GROUP SUNSPOT NUMBERS: SUNSPOT CYCLE CHARACTERISTICS

DAVID H. HATHAWAY, ROBERT M. WILSON and EDWIN J. REICHMANN
NASA/Marshall Space Flight Center, Huntsville, AL 35812, U.S.A.

(Received 26 February 2002; accepted 17 April 2002)

Abstract. We examine the 'Group' sunspot numbers constructed by Hoyt and Schatten to determine their utility in characterizing the solar activity cycle. We compare smoothed monthly Group sunspot numbers to Zürich (International) sunspot numbers, 10.7-cm radio flux, and total sunspot area. We find that the Zürich numbers follow the 10.7-cm radio flux and total sunspot area measurements only slightly better than the Group numbers. We examine several significant characteristics of the sunspot cycle using both Group numbers and Zürich numbers. We find that the 'Waldmeier Effect' – the anti-correlation between cycle amplitude and the elapsed time between minimum and maximum of a cycle – is much more apparent in the Zürich numbers. The 'Amplitude–Period Effect' – the anti-correlation between cycle amplitude and the length of the previous cycle from minimum to minimum – is also much more apparent in the Zürich numbers. The 'Amplitude–Minimum Effect' – the correlation between cycle amplitude and the activity level at the previous (onset) minimum is equally apparent in both the Zürich numbers and the Group numbers. The 'Even–Odd Effect' – in which odd-numbered cycles are larger than their even-numbered precursors – is somewhat stronger in the Group numbers but with a tighter relationship in the Zürich numbers. The 'Secular Trend' – the increase in cycle amplitudes since the Maunder Minimum – is much stronger in Group numbers. After removing this trend we find little evidence for multi-cycle periodicities like the 80-year Gleissberg cycle or the two- and three-cycle periodicities. We also find little evidence for a correlation between the amplitude of a cycle and its period or for a bimodal distribution of cycle periods. We conclude that the Group numbers are most useful for extending the sunspot cycle data further back in time and thereby adding more cycles and improving the statistics. However, the Zürich numbers are slightly more useful for characterizing the on-going levels of solar activity.

1. Introduction

The single most important index of solar activity has been the Zürich or Wolf sunspot number (now referred to as the International sunspot number). This index, first introduced in 1848 by Rudolf Wolf, provides the longest continuous measure of solar activity over time (Kiepenheuer, 1953; Waldmeier, 1961; McKinnon, 1987). Initially it was provided and maintained by the Swiss Federal Observatory in Zürich, Switzerland. Today the index is provided and maintained by the Sunspot Index Data Center in Brussels, Belgium, where monthly updates are available online.

The Zürich number has proven invaluable in studies of long-term changes in solar activity, especially as related to terrestrial climate (e.g., Eddy, 1980; Hoyt and Schatten, 1997; Wilson, 1998a). However, certain deficiencies have been recognized in the record for specific intervals of time. For example, during the earliest

Solar Physics **211:** 357–370, 2002.

portion of the record the shapes and amplitudes of some of the cycles appear highly questionable (e.g., Baiada and Merighi, 1982; Hoyt, Schatten, and Nesmes-Ribes, 1994; Hoyt and Schatten, 1995a–d; Wilson, 1998b).

In a recent series of papers Hoyt and Schatten (1995a–d, 1998a, b) describe their fruitful efforts at uncovering early historical records of sunspot observations. From this work they construct a 'Group' sunspot number (Hoyt and Schatten, 1998a) that is designed to be a consistent replacement or alternative to the Zürich sunspot number. As the name implies, this index is based purely on the number of sunspot groups identified on the Sun. The number is normalized with a multiplicative factor to produce an activity index that closely mimics the Zürich number. By using this index coupled with the early solar observations they uncovered, Hoyt and Schatten provide a more complete record of sunspot numbers dating back to Galileo's observations in 1610.

Here we examine the Group sunspot number and its characteristics relative to other datasets to assess its value as an indicator of solar activity and as a tool for understanding the solar activity cycle. Because the Group sunspot number is based upon a much larger set of observations than the Zürich sunspot number, especially during the early years, it is expected to provide a better description of overall solar activity.

2. Datasets and Data Preparation

In producing the Zürich sunspot number index Wolf recognized the difficulty in identifying individual spots and the importance of sunspot groups. His 'relative' sunspot number index, R_Z, is given by

$$R_z = k(10g + n), \tag{1}$$

where k is a correction factor for the observer, g is the number of identified sunspot groups, and n is the number of individual sunspots. In spite of the apparent arbitrary nature of this formula, it has been found to correlate extremely well with other, more physical measures of solar activity such as sunspot area, 10.7-cm radio flux, X-ray flare frequency, and magnetic flux. Monthly values for R_Z are available from 1749 onward but many of the values prior to 1849 are based on incomplete or missing data.

Hoyt and Schatten (1998a) introduced the Group sunspot number, R_G, as an alternative to the Zürich sunspot number. It uses only the number of sunspot groups but is normalized to make it agree with the Zürich numbers during the years from 1874 to 1976 when the Royal Greenwich Observatory provided daily reports on the number and characteristics of sunspot groups. With this normalization the Group sunspot number is given by

$$R_G = \frac{1}{N}\sum_{i=1}^{N} k_i \, 12.08 g_i, \tag{2}$$

where N is the number of observers, k_i is the correction factor for observer i, and g_i is the number of sunspot groups reported by observer i. Through the diligent efforts of Hoyt and Schatten this dataset is more complete than the Zürich dataset. Monthly values of R_G are available from 1610 albeit with missing values up to 1795. Their dataset ends in 1995 but for this study we extend it to 2002 using the number of groups reported daily by the US National Oceanic and Atmospheric Administration.

