Stability of Steep Slopes in Cemented Sands

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Abstract: The analysis of steep slope and cliff stability in variably cemented sands poses a significant practical challenge as routine analyses tend to underestimate the actually observed stability of existing slopes. The presented research evaluates how the degree of cementation controls the evolution of steep sand slopes and shows that the detailed slope geometry is important in determining the characteristics of the failure mode, which in turn, guide the selection of an appropriate stability analysis method. Detailed slope-profile cross sections derived from terrestrial lidar surveying of otherwise inaccessible cemented sand cliffs are used to investigate failure modes in weakly cemented [unconfined compressive strength (UCS) <30 kPa] and moderately cemented ($30<\text{UCS}<400$ kPa) sands and their role in the evolution of the geometry of the slopes. The results show that high-resolution slope topography, such as can be obtained with terrestrial lidar, is essential for identifying altogether different failure modes in weakly cemented (shear-mode) and moderately cemented (tensile-mode) sand slopes. Analyses show that the standard Culmann method for steep slopes is inappropriate for modeling the stability of cemented sand slopes since it tends to overpredict expected crest retreat and underestimate failure plane angle. Instead, a simplified analysis using infinite slope assumptions, but applied to a slope with finite dimensions subject to changing geometric conditions, such as toe erosion and slope steepening, is suggested for analysis of weakly cemented sand slopes. For moderately cemented sand slopes, a limit equilibrium analysis directly comparing the cliff tensile stress and cemented sand tensile strength is shown to reasonably predict failure conditions and timing as a result of either slope steepening or tensile strength loss, presumably from wetting in most cases.

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Introduction

Steep slopes and cliffs composed of variably cemented sands are common along emergent coastal terraces throughout the world and are particularly common along the coasts of the western North and South American continents. In California, Oregon, and Washington, for example, coastal bluffs extending kilometers in length and composed of variably cemented sands support residences and infrastructure (Fig. 1) and are subjected to a suite of natural processes that trigger failures each year (Komar and Shih 1993; Benumof et al. 2000; Collins and Sitar 2008). The ability to form steep outcrops is a testament to cemented sand's intrinsic material strength; without cementation and resulting cohesion, these materials would only form slopes as steep as the angle of repose in the absence of any other minor apparent cohesion from water suction. However, despite their strength from cementation, the tendency for brittle and sudden failure make the need for accurate assessment of slope stability imperative, especially when lives and infrastructure are affected.

In recent years, there has been considerable research aimed at the geotechnical behavior of weakly and moderately cemented granular soils, including their strength characteristics (e.g., Clough et al. 1981; O'Rourke and Crespo 1988; Lade and Overton 1989; Das et al. 1995; Fernandez and Santamarina 2001; Dittes and Labuz 2002; Collins and Sitar 2009), liquefaction resistance (Frydman et al. 1980; Clough et al. 1989; Saxena et al. 1988; Baig et al. 1997), CPT resistance and foundation design (Rad and Tumay 1986; Puppala et al. 1995; Randolph et al. 2000), effect of seepage (Nikraz 1998), and arching ability (Abdulla and Goodings 1996). Similarly, the general processes affecting slope stability have received significant attention such as the work of Lohnes and Handy (1968) on loess slopes, Barton and Cresswell (1998) on quarry slopes in locked sands, and Arkin and Michaeli (1985), Sitar (1983, 1990), Sitar and Clough (1983), Ashford and Sitar (2002), and Hampton (2002) on coastal cliffs. Whereas these studies have addressed many important aspects of weakly and moderately cemented sand slope behavior, including their moisture sensitivity, the role of tensile stress generation, and the influence of depositional fabric, the adequacy of existing analysis techniques to accurately predict failure has not been satisfactorily addressed. In fact, one of the major challenges facing practicing engineers and planners in evaluating the viability of proposed developments in the coastal zone, for example, has been the lack of reliable guidance on how to approach slope stability assessment in these types of materials. In some instances, existing standard limit equilibrium analysis methods give very conservative solutions, suggesting that slopes should be failing (e.g., Hampton 2002 showed that some cemented sand and sea-cliff slopes in northern California are theoretically unstable although observed to be stable). In other cases, existing methods suggest that slopes should be stable, whereas they have been observed to fail (e.g., other areas observed by Hampton 2002 and some friable

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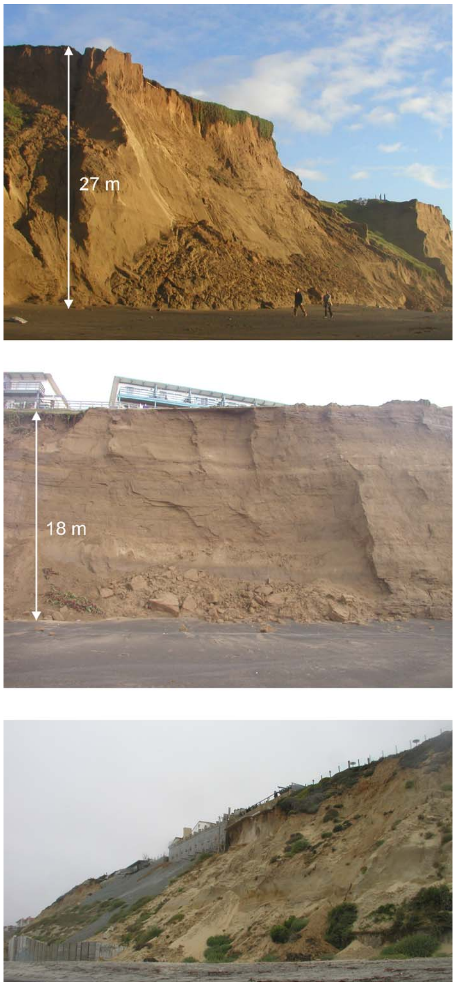

