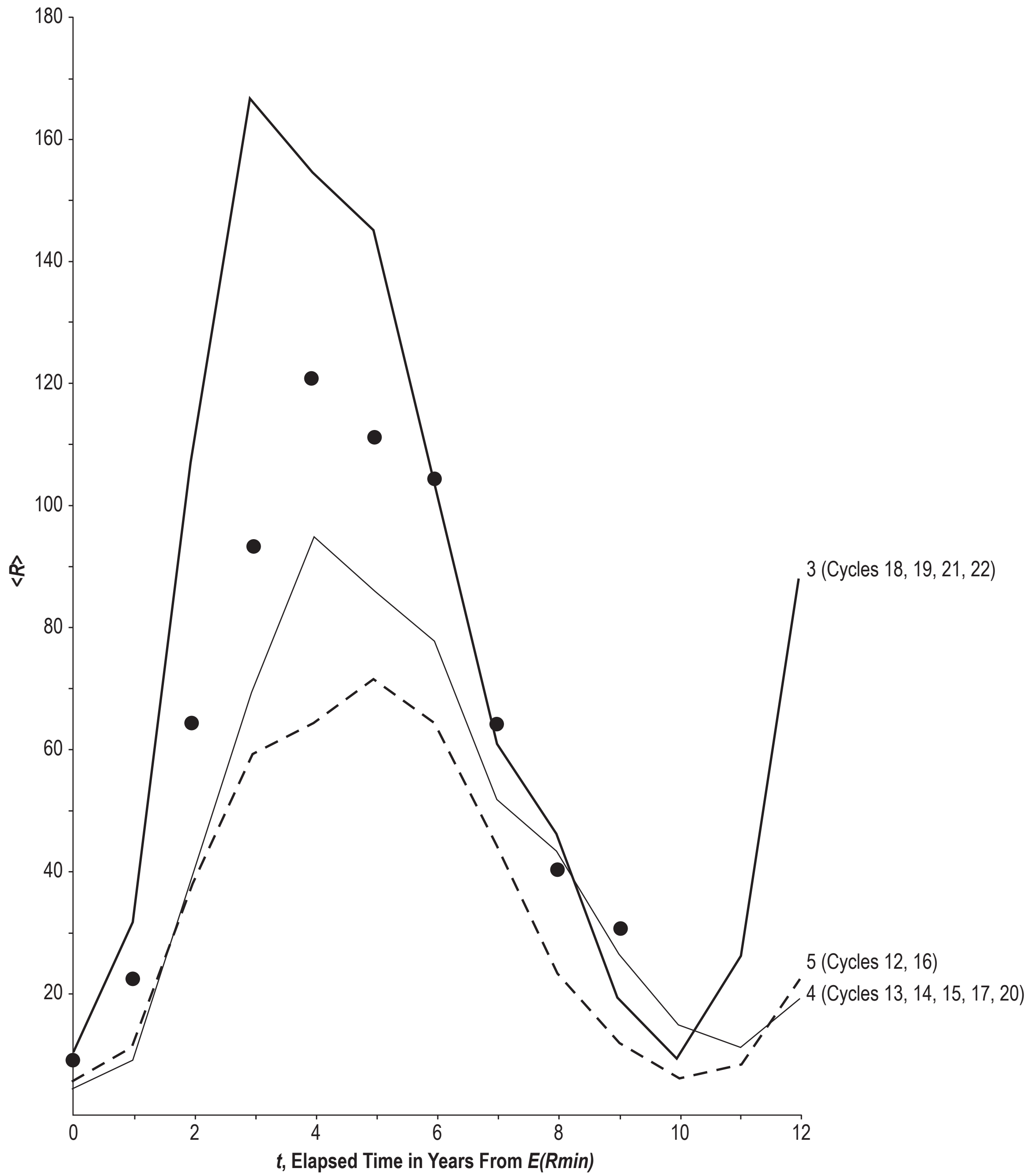

Figure 5. Comparison of cycle 23 *R*-values (filled circles) against the mean sunspot number (<*R*>) for cycles of *ASC*=3 yr (thick line), 4 yr (thin line), and 5 yr (dashed line) relative to the elapsed time (*t*) in years from the epoch of sunspot minimum (*E(Rmin)*).