In addition to the Zürich and Group sunspot numbers we also consider the 10.7-cm radio flux and sunspot area measurements as alternative solar activity indicators that cover several cycles. The 10.7-cm radio flux has been measured on a nearly daily basis since early 1947. Monthly values are available from February 1947 to the present – covering nearly 5 cycles. Although the radio receivers were moved from Ottawa, Ontario to Penticton, British Columbia in 1990, this dataset remains very uniform and is often preferred as an indicator of solar activity. Sunspot area measurements are available from the Royal Greenwich Observatory from 1874 to 1976. We augment these data with data from the NOAA/USAF SOON network as reported in the *Region Reports* at the NOAA web site. An inter-comparison between the Greenwich, NOAA, and overlapping Mt. Wilson data (Howard, Gilman, and Gilman, 1984) indicates that the NOAA sunspot areas need to be increased by 40% to match the earlier Greenwich data. Including this 'correction' gives a reasonably uniform dataset extending from 1874 to the present.

These records of solar activity all display the inherent noisiness of the solar cycle. Daily and even monthly values vary widely. The underlying characteristics of the solar cycle are more evident when the data is smoothed by more than just monthly averages. The usual smoothing is the 13-month running mean which is centered on a given month and averages over that month and the six months before and after with half weights given to the monthly values on either end. Unfortunately this temporal filter allows many high frequencies to pass which in turn can influence the statistics related to the solar cycle. For example, high-frequency peaks occurring near solar minimum or maximum can give ambiguous results for the times and values of these extrema. Gaussian shaped filters are well known to have cleaner frequency responses. In order to smooth monthly values to see the solar cycle behavior we prefer a 24-month Gaussian average with relative weights given by

$$W(\Delta t) = \exp[-2\Delta t^2/b^2] - \exp[-2](3 - 2\Delta t^2/b^2), \tag{3}$$

where Δt is the number of months from the center and b (= 24 months) is the full width at half maximum. Both the filter weight and its first derivative vanish at $\pm b$ months. In Figure 1 we compare the frequency response of this 24-month Gaussian to that of the 13-month running mean. The 13-month running mean passes significant signal at frequencies much greater than a cycle-per-year while the 24-month Gaussian suppresses all of these higher frequencies as well as those with frequencies as low as one cycle in two years.

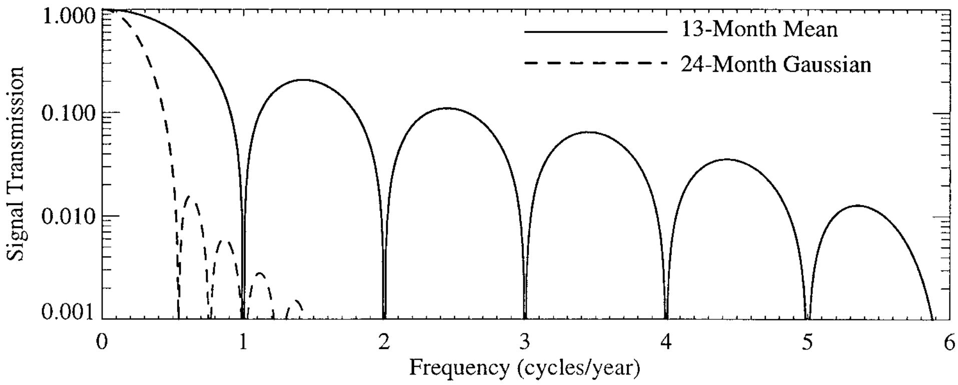

Figure 1. Signal transmission factors for the 13-month running mean (*solid line*) and the 24-month Gaussian average (*dashed line*) as functions of signal frequency. The 13-month running mean passes 20% of the signal with frequencies near 1.5 cycles per year and about 4% of the signal with frequencies near 4.5 cycles per year. The 24-month Gaussian passes less than 0.3% of all signals with frequencies greater than 1 cycle per year.

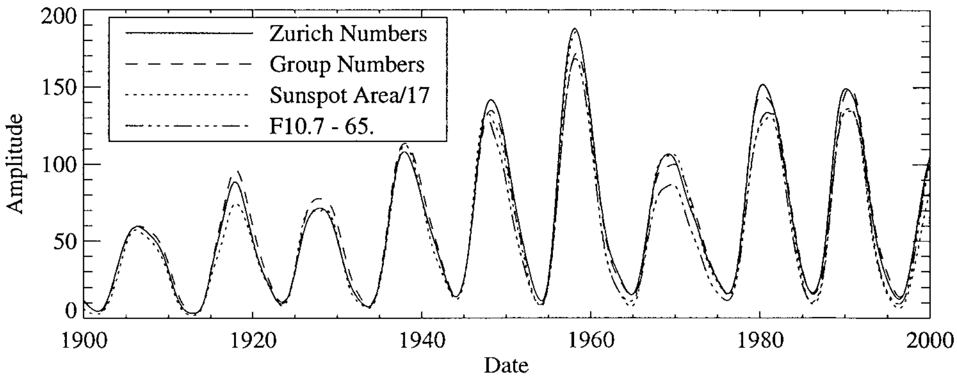

Figure 2. Zürich sunspot numbers (*solid line*), Group sunspot numbers (*dashed line*), total sunspot area (*dotted line*), and 10.7-cm flux (*dash-dotted line*) smoothed with the 24-month Gaussian filter for the last century. A strong correlation between these indices is evident in how closely they follow each other. The utility of the 24-month Gaussian filter for solar cycle studies can be seen by the lack of high-frequency oscillations and the production single-peaked cycle maxima and minima.

