



**US Army Corps
of Engineers**

Hydrologic Engineering Center

Modified-Puls Routing in Chuquatonchee Creek

September 1980

REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-0188

The public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to the Department of Defense, Executive Services and Communications Directorate (0704-0188). Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.

PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ORGANIZATION.

1. REPORT DATE (DD-MM-YYYY) September 1980		2. REPORT TYPE Research Document		3. DATES COVERED (From - To)		
4. TITLE AND SUBTITLE Modified-Puls Routing in Chuquantonchee Creek				5a. CONTRACT NUMBER		
				5b. GRANT NUMBER		
				5c. PROGRAM ELEMENT NUMBER		
				5d. PROJECT NUMBER		
				5e. TASK NUMBER		
				5f. WORK UNIT NUMBER		
6. AUTHOR(S) Theodor Strelkoff, Hydraulic Engineer 43 Liberty Street San Francisco, CA 94110						
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) US Army Corps of Engineers Institute for Water Resources Hydrologic Engineering Center (HEC) 609 Second Street Davis, CA 95616-4687				8. PERFORMING ORGANIZATION REPORT NUMBER RD-23		
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/ MONITOR'S ACRONYM(S)		
				11. SPONSOR/ MONITOR'S REPORT NUMBER(S)		
12. DISTRIBUTION / AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.						
13. SUPPLEMENTARY NOTES This work was performed under contract (DACW05-80-P-0324) to the Hydrologic Engineering Center						
14. ABSTRACT This report describes the application of modified-puls for evaluating the relative accuracy of flood routing methods to a natural river. The modified-puls method has at its core the postulate that storage depends only on outflow rate. The relationship S(0) is presumed known for the reach of interest and for the range of outflow to be encountered. The hope that the method will yield at least approximately correct outflow hydrographs rests on the assumption that the storage depends primarily, if not only, on outflow rate. If only certain features of the outflow hydrograph, such as peak and time of peak, are desired with reasonable precision, then the dependence of S on 0 perhaps could be relaxed further, with dependence existing only for some particularly important range of 0.						
15. SUBJECT TERMS flood routing, modified-puls, storage routing, hydrologic routing, river channels, outflow						
16. SECURITY CLASSIFICATION OF:				17. LIMITATION OF ABSTRACT UU	18. NUMBER OF PAGES 46	19a. NAME OF RESPONSIBLE PERSON
a. REPORT U	b. ABSTRACT U	c. THIS PAGE U	19b. TELEPHONE NUMBER			

Modified-Puls Routing in Chuquatonchee Creek

September 1980

Prepared by:
Theodor Strelkoff, Hydraulic Engineer
43 Liberty Street
San Francisco, CA 94110

Prepared for:
US Army Corps of Engineers
Institute for Water Resources
Hydrologic Engineering Center
609 Second Street
Davis, CA 95616

(530) 756-1104
(530) 756-8250 FAX
www.hec.usace.army.mil

RD-23

PREFACE

The work reported herein is a product of the Hydrologic Engineering Center's continuing effort to improve both the accuracy and efficiency of the analytic techniques used by the Corps of Engineers to route floods through natural and modified river channels. This report describes the application of a technique for evaluating the relative accuracy of flood routing methods to a natural river. The theoretical development of that technique is presented in the companion report "Comparative Analysis of Flood Routing Methods."

TABLE OF CONTENTS

	<u>Page</u>
1. Introduction and summary.....	1
2. Schematization of the Chuquatonchee test reach and flood events.....	6
3. A preliminary view of Modified-Puls in the Chuquatonchee.....	8
4. The role of the storage-outflow relation in the Modified-Puls method..	12
5. Deduction of tributary inflow.....	21
6. Conclusions.....	34
References.....	37

LIST OF FIGURES

	<u>Page</u>
Fig. 1 Hydrographs of event No. 1 (in-bank).....	10
Fig. 2 Hydrographs of event No. 2 (out-of-banks).....	11
Fig. 3 Typical storage-outflow relation.....	12
Fig. 4 Relation between inflow and outflow hydrographs with "typical" storage-outflow relation (Fig. 3).....	12
Fig. 5 Dimensionless storage-outflow relation, event No. 1.....	14
Fig. 6 Dimensionless storage-outflow relation, event No. 2.....	15
Fig. 7 Dimensionless hydrographs with linear storage-outflow relation.	16
Fig. 8 Influence of outflow-hydrograph timing on deduced tributary inflow.....	22
Fig. 9 Longitudinal distributions of tributary inflow.....	24
Fig. 10a Outflow hydrographs with tributary inflow, event No. 1.....	25
Fig. 10b Deduced apparent tributary inflow, event No. 1.....	26
Fig. 11a Outflow hydrographs with tributary inflow, event No. 2.....	28
Fig. 11b Deduced apparent tributary inflow, event No. 2.....	29
Fig. 12a Time rate of change of storage with and without downstream tributary inflow, event No. 2.....	31
Fig. 12b Apparent tributary inflows as functions of storage-outflow relation, event No. 2.....	32

1. INTRODUCTION AND SUMMARY

The principle of mass conservation can be expressed for a reach of river by the equation

$$I + T - O = \frac{dS}{dt} \quad (1)$$

in which the terms on the left represent volumetric rates of flow, I the inflow into the reach at its upper end, T the instantaneous sum of all tributary inflow over the length of the reach,¹ and O the outflow at the lower end. The term S represents the volume of water in the reach, and t is time. Equation 1 is exact, and applies at every instant during the total time of interest. With the inflow hydrograph I(t) known, if also T(t) and S(t) were known, determination of O(t), the outflow hydrograph, would be trivial. Of course, S(t) for any given inflow hydrograph is not known. So, the factors influencing S and a mathematical expression of the dependency are sought to allow, ultimately, solution of Eq. 1.

The Modified-Puls method has at its core the postulate that storage depends only on outflow rate. The relationship S(O) is presumed known for the reach of interest and for the range of outflow to be encountered. Then, Eq. 1 takes the form

$$\frac{dS}{dO} \frac{dO}{dt} + O = I + T \quad (2)$$

an ordinary differential equation for the outflow hydrograph. With I(t) and $\frac{dS}{dO}(O)$ known, and the tributary hydrograph known, assumed, or neglected, the outflow hydrograph O(t) can be computed. The Modified-Puls method provides a numerical technique for solving this differential equation.

¹Understood as net inflow: tributary inflow + overland inflow + precipitation - evaporation - seepage - lateral outflow through side weirs, pump intakes, diversions, etc.

The hope that the method will yield at least approximately correct outflow hydrographs rests on the assumption that the storage depends *primarily*, if not only, on outflow rate. If only certain features of the outflow hydrograph, such as peak and time of peak, are desired with reasonable precision, then the dependence of S on O perhaps could be relaxed further, with dependence existing only for some particularly important range of O.

A key factor in the application of the Modified-Puls method to any given reach is the determination of this relationship S(O) for the reach. Two techniques of arriving at this function are employed extensively: (a) computing steady-flow profiles in the given reach for a succession of discharges O, from given topographical and roughness information, then computing water volume under the profile for each value of discharge, and (b) deriving S(O) for historical events from given inflow and outflow hydrographs. The former procedure is complicated by the difficulty of obtaining accurate topographical and roughness data, and by the fact that these can vary from event to event. The latter is complicated by the existence of unengaged tributary inflow, with only $\int T dt$ derivable from the inflow and outflow hydrographs,

$$\int_{t_1}^{t_2} T dt = \int_{t_1}^{t_2} O dt - \int_{t_1}^{t_2} I dt \quad (3)$$

with t_1 and t_2 taken, at the beginning and end of the event, such that $S(t_1) = S(t_2)$. With T(t) and S(O) independent functions, Eq. 2, alone, is insufficient to solve for either, if only I(t) and O(t) are known. In one variant (reference 1), an "optimum" S(O), common to several (three) calibration events, was found by minimizing the negative portions of the resultant computed T(t).

Whichever procedure is used, it relies on the validity of the assumption that storage is truly a function of outflow alone, at least approximately.

In the given reach of Chuquatonchee Creek, the two procedures led to very different $S(0)$ functions. It was the purpose of the present investigation to determine the most likely causes of the disagreement.

The given reach of the Chuquatonchee was schematicized by a simple prismatic geometry, with a rectangular main channel for in-bank flows, and a wide, plane, slightly sloping floodplain bordered by vertical bluffs to receive the large flood flows. Specific flood events of record were also schematicized, by simple, single-peak inflow hydrographs, with continuously variable time-rate of change. Test-channel geometry was selected consistent with the topographical features of the Chuquatonchee valley. Peak and rise time of test inflow hydrographs were consistent with peaks and rise times of selected historical events.

The unsteady flows generated by the test floods in the test channel were modeled by solution of the complete one-dimensional Saint-Venant equations. Water-surface profiles (and, hence, stored volumes), as well as outflow, were monitored during the course of the calculation. Time and distance steps in the numerical procedures were taken small enough that their size played no significant role in the resulting solutions. Because test-channel geometry and roughness are known precisely, errors in input data are not of concern, and solutions of the Saint-Venant equations can be considered, except for one reservation, very close to what would be observed in real water flows. To be kept in mind is that the model assumes lateral flooding of the floodplain to take place instantaneously, in keeping with the one-dimensional character of the model, which assumes, at all times, a horizontal water surface transverse to the main channel. In actual fact, water flows up onto the floodplain in consequence of a transverse sloping water surface, and a finite length of time is required to inundate the plain to any given elevation.

In an opposing set of calculations, stored volumes were determined as a function of outflow for a succession of steady states at various discharges. The resultant $S(0)$ function was placed into a numerical solution procedure for Eq. 2, and Modified-Puls-routed hydrographs determined. As a variation of this procedure, routing could be performed over a series of subreaches, the outflow hydrograph from one subreach serving as the inflow hydrograph for the next subreach downstream. The $S(0)$ relation for each subreach was determined by apportioning the storage determined for the entire reach amongst the subreaches in proportion to their length. For the uniform flows developed for each steady-state discharge (in consequence of the prismatic channel and normal-depth rating curve postulated at the downstream end), this is a precisely correct distribution of storage.

Tests were performed without tributary inflow, with tributary inflow distributed uniformly and nonuniformly over the length of the reach, and also lumped at the upstream and downstream ends.

The most significant finding was that, in this reach, storage is only very loosely tied to outflow. Mathematically $S \neq S(0)$, but instead, $S = S(0, \dots)$, and that 0 is not even a very important argument during much of the event. Physically, the shape of the flood wave, and hence the storage therein, is very different from the shape of the profile in steady flow. The key feature of the Chuquatonchee channel leading to this result is the small bottom slope and broad, flat floodplain in which the in-bank channel meanders.

Furthermore, it was found that the time of arrival of the routed outflow hydrograph, in comparison with a measured one, plays a significant role in computing thereby $T(t)$; consequently computation of $S(0)$ should not be based on the behavior of $T(t)$, without taking the timing of the routed hydrograph into account. In addition, the assumption that all tributary inflow is concentrated at the lower end of the reach can significantly

affect the shape of the desired $T(t)$, though this factor is not likely of great importance in the given reach. Probably, the most significant factor in the deduction of negative tributary inflow is the assumption of an incorrect storage-outflow relation with too narrow a loop in the Modified-Puls method.

In conclusion, the Modified-Puls method with a storage-outflow based on steady-flow profiles is not suitable for this reach, primarily because of the great size of the depth gradient relative to other forces driving or retarding the flow, and because the depth gradient is strongly a function of the unsteadiness in a channel of small slope. Further, the effect of depth on storage is highly exaggerated by the broad flat floodplains of this reach. By the same token, $S(t)$ is highly event dependent, so that no single function, even approximately valid for a wide range of peaks, exists.

