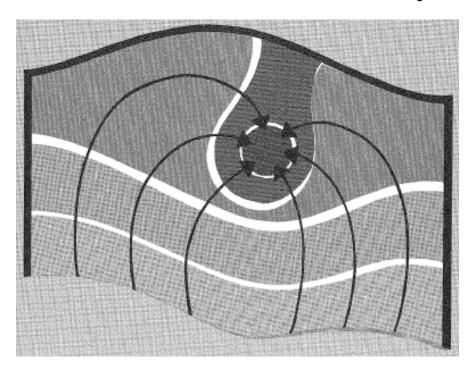


A Comparative Analysis of Groundwater Model Formulation

The San Andres-Glorieta Case Study



June 1984

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14. ABSTRACT This report is concerned primarily with the technical choices which influence the predictions made by groundwater models. In order to keep the discussion specific, attention is focused on a typical modeling case study. This case study involves a public utility wanting permission to pump groundwater from an aquifer, and provided model results that suggested the impacts of its proposed pumpage plan would be acceptable. Local water users responded by sponsoring several independent modeling studies using the same ground water model, with varying results. Overall, the introduction of computer modeling seemed to create more confusion that it dispelled. The case study analysis in this report was undertaken with the belief that the case study might reveal some important insights about the basic strengths and weaknesses of modeling technology.						
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June 1984

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FOREWORD

The idea for this report originated during a routine review of several water supply modeling studies. These studies were all concerned with the impacts of a proposed increase in groundwater withdrawals from the San Andres-Glorieta aquifer in northwestern New Mexico. As the review progressed it became apparent that the drawdowns predicted by the various studies differed dramatically, even though they all relied on the same data base and computer model. This prompted a number of questions – Why were the modeling results so different? Is such variabilty typical? Can any conclusions about groundwater modeling in general be drawn from this example?

This report addresses the questions which grew out of the San Andres-Glorieta modeling review. It also proposes some general modeling guidelines which are intended to deal with the uncertainties and ambiguities inherent in the modeling process. The report is designed for use in training courses and workshops and so is written in a tutorial style. Report preparation was funded under Contract No. DACW05-83-P-1410 from the U.S. Army Corps of Engineers, Hydrologic Engineering Center, Davis, CA. 95616.

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1. INTRODUCTION

Over the last few decades computerized groundwater models have moved from the research laboratory into the offices of consulting hydrologists and government planners. The wide accessibility of computer models and the equipment needed to run them has brought about dramatic changes in the way groundwater studies are conducted. This is particularly true in water supply investigations. Computergenerated maps are commonly found in water supply reports and computer results are frequently entered as evidence in court proceedings dealing with water rights.

Computer modeling has undoubtedly helped to make water supply planning more reliable. But the widespread use of computerized technology has been, in some ways, a mixed blessing. The highly quantified and voluminous outputs generated in modeling studies tend to give the impression that a problem is better understood than it really is. Sometimes computer results can actually obscure the key factors influencing groundwater flow patterns, particularly if these results are poorly presented. Computer modeling is a powerful tool but it needs to be used with discretion. In particular, the uncertainties associated with the modeling process should be recognized and honestly acknowledged.

Perhaps the best way to examine the role of uncertainty in a modeling study is to review the technical decisions which must be made when a model is formulated and applied. Important assumptions are required at every stage of the process—when the model's equations are derived, when a solution procedure is selected, and when inputs are estimated. Many of these assumptions are ultimately based on subjective interpretation of limited amounts of ambiguous field data. Different interpretations lead, of course, to different predictions, making the modeling process more dependent on the judgement of the individual modeler than is generally recognized.

This report is concerned primarily with the technical choices which influence the predictions made by groundwater models. In order to keep the discussion specific, attention is focused on a typical modeling case study. The particular example selected involves an application by Plains Electric Generation and Transmission Cooperative, Inc. (Plains Electric), a public utility, for permission to pump groundwater from the San Andres-Glorieta aquifer in northwestern New Mexico. As part of its application, Plains Electric introduced model results which suggested that the impacts of its proposed pumpage plan would be acceptable. Local water users responded by sponsoring several independent modeling studies. These studies all used the same groundwater model (developed by the U.S. Geological Survey) but the inputs and general approaches to model application differed considerably. Some of the modeling studies predicted rapid dewatering in the vicinity of the Plains well field while others predicted only minor effects on local water levels. Overall, the introduction of computer modeling seemed to create more confusion than it dispelled.

The case study analysis presented in this report was undertaken with the belief that the San Andres-Glorieta experience might reveal some important insights about the basic strengths and weaknesses of modeling technology. Since the report is designed to be used in groundwater training courses, it is written in a tutorial style which focuses on general principles rather than technical details. The references cited throughout the text provide background information on particular subject areas which some readers may wish to cover in more depth.

The next chapter of the report presents background material on the San Andres-Glorieta aquifer system and briefly summarizes the management problem considered in the case study. Chapter 3 reviews basic groundwater modeling concepts, with emphasis placed on the technical decisions which must be made in a typical groundwater impact study. Chapter 4 describes the various modeling approaches used in the case study and considers how each modeler dealt with the technical issues identified in Chapter 3. The report concludes with a suggested set of modeling guidelines for groundwater impact investigations. These guidelines, which are based both on the case study and on general principles, attempt to address the issues of data interpretation and model accuracy in a systematic way.

