Calculation of stage-discharge relations for gravel bedded channels

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[1] A method for calculating stage-discharge relations (rating curves) in gravel bedded streams is presented and applied to five reaches at U.S. Geological Survey (USGS) gaging stations. The approach, which builds on the work of Kean and Smith (2005), uses a fluid-mechanically based model to convert measurements of stage into flow and boundary shear stress fields as appropriate for determining water discharge. The model does not use site-specific empirical roughness coefficients, such as the Manning coefficient, but rather determines channel roughness from field measurements of the channel geometry and the dominant physical and biological roughness elements in the modeled reach. The approach is fully compatible with current USGS-style river gaging procedures and can be used in conjunction with or instead of the standard empirical gaging methods. When used in parallel, the theoretical rating curves produced by the model are in good agreement with direct measurements of discharge made by the USGS. The results of our analyses indicate that the theoretical rating curve approach has the potential to substantially reduce the number of measurement visits, and therefore costs, required to develop rating curves for gaging stations. Owing to this lower cost, it is well suited for regional hydrologic studies in which rainfall distributions are measured in space and time and are compared with measured discharges on the links of the river network. In addition, our method is ideally suited for sites, such as remote locations, where it is difficult or impossible to define a complete rating curve using conventional methods alone.

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1. Introduction

[2] The current U.S. Geological Survey (USGS) procedure for gaging discharge in steams and rivers is to measure stage and then to calculate discharge from an empirically generated stage-discharge relation (rating curve) [*Rantz*, 1982a, 1982b]. The rating curve is defined by making paired measurements of stage and discharge over the full range of flow conditions and then fitting a curve to the stage-discharge data field. This approach has been applied to a wide variety of streams and rivers by the USGS and other agencies for more than a century, attesting to its accuracy and robustness. Despite the success of the approach, there are some drawbacks to the method, which have not yet been completely addressed. This paper presents an alternative fluid-mechanically based approach to gaging and rating curve development, which builds on the work of *Kean and Smith* [2005]. Our new approach avoids some of the problems associated with the conventional gaging method and is suitable for application in many situations where it is impractical or impossible to accurately gage discharge by empirical methods alone. Moreover, our new gaging method is fully compatible with the current USGS approach and can be used to supplement the empirical rating measurements and generate confidence in the accuracy with which the site is being gaged.

[3] Measuring stage is relatively easy, accurate, and not particularly expensive. Obtaining the discharge measurements necessary to generate the empirical rating curve, however, is time consuming, expensive, and sometimes dangerous. These factors contribute to the fact that there are many sites for which it is difficult to define all, or even part, of the rating curve through direct measurements. These include (1) reaches experiencing flows that are too dangerous to measure with a current meter, (2) reaches with highly unsteady flows, (3) reaches that are remote or difficult to access, and (4) sites requiring immediate information on discharge but lack a valid rating, such as a new station on a previously ungaged channel or an existing station on a reach that has been substantially altered by high flows. For unsteady flow conditions common in small streams, the discharge can change substantially while the measurement is being made. For remote gages, the travel costs associated with procurement of the discharge data can be prohibitively expensive. Moreover, it can be extremely difficult to schedule site visits to coincide with the

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high flows required to define the upper end of the rating. These problems are not present for channels that are easily accessible and experience safe, near-steady high flows relatively often; however, even under such ideal circumstances, it might take several months or years to develop a complete new rating curve (i.e., one with high- and flood-flow definition) following the conventional protocol (as delineated, for example, by *Rantz*, 1982a). The time lag between the start of data collection and the availability of a complete rating is often too long to be useful for many engineering and research needs. For example, in a recently burned watershed, which is left vulnerable to flash flooding as a result of removal of the vegetation by fire, it is often desirable to establish gaging stations throughout the basin to provide flood and debris flow warnings for communities located downstream. While it is generally possible to establish a network of stage gages shortly after the fire, there is no way to develop an empirical stage-discharge relationship for these gages before the first runoff event. Stage data, by themselves, are useful for warning purposes; however, it is impossible to fully assess the magnitude and potential hazard of a flood without an accurate estimate of the discharge.

[4] The most common method to estimate discharge when direct measurements for channel-controlled conditions cannot be made is to compute discharge using a simple one-dimensional model based on the Manning equation [*Dalyrmple and Benson*, 1967; *Rantz*, 1982b]. The calculation is typically constrained by three to five cross sections, a measure of the water surface slope obtained from high water marks, and an estimate of channel roughness. A similar approach using step-backwater models (e.g., Hydrologic Engineering Centers River Analysis System, HEC-RAS) and estimated roughness coefficients is also frequently taken to define all or part of a rating curve when measurements are unavailable [e.g., *Bailey and Ray*, 1966; *Davidian*, 1984]. The accuracy of discharge estimates based on either of these two methods is generally considered less than the accuracy of a direct discharge measurement [*Tillery et al.*, 2001]. The primary reason that these approaches are less accurate is related to the uncertainty associated with estimating the channel roughness coefficient (e.g., Manning or Chezy coefficient), which affects the uncertainty of the discharge estimate at that stage in direct proportion to the error in estimating either the Manning or the Chezy coefficient. Furthermore, roughness coefficients usually vary with stage depending on the local characteristics of the reach. Manning coefficients are estimated typically from experience, by the results of previous studies in other channels [e.g., *Barnes*, 1967; *Limerinos*, 1970; *Hicks and Mason*, 1998], or by extrapolation from a known value at another stage. Despite a large body of work available to aid in the selection of roughness coefficients, it is still not possible to estimate bulk roughness coefficients indirectly with an accuracy comparable to that of a direct measurement (i.e., with an error <10% and preferably <5% [*Rantz*, 1982a; *Tillery et al.*, 2001]). It is also difficult to extrapolate known roughness coefficients to other stages with such accuracy, and the extrapolation process leads to an unquantifiable error. Finally, in many reaches, bulk channel roughness and channel geometry do not increase smoothly with stage. As a consequence, discharge does not increase smoothly with stage, making smooth empirical fits to the rating curve inaccurate. This characteristic has been shown for the Whitewater River (Kansas) in the study of *Kean and Smith* [2005].

[5] In recent years, several alternative approaches to the above mentioned methods of defining stage-discharge relations have been developed. One such method is to use measurements of surface velocity obtained either through video imagery [*Bradley et al.*, 2002; *Creutin et al.*, 2003; *Hauet et al.*, 2008] or through radar [*Costa et al.*, 2006] to constrain the determination of discharge. Discharge is computed by first converting the surface velocity measurement(s) to a mean velocity for the cross section (using either an empirical or a theoretical relation) and then multiplying the mean velocity by an independently determined cross-sectional area. This approach has the following advantages: the surface velocity measurement does not require contact with the flow and the velocity measurements can be made remotely and automatically. The approach of *Costa et al.* [2006] has the additional advantage of being able to measure discharge in channels with changing cross sections because the cross-sectional shape is simultaneously determined with ground-penetrating radar. This advantage, however, is partly offset by the high cost of the measurement equipment and large power requirements, which make the approach not yet practical for gaging remote sites. Moreover, one has to be careful in using radar, which is reflected from irregularities on the stream surface (waves and eddies) that propagate at speeds different from the surface velocity of the flow.

[6] Another alternative to the conventional methods for streamflow gaging is to determine the flow resistance as a function of stage explicitly by accounting for the drag on all of the physical and biological roughness elements on the entire potentially wetted area of a reach. This approach, which is the focus of this paper, makes it possible to develop a completely predictive model (namely, one with no site-specific empirically adjusted coefficients) for the flow in the reach at all stages. Using such a model, a "theoretical rating curve" can be constructed. *Kean and Smith* [2005] developed such a model and evaluated the rating curves produced by it against the empirical rating curves at two USGS streamflow gaging stations in Kansas having relatively stable gravel beds. In that paper, the variance of the discharge measurements around the theoretical rating curve was shown to be even less than that around the empirical rating curves generated from the direct measurements.