While doubt has been cast on the accuracy of the topographical data gathered for Chuquatonchee Creek, it was found that the errors stemming from application of Modified-Puls to events in reaches for which it is theoretically unsuitable, were of sufficient magnitude that they alone could explain the noted discrepancy.

2. Schematization of the Chuquatonchee test reach and flood events.

An examination of plots of valley cross sections derived from large-scale topographical maps of the Chuquatonchee tributary valley from Abbott to West Point, Mississippi (drawn by the Mobile District of the Corps of Engineers) led to schematization of the given 12.4-mile reach by a prismatic channel characterized by the following geometrical parameters.

In-bank channel of rectangular section, width $B = 70$ feet

depth $D = 13$ feet

bottom slope $S_0 = 0.000462 = 2.4$ feet/mile

Manning $n = 0.07 \text{ ft}^{1/6}$

length $L = 12.4$ miles

Single flood plain, characterized by

lateral slope $S_{FP} = 0.0002$

width $w_{FP} = 7000$ feet

Manning $n_{FP} = 0.15 \text{ ft}^{1/6}$

and vertical bluffs.

Historic flood events also were schematicized, two in particular: (1) the small in-bank event of March 3-9, 1970, and (2) the large flood of March 18-24, 1970. In both cases, inflow-hydrograph shape was assumed of Pearson type III, namely,

$$I(t) = Q_0 + (I_P - Q_0) * \left(\frac{t}{t_{I_P}} \right)^{\frac{1}{s-1}} * e^{-\frac{1 - t/t_{I_P}}{s-1}}$$

in which Q_0 is the base flow, I_P is the inflow peak value, t_{I_P} is the time to peak for the inflow hydrograph, and s is the so-called skew, held constant in this study at

$$s = 1.2$$

The in-bank event was characterized herein by the peak value

$$I_{P_1} = 1800 \text{ cfs}$$

with the time to peak

$$t_{I_{P_1}} = 48 \text{ hours}$$

values similar to those measured for event No. 1.

The out-of banks flood was given the values

$$I_{P_2} = 25,000 \text{ cfs}$$

$$t_{I_{P_2}} = 28 \text{ hours}$$

again close to the measured inflow characteristics, for event No. 2.

Base flow in both cases was given by

$$Q_0 = 300 \text{ cfs.}$$

The downstream boundary condition in both cases was a normal-depth rating curve.

3. A preliminary view of Modified-Puls in the Chuquatonchee.

An approximate prediction of the efficacy of the Modified-Puls method in the test reach is made by examining the dimensionless comparative solution curves in reference 2. For a steady-state (reference) base flow $Q_0 = 300$ cfs, the normal (reference) depth is $Y_0 = 4.00$ feet, and the characteristic (reference) distance $X_0 = Y_0/S_0 = 1.64$ miles, so that dimensionless channel length is $L^* = L/X_0 = 7.56$. The reference time is $T_0 = X_0 Y_0 B_0 / Q_0 = 2.25$ hours.

For the in-bank event, No. 1, with $I_P = 1800$ cfs, the ratio of peak to base flow is $Q_P^* = 6.0$, and the dimensionless rise time $t_{Q_P^*} = T_{I_P} / T_0 = 21.3$. Examination of Fig. 10e of ref. 2 suggests that Modified-Puls, with steps $\delta x^* = 4.0$ (in the given reach, with the number of steps, $N = 2$, $\delta x^* = L^*/N = 3.78$) would lead to outflow hydrograph peaks less than 2-3% low (also, that very little attenuation is to be expected).

For the out-of-banks event, No. 2, $Q_P^* = 83.3$, and $t_{Q_P^*} = 12.5$. Examination of Figs. 13e and f of ref. 2, for $Q_P^* = 5$, suggests that Modified-Puls outflow peaks would be about 30% low. Comparison of Figs. 13d, for $Q_P^* = 5.0$, and 13g, for $Q_P^* = 40.0$, shows that the percent error would increase beyond the aforementioned 30%, for $Q_P^* = 83$. It should be borne in mind that the curves of ref. 2 were obtained for a base flow just bank full, for floodplains with Manning n 5 times that of the main channel, and for a cross slope of 0.0019 ($0.01 * Y_0/B_0$). In the given test reach, the flood-plain n is just over twice that of the main channel, and the cross slope is 0.0002 . The percent-error figures must thus be considered rough estimates. An indication of the effect of the given base flow, as opposed to bank full, can be seen by postulating a base flow of 1800 cfs, instead of 300 cfs. In this case, $Y_0 = 12.7$ feet, $X_0 = 5.2$ miles, $T_0 = 3.78$ hours, so that $L^* = 2.38$, $Q_P^* = 13.9$, $t_{Q_P^*} = 7.4$. With $\delta x^* = 1$, as in Figs. 13d,e

(as opposed to $L^*/N = 1.2$), Modified-Puls peaks would be expected to run low by something over 20%. Again, this can only be considered an estimate because of the effects of flood-plain roughness and cross slope.

An examination of Figs. 1 and 2 shows that these estimates are generally borne out.

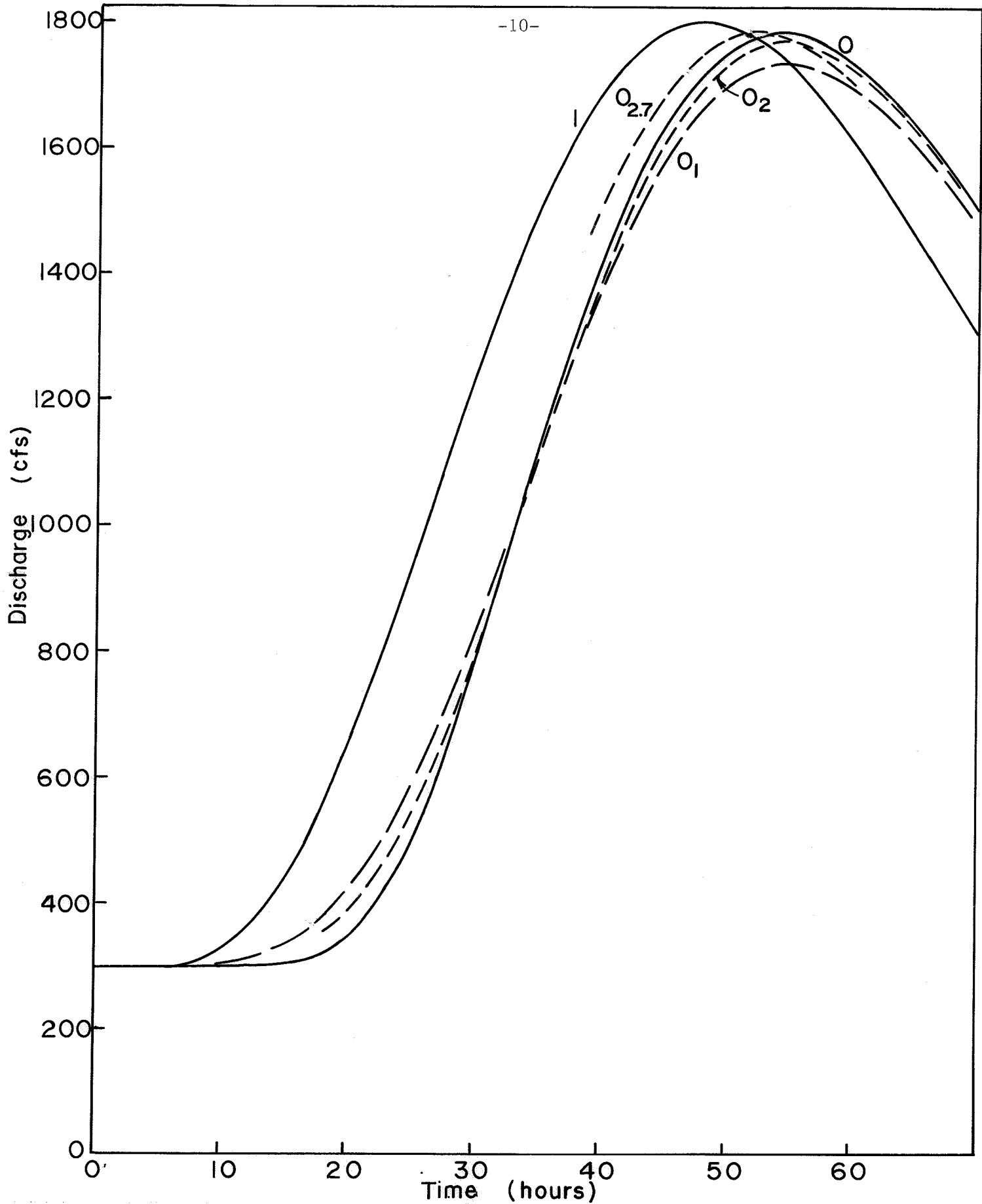


Fig. 1. Hydrographs of event No. 1 (in-bank)

- I : Inflow
- O : Outflow (Saint-Venant)
- O₁ : Outflow via Modified-Puls, N = 1
- O₂ : Outflow via Modified-Puls, N = 2
- O_{2.7}: Outflow via Modified-Puls, N = 2; storage reduced to 70%.

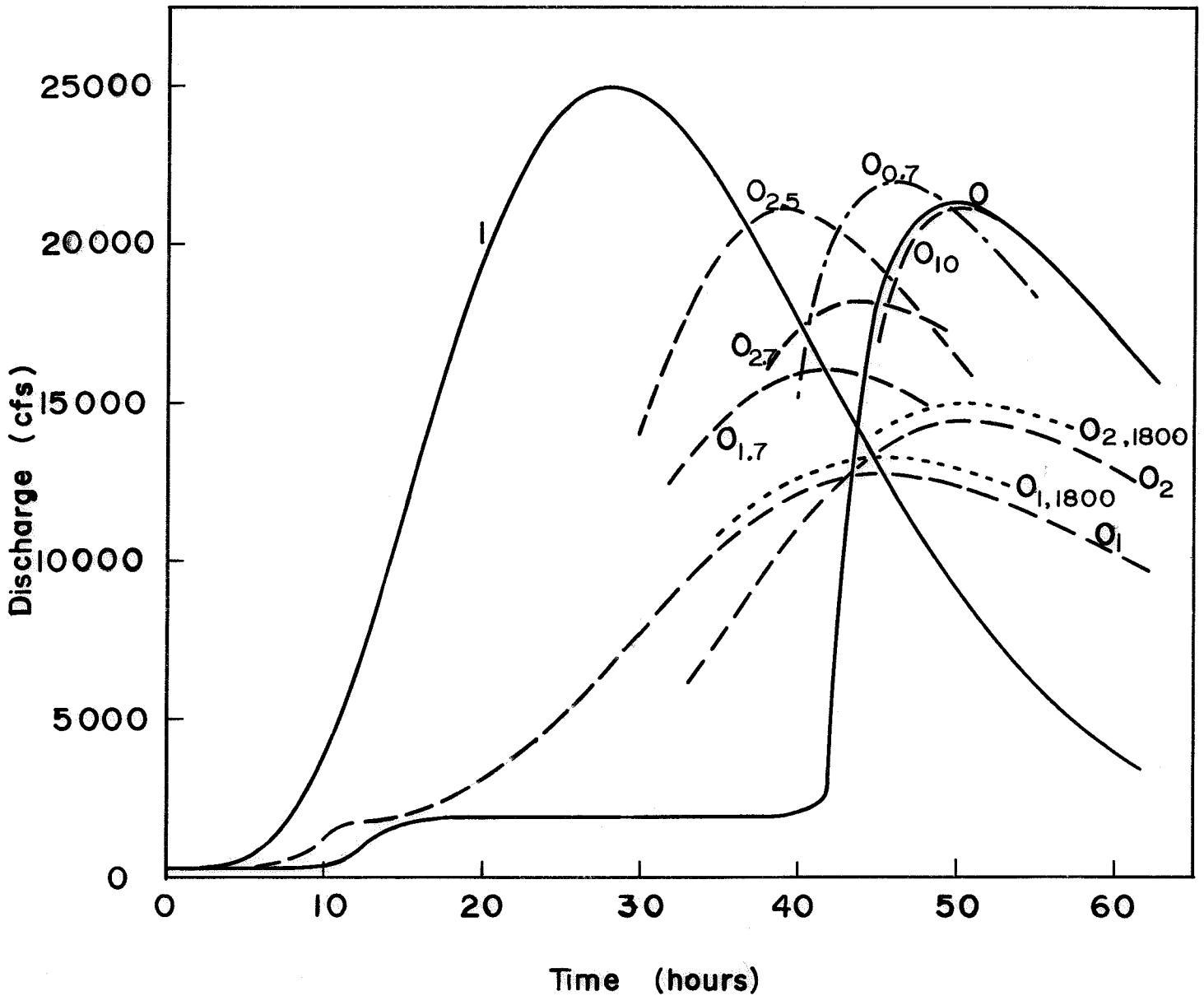


Fig. 2. Hydrographs of event No. 2 (out-of-banks)