2. WATER SUPPLY IN THE SAN ANDRES-GLORIETA AQUIFER

2.1 THE MANAGEMENT PROBLEM

The San Andres-Glorieta aquifer is part of the San Juan basin, a layered geological system which extends throughout much of northwestern New Mexico. The portion of this system of most interest here lies between Gallup and Grants along the northern flank of the Zuni mountains (see Figure 2-1). The San Andres-Glorieta is in many ways a typical aquifer of the Colorado plateau. It is a sandstone-limestone formation, confined by adjoining beds of shale and clay, which is recharged from an outcrop area in the Zuni range. The aquifer was once the source of many springs and flowing wells but water levels have gradually dropped in recent years and most groundwater must now be pumped.

Groundwater development in New Mexico is closely regulated by the state, particularly in areas which are declared to be "underground water basins" by the State Engineer. The study area shown in Figure 2-1 covers two declared basins, the Gallup basin to the northwest and the Bluewater basin to the southeast, which are hydrologically continuous but are separated for jurisdictional purposes by the Continental Divide. In 1980 Plains Electric filed an application to withdraw water from a well field located near the existing Shell refinery site at Ciniza. This water is to be pumped exclusively from the San Andres-Glorieta formation. Local water users who believe the Plains proposal will have adverse impacts on their water supply have filed protests with the State Engineer. The pumping schedule proposed by Plains Electric is keyed to projected power plant operations and averages about 3600 acre-feet/year over the 38 year plant life (see Figure 2-2). Since this represents an increase in total annual pumpage of approximately 150 percent above 1982 levels it is not surprising that local residents are concerned.

The water supply management problem posed by the Plains Electric application ultimately involves a decision to accept, reject, or add stipulations to the Plains' proposal. This decision will be based in part on an evaluation of the hydrologic effects of additional pumpage. If projected water level declines are moderate, remedial measures such as the lowering of existing well pumps may be sufficient to protect existing water rights. If projected declines are more severe, such measures may not be adequate and the pumping plan will have to be modified.



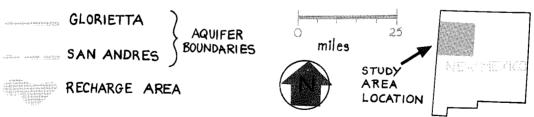


FIGURE 2–1 LOCATION MAP OF THE SAN ANDRES-GLORIETA AQUIFER

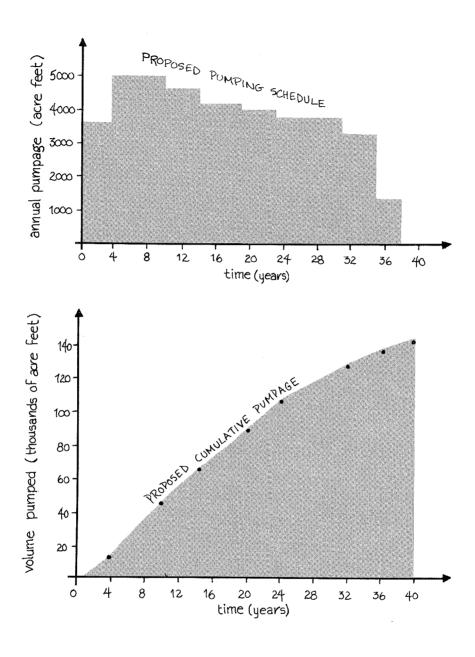


FIGURE 2-2 PROJECTED WATER DEMANDS FOR THE PLAINS ELECTRIC WELL FIELD

2.2 GEOHYDROLOGY OF THE SAN ANDRES-GLORIETA AQUIFER

The geohydrology of the San Andres-Glorieta aquifer has been briefly reviewed in most of the modeling studies of the region (see, for example, Geohydrology, 1982, Geotrans, 1982, and HEC, 1982). Some of the more important primary references include Shomaker (1971), Baars and Stephenson (1977), and Read and Wanek (1961). The primary objective of this section is to summarize available information about the following aquifer properties:

- 1. The geologic structure of the aquifer and its surroundings
- 2. The extent of subsurface leakage into or out of the aquifer
- 3. The location and significance of areas where groundwater flows from the surface into the aquifer (recharge) or from the aquifer to the surface (discharge).
- 4. The distribution of hydraulic head throughout the aquifer
- 5. The range of likely values for aquifer transmissivity and storage coefficient.

This is the basic information needed to construct flow nets, water budgets, and other more complex models of groundwater flow in the aquifer.

The geologic strata in the San Juan basin can be divided, for present purposes, into the following groups (see Figure 2-3 for a typical geological cross section):

- 1. Older strata bounded on the top by the Yeso formation and extending down to Precambrian rock.
- 2. Strata of intermediate age, including the San Andres limestone, the Glorieta sandstone, and the Chinle formation.
- 3. Newer strata composed of alternating layers of shale and sandstone extending up to recent alluvium.

The uplift of the Zuni mountains has resulted in the progressive exposure of each of these strata along an axis extending southwest to northeast perpendicular to the range.