The monthly values for R_Z, R_G, sunspot area, and 10.7-cm radio flux (smoothed with the 24-month Gaussian) are shown in Figure 2 for the last century. This figure shows how this temporal filter retains the basic cycle shape with uniquely defined maxima and minima. A 12-month Gaussian (while attractive because of its shorter length) passes enough signal with periods near 18 to 24-months to produce double peaked cycles with non-unique maxima.

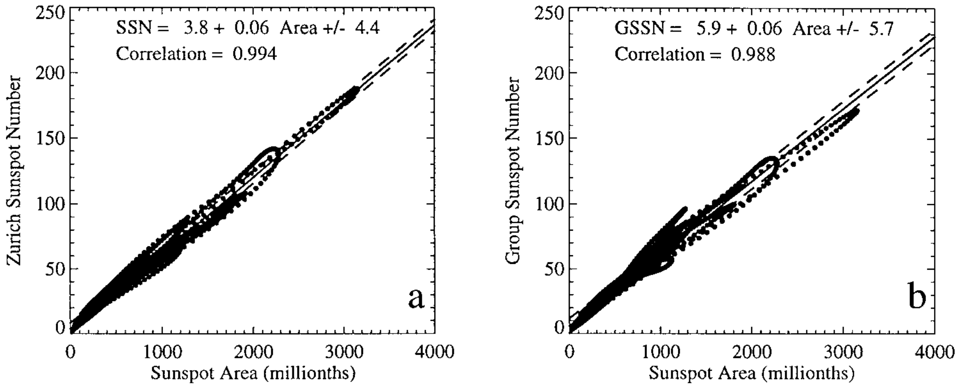

Figure 3. The relationship between R_Z and sunspot area is shown in (a). The relationship between R_G and sunspot area is shown in (b). R_Z has a slightly stronger correlation with sunspot area than does R_G and a slightly tighter fit to a linear relationship between the two.

3. Comparison of Sunspot Cycle Characteristics

The four indices plotted in Figure 2 track each other quite closely through time but nonetheless display some differences. We compare R_Z and R_G to both sunspot area and 10.7-cm radio flux. We find that the correlation between R_Z and sunspot area is 0.994, only slightly better than the correlation between R_G and sunspot area (0.988). A linear relationship between R_Z and sunspot area also has a smaller standard deviation (4.4) than the linear relationship between R_G and sunspot area (standard deviation of 5.7). These relationships are shown in Figure 3 where the tighter fit between R_Z and sunspot area is evident by eye as well. The same situation is found when we compare R_Z and R_G to the more recent 10.7-cm radio flux data. We find that the correlation between R_Z and radio flux is 0.997 while the correlation between R_G and radio flux is 0.994. The linear relationship between R_Z and radio flux has a smaller standard deviation of 4.0 while the linear relationship between R_G and radio flux has a standard deviation of 5.1. These relationships are shown in Figure 4 where the tighter fit between R_Z and radio flux is again evident.

Several statistically significant features of the sunspot cycle have been found in the sunspot number record. (These features are important in helping us to understand the nature of the sunspot cycle and should be reflected in faithful reproductions of the cycle by theoretical models.) We examine several of these features using both of the sunspot number indices. Figure 5 shows the Zürich and the Group sunspot numbers for the last three centuries after smoothing the monthly averages with the 24-month Gaussian filter. The values agree very closely after about 1870. (Hoyt and Schatten normalized the Group sunspot numbers to the Zürich sunspot numbers using data from the time period 1874 to 1976.) Differences can be seen in the earlier cycles where the Group numbers fall significantly below the Zürich numbers. It is these early cycles that are better represented in the Group sunspot numbers as previously indicated by Hoyt and Schatten.

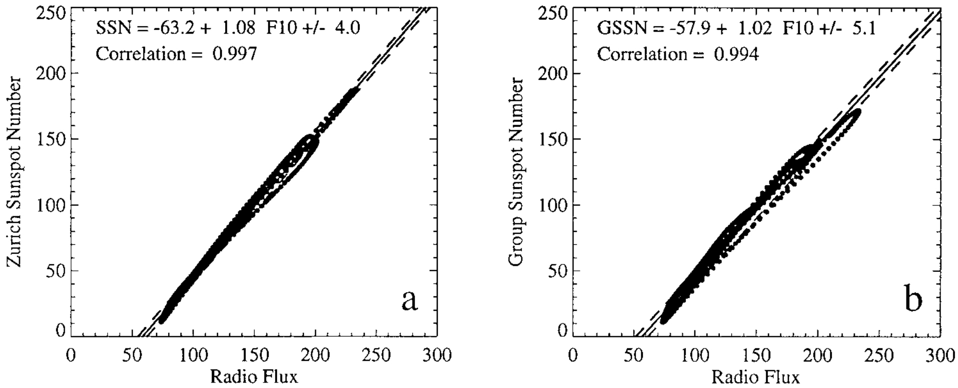

Figure 4. The relationship between R_Z and 10.7-cm radio flux is shown in (a). The relationship between R_G and radio flux is shown in (b). R_Z has a slightly stronger correlation with radio flux than does R_G and a slightly tighter fit to a linear relationship between the two.