- I : Inflow
- O : Outflow (Saint-Venant)
- O_{0.7} : Outflow (Saint-Venant) with all breadths reduced 30%
- O₁ : Outflow via Modified-Puls, N = 1
- O₂ : Outflow via Modified-Puls, N = 2
- O₁₀ : Outflow via Modified-Puls, N = 10
- O_{1.7} : Outflow via Modified-Puls, N = 1, storage reduced to 70%
- O_{2.7} : Outflow via Modified-Puls, N = 2, storage reduced to 70%
- O_{2.5} : Outflow via Modified-Puls, N = 2, storage reduced to 50%
- O_{1,1800} : Outflow via Modified-Puls with base flow 1800 cfs, N = 1
- O_{2,1800} : Outflow via Modified-Puls with base flow 1800 cfs, N = 2.

4. The role of the storage-outflow relation in the Modified-Puls method.

A typical storage-outflow curve presents the former as a monotonically increasing function of the latter, as in Fig. 3.

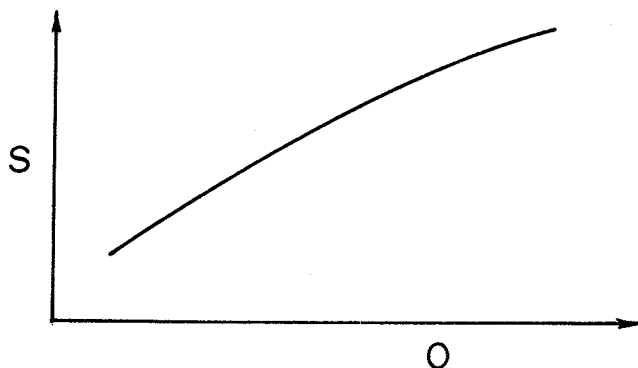


Fig. 3. Typical storage-outflow relation.

With Eq. 2 in the following form (tributary contribution negligible),

$$\frac{dO}{dt} = \frac{I - O}{dS/dO} \quad (4)$$

and $0 < dS/dO < \infty$ as in Fig. 3, it is clear that the outflow hydrograph must start to rise *immediately*, without any lag, upon a rise in inflow. This, in principle, is antithetical to the notion of a *wave*, in which an event occurs at one point in a stream, and only some time later is perceived at a downstream point. Furthermore, it is evident that the peak outflow *must* occur on the falling limb of the inflow hydrograph, as in Fig. 4.

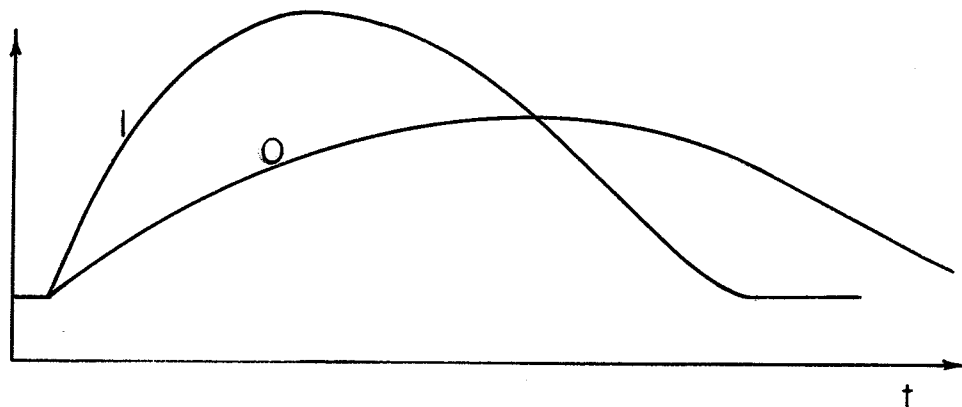


Fig. 4. Relation between inflow and outflow hydrographs with "typical" storage-outflow relation (Fig. 3).

In actuality, not only does the outflow hydrograph start its rise some time after the inflow hydrograph, especially with extensive floodplain storage, but the outflow peak generally occurs somewhat later than at the time outflow equals inflow (see, e.g., Figs. 1 and 2).

A delayed rise in outflow can occur (see Eq. 4) only if dS/dO is infinitely great during the early stages of flood inflow. Similarly, an outflow peak at $O \neq I$, say $O > I$, which is usually the case, can also occur only if $dS/dO \rightarrow \infty$ at that time. Finally, for the outflow and inflow hydrographs to cross without an outflow peak, i.e., $O = I$, $dO/dt \neq 0$, can occur only if $dS/dO = 0$ at that time. These necessary characteristics of the $S(O)$ function are, of course, precisely what are exhibited by the relations computed by solution of the Saint-Venant equations for the schematicized Chuquatonchee reach and events (Figs. 5, 6).¹

In general, with a given inflow hydrograph, it is possible to construct any desired outflow hydrograph by manipulating the storage-outflow relation. Some appreciation of the control exerted by the $S(O)$ function upon $O(t)$ can be acquired by a study of Fig. 7, which shows in dimensionless form the entire range of outflow hydrographs that can be obtained with a given *shape* of inflow hydrograph by varying the slope of a straight-line $S(O)$ curve. All discharges I^* , O^* are referenced to the inflow peak discharge, and all times t^* are referenced to the time of peak inflow. Dimensionless storage S^* is consequently referenced to the product of peak inflow discharge and time to

¹ Longitudinal depth, area, and discharge profiles were obtained in the course of solution of these one-dimensional hydrodynamic equations (by the model described in ref. 2). This permits calculation of the volume stored in the reach at any instant. Coordination with the corresponding outflow leads to the $S(O)$ function pertinent to the given event. While $S(O)$ could also have been deduced by integrating $dS/dt = I - O$, the former method illustrates quantitatively how the storage is distributed along the reach.

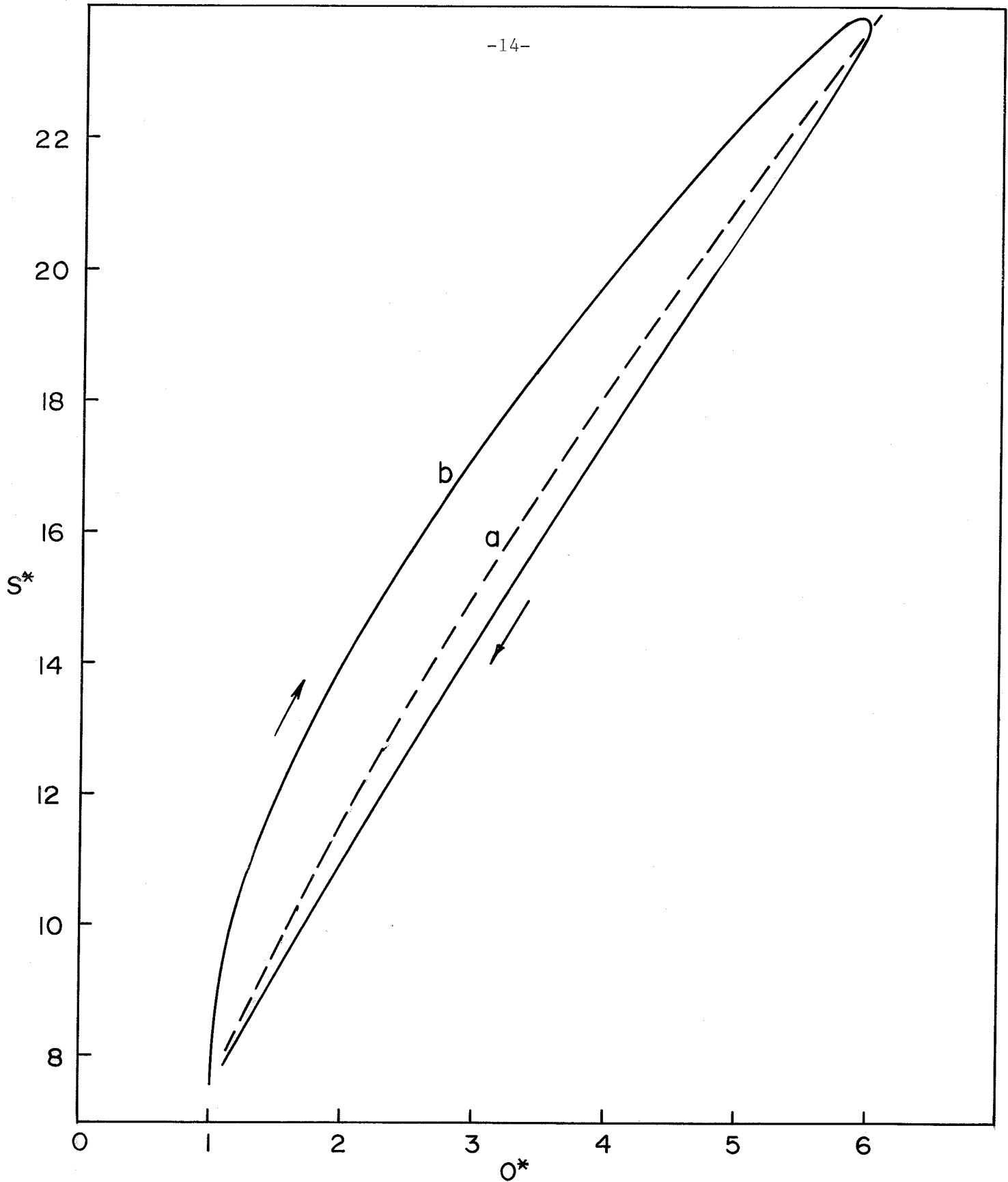


Fig. 5. Dimensionless storage-outflow relation, event No. 1
a. Computed from a succession of steady states
b. In effect during event No. 1
For storage in acre-feet, multiply S^* by 55.8; for outflow in cfs, multiply O^* by 300 (ref. 2).