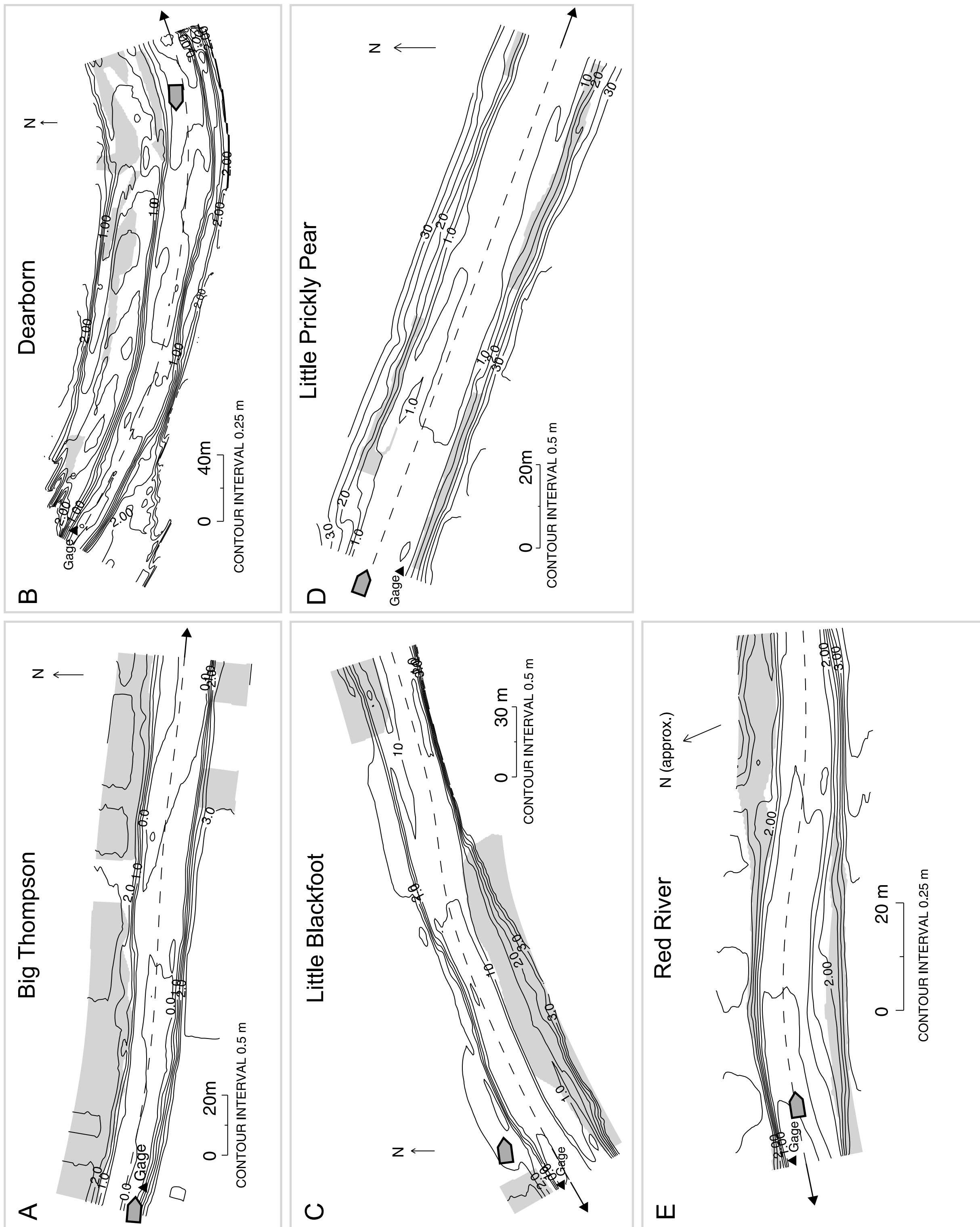
[7] In this paper, we continue development and evaluation of the theoretical rating curve method using a version of the model appropriate for gravel bedded channels with bank and floodplain vegetation. The method is evaluated at five reaches near USGS gages in Colorado, Montana, and New Mexico. Such channels have geometric and roughness characteristics representative of a much broader class of channels than the narrow reaches investigated by *Kean and Smith* [2005]. They can also be modeled using a simpler algorithm than the one described by *Kean and Smith* [2005] because they do not require special procedures to quantify the flow resistance of the banks as were needed to address the flows in the atypically narrow, tree-lined Kansas channels.

2. Study Sites

[8] Photographs and maps of the five study reaches adjacent to USGS gages are shown in Figures 1 and 2,

Figure 1. Photographs of the five study reaches at USGS gaging stations. (a) Big Thompson River at Loveland, CO; (b) Dearborn River near Craig, MT; (c) Little Blackfoot River near Garrison, MT; (d) Little Prickly Pear Creek at Wolf Creek, MT; and (e) Red River below Fish Hatchery near Questa, NM.

Figure 2. Maps of the five study reaches: (a) Big Thompson, (b) Dearborn, (c) Little Blackfoot, (d) Little Prickly Pear, (e) Red River. The maps show the approximate location where each photo was taken (shaded arrows), the centerline of the main channel (dashed lines), the flow direction (solid black arrows), the location of each USGS stage gage (black triangles), and the presence of woody vegetation (shaded areas). Map contour elevations are relative to the gage datum. The surveys from which the maps were made are the basis for the model calculations of discharge.



A
Big Thompson
Gage
0 20m
CONTOUR INTERVAL 0.5 m
B
Dearborn
Gage
0 40m
CONTOUR INTERVAL 0.25 m
C
Little Blackfoot
Gage
0 30 m
CONTOUR INTERVAL 0.5 m
D
Little Prickly Pear
Gage
0 20m
CONTOUR INTERVAL 0.5 m
E
Red River
Gage
N (approx.)
0 20 m
CONTOUR INTERVAL 0.25 m

Figure 2

Table 1. Summary of Site Characteristics

Site Name and USGS Station Number	Date Gage Established at Present Location	Date of Channel Survey	Drainage Area (km^2)	Reach Length (m)	Sinuosity	Bankfull Depth (m)	Bankfull Width (m)	Measured Water Surface Slope	
								Low Flow	High Flow
Big Thompson, 6741510	3 Jul 1979	17 May 2006	1390	186	1.00	2.5	25	0.0014 (2.6 cm)	-
Dearborn, 06073500	13 Aug 1993	27 Oct 2004	842	323	1.03	1.4	33	0.0029 (1.2 cm)	0.0033 (95 cm)
Little Blackfoot, 12324590	9 Sep 1992	28 Oct 2004	1050	241	1.01	1.0	17	0.0037 (1.9 cm)	-
Little Prickly Pear, 06071300	10 Oct 1991	26 Oct 2004	987	132	1.00	3.0	32	0.0033 (1.1 cm)	0.0035 (29 cm)
Red River, 08266820	5 May 1999	14 Sep 2005	479	100	1.01	1.2	14	0.0076 (1.4 cm)	-

respectively. The gages are Big Thompson River at Loveland, CO (Big Thompson); Dearborn River near Craig, MT (Dearborn); Little Blackfoot River near Garrison, MT (Little Blackfoot); Little Prickly Pear Creek at Wolf Creek, MT (Little Prickly Pear); and Red River below Fish Hatchery near Questa, NM (Red River). A summary of important characteristics for each site is given in Tables 1 and 2. The reaches are relatively straight, have gravel beds, and contain various types of woody and herbaceous (mainly grass) vegetation along the banks. These reaches are locally straighter than the regional sinuosity because gaging operations are typically conducted on straight reaches to avoid complications associated with curvature, such as the instability of the channel geometry due to bank erosion and movement of point bars. Curved channel rating curves could be calculated following approaches similar to those outlined in this paper, provided the additional sources of flow resistance from drag on the bar and bank forms is determined. About channel geometry, it should also be noted that, like many channels throughout the country, engineering bank stabilization measures have been undertaken in two of the study reaches (Big Thompson and Little Prickly Pear). As a result, these channels have higher banks than are typical of self-formed gravel channels. Despite the higher banks, the shear stresses on the banks are sufficiently small to permit the two engineered reaches to be modeled in the same manner as the other three sites.