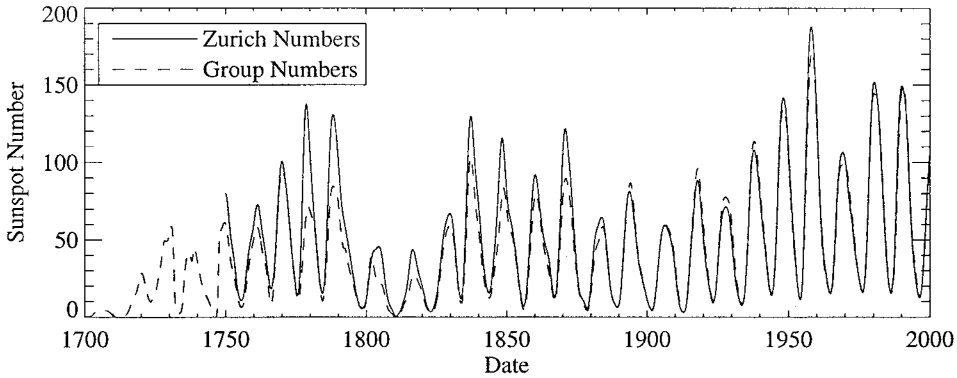

Figure 5. Smoothed (24-month Gaussian) sunspot numbers for the last three centuries. The Zürich (*solid line*) and Group (*dashed line*) sunspot numbers follow each other closely from about 1870 on but deviate from each other significantly in earlier cycles.

The Waldmeier Effect (Waldmeier, 1935, 1939) is seen in the relative timing of the cycle maxima and minima. Stated simply, large-amplitude cycles rise to maximum in less time than small-amplitude cycles. We use the smoothed data as shown in Figure 5 for the complete cycles covered by both datasets (from the minimum in May of 1755 to the minimum in July of 1996). We obtain the month, year, and value for each maximum and minimum directly from this smoothed data. We measure the Waldmeier Effect by comparing the amplitude of the maximum to the rise time for the cycle (the number of months from minimum to maximum). These quantities are plotted against each other in Figure 6 for both the Zürich sunspot numbers and the Group sunspot numbers. We find a much more significant effect using the Zürich values. The correlation coefficient between amplitude and rise time is −0.73 with the Zürich values but only −0.34 with the Group values.

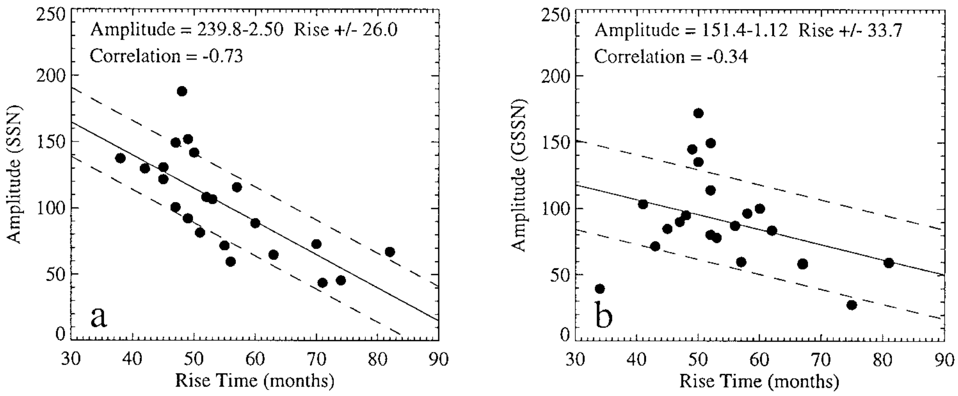

Figure 6. The Waldmeier Effect as seen in the Zürich sunspot numbers (a) and the Group sunspot numbers (b). The Zürich numbers show a much more significant effect with less scatter about a linear relationship between cycle amplitude and rise time.

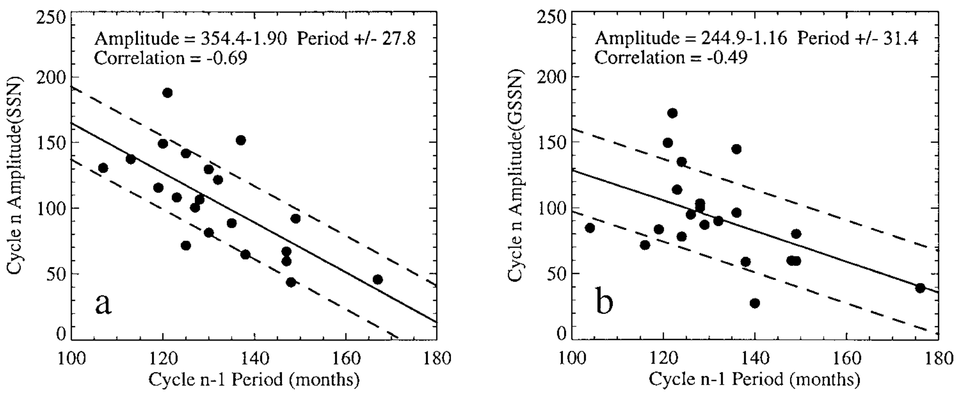

Figure 7. The Amplitude–Period Effect as seen in the Zürich sunspot numbers (a) and the Group sunspot numbers (b). The Zürich numbers show a much more significant effect with less scatter about a linear relationship between cycle amplitude and the period of the preceding cycle.

The linear fits to the data also show less scatter with the Zürich values (a standard deviation of 26.0) than with the Group values (a standard deviation of 33.7).

The Amplitude–Period Effect (Chernosky, 1954; Wilson, Hathaway, and Reichmann, 1998) is seen when the cycle amplitudes are compared to the period of the *preceding* cycle. The amplitudes and periods (time from minimum to minimum) as obtained from the smoothed data are shown in Figure 7 for both the Zürich and Group numbers. We find a much more significant effect with the Zürich numbers. The correlation coefficient between the amplitude and the period of the previous cycle is -0.69 with the Zürich values but only -0.49 with the Group values. The linear fits to the data also show less scatter with the Zürich values (a standard deviation of 27.8) than with the Group values (a standard deviation of 31.4).

