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ESTIMATION OF CONTROL ROD WORTH FROM MEASUREMENTS AT THE BR2 REACTOR USING KINETICS EQUATIONS WITH INCLUDED PHOTONEUTRONS

S. Kalcheva and E. Koonen
SCK•CEN, BR2 Reactor Department
Boeretang, 2400 Mol – Belgium

ABSTRACT

Ihe reactivity worth of the control rods in the BR2 reactor is strongly influenced by the delayed photoneutrons, emitted after (γ, n) – reactions of photons with the beryllium matrix. To analyze this reactivity worth, important to evaluate the correct value of the shut down margin, refined estimations of the control rod worth are performed using the reactor kinetics equations including delayed photoneutron groups. The reactivity worth of the control rods are derived from nuclear measurements, which have been performed during several BR2 shutdowns in 2009. The experimental program included a series of rod-drop tests and asymptotic reactor period measurements. The importance of the photoneutrons for the reactivity worth determination depends on the chosen analysis method. The reactivity worth from the rod-drop tests is estimated using an approximate equation for the neutron density decay, the validity of which was verified by comparison with the numerical solutions, obtained by the transient code PARET/ANL V7.5. The contribution of the photoneutrons into the reactivity worth, analyzed by this method for the BR2 reactor conditions is 2%. Neglecting the photoneutrons contribution in the inhour equation leads to underestimation of the rod worth by 18%. A comparison with reactivity worths, calculated by MCNPX are presented.

1. INTRODUCTION

The BR2 reactor is a heterogeneous high flux engineering test reactor at SCK-CEN (Centre d'Etude de l'energie Nucléaire) in Mol at a thermal power 60 to 100 MW, cooled by light water. The reactor is reflected and moderated by a beryllium matrix, positioned inside the core. The BR2 core is composed from skew beryllium hexagonal prisms, loaded with highly enriched uranium fuel elements (93% U5), control rods and experimental devices in test holes, which are arranged in a twisted hyperboloid bundle. The control rods are six shim-safety rods, which use cadmium as absorber material. The detailed description of the BR2 reactor can be found in [1-2].

In the framework of the BR2 optimization project for the replacement of the presently used reference cadmium control rods with foot-end of cobalt by the new hafnium rods with stainless steel foot-end [1], several experimental programs have been developed and executed during the shutdowns of the BR2 cycles in 2009. A series of measurements, including more than 30 rod-drop tests (scrams), asymptotic reactor period measurements, based on the doubling time method and measurements of the axial form of the reactivity rod worth by compensation movement of a set of other rods have been performed. The purpose of these measurements was to compare the absolute values and the axial form of rod reactivity worth obtained by independent experimental methods and calculations: by scram; from reactor period measurements in combination with perturbation method [2]; Monte Carlo evaluations (MCNPX 2.7.A, [3]). The contribution of the photoneutrons from (γ, n) – reactions, generated in the beryllium matrix by the delayed gammas, is taken into account in the estimation of the reactivity worth by all methods. The effective beta fractions for 6 delayed neutron and 16 delayed photoneutron groups were determined experimentally in the BR02 mock-up reactor [4]. The reactivity worth from the rod-drop tests is estimated using an equation for the neutron density decay expressed as a sum of exponentials with included photoneutrons. The validity of this equation is verified on comparison with numerical solutions, obtained by the transient code PARET/ANL V7.5 [5]. The rod reactivity worth can be derived also from the reactor period measurements using the inhour equation with included 6 delayed neutron groups and only short-lived (4, 5) or all 16 delayed photoneutrons groups. The uncertainties in the estimation of the rod reactivity worth by all presented methods are discussed. Comparing the different experimental methods we conclude that the rod-drop test (scram) with correct interpretation of the experimental data gives more reliable results for the absolute values of the rod reactivity worth vs. the reactor period measurement, since the uncertainties in the latter are higher. The experimental program included the following measurements: individual drop of each rod from the highest control rod position, $Sh = 900$ mm and estimation of the individual total rod worth; individual drop of each rod from $Sh = Sh_{crit}$ and estimation of the individual rod worth from the critical position; drop of all 6 rods from $Sh = Sh_{crit}$ and estimation of the mutual rod worth from the critical position, taking into account the rods interaction; estimation of the axial form of the control rod worth by individual drop of a single rod from different axial positions; estimation of the axial form of the same single rod by compensation movement of a set of other rods; measurement of the asymptotic period with all 6 rods.

2. MEASUREMENTS OF THE EFFECTIVE BETA FRACTIONS OF DELAYED NEUTRONS AND DELAYED PHOTONEUTRONS IN BR02 MOCK-UP

The beryllium, which is used in the beryllium matrix of the BR2 reactor core, has a significant (γ, n) – reaction cross section with a low gamma–rays threshold energy of $Q_\gamma(Be) \approx 1.6$ MeV. The typical gamma radiation in the BR2 reactor has continuous energy spectrum with a maximum energy below 5 MeV. The evaluated by MCNPX photon spectra from different photon sources are depicted in Fig. 1. All gamma irradiation above the threshold can give rise to photoneutrons. In the subsequent analysis of the kinetic behaviour of the reactor, we are most interested in the delayed photons giving rise to photoneutrons. The delayed neutron and delayed photoneutron parameters, which have been determined by precise scram measurement in the BR02 mock-up reactor [4], are summarized in Table I. The effective delayed beta fraction, including the photoneutrons is defined by the following equation:

$$\beta^{eff} = \sum_{i=1}^{6} \beta_i^{eff} + \sum_{j=1}^{16} \beta_j^{eff} = \bar{\gamma}\beta = \sum_{i=1}^{6} \gamma_i \beta_i \left[1 - \exp(-\lambda_i t_0)\right] + \sum_{j=1}^{16} \gamma_{p\,j} \beta_j \left[1 - \exp(-\lambda_j t_0)\right]. \quad (1)$$

In Eq. (1) index i refers to delayed neutrons and index j – to delayed photoneutrons, t_0 is the time interval during which the reactor is maintained at a steady power level prior to the reactivity insertion transient, thus $\left[1 - \exp(-\lambda t_0)\right]$ is a correction factor to the unsaturated precursor concentrations.

Figure 1. Photon spectra in the beryllium matrix of the BR2 reactor, caused by prompt and delayed photons and by photons, escaped in (n,γ) – reactions. Normalization: to the total number of the photons of type "*i*", *i*=prompt, delayed, captured, produced in 1 fission.