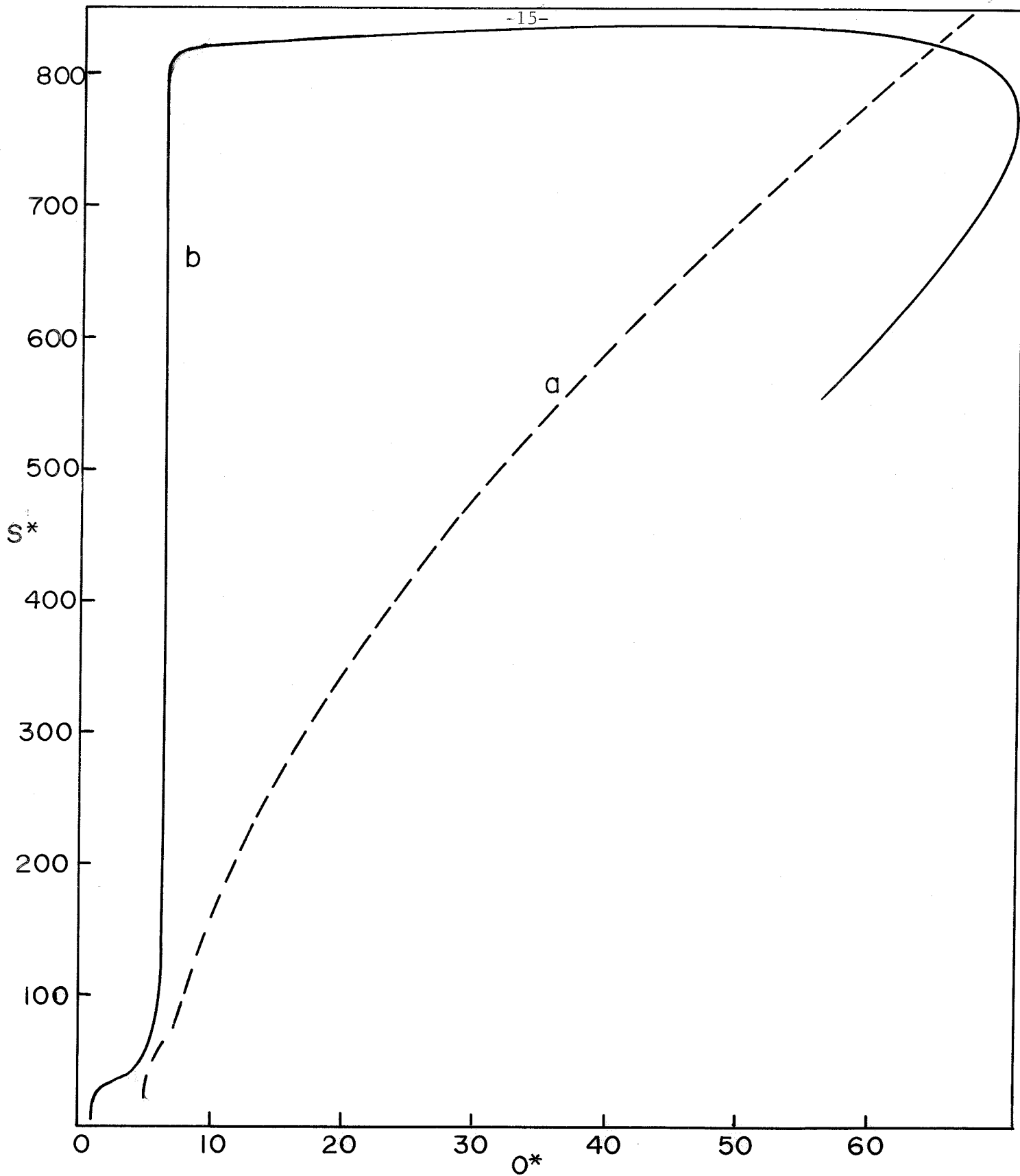


Fig. 6. Dimensionless storage-outflow relation, event No. 2
a. Computed from a succession of steady states
b. In effect during event No. 2
For storage in acre-feet, multiply S* by 55.8; for outflow in cfs,
multiply O* by 300 (ref. 2).

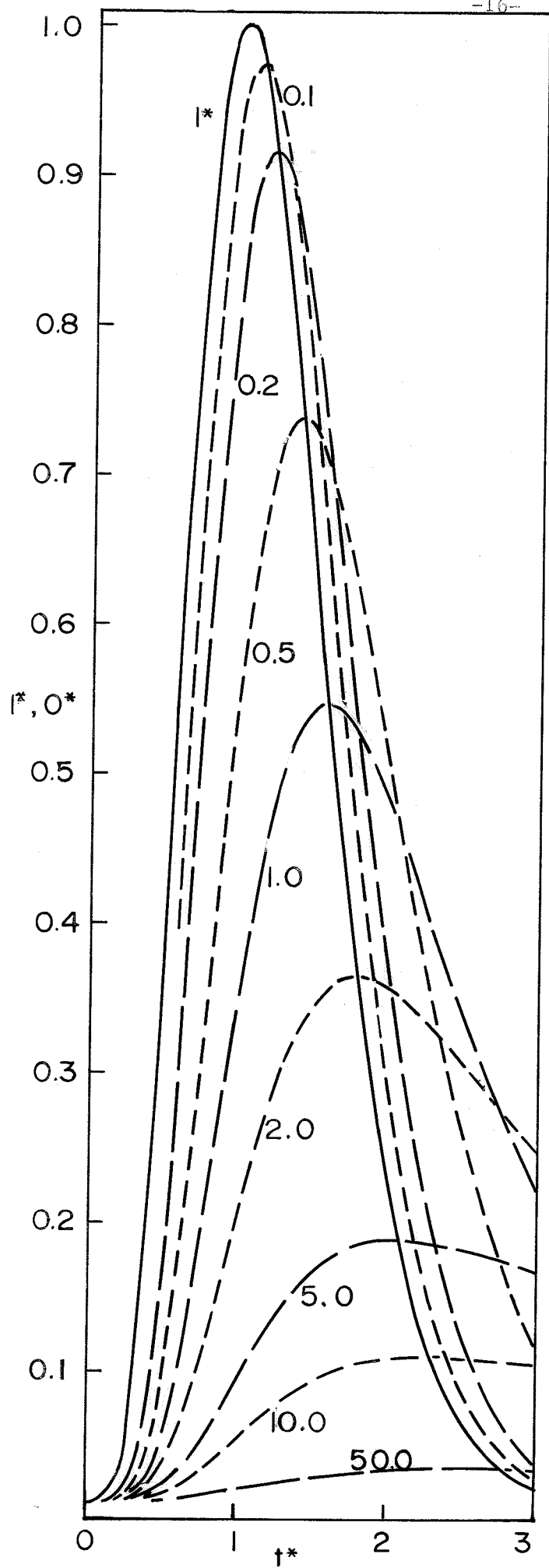


Fig. 7. Dimensionless hydrographs with linear storage-outflow relation
I: Inflow
Outflow curves are distinguished by the value of the parameter $t_T^* = dS^*/dO^*$, in which reference values are T_{Ip} time to peak of the inflow hydrograph and I_p peak inflow.

peak. The dimensionless slope of the storage-outflow relation dS^*/dO^* is the theoretical travel time of the wave in the reach, relative to the time to peak of the inflow hydrograph. This value, t_T^* , is the parameter distinguishing the various outflow hydrographs in Fig. 7. The shape of the inflow hydrograph is again of Pearson type III with skew $s = 1.2$, and a base flow 1.2% of the peak. Relatively large increases in storage for a given increase in outflow, i.e., large values of dimensionless travel time t_T^* , lead to small, late outflow hydrograph peaks. The smaller is t_T^* , the more nearly do the inflow and outflow hydrographs coincide. In every case, of course, the area under the complete hydrograph is unity. That the locations of the outflow peaks are confined to the falling limb of the inflow hydrograph is a consequence of the assumption of a unique relation between storage and outflow, applied both to the rising and falling limbs of the outflow hydrograph. Only a looped $S(O)$ curve, with $dS/dO = 0$ at the time that $I = 0$, will allow the outflow hydrograph to continue rising at $I = 0$. At the time that the outflow ultimately peaks, $dS/dO \rightarrow \infty$; the subsequent descent of the outflow hydrograph is subject to that branch of the loop that descends monotonically to the starting value of S , at the end of the event.

As pointed out by Slocum and Dandekar (ref. 1), a single-valued storage-outflow relation can be made into a loop, by applying the Modified-Puls method as previously mentioned, herein, to a series of subreaches comprising, together, the given reach. The storage-outflow relation deduced from the resulting reach-outflow and inflow hydrographs, say through summation of the increments

$$\delta S = (I - O)\delta t \quad (5)$$

exhibits a loop, the width of the configuration depending upon the number of subreaches N .

With the in-bank event, No. 1, the relatively narrow loop which exists in fact (Fig. 5) is fairly well modeled by the loop obtained with an original, single-valued curve stemming from the assumption of normal depth for all discharges, followed by application of Modified-Puls to subreaches, with $N = 2$. The results of the technique for $N = 1$ and 2 are shown in Fig. 1.

For the large, out-of-banks flood, event No. 2, the loop obtained with $N = 2$ from a similar, normal-depth, original storage-outflow curve is far narrower than the true loop, shown in Fig. 6. Evidently, from the results presented in Fig. 2, $N = 10$ results in a loop of about the right proportions.

As pointed out in ref. 2 it is usually possible to find, empirically, a value of N , that will produce an outflow that matches a given one in peak and time of peak. However, because the storage-outflow loop is produced mathematically, rather than physically, it is not possible to predict the optimum N from physical considerations, short of already knowing the outflow hydrograph.

When comparing with an actual outflow hydrograph the results of Modified-Puls coupled with a storage-outflow relation based on area profiles derived for a series of steady flows and $N = 1$, the peaks of the latter are seen to be generally too low and, of course, situated on the descending limb of the inflow hydrograph, rather than somewhat later. In ref. 2, it is shown that increasing N both raises the outflow peak and delays it. Note how, in Fig. 7, the computed travel time of the flow peak is less than t_T , a figure very close to the actual travel time (ref. 2). This circumstance, essentially unnoticeable for small travel times ($t_T^* = 0.1, 0.2$) that result in little attenuation, is greatly in evidence at travel times equal to the time to inflow peak and greater. Increasing N , as in ref. 2, causes the computed travel time of the peak, to approach the theoretical travel time t_T .

The computed magnitude of the peak outflow can also be increased by decreasing the storage assumed at each value of outflow. Slocum and Dandekar (ref. 1) found that uniformly decreasing the storage by a factor of 30% from the values computed with HEC-2 for the given reach of the Chuquatonchee yielded about the right values of peak outflow, computed by Modified Puls with $N = 2$. In the schematicized reach studied by the writer, 70% of backwater-curve computed storage brought the outflow peak up somewhat, but a factor of 50% (with $N = 2$) was necessary to match the correct peak (see Fig. 2). Worthy of note, decreasing the storage values for a given outflow, raises the peak outflow, but also speeds up the time of arrival of that peak. This plays a significant role in the deduction of the tributary-inflow hydrograph, as will be discussed in section 5.