Fig. 1. Examples of (a) weakly; (b) moderately cemented sand cliffs in northern California; and (c) variably cemented sand cliffs in southern California. Note proximity of homes to cliff edge in (b) and (c).

loess slopes investigated by Lohnes and Handy 1968). In these latter cases, the slopes were generally assumed to be failing due to other unrecognized failure modes. Consequently, the use of existing methods lack credibility with the profession and the general public, and even more so, on predicting the magnitude of failures.

Thus, the objective of our research has been to obtain a better understanding of the factors controlling slope stability in these materials and to develop slope analysis methods consistent with their observed behavior. To this end, a detailed long-term study of a series of slopes was undertaken. This study included extensive measurements of material properties (Collins and Sitar 2009), long-term documentation of the geomorphic processes (Collins and Sitar 2008), and high resolution slope profile measurements using terrestrial lidar surveying. In this paper the results of these latter studies along with cemented sand specific slope stability analysis approaches using existing and simplified limit equilibrium techniques are presented. A case study of cliffs located in a coastal setting subject to a variety of possible failure modes and a long history of slope instability is used to show the application of these proposed approaches in a practical setting.

Failure Modes of Cemented Sand Cliffs

Assessing the stability of cemented-sand cliffs presents numerous challenges including the proper identification of the failure mode which governs the selection of an appropriate analysis method. Many cliffs, by their exposed nature, are subject to a suite of erosional forces including surface weathering (Carson 1971), groundwater or surface-water seepage (Norris and Back 1990; Hampton 2002), toe erosion (Sunamura 1982), and slope modification (Barton and Cresswell 1998). In areas of high seismicity, inertial forces may also play a role in decreasing relative slope stability (Sitar and Clough 1983; Ashford and Sitar 2002). Each of these processes is related to a particular failure mode (planar shear, rotational shear, rotational tension, etc.) which requires a specific method of analysis (e.g., planar translational infinite slope analysis, deep-seated rotational analysis, exfoliation tensile-stress analysis, cantilever block analysis, seepage-induced pore-pressure analysis, etc.). Thus, the correct identification of the failure mode dictates the degree of certainty that the slope stability analysis can be performed. Further, the specific mode must also be carefully assessed in concert with the actual cliff geometry and local material properties.

In a detailed, long-term study on the failure modes of coastal cliffs in northern California, Collins and Sitar (2008) showed that cliff geometry and failure mode are directly dependent on cliff material strength and yet are distinctively different in neighboring cliff sections despite similar providence of the cliff forming materials. Here, weakly cemented sands [e.g., Fig. 1(a), with unconfined compressive strength (UCS) <30 kPa] were observed to collapse in parallel-sided planar failures due to toe erosion from wave action, whereas cliffs formed in moderately cemented sands [e.g., Fig. 1(b), with UCS $=30$ to 400 kPa] collapsed in exfoliation slab-type failures as a result of groundwater seepage and resulting tensile-strength loss. As a part of the present study, the evolving geometry of these cliffs was analyzed from 2002 to 2008 through the collection of terrestrial lidar data of the cliff face, herein presented specifically for the purpose of performing stability analyses. From previous studies of slope stability of these bluffs, it was apparent that the exact geometry of the bluff face had an important influence on the predicted stability, and yet, until recently, traditional surveying techniques did not provide the capability to rapidly obtain high-accuracy topographic detail on inaccessible slopes or cliffs. With the advent of terrestrial lidar methods in the engineering geology field (e.g., Collins and Sitar 2004, 2008; Rosser et al. 2005; Oppikofer et al. 2009), this technology is now, for the first time, allowing the collection of detailed pre- and postgeometry of evolving cliffs in space and time.

Typical cross sections generated from the terrestrial lidar data indicate the key differences in failure mode between sands of varying degree of cementation. In weakly cemented sands [Fig. 2(a)], fairly specific, erosion-induced geometric conditions are required for failure. During the intervals October 2002–January 2003 and January 2003–March 2003, failures occurred with cliff-slope inclinations upwards of 60° and with vertical cliff-toe heights of between 0.7 and 1.4 m. Note that in Fig. 2, the cross section lines do not necessarily show the topography immediately before or following failure; some interpretation must be made to estimate the geometry of the slope and the related failure plane exactly at the time of failure. In moderately cemented sands [Fig. 2(b)], the geometric signature of the cliff is vastly different than its weakly cemented counterpart. Here, slope inclinations are nearer to vertical and the cliff does not undergo appreciable change at the toe as a result. Whereas the lack of debris at the cliff toe is due to removal by ongoing wave action, the measured slope inclinations make the case for the dependence and requirement of a stress-related failure condition; the cliff geometry is undoubt-