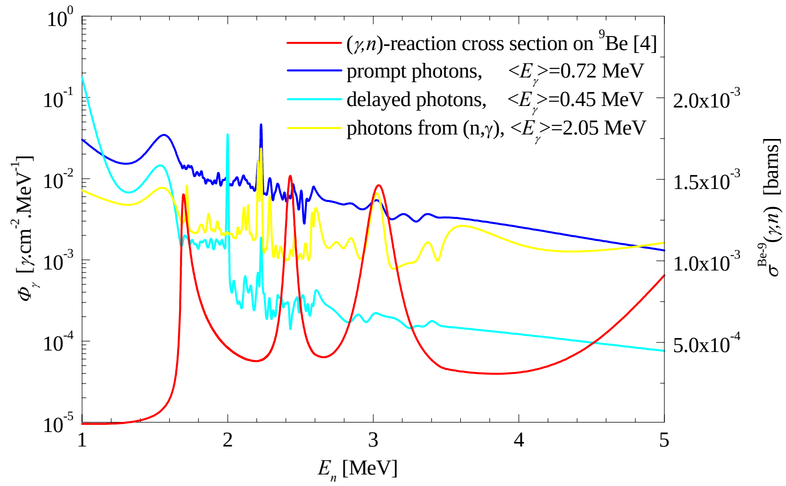


Table I. Measured effective fractions beta of delayed neutrons and delayed photoneutrons in the BR02 mock-up reactor [4][1].

	N° gr.	Period $T_{i,j}$, [sec]	Measured $\lambda_{i,j}$, [sec-1]	"Ordinary" beta, $\beta_{i,j}$	Effective beta (core), $\beta_{i,j}^{eff}$	Effective beta (Be-matrix), β_{j}^{eff}
Photon	16	$1.106\ 10^{6}$	$6.266\ 10^{-7}$	$1.440\ 10^{-7}$	$1.680\ 10^{-7}$	0
	15	$2.797\ 10^{5}$	$2.478\ 10^{-6}$	$1.823\ 10^{-7}$	$2.120\ 10^{-7}$	$4.200\ 10^{-7}$
	14	$6.120\ 10^{4}$	$1.132\ 10^{-5}$	$1.453\ 10^{-7}$	$1.690\ 10^{-7}$	$3.390\ 10^{-7}$
	13	$2.405\ 10^{4}$	$2.882\ 10^{-5}$	$2.523\ 10^{-6}$	$2.940\ 10^{-6}$	$4.330\ 10^{-6}$
	12	$1.296\ 10^{4}$	$5.347\ 10^{-5}$	$3.070\ 10^{-7}$	$3.580\ 10^{-7}$	$6.520\ 10^{-7}$
	11	$9.972\ 10^{3}$	$6.950\ 10^{-5}$	$2.728\ 10^{-6}$	$3.180\ 10^{-6}$	$6.370\ 10^{-6}$
	10	$4.620\ 10^{3}$	$1.500\ 10^{-4}$	$5.473\ 10^{-6}$	$6.380\ 10^{-7}$	$1.324\ 10^{-5}$
	9	$3.120\ 10^{3}$	$2.221\ 10^{-4}$	$3.263\ 10^{-6}$	$3.800\ 10^{-6}$	$5.700\ 10^{-6}$
	8	$1.920\ 10^{3}$	$3.609\ 10^{-4}$	$2.070\ 10^{-6}$	$2.410\ 10^{-6}$	$4.970\ 10^{-6}$
	7	$0.900\ 10^{3}$	$7.700\ 10^{-4}$	$8.107\ 10^{-6}$	$9.450\ 10^{-6}$	$1.985\ 10^{-5}$
	6	$0.246\ 10^{3}$	$2.817\ 10^{-3}$	$8.724\ 10^{-6}$	$1.016\ 10^{-5}$	$2.123\ 10^{-5}$
	5	$0.138\ 10^{3}$	$5.022\ 10^{-3}$	$1.482\ 10^{-5}$	$1.730\ 10^{-5}$	$9.060\ 10^{-5}$
	4	86.0	$8.058\ 10^{-3}$	$1.136\ 10^{-5}$	$1.320\ 10^{-5}$	$2.800\ 10^{-5}$
	3	55.6	$1.246\ 10^{-2}$	$1.716\ 10^{-5}$	$2.000\ 10^{-5}$	$4.630\ 10^{-5}$
	2	33.0	$2.100\ 10^{-2}$	$4.366\ 10^{-5}$	$5.090\ 10^{-5}$	$1.037\ 10^{-4}$
	1	16.0	$4.331\ 10^{-2}$	$3.494\ 10^{-5}$	$4.070\ 10^{-5}$	$8.030\ 10^{-5}$
Del. n	6	55.6	0.01246	0.00021	0.00025	0
	5	22.7	0.03053	0.00142	0.00166	0
	4	6.22	0.11142	0.00128	0.00149	0
	3	2.30	0.30130	0.00257	0.00299	0
	2	0.61	1.13607	0.00075	0.00087	0
	1	0.23	3.01304	0.00027	0.00032	0
				$\beta = 0.00666$	$\beta_{core}^{eff} = 0.00776$	$\beta_{refl.}^{eff} = 0.00042$

[1] The maximum error in the measured photoneutron effectiveness is ±12% and in the measured decay constants ±2.3% for short lived, and less than ±1% for long-lived delayed photoneutrons [4].

The effectiveness $\gamma_i, \gamma_{p\,j}$ of the delayed neutrons and delayed photoneutrons is different from that of the prompt neutrons. The delayed neutrons and delayed photoneutrons have a lower average energy than prompt neutrons which results in lower leakage out of the reactor. The precise evaluation of the control rod worth requires including the delayed photoneutrons in the reactor kinetics equations and determination of the relative effectiveness of the delayed neutrons $\gamma_{p\,j}/\gamma_i$. The exact values of β_i^{eff}, β_j^{eff}, which have been determined by measurements for fuelled beryllium assemblies inside the core and in the surrounding reflector (non-fuelled) channels, are given in Table I. The average effectiveness of the photoneutrons relatively to the effectiveness of the delayed neutrons is $\left(\overline{\gamma_{p\,j}}/\overline{\gamma_i}\right)^{core} = 1.0$ for the fuelled core channels and $\left(\overline{\gamma_{p\,j}}/\overline{\gamma_i}\right)^{reflector} = 1.9$ for the reflector channels.