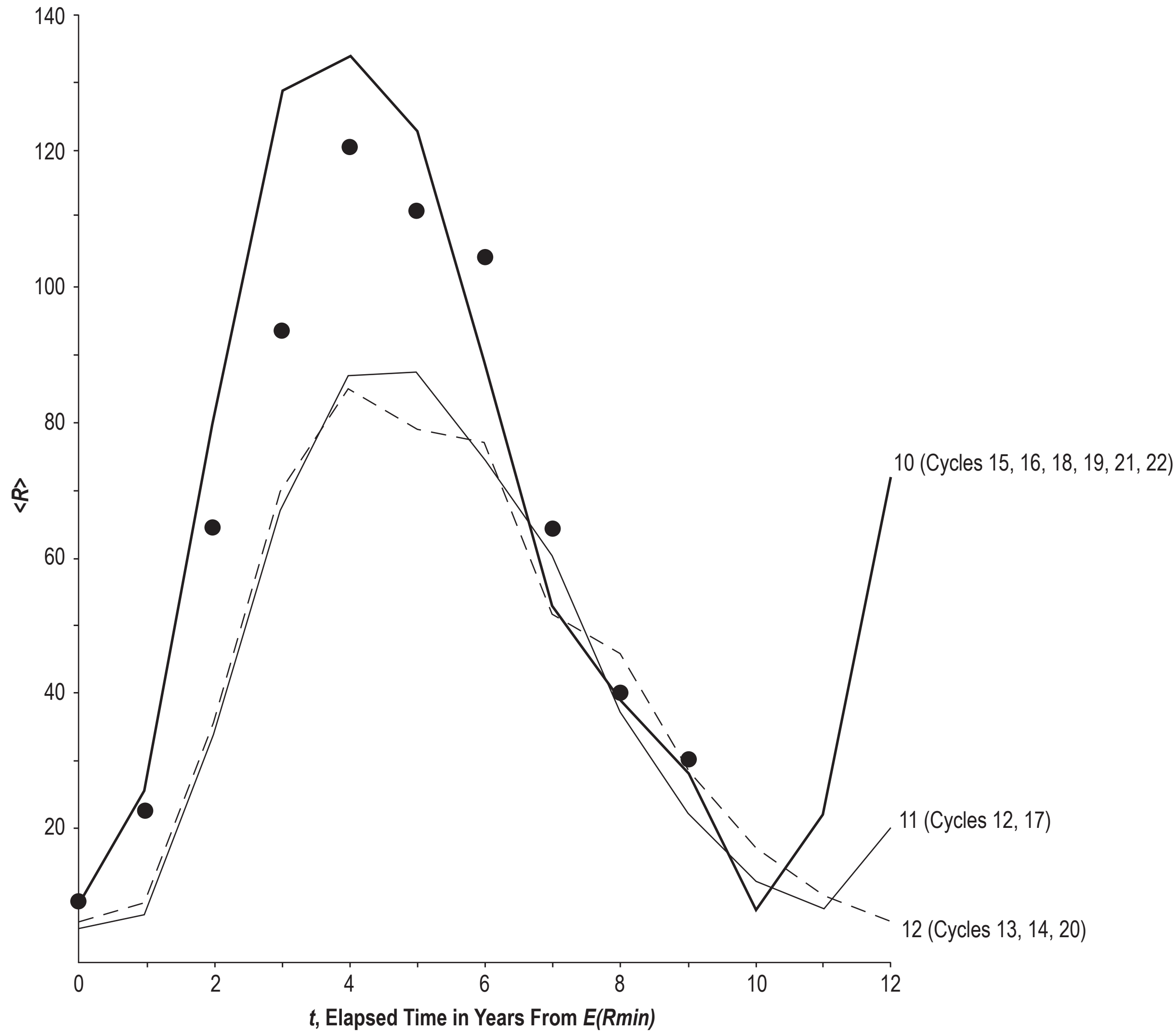

Figure 6. Comparison of cycle 23 *R*-values (filled circles) against the mean sunspot number (<*R*>) for cycles of *PER*=10 yr (thick line), 11 yr (thin line), and 12 yr (dashed line) relative to the elapsed time (*t*) in years from the epoch of sunspot minimum (*E(Rmin)*).

Once it is clear that sunspot minimum has occurred, one can readily employ observed sunspot number values to predict *Rmax*. Figure 7 displays scatterplots of *Rmax* versus *Rmin,* left panel; *Rmax* versus ΔR_{GPV}, center panel; and *Rmax* versus *Rmax,* (y_{x1x2}), right panel, where *Rmax*(y_{x1x2}) is a bivariate fit of *Rmax* against both *Rmin* ($x1$) and ΔR_{GPV} ($x2$). Plainly, *Rmin* provides a crude first-order prediction some 2–4 yr in advance for the later occurring *Rmax*. A much better prediction can be made following the occurrence of ΔR_{GPV}, which usually precedes *Rmax* occurrence by 1–2 yr (usually occurring in years 2 or 3 following *Rmin* occurrence). ΔR_{GPV} represents the inflection point during the rising portion of the ongoing sunspot cycle. The bivariate fit is found to further improve upon the prediction for *Rmax* of the growing sunspot cycle.[24]

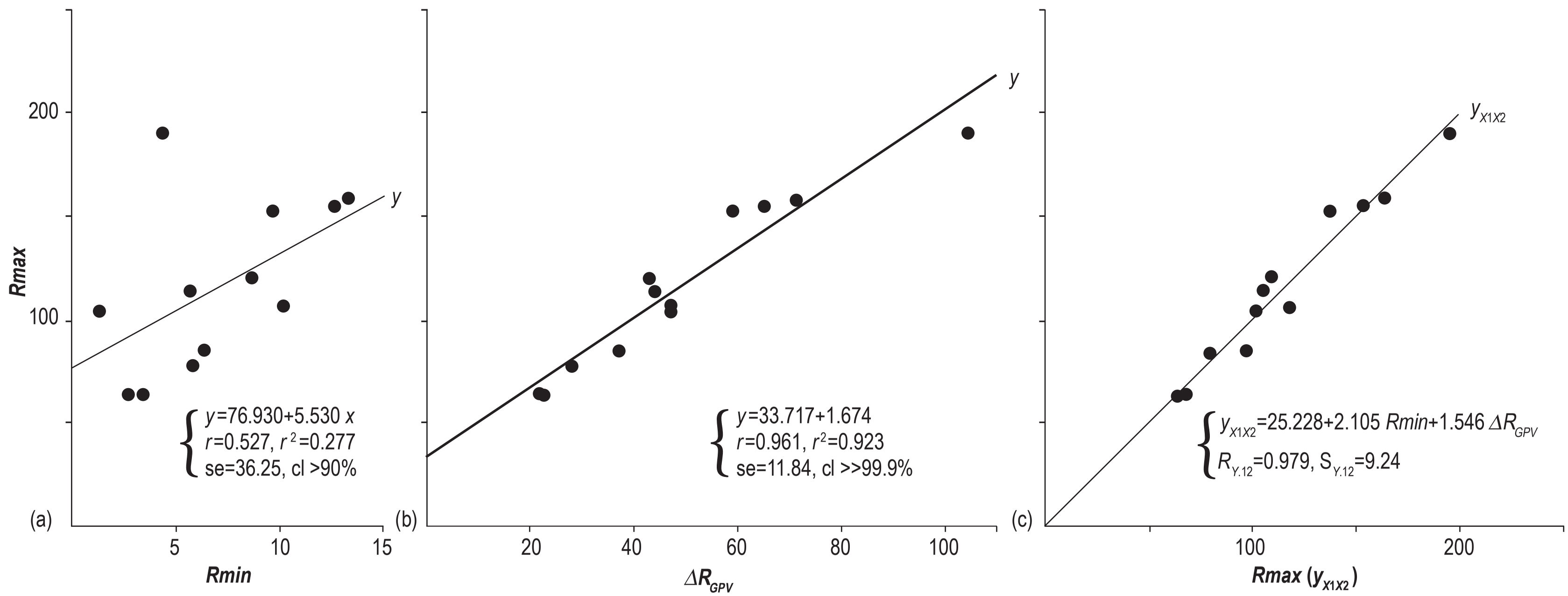

Figure 7. Scatterplots of (a) *Rmax* versus *Rmin*, (b) *Rmax* versus ΔR_{GPV}, and (c) *Rmax* versus *Rmax* (y_{x1x2}).

Figure 8 depicts the scatterplot of *Rmax* versus $SLOPE_{ASC}$, where $SLOPE_{ASC}$ is the average rate of growth during the rising portion of the cycle. As noted before, it is computed as (*Rmax*-*Rmin*)/*ASC*. Unfortunately, one cannot compute $SLOPE_{ASC}$ until *Rmax* has been observed. While strictly true, one can examine the evolving average rate of growth as the cycle progresses from *Rmin* occurrence to estimate the later occurring *Rmax*.

