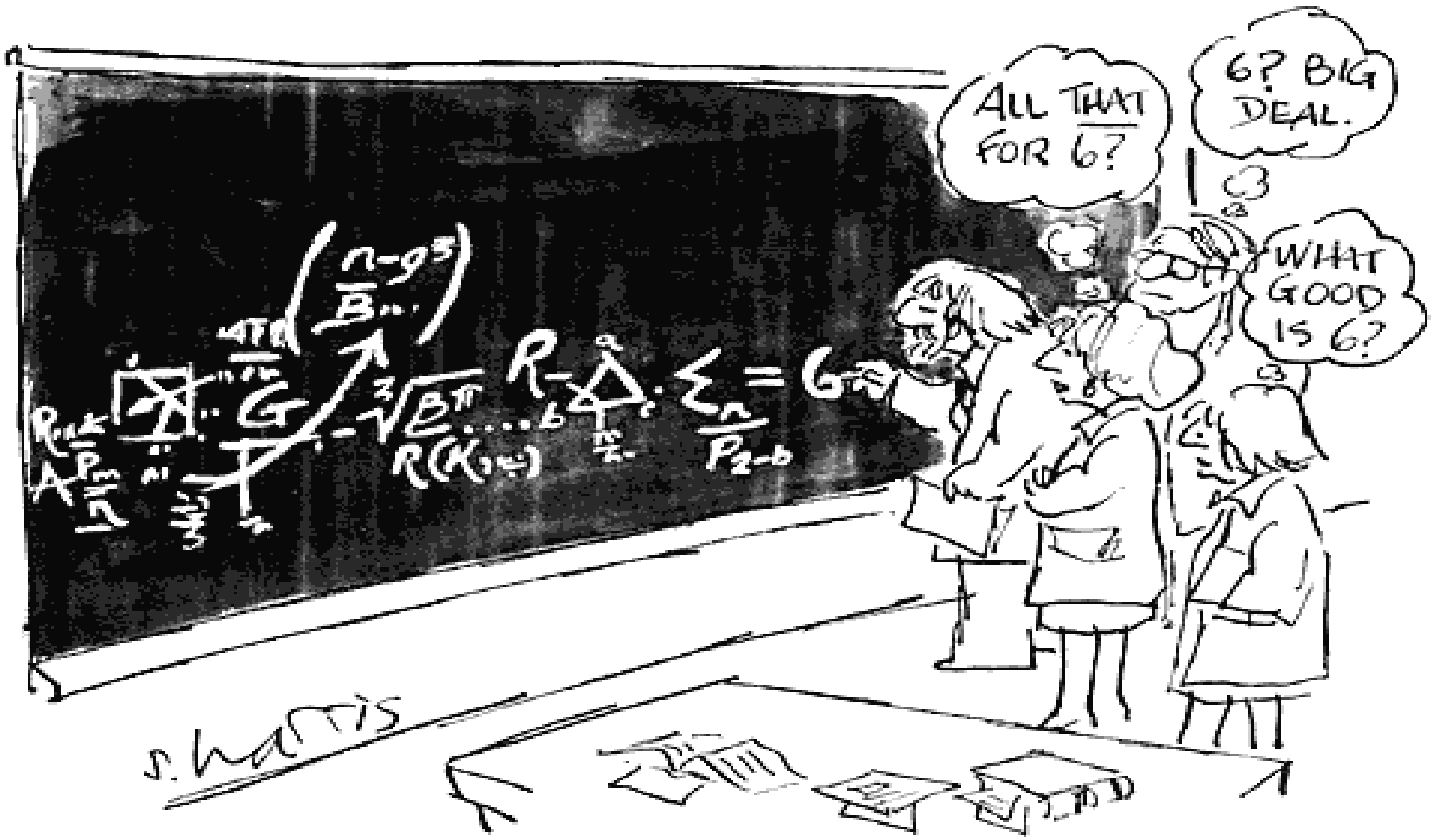
Math Review



Topics

In this self-study, you will review mathematical concepts that will be used to solve problems during the Advanced Health Physics (H-201) course, including:

- **Scientific Notation**
- **Exponents**
- **Logarithms**
- **Dimensional Analysis**
- **Geometry**
- **Trigonometry**
- **Linear Interpolation**
- **Solving for Embedded Variables**

Scientific Notation

- If you look in a reference table and find the number "0.0054," you know that this value has been measured to a precision of two significant figures (the leading and trailing zeros are merely placeholders to indicate the scale of the number).
- This value can also be written in scientific notation as:

$$5.4 \times 10^{-3} \quad \text{or} \quad 5.4\text{E-}3$$

Note on Calculations

When performing calculations in this course, you should only answer to 2 decimal places unless you are provided with the precision to carry more.

EXPONENTS

Exponents

- Exponentiation is a mathematical operation involving two numbers: a base and an exponent or “power.” A simple exponential expression is shown below:

$$x^n$$

- In the expression above, *x* represents a base number and *n* represents the exponent.

- Exponents have mathematical properties that make them useful for solving and simplifying equations.