[9] Calculations show that the two primary sources of flow resistance in the study reaches are the surface roughness of the gravel bed and the drag on the stems and branches of woody bank vegetation. A summary of the geometric characteristics of these main roughness elements is given in Table 2. The contribution to the total flow resistance of herbaceous vegetation is not considered here, because it tends to lay flat during most high flows, and consequently offers little resistance to the flow. We consider the relative roughness of the bed material to the flow depth in the study reaches to be "normal" over most of the stage range. Relative roughness is defined here to be the ratio D_{84Z}/h, where D_{84Z} is the 84th percentile of the grain size distribution for the vertically oriented axes and h is the local flow depth. We have found that the relative roughness of a channel is normal when D_{84Z}/h is less than 0.5 over most of the reach. Mountain channels often have high relative roughness ($D_{84Z}/h > 0.5$) over most, if not all, of the stage range, and procedures different from the ones described in this paper are required to determine the flow resistance of these bed surfaces (see, for example, *Wiberg and Smith* [1991]). Calculation and verification of theoretical rating curves for channels with high relative roughness are the subject of a paper currently in preparation by the authors.

[10] Owing to the relatively coarse bed material and presence of bank vegetation, the geomorphology of the five study channels is fairly stable. Substantial channel change only occurs during large flows with recurrence intervals of a decade or more. The relative stability of these channels compared with sand bedded streams means that a single rating curve might remain valid for a number of years.

3. Model Components

[11] As developed in the study by *Kean and Smith* [2005], the model for calculating a theoretical stage-discharge relation is composed of two parts: a set of procedures for quantifying the various contributions to the total flow resistance in the channel and a flow model into which the results of these procedures are embedded. The description in this section is for the components necessary to calculate rating curves for channels similar to the five study sites. Different components are required to compute theoretical rating curves for channels with different roughness characteristics, such as high relative roughness or different geometric characteristics, such as narrow channels [*Kean and Smith*, 2005] or curved channels [*Kean*, 2003].

Table 2. Summary of Roughness Characteristics for the Study Sites[a]

	Gravel					Woody Vegetation		
	D_{50N} (mm)	D_{84N} (mm)	D_{50Z} (mm)	D_{84Z} (mm)	z_o (m)	Mean D_s (m)	Mean λ (m)	D_s/λ^2 (m^{-1})
Big Thompson	61	89	34	62	0.013	0.010	0.30	0.11
Dearborn	55	110	26	57	0.012	0.010	0.31	0.11
Little Blackfoot	52	86	25	53	0.011	0.009	0.24	0.16
Little Prickly Pear	43	87	20	49	0.009	0.018	0.26	0.27
Red River	80	140	41	70	0.015	0.022	0.23	0.42

[a]N, nominal diameter; Z, vertical axis.

3.1. Bed Roughness

[12] In channels with gradually varied flow conditions, gently sloping banks, width-to-bankfull-depth ratios approximately greater than 10, and gravel beds with normal relative roughness ($D_{84Z}/h < 0.5$), the vertically averaged velocity at any position in the channel ($\bar{u}$) can be related directly to the local boundary shear stress (τ_b) through the expression

$$\bar{u} = (\tau_b/\rho)^{1/2}\beta = u_*\beta \quad (1)$$

where ρ is the density of water, u_* is the local shear velocity, and β is a nondimensional roughness coefficient that is a function of the size of the bed material, the local flow depth, and the shape of the velocity profile. In this simple situation, the boundary shear stress is that appropriate for calculating bed material transport and is given by $\rho g h S_f$. Here g is the acceleration of gravity and S_f is the friction slope. Under these conditions, the velocity profile will be quasilogarithmic in shape [see *Wiberg and Smith*, 1991], and β will have the form

$$\beta = [\ln(h/z_o) - 0.74]/\kappa \quad (2)$$

where κ is the von Karmann constant equal to 0.408 [*Long et al.*, 1993]. The roughness height, z_o, can be related to the moments of the particle size distribution for the gravel composing the bed. A simple but adequate approximation is $z_o = 0.2D_{84Z}$ [*Wiberg and Smith*, 1991]. Alternatively, if the diameters of the vertically oriented axis are approximately half of the nominal diameters (D_N), the approximation $z_o = 0.1D_{84N}$ [*Whiting and Dietrich*, 1990] can be used.

3.2. Vegetation Roughness

[13] In vegetated portions of the channel (typically edges), the velocity and boundary shear stress are reduced by drag on the plant stems, which can be calculated using the method of *Smith* [2001, 2007]. The stems are modeled as a randomly distributed array of circular cylinders that have a mean stem diameter (D_s) and mean spacing (λ) specified from field measurements. In this application, the stems are assumed to be rigid and extend throughout the entire flow depth. The flow resistance of submerged flexible vegetation can be addressed using the method of *Smith* [2007]. Drag on the field of stems acts as body force on the fluid.

[14] The average drag force on an individual stem (F) is given by

$$F = \frac{1}{2}\rho C_D D_s h (u_{ref})^2 \quad (3)$$

where u_{ref} is the reference velocity and C_D is the drag coefficient of a single stem, which, for the flows of interest here, is essentially constant at a value of 1.2. Modeling drag on stems using a single fixed value of C_D is appropriate provided the reference velocity is defined locally [see, for example, *Kean and Smith*, 2004]. The reference velocity is defined as

$$u_{ref} = \left[\frac{1}{h}\int_{z_o}^{h} u(z)^2 dz\right]^{1/2} \quad (4)$$

where $u(z)$ is the local velocity within the field of stems. Given a field of stems that extend throughout the entire flow depth, the primary difference between the velocity profile in vegetated and nonvegetated areas of the channel is associated with a reduction in the local shear velocity within the stems. Although the local shear velocity within a field of free-surface-penetrating stems is reduced because of the drag on the stems, the nondimensional shape of the velocity profile is similar in shape to the velocity profile in an unvegetated part of the channel. Thus, a suitable approximation for the reference velocity can be made using (1), (2), and a shear velocity that reflects the reduced boundary shear stress caused by the form drag on the stems [*Smith*, 2001, 2004, 2007].

[15] By converting the drag force on a single stem into a drag stress [i.e., by dividing (3) by λ^2] and separating it from the total boundary shear stress, *Smith* [2001, 2007] obtained an expression for the boundary shear stress among a field of free-surface-penetrating stems, which is given by

$$\tau_b = \frac{\rho g h S_f}{1 + \sigma_D} \text{ where } \sigma_D = \frac{1}{2} C_D \beta^2 \frac{h D_s}{\lambda^2} \quad (5)$$

It is important to note that equations (3)–(5) are applied locally and only at grid points that contain submerged stems. At low flows, there may not be any contribution from plant stems to the total flow resistance.

3.3. Channel Flow Model

[16] Discharge is calculated using a flow model for a reach that is usually 7 to 10 times longer than the bankfull width. Long reaches permit the average water surface slope to be approximated by the average bed slope, whereas in a short reach, the average water surface slope must be measured as a function of stage. Doing the calculation for a reach, as opposed to a single cross section or limited number of cross sections, also averages hydraulic effects over a greater domain and improves the accuracy of the result. The surveyed topography is interpolated onto a curvilinear grid constructed about the centerline of the channel [e.g., *McDonald et al.*, 2005]. The computation grid is spaced equally in the cross-stream and streamwise directions. Grid points are spaced approximately 30 cm apart for the size of channels investigated here. A velocity profile [with a vertical average given by (1) and (2)] is determined for every submerged grid point in the model channel.