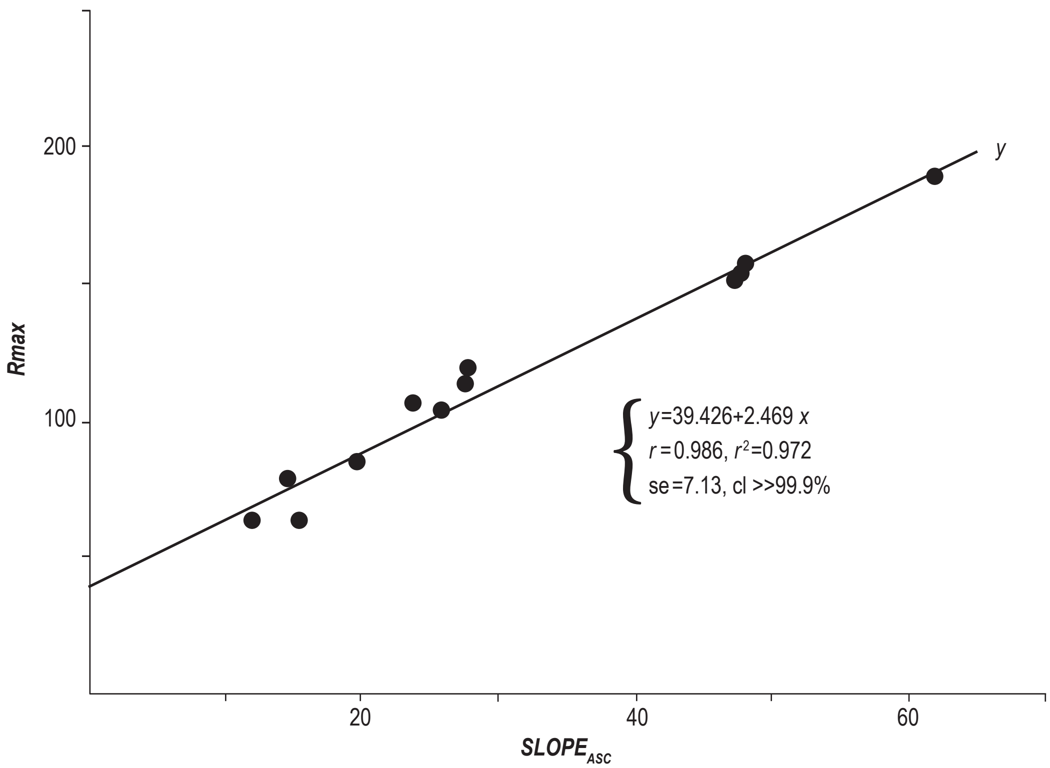

Figure 8. Scatterplot of *Rmax* versus $SLOPE_{ASC}$.

Figure 9 shows *Rmax* versus $SLOPE_{ASC}$(1), left panel; *Rmax* versus $SLOPE_{ASC}$(2), center panel; and *Rmax* versus $SLOPE_{ASC}$(3), right panel, where

- $SLOPE_{ASC}$(1) is the difference in *R* between year 0 (sunspot minimum year) and year 1 (year after sunspot minimum year).
- $SLOPE_{ASC}$(2) is the difference in *R* between year 0 and year 2 divided by 2.
- $SLOPE_{ASC}$(3) is the difference in *R* between year 0 and year 3 divided by 3.

As an example, cycle 23 had—

- *R*=8.6 in year 0, 1996, the sunspot minimum year.
- *R*=21.5 in year 1, 1997.
- *R*=64.2 in year 2, 1998.
- *R*=93.2 in year 3, 1999.

Hence, for cycle 23 $SLOPE_{ASC}$ (1)=12.9, $SLOPE_{ASC}$ (2)=27.8, and $SLOPE_{ASC}$ (3)=28.2.

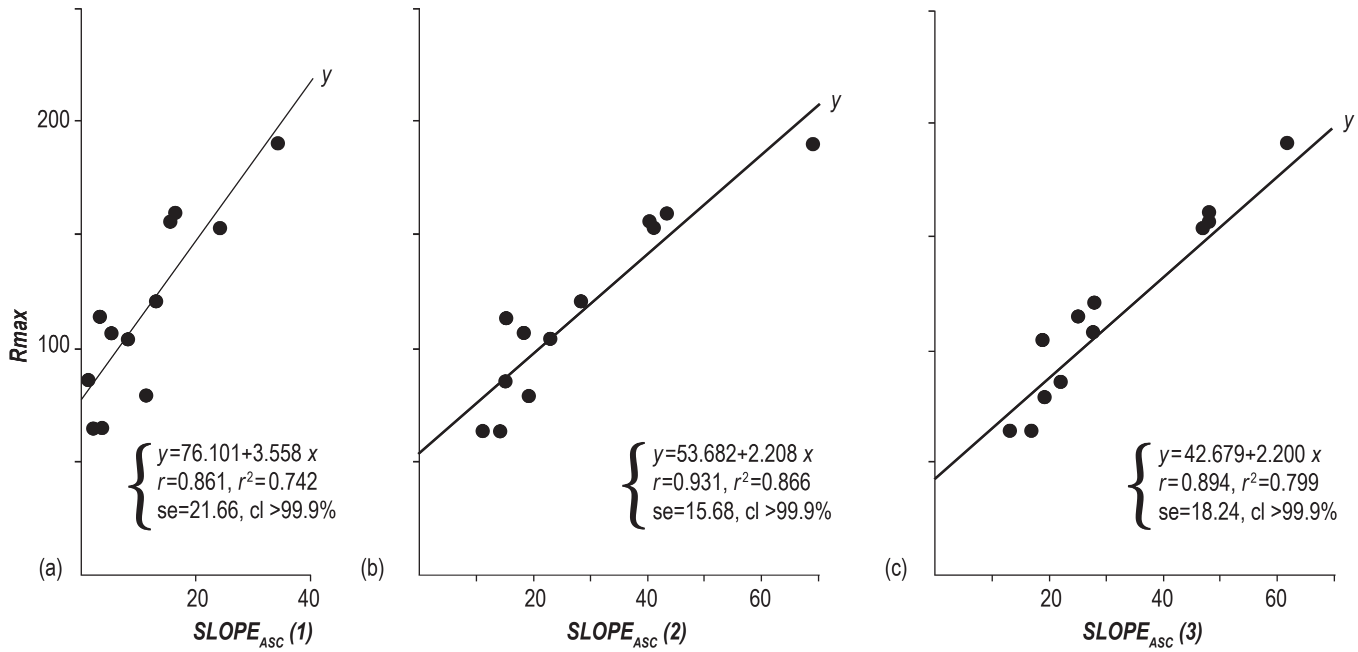

Figure 9. Scatterplots of (a) *Rmax* versus $SLOPE_{ASC}$ (1), (b) *Rmax* versus $SLOPE_{ASC}$ (2), and (c) *Rmax* versus $SLOPE_{ASC}$ (3).

Plainly, beginning at one year past *Rmin* occurrence, one can estimate *Rmax* considerably better (standard error (se) of 21.7 units of sunspot number) than at *Rmin* (se=36.3 units of sunspot number), and better yet at two years past *Rmin* occurrence (se=15.7 units of sunspot number).