3. APPROXIMATE SOLUTION OF REACTOR KINETIC EQUATIONS WITH INCLUDED DELAYED PHOTONEUTRONS

The response of the reactor to a reactivity step insertion Δk is given by an expression for the variation of the neutron flux density (or reactor power) as function of time. The point-reactor kinetic equations for a nuclear reactor including photoneutrons can be written as:

$$\frac{dn}{dt} = \frac{k\left(1-\bar{\gamma}\beta\right)-1}{l}n + \sum_i \gamma_i \lambda_i C_i + \sum_j \gamma_{p\,j} \lambda_j C_j\,, \tag{2}$$

whereby the delayed neutrons precursors C_i are described by:

$$\frac{dC_i}{dt} = \frac{nk\beta_i}{l} - \lambda_i C_i\,, \tag{3}$$

and the photoneutron precursors C_j and their parent isotopes P_j by

$$\frac{dP_j}{dt} = \frac{nk\beta_j}{l} - \lambda_{p\,j} P_j \qquad \frac{dC_j}{dt} = \lambda_{p\,j} P_j - \lambda_j C_j\,. \tag{4}$$

The above equations for the neutron density $n(t)$ as function of time t can be solved numerically, or in some special cases, by Laplace transform (e.g., for step reactivity insertions). In this paper we will use two important relations derived by classical methods, i.e. by Laplace transform on the point-kinetic equations listed above (see Appendix A). First, the relation between the stable reactor period τ and a reactivity insertion Δk for systems with photoneutrons is represented by the inhour equation:

$$\rho = \frac{\Delta k}{k\bar{\gamma}\beta} = \frac{l}{k\bar{\gamma}\beta\tau} + \sum_i \frac{\left(\gamma_i\beta_i/\bar{\gamma}\beta\right)}{1+\lambda_i\tau} + \sum_j \frac{\left(\gamma_{p\,j}\beta_j/\bar{\gamma}\beta\right)}{1+\lambda_j\tau}\left[1+\frac{\lambda_j}{\lambda_{p\,j}+t_0^{-1}}\right]. \tag{5}$$

In general, the photoneutrons can be treated as individual groups formed directly in fission, then $\lambda_{p\,j} \to \infty$ and the factor $1+\lambda_j/\left(\lambda_{p\,j}+t_0^{-1}\right) \to 1$. In this paper, we consider the 16-group Be-photoneutron parameters as additional groups of delayed neutrons with corresponding decay constants λ_j and fractions β_j, buildup factors $\left[1-\exp\left(-\lambda_j t_0\right)\right]$ and photoneutron effectiveness $\gamma_{p\,j}$.

Second, we will use the evolution of the neutron density $n(t)$ as a function of time t after a negative step reactivity insertion, e.g. as would be observed after a reactor scram or control rod drop. This evolution is expressed by a sum of exponentials [6] and can be approximately written as:

$$\frac{n(t)}{n_0} = \sum_{i}^{\substack{delayed\\neutrons}} \frac{\gamma_i \beta_i}{\overline{\gamma\beta} + \Delta k}\left[1 - \exp\left(-\lambda_i t_0\right)\right]\exp\left[-\lambda_i\left(1 - \frac{\gamma_i \beta_i}{\overline{\gamma\beta} + \Delta k}\right)t\right] + \\ + \sum_{j}^{\substack{photo-\\neutrons}} \frac{\gamma_{p\,j} \beta_j}{\overline{\gamma\beta} + \Delta k}\left[1 - \exp\left(-\lambda_j t_0\right)\right]\exp\left[-\lambda_j\left(1 - \frac{\gamma_{p\,j} \beta_j}{\overline{\gamma\beta} + \Delta k}\right)t\right], \quad (6)$$

with n_0 the neutron density before the rod drop. In Eq. (5) and Eq. (6) γ_i is the delayed neutron effectiveness and $\gamma_{p\,j}$ is the delayed photoneutrons effectiveness, which we determine from Table I; β_i, λ_i are the fractional yield and decay constant of the ith delayed-neutron group and β_j, λ_j are the fractional yield and decay constant of the jth photoneutron group. In this notations $\overline{\gamma\beta}$ represents the total delayed neutron and photoneutron effective fraction of all neutrons from fission, given with Eq. (1). The validity of the Eq. (6) for reactivity insertion times up to ~ 0.5 sec. is discussed in the following Sect. 4.1. It is interesting to observe that for $t = 0$, Eq. (6) reduces to the 'prompt jump' approximation

$$\frac{n(0)}{n_0} = \frac{\overline{\gamma\beta}}{\overline{\gamma\beta} + \Delta k}. \quad (7)$$

4. METHODOLOGY FOR DETERMINATION OF REACTIVITY WORTH FROM MEASUREMENTS

1.1 4.1. Rod Drop Tests (Scram)

The experimental data for the neutron density decay following rod-drop measurement (scram) have been used to derive the rod reactivity worth of the BR2 control rods. A typical curve of neutron density decay, measured with the detecting chambers L1 and L2 after scram of a single rod from the highest rod position, $Sh = 900$ mm (top of the core) to zero (bottom of the core) is given in Fig. 2. The estimated transient rod-drop times from the measurements from various axial positions to zero, with and without flow, are presented in Table II.

In this paper we use the Eq. (6) to evaluate the reactivity values from the measured neutron decay curves after rod-drop test. The calculation procedure includes the following steps: a. the values of the decay constants λ_i, λ_j, the fractions β_i, β_j and the effectiveness γ_i, $\gamma_{p\,j}$ are determined from the measured data, presented in Table I; b. then, we solve Eq. (6) for the neutron density for different values of the inserted reactivity, $\rho[\$]$; c. we repeat ‘b’ for the different rod-drop times, corresponding to drop from different axial positions Sh_i ($i = 1,2,...,9$), given in Table II; d. then we present graphically in Fig. 3a the solutions of Eq. (6), $n(t)/n_0 = f(\rho[\$])$, for different rod-drop times, which corresponds to drop of the rod from different axial positions, Sh_i; e. finally, the “true” reactivity values for a given rod-drop time, we determine from Fig. 3a and from the measured neutron decay curve back to the time of the drop.

Figure 2. Measured neutron density decay after scram of a single rod.

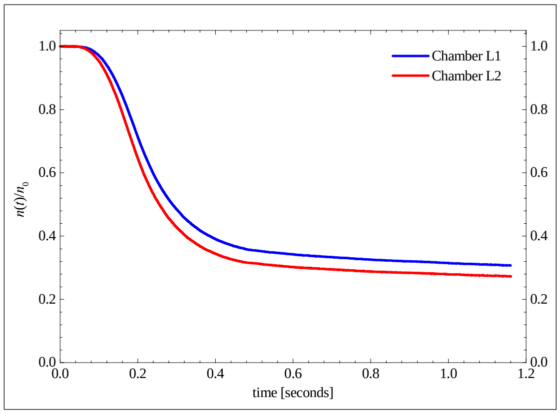


Table II. Estimated rod-drop times from different axial positions Sh_i to zero.