The San Andres and Glorieta formations are the major sources of groundwater in the region—they form a single hydrologically connected unit generally referred to as the San Andres-Glorieta aquifer. The Glorieta sandstone has the largest areal extent of the two formations (see Figure 2-1). It is exposed along the northern flank of the Zuni mountains and gradually pinches out about 80 miles north. The San Andres consists of two limestone beds separated by an intervening sandstone layer similar in composition to the Glorieta. The San Andres outcrops along portions of the northern flank of the Zunis—in these areas it is eroded and has a relatively high transmissivity. The San Andres has a maximum thickness of about 200 feet, as compared to 300 feet for the Glorieta, and disappears altogether in some areas north of the Zuni mountains. It is worth noting that, although the northern and southern limits of the San Andres-Glorieta aquifer are relatively well-defined, it's

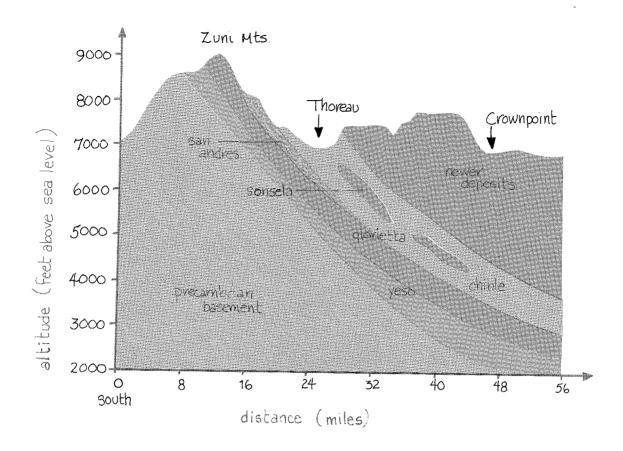


FIGURE 2-3
TYPICAL GEOLOGICAL CROSS SECTION THROUGH THE SAN JUAN BASIN

eastern and western boundaries are less clear. According to information presented by Baars and Stephenson (1977), these boundaries appear to lie well beyond the area covered by Figure 2-1.

Available evidence on the San Juan basin suggests that the San Andres-Glorieta is confined, except near outcrop areas, by the Chinle shale above and the siltstone and claystone beds of the Yeso formation below. The Chinle is a large formation found throughout most of northern Arizona and New Mexico. The areal extent of the Yeso appears to be about the same as the Glorieta. Although the San Andres-Glorieta is surrounded by less permeable geologic formations, there is probably some subsurface movement of water (leakage) into or out of the aquifer. Both the Chinle and Yeso formations include sandstone beds which are sufficiently permeable to supply at least modest amounts of water.

The direction and magnitude of leakage between the San Andres-Glorieta and

adjoining formations depends on the direction and magnitude of the head gradient across the formation boundaries as well as on the leakance value at the boundary. Since both the head gradient and leakage vary over the aquifer it is difficult to make general statements about leakage. Investigators appear to disagree about the direction of vertical gradients between the San Andres-Glorieta and its neighboring strata. Assumed or estimated leakance values vary from 0.0 (HEC, 1982) to 1.0×10^{-11} sec⁻¹. (Geotrans, 1982). An approximate estimate of net leakage flux can be obtained by computing a water budget for the aquifer. The budget presented at the end of this section suggests that the net direction of leakage is out of the San Andres-Glorieta into adjoining formations.

Surface recharge to the San Andres-Glorieta comes mostly from areas where the limestone or sandstone layers of the aquifer are exposed or where these layers are overlain by permeable beds of the Chinle formation, The major recharge area lies along the higher elevations of the Zuni mountains, in the shaded region indicated on Figure 2-1. Note that the aquifer is unconfined (has a free surface for an upper boundary) where recharge takes place. This implies that the hydraulic head response near the Zuni mountain recharge area will be slower and more damped than in interior regions of the aquifer where artesian conditions prevail.

Recharge in the Zuni mountain outcrop region comes mostly from snowmelt and does not appear to vary greatly from year-to-year. Most investigators seem to accept Shomaker's (1971) estimate that the long-term average (approximately steady-state) annual infiltration rate in the recharge zone is about 1.0 inch/year. It should be pointed out that this value is speculative since supporting field observations are limited (Shomaker, 1971; Geotrans, 1982). If the 1.0 inches/year infiltration rate is applied uniformly over a recharge area of 150 square miles (a conservatively low value) the estimate obtained for total annual recharge is 8040 acre-feet/year. Head-dependent leakage from Bluewater Lake and other surface waters could contribute additional recharge, although these sources are probably small.

Groundwater discharge from the San Andres-Glorieta is even more difficult to estimate than recharge, partly because it is less localized and varies more over time. Temporal variations in total annual discharge from the San Andres-Glorieta are illustrated by Shomaker's (1971) estimates for the period 1929-1969 (see Figure 2-4). These estimates clearly demonstrate the increase in groundwater usage which has occurred over the last few decades.

The most convenient way to summarize available discharge information is to distinguish two discharge categories – (1) small wells and springs with nearly constant discharges and (2) large wells which account for the observed increase in regional pumpage. The first category includes about 90 stock wells scattered throughout the aquifer and a few springs on the northern flank of the Zuni mountains. Each of these small sources discharges an average of about 1.5 acre-feet/year (Shomaker, 1971). Table 2-1 summarizes estimated discharges for the second category for two years, 1968 and 1982. The values given are rough estimates intended for comparative purposes only and do not necessarily represent the water rights

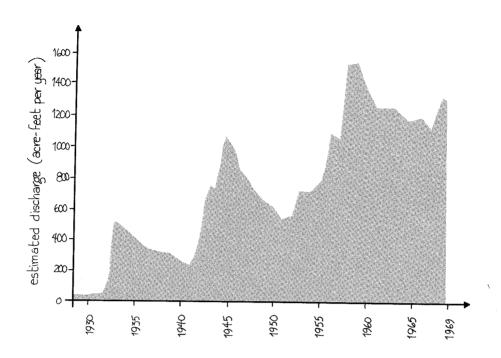


FIGURE 2-4
ESTIMATED TOTAL DISCHARGE FOR THE SAN ANDRES-GLORIETA AQUIFER
(After Shomaker,1971)

claimed by the water users listed (claimed rights are generally larger than historical pumpage values). Table 2-1 shows that the larger wells in Category 2 are responsible for most of the discharge from the aquifer. These wells tend to be concentrated in a narrow strip running along Interstate 40 between Grants and Gallup.