Another parameter of interest is ΔR_{GNV}, the inflection point during the cycle decline from *Rmax* occurrence of the ongoing cycle to the *Rmin* occurrence of the following cycle. ΔR_{GNV} usually precedes *Rmin* occurrence of the succeeding cycle by about three to four years, usually occurring in year seven following *Rmin* occurrence of the ongoing cycle. Figure 10 displays scatterplots of ΔR_{GNV} versus ΔR_{GPV} (left panel) and ΔR_{GNV} versus *Rmax* (right panel). Both the inflection amplitude during the rise and the actual maximum sunspot number amplitude provide a reliable prediction for the inflection amplitude during the fall of the ongoing sunspot cycle. A bivariate fit employing both ΔR_{GPV} and *Rmax* does not significantly improve the estimate for ΔR_{GNV}.

Figure 11 depicts scatterplots of *Rmin* for cycle *n*+1 versus *Rmax* for cycle *n* (left panel), and *Rmin* for cycle *n*+1 versus ΔR_{GNV} for cycle *n*. Of the two, only the first is marginally statistically significant. Because *Rmax* for cycle 23 (119.5 denoted by the small downward pointing arrow along the *x*-axis) was above the median for cycles 12–22 (thin vertical line), the indication is that *Rmin* for cycle 24 will lie above the median for *Rmin* (thin horizontal line) in the upper right quadrant. On the basis of the inferred regression, cycle 24 *Rmin* will probably measure about 7.6±3.4. Similarly, on the basis of cycle 23 ΔR_{GNV} (–40.5 denoted by the small downward pointing arrow along the *x*-axis), cycle 24 *Rmin* is expected to lie within the upper left quadrant, having a value of about 7.9±3.5. A bivariate fit employing both parameters does not significantly improve the estimate of *Rmin* for cycle *n*+1.

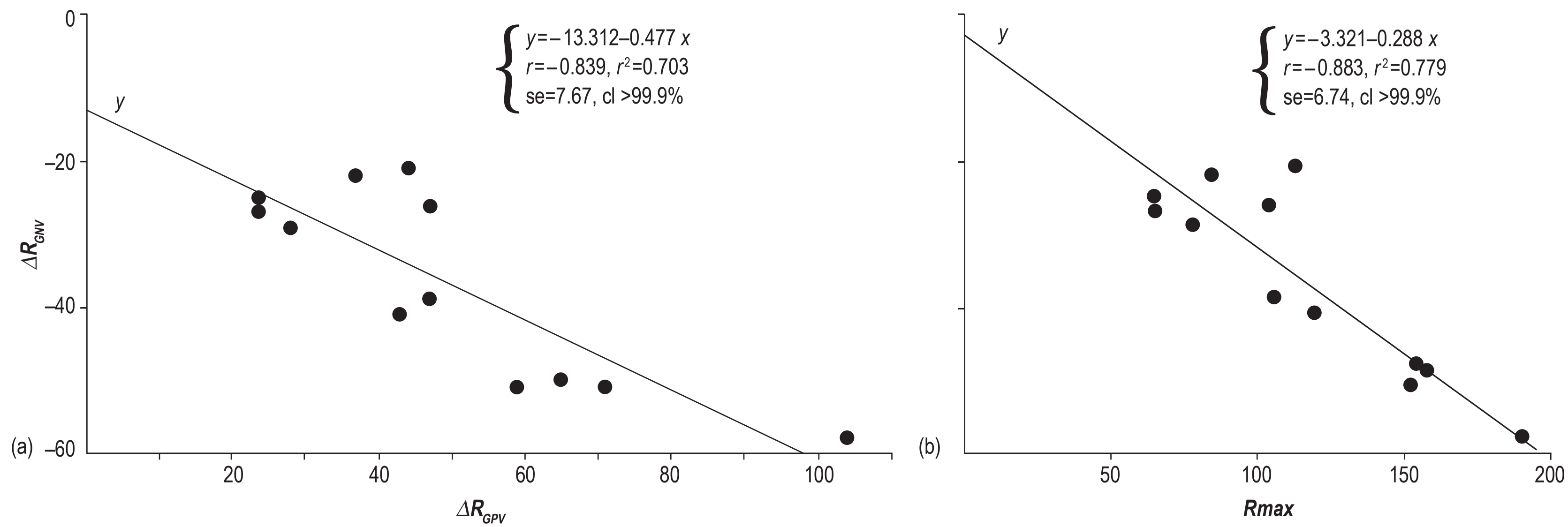

Figure 10. Scatterplots of (a) ΔR_{GNV} versus ΔR_{GPV} and (b) ΔR_{GNV} versus $Rmax$.

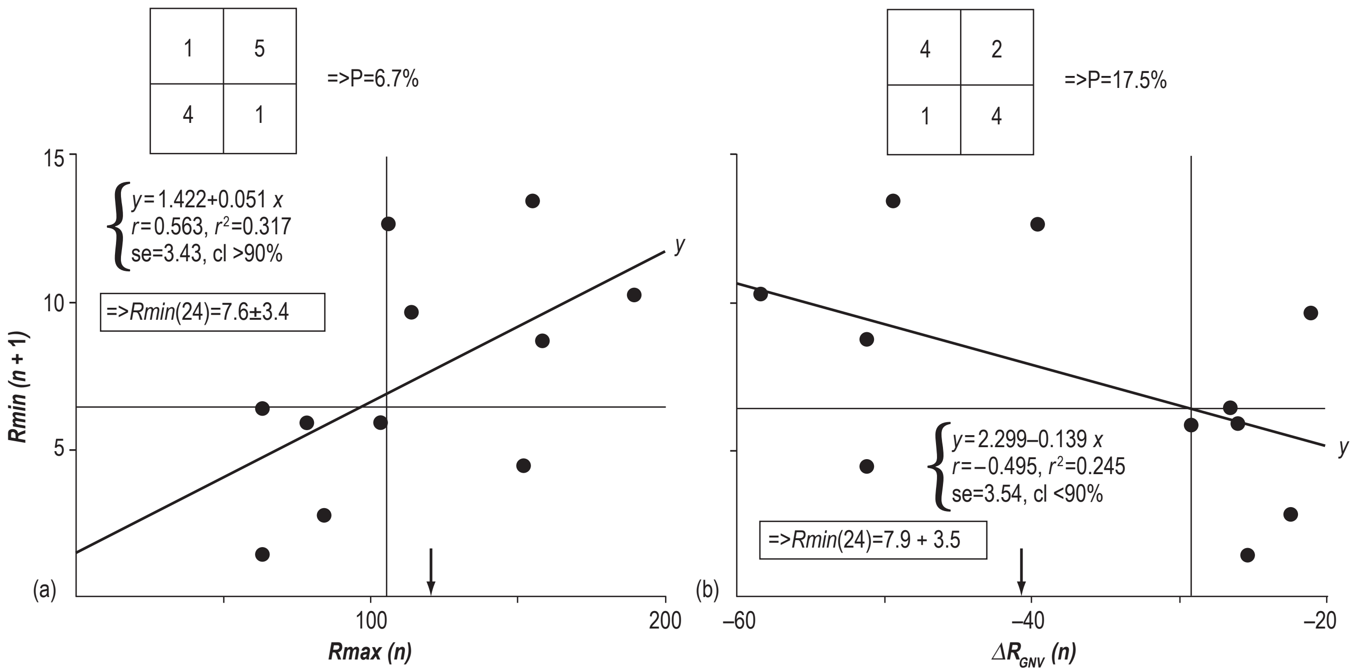

Figure 11. Scatterplots of (a) *Rmin* (*n*+1) versus *Rmax* (*n*) and (b) *Rmin* (*n*+1) versus ΔR_{GNV} (*n*).