Drop from position Sh_i [mm]	Drop time without flow [ms]	Drop time with flow [ms]
0	20	10
100	195	145
200	380	290
300	500	350
400	610	415
500	710	470
600	810	525
700	910	580
800	1020	635
900	1130	690

The described methodology has been developed, assuming step reactivity change Δk, which means that the negative reactivity of the control rods is inserted instantly into the reactor and remains after that constant. However, in the rod-drop experiment, the rate of the reactivity insertion is some function of time and rod position, $\Delta k(t, Sh)$. In order to verify the validity of Eq. (6) for this case, we have performed simulation calculations by the transient code PARET/ANL V7.5 [5] for the conditions of the BR2 reactor. The input data for PARET, necessary to simulate the scram include initial power level, time during which the reactor has been maintained at steady power level prior the reactivity insertion, the reactivity insertion rate ($/sec.) and effective delayed (photo)neutron fractions. The reactivity rate we determine from the known (measured or calculated) axial integral control rod worth (see Fig. 6 in Sect. 5) and using the measured rod-drop times in Table II (i.e., the reactivity is inserted as a linear ramp, $\Delta k_i(\Delta Sh_i)/\overline{\gamma\beta}\Delta t_i$ in the interval $\Delta Sh_i = Sh_i - Sh_{i-1}$, i=1, …, 9). The code calculates the time evolutions of the power decay, reactivity, etc. after a scram. The numerical solutions by PARET for the relative post-drop power level have been compared with Eq. (6). The difference between the both methods are within ±2%.

To analyze the influence of the photoneutrons on the reactivity worth derived from rod-drop test, we have performed calculations by both PARET and Eq. (6) for different values of the delayed neutron and delayed photoneutron effectiveness's. The neutron density decay curves after scram from the highest rod position $Sh = 900$ mm to $Sh = 0$ are compared in Fig. 3b for different values of the used effective beta fractions. The curve (2) is obtained for the case of 'ordinary' beta, $\beta = 0.00666$ with $\gamma_i = \gamma_{p_j} = 1.0$. The curve (3) is calculated using the 'effective' beta, $\overline{\gamma\beta} = 0.00776$ and $\gamma_i = \gamma_{p_j} = 1.165$ for the fuelled beryllium assemblies inside the core. The curve (4) represents the neutron density decay, evaluated for the total 'effective' beta $\overline{\gamma\beta} = 0.00818$ with the values γ_i, γ_{p_j}, determined from Table I for the fuelled beryllium assemblies inside the core and for the surrounding reflector (non-fuelled) channels. The contribution of the photoneutrons into the reactivity worth, estimated by both, Eq. (6) and PARET is about 2% for the BR2 reactor. For comparison, the neutron density decay, calculated using the 'prompt jump' approximation, represented by Eq. (7) is given with curve (1) in Fig. 3b.

Finally, additional calculations by PARET have been simulated to analyze the dependence of the post-drop power level on the reactivity insertion rate. For this purpose, the reactivity worth of the control rod is inserted into the core at different rates as shown in Fig. 4a. The calculated time evolutions of the post-drop power level, corresponding to these reactivity rates are given in Fig. 4b. The time denoted by t' is the time after the scram at which the total reactivity of the rod is inserted into the reactor. The time $t'=0.0001$ sec. corresponds to almost instantly inserted reactivity, i.e. 'prompt jump'. We observe that for times $t'\sim 1$ second, the post-drop power level is higher by about 5% in comparison with the 'prompt jump' level and for $t'\sim 0.5$ seconds this difference reduces to 2%. If we look back to Fig. 2, we see that the rod which drops during ~ 1 second from the highest rod position, $Sh=900$ mm, inserts its negative reactivity at ~ 0.5 seconds after the drop. We can conclude that the methodology, described in this section 'overestimates' the post-drop power level (and the reactivity of the rod, respectively) by 2% for drop times up to about 1 second, which represents the case of the maximum rod drop times in the BR2 reactor.

Figure 3. a) Neutron density decay following reactor scram vs. reactivity worth, calculated by Eq. (6) for different rod-drop times, corresponding to drop from different axial positions; b) influence of the photoneutrons on the neutron density decay after rod drop from $Sh=900\,\mathrm{mm}$ to zero for different values of the delayed neutron, γ_i and delayed photoneutron effectiveness, γ_{p_j}.

a) b)

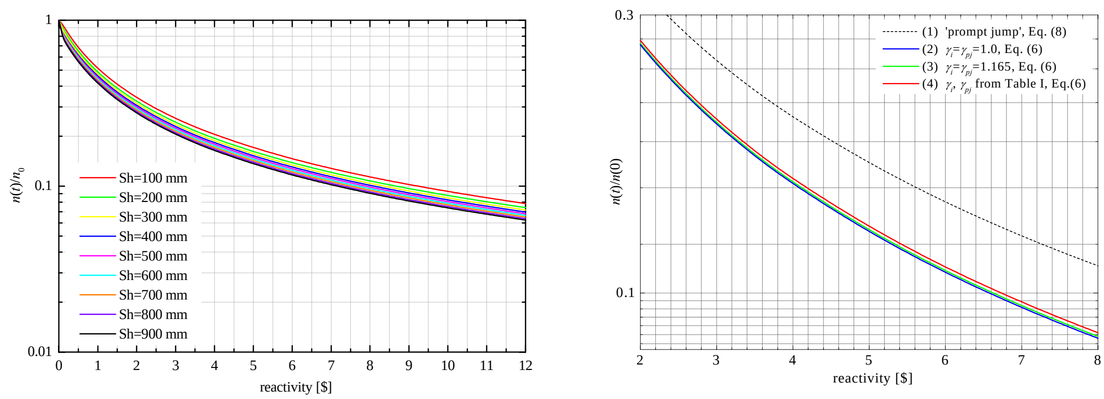
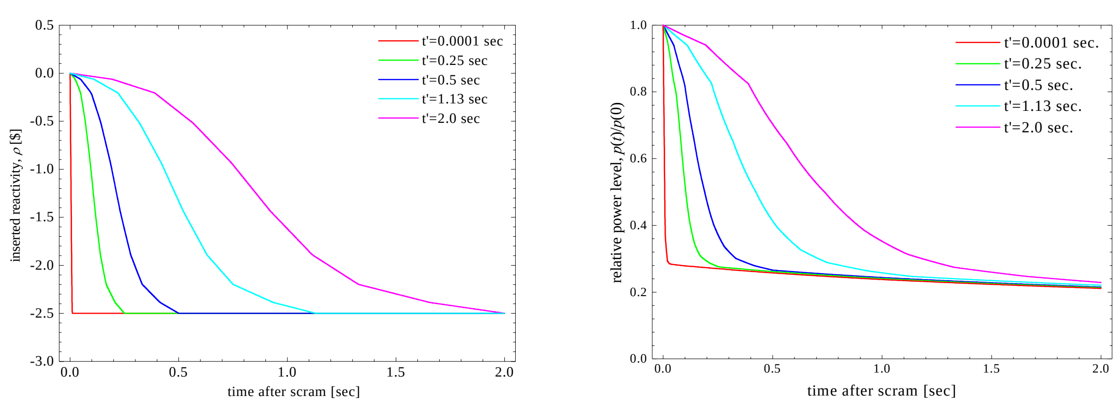