It has been suggested that the topographical data for the Chuquatonchee could be in error, leading to HEC-2-computed volumes too great for the given outflows. As reported in ref. 1, a 70% factor was applied to storage as used in the Saint-Venant routings as well. In the present studies, in the schematic channel, a 30% reduction in width at every depth, led, in the out-of-banks event, 2, to a 3% increase in outflow peak computed with the Saint-Venant equations.¹ In the physically less consistent procedure applied to the solution of the Saint-Venant equations reported in ref. 1, only the top width B in the partial derivative of area with respect to time, $B\partial y/\partial t = \partial A/\partial t$, in the continuity equations was reduced by the 30%. All other terms in the equations were left undisturbed. This procedure, applied for the present study by HEC personnel to both the schematic channel with event No. 2

¹When the same width reduction was applied to the in-bank event, the latter becomes an out-of-bank flood with, of course, totally different characteristics.

and the original Chuquatonchee topography and corresponding event, led to a 10% increase in peak outflow. Thus, the effect of the topographical adjustments suggested in ref. 1 are far greater on the Modified-Puls results, than on the results of solution of the Saint-Venant equations.

While the change in Modified-Puls results due to the 70% storage factor with real Chuquatonchee topography was only about 2/3 of that noted in the schematic channel, it seems clear that this factor is less a correction of faulty topographical data than an attempt at adjusting a steady-flow storage-outflow relationship, so it could be used with an unsteady flow. Further evidence in support of this premise is found upon examination of the Saint-Venant-computed outflow hydrographs for all five events on the Chuquatonchee reported in ref. 1: most of the peaks shown, computed with the 70% factor in force, run about 10% higher than the measured peaks; troublesome negative tributary inflow is deduced. While other factors can contribute to apparent negative tributary inflow, it is unlikely that peaks correctly computed with no tributary inflow would be higher than observed peaks, especially if this inflow was in fact significant.

5. Deduction of tributary inflow.

From the foregoing and Eq. 2 written in the form

$$\frac{dS}{dO} \frac{dO}{dt} - T = I - O \quad (6)$$

it is clear that given inflow and outflow hydrographs for any particular event can yield *any* $T(t)$ function by choosing a commensurate $S(O)$ function, and vice versa. A characteristic feature of $T(t)$ hydrographs deduced in ref. 1 from chosen $S(O)$ relations was the tendency of the former to exhibit negative values during certain portions of the total time of interest. Several possible causes of this phenomenon were investigated.

The particular deduction technique employed in ref. 1 is based on the assumption that all of the ungaged local flow is concentrated in a single tributary entering the given reach just above the downstream gage. Then, that tributary inflow hydrograph is found by subtracting the routed hydrograph O_R based on zero local flows from the observed outflow hydrograph O_0 .

Not only the relative magnitudes of computed and measured outflow hydrographs influence the inferred tributary inflow, but also their relative timing. Figure 8 shows the tributary hydrographs deduced from three routed hydrographs O_{R1} , O_{R2} , O_{R3} , differing only in arrival time, relative to an observed outflow O_0 . For simplicity, the hydrographs are given triangular shapes. If O_{R2} is assumed in the correct position relative to O_0 , with the resulting tributary hydrograph T_2 , a routed hydrograph which arrives too early, such as O_{R1} (or a measured hydrograph that appears to arrive later than in fact) can yield a tributary hydrograph that is low or negative in its earlier portion, and, in compensation, too high in its later portions. Of course the net area under each of the deduced tributary hydrographs is the same. With a routed hydrograph arriving later than it should, relative

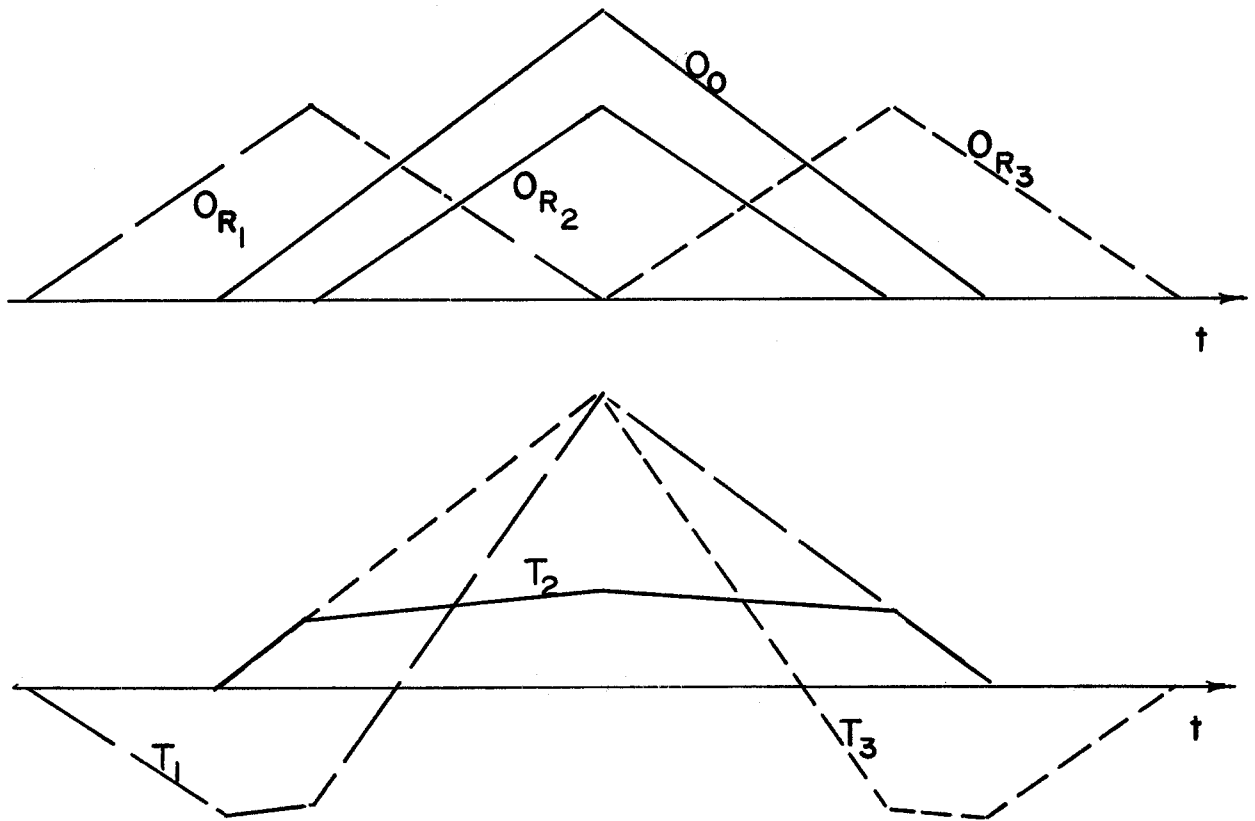


Fig. 8. Influence of outflow hydrograph timing on deduced tributary inflow
 O_0 : "Observed" outflow hydrograph
 O_R : Routed hydrograph.

to the observed, excessively high values of tributary inflow are inferred at small times, and low or negative values, as with O_{R3} , at large times. Modified-Puls routings tend to arrive too soon, so they can be expected to yield tributary inflow patterns more like O_{R1} than O_{R3} . "Observed" outflow hydrographs tend to peak later than in fact, because they are generally based on single-valued rating curves. The actual observation is of stage, which is translated by the rating curve to outflow, and in an unsteady flow, the stage peak arrives actually a little later than the discharge peak. This phenomenon,

too, leads to tributary-hydrograph errors of the T_1 type, but with overbank events in the Chuquatonchee, the magnitude of error from this source is relatively small. *The width of the loop in the stage-discharge relation is far narrower than that of the storage-outflow loop.* Furthermore, the errors are somewhat compensated in O_R , because a gaged inflow hydrograph is subject to the same kind of error. Similarly, the underestimation of O_0 (by a steady-flow rating curve) during its rise in a flood, and overestimation during the fall leads to a relatively small $T(t)$ error, also negative early, and positive later.

Topographic maps of the given reach of the Chuquatonchee show several tributaries entering the main stream at various points along the way. In particular, a large tributary appears to enter just downstream of the apparently smaller, but gaged, Houlka Creek. The contribution of the Houlka was lumped in together with the inflow to the upper end of the Chuquatonchee reach to comprise the inflow hydrographs used in ref. 1. Thus, there is some concern about the possibility that perhaps most of the tributary inflow occurs near the upstream end of the reach, rather than at the downstream end as assumed in ref. 1 when deducing the tributary-inflow hydrograph.

The influence of this assumption was investigated by distributing a given tributary hydrograph $T(t)$ in various ways over the length of the reach and noting the effect on the outflow hydrograph. The mathematical model used to depict this behavior was again that based on the Saint-Venant equations and described in ref. 2. The model was programmed for a full range of uniformly varying (straight-line) longitudinal distributions of tributary inflow. Examples of these are shown in Fig. 9. The ordinate scale therein is the ratio

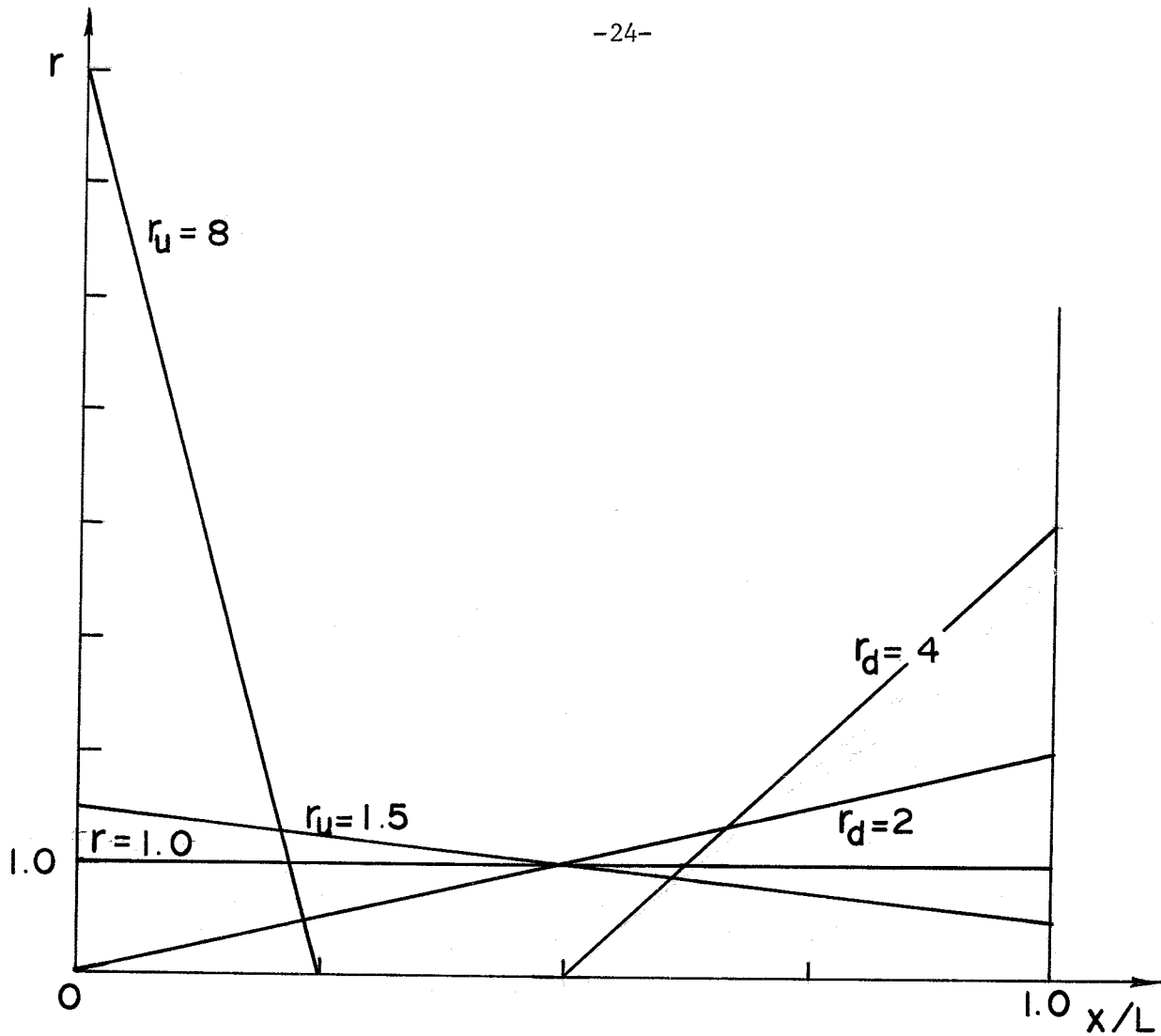


Fig. 9. Longitudinal distributions of tributary inflow.

$$r = \frac{dT/dx}{T/L} \quad (7)$$

and the parameter identifying the distributions is the upstream intercept r_u or downstream intercept r_d . Of course, the area under all the lines is unity. In addition, concentrated inflow at either the upstream or downstream ends ($r_u = \infty$, $r_d = \infty$, respectively) was also allowed in the model.