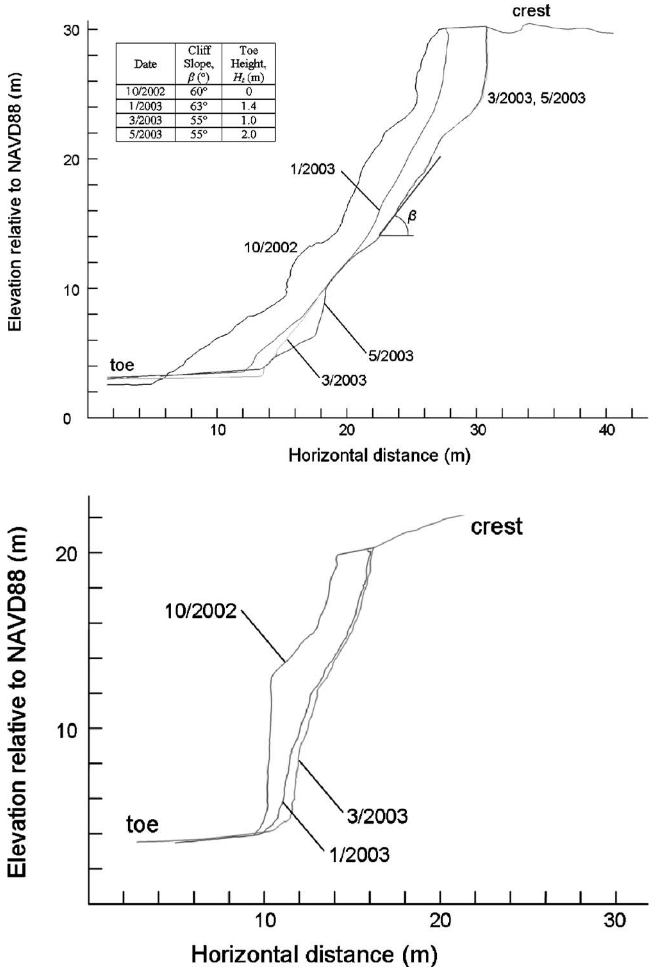

Fig. 2. Topographic expression of cliffs in (a) weakly cemented sand; (b) moderately cemented sand from selected dates of terrestrial lidar data collection. The cliff slope angle (β) is measured at the midslope of the cliff, omitting the influence of debris fans from previous failures at the toe. In (a), inset table shows data used for calculation of Eq. (7).

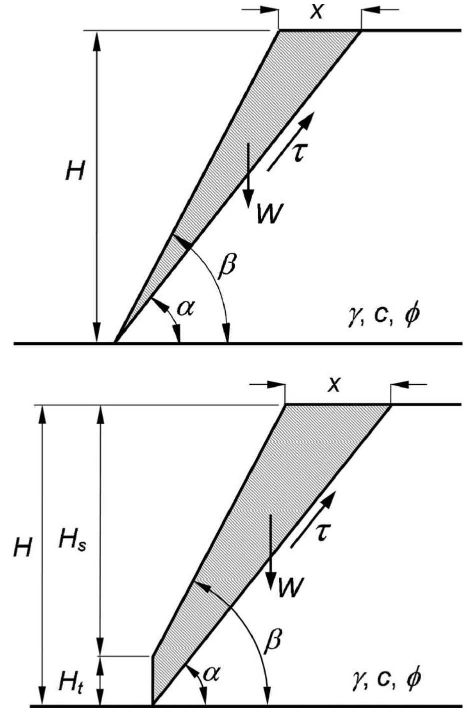

Fig. 3. Geometric assumptions made using the (a) conventional Culmann method; (b) modified Culmann method for vertical toe

edly generating tensile stress at the cliff face as a result of the near-vertical slope inclinations.

The dependence of the failure mode is not isolated to this particular case study. Although not explicitly highlighted, Hampton (2002) also observed variations in failure mode over a range of material strengths and localities from northern California (cohesion=200 to 800 kPa) to southern California (cohesion =400 to 950 kPa). Elsewhere, Arkin and Michaeli (1985) found strength and predominant failure mode variations in adjoining cemented sand units from studies of coastal cliffs in Israel. Thus, the role of cementation on failure mode is important to any study of slope stability in these materials.

Existing Methods of Slope Stability Analysis

Stability of steeply inclined slopes and vertical cliffs is typically modeled using either limit equilibrium or deformation (constitutive modeling) techniques. The choice between these techniques is based on the nature of the failure mode and material properties. For slopes that collapse suddenly with minimal strain potential, such as those composed of cemented sands, limit equilibrium techniques are preferred (Ashford and Sitar 2002). Additionally, the relative ease of implementation and the ability to analyze the development of failure planes from changing geometric (slope) conditions lends itself to these techniques. On the basis of the discussion of failure modes summarized in the previous section detailing planar failures in weakly cemented sand slopes and cliffs, existing analysis methods applicable to these materials are summarized here. Note that analysis methods for evaluating the effects of seismic events on cliff stability are purposely not included in the ensuing discussion as they have been previously investigated (Sitar and Clough 1983; Sitar 1990; Ashford and Sitar 2002).