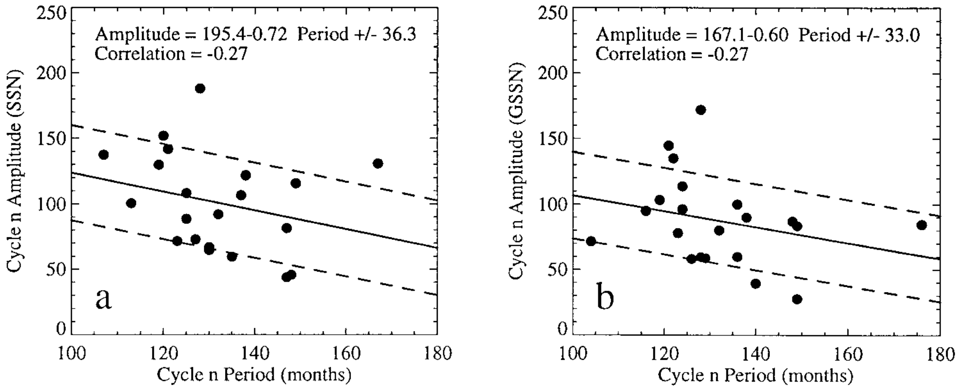

Figure 8. Relationship between cycle amplitude and period for Zürich numbers (a) and Group numbers (b). Both datasets show only a weak correlation between these two quantities.

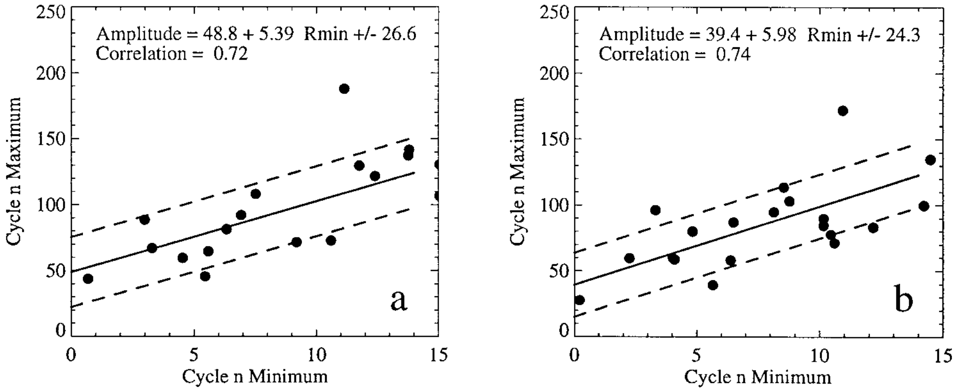

Figure 9. The Amplitude–Minimum Effect as seen in the Zürich sunspot numbers (a) and the Group sunspot numbers (b). Both sets of numbers show a significant effect. The Group numbers show a marginally more significant effect with less scatter about a linear relationship between cycle amplitude and the previous minimum.

An alternative relationship between amplitude and period has also been suggested. Friis-Christensen and Lassen (1991) suggest a relationship between the amplitude (in total irradiance) and the period of the *same* cycle. Baliunas and Soon (1995) suggest a similar relationship from studies of solar-type stars. This relationship is shown in Figure 8 for both the Zürich and the Group numbers. Neither set of data shows a significant relationship. The correlation coefficient between cycle amplitude and period is only -0.27 for both datasets and the linear fits show significant scatter (standard deviations of 36.3 and 33.0).

The Amplitude–Minimum Effect (Wilson, Hathaway, and Reichmann, 1998) is seen when the cycle amplitudes are compared to the minima that precede each cycle. The amplitudes and minima as obtained from the smoothed data are shown in Figure 9 for both the Zürich and Group numbers. We find a significant effect with

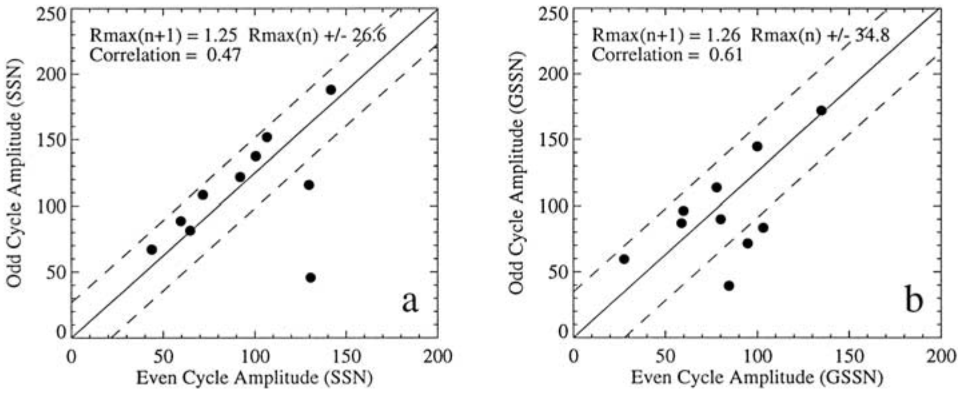

Figure 10. The Even–Odd Effect as seen in the Zürich sunspot numbers (a) and the Group sunspot numbers (b). Both sets of numbers show only a weak effect. The Group numbers show a more significant effect but with more scatter about a linear relationship between odd-cycle amplitude and the previous even-cycle amplitude.

both datasets. The Group numbers have a marginally better correlation coefficient and slightly less scatter about a linear fit. The correlation coefficient between the amplitudes and the minima for the cycles is −0.72 with the Zürich values and −0.74 with the Group values. The linear fits to the data give a standard deviation of 26.6 for the Zürich values and a standard deviation of 24.3 for the Group values.