Figure 10a shows the outflow hydrographs for various distributions of tributary inflow augmenting the upstream inflow of event No. 1. The instantaneous sum T of tributary flows is given by a Pearson type III distribution with skew of 1.2, initial flow 30 cfs, 10% that of the channel base flow, a

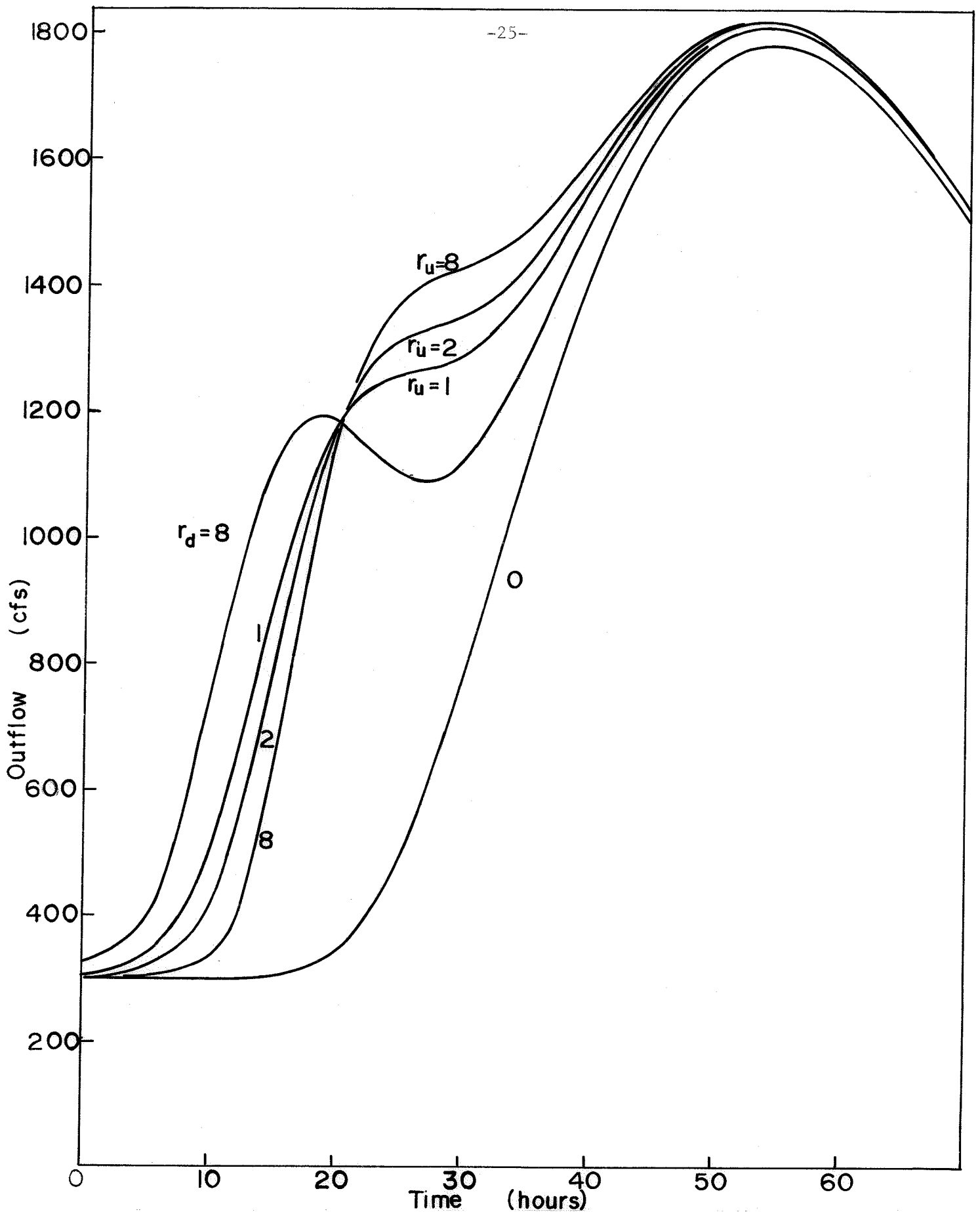


Fig. 10a. Outflow hydrographs with tributary inflow, event No. 1
0 : Outflow with no tributary inflow
 r_u : Upstream intercept of normalized tributary distribution
(see Fig. 9)
 r_d : Downstream intercept of normalized tributary distribution.

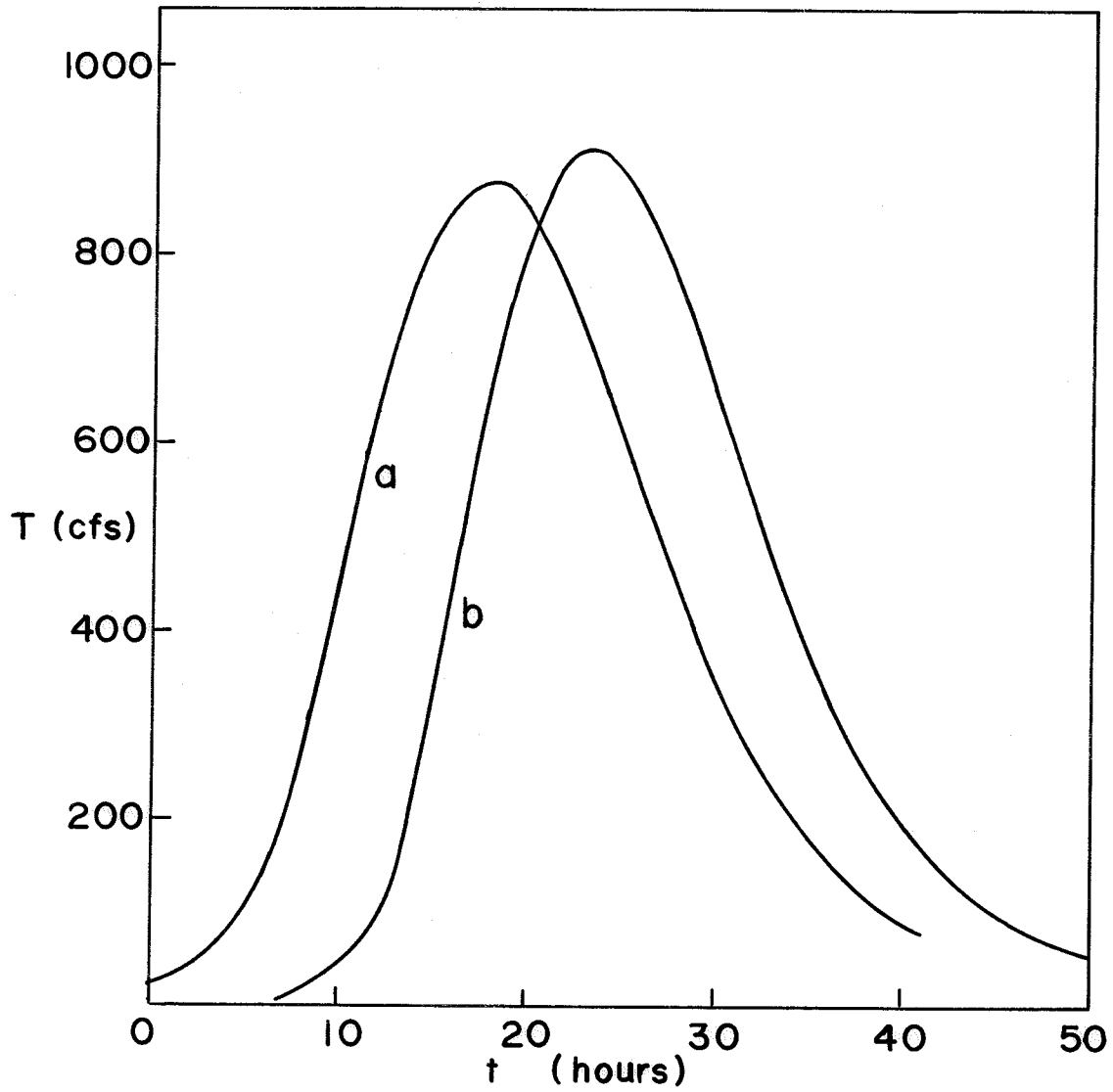


Fig. 10b. Deduced apparent tributary inflow, event No. 1
a: With $r_d = 8$
b: With $r_u = 8$.

peak of 900 cfs, 3 times that of the base flow and half the upstream-inflow peak, and a time to peak of 16 hours, in contrast to the 48-hour time to peak of the upstream inflow hydrograph. These characteristics of the hypothetical tributary hydrograph reflect those of the tributary hydrograph deduced in ref. 1 by use of the UFP model on the March 3, 1970 flood. The parameter distinguishing one outflow hydrograph from another is the intercept of the straight-line distribution, r_u or r_d . These curves illustrate the effect of tributary distribution on the outflow hydrograph, and can be used to show the influence of this actual distribution on the *deduced* tributary hydrograph, computed, as in ref. 1, on the assumption that all tributary flow is concentrated at the downstream end of the reach. Subtraction of the outflow hydrograph obtained without tributary inflow, also shown in Fig. 10a, from the two hydrographs with the most nonuniform tributary distributions results in the two curves of deduced tributary inflow shown in Fig. 10b. Evidently, in the translation of the tributary flow downstream, the peak is moved slightly later in time and higher in magnitude.

These effects are far more evident in the overbank event. For this case, the hypothetical tributary hydrograph was constructed similar to that deduced in ref. 1 for event No. 2, the flood of March 18, 1970. Initial tributary flow was 20 cfs, 6 2/3% of the base flow, the peak was 7000 cfs, 28% of the upstream inflow peak, and the time to peak was 18 hours, in contrast to the 28 hours to peak of the upstream inflow. The tributary hydrograph was again given the form of a Pearson type III with skew of 1.2.