[17] When the study reach is chosen to be relatively straight and not to contain large amplitude bars, which would steer the high-velocity core of the flow from side to side in the channel, we consider that the dominant flow accelerations may be resolved sufficiently in the same manner as is done in a simple, steady, one-dimensional (step-backwater) model. Application of a one-dimensional flow acceleration model requires that the average velocity for each cross section, $(u)_{av}$, be related to the perimeter-averaged shear stress, $(\tau_b)_{av}$. This calculation is done using an expression analogous to (1), given by $(u)_{av} = \beta_r[(\tau_b)_{av}/\rho]^{1/2}$, where β_r is the nondimensional roughness coefficient for the cross section. The value of β_r varies as a function of stage and streamwise position, and this variation can be specified completely a priori using the roughness methods outlined in the previous

sections. Note that β_r is different from β, the nondimensional roughness coefficient for a vertical within a cross section [equation (2)]. For a given stage and cross section, β_r is calculated by integrating the unit discharge ($\bar{u}h$), obtained from (1), (2), and (5), across each grid point in the cross section and dividing this value by the area of the cross section and the shear velocity computed from $(\tau_b)_{av}$.

[18] This type of channel flow model differs from standard one-dimensional models in two respects. First, despite the fact that flow accelerations are only resolved in one dimension in the streamwise direction, a velocity profile for every submerged point in the two-dimensional computational grid is specified. This is accomplished by scaling the local velocity profiles by the friction slope, S_f, computed for each streamwise position in the computational grid. For an unvegetated grid point, the vertically averaged velocity is given by $\bar{u} = (ghS_f)^{1/2}\ \beta(h, z_o)$. For a vegetated grid point, the vertically averaged velocity is given by $\bar{u} = [ghS_f/(1 + \sigma_D)]^{1/2}\ \beta\ (h,z_o)$. Note that, in these expressions, S_f varies with streamwise position in the channel, and h, z_o, and σ_D may vary in both cross-stream and streamwise directions.

[19] The second and more fundamental difference between this model and standard one-dimensional models is that the roughness is fixed based on the field measurements of the geometry of the roughness elements (the size of the gravel and the diameter and spacing of rigid plant stems). In most applications of one-dimensional models, β_r, or its equivalent Manning ($n = R^{1/6} g^{-1/2} \beta_r^{-1}$) or Chezy ($C = g^{1/2}\beta_r$) coefficient are either determined empirically by measuring the water discharge and water surface elevations [e.g., *Wiele and Smith*, 1996] or estimated from experience or the results of previous studies [e.g., *Barnes*, 1967; *Limerinos*, 1970; *Hicks and Mason*, 1998].

[20] The discharge for a given stage is determined iteratively by solving the channel flow model for the water surface profile that matches both the stage and a measured water surface elevation drop (fall) through the reach. As mentioned above, in addition to discharge, the solution yields a quasi three-dimensional representation of the velocity field, meaning that values of velocity and shear stress are provided by the solution for all points in the flow field. The theoretical rating curve for the reach is generated by repeating the calculation for different stages over the range of flows that can occur at the site. The approach is fully predictive in that once the physical and biological roughness elements are characterized, there are no site-specific empirical coefficients to be adjusted or that can be adjusted. In addition to discharge, the theoretical rating curve model also determines the distribution of the local boundary shear stress with the effects of form drag removed. This procedure provides a foundation for determining rates of bed load and suspended sediment transport in the measurement reach. An example of how these sediment transport calculations can be used to gage suspended sediment at the Little Prickly Pear site is given in the study by *Kean and Smith* [2006].

4. Field Methods

[21] This section describes the field steps required to develop theoretical rating curves for gravel bedded channels in general and for the five study sites in particular.

4.1. Stage

[22] Like the conventional empirical gaging method, the theoretical rating curve approach requires a measurement of stage to which the discharge rating curve is referenced. In principle, the stage gage can be located anywhere within the measurement reach (i.e., the reach that is being modeled). The standard USGS procedure is to locate the gage upstream of a well-defined local hydraulic control (e.g., riffle) to increase the sensitivity (accuracy) of the rating at low flow. The theoretical rating curve approach is compatible with locating the gage in this manner, provided the geometry and roughness of the hydraulic control are resolved. In the five test applications presented here, the existing USGS gage was the source of the stage data, and Figure 2 shows the location of the gage within each measurement reach. One trade off to locating a gage upstream of a hydraulic control is that the low-flow rating is also sensitive to minor changes to the control, such as can occur by limited channel rearrangement or lodged debris. These complicating effects can be circumvented by using a second-stage gage or a surface-velocity measurement and the fully predictive model that produces the theoretical rating curves. If low-flow sensitivity in the rating is not required, the problems associated with flow-caused changes to the hydraulic control can be avoided by locating the stage gage in a position where the stage-discharge relation is controlled primarily by the overall roughness of the reach.

[23] Although only a single-stage gage was used in the applications presented here, it is best to use two-stage gages located at opposite ends of the measurement reach. In addition to providing measurement redundancy and a means of identifying and correcting the discharge time series for shifts, the auxiliary stage gage can be used to determine the water surface elevation drop (fall) across the reach, which is a required boundary condition in the model. In some cases, such as the ones presented here, fall is nearly constant with stage, and the fall can be determined without a second-stage measurement by the surveying water surface profile, as discussed in the next paragraphs. In general, however, the fall across the reach is a function of stage, which is controlled by the flow conditions in vicinity of the measurement reach. For example, fall can vary systematically with stage if there is a hydraulic control within or nearby the measurement reach, such as a natural riffle or bridge constriction. These geometric variations can affect the water surface profile (and fall) differently at low flow than at high flow. Fall can also vary with time and stage as a result of rapidly changing flow conditions associated with a passing flood wave. A flood wave increases the water surface slope on the rising limb of the hydrograph and decreases the slope on the falling limb relative to steady flow conditions. Despite the fact that the theoretical rating curve model is formulated here with the assumption of steady flow, these unsteady effects on discharge can be included in the calculation of discharge, provided the changes in fall are resolved by the measurements. If resolving unsteady effects are not required to achieve the desired accuracy, then a pair of crest-stage gages located at either end of the reach provides a less expensive alternative for determining moderate- and high-stage fall. Readings from the paired crest-stage gages for several moderate- to high-flow events yield the

data necessary to develop an accurate stage-fall relation for the reach.

4.2. Channel Shape and Roughness

[24] The remaining field observations required to construct a theoretical rating curve model for a site are divided between making measurements of channel shape and channel roughness. These measurements are best made at times of low flow to have easy access to all parts of the channel by wading. The shape of the channel is determined by surveying at least 20 cross sections in a reach from 7 to 10 times longer than its width. Additional, closely spaced cross sections may be necessary to resolve better the geometry of hydraulic controls. A high-precision GPS system was used to survey the three Montana sites, whereas the Colorado and New Mexico sites were surveyed using a standard total station. During each survey, the locations of the areas of each measurement reach containing woody vegetation were mapped (Figure 2) and assigned a representative stem density determined from measurements made within each area.

[25] The measurements of channel roughness consist of characterizing the size of the bed material and the size and spacing of the bank vegetation. A summary of these measurements for each site is given in Table 2. The size of the bed material was determined from Wolman pebble counts made at multiple locations in the reach. The long (D_A), intermediate (D_B), and short (D_C) axes and the protrusion height (D_Z) of each pebble were recorded. While only the size distribution of the protrusion height is required to specify the bed surface roughness, it is desirable to measure all four lengths to (1) determine the size distribution of the nominal diameter [$D_N \approx (D_A D_B D_C)^{1/3}$] for sediment transport calculations and (2) estimate the diameter of the average cross-stream axis [$D_Y \approx (D_A D_B)^{1/2}$], which is a necessary parameter for making high relative roughness calculations (when $D_{84Z}/h > 0.5$) using the approach of *Wiberg and Smith* [1991]. With very few exceptions, the natural orientation of the pebbles at each study site was such that the short axis was equivalent to the protrusion height ($D_C = D_Z$). Multiple pebble counts were made to determine whether there was significant spatial variation of the bed material throughout the reach. At all sites, no significant differences between the size distributions of the individual counts were found, so all of the samples were grouped together to obtain a representative size distribution for the entire reach.