There are relatively few surveys of the regional hydraulic head distribution in the San Andres-Glorieta aquifer. Shomaker (1971) conducted a study of well observations in the Fort Wingate vicinity (east of Gallup) which shows declines of from 40 to 150 feet over the period 1958 to 1968. Geohydrology (1982) extrapolated Shomaker's 1968 data several miles east using more recent information obtained from scattered well observations. The extrapolated distribution is shown in Figure 2-5 (in blue) together with some 1958 contours compiled by Gordon (1961) for the Grants-Bluewater area (in red). Geotrans (1982) and HEC (1982) assumed that the 1968 heads represent an approximate steady-state. This assumption is supported by data which indicate that annual aquifer discharge remained nearly constant

TABLE 2-1

ESTIMATED HISTORICAL DISCHARGES FROM THE SAN ANDRES-GLORIETA AQUIFER

(all values in acre-feet/year)

CATEGORY I-Small Wells and				105
90 sources @ 1.5 acre-fe	eet			135
	Subt	total		135
CATEGORY II-Large Wells a	nd Well Fields			
			Annual	Pumpage
WATER USER	TOWNSHIP	RANGE	<u>1968</u>	1982
El Paso Natural Gas	13	13	70	80
Independent users	13	13	120	220
Bureau of Indian Affairs	13	14	50	90
Transwestern Pipeline	14 -	13	75	75
Town of Thoreau	14	13	40	75
Independent users	14	14	90	200
Whispering Cedars	14	15	50	200
Independent users	14	15	135	250
Shell Oil Refinery	15	15	600	60 0
El Paso Natural Gas	15	16	200	225
El Paso Natural Gas	15	17	200	275
U.S. Army (Ft. Wingate)	15	17		200
Subtotal			<u>1630</u>	<u>2490</u>
AQUIFER TOTALS			<u>1765</u>	<u> 2625</u>

for several years prior to 1968 (see Figure 2-4). Although the 1968 head data do not necessarily reflect current water levels in the San Andres-Glorieta, they do give a good qualitative picture of groundwater movement. Figure 2-5 clearly shows that the general direction of flow is northward, away from the recharge area, with a gradient of approximately 60 feet/mile at the northern edge of the aquifer outcrop.

Practical estimates of aquifer transmissivity and storage coefficient are usually based on several different sources of data. The following paragraphs briefly review some relevant geologic and hydrologic information available for the San Andres-Glorieta. An analysis of the way this information may be used to develop model inputs for the case study is provided in Chapter 4.

Transmissivity and storage coefficient are vertically averaged aquifer parameters which may be obtained by multiplying the hydraulic conductivity and specific storage, respectively, by the groundwater flow depth (see Section 3.2.2). Under artesian conditions, this depth is equal to the aquifer thickness. Since the San Andres-Glorieta is composed of two geological formations which have different hydrologic properties it is best to consider the thickness of each formation when estimating vertically averaged parameters. Aquifer thickness maps computed by Geotrans (1982) indicate that the Glorieta sandstone gradually decreases in thickness from the Zuni mountains north with a moderate bulge occurring east of Crown Point. The thickness of the San Andres limestone varies significantly, primarily as a result of the effects of erosion and fracturing. Available well logs suggest that the San Andres may vanish altogether in some areas near the Plains well field. The major contribution of this formation probably occurs near the Zuni mountain recharge area.

There appear to be significant regional variations in the hydraulic conductivity and specific storage of the San Andres and Glorieta formations. The porosity and conductivity of the San Andres increases where there has been erosion, fracturing and minor faulting, as has been observed in the Grants-Bluewater region. The Glorieta sandstone is generally less heterogeneous than the San Andres formation, although its conductivity and porosity apparently increase where the limestone has eroded. This tends to compensate somewhat for the disappearance of the more conductive San Andres. Some faults and anomalies found near the Zuni mountains may block groundwater flow, particularly in the direction parallel to the axis of the Zuni range. Although these structural features could influence the results of local pump tests they probably have little effect on regional (aquifer scale) flow patterns, at least north of the Zuni outcrop.

When the above factors are considered together a qualitative description of aquifer parameter variations emerges. Transmissivity is highest in the southern portion of the aquifer, particularly near Grants, and gradually decreases to the north. A similar but less dramatic variation probably applies to the storage coefficient. Locally high values of transmissivity may be observed where the San Andres is present if fracturing or solution porosity are significant. Finally, aquifer properties are relatively homogeneous in areas where the San Andres is absent.