Figure 11a shows hydrographs of upstream inflow (solid curve), inflow with tributary flow entering at the upstream end (dashed curve), outflow without tributary flow (solid), outflow with tributary flow entering upstream (dashed), and outflow with tributary flow entering just above the downstream

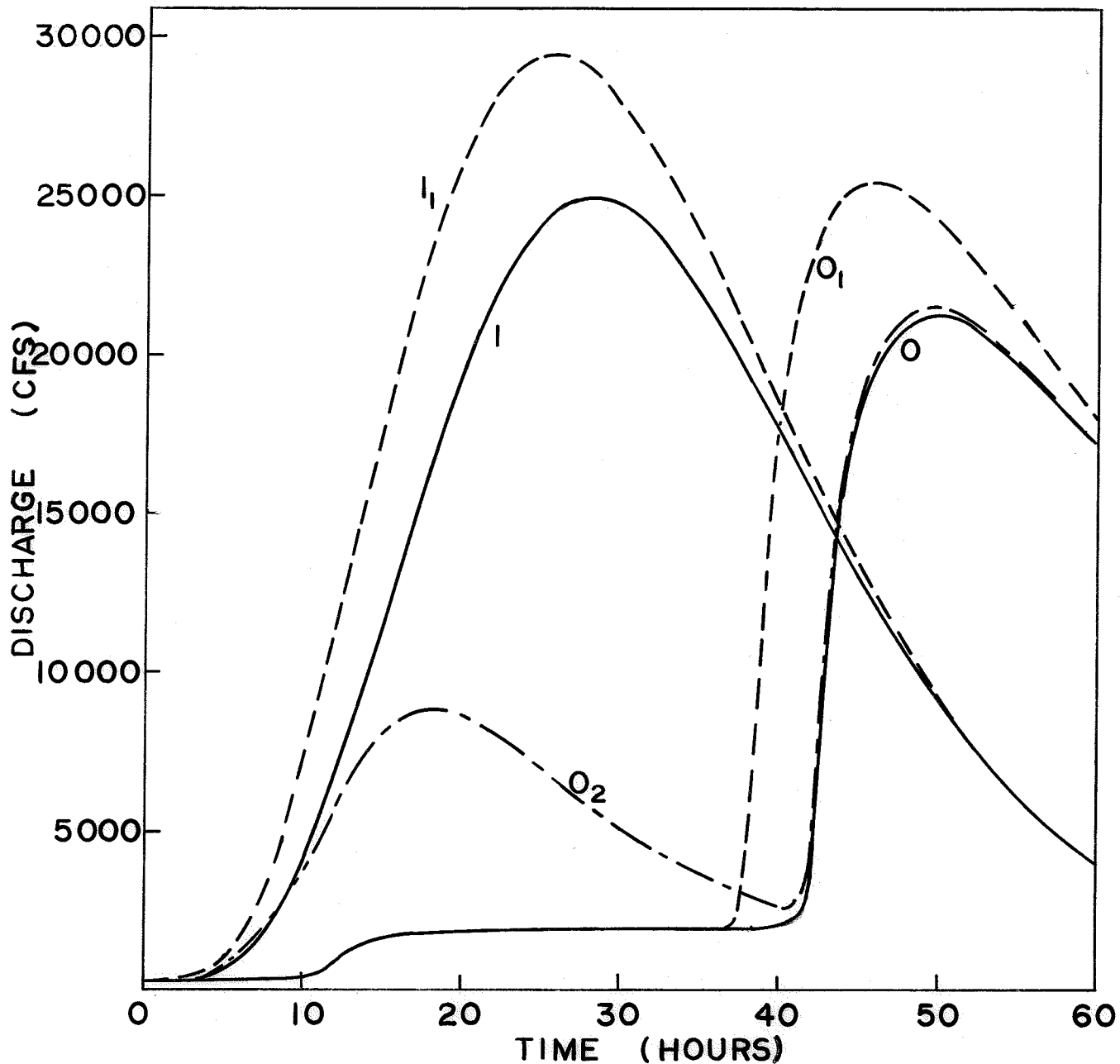


Fig. 11a. Outflow hydrographs with tributary inflow, event No. 2
I : Inflow at upstream end of reach
I₁: Inflow at upstream end plus all tributary inflow
O : Outflow without tributary inflow
O₁: Outflow with tributaries actually entering upstream
O₂: Outflow with tributaries actually entering downstream.

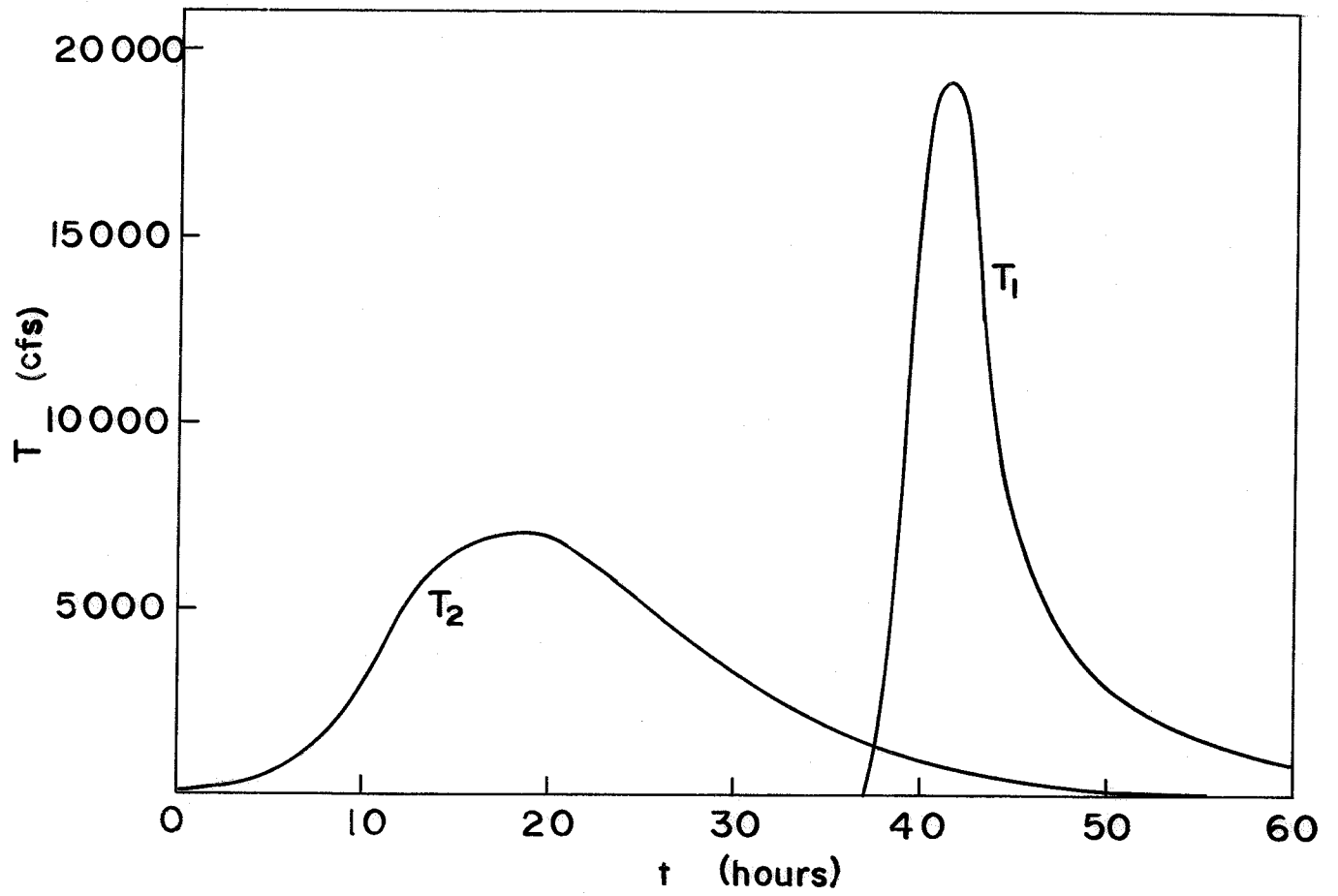


Fig. 11b. Deduced apparent tributary inflow, event No. 2
 T_1 : With tributaries actually entering upstream
 T_2 : With tributaries actually entering downstream.

gage (dot-dash). The resulting deduced tributary hydrographs (assumed lumped downstream) are shown in Fig. 11b.

It is evident that the longitudinal distribution of tributary inflow assumed plays a great role in determining the shape of tributary hydrograph deduced, but no actual, wholly positive distribution can lead to deduced negative values. The source of the latter must come from other factors influencing the routed hydrograph.

The most important such factor in the given reach of the Chuquatonchee, is the assumed storage-outflow relation. If an inappropriate relation is used to route the inflow hydrograph downstream, the deduced tributary hydrograph can exhibit all kinds of peculiar behavior. To test this premise, the tributary hydrograph previously coupled to event No. 2 was entered, in fact, at the downstream end of the channel and the outflow hydrograph "observed" (i.e., computed by the mathematical model). This hydrograph O_0 subtracted from the inflow hydrograph, yields the value of time-rate of change of storage minus tributary inflow,

$$\frac{dS}{dt} - T = I - O_0 \quad (8)$$

from Eq. 1. The left side of Eq. 8 is depicted by a solid curve on Fig. 12a. Since the tributaries are in fact all at the downstream end, they do not influence the storage, hence dS/dt in Eq. 8 can be calculated by routing the inflow hydrograph without regard for tributary inflow. When this is done by the mathematical model to yield a correct outflow hydrograph O_{R_1} , the resulting

$$\frac{dS}{dt} = I - O_{R_1} \quad (9)$$

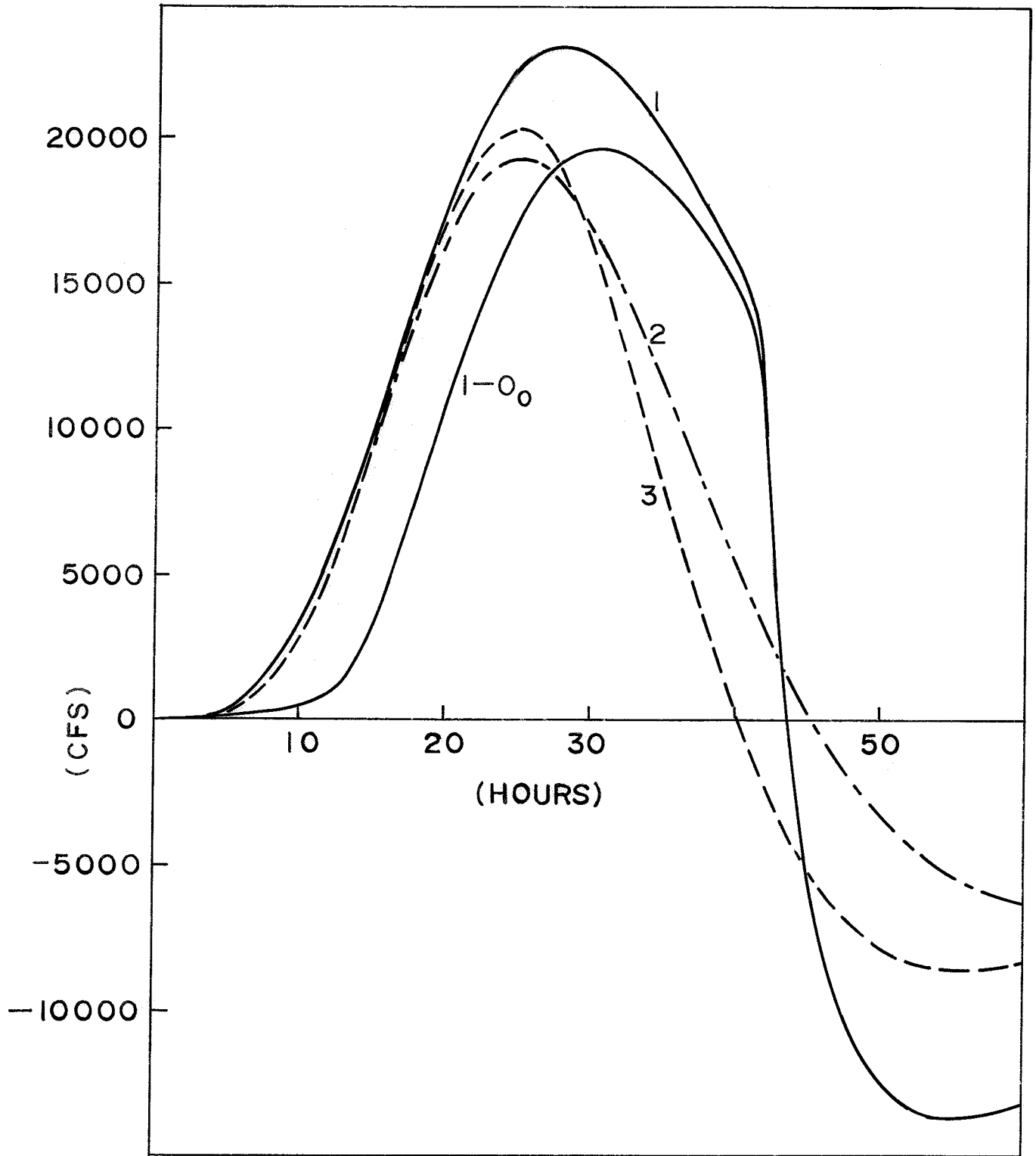


Fig. 12a. Time rate of change of storage with and without downstream tributary inflow, event No. 2