Figure 4. Evaluated by PARET [5] time evolutions of the inserted negative reactivity (a) and power level (b) after drop of a single control rod from $Sh=900\,\mathrm{mm}$ to zero for different reactivity insertion rates (the time for the total reactivity insertion is denoted with t').

a) b)



4.2. Measurement of the Asymptotic Reactor Period

The Eq. (5), which is the inhour equation with included delayed photoneutron groups is used to derive the reactivity worth from the measured asymptotic reactor period τ, when a small positive reactivity is inserted by slight withdrawal of the 6 control rods from the critical position (about 8 mm for typical BR2 core loadings). The Eq. (5) has been solved for the following cases of included delayed-neutron parameters:

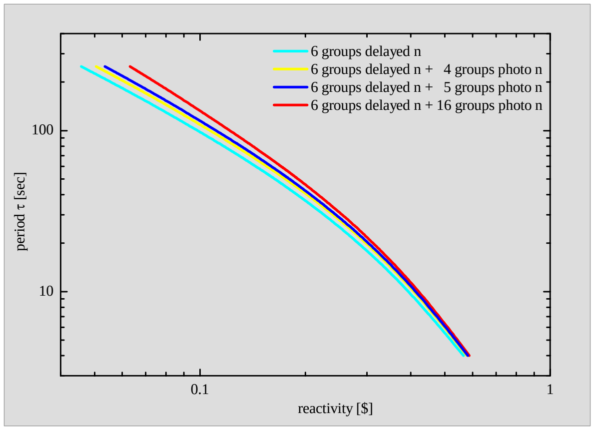
a. for 6 delayed neutron groups;
b. for 6 delayed neutron groups + 16 delayed photoneutron groups (only for the core fuelled channels);
c. for 6 delayed neutron groups + 16 delayed photoneutron groups (for the fuelled core channels + periphery, non-fuelled beryllium channels);
d. for 6 delayed neutron groups + 4 short-lived delayed photoneutron groups (for the fuelled core channels + periphery, non-fuelled beryllium channels);
e. for 6 delayed neutron groups + 5 short-lived delayed photoneutron groups (for the fuelled core channels + periphery, non-fuelled beryllium channels).

The estimated dependence of the asymptotic period on the inserted positive reactivity by the withdrawal of the control rods, is given in Table III and Fig. 5. The data in Table III and Fig. 5 (as well as the data in Fig. 3) are valid for an arbitrary type of control rod. The curves in Fig. 5 are obtained from the solution of Eq. (5) for different reactor periods. The contribution from the photoneutrons into the reactivity values for included different number delayed-photoneutron groups is presented. It is seen, that the photoneutrons give an essential contribution into the rod reactivity worth estimated by the inhour equation, Eq. (5). For a typical BR2 reactor period of about 54 seconds, the contribution from the four short-lived photoneutron groups is 7.4%, from 5 short-lived groups is 10% and from all 16 groups is 18%, respectively.

Table III. Estimated by Eq. (5) dependence of reactivity worth [$] of 6 control rods in BR2 as function of asymptotic reactor period. The measured asymptotic period for the load of cycle 01/2009A is T=53.6 sec for withdrawal $\Delta Sh = 8$ mm (from $Sh = 487$ mm to $Sh = 495$ mm).

Asymptotic period [sec]	Reactivity worth for 6 control rods [$]				
	6 groups del. n	6 groups del. n + 16 groups photo n (core)	6 groups del. n + 4 groups photo n (core+periphery)	6 groups del. n + 5 groups photo n (core+periphery)	6 groups del. n + 16 groups photo n (core+periphery)
4	0.5656	0.5728	0.5783	0.5817	0.5877
10	0.3991	0.4076	0.4143	0.4184	0.4266
20	0.2850	0.2938	0.2999	0.3044	0.3139
30	0.2266	0.2352	0.2405	0.2452	0.2553
40	0.1897	0.1980	0.2025	0.2072	0.2176
50	0.1637	0.1718	0.1756	0.1803	0.1908
53.6	**0.1561**	**0.1641**	**0.1677**	**0.1723**	**0.1839**
60	0.1444	0.1521	0.1554	0.1600	0.1706
70	0.1293	0.1367	0.1395	0.1440	0.1546
80	0.1171	0.1244	0.1267	0.1312	0.1418
100	0.0987	0.1056	0.1073	0.1115	0.1220
150	0.0711	0.0772	0.0778	0.0816	0.0918
200	0.0557	0.0612	0.0612	0.0646	0.0745
250	0.0458	0.0509	0.0506	0.0536	0.0631

Figure 5. Dependence of the reactor asymptotic period vs. reactivity worth, estimated from the inhour equation, Eq. (5).



5. CALCULATION RESULTS

In Tables IV we compare the reactivity values, determined by Eq. (6) from the rod-drop tests, which have been performed in the shutdown of the BR2 operating cycle 01/2009A, with the results, derived from the period measurements in combination with perturbation method [2], and MCNPX evaluations. The data in Table IVa and Table IVb refer correspondingly to total rod worth, $\Delta\rho_0$, and to control rod worth, $\Delta\rho_{crit}$, defined as:

$$\Delta\rho_0[\$] = \rho(900) - \rho(0), \qquad \Delta\rho_{crit}[\$] = \rho(Sh_{crit}) - \rho(0). \tag{8}$$

The reactivity worth, denoted as "independent" in the foot-note [a)] in Tables IV, is estimated using the measurements for drop of individual rods in positions S1, S2, …, S6 from $Sh = 900$ mm to 0 mm (Table IVa) and from $Sh_{crit} = 483$ mm to 0 mm (Table IVb). The mutual control rod worth is estimated from both types of measurements: rod-drop tests and period measurements, taking into account the mutual interaction of six rods (see foot-notes [b)] in Table IVa and Table IVb).