[26] The salient properties of the sparse woody vegetation are determined by counting and recording the diameters of the stems within one or more plots of known area. For reference, the stem measurements in a plot can be completed in approximately the same amount time of time as a pebble count. The sizes of the plots depend on the vegetation density. Sparse woody vegetation requires larger plots (typically 10×10 m^2) to obtain a representative sample, whereas smaller plot sizes (typically 2×2 m^2) are adequate to sample dense woody vegetation. Generally, the stem density measurements are made at only one elevation above the ground (about 20 cm). The model can accommodate vertical variations in stem density; however, for most species of woody vegetation, stem density does not change substantially with height because stem diameters tend to become smaller after branching. Despite the fact that a variety of species of woody vegetation was present at the study sites, the stem densities in the vegetated portions of the reach were fairly uniform throughout each reach. This uniformity permitted use of a single value for stem density to be used at each site.

[27] The fieldwork required to characterize the channel shape and the roughness at the five test sites was completed by small crews (usually two people) in 1 to 2 days. Individual pebble counts and stem density measurements each took about 30 minutes to complete, and the remaining time was devoted to surveying the topography of the reach.

4.3. Water Surface Profiles

[28] As part of a survey, low-flow water surface profiles are measured along the left and right banks of the channel. These data are used (1) to provide a low-flow measure of the fall across the reach and (2) to verify the estimated bed surface roughness. As mentioned at the beginning of this section, the fall across the five study reaches is assumed to be constant as a function of stage. This assumption is supported by the facts that the reaches are moderately steep ($|S| > 0.001$) and that the average low-flow water surface slopes are nearly equal to the average bed gradients. The assumption of a constant stage-fall relation was tested at two of the study sites by resurveying the water surface profiles at high flow. As shown in Table 1, the high-flow water surface slopes are nearly equal to the low-flow slopes, supporting the assumption of a constant stage-fall relation.

[29] The water surface profile can also be used in combination with a single discharge measurement to verify the measured surface roughness obtained from the pebble count and, if necessary, make small corrections for errors in the measured size distribution. This technique can also be used to specify empirically the bed surface roughness for a characteristic low flow in the event that the lowest available flow is too deep or too swift to perform accurate pebble counts [e.g., *Kean and Smith*, 2005]. The measured bed surface roughness is evaluated by first calculating a water surface profile using the measured discharge and the measured D_{84Z} and then comparing it with the measured water surface profile. For the five study sites investigated here, the calculated water surface profiles based on the measured D_{84Z} were in very close agreement with the measured water surface profiles. This close agreement indicates that the field-measured bed roughness adequately represented the effective roughness for the measured low flows. Consequently, no empirical adjustments to the measured grain size distributions were made.

[30] If there are substantial differences between the calculated and measured fall across the reach, then it can be assumed that the overall bed roughness was different from that measured. In contrast, if there are differences in the shape of the calculated and measured water surface profiles, then it is likely that there is additional spatial structure to the bed surface roughness, such as larger clasts in a riffle that was not resolved during sampling. Measuring the entire water surface profile (as opposed to just the fall across the reach) provides the necessary data to diagnose the cause of the discrepancy. When this type of water surface fall comparison indicates a discrepancy, undersampling for the particle size distribution is the likely culprit. If so, the D_{84Z} from the pebble counts can be tweaked to make these profiles conform. Subsequent to this minor adjustment, no changes in the bed composition can be made until the bed begins to

move. As was mentioned previously with regard to the five study sites, no empirical adjustments were made to the measured D_{84Z} used in the models. Owing to the fully predictive nature of an appropriately constructed model for the measurement reach, the low-flow version of the model can be thought of as a transfer function between the D_{84} of the bed and the fall. Either can be used as measured low-flow input.

5. Results and Discussion

5.1. Model Results and Comparison to USGS Gage

[31] Comparisons of theoretical rating curves with USGS discharge measurements and empirical rating curves for the five sites are shown in Figures 3 and 4. Before discussing the comparison, there are several things to note about the measurements and empirical ratings shown in the figures. Low flows occur more often and are sampled more frequently to identify minor shifts in the rating, whereas high flows are sampled infrequently owing to their longer recurrence intervals. Consequently, the low flow end of an empirical rating curve is defined by recent measurements (to reflect current flow conditions), and the middle and upper portions of the curve are defined by a combination of old measurements and available recent measurements, if any. Fortunately, many of the minor shifts that occur at low flows do not significantly affect the high-flow data and rating. Owing to this nonuniformity in sampling, the choice of which measurements to use to generate an empirical rating curve is, in part, subjective and generally guided by the hydrographer's experience and familiarity with the site.

[32] Owing to the subjective nature of excluding any points, all measurements made between the time the gage was established at its present location (Table 1) and 14 July 2007 (the date the figures were created) are shown in Figures 3 and 4. The empirical rating curves shown in Figures 3 and 4 are the shift-adjusted ratings that were in use by the USGS at the time of the topographic survey. Recent measurements are denoted by the diamond symbols and were made between 2 years before the date of the topographic survey (Table 1) and 14 July 2007. Ice-affected measurements are not shown because ice is not considered in this study. It should be noted that, at all sites except Red River, there are many measurements that plot significantly away from the trend defined by the most recent measurements. Most of these outlying measurements were made at times when the reach had different stage-discharge relations, whereas others are the result of temporary shifts away from the long-term rating due to minor variations in channel morphology and gage-pool control. The latter types of problems can be addressed by identifying the exact time when each shift occurs and correcting the discharge time series appropriately when it does. Investigation of this latter subject is beyond the scope of this paper. In the data sets presented in Figures 3 and 4, the stage-discharge relation likely became effective following the last channel-rearranging high flow at that site (for example, on the Little Blackfoot River, this was the ice-jam flood of 1996). The measurements at Red River do not exhibit the typical, above-discussed scatter of measurements as clearly because the gage has only been at its present location for a short period.

[33] In general, there is good agreement between the theoretical rating curve and the most recent measurements. In all cases, the theoretical rating curves have been calculated to a stage above the stage of the highest direct measurements and the top of the empirical rating. The theoretical ratings do not extend to the very lowest end of the stage range because, at these stages, the flow depth is comparable to the size of the bed material. This hydraulic situation is more accurately addressed by a high relative roughness model, which is the subject of a paper currently in preparation by the authors. The theoretical ratings also are in good agreement with the empirical ratings at three of the five sites (Dearborn, Little Blackfoot, and Little Prickly Pear). At Big Thompson River and Red River, however, the high-flow portions of the two curves are appreciably different. In the case of Big Thompson, it is difficult to assess which rating is more accurate at high flow because both curves are in reasonable agreement with different clusters of high-flow measurements. In the case of Red River, there are no high-flow measurements available to evaluate which rating is more accurate. To provide more insight into the stage-discharge relation at high flow for the Red River site, a separate rating was calculated based on the assumption that the mean flow is critical at the gage cross section (i.e., $(u)_{av}/\sqrt{gR} = 1$). A comparison of the critical flow rating to the other rating curves indicates that the empirical rating is supercritical at high flow, whereas the theoretical rating is subcritical. Although supercritical flows can occur in natural gravel bed streams, the enhanced erosion under supercritical flow conditions would eventually transform the channel into a rougher, topographically more complex geometry, which would cause the time- and space-averaged flow to return to critical or subcritical conditions. Consequently, it is likely that the theoretical rating curve for this site is correct and the empirical one is in error.

[34] Theoretical rating curves calculated without the effects of drag on vegetation are also shown in Figures 3 and 4. These curves show that the flow resistance of the vegetation at these sites is only important at very high stages. In other channels, such as the forested midwestern streams investigated by *Kean and Smith* [2005], a much greater fraction of the total flow resistance can come from woody riparian vegetation, which produces deep, narrow streams and rivers, more prone to overbank flow.