Figure 12 shows scatterplots of $SLOPE_{DES}$ versus $SLOPE_{ASC}$ (left panel) and $SLOPE_{DES}$ versus *Rmax* (right panel). Both plots reveal strong linear negative correlation between the parameters. $SLOPE_{DES}$ provides a simple way to estimate the *Rmin* year of occurrence for the following cycle. For example, cycle 23 had $SLOPE_{ASC}$=(119.5–8.6)/4=27.73, as denoted by the small downward pointing arrow along the *x*-axis. Using this value, one estimates cycle 23 $SLOPE_{DES}$ to be about –14.67±1.97. Also, using cycle 23 *Rmax*, 119.5 (denoted by the small downward pointing arrow along the *x*-axis), one estimates $SLOPE_{DES}$ to be about –16.27±1.92. Presuming cycle 24 *Rmin* will measure about 9.8±3.2, the average for cycles 18–23, one finds that cycle 24 *Rmin* should follow cycle 23 *Rmax* occurrence by about 7 yr, indicating cycle minimum in the year 2007. For cycle 24 minimum to occur in 2006, cycle 23 $SLOPE_{DES}$ must measure at least –17 (or more negative) in value.

Strictly speaking, one cannot measure $SLOPE_{DES}$ until after the minimum for the following cycle has occurred. However, in addition to using $SLOPE_{ASC}$ and *Rmax* to estimate the value of $SLOPE_{DES}$, another simple way for estimating $SLOPE_{DES}$ is based on the evolving values during the decline of the cycle.

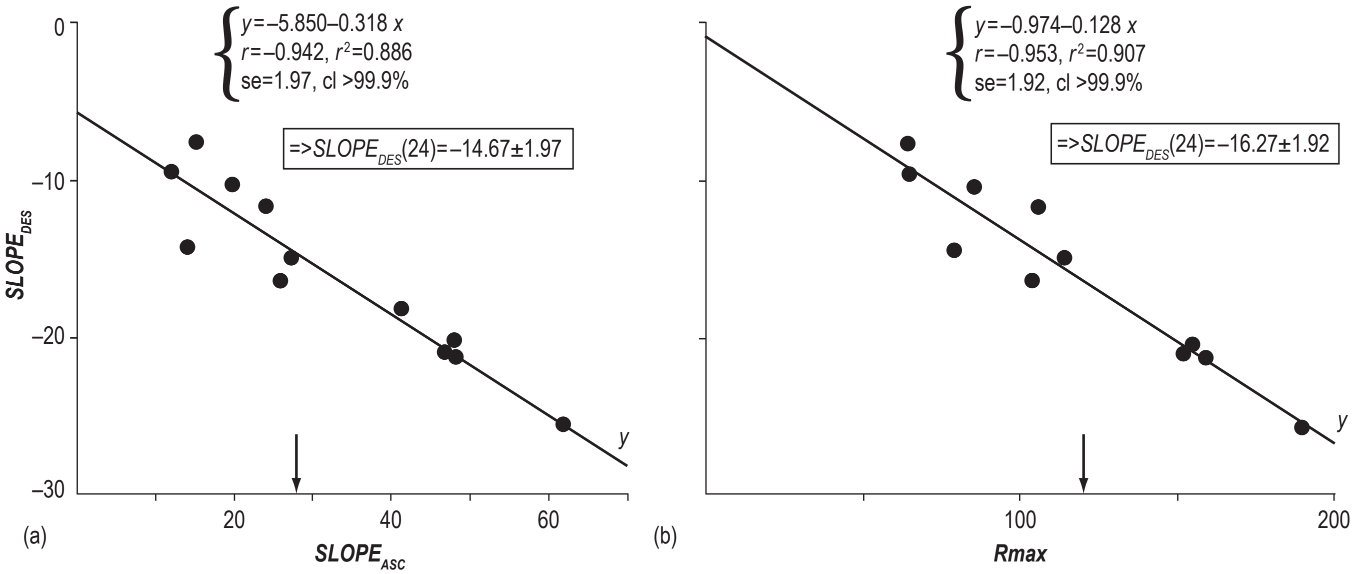

Figure 12. Scatterplots of (a) $SLOPE_{DES}$ versus $SLOPE_{ASC}$ and (b) $SLOPE_{DES}$ versus *Rmax*.

Figure 13 displays the scatterplot of $SLOPE_{DES}$ versus $SLOPE_{DES}$ (*GNV*), where $SLOPE_{DES}$ (*GNV*) is the greatest negative value of the evolving slope during the sunspot cycle decline. As an example, cycle 23 *Rmax* measured 119.5 in the year 2000. For 2001–2005, *R* measured 110.9, 104.1, 63.6, 40.4, and 29.9, respectively. The evolving $SLOPE_{DES}$ has values of –8.60 (the difference of 119.5–110.9), –7.70 (the difference of (119.5–104.1)/2, −18.63 (the difference of (119.5–63.6)/3, −19.78 (the difference of (119.5–40.4)/4, and –17.92 (the difference of (119.5–29.9)/5. The greatest negative value of the evolving $SLOPE_{DES}$ is –19.78, shown in the plot as the small downward pointing arrow along the *x*-axis.

While for the general scatterplot, one infers a strong linear positive correlation between the parameters at <0.1-percent level of significance, having the form y=–0.393+0.773x, a correlation coefficient of r=0.959 and an se of 1.76, a more interesting result is that the cycles appear to be distributed along two different regression lines—cycles 12–16 along y_U and cycles 17–22 along y_L. As stated earlier, cycle 23 $SLOPE_{DES}$(*GNV*) equals −19.78, hence, cycle 23 $SLOPE_{DES}$ will either be equal to −13.87±0.4, based on the y_U fit, or −17.36±1.03, based on the y_L fit. Ignoring the cyclic split and using the general regression, cycle 23 $SLOPE_{DES}$ equals −15.68±1.76. If cycle 23 continues the trend characterized by y_L for cycles 17–22, then cycle 24 *Rmin* occurrence will be in 2006; however, if cycle 23 reverts to the cycle 12–16 trend, then cycle 24 *Rmin* occurrence will be delayed until 2007.

It should be noted that the sunspot minimum year is closely related to the peak in the number of reported spotless days during that year. Figure 14 shows the envelope (lines 1 and 4) and means of cycles 12–16 (line 2) and 17–23 (line 3) of the number of spotless days (top panel) and sunspot number (bottom panel) relative to *Rmin* occurrence. Plainly, as one approaches sunspot minimum, the sunspot number decreases and the number of spotless days (NSD) increase. Table 1 gives the *NSD* and *R* relative to *E(Rmin*) for the elapsed time of 3 yr before cycle minimum to 2 yr after sunspot minimum.