Table IVc presents estimated axial reactivity worth of a single control rod in position S3 for the cycle 01/2009A, which is determined by Eq. (6) using the measured neutron density decay curves for different rod-drop times, corresponding to drop from different axial positions. The results from the rod-drop tests are compared with those, estimated from the measured asymptotic period. In order to obtain the absolute values of the rod worth for different axial positions, we need a curve for the relative control rod worth. This curve, which is given in Fig. 6, is obtained by measurement of the axial form of a control rod worth by compensation movement of a set of other rods. For comparison, in Tables IV we present the reactivity worth values, estimated by the prompt jump approximation and by MCNPX. The reactivity worth, estimated by the prompt jump approximation, given with the Eq. (7) is highly overestimated.

The data in Tables IV, which can be compared are highlighted in corresponding color. All data, estimated from the period measurements, refer to mutual rod interaction, therefore they can be compared only with the corresponding data for "interaction of rods" obtained from the rod-drop tests. These data are highlighted in yellow color. Another set of comparable data, which are highlighted in blue color, refer to "independent" reactivity worth. The period was measured for 6 rods together and therefore we can compare the "independent" rod worth only from the rod-drop tests and MCNPX.

Table IVa. Total individual rod worth and worth of 6 rods between 0 and 900 mm, estimated by three methods: determined with Eq. (6) and Fig. 3 (rod-drop tests from Sh=900 mm); determined with Eq. (5) and Table III, Fig. 5 (period measurements) for BR2 cycle 01/2009A; MCNPX.

Control rod channel	n/n_0			ρ [$] (Eq. 7)	ρ [$] from scram, (Eq. 6, Fig. 3a)			ρ [$] from period, (Eq. 5, Table III, Fig. 5)			ρ [$] MCNPX
	L1	L2	(L1+L2)/2	(L1+L2)/2	L1	L2	(L1+L2)/2	6 gr. del n	6 del. + 5 ph.	6 del. + 16 ph.	
S1	0.298	0.269	0.284	2.521	1.818	2.118	1.968[a)] 2.094[b)]	1.721	1.917	2.038	2.12
S2	0.313	0.228	0.270	2.704	1.667	2.623	2.145[a)] 2.282[b)]	1.864	2.077	2.208	2.01
S3	0.308	0.272	0.290	2.448	1.724	2.078	1.901[a)] 2.022[b)]	1.660	1.849	1.965	1.91
S4	0.272	0.313	0.292	2.425	2.077	1.670	1.874[a)] 1.994[b)]	1.632	1.819	1.933	1.97
S5	0.205	0.297	0.251	2.984	3.060	1.813	2.436[a)] 2.592[b)]	2.118	2.359	2.507	2.29
S6	0.227	0.273	0.250	3.000	2.664	2.054	2.359[a)] 2.510[b)]	2.063	2.299	2.442	2.15
Sum of 6 rods				16.08	13.01	12.36	12.68[a)]				12.45
6 rods together					13.84	13.15	13.49[b)]	11.06	12.32	13.10	13.38

Table IVb. Individual rod worth and worth of 6 rods (between 0 and $Sh_{crit.}$), estimated with Eq. (6) and Fig. 3 (rod-drop tests from $Sh_{crit.}$=483 mm); determined with Eq. (5) and Table III, Fig. 5 (period measurements) for cycle 01/2009A.

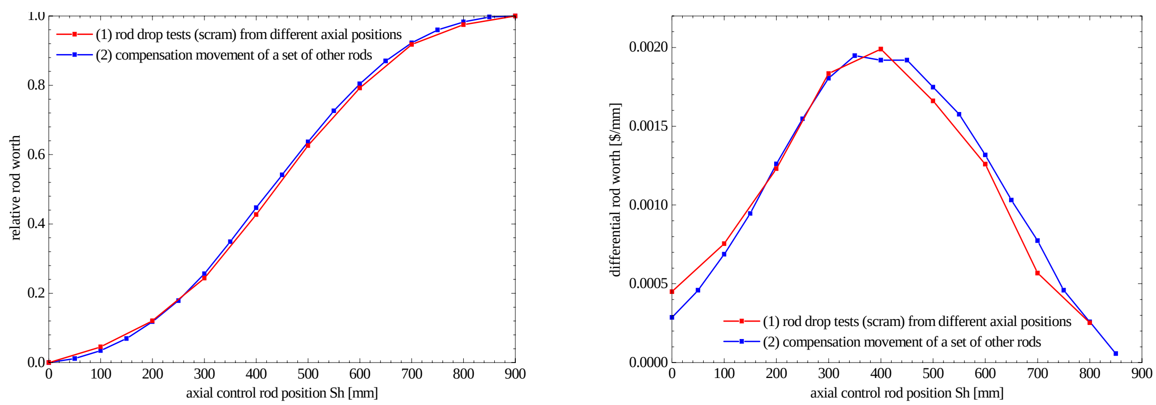
control rod	n/n_0			ρ [$] (Eq. 7)	ρ [$] from scram, (Eq. 6, Fig. 3a)			ρ [$] from period, (Eq. 5, Table III, Fig. 5)		
	L1	L2	(L1+L2)/2	(L1+L2)/2	L1	L2	(L1+L2)/2	6 gr. del n	6 del. + 5 ph.	6 del. + 16 ph.
S1	0.402	0.384	0.393	1.544	1.258	1.360	1.301[a)] 1.384[b)]	1.235	1.378	1.462
S2	0.452	0.402	0.427	1.342	1.006	1.257	1.128[a)] 1.200[b)]	1.059	1.181	1.253
S3	0.444	0.418	0.431	1.320	1.043	1.172	1.100[a)] 1.196[b)]	1.005	1.113	1.190
S4	0.435	0.455	0.445	1.247	1.087	0.994	1.045[a)] 1.112[b)]	0.992	1.106	1.174
S5	0.370	0.421	0.396	1.525	1.449	1.158	1.302[a)] 1.385[b)]	1.176	1.311	1.391
S6	0.378	0.405	0.391	1.557	1.399	1.237	1.322[a)] 1.406[b)]	1.255	1.400	1.486
Sum of 6 rods				8.535	7.242	7.178	7.198[a)]			
6 rods together	0.101	0.106	0.103	8.709	7.829	7.404	7.683[b)]	6.722	7.489	7.956

[a)] independent rod worth → blue color
[b)] interaction of rods → yellow color

Table IVc. Axial rod worth of control rod in position S3, estimated with Eq. (6) and Fig. 3 (rod-drop tests from Sh=100, 200, …, 900 mm); determined with Eq. (5) and Table III, Fig. 5 (period measurements) for BR2 cycle 01/2009A.