Properties of Exponents

$$x^{-1} = 1/x$$

$$x^{-a} = 1/x^{a}$$

$$\exp(x) = e^{x}$$

$$e^{-1} = 1/e$$

$$x^{a}\,x^{b} = x^{a+b}$$

$$(x^{a})^{b} = x^{ab}$$

$$x^{a}/x^{b} = x^{a-b}$$

$$10^{0} = 1$$

$$x^{0} = 1$$

$$e^{0} = 1$$

Note: "e" is just a mathematical constant approximately equal to 2.71828. We will use this constant in calculations throughout the H-201 course.

Properties of Exponents

The following expressions cannot be simplified any further:

$$x^a \, y^a$$

$$x^a / y^a$$

More Properties of Exponents

$$x^{1/2} = \sqrt{x}$$

$$x^{1/3} = \sqrt[3]{x}$$

$$(x^{1/2})^{1/3} = x^{1/6} = \sqrt[6]{x}$$

$$e^{-\mu x} = 1/e^{\mu x}$$

$$e^{-\mu x}\, e^{a} = e^{(a-\mu x)}$$

$$e^{-\mu x}\, e^{-\mu x} = e^{-2\mu x}$$

Here the exponent "$-\mu x$" is shown because it is a common exponent seen in shielding equations.

Simplifying Expressions with Very Large Exponents

- As an exponent gets larger and larger, we say the exponent "approaches" infinity, (which is mathematically written using an arrow as shown below). As a *positive* exponent approaches infinity, the exponential itself also approaches infinity.

- For example, for $e^{\mu x}$: $\mu x \rightarrow \infty, \quad e^{\mu x} \rightarrow \infty$

  (read "as μx approaches infinity, $e^{\mu x}$ also approaches infinity)

- However, when a *negative* exponent approaches infinity, the exponential approaches a zero value:

$$\mu x \rightarrow \infty, \quad e^{-\mu x} = 1/e^{\mu x} \rightarrow 0$$

Simplifying Expressions with Very Small Exponents

- **As an exponent gets smaller and smaller, we say that exponent "approaches" zero. As a *positive* exponent approaches zero, the exponential itself approaches a value of one.**

- **For example, for $e^{\mu x}$:** $\mu x \to 0 , \quad e^{\mu x} \to 1$

- **When a *negative* exponent approaches zero, the exponential also approaches value of one:**

$$\mu x \to 0 , \quad e^{-\mu x} \to 1$$

LOGARITHMS

Logarithms

- Logarithms are closely related to exponentials.
- The logarithm of a number is the exponent by which a base value must be raised to produce that number.
- There are different logarithmic functions, depending on the base that is used.

Logarithms

In this course, we will use the following logarithms:

- **<u>Common Logarithm</u> (Base 10),**

  written as "$\log_{10}$"or just "log"

- **<u>Natural Logarithm</u> (Base *e*),**

  written as "ln"

  (recall that $e \approx 2.7183$)

Logarithms

The answer "Y" to "$\log_{10} X = Y$" can be thought of as:

"What power must 10 be raised to get X?"

or $10^Y = X$

Common Logarithm Example

$$\log_{10}(100) = 2$$

Because $10^2 = 100$

Natural Logarithms (ln)

Similarly,

"ln X = Y" can be thought of as:

"What power must "e" be raised to get X?"

or $e^Y = X$

Natural Log (ln) Example

Example: $\ln_e(10) = 2.3$

e must be raised to a power of 2.3 to get 10:

$$(e)^{2.3} = (2.7183)^{2.3} = 10$$

NOTE: Since e is rounded and the answer (2.3) is also rounded, this is an approximation

Log and Natural Log

Number	log	ln
0	-------	-------
e	0.4343	1
10	1	2.3026
100	2	4.6052
1000	3	6.9078
10000	4	9.2103

Properties of Logs

$\ln(X^a) = a \ln(X)$

$\ln(A * B) = \ln(A) + \ln(B)$

$\ln(A/B) = \ln(A) – \ln(B)$

$\ln(A/B) = - \ln(B/A)$

$\ln(e^A) = A$

Logs and Your Calculator

On your calculator you should have two keys:

- one labeled *log* (base 10)
- one labeled *ln* (base e)

Each key may be associated with an inverse which might require you to press a 2^{nd} or Shift key

DIMENSIONAL ANALYSIS and UNIT CONVERSION

Dimensional Analysis

The ability to keep track of units, cancel them out properly, and ensure that your final answer has the correct units is essential for solving word problems like the ones you will be working throughout the H-201 course.

Dimensional Analysis Problem

Attenuation equations are typically expressed in terms of the linear attenuation coefficient (μ) or the mass attenuation coefficient (μ/ρ).