Culmann (1866) was among the first to develop an analysis method for the stability of steep slopes, although it has been little appreciated that Français (1820) developed a similar methodology independently from Culmann some 46 years earlier. Both methods were based on even earlier work by Coulomb (1773) and were developed to calculate the maximum excavation depth, H in steep cut slopes at angle β [Fig. 3(a)]. The analyses presented a static solution for a rigid body of soil moving along a Mohr-Coulomb type shear-plane and most commonly denoted in the form

$$H = \frac{4c}{\gamma}\left[\frac{\sin\beta\cos\phi}{1-\cos(\beta-\phi)}\right] \tag{1}$$

where c=soil cohesion; ϕ=soil friction; and γ=soil unit weight. Beginning from the original geometric problem and disregarding trigonometric identities involving the shear plane inclination angle α, Eq. (1) can be expressed in terms of a factor of safety (F_s) of the shear strength parameters

$$\frac{c}{F_s}=\frac{\gamma}{2H}\left(\frac{H^2}{\tan\alpha}-\frac{H^2}{\tan\beta}\right)\left(\sin^2\alpha-\sin\alpha\cos\alpha\frac{\tan\phi}{F_s}\right) \quad (2)$$

where $\alpha=(\beta+\phi)/2$. In general, this method predicts reasonable results when the slope angle is steep, becoming more exact, especially for cohesive soils, as the slope approaches vertical (Taylor 1948). However, in the application of this analysis to typical cemented sand cliffs, the magnitude of crest retreat (as calculated by H, α, and β) is grossly overpredicted by up to five times its expected value and the failure plane inclination is underpredicted by up to 10° (Collins 2004). Hampton (2002) also found similar discrepancies with the predicted cliff height and slope inclination using the Culmann method when applied to cemented sand cliffs.

A slope stability model that more accurately simulates evolving cliff geometry, and particularly the presence of a vertical toe, was investigated by Carson (1971) for actively down-cutting rivers. This was later refined by Sunamura (1992) for wave-action induced failures in coastal settings. These methods assume a variable vertical toe height (H_t) and appropriately adjust the standard Culmann expression through use of an effective inclination angle as defined by the vertical toe geometry. Here, the total cliff height H is equivalent to the sum of the slope height (H_s) and H_t [Fig. 3(b)]. Through algebraic analysis, Sunamura's (1992) formula can be expressed in the same format as Eq. (2)

$$\frac{c}{F_s}=\frac{\gamma}{2H}\left(\frac{H^2}{\tan\alpha}-\frac{H_s^2}{\tan\beta}\right)\left(\sin^2\alpha-\sin\alpha\cos\alpha\frac{\tan\phi}{F_s}\right) \quad (3)$$

where

$$\alpha=\frac{1}{2}\left[\phi_N+\tan^{-1}\left(\frac{H^2}{H_s^2}\tan\beta\right)\right]$$

and

$$\phi_N=\tan^{-1}\left(\frac{\tan\phi}{F_s}\right)$$

Whereas this expression provides a slight improvement in the prediction of actual crest retreat and better simulates the true geometric conditions of evolving slopes, it still is not adequately accurate for the types of slopes studied herein especially when considering predictions of the slope and failure plane angle. Darby and Thorne (1996) successfully introduced improvements to this methodology for river-bank stability analysis including the use of variable pore pressure and a failure plane that is not constrained to pass through the toe. However, the resulting formula is generally only applicable to cohesive soils that undergo a magnitude of crest retreat greater than the shear plane surface depth (i.e., $\alpha<\beta$).

The preceding discussion is most specifically applicable to weakly cemented deposits where a clearly identifiable shear plane develops at depth over the length of the slope. In moderately cemented materials (i.e., those with UCS > 30 kPa), increased cementation and cohesion results in steeper slopes than weakly cemented sands and an altogether different failure mode may dominate. In general, shear failures that extend the entire height of the bluff in moderately cemented sand slopes are not as common whereas shallow, discontinuous brittle failures are more prevalent. Several analysis methods proposed for moderately cemented sand cliffs depend on the mode of failure and generally follow those developed for rock mechanics analysis rather than that for soil mechanics. For example, when planar joint- or bedding-controlled failure is suspected, analytical solutions for stability can be obtained using principals related to rock-block analysis (e.g., Cliche 1999). Moderately cemented sands may also form undercut sections from toe erosion, although in general, this occurs most often in strongly cemented sands (UCS > 400 kPa). However, when the cliff material is sufficient in strength to form these geometries, cantilever section analysis can be used (e.g., Sunamura 1982; Hampton 2002). Finally, fracture mechanics approaches have also proved to be useful for explaining cemented sand behavior (i.e., Sture et al. 1999). However, slope stability analysis using this method is still overly complicated with respect to the required material parameters needed for case-study application (Cai et al. 1990).

Failures resulting from a loss of tensile strength form an additional, although uncommonly identified, failure mode. This may result from a suite of processes, including fracturing and stress relief from cliff-face unloading or from a reduction of tensile strength due to wetting. Analyses have shown that cliffs often exist in a state of tension near the slope face (Sitar and Clough 1983; Ashford and Sitar 2001) and are therefore held in a pseudostate of stability by the tensile strength of the cemented sand. Fernandez and Santamarina (2001) showed that cemented sands are subject to tension forces when unloaded, especially when the unloading follows the development of cementation. This is typically the case for cliffs subjected to ongoing erosion at the slope face. With the exception of only minimal previous research (e.g., Hampton 2002; Sitar and Clough 1983), these failure modes have not been quantitatively explored with respect to moderately cemented sands and are therefore herein presented. Notably, these methods (and mode of failure) are not generally applicable to weakly cemented sands due to their negligible tensile strength.