From Sh_i [mm] to 0	n/n_0			ρ [\$] (Eq. 7)	ρ [\$] from scram, (Eq. 6, Fig. 3a)			ρ [\$] from period, (Eq. 5, Table III, Fig. 5)		
	L1	L2	(L1+L2)/2	(L1+L2)/2	L1	L2	(L1+L2)/2	6 gr. del n	6 del. + 5 ph.	6 del. + 16 ph.
100	0.934	0.930	0.932	0.073	0.101	0.106	0.086[a] 0.092[b]	0.057	0.064	0.068
200	0.806	0.798	0.802	0.247	0.222	0.233	0.229[a] 0.244[b]	0.196	0.218	0.232
300	0.657	0.641	0.649	0.541	0.450	0.486	0.463[a] 0.493[b]	0.426	0.474	0.504
400	0.525	0.503	0.514	0.946	0.763	0.838	0.812[a] 0.864[b]	0.742	0.826	0.878
500	0.431	0.404	0.418	1.392	1.099	1.236	1.190[a] 1.266[b]	1.057	1.178	1.251
600	0.372	0.341	0.357	1.801	1.378	1.597	1.506[a] 1.602[b]	1.335	1.487	1.580
700	0.333	0.299	0.316	2.165	1.607	1.749	1.745[a] 1.856[b]	1.531	1.705	1.812
800	0.314	0.279	0.296	2.378	1.708	2.039	1.853[a] 1.971[b]	1.631	1.817	1.931
900	0.308	0.272	0.290	2.448	1.712	2.058	1.901[a] 2.022[b]	1.660	1.849	1.965

Figure 6. Estimated relative axial and differential rod reactivity worth of control rod in position S3 derived from rod-drop test and from measurement of the axial form of the rod worth by compensation movement of a set of other rods. The data are normalized to total rod worth 1\$.



The notations L1 and L2 are used for the chambers, located around the reactor vessel and measuring the relative neutron flux level $n(t)/n_0$ during the rod drop test; CR is control rod; S1, .., S6 location channel of the control rod. The data are compared for different number of delayed neutron and photoneutron groups, used in the inhour equation.

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[a] independent rod worth → blue col
[b] interaction of rods → yellow color

6. UNCERTAINTY ANALYSIS

In the present study, we use the delayed neutron and delayed photoneutron parameters, which were experimentally determined in the BR02 mock-up to estimate the rod worth derived from measurements. The BR02 mock-up configuration differs from the current BR2 core loadings. In general, rigorous calculation of the kinetics of both Be-moderated and Be-reflected system as it is the case of the BR2 reactor, should include computation of the effective delayed photoneutron group fractions $\gamma_{p\,j}$ and β_j for each specific fuel-moderator configuration. Also, due allowance should be given to the fact, that the current BR2 fuel loadings contain mixed fuel assemblies with variable U5 burnup between 0% and 50%, which will affect the value of the effective delayed neutron fraction, β^{eff}. Therefore, a series of MCNPX evaluations with included photoneutrons have been performed similarly to the described methodology in [7] for the current BR2 core configurations. The total effective delayed neutron fraction obtained was:

$$\beta^{eff}(MCNPX) = \beta^{eff}_{del.n} + \beta^{eff}_{del.ph} = 0.00680 + 0.00065 = 0.00745 \ . \qquad (9)$$

In Table V we list the possible uncertainties, related to the estimated reactivity by the two experimental methods: rod-drop tests (scram) and period measurements. As can be seen, neglecting the delayed photoneutron groups in the inhour equation, Eq. (5) underestimates significantly the reactivity rod worth (up to 18%), obtained from the measured asymptotic period. The uncertainties in the rod-drop measurement are less sensitive to the photoneutrons contribution: we observe only ~ 2% higher worth with included photoneutrons.

Table V. Errors in the estimated reactivity values from rod-drop test and from period measurement. Signs '+' and '-' are used to denote overestimated and underestimated reactivity.

Errors related to	Effect on the control rod worth			
	Eq. (6) (rod-drop test)		Eq. (5) (period measurements)	
	5 del photo n (short-lived)	16 del photo n (all)	5 del photo n (short-lived)	16 del photo n (all)
Excluded delayed photoneutron groups	– 1.5%	– 2.0%	– 10%	– 18%
Measured (photo)neutron decay constant: maximum error ± 2% [4]	–/+ 0.3%	–/+ 0.3%	–/+ 1.3%	–/+ 1.8%
Measured photoneutron effectiveness: maximum error ± 12% [4]	± 0.3%	± 0.3%	± 1.0%	± 1.6%
Reactivity insertion times up to ~ 0.5 sec. (corresponds to drop time ~1 sec.)	+ 2.0%	+ 2.0%	–	–
Measured asymptotic period (from 2 or 3 successive measurements), ± 2%	–	–	–/+ 1.6%	–/+ 1.6%
Measured neutron density decay on chambers L1 and L2, ± 3%	–/+ 5.5%	–/+ 5.5%	–	–
Total	–5.4% to +5.7%	–5.4% to +5.7%	–10.3% to –9.7%	–18.2% to –17.8%

The uncertainties, related to the counts on the two chambers L1 and L2 during the rod-drop tests are within +/–3% of the measured neutron density decay following drop of six control rods together from the critical position, which introduces uncertainty of –/+ 5.5% in the estimated rod worth by Eq. (6). We should note here, that the difference in the counts between L1 and L2 chambers is quite high (see Tables IV) for the drop of the individual rods, but this can be explained with the geometrical effect, caused by the relative position of the counting chamber and the rod-drop channel. Therefore, it is reasonable to estimate the uncertainties in the rod-drop method, using the data for the drop of the six rods together (then the chambers L1 and L2 will be in almost equivalent conditions to 'see' the neutron density decay). More over, these data are comparable with the worth, obtained from the asymptotic period, which is measured by withdrawal of the six control rods. The validity of Eq. (6) for reactivity insertion times up to ~ 0.5 sec., which corresponds to drop time ~ 1 second at the BR2 conditions, was verified by numerical solutions and discussed in Sect. 4.1. If we statistically distribute the listed in Table V uncertainties, we obtain that the uncertainties of 'rod-drop' worth, estimated with Eq. (6) are within ~ –6% to +6%, i.e., the probability that the worth is underestimated or overestimated is almost equivalent. The uncertainties of the worth, estimated from the period measurements by the inhour equation, Eq. (5), are within ~ –10% to –18%, and the worth is always underestimated.