$e^{-\mu x}$ may also be written as $e^{-(\mu/\rho)(\rho)(x)}$

Let $x = 3$ cm, $\mu/\rho = 0.021$ cm^2/g and $\rho = 0.01$ g/cm^3

Solve $e^{-\mu x}$

Dimensional Analysis Solution

$$(\mu/\rho)(\rho)(x) = (-\ 0.021\ \frac{\cancel{cm^2}}{\cancel{g}}) * (0.01\ \frac{\cancel{g}}{\cancel{cm^3}}) * (3\ \cancel{cm})$$

The grams cancel and the cm^2 in the numerator cancels out 2 of the cm^3 in the denominator leaving only one cm in the denominator. But that remaining cm cancels out the cm from the "x" value. The net result is that no units are left, which is what we want since an exponent should not have any dimensions.

$\exp(-\ 0.00063) = 0.99937 \cong 1$

Unit Conversion

The reference texts you will use during the course will include conversion tables for:

- **Length, Area, Volume**
- **Mass, Density, Time**
- **Energy, Dose, Activity**

You should be able to convert between any given units of the same measurement.

Unit Conversion Problem

- **Protective clothing is often sold with units of areal density, e.g., ounces per square yard (oz/yd^2). The protective clothing used at your facility is advertised to have an areal density of 40 oz/yd^2.**

  Calculate what this corresponds to in units of mg/cm^2.

Unit Conversion Problem (cont)

- 1 oz = 28.35 g
- 1 yd = 3 feet
- 1 foot = 12 inches
- 1 inch = 2.54 cm
- 1 g = 10^3 mg

Unit Conversion Solution

We start out with units of oz/yd^2 and we want to get to units of mg/cm^2:

$$\begin{array}{c|c|c|c|c|c} 40\ \cancel{\text{oz}} & 28.35\cancel{\text{g}} & 10^3\ \text{mg} & 1\ \cancel{\text{yd}^2} & 1\ \cancel{\text{ft}^2} & 1\ \cancel{\text{in}^2} \\ \hline \cancel{\text{yd}^2} & \cancel{\text{oz}} & \cancel{\text{g}} & 3^2\ \cancel{\text{ft}^2} & 12^2\ \cancel{\text{in}^2} & 2.54^2\ \text{cm}^2 \end{array}$$

The oz's cancel, the g's cancel, the yd^2 cancel, the ft^2 cancel, the in^2 cancel and we're left with mg in the numerator and cm^2 in the denominator

$$40\ \text{oz/yd}^2 = 136\ \text{mg/cm}^2$$

Extracting Information from Tables

To solve exam problems you will be required to look up numbers from reference tables. If you look up the wrong value, you will surely get the wrong answer. To avoid this, always be sure of the following:

- **you are on the correct page**
- **you are looking down the correct column**
- **you are looking across the correct row**

The following page shows a sample conversion table from the reference manual.

Conversion Tables

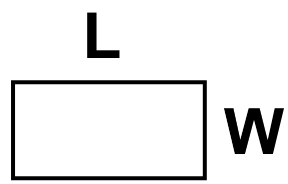
LENGTH							
µm	mm	cm	m	in	ft	yd	mi
1	1.000E-03	1.000E-04	1.000E-06	3.937E-05	3.281E-06	1.094E-06	6.214E-10
1.000E+03	1	1.000E-01	1.000E-03	3.937E-02	3.281E-03	1.094E-03	6.214E-07
1.000E+04	1.000E+01	1	1.000E-02	3.937E-01	3.281E-02	1.094E-02	6.214E-06
1.000E+06	1.000E+03	1.000E+02	1	3.937E+01	3.281E+00	1.094E+00	6.214E-04
2.540E+04	2.540E+01	2.540E+00	2.540E-02	1	8.333E-02	2.778E-02	1.578E-05
3.048E+05	3.048E+02	3.048E+01	3.048E-01	1.200E+01	1	3.333E-01	1.894E-04
9.144E+05	9.144E+02	9.144E+01	9.144E-01	3.600E+01	3.000E+00	1	5.682E-04
1.609E+09	1.609E+06	1.609E+05	1.609E+03	6.336E+04	5.280E+03	1.760E+03	1

Problem: Convert 64 feet to mm

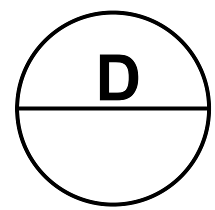
GEOMETRY

Length

Perimeter of a rectangle = (2 x Length) + (2 x Width)