Proposed Analysis Methods

Weakly Cemented Sand Slopes

To accurately account for crest retreat, slope inclination, failure-plane inclination, and toe geometry in weakly cemented sand slopes, a limit equilibrium analysis method is presented that allows for modeling variable cliff toe height, midslope inclination angle, and vertical crest with tension crack. The key assumption is that the failure plane inclination is parallel to the slope inclination (i.e., the infinite slope assumption). Although greatly simplifying the analysis, this assumption is also supported by the terrestrial lidar measurements previously presented.

The formulas combine conventional infinite slope stability concepts with a finite slope method using variable toe and crest heights to directly solve for force equilibrium (Fig. 4). No additional Culmann-style mathematics involving iteratively or derivatively solving for the appropriate inclination angle (α) at failure (i.e., $\alpha=(\beta+\phi)/2$) are necessary since α is equivalent to the changing slope angle (β) through the use of infinite slope analysis assumption.

Given the outline of a cliff profile with evolving toe (H_t), slope (H_s), and vertical, tension-cracked crest (H_{tc}) geometry and slope inclination (β) (Fig. 4), the weight (W) of the sliding wedge denoted by the shaded region is

$$W=\frac{\gamma}{2\tan\beta}(H^2-(H_s+H_{tc})^2) \quad (4)$$

where H = total cliff height and γ = total unit weight. The resisting (τ) and driving forces (W_T) are

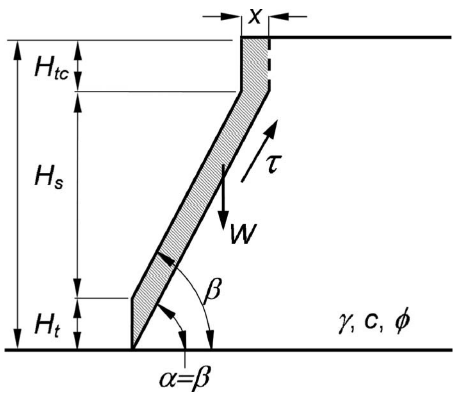

Fig. 4. Finite slope formulation with parallel shear plane. The toe (H_t), midslope (H_s, β), and crest (H_{tc}) geometry are allowed to vary. A nonvertical crest can be defined by assuming $H_{tc}=0$, but the expression requires at least some minimum value for H_t.

$$\tau = cL + W\cos\beta\tan\phi \tag{5}$$

and

$$W_T = W\sin\beta \tag{6}$$

where $L=(H_s+H_t)/\sin\beta$. Equating forces τ and W_T and keeping the same form of the expression as previously developed, the formulation in terms of the factor of safety is

$$\frac{c}{F_s} = \frac{\gamma}{2\tan\beta(H_s+H_t)}(H^2-(H_s+H_{tc})^2) \cdot \left(\sin^2\beta - \sin\beta\cos\beta\frac{\tan\phi}{F_s}\right) \tag{7a}$$

Because the traditional Culmann expression written in terms of a variable and unknown failure plane inclination angle (α) is not required, the expression can also be written explicitly for F_s

$$F_s = \frac{2c\left[\frac{(H_s+H_t)}{H^2-(H_s+H_{tc})^2}\right] + \gamma\cos^2\beta\tan\phi}{\gamma\sin\beta\cos\beta} \tag{7b}$$

for a nearly similar expression to that of infinite slope analysis.

Note that in Eq. (7*a*), for $H_t=0$ and $H_{tc}=0$ (i.e., $H_s=H$) and by excluding the assumption that $\alpha=\beta$ earlier in the derivation, the expressions for the original Culmann analysis [Eq. (2)] is obtained. However, in this form, a nonzero value for H_t must be assumed to generate a discrete failure plane depth. This is identical to selecting a fixed shear plane depth using infinite slope stability analysis, but here is tied directly into geometric considerations offered by the observations and cemented sand strength (i.e., a vertical toe and crest can exist). Eq. (7) can be used to identify the critical slope geometric conditions required for failure. That is, if H_t, H_s, and β change for whatever reason (toe erosion, excavation, etc.) the factor of safety can be explicitly computed. Change in vertical crest or tension crack formation can also be calculated, as can the horizontal crest retreat ($x=H_t/\tan\beta$, see Fig. 4) resulting from a given failure. The mathematical solution can therefore be easily implemented for rapid stability evaluation, especially over long lengths of cliffs.

Moderately Cemented Sand Slopes

Field observations of the evolution of steep slopes in higher-strength, moderately cemented sands indicate that slope steepening and tensile-strength reduction by wetting (e.g., from groundwater and surface water seepage) can be the primary causes of failure (Collins and Sitar 2008). Hence a separate analysis method is required for these materials.

Because tensile stresses in a near-vertical slope cannot be explicitly determined from a limit equilibrium analysis, a departure from LEM techniques must be made, despite their applicability to brittle-type failures such as those investigated here. Linear-elastic, finite-element method (FEM) analyses can be used to evaluate the stresses (tensile and otherwise) in a cliff face such that overall stability can be then evaluated back into an LEM format, for variable cliff geometries. Whereas other constitutive models for cemented sands have been proposed (Reddy and Saxena 1992; Rumpelt and Sitar 1993; Vatsala et al. 2001), Ashford and Sitar (2001) showed that linear elastic models can adequately simulate the tensile stresses that are important to steep slope stability if the mesh element size is selected to be less than approximately 3% of the slope height (element height $\leq H/32$).