7. CONCLUSIONS

The reactivity worth of the control rods is important to evaluate the correct value of the shutdown margin. To analyse this worth, refined estimations of the control rod worth have been performed using the kinetics equations with included delayed photoneutron groups. The reactivity worths are derived from rod-drop tests and asymptotic reactor period measurements in the beryllium reflected and beryllium moderated BR2 reactor. The photoneutron parameters, which we use in the kinetics equations, were experimentally determined in the BR02 mock-up reactor.

The importance of the photoneutrons for the reactivity worth determination depends on the chosen analysis method. The reactivity worth from the rod-drop tests is estimated, using an approximate equation for the neutron density decay after scram, obtained for the step reactivity change by applying Laplace transform on the classical point-kinetics equations. The validity of the presented methodology for the case of time dependent negative reactivity insertions $\Delta k(t)$ has been verified by comparison with numerical solutions for the neutron density decay after scram, obtained by the transient code PARET V7.5/ANL. The contribution of all photoneutrons into the reactivity worth, analyzed by this method for the BR2 reactor conditions is 2%. Another way to estimate the rod worth is to use the measured asymptotic period in the inhour equation including the delayed photoneutron groups. The reactivity worth determined by this method depends on the number of the included delayed photoneutron groups. The contribution into the reactivity worth from all delayed photoneutron groups is equal to 18%.

On the basis of the performed uncertainty analysis, we have concluded that the rod reactivity worth, estimated from the period measurements is always underestimated by ~ 10% if neglect only the short-lived photoneutrons, and up to 18% if neglect all delayed photoneutron groups. The correct interpretation of the 'rod-drop' tests data has shown that the uncertainties in the worth, estimated by this method are within ±6%. The MCNPX results are close to the 'rod-drop' worth and to the worth estimated from the period measurements by the inhour equation with included all 16 delayed photoneutron parameters.

The main results for the total control rod worth of a single rod, derived from the rod-drop tests, period measurements and MCNPX are summarized in Table VI.

Another conclusion from the rod-drop tests, which confirmed the predicted by MCNPX evaluations [2], is that the mutual rods interaction worth is higher by about 7 % than the sum of the separated rod worths.

This so called 'anti-shadowing' effect can be observed when the control rods are located in reactor channels, which are relatively far from each other, which is the case of the typical current BR2 reactor core loadings. Using the approximate equation for the neutron density decay after scram, expressed as a sum of exponentials, and the inhour equation with included delayed photoneutron groups, we have estimated two curves – neutron density decay following scram vs. rod reactivity worth and dependence of asymptotic period on the inserted rod reactivity worth, which are currently used at the BR2 reactor for automatic conversion of the measured neutron density or asymptotic period into reactivity dollars.

Table VI. Summary results for the absolute values of the total control rod worth of a single rod, obtained by three methods: rod-drop tests (scram), period measurements and MCNPX.

Control rod	Rod-drop tests (scram)		Period measurements, Eq. (5)			MCNPX
	Eq. (6)	Numerical solution PARET [5]	6 del. n gr.	6 del. n + 5 photo n gr.	6 del. n + 16 photo n gr.	
S1	2.09[*)] (1.97)[**)]	(1.93)[*)]	1.72[*)]	1.92[*)]	2.04[*)]	2.28[*)] (2.12)[**)]
S2	2.28 (2.14)	(2.10)	1.86	2.08	2.21	2.16 (2.01)
S3	2.02 (1.90)	(1.86)	1.66	1.85	1.96	2.05 (1.91)
S4	1.99 (1.87)	(1.83)	1.63	1.82	1.93	2.12 (1.97)
S5	2.59 (2.44)	(2.39)	2.12	2.36	2.51	2.46 (2.29)
S6	2.51 (2.36)	(2.31)	2.06	2.30	2.44	2.31 (2.15)
6 rods	13.49 (12.68)	(12.42)	11.06	12.33	13.09	13.38 (12.45)

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[*)] The data without brackets refer to estimated control rod worth taking into account the mutual rods interaction

[**)] The data in the brackets refer to control rod worth, estimated with no interaction between the separated rods

APPENDIX A

The general solution for the neutron density as function of time can be obtained applying the Laplace transform on the reactor kinetic equations and by determination of the reactor transfer function. The Laplace transform of the neutron response, $n(t)$, for the case of step reactivity insertions, Δk, from initial equilibrium neutron level n_0 is:

$$L[n(t)] = n_0 \frac{l + \sum_i \left[\gamma_i \; \beta_i / (s + \lambda_i)\right]}{sl + ks \sum_i \left[\gamma_i \beta_i / (s + \lambda_i)\right] - \Delta k}, \qquad \text{(A.1)}$$

where s is the reactor transform variable or reactor transfer function. The general solution for $n(t)$ for step change Δk reduces to superposition of exponentials:

$$n(t) = \sum_k N_k \exp(\omega_k t), \qquad \text{(A.2)}$$

where ω_k are the roots of the characteristic equation:

$$\omega_k = \frac{\Delta k}{l} - \frac{\omega_k k}{l} \sum_i \frac{\gamma_i \beta_i}{(\omega_k + \lambda_i)}. \qquad \text{(A.3)}$$

The coefficients N_k are given with:

$$N_k = \frac{l + \sum_i \frac{\gamma_i \beta_i}{\omega_k + \lambda_i}}{l + k \sum_i \frac{\gamma_i \beta_i \lambda_i}{(\omega_k + \lambda_i)^2}}. \qquad \text{(A.4)}$$

The periods are defined as $\tau_k = \omega_k^{-1}$ and expressing the reactivity in dollars, we have:

$$\rho = \frac{\Delta k}{k \bar{\gamma} \beta} = \frac{l}{k \bar{\gamma} \beta \tau_k} + \sum_i \frac{(\gamma_i \beta_i / \bar{\gamma} \beta)}{1 + \lambda_i \tau_k}. \qquad \text{(A.5)}$$

For systems with photoneutrons and for the stable period, Eq. (A.5) can be written as [6]:

$$\rho = \frac{\Delta k}{k \bar{\gamma} \beta} = \frac{l}{k \bar{\gamma} \beta \tau} + \sum_i \frac{(\gamma_i \beta_i / \bar{\gamma} \beta)}{1 + \lambda_i \tau} + \sum_j \frac{(\gamma_{p\,j} \beta_j / \bar{\gamma} \beta)}{1 + \lambda_j \tau} \left[1 + \frac{\lambda_j}{\lambda_{p\,j} + t_0^{-1}}\right]. \qquad \text{(A.6)}$$