The Even–Odd (or Gnevyshev) Effect (Gnevyshev and Ohl, 1948; Vitinskii, 1965; Wilson, 1992) is seen when the odd-numbered cycle amplitudes are compared to the amplitudes of the preceding even-numbered cycles. The amplitudes obtained from the smoothed data are shown in Figure 10 for both the Zürich and Group numbers. We find mixed results. The Group numbers have a much better correlation coefficient but with more scatter about a linear fit. The correlation coefficient between the amplitudes and the minima for the cycles is 0.47 with the Zürich values and 0.61 with the Group values. Much of this difference is undoubtedly due to the single outlier in the Zürich data for cycle pair 4/5. The linear fits to the data give a standard deviation of 26.6 for the Zürich values and a standard deviation of 34.8 for the Group values. Note that these data do not include the amplitude of the current cycle (Cycle 23) which apparently will deviate very significantly from this relationship. (As of this writing Cycle 23 had an amplitude of 118.1 with the 24-month Gaussian filter while Cycle 22 had an amplitude of 156.5.)

A Secular Trend (Wilson, 1988) is seen when the cycle amplitudes are compared to the cycle numbers. The amplitudes obtained from the smoothed data are shown in Figure 11 for both the Zürich and Group numbers. The Group numbers have a much better correlation coefficient with less scatter about a linear fit. The correlation coefficient between the amplitudes and the cycle number is only 0.41 with the Zürich values but rises to 0.70 with the Group values. The linear fits to the data give a standard deviation of 34.9 for the Zürich values and a standard

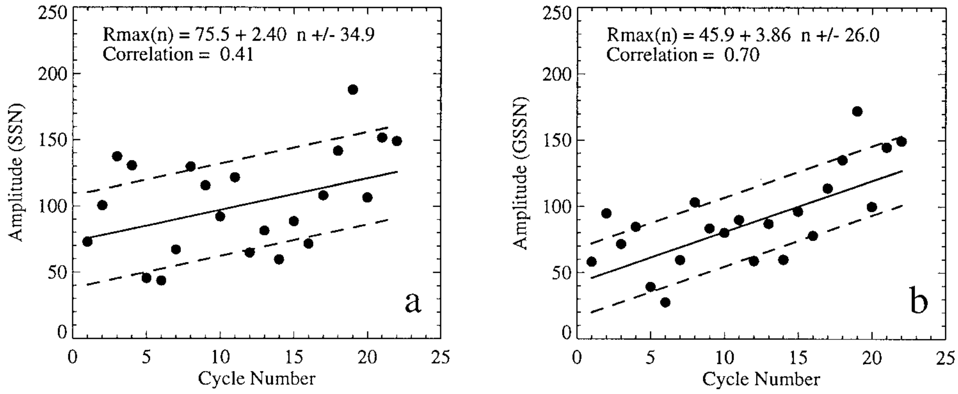

Figure 11. The Secular Trend in cycle amplitudes as seen in the Zürich sunspot numbers (a) and the Group sunspot numbers (b). The Group numbers show a much more significant effect with less scatter about a linear relationship between cycle amplitude and the cycle number.

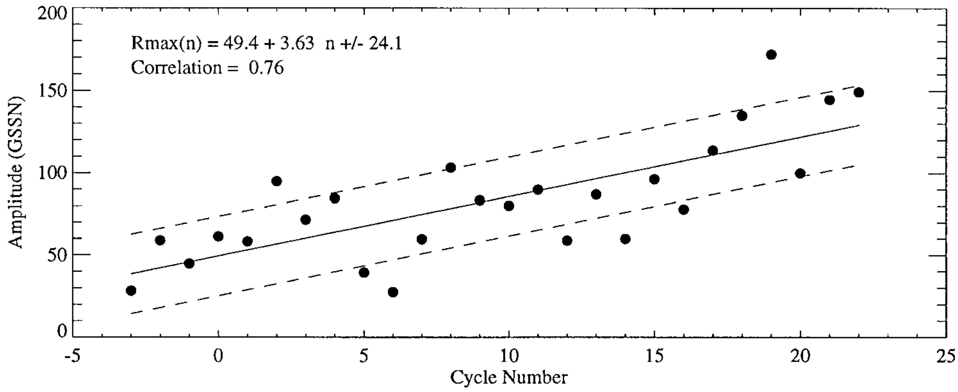

Figure 12. The Secular Trend for the Group sunspot numbers including cycles −3 through 22. The correlation between cycle amplitude and cycle number is stronger when the early cycles are included and the variation about a linear fit is smaller.

deviation of 26.0 for the Group values. The lower Group sunspot numbers for the early cycles clearly produces a strong secular trend since the Maunder Minimum (1645–1715) for that dataset.

The Group sunspot numbers cover 4 early cycles that are not included in the Zürich dataset. Although the observations for the Group numbers are not complete, they nonetheless provide useful information on the long-term behavior of the sunspot cycle going back to the end of the Maunder Minimum in 1715. Figure 12 shows the Secular Trend when these early cycles are included. The correlation between cycle amplitude and cycle number is even stronger and the scatter about the linear fit is smaller.

We examine the residual variations about this linear trend for evidence of periodicities like the 8-cycle (Gleissberg, 1939), 3-cycle (Ahluwalia, 1998), and 2-

cycle (even–odd) variations. The best fit to a several cycle periodicity (7-cycle to 10-cycle periods) is a variation with a period of 9.1-cycles. This fit to a 'Gleissberg Cycle' gives a small reduction in the standard deviation (from 24.1 to 20.1) and has a correlation of only 0.51 with the variations about the secular trend. The best fit to a 3-cycle periodicity gives an insignificant reduction in the standard deviation (from 24.1 to 23.6) and an insignificant correlation coefficient (0.19). Similarly, the best fit to a 2-cycle periodicity reduces the standard deviation from 24.1 to 23.7 and has a correlation coefficient of only 0.18. Thus, these multi-cycle periodicities that have been touted as being significant and having predictive capabilities actually may not be as statistically important as originally believed.