The analysis method developed for moderately cemented sand cliffs compares the tensile strength distribution (σ_3, here taken as a negative value for tension) of a given slope to the soil's tensile-strength parameters. When the insitu tensile strength (σ_t) is greater (i.e., more negative) than the tensile stress, the slope is considered stable:

$$|\sigma_t| > |\sigma_3| \text{ for } \sigma_3 < 0(\text{stable}) \tag{8}$$

Two possibilities exist that may lead to instability: (1) the slope geometry changes such that the tensile stresses increase

$$|\sigma_t| < |\sigma_3| \text{ for } \sigma_3 < 0(\text{unstable}) \tag{9a}$$

or (2) the material tensile strength decreases, for example, as a result of wetting

$$|\sigma_{\text{t-wetted}}| < |\sigma_3| \text{ for } \sigma_3 < 0(\text{unstable}) \tag{9b}$$

Increases in tensile stresses at the cliff face upon slope steepening can be shown using FEM models (e.g., Sitar and Clough 1983). Additionally, decreases in tensile strength due to wetting has been verified by laboratory tests on cylindrical samples (Collins and Sitar 2009), and comparisons of data from other studies (e.g., O'Rourke and Crespo 1988) show a similar decrease between insitu and wetted tensile strength. Thus, either possible mechanism [Eq. (9*a*) or Eq. (9*b*)] can lead to failures of moderately cemented sand cliffs with the former being particularly important in cases where the slope profile is constantly being steepened from toe erosion (e.g., in stream banks, coastal cliffs or in excavated slopes). Note that, despite the use of FEM analyses to obtain the tensile stress distribution in a cliff, this approach is still considered a limit equilibrium analysis, albeit one that depends on the stress distribution to identify the failure surface. Although the proposed analysis does not provide an exact indication of the failure magnitude, it does give an explicit explanation of the observed failure mode which is considered an improvement over existing analysis methods with regard to failure timing.

Application to a Case Study

The analytical solutions presented above were applied to a case study consisting of 18 to 27 m variably cemented sand coastal bluffs in northern California. This area was chosen because it is relatively easily accessible and it has been under more or less constant surveillance since the late 1970s (e.g., Sitar 1983; Hampton 2002; Collins and Sitar 2008). Also, the area is relatively

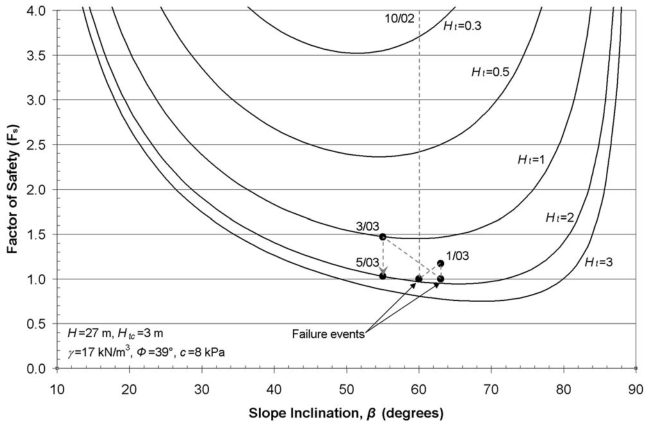

Fig. 5. Factor of safety as a function of slope inclination (β) and cliff toe height (H_t) for parallel shear plane cliff analysis [Eq. (7)] for the weakly cemented sand slope case study. Dashed line shows path of cliff in Fig. 1(a) during 2002–2003 according to geometry in Fig. 2(a) with two documented failures. Failure eventually occurred again in December 2003.

densely developed up to the cliff edge and hence the question of slope stability is more than academic, being the subject of intense scrutiny over the past 10 years.

Analysis of Weakly Cemented Sand Cliffs

During the winter of 2002–2003, the weakly cemented sea-cliff shown in Fig. 1(a) underwent several cycles of failure with toe heights reaching as high as 2 m. Geologic mapping of the cliff reveals that the upper 3 m of the cliff (~10% of the cliff height) is composed of moderately cemented sand (Collins and Sitar 2008) and it is assumed that a tension crack may develop over this height (i.e., $H_{tc}=3$ m). Whereas the following analysis makes the simplified assumption of a homogenous slope in applying Eq. (7), an expression for a heterogeneous slope can also be developed using similar limit equilibrium principles but is not presented here. In the subsequent analysis, soil parameters are selected from Collins and Sitar (2009) for the average unit weight and midrange confining stress (20–50 kPa) nonlinear shear-strength parameters ($\gamma=17.0$ kN/m^2, $\phi=39°$, $c=8$ kPa). The applicability of this stress range to the case study cliffs has been previously confirmed by finite-element analysis by Sitar and Clough (1983) and Collins (2004).

Given this information, the factor of safety in Eq. (7) is plotted for a variety of toe heights (H_t) and slope angles (β) and depicts the evolving stability envelope of the slope from this failure mode (Fig. 5). The pseudoparabolic shape of the stability lines highlights the interplay of the tangential stress and strength components in Eq. (7). With increasing slope angle, the normal stress component (first third of the right hand side of the expression) decreases, whereas the shear-strength component (last third of the expression) increases. The bounds of the expression represent a flat slope ($\beta=0°$, with no shear plane tangential stress) and a perfectly vertical slope ($\beta=90°$, with zero shear plane normal stress and with tangential stress acting directly downward). Thus, assuming a nonzero value of cohesion, an increase in stability is predicted at very high slope inclinations using this failure mode analysis, identical to that found for infinite slope analysis (Duncan and Wright 2005). The expression should therefore only be used for calculating the failure geometry when failure will first occur in an evolving, steepening slope. Although the expression also technically explains how a cliff can become less stable through initial excavation to a very steep inclination (80°+) and subsequent slope flattening, with the shear stresses overcoming the shear-strength components, other potential failure modes may occur first.