5.2. Statistical Comparison of Theoretical and Empirical Ratings

[35] A quantitative comparison of the agreement of the empirical and theoretical rating curves with the measurements can be made by comparing the sum of the squared weighted error (SSWE) of the two curves about the measurements within the stage range shared by the two curves. In order for the comparison to be unbiased by the high discharge measurements, the squared difference between each measurement and the value from the rating is weighted to reflect the stated uncertainty of each measurement [see *Hill*, 1998; *Kean and Smith*, 2005]. Specifically, the weights are defined as the inverse of the variance of the measurement error [*Draper and Smith*, 1981]. The variance of the measurement error is estimated by assuming that the errors are normally distributed and that it is 95% certain that the true discharge is within the percentage error of the measurement. Measurement error is quantified as being 2%, 5%, 8%, or 11% (>10%) corresponding to the USGS measurement quality ratings excellent, good, fair, or poor. Figure 5

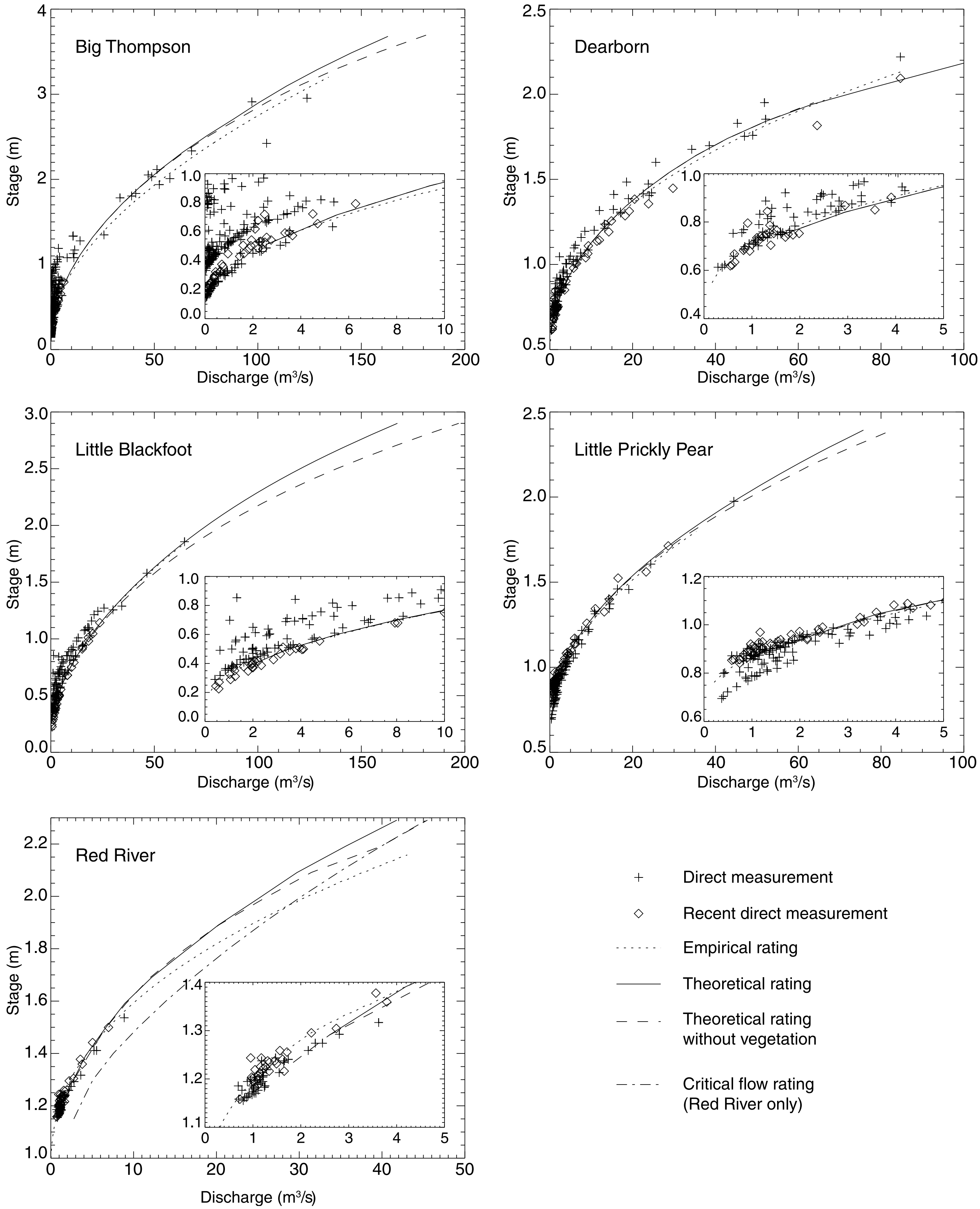


Figure 3. Comparison of rating curves and measurements for the five study sites (linear scale for stage and discharge). Stage is relative to the USGS gage datum. Direct measurements of discharge (excluding measurements with ice cover) are shown for the station's present location. Recent measurements (July 2005 to July 2007) are shown with the diamond symbol. The additional critical flow rating for Red River is calculated assuming a unit channel Froude number (Fr = 1) at the gage cross section.

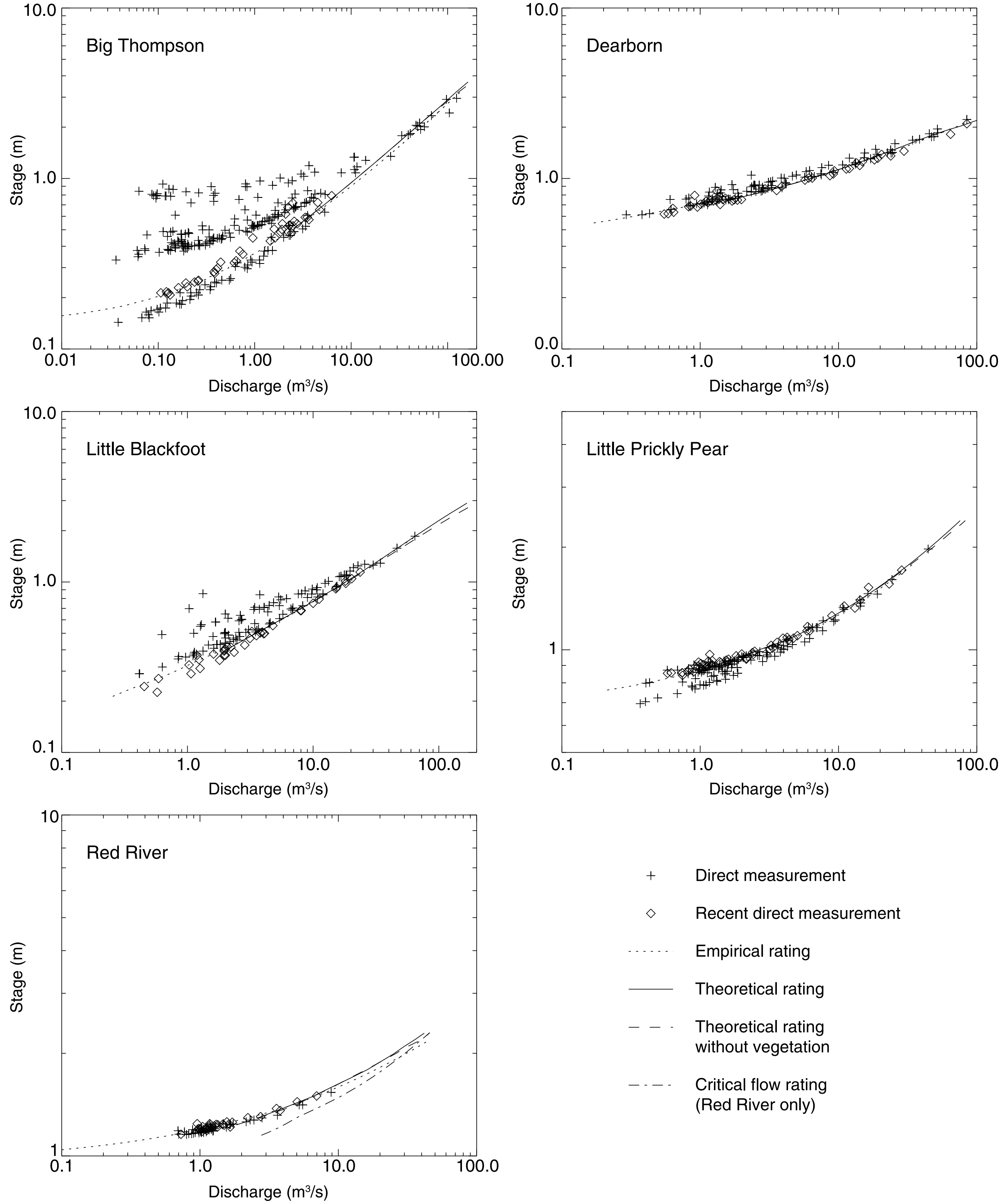


Figure 4. Comparison of rating curves and measurements for the five study sites (logarithmic scale for stage and discharge).

shows the SSWE of both the theoretical and the empirical ratings computed for different cumulative periods. The comparison begins with the most recent measurement (made no later than 14 July 2007) and goes back in 1-year increments to 12 years before the date of the topographic survey. The total weighted variance is normalized by the number of measurements (N) used in the comparison.