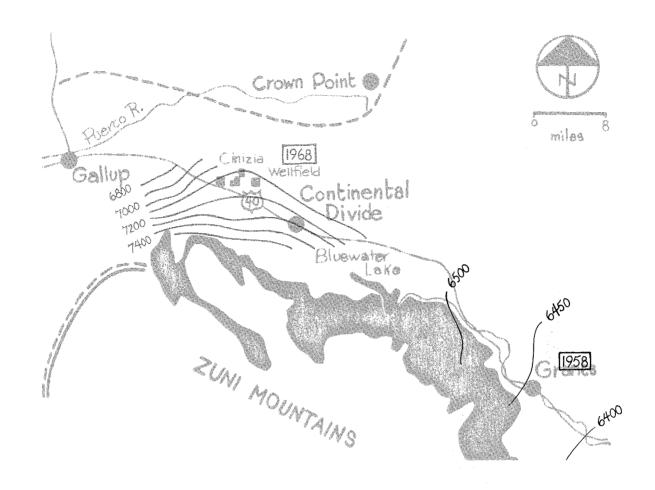


Figure 2-5
MEASURED HEAD CONTOURS IN THE SAN ANDRES-GLORIETA AQUIFER

Although this qualitative description is useful, it does not provide actual values for the aquifer parameters. These values must be estimated using one of the procedures mentioned in Chapter 3. For an aquifer the size and complexity of the San Andres-Glorieta the most practical estimation techniques are pump test analysis and regional model calibration. These methods are not mutually exclusive but may be combined to make best use of available information.

Pump test results for the San Andres- Glorieta are available from Shomaker's (1971) report on the Ft. Wingate vicinity and from Geohydrology (1982). The Shomaker results give transmissivities ranging from 5 to 3700 ft²/day, depending on location. The storage coefficients (based only on a few wells) range between 3.0×10^{-5} and 1.3×10^{-4} (unitless). It should be noted that these estimates come from a variety of sources and are often based on short-term (i.e., limited range) tests on wells which may be partially or multiply completed. They are useful primarily for defining an approximate range of likely values.

The pump tests reported by Geohydrology (1982) were conducted at the pro-

posed Plains Electric well field and have therefore received considerable scrutiny by modelers concerned with the Plains pumping proposal. Transmissivity estimates based on these tests vary from 80 ft²/day (Dames and Moore, 1982) to 3000 ft²/day (Geohydrology, 1982) although most investigators appear to now agree on the narrower range of 150 to 450 ft²/day. Storage coefficient estimates vary from 1.0×10^{-4} (Dames and Moore, 1982) to 5.0×10^{-4} (Geohydrology, 1982).

The primary data available for regional model calibration are the 1968 head contours shown in Figure 2-5. These heads were used by both Geotrans (1982) and HEC (1982) to guide adjustments of transmissivity and other model inputs such as recharge and leakage. Geohydrology (1982) used head observations from the area near the Shell refinery well field for a similar purpose. While the Shell observations are not regional in nature, they do provide useful information about the long-term response to stress in an important part of the study area.

Geohydrologic information presented in this section is conveniently summarized with a simplified water budget for the San Andres-Glorieta aquifer. The major components of the water budget are recharge (supply), discharge and leakage (demand), and storage change. The recharge, discharge, and head data described earlier provide the information needed to estimate two water budgets – a quasi-steady-state budget for 1968 and a non-steady-state budget for 1982. Each of these is summarized in Table 2-2.

The recharge is based on the 1.0 inch/year infiltration rate suggested by Shomaker (1971) and is assumed to be relatively constant. Recharge from Bluewater Lake and other surface waters (if any) is not included. Discharge values are obtained from Table 2-1. Leakage is selected as the unknown element of the 1968 steady-state water budget since it is the most difficult to estimate. If the 1968 leakage is adjusted to give a water balance the result is 6475 acre-feet /year. This is an estimate of the net amount of subsurface water leaving the San Andres-Glorieta aquifer during 1968. A water balance can be obtained for 1982 if the 1968 leakage value is assumed to apply and storage change is selected as the unknown. This gives an estimated 1982 storage decrease of 860 acre-feet. This decrease is due both to compressibility (artesian) effects and to water table declines in unconfined portions of the aquifer.

Although the estimates given in Table 2-2 are speculative, they clearly suggest that leakage is an important component of the water budget. The estimated recharge would have to be decreased significantly (by a factor of five) to change this conclusion. Such a decrease could be obtained only by limiting infiltration to unreasonably low values or by greatly reducing the generally accepted area of the recharge region. A rough check on the recharge estimate can be obtained by using the head gradient of 60 feet/mile observed at the northern edge of the Zuni mountain outcrop. If this value is multiplied by the length of the recharge area (approximately 50 miles) and a conservatively low transmissivity value of 300 ft²/day, the resulting estimate of subsurface flux entering the aquifer from the Zuni outcrop is 7500 acre-feet/year. This is consistent with Table 2-2.

TABLE 2-2

SIMPLIFIED WATER BUDGETS FOR THE SAN ANDRES-GLORIETA AQUIFER

(all values in acre-feet/year)

SUPPLY (into aquifer)	$\begin{pmatrix} \text{water levels} \\ \text{steady} \end{pmatrix}$	1982 (water levels) falling)
Recharge from Zuni mountain outcrop	8040	8040
Subtotals	8040	8040
DEMAND (out of aquifer)		
Discharge from wells and springs	1765	2625
Leakage to adjoining aquifers	6275	6275
Subtotals	8040	8900
STORAGE CHANGE		
Annual decrease in storage		860

3. BASIC MODELING CONCEPTS

The case study described in the preceding chapter has a number of features which are commonly encountered in ground water management. The basic problem is to predict impacts relatively far into the future (forty years) using a limited amount of ambiguous data. The economic stakes are large, both for the power company and for local residents who may be adversely affected by increased groundwater pumpage. Mathematical modeling seems to be a reasonable approach to use, although the limitations of the available data base cast some doubt on the accuracy of the any long-term prediction.