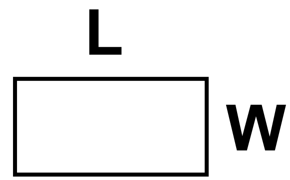


Circumference of a circle = π x Diameter
or π x 2 x radius
= $2\pi r$

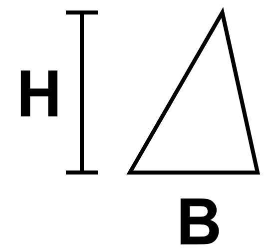


Area

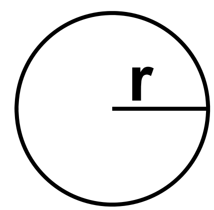
Rectangle = Length x Width



Triangle = ½ Base x Height

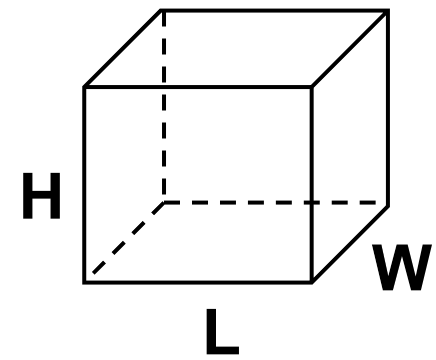


Circle = π x radius^2

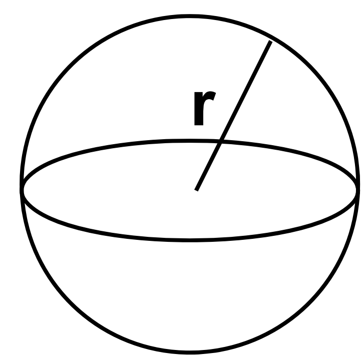


Surface Area

Box = 2 [(L x W) + (L x H) + (W x H)]

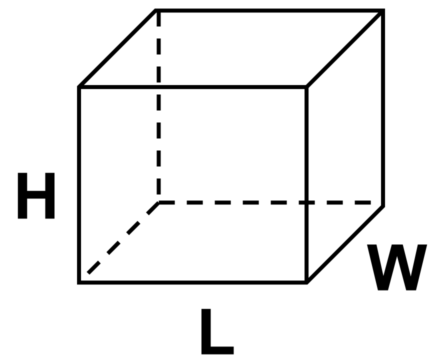


Sphere = 4 π r^2

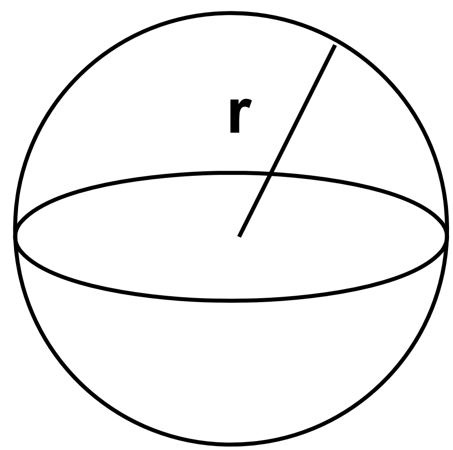


Volume

Box = L x W x H

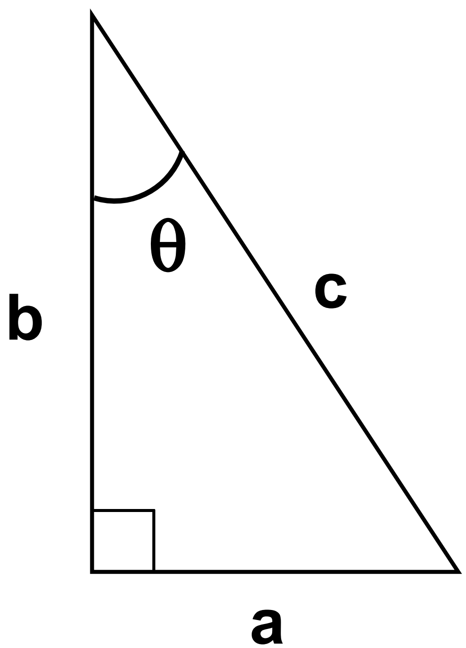


Sphere = $\frac{4}{3}\pi r^3$



TRIGONOMETRY AND INVERSE TRIGONOMETRIC FUNCTIONS

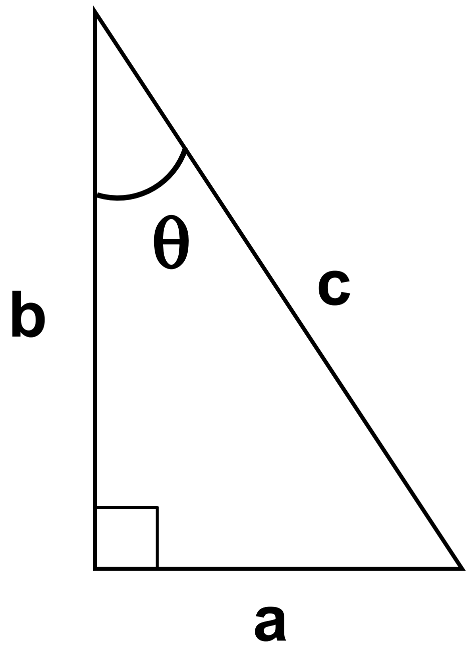
Trigonometry



- Trigonometry is the mathematical study of triangles and the relationships between their sides and angles.
- If you have a right triangle with sides of length a, b and c, you can use simple trigonometric functions (see next slide) to relate the measures of its sides to the angle θ.
- The following equation, called the Pythagorean equation, also relates the length of the sides of this triangle:

$$c^2 = a^2 + b^2$$

Trigonometric Functions



$$\text{Sine } \theta = \frac{a}{c}$$

$$\text{Cosine } \theta = \frac{b}{c}$$

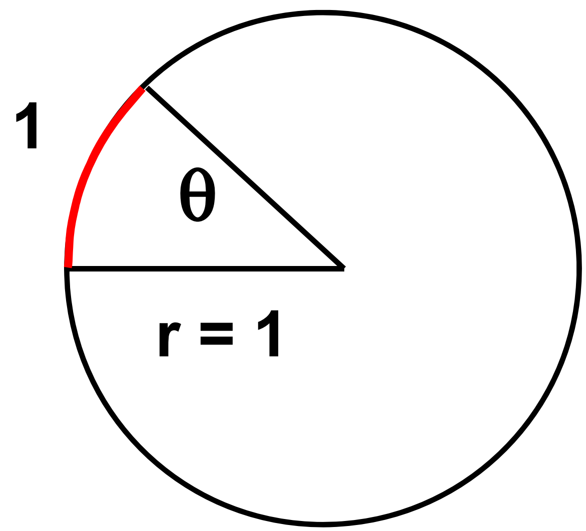
$$\text{Tangent } \theta = \frac{a}{b} = \frac{\text{Sine } \theta}{\text{Cosine } \theta}$$

Measuring Angles

Imagine a circle with radius = 1.

Now imagine an arc on this circle whose length is equal to 1 as well.

The measurement of the angle that subtends this arc is defined as one radian (abbreviated rad).

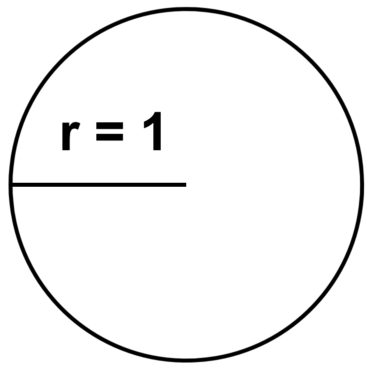


$\theta = 1$ rad

Measuring Angles

If you measured around a circle in degrees, one trip around the complete circle is 360°

We also know that the circumference of a circle is $2\pi r$



So if we go around our circle of radius 1, we have traveled a distance equal to 2π, But we have also traveled 360°

Thus 360° = 2π radians = 6.2832 radians

Degrees and Radians

Degrees	Radians		
0	$0\,\pi$	=	0.0000
30	$(1/6)\,\pi$	=	0.5236
45	$(1/4)\,\pi$	=	0.7854
60	$(1/3)\,\pi$	=	1.0472
90	$(1/2)\,\pi$	=	1.5708
120	$(2/3)\,\pi$	=	2.0944
135	$(3/4)\,\pi$	=	2.3562
150	$(5/6)\,\pi$	=	2.6180
180	π	=	3.1416
270	$(3/2)\,\pi$	=	4.7124
360	$2\,\pi$	=	6.2832

sine (180°) =
sine (π radians)

Trigonometric Functions

Angle (degrees)	Angle (radians)	Sine	Cosine	Tangent
0	0	0	1	0
30	0.5236	0.5	0.866	0.577
45	0.7854	0.707	0.707	1
60	1.0472	0.866	0.5	1.732
90	1.5708	1	0	-

Inverse Trigonometric Functions

The calculated sine of an angle is just a number with no dimensions.

If you know that number and want to find the angle corresponding to that number, you can use an inverse trigonometric function like the *inverse sine*, *inverse cosine* or *inverse tangent*. These are sometimes called the arcsine, arccosine and arctangent.

Example:

sine 30° = 0.5

inverse sine (0.5) = 30° or 0.5236 radians

Inverse Trigonometric Functions

The inverse sine is probably displayed as the secondary function $\sin^{-1}$ on your calculator so you may have to hit a *2nd* key or *inv* key before hitting the sin key

IMPORTANT: Remember that if you input a number and ask for the inverse sine, cosine or tangent, you will get an answer in either degrees or radians.

You must check what your calculator is set for. Most calculators have a DRG key which toggles between DEG, RAD and GRAD. If you want your answer to be in radians (as we will for the purposes of the course) be sure your calculator is set for RAD.

Problem

What is the arctan of 0.06?