[36] If the empirical ratings were generated by an objective statistical analysis, then they would represent the best smooth fits to the rating measurements. Under these conditions, the

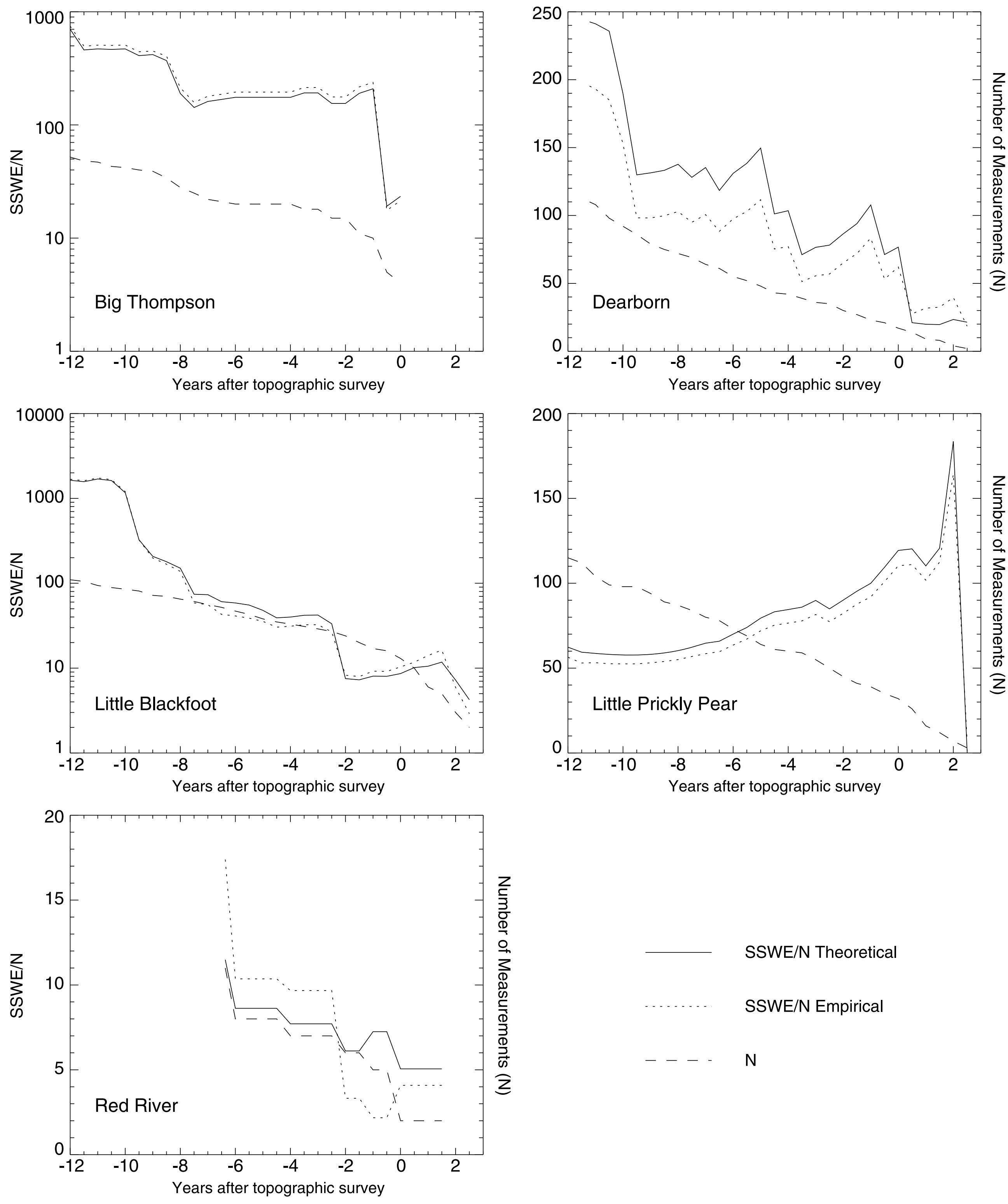


Figure 5. Comparison of the sum of the SSWE between measurements and rating curves as a function of time. N is the number of measurements in the comparison.

best that the theoretical rating curves could do is to have the same variance of the rating data around them, as is the case for the equivalent empirical curve. Owing to the facts that the rating curves are not entirely smooth and they were not fit in a least squares manner in a Cartesian space, it is possible for the theoretical rating curves to sometimes fit the data slightly better. By examining Figure 5, it can be seen that the sum of the SSWE for the empirical and theoretical rating curves is indeed very similar. At some sites and periods, the empirical rating has lower error, and in others, the theoretical

Table 3. Comparison of the SSE Between Measurements and the Lower and Upper Halves of the Rating Curves[a]

		Lower Rating ($H_{meas} < H_{50}$)			Upper Rating ($H_{50} < H_{meas}$)		
	H_{50} (m)	n	SSE/n Theoretical	SSE/n Empirical	n	SSE/n Theoretical	SSE/n Empirical
Big Thompson	1.86	15	1.70	2.07	9	195	134
Dearborn	1.39	28	1.75	1.33	16	41.1	43.8
Little Blackfoot	1.11	23	0.15	0.14	8	16.8	17.5
Little Prickly Pear	1.34	40	0.24	0.23	11	2.67	2.88
Red River	1.70	6	0.09	0.07	0	-	-

[a]H_{50} is the stage transition between the halves of the rating and H_{meas} is the stage for a measured discharge. Only recent measurements (as defined in Figures 3 and 4) are used in the analysis of the lower rating.

rating is lower. These two statistical measures are particularly close at all time intervals for the Big Thompson, Little Blackfoot, and Little Prickly Pear rivers, whereas the discrepancies are slightly larger for the Dearborn River and Red River.

[37] An additional statistical comparison can be made between the measurements and both the upper and lower halves of the rating. Here the halves of the ratings are distinguished by the midpoint between the highest and the lowest stage shared by the both the empirical and the theoretical rating curves. Table 3 summarizes the sum of squared error (SSE) between the half of the rating and the measurements. Note that no weighting is used because the range of discharge in each comparison is relatively narrow. As in the previous statistical analysis, the errors for the empirical and theoretical rating curves are similar. For the Montana sites, all of the results are very close, with the empirical curves fitting slightly better at the lower stages and the theoretical curves fitting somewhat better at the higher stages. The empirical curve also fits slightly better at low stage for the Red River. The opposite is true for the Big Thompson River. Nevertheless, the statistical results indicate that the theoretical rating curves are essentially as accurate and are sometimes more accurate than the empirical ones. In no cases were the theoretical rating curves substantially less accurate than the empirical ones. When examining Table 3 and Figures 3, 4, and 5, it is important to note that the theoretical rating curves are produced directly from the field measurements of channel geometry and roughness. They have no site-specific empirically adjusted coefficients, so in their purest form (as presented in this paper), nothing can be adjusted to improve or degrade the fits. Discrepancies must be accounted for by adding to the model, in a completely predictive manner, fluid-mechanical processes and effects that were not incorporated in the original calculation. The models used in this paper have been constructed without including any secondary processes or effects. As a consequence of the absence of site-specific empirically adjusted coefficients in the model from which the theoretical rating curves were calculated, and the close statistical agreement between the empirical and theoretical rating curves, one can conclude that both of these methods, at least for several of these rivers, are not only precise but also accurate for moderate and high flows. This result means that the main errors in both of the methods are the result of poorly resolved, nondischarge-related shifts in stage, which are associated with minor channel perturbations, such as debris getting stuck on the control. Therefore, to improve the monitoring of water discharge better procedures for identifying and correcting the discharge time series are required. This can be done using the model from which the theoretical rating curves are calculated in conjunction with subsidiary stage or surface-velocity measurements.