This chapter considers in detail how groundwater models may be applied in case studies such as the San Andres-Glorieta. Individual sections describe the various tasks which are required to derive credible predictions from uncertain field data. These tasks were carried out in all of the modeling studies examined in Chapter 4, although the specific methods used and answers obtained differed considerably. As the discussion moves from general concepts to specific criticisms it should become apparent how the methods outlined here could be used to evaluate other modeling studies directed toward other management problems.

A typical modeling study includes three basic elements – definition of the problem to be investigated, formulation of a modeling strategy for solving this problem, and actual application of the model. These can be divided into more specific tasks as follows:

Problem Definition

- 1. Identify the management problem to be investigated (e.g., evaluation of the impacts of aquifer development) and formulate a set of preliminary specifications for the modeling study.
- 2. Review existing information on the site (e.g., geological reports, well data, etc.). Develop a feeling for the quantity and quality of the data available. Construct a qualitative description of groundwater flow in the aquifer of interest.

Model Formulation

1. Develop a detailed mathematical description of the aquifer system. This description should account for the physical factors most relevant to the problem of interest and should be specific enough to indicate the type of computer program required.

Model Application

- 1. Select a computer program for solving the modeling problem.
- 2. Determine the level of spatial and temporal discretization required in the model's inputs and outputs and set up a simulation network.
- 3. Estimate all model inputs from available data sources.
- 4. Check the model's prediction accuracy using comparisons of model predictions with field data and/or sensitivity analyses.
- 5. Use the model to investigate the management problem posed at the beginning of the study.

The discussion of modeling concepts presented in this chapter follows the organization outlined above. Section 3.1 defines the impact evaluation problem and suggests how a qualitative description of groundwater flow can be constructed from commonly available data sources. Section 3.2 reviews the general principles used to develop a mathematical groundwater model. The modeler should have a working knowledge of these principles even if he plans to use an "off-the-shelf" computer program since many practical modeling decisions depend on the properties of the governing equations. Section 3.3 examines a number of application-oriented issues, including the important problems of input estimation and accuracy evaluation. These three sections provide most of the technical background needed for the case study analysis of Chapter 4.

3.1 PROBLEM DEFINITION

The management issue addressed in the San Andres-Glorieta case study is the problem of evaluating the impact of pumpage from a limited groundwater resource. This problem arises frequently in water supply planning, particularly in arid regions such as the San Juan basin, where groundwater is the major supply source. A comprehensive impact evaluation should probably include some consideration of the economic, social, water quality, and hydrologic consequences of development. In practice, attention is usually focused on those impacts which can be most readily measured – changes in aquifer water levels and changes in groundwater quality. This report considers only hydrologic impacts, primarily because of a need to limit the scope of the presentation. It should be noted, however, that most of the general concepts discussed here are also relevant to investigations of water quality.

When a groundwater hydrologist decides to use a mathematical model for impact analysis, he should identify as clearly as possible the type of information he expects to obtain. One way to do this is to attempt to answer the following questions:

1. What variables should be used to measure hydrologic impact? Depth-to-water, change in storage, drawdown, or other less common indices?

- 2. Where are the boundaries of the aquifer? Do they extend beyond the region of most interest for impact evaluation?
- 3. Where are predictions required? How much spatial resolution is needed? Do some areas warrant a more detailed analysis than others?
- 4. How far in the future are predictions required? How much temporal detail is needed? Are certain seasons or years particularly important?
- 5. How accurate must the model's predictions be in order to be useful for decision-making?

The answers to these questions constitute a set of preliminary specifications for the modeling study.

It is generally helpful to review available aquifer data before a mathematical model is formulated. In the San Andres-Glorieta case study, useful sources of information included geological reports and maps prepared by the U.S. Geological Survey, consultant or government reports documenting earlier studies, and well logs or water level observations compiled locally. A preliminary "desk-top" analysis of some of this information provides a good feeling for regional groundwater behavior and may even indicate that a detailed modeling investigation is not necessary.

A good example of a valuable desk-top calculation is the San Andres-Glorieta aquifer water budget presented in Table 3-2. This type of simple water budget quickly identifies serious data gaps which must be filled before a complete modeling effort can be undertaken. If water level data are available at two or more times they can be used to check the storage change computed by subtracting supply and demand figures. This can reveal inconsistencies in data and assumptions which are best discovered early in the study.

There are a number of other simple analyses that can be performed prior to modeling. These include construction of flow nets and calculations of well field drawdowns using Theis curves or other graphical methods (see Freeze and Cherry, 1979, and Chapter 4 of this report for some examples). The results of several different desk-top computations can be combined to give an overall picture of groundwater flow in the aquifer of interest. This may later be used to check the credibility of the model's predictions.