A bimodal distribution in the cycle periods and their behavior has previously been reported. Wilson (1987) found evidence for two distributions of cycle periods – short-period cycles with a mean period of about 120 months and long-period cycles with a mean period of about 140 months. A separation between the two was particularly evident in the more recent cycles (cycles 8 through 21). Rabin, Wilson, and Moore (1986) also suggested a bimodal distribution in which short-period cycles occur as the cycle amplitudes increase and long-period cycles occur as the cycle amplitudes decrease. We examine both datasets for evidence of a bimodal distribution of cycle periods. The individual cycle periods are shown in Figure 13. There is little evidence for a separation between short-period and long-period cycles – even for the more recent cycles in the Zürich data. The distribution of cycle periods is shown in Figure 14. Following Wilson (1987), we bin the cycle periods into bins that are one standard deviation wide centered on the mean period, ± 1 standard deviation, and ± 2 standard deviations to either side of the mean. The resulting distributions are fully consistent normal distributions (shown by the dotted lines in Figure 14). The cycle periods used by Wilson (1987) were based on the Zürich data smoothed with the traditional 13-month running mean. We find that the same data smoothed with the 24-month Gaussian gives slightly different dates for the cycle minima. The gap seen by Wilson between short-period and long-period cycles does not appear when the data are smoothed in this manner. Note, however, that while a bimodal distribution of cycle periods is not suggested in these data, it is true that series of large amplitude cycles have shorter periods and faster rises.

4. Conclusions

We examined the Group sunspot numbers reported by Hoyt and Schatten (1998a, 1998b) and compared the sunspot cycle characteristics they exhibit to those exhibited by the Zürich sunspot numbers. We found that the Zürich numbers have a slightly stronger correlation with both sunspot area and 10.7-cm radio flux and exhibit less scatter about the linear fits. We also found that some cycle characteristics, namely the anti-correlation between cycle amplitude and rise time (the Waldmeier Effect) and the anti-correlation between cycle amplitude and preceding

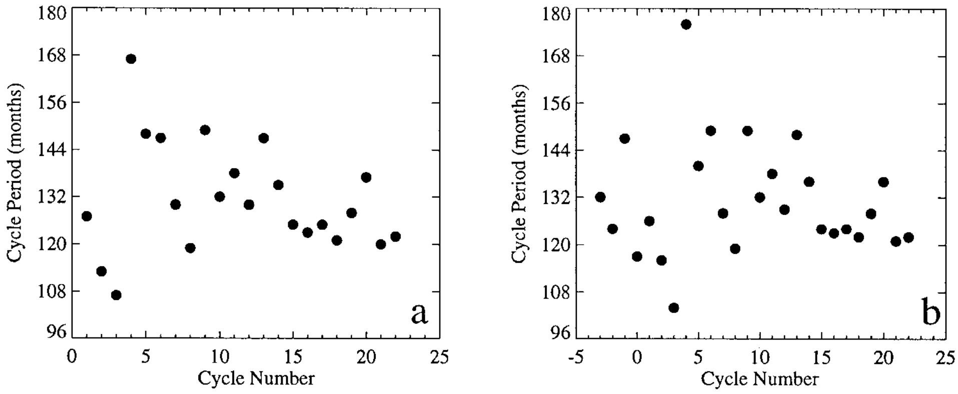

Figure 13. Cycle periods as functions of cycle number for the Zürich data (a) and the Group data (b). There is little evidence for a separation into short-period and long-period cycles.

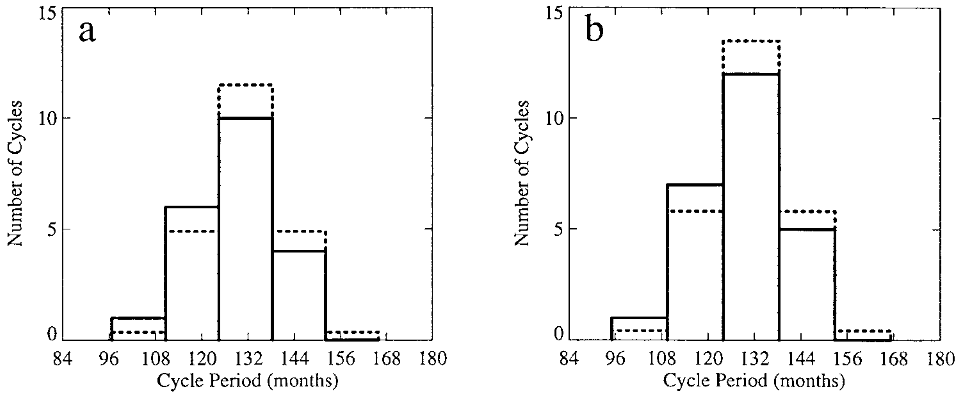

Figure 14. Cycle period distributions for the Zürich data (a) and the Group data (b). The bins are one standard deviation wide centered on the mean period (131.4 months for the Zürich data and 131.2 months for the Group data), ± 1, and ± 2 standard deviations from the mean. Normal distributions are indicated by the *dotted lines*. Both distributions are close to, and consistent with, a single, normal distribution for cycle periods.

cycle period (the Amplitude–Period Effect) were stronger in the Zürich dataset. Other characteristics gave mixed results. The correlation between odd-cycle amplitudes and preceding even-cycle amplitudes (the Even–Odd Effect) was stronger with the Group numbers but showed less scatter about a linear fit with the Zürich numbers. Of course, the fact that an effect is stronger in one index than in the other does not imply that one index is better than the other since the effect itself may be a spurious result from the statistics of small numbers.