On my calculator, if I enter .06, 2nd, then tan^{-1}, I get 3.43

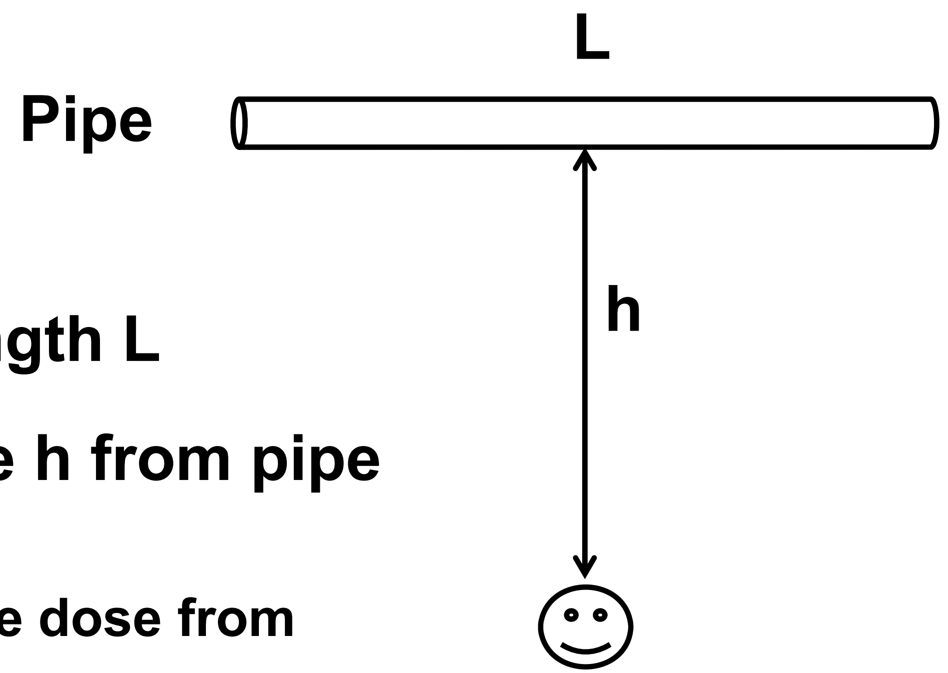
Q: Is this answer degrees or radians?

A: This is in degrees. I need to toggle my calculator to RAD and I would get an answer of 0.06 radians.

To avoid problems always check your DRG key setting!

HP Application of Trigonometry

Radians and inverse tangents are used to calculate dose from "line sources" (see illustration below):



- **Radioactive Pipe with length L**
- **Person standing distance h from pipe**

(You will learn how to calculate dose from such a line source during the course)

Linear Interpolation

Linear Interpolation

Let's say you have a set of data like that shown below:

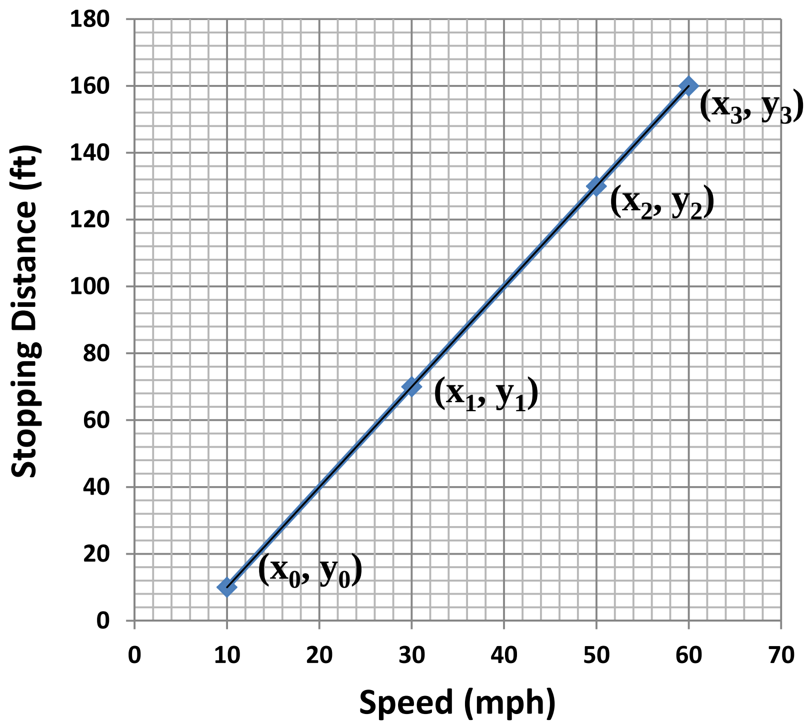
Speed (mph)	Stopping Distance (feet)
10	10
30	70
50	130
60	160

If you are asked for the stopping distance for a car travelling at 45 mph, how can you find this value?

Linear Interpolation

You can think of your data as points on a line:

Speed (X values)	Stopping Distance (Y values)
10 (x_0)	10 (y_0)
30 (x_1)	70 (y_1)
50 (x_2)	130 (y_2)
60 (x_3)	160 (y_3)