3.2 MODEL FORMULATION

3.2.1 Basic Concepts of Groundwater Flow

An aquifer-scale water budget provides a useful but highly aggregated description of subsurface flow. A more detailed analysis is needed to predict local changes in water levels and to account for spatial heterogeneity in applications such as the San Andres-Glorieta case study. A detailed analysis requires the water budget approach to be applied individually to many small pieces (or elements) of the aquifer. The smallest aquifer element which can be analyzed with traditional groundwater flow theory is the so-called "representative elementary volume", commonly abbreviated REV (see Freeze and Cherry, 1979). This can be thought of as a volume element whose dimensions are small compared to the entire aquifer but large compared to individual pores in the aquifer soil matrix. The actual size of an elementary volume depends on the type of aquifer—typical dimensions for the San Andres-Glorieta vary from a few inches for a fine sandstone to hundreds of feet for a fractured limestone. It is mathematically convenient to derive water belances and other equations describing groundwater flow at the REV scale and then to integrate these equations to obtain solutions for larger regions.

Some of the basic definitions used in groundwater flow derivations are illustrated in Figure 3-1. This figure shows a hypothetical experimental apparatus designed to measure the properties of a saturated porous medium (e.g., sand). The sand is confined in a cylinder whose dimensions are large compared to the representative elementary volume illustrated in the detail. Water flows from an inlet tank with a surface elevation h_1 through the sand into an outlet tank with a lower surface elevation h_2 . The respective inflow and outflow rates (in units of l^3/t) are designated Q_{in} and Q_{out} .

The water balance for the cylinder simply states that the difference between the inflow and outflow at any time is equal to the rate of change in the mass of water stored in the sand. This may be expressed mathematically in the following way:

$$\frac{d[\rho nV]}{dt} = \rho Q_{in} - \rho Q_{out} \tag{3-1}$$

where ρ is the density of water (with units m/l^3), n is the unitless porosity defined in Figure 3-1, and V is the volume of the cylinder. The product ρnV is the total mass of water in the cylinder at a given time.

l = length

m = mass

t = time

 $f = force(ml/t^2)$

¹In this discussion, the following symbols are used to define units:

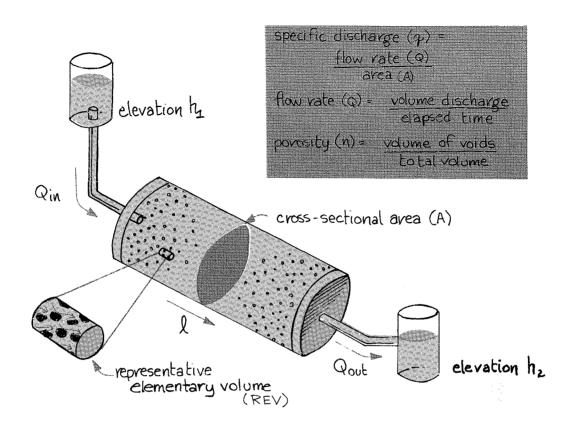


FIGURE 3-1
EXPERIMENTAL APPARATUS USED TO ILLUSTRATE BASIC CONCEPTS
OF GROUNDWATER FLOW

At the smaller REV scale an equivalent water balance equation may be written in differential form:

$$\frac{\partial(\rho n)}{\partial t} = -\frac{\partial[\rho q(l,t)]}{\partial l} \tag{3-2}$$

Here q(l,t) is defined as the specific discharge, or flow per unit area, through the REV at location l and time t (see Figure 3-1). Partial derivatives are used because the discharge depends on more than one independent variable. Note that the minus sign is required to insure that the mass decreases if the flow increases in the direction of positive l.

The dependence of ρ and n on pressure are accounted for reasonably well by the following expression (Freeze and Cherry, 1979):

$$\frac{\partial(\rho n)}{\partial t} = \rho S_s \frac{\partial h(l, t)}{\partial t} \tag{3-3}$$

where h is the hydraulic head (with units l) and S_s is a soil parameter called the specific storage (with units l^{-1}). Equations (3-2) and (3-3) may be combined to give a compact expression for the dynamic mass balance at the REV scale:

$$S_s \frac{\partial h(l,t)}{\partial t} = -\frac{\partial q(l,t)}{\partial l} \tag{3-4}$$

This is sometimes called the mass continuity equation.

The derivation leading to Equation (3-3) provides a convenient expression for the specific storage:

$$S_s = \rho g(\alpha + n\beta) \tag{3-5}$$

Here ρg is the specific weight of water (with units f/l^3), g is the acceleration of gravity (with units l/t^2), and α and β are the aquifer and fluid compressibility coefficients (with units l^2/f). Spatial variations in specific storage are due primarily to variations in aquifer porosity and compressibility; the other parameters in Equation 3-5 are nearly constant

The unknowns in the continuity equation are related by Darcy's law, an empirical principle which states that specific discharge is directly proportional to the head gradient:

$$q(l,t) = -K\frac{\partial h(l,t)}{\partial l} \tag{3-6}$$

Here K represents the <u>hydraulic conductivity</u> of the soil (with units l/t). This parameter depends on the structure of the soil or rock matrix and may vary over many orders of magnitude. It can be estimated from the apparatus shown in Figure 3-1 if the specific discharge and hydraulic head are held constant.