1 - 0₀: $dS/dt - T$ (Saint-Venant)

1: dS/dt (Saint-Venant)

2: dS/dt via Modified-Puls, $N = 2$

3: dS/dt via modified-Puls, $N = 2$, storage reduced to 70%.

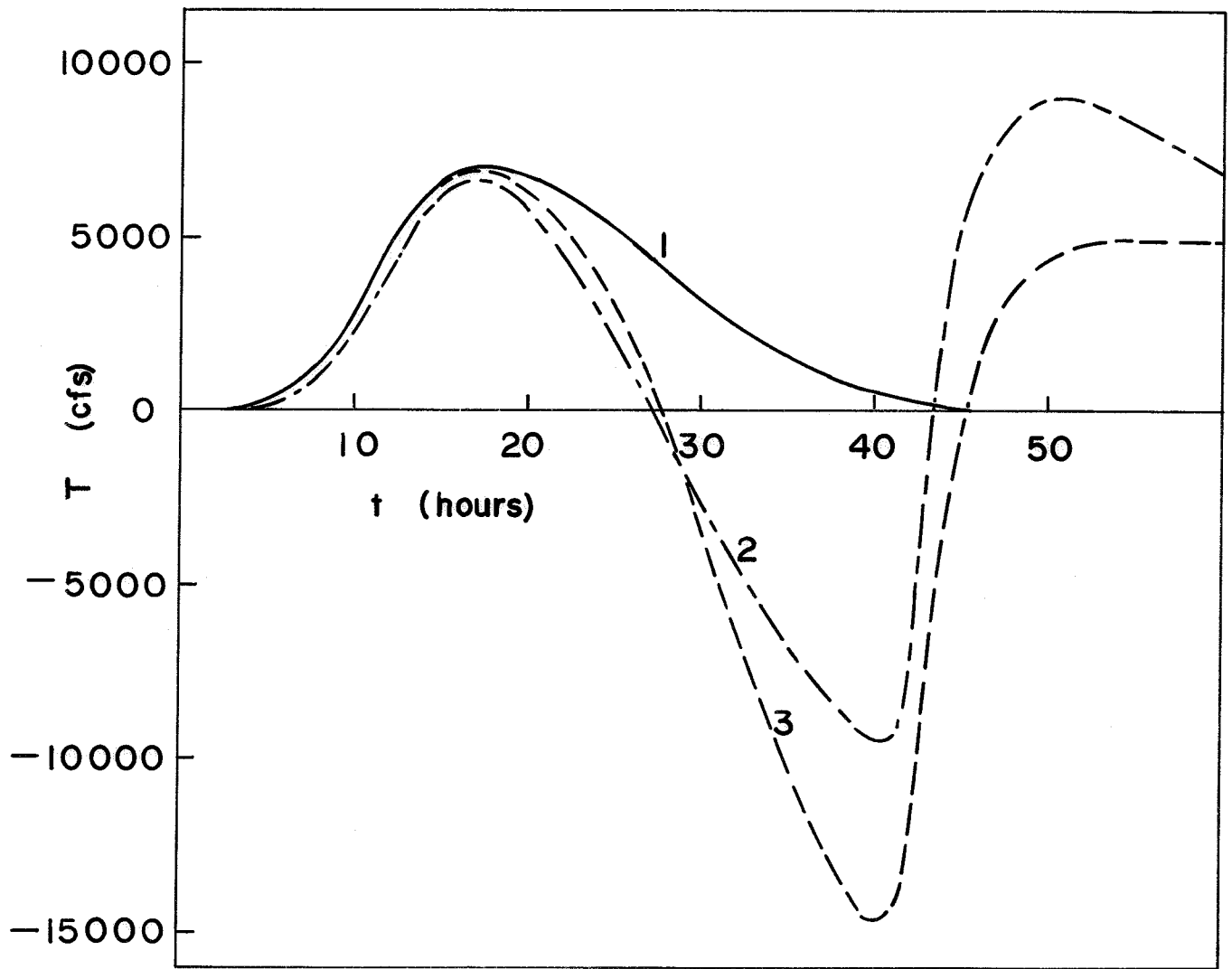


Fig. 12b. Apparent tributary inflows as functions of storage-outflow relations, event No. 2

- 1: With correct storage-outflow relation for event (Saint-Venant)
- 2: With normal-depth storage-outflow relation, looped by Modified-Puls, $N = 2$
- 3: With 70 % of normal-depth storage-outflow relation, looped by Modified-Puls, $N = 2$.

can be substituted in Eq. 8 to reproduce the correct tributary-inflow hydrograph. The left side of Eq. 9 is plotted as a solid curve (1) in Fig. 12a. The resulting deduced $T(t)$ is shown solid, (1), in Fig. 12b.

A second routed hydrograph O_{R_2} was found by the Modified-Puls method using the storage-outflow relation based on steady-flow profiles and $N = 2$. The resulting $I - O_{R_2} = dS_2/dt$ is plotted by dots and dashes (2) in Fig. 12a, and the resulting $T_2(t)$ by dots and dashes (2) in Fig. 12b. The application of the aforementioned 70% storage factor to the Modified-Puls method led to the dashed curves (3) of Figs. 12a and b.

6. Conclusions.

When the storage-outflow relationship for use in the Modified-Puls method is computed in the reach of Chuquatonchee Creek between Abbott and West Point, Mississippi, by two different methods, the results do not agree. One method computes the storage under steady-flow water-surface profiles and associates this with the outflow. The other takes several past records of inflow and outflow and seeks such a storage-outflow relationship common to all these calibration events, that produces reasonable deduced tributary-inflow hydrographs, specifically, one that minimizes their negative portions.

The investigation of this disagreement was performed through a series of easily controlled numerical experiments in a prismatic schematicized channel similar in its principal features to the given Chuquatonchee reach, and with schematic hydrographs similar to the historic events. Real flows in this channel were mathematically modeled by the one-dimensional Saint-Venant equations. By this means, water-surface profiles extant during the floods were computed and storage at any instant readily determined. The conclusions drawn from the results of the study in the prismatic channel were supported by computer runs using the HEC-1, HEC-2, and UFP programs with real topographical and flood data for the Chuquatonchee reach, as well as for the schematic reach and events. This suggests that schematization is a valid procedure for investigating differences between approaches and that the nonprismatic character of the real channel does not affect such results significantly.

The principal reason for the disagreement between the two methods of determining a storage-outflow relationship lies in the fact that no such (unique) relationship exists for the reach. The key postulate in the Modified-Puls approach is that there exists a physical relationship between storage

and outflow, dependent only upon the topography and roughness of the channel. If this were true, the mathematical expression of this relation, $S(O)$, would transcend a particular event, and once found can be used for all events, within the range of calibration. Instead, it was found that the storage-outflow relation is *highly event dependent*.

The storage-outflow relation computed through steady-flow profiles is in error, gross error in the case of overbank events, because the flow profiles in unsteady flow are very different from the unsteady ones at the same outflow. The very small bottom slope of the Chuquatonchee requires a substantial depth gradient to move the flood flow. These relatively large depths at the upstream end of the reach (as opposed to the small depths corresponding to the outflow at the downstream end) engender an enormous amount of storage, due to the broad flat flood plains present in the Chuquatonchee valley. Thus, in the early stages of the flood, the storage in the valley grows extensively while there is practically no change in outflow. Then, eventually, when outflow (and downstream stages) increase rapidly as the flood front finally arrives, relatively little storage is added, and the storage-outflow relation is nearly flat,¹ despite large changes in outflow. The trailing limb of the outflow hydrograph reflects a relatively gradual decrease in storage as the outflow falls. This results in a very "obese" looped storage-outflow relation. The loop obtained with a single-valued $S(O)$ and $N = 2$ is not nearly fat enough to model overbank events. Further, while Modified-Puls outflow peaks can be increased by arbitrarily reducing the steady-flow-computed storage for every outflow, the loop is still too narrow and leads

¹Storage plotted as ordinate, outflow as abscissa.

to peculiar behavior of deduced tributary inflow hydrographs, including double-humps, regions of negative inflow, and so on.

Storage-outflow relationships computed from observed inflow and outflow hydrographs also do not serve the purpose, in principle, because each different event is characterized by a different relation, and in application, because of the single-valuedness or narrow loop assumed for the sought-after relation. Any desired tributary-hydrograph behavior can be achieved with given inflow and outflow hydrographs, by adjusting the $S(0)$ relation up to the point of maximum outflow. Then the die, so-to-speak, has been cast, and the trailing limb of the routed hydrograph is "at the mercy" of the previously set $S(0)$, with attendant loss of control over that portion of the tributary hydrograph.

Concern over the accuracy of the topographical data for the Chuquatonchee became secondary, when it was noted that these theoretical considerations were sufficient in themselves to produce the difficulties encountered in routing floods in this reach. When the geometrical data was adjusted to decrease storage by 30%, in an earlier study, then used in the UFP program, the resultant outflow peaks computed exceeded the measured values, by about 10%. The tests performed for the present study suggest that use of the original topographic data might well reduce these to observed levels and reduce the incidence of deduced negative tributary inflow.

Similarly, errors in estimation of roughness would be relatively insignificant. Other factors, such as longitudinal distribution of tributary inflow, and errors in inflow and outflow measurements due to unsteadiness were investigated and found to be relatively unimportant, compared to the clear event dependency of the storage-outflow relation.

REFERENCES

1. Slocum, Anthony H. and Ramesh A. Dandekar, "Evaluation of Streamflow Routing Techniques with Special Emphasis on Determining Nonlinear Routing Criteria by Optimization." Final Report to the Hydrologic Engineering Center, Sacramento District, Army Corps of Engineers, 609 2nd Street, Davis, California, under Contract No. DACW05-75-C-0027. October 1975.
2. Strelkoff, Theodor, "Comparative Analysis of Floodrouting Methods." Prepared for the Hydrologic Engineering Center, U.S. Army Corps of Engineers, 609 2nd Street, Davis, California, under Contract No. DACW05-80-P-0323. September 1980.