When several stages of the evolving slope geometry are known, they can be plotted so that impending failure can be predicted. The formula provides both a method for the relative stability to be assessed and for the slope conditions required for failure to be identified. For the case study described here [Figs. 1(a) and 2(a)], observations from the 2002–2003 winter indicate that this cliff failed when the toe reached about 2 m in height and the midslope reached an inclination of about 60° (dashed line, Fig. 5). Here, the terrestrial lidar slope-geometry data are used to plot several stages of the cliff evolution during this time. As the plot indicates, stability is rapidly reduced with the growth of a vertical toe section. The two failures from 2002–2003 ($F_s=1$), each with approximately 2–3 m of crest retreat [Fig. 2(a)], are easily identified and additional failures similar to them (e.g., following an erosive event soon after May 2003) could be presumably anticipated given this level of high resolution data.

Analysis of Moderately Cemented Sand Cliffs

The tensile stress analysis developed for moderately cemented cliffs is tested here in a case study comparison relying on observations provided from the present study. A three-material, finite-element analysis constructed of nine-noded, unstructured elements (Geo-Slope 2009) was performed to model the distribution of tensile and compressive stresses using the cliff geometry of the left-most cross section shown in Fig. 2(b). An element height of $H/40$ (with H=total cliff height) was selected, consistent with that needed to model tensile stresses (Ashford and Sitar 2001). Elastic and tensile strength soil parameters for the moderately cemented sand (Table 1) were obtained from site-specific geotechnical testing (Collins and Sitar 2009); parameters for dense beach sand and Franciscan bedrock (greenstone/metabasalt) (Table 1) were taken from Das (2002) and West (1995), respectively.

The linear elastic analysis results (Fig. 6) show that the back-crest area and the majority of the cliff face exist in a state of tension extending a few meters into the cliff. Note that the tension zone in the back-crest area does not result in slope failure but

Table 1. Material Properties for FEM Tensile Stress Modeling of Moderately Cemented Sand Cliff

Layer	Elastic modulus, E (kPa)	Poisson's ratio, ν	Unit weight γ (kN/m^3)	Void ratio, e	Tensile strength, σ_t (kPa)
Moderately cemented sand	$1.15\cdot10^5$	0.295	18.7	0.6	32 (dry), 6 (wet)
Dense beach sand	$3.5\cdot10^4$	0.3	17.2	0.45	n/a
Franciscan bedrock	$5.74\cdot10^7$	0.2	28.3	0.2	n/a

Note: n/a=not applicable.

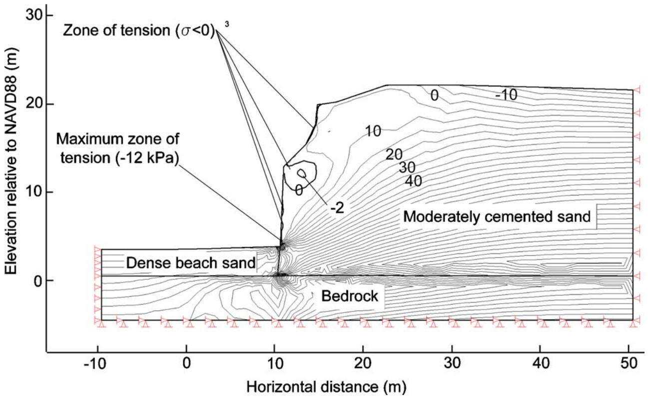

Fig. 6. Distribution of minor principal stress (σ_3) at the bluff face in typical moderately cemented slope. All stresses are in kilopascals.

rather contributes to overall soil mass weakening. Additionally, it may potentially increase surface-water inflow to the slope through the formation of tension cracks, often observed in the field. At the cliff face, the predicted tensile stresses are approximately −2 kPa but increase up to −12 kPa at the bottom of the cliff. Thus, using the tensile stress mechanism approach [Eqs. (8) and (9)], the slope is determined to be stable at its in situ (dry) condition with tensile strength (32 kPa, Table 1) greater than the tensile stresses. Upon wetting, the tensile strength (6 kPa, Table 1), is less than the tensile stresses and failure is predicted to occur according to Eq. (9*b*).

Discussion

Weakly Cemented Sand Cliffs

To evaluate the effectiveness of Eq. (7) in estimating both the failure geometry and associated crest retreat of typical weakly cemented sand cliffs, the results of several other methods are compared (Table 2) using the case-study soil ($\gamma=17$ kN/m^3, $\phi=39°$, $c=8$ kPa) and slope ($H=27$ m, $H_t=2$ m, $H_{tc}=3$ m) parameters used in the previous section. Here, the slope and failure plane inclinations along with the resultant crest retreat are back-calculated to achieve a factor of safety of 1.0 with each method. The slope stability program SLOPE/W (Geo-Slope 2009) was used for Spencer's method-of-slices analysis by forcing the slip surfaces through the cliff toe; other methods were programmed and solved analytically. For the Culmann analysis [Eq. (2)], no value for H_t or H_{tc} is used; for the Culmann with vertical toe [Eq. (3)], no value for H_{tc} is used; and for the finite slope method of slices, the failure plane follows that directly determined by Fig. 4. Finally, for the upper bound limit analysis (e.g., Chen 1975; Donald and Chen 1997), associated flow law for plastic flow along the slip surface with Mohr-Coulomb failure criterion separating two rigid bodies was assumed and developed specifically for the proposed collapse mechanism shown in Fig. 4

$$F_s=\frac{2c\left[\frac{(H_s+H_t)}{H^2-(H_s+H_{tc})^2}\right]\cos\phi}{\gamma\sin(\beta-\phi)\cos\beta} \tag{10}$$