The real value of the Group numbers was seen in the long-term behavior. The correlation between cycle amplitude and cycle number (the Secular Trend) was much stronger in the Group dataset. From this we conclude on one hand that the Zürich numbers continue to be valuable in capturing characteristics of the recent cycles that are not quite as well-reflected in the Group numbers. On the other hand, the Group numbers are valuable for capturing the behavior in the earliest cycles that help to reveal long-term behavior.

Some of the sunspot cycle characteristics we examined showed little significance. Little correlation was found between the amplitude of a cycle and its period in either dataset. After removing the long-term secular increase in cycle amplitudes we found no evidence for multi-cycle periodicities with 2- and 3-cycle periods. Even the evidence for the much touted Gleissberg cycle (7- to 8-cycle period) and the Even–Odd Effect was weak. We also did not find support for a bimodal distribution in cycle periods but instead found the distribution to be very close to a normal distribution.

Several of the significant characteristics have their source in a single, simple rule: large-amplitude cycles grow fast. By growing fast they start early and quickly dominate the remnant activity from the previous cycle. This gives the previous cycle a short period (Amplitude–Period Effect) and produces a high level of activity at minimum (Amplitude–Minimum Effect). The rapid growth directly produces the Waldmeier Effect in which large-amplitude cycles reach maximum quicker than small-amplitude cycles. This simple rule also helps to explain the bimodality reported by Rabin, Wilson, and Moore (1986). During times when the amplitudes of the cycles are increasing we would expect short-period cycles and conversely, during times when the cycle amplitudes are decreasing we would expect long-period cycles. This behavior should be reflected in dynamo models for the solar cycle and may help to discriminate between opposing models.

Acknowledgements

This work was supported in part by NASA's Sun–Earth Connection Enterprise through a grant from the Solar and Heliospheric Physics Supporting Research and Technology Program. It was also supported in part by the National Science Foundation under Grant No. PHY99-07949 to the Institute for Theoretical Physics at the University of California in Santa Barbara where DHH was a visitor during the writing of this paper. We, and the community, are indebted to Douglas Hoyt and Kenneth Schatten for their efforts in acquiring and compiling that data that comprise the Group sunspot numbers. These sunspot number data are available at: *ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/*. We would also like to thank Kenneth Schatten for his many useful comments on the manuscript.

References

Ahluwalia, H. S.: 1998, *J. Geophys. Res.* **103**, 12 103.
Baiada, E. and Merighi, R.: 1982, *Solar Phys.* **77**, 357.
Baliunas, S. and Soon, W.: 1995, *Astrophys. J.* **450**, 896.

Chernosky, E. J.: 1954, *Publ. Astron. Soc. Pacific* **66**, 241.
Eddy, J. A.: 1980, in R. O. Pepin, J. A. Eddy, and R. B. Merrill (eds.), *The Ancient Sun*, Pergamon Press, New York, p. 119.
Friis-Christensen, E. and Lassen, K.: 1991, *Science* **254**, 698.
Gleissberg, M. N.: 1939, *The Observatory* **62**, 158.
Gnevyshev, M. N. and Ohl, A. I.: 1948, *Astron. Zh.* **25**, 18.
Hathaway, D. H., Wilson, R. M., and Reichmann, E. J.: 1999, *J. Geophys. Res.* **104**, 22 375.
Howard, R. F., Gilman, P. A., and Gilman, P.: 1984, *Astrophys. J.* **283**, 373.
Hoyt, D. V. and Schatten, K. H.: 1995a, *Solar Phys.* **160**, 371.
Hoyt, D. V. and Schatten, K. H.: 1995b, *Solar Phys.* **160**, 379.
Hoyt, D. V. and Schatten, K. H.: 1995c, *Solar Phys.* **160**, 387.
Hoyt, D. V. and Schatten, K. H.: 1995d, *Solar Phys.* **160**, 393.
Hoyt, D. V. and Schatten, K. H.: 1997, *The Role of the Sun in Climate Change*, Oxford University Press, New York, 279 pp.
Hoyt, D. V. and Schatten, K. H.: 1998a, *Solar Phys.* **179**, 189.
Hoyt, D. V. and Schatten, K. H.: 1998b, *Solar Phys.* **181**, 491.
Hoyt, D. V., Schatten, K. H., and Nesmes-Ribes, E.: 1994, *Geophys. Res. Lett.* **21**, 2067.
Kiepenheuer, K. O.: 1953, in Kuiper, G. P. (ed.) *The Sun*, The University of Chicago Press, Chicago, p. 322.
McKinnon, J. A.: 1987, *Rep. UAG-95*, World Data Ctr. A for Solar-Terr. Phys., Boulder, 112 pp.
Rabin, D., Wilson, R. M., and Moore, R. L.: 1986, *Geophys. Res. Lett.* **13**, 352.
Vitinskii, Yu. I.: 1965, *Solar Activity Forecasting*, NASA TTF-289, NASA, Washington, D.C., 129 pp.
Waldmeier, M.: 1935, *Astron. Mitt. Zürich* **14**, 105.
Waldmeier, M.: 1939, *Astron. Mitt. Zürich* **14**, 439 and 470.
Waldmeier, M.: 1961, *The Sunspot-Activity in the Years 1610–1960*, Schulthess, Zürich, Switzerland, 171 pp.
Wilson, R. M.: 1987, *J. Geophys.Res.* **92**, 10 101.
Wilson, R. M.: 1988, *Solar Phys.* **115**, 397.
Wilson, R. M.: 1992, *Solar Phys.* **140**, 181.
Wilson, R. M.: 1998a, *J. Geophys. Res.* **103**, 11 159.
Wilson, R. M.: 1998b, *Solar Phys.* **182**, 217.
Wilson, R. M., Hathaway, D. H., and Reichmann, E. J.: 1998, *J. Geophys. Res.* **103**, 6595.