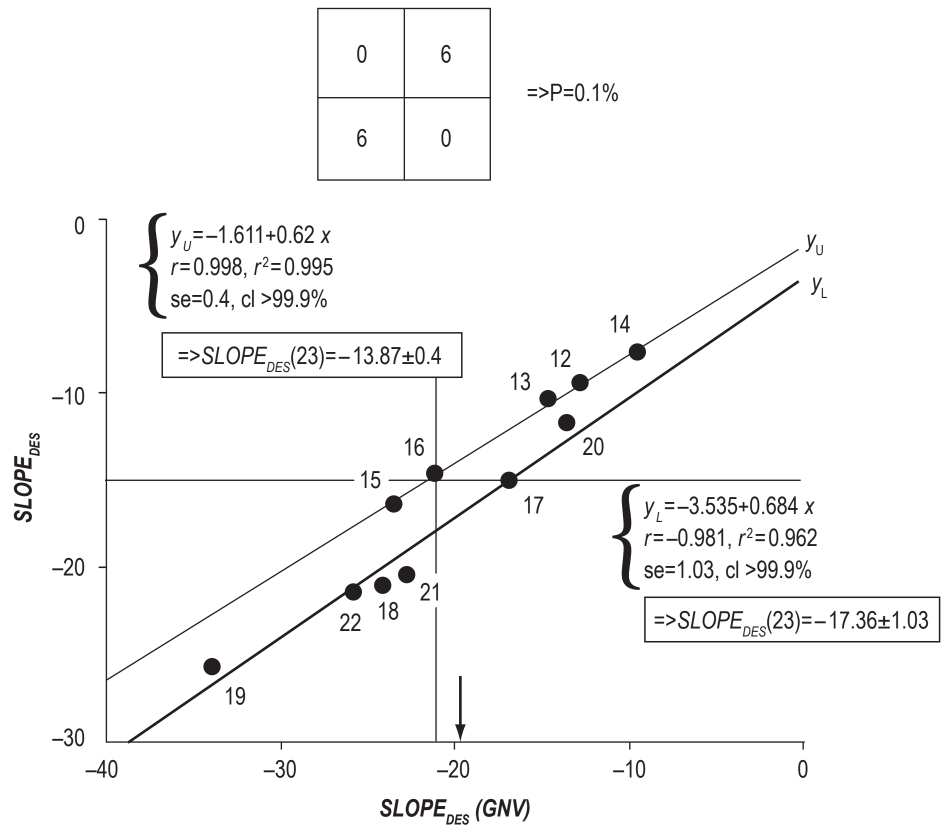

Figure 13. Scatterplot of $SLOPE_{DES}$ versus $SLOPE_{DES}(GNV)$.

Cycle 23 experienced its first spotless days during its decline in the year 2004. The total number of spotless days in 2004 numbered three and *R* measured 40.4. For 2005, the number of spotless days numbered 13 and *R* measured 29.9. Now, in 2006 (through February) there have been 17 spotless days and *R* has averaged only 10.5. Such values are suggestive that, for cycle 24, the sunspot minimum year will be either 2006, especially if cycle 24 has an unusually high *Rmin* value and is a robust cycle, or 2007.

Figure 15 displays the cyclic variation of the number of spotless days during the sunspot minimum year for cycles 12–23. A strong downward decrease is noticeable in the number of spotless days, which is statistically significant at the 0.5-percent level of significance. Presuming the validity of the inferred regression and extrapolating it to cycle 24 suggests that cycle 24 will have 107±48 spotless days in the sunspot minimum year. Cycles 18–23 have averaged 152±50 spotless days, significantly less than the 255±44 average of cycles 12–17.

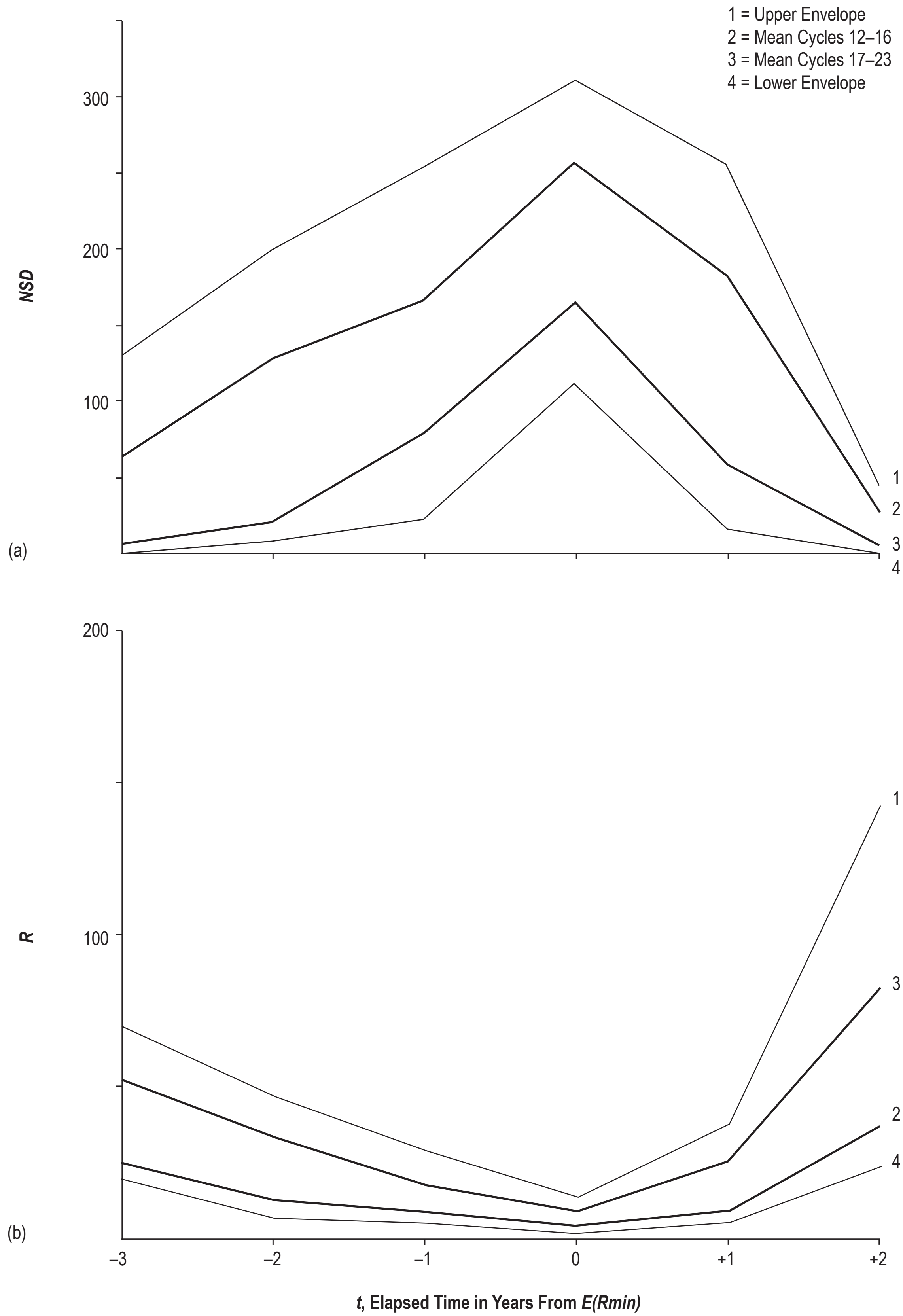

Figure 14. Scatterplots of (a) *NSD* versus *t* and (b) *R* versus *t*.