5.3. Equivalent Manning Coefficient

[38] The Manning coefficient (n) can be back-calculated from the theoretical rating curve as a function of stage using the mean friction slope and the hydraulic radius for the reach. As seen in Figure 6, the calculated Manning coefficients vary considerably with stage. Moreover, it is important to note that the Manning coefficients do not vary similarly or smoothly with stage. For comparison, an estimate of the Manning coefficient based on the empirical equation of *Limerinos* [1970] is also shown. That equation is given by

$$n = \frac{0.0926\ R^{1/6}}{[1.16 + 2.0 \log_{10}\ (R/D_{84B})]} \tag{6}$$

where the dimensions of R and D_{84B} are given in feet. The values of R used in Limerinos' empirical analysis ranged from 0.3 to 1.8 m, and D_{84B} ranged from 1.5 to 250 mm. It should be noted that the streams used to develop equation (6) were relatively wide and straight, had minimal irregularities, and had negligible bank vegetation. Equation (6) predicts a greater Manning coefficient for four of the five sites, despite the fact that the empirical equation does not contain sources of channel roughness other than the surface roughness of the uniformly distributed bed material. The different magnitudes and shapes of the curves in Figure 6 indicate that it is unlikely that a simple but accurate expression for Manning coefficients can be derived to fit broad classes of streams and that their use should be considered to give only a rough approximation of the actual discharge.

6. Summary and Conclusions

[39] This paper has presented a model-based approach to define the stage-discharge relations for common gravel bedded channels with or without bank and floodplain vegetation. The approach, which builds on the work of *Kean and Smith* [2005], uses simple field measurements of the topography and roughness of the reach to predict the effective roughness of the channels and floodplains over the full range of flow depths. The effective roughness is determined without using site-specific empirically adjusted roughness parameters (such as the Manning coefficient) but by calculating explicitly the drag on the dominant topographic and biological roughness elements in the channel. A fully predictive, fluid-mechanically based channel flow model is then used to calculate the theoretical stage-discharge relation for the

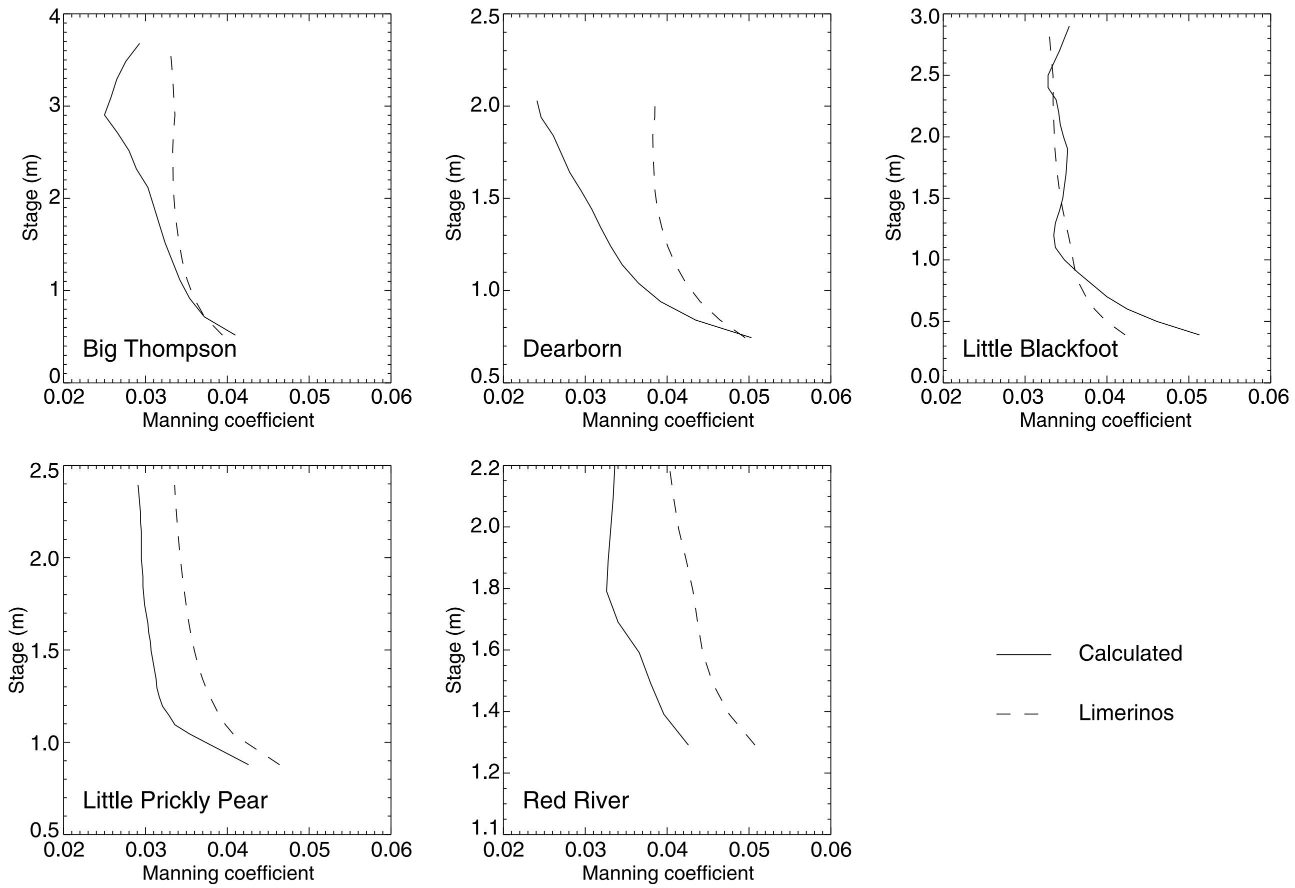


Figure 6. Variation of the Manning coefficients with stage. The additional resistance from vegetation, which is not included in the Limerinos equation, only slightly increases the back-calculated Manning coefficient at high stages for the Big Thompson and Little Blackfoot.

site using the calculated effective roughness and a water surface boundary condition.

[40] Tests of the method at five sites adjacent to USGS streamflow gaging stations show the theoretical rating curves to be in good agreement with direct measurements of discharge. These results indicate that the theoretical rating curve approach can be used by itself with good confidence at sites with hydraulic characteristics in the range of those tested in this study. Owing to its design, the theoretical rating curve method can also be used in conjunction with the standard USGS empirical rating curve technique to reduce the number of gage site visits and therefore significantly reduce the cost of operation of most gaging stations on gravel bedded rivers. Agreement between the theoretical and empirical rating curve methods also provides a foundation for assessing the accuracy, not just precision, of discharge time series, assuming that the stage is being measured accurately. Examination of data from these five, as well as other gaging stations, suggests that the largest source of error in gaging gravel bedded rivers is the occurrence of non-discharge-related shifts in stage, such as can be caused by debris lodged in the channel, seasonal growth of aquatic vegetation, or minor channel rearrangement.

[41] Despite the need for further testing of our approach, the results presented in this paper, together with those presented earlier in the study by *Kean and Smith* [2005], suggest that the "theoretical rating curve method" is a viable alternative to the robust and longstanding empirical method for defining stage-discharge relations. In addition, the calculations show that the theoretical rating curve approach can be used reliably in many situations where it is difficult to define all or part of the stage-discharge relation by empirical methods. These situations include (1) high flows that are too dangerous to measure; (2) remote sites on small streams, where measurement visits rarely coincide with high flows; and (3) new sites, such as those in burned areas, which require a rating curve to be established rapidly before the first postfire runoff event. Application of the theoretical rating curve approach to these situations can reduce the uncertainty of discharges relative to those obtained from standard indirect methods (e.g., the slope-area method), which rely on estimated roughness coefficients.

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