Each of these assumptions is consistent with its associated methodology. The results show the proposed limit equilibrium method calculations are consistent with those obtained using method-of-slices analysis, with a similar slope and failure plane inclination predicted for the given geometry for $F_s=1$ (i.e., $57° \approx 59°$). However, comparisons with other methods show their limitation by predictions of excessive crest retreat and shallower than observed shear-plane inclination-angle (Table 2). Furthermore, the proposed method is shown to be equivalent with the upper bound limit analysis approach at failure [Eq. (10)], which by definition indicates that the postulated mechanism of collapse (Fig. 4) is an upper bound to the true collapse load condition. The proposed approach therefore provides an easily tractable, accurate, analytic solution for evaluating the stability of weakly cemented sand cliffs subject to changing geometric conditions.

Moderately Cemented Sand Cliffs

The most important features of the tensile stress mechanism analysis are that failure is localized at the lower cliff-face area, failure does not propagate very far into the cliff profile, and that failure only occurs when the wetted soil strength parameters are used. For this slope geometry, the predicted depth of failure into the slope is 0.5–1 m, consistent with field observations (Collins and Sitar 2008). Given this agreement between observed and modeled failure modes, limit equilibrium methods that utilize a wedge or other such form of shear plane are judged inappropriate for these types of slopes. This conclusion is reinforced by application of the standard Culmann method [Eq. (2)]. For identical cliff height and actual soil parameters under wetted conditions for this cemented sand ($\gamma_{sat}=19.6$ kN/m^3, $\phi=47°$, $c=34$ kPa; Collins and Sitar 2009), the Culmann expression predicts an 89° slope angle at failure, a 68° failure plane inclination, and an expected crest retreat of 7 m. Although the slope angle at failure is approximately correct, the magnitude of crest retreat is much larger than expected or observed in these cliffs (Collins and Sitar 2008) and the failure plane inclination is much too shallow. In general, it is the inability of this method and others based on it, to

Table 2. Comparison of Analysis Results for Typical Weakly Cemented Sand Cliff Failure

Analysis method	Slope angle, β (degrees)	Failure plane angle, α (degrees)	Crest retreat, x (m)
Field observations	60°–65°	60°–65°	0.5–3
Proposed method [Eq. (7)]	57	57	1.3
Spencer's method of slices—finite slope verification	59	59	1.2
Conventional Culmann [Eq. (2)]	56	48	6.6
Culmann with vertical toe [Eq. (3)]	52	48	5.2
Spencer's method of slices—circular	48	n/a[a]	3.1
Upper bound limit analysis [Eq. (10)]	57	57	1.3

[a]No estimate of a constant failure plane inclination is possible due to a circular slip surface.

correctly model the predominant failure mode that makes these approaches inappropriate for moderately cemented sand slopes such as those investigated here.

Conclusions and Recommendations

Failure modes of steep, variably cemented sand slopes were investigated to develop an approach to slope stability that would adequately capture the predominant slope behavior (i.e., geometry) in these materials. Detailed topographic measurements obtained using terrestrial lidar scans were used to guide the development of the analyses. Incorporating high resolution topography in the slope stability analyses was found to be essential to obtaining results that are representative of the observed slope response.

In general, due to the low shear strength (and lack of tensile strength) of weakly cemented sand cliffs, the details of the evolving site-specific geometry (e.g., an oversteepened slope toe) must be considered. Existing analysis approaches, such as the Culmann method, typically underpredict the shear plane inclination and overpredict the amount of crest retreat at the top of the slope, in some cases, by an order of magnitude. As a result, only limit equilibrium method analyses which model the primary geometric features of the slope profile should be used. The approach presented herein uses the infinite slope assumption coupled with geometric considerations that account for oversteepened or vertical slope sections at the slope toe and crest. A logical conclusion that results from the presented analyses mandates that when slope instability mitigation is necessary, measures should be aimed at preventing those geometric changes (i.e., toe erosion, slope steepening) that will lead to failure. This may include either slope grading, retaining-wall construction, or toe protection from erosive sources.

Due to their high cementation strength, moderately cemented sands may form steeper slopes and cliffs, compared to their weakly cemented counterpart, where the likely failure mode is due to tensile stresses rather than shear stresses. In the absence of an existing analysis method that captures the observed slope failure mode correctly, a new tensile stress approach was developed. The approach compares the in situ slope tensile stresses to the potentially water-content variable, soil tensile-strength and can be used as an effective guide for assessing relative stability. If needed, linear elastic FEM analysis can be used to provide quantification of realistic stress values. When general instability is a result of tensile failure, appropriate mitigation measures may include groundwater and surface-water management (to maintain tensile strength) or slope grading and layback (to prevent the development of tensile stresses). However, other failure mechanisms (such as toe erosion or bedrock-controlled fracturing) should also be addressed, if present, regardless of the specific material properties.

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