Table 1. Comparison of *R* and *NSD* values relative to *E(Rmin)*.

Cycle	–3		–2		–1		0		+1		+2	
	R	*NSD*	*R*	*NSD*	*R*	*NSD*	*R*	*NSD*	*R*	*NSD*	*R*	*NSD*
12	17.0	130	11.3	190	12.4	139	3.4	280	6.0	217	32.2	32
13	25.4	62	13.1	104	6.8	150	6.3	212	7.1	171	35.6	24
14	26.7	39	12.1	104	9.5	158	2.7	287	5.1	257	24.4	45
15	18.6	75	5.7	200	3.6	254	1.4	311	9.6	153	47.4	12
16	37.6	7	26.1	46	14.2	134	5.8	200	16.7	116	44.3	27
17	35.7	3	21.2	42	11.1	108	5.7	240	8.7	154	36.0	19
18	47.5	5	30.6	24	16.3	64	9.6	159	33.1	16	92.5	0
19	69.4	0	31.4	23	13.9	131	4.4	241	38.0	48	141.7	0
20	53.9	6	37.6	9	27.9	21	10.2	111	15.1	67	46.9	8
21	38.2	27	34.4	20	15.5	95	12.6	105	27.5	25	92.7	0
22	66.6	4	45.9	13	17.9	82	13.4	129	29.2	44	100.0	0
23	54.7	0	29.4	19	17.5	59	8.6	165	21.5	60	64.2	3

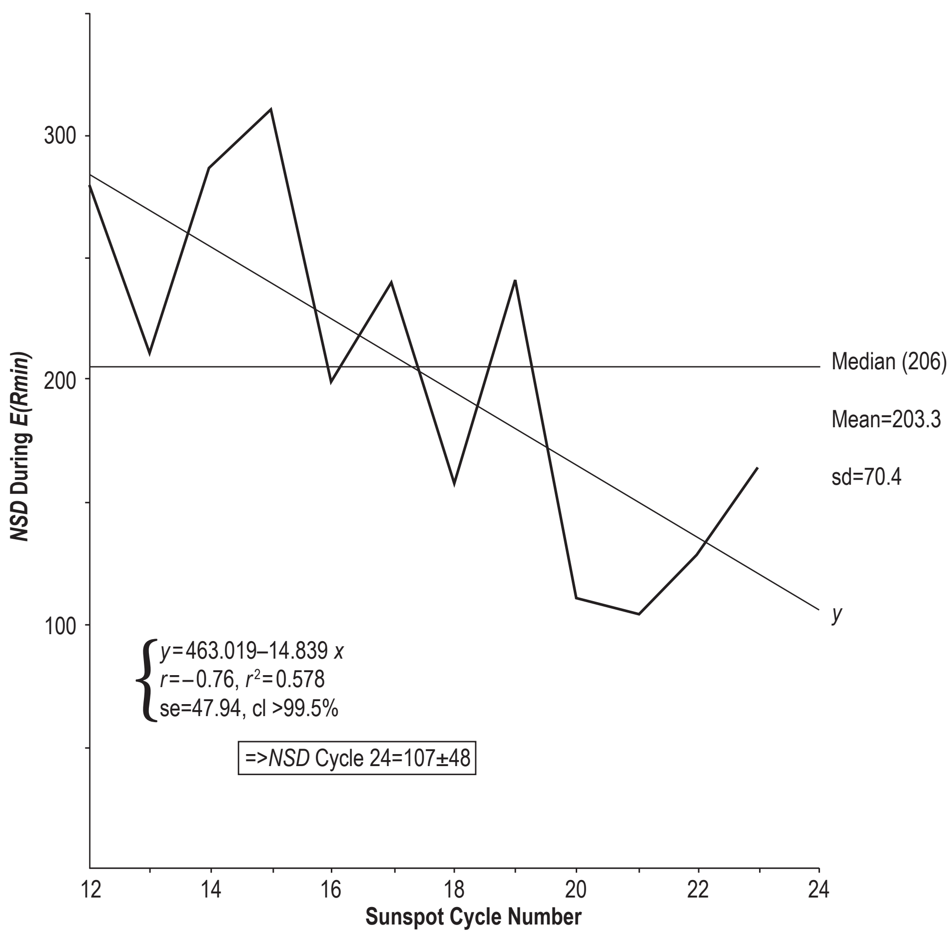

Figure 15. Cyclic variation of the number of spotless days (NSD) during the sunspot minimum year (E(Rmin)) for cycles 12–23.

Figure 16 depicts the scatterplot of *Rmin* versus *NSD(E(Rmin))*, the latter term meaning the number of spotless days during the sunspot minimum year. The inferred regression is statistically significant at the 0.1-percent level of significance. If cycle 24 has *NSD((E(Rmin))*=107±48, then cycle 24 *Rmin* would be expected to be about 12.1±2.5. Such a value, when applied using the y_L regression (fig.13) suggests an expected $SLOPE_{DES}$ for cycle 23 that implies cycle 24 sunspot minimum year to be 2006.