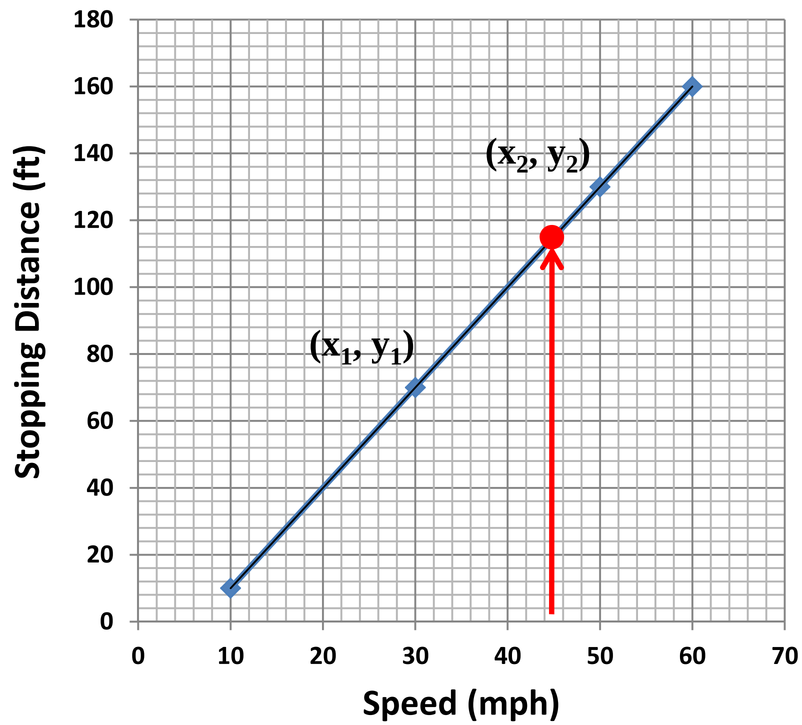
The slope of the middle segment of this line is:

$$y_2 - y_1 \; / \; x_2 - x_1$$

Linear Interpolation

You can use the slope equation to solve for an unknown value, like the stopping distance for a speed of 45 mph:

Speed (X values)	Stopping Distance (Y values)
10 (x_0)	10 (y_0)
30 (x_1)	70 (y_1)
45	?
50 (x_2)	130 (y_2)
60 (x_3)	160 (y_3)



Linear Interpolation

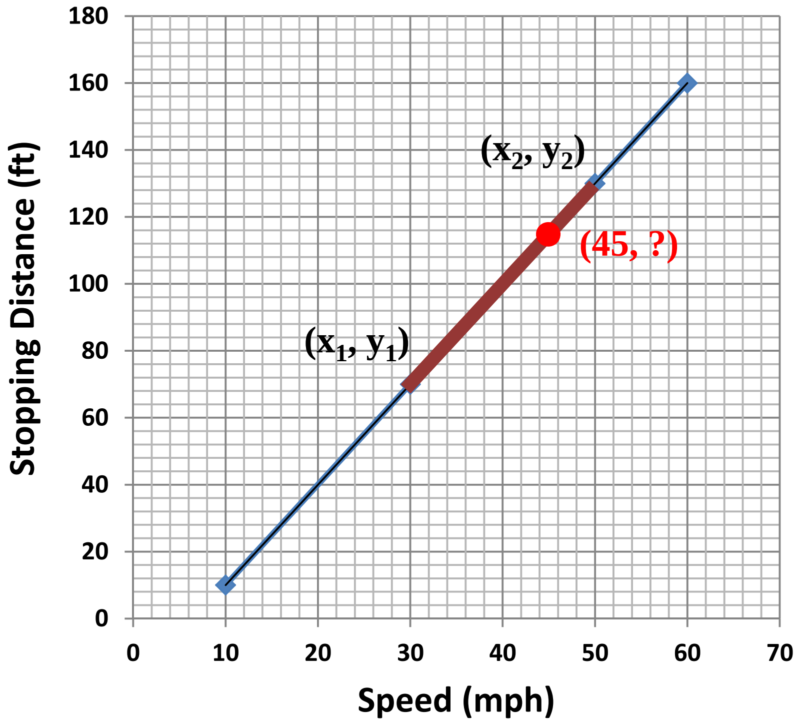
Since our new point is on the line segment between (x_1,y_1) and (x_2,y_2), we can use the slope equation to solve for our unknown y-value.

$$y_2 - y_1/x_2 - x_1 = y_2 - ?/x_2 - 45$$

Solving this equation for ?:

$$? = 115$$

So we know that the stopping distance for a speed of 45 mph is 115 ft.



Linear Interpolation

Let’s try using real HP data to do an interpolation.

Look at the table at the bottom the following slide. The values listed in the first column are Photon Energy (in MeV) and the values listed in the second column are values for a quantity known as the “Mass Energy Absorption Coefficient” or μ_{en}/ρ. This value can be used to perform dose calculations for photons of different energies.

Linear Interpolation Problem

In the table, we see values of μ_{en}/ρ for 6 MeV and 8 MeV photons, but the tables don't list a value for 7 MeV photons. Using the data in the table, interpolate to find μ_{en}/ρ for 7 MeV photons:

MeV	μ_{en}/ρ
6	0.0178
7	?
8	0.0167

Linear Interpolation Solution

$$\frac{0.0167 - 0.0178}{8-6} = \frac{0.0167 - ?}{8-7}$$

$$\frac{-0.0011}{2} = 0.0167 - ?$$

$$? = 0.0167 + 0.00055 = 0.0173$$

Quick check:
Since 7 MeV is half way between 6 MeV and 8 MeV:

$$\frac{0.0178 + 0.0167}{2} = 0.0173$$

Solving for Embedded Variables

Solving for Embedded Variables

- During the H-201 course, you will have to manipulate algebraic equations and solve for different variables found within equations.

- For example, we will use the neutron activation equation:

$$A = \sigma\phi n(1-e^{-\lambda t})$$

- If you needed to solve for the variable ϕ (which happens to stand for neutron flux), you would have to get this "embedded variable" by itself on one side of the equation.

- Certain simple rules will help you manipulate algebraic equations to solve for embedded variables. These are listed on the following slide.

Solving for Embedded Variables

1. Multiply or divide both sides of an equation by the same value

2. Add or subtract the same value to both sides of an equation

3. Multiply any side of an equation by 1 or any ratio that equals 1, (e.g. a / a)

4. Flip both sides of an equation (i.e. reverse numerator and denominator) provided there is only one denominator on each side, e.g. ($1/a = 1/b \rightarrow a = b$) but ($1/a + 1/b = 1/c \rightarrow a + b = c$)

5. $\ln(b^a) = a \ln(b)$ {if b = e, ln e = 1 so $\ln(e^a) = a$} also $e^{(\ln b)} = b$

6. If sine a = b then inverse sine b = a

Example 1

$$A = \frac{B \times D}{C \times E}$$

Solve for C

Example 1 Solution

Isolate “C” on one side all by itself

Multiply both sides by C

$$C \times A = \frac{B \times D}{\cancel{C} \times E} \times \cancel{C}$$

Divide both sides by A

$$\frac{C\cancel{A}}{\cancel{A}} = \frac{B \times D}{E} \Big/ A$$

$$\boxed{C = \frac{B \times D}{A \times E}}$$

Example 2

$$A = \frac{B}{C} + \frac{D}{E}$$

Solve for C

Example 2 Solution

$$A = \frac{B}{C} + \frac{D}{E}$$

Subtract D/E from both sides.

$$A - \frac{D}{E} = \frac{B}{C} + \cancel{\frac{D}{E}} - \cancel{\frac{D}{E}}$$

$$A - \frac{D}{E} = \frac{B}{C}$$

Multiply the “A” by 1 or (E/E)

$$\frac{AE}{E} - \frac{D}{E} = \frac{B}{C}$$

Now we have a common denominator on the left so we can add the numerators

Example 2 Solution

$$\frac{AE - D}{E} \times C \times E = \frac{B}{C} \times C \times E$$

To isolate C, we multiply both sides by C and by E. Then we divide both sides by (AE - D).

$$C \times (AE-D) = BE$$

$$\frac{E}{AE - D} = \frac{C}{B}$$

This produces the same effect as flipping both sides of the equation and then multiplying both sides by B.

$$\boxed{\frac{BE}{AE - D} = C}$$

Example 3

$$A = B \times e^{\frac{C \times D}{E}}$$

Solve for C

Example 3 Solution

First isolate the exponential by dividing both sides by B

$$\frac{A}{B} = e^{\frac{C \times D}{E}}$$

Next we get rid of the exponential by taking the natural logarithm (ln) of both sides

$$\ln\left[\frac{A}{B}\right] = \ln\left[e^{\frac{C \times D}{E}}\right]$$

But we know that the natural logarithm of e raised to an exponent is just the exponent (recall properties of logs).

Example 3 Solution

$$\ln \left[\frac{A}{B}\right] = \frac{C \times D}{E}$$

Isolate C by multiplying both sides by E and dividing both sides by D:

$$\boxed{\frac{E}{D} \ln \left[\frac{A}{B}\right] = C}$$

If you wished, you could expand the ln of the fraction since we know that ln (A/B) = ln A - ln B.

Example 4

The following expression is the basic equation for radioactive decay that you will be using during the course.

$$A = A_0 e^{-\lambda t}$$

How would you solve for *t* ?

Example 4 Solution

$$A = A_0 e^{-\lambda t}$$

First rearrange to get: $A/A_0 = e^{-\lambda t}$

Then take the natural log of both sides:

$$\ln (A/A_0) = \ln (e^{-\lambda t})$$

Example 4 Solution

Now we have:

$$\ln (A/A_0) = \ln (e^{-\lambda t})$$

ln $e^{-\lambda t}$ is just the exponent, or $-\lambda t$

Simplifying and solving for t, you get:

$$\boxed{t = -\frac{\ln (A/A_0)}{\lambda}}$$

Example 5

$$A = B \times \ln\left[\frac{C}{D}\right]$$

Solve for C

Example 5 Solution

To isolate C we must get rid of the logarithm. To do this, we employ the inverse which is the exponential. But first we must remove the B by dividing both sides by B.

$$\frac{A}{B} = \ln\left[\frac{C}{D}\right]$$

Now we apply an exponential to both sides.

$$e^{\frac{A}{B}} = e^{\ln\left[\frac{C}{D}\right]}$$

Example 5 Solution

e raised to an exponent and the natural logarithm are inverse functions so they cancel, leaving only the variables on the right side.

$$e^{\frac{A}{B}} = \frac{C}{D}$$

Finally, we isolate the C by multiplying both sides by D:

$$\boxed{D \times e^{\frac{A}{B}} = C}$$

END OF
MATH REVIEW