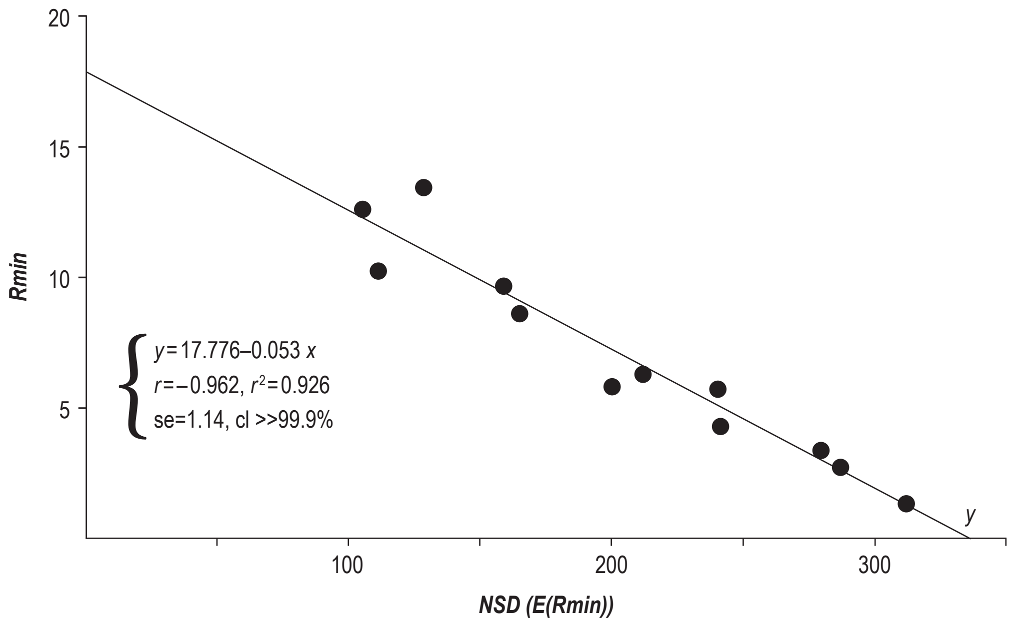

Figure 16. Scatterplot of *Rmin* versus *NSD(E(Rmin))*.

Table 2 provides a summary of the values and times of occurrences for the various parameters discussed in this section. Temporal parameters (*ASC* and *PER*) are expressed in years, and *t* and *T* refer, respectively, to the elapsed time in years from the epochs of sunspot minimum (*E(Rmin)*) and sunspot maximum (*E(Rmax)*).

Table 2. Selected solar cycle parametric values and times of occurrence based on annual averages of sunspot number.

Cycle	Rmin	E(Rmin)	Rmax	ASC	PER	ΔR_{GPV}	t (E(ΔR_{GPV}))	ΔR_{GNV}	t (E(ΔR_{GNV}))	$SLOPE_{ASC}$	$SLOPE_{DES}$	$SLOPE_{DES}$(GNV)	T (E($Slope_{DES}$(GNV)))	NSD (E(Rmin))
12	3.4	1878	63.6	5	11	22.1	3	–26.6	8	12.04	–9.55	–12.73	3	280
13	6.3	1889	85.1	4	12	37.3	3	–22.2	7	19.70	–10.30	–14.73	4	212
14	2.7	1901	63.5	4	12	21.5	4	–25.3	9	15.20	–7.76	–9.63	6	287
15	1.4	1913	103.9	4	10	46.8	4	–26.0	7	25.63	–16.35	–23.30	1	311
16	5.8	1923	77.8	5	10	27.6	2	–29.2	7	14.40	–14.42	–21.05	2	200
17	5.7	1933	114.4	4	11	43.7	3	–21.0	7	27.18	–14.97	–16.76	5	240
18	9.6	1944	151.5	3	10	59.4	2	–51.2	6	47.30	–21.01	–24.02	5	159
19	4.4	1954	189.9	3	10	103.7	2	–58.4	7	61.83	–25.67	–34.00	4	241
20	10.2	1964	105.9	4	12	46.8	3	–38.9	7	23.93	–11.66	–13.54	5	111
21	12.6	1976	155.3	3	10	65.2	2	–49.7	7	47.57	–20.27	–22.90	6	105
22	13.4	1986	157.8	3	10	70.8	2	–51.3	6	48.13	–21.31	–25.78	4	129
23	8.6	1996	119.5	4	–	42.7	2	–40.5	7	27.73	–	–19.78	4	165

3. CONCLUSION

The preceding sections have shown that cycles of late have been rather robust in comparison to earlier cycles in the span of cycles 12 through present, the most reliably known sunspot cycles (corresponding to the interval 1878 to present). In particular, five of the past six cycles have had minimum (*Rmin*) and maximum (*Rmax*) amplitudes that are above both the yearly median (6.1 and 110.2, respectively) and mean (7.0 and 115.7) values. Cycle 23, the current ongoing sunspot cycle, ranks fifth in size in terms of its observed *Rmin* and *Rmax*. Comparison of its yearly sunspot number averages against the mean of cycles 12–22 strongly suggests that onset for cycle 24 will likely occur in year 10 of the sunspot cycle from sunspot minimum occurrence, corresponding to the year 2006. However, using *Rmax* occurrence as the epoch of comparison, it is difficult to strictly determine the onset year for cycle 24, being either year six (2006) or year seven (2007), following sunspot maximum amplitude. For nearly its entire life, cycle 23 behavior, in terms of yearly averages of sunspot number, seems to be more like the mean of 10-yr length sunspot cycles, which, if true, indicates that onset for cycle 24 will be 2006.

Various techniques were examined to determine the predictive capabilities regarding *Rmax*, ΔR_{GNV}, $SLOPE_{DES}$, *Rmin*, and *NSD*. Very strong positive correlations are found to exist between *Rmax* and ΔR_{GPV} and *Rmax* and both ΔR_{GPV} and *Rmin* (a bivariate fit). Likewise, very strong positive correlations are found between *Rmax* and $SLOPE_{ASC}$ and inferred growth rates after one, two, and three years. ΔR_{GNV} is found to strongly and negatively correlate against ΔR_{GPV} and against *Rmax*. Also, $SLOPE_{DES}$ is found to strongly and negatively correlate against $SLOPE_{ASC}$ and *Rmax*. A rather interesting finding seems to exist for $SLOPE_{DES}$ when compared against $SLOPE_{DES}(GNV)$, which is the greatest negative value of the evolving slope during the declining phase of the sunspot cycle. Namely, cycles 12–16 appear to prefer a regression line that differs from the preferred regression line for cycles 17–22. If cycle 23 $SLOPE_{DES}$ is similar to those of recent cycles 17–22, then onset for cycle 24 will occur in 2006; on the other hand, if cycle 23 $SLOPE_{DES}$ is similar to those of earlier cycles 12–16, then onset for cycle 24 will be delayed until 2007. It should be noted that the general distribution of $SLOPE_{DES}$ versus $SLOPE_{DES}(GNV)$, ignoring the apparent division of cycles into two distinct groupings, has a strong positive correlation at the 0.1-percent level of significance. Finally, the number of spotless days has been increasing since 2004, this being a sign of the approach of onset for cycle 24. The number of spotless days is at maximum during the sunspot minimum year. Because five of the past six cycles have had *NSD* <206 days (the median) and because there appears to exist a strong negative correlation (at the 0.5-percent level of significance) between *NSD* at *E(Rmin)* against sunspot cycle number, one predicts cycle 24 to have 107±48 days (the mean and standard deviation of *NSD* for cycles 18–22 is 152±50). This suggests that *NSD* at *E(Rmin)* for cycle 24 will be <206 days and that *Rmin* will be >6.1, indicating further that cycle 24 should be expected to be another robust cycle, probably of larger than average maximum amplitude (*Rmax*), shorter than average ascent duration (*ASC*) and shorter than average length (*PER*).[25–27]

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