Darcy's law allows the REV mass balance to be written as a single differential equation in one unknown, the hydraulic head:

$$S_s \frac{\partial h(l,t)}{\partial t} = \frac{\partial}{\partial l} \left[K \frac{\partial h(l,t)}{\partial l} \right]$$
 (3-7)

This expression may be generalized to three-dimensional situations to give the fundamental equation of groundwater flow in various coordinate systems. The well-known cartesian version of this equation obtained for isotropic (direction-independent) hydraulic conductivity and specific storage may be written:

$$S_s \frac{\partial h}{\partial t} = \frac{\partial}{\partial x} \left[K \frac{\partial h}{\partial x} \right] + \frac{\partial}{\partial y} \left[K \frac{\partial h}{\partial y} \right] + \frac{\partial}{\partial z} \left[K \frac{\partial h}{\partial z} \right]$$
(3-8)

Here it should be understood that the head depends on the coordinates of the REV (x, y, and z) as well as on time. In saturated soils the isotropic parameters S_s and K are functions of location only.

3.2.2 The Vertically Averaged Flow Equation

Many aquifers used for water supply are thin geological strata which extend horizontally for many miles but may be only a few hundred feet thick. The San Andres-Glorieta is a typical example, as is illustrated by the geological cross section shown in Figure 2-3. In such aquifers, variations in head over the smaller vertical dimension may be negligable compared to variations over the larger horizontal dimension. In order to consider some of the practical implications of this, it is useful to distinguish two types of head observations. The three-dimensional head h(x, y, z, t) appearing in Equation (3-8) can be thought of as the water level measured in a well located at horizontal coordinates x and y which has a single very short well screen installed at vertical elevation z. If the screen elevation were changed to z', the water level in the well could also change, although the difference may be negligable. If the well screen extended throughout the entire depth of the aquifer, from a lower elevation of $Z_L(x,y)$ to an upper elevation of $Z_U(x,y)$, the water level observed in the well would be an average of the heads at all elevations from Z_L to Z_U . This vertically averaged head (written $\overline{h}(x,y,t)$) may be defined mathematically as:

$$\overline{h}(x,y,t) = \frac{1}{D(x,y,t)} \int_{Z_L(x,y)}^{Z_U(x,y,t)} h(x,y,z,t) dz$$

where $D(x, y, t) = Z_U(x, y, t) - Z_L(x, y)$ is the saturated flow depth. Note that Z_U and D will depend on time if the aquifer is unconfined (i.e. if the upper boundary surface is free to rise or fall). If the aquifer is confined, Z_U and D will be constant at any given location.

If vertical variations throughout the aquifer depth are small compared to horizontal variations across the aquifer, then the unaveraged head h(x, y, z, t) will be close to the averaged head $\overline{h}(x, y, t)$ at any elevation z. In this case, there is really no need to solve the three-dimensional flow equation to obtain the unaveraged head. Instead, we can solve a simpler two-dimensional equation which gives the vertically averaged head at any horizontal location (x, y). This vertically averaged head is effectively treated as an approximation to the unaveraged head at any depth between Z_L and Z_U .

The relationship between averaged and unaveraged head is illustrated in Figure 3-2 for a simple wedge-shaped aquifer. The upper three-dimensional portion of the figure shows the five and ten foot contours plotted as functions of all three spatial coordinates. Note that the head decreases slightly as z increases (for constant x and y), suggesting a small upward flow (from high to low head). The vertical variation in head is clearly small compared to the large horizontal variation observed in the x direction. The lower two-dimensional portion of the figure shows the five and ten foot contours obtained by integrating the three-dimensional head over the z coordinate. This two-dimensional description captures the major horizontal variations which are of primary concern in a regional impact evaluation.

The two-dimensional flow equation which gives the vertically averaged head may be obtained by integrating Equation (3-8) over the z coordinate (Pinder and Gray, 1977). The result is the following equation, which applies at any location (x,y) in the horizontal plane:

$$S\frac{\partial \overline{h}}{\partial t} = \frac{\partial}{\partial x}T\frac{\partial \overline{h}}{\partial x} + \frac{\partial}{\partial y}T\frac{\partial \overline{h}}{\partial y} + q_h - Q_w$$
 (3-9)

The new variables appearing in this expression are defined as follows:

 \overline{h} =vertically averaged hydraulic head(l)

 $S = S_s D = \text{storage coefficient (unitless)}$

T = KD = transmissivity (l/t)

D =saturated flow depth (l)

 q_h =net vertical flux per unit area crossing the upper and lower boundaries of the three-dimensional aquifer (l/t)

 Q_w =vell pumpage per unit area (l/t)

All of the groundwater models discussed in this report are based on Equation (3-9).

The vertical boundary flux q_h accounts for all water entering or leaving the aquifer across its upper or lower boundaries. This includes surface recharge, evapotranspiration, and leakage, as well as the change in storage which results if the upper aquifer boundary is a free surface (i.e., if the aquifer is unconfined). When all of these flux components are considered q_h may be written as follows:

$$q_h = q_i + q_u + q_l + q_f (3-10)$$

where q_i is the net infiltration rate (surface recharge minus evapotranspiration), q_u and q_l are the leakage fluxes across the upper and lower boundaries, and q_f is the effective flux due to changes in the free surface at the upper aquifer boundary.

Leakage may be treated as if it were a recharge flux, or it can be described by the following version of Darcy's law (written for the upper boundary):

$$q_u = K_{vu} \frac{h_u - \overline{h}}{\Delta z_u} \tag{3-11}$$

This equation assumes that the leakage passes through a confining bed of thickness Δz_u (with units l) and vertical hydraulic conductivity K_{vu} (with units l/t). The head on the far side of the upper confining bed is h_u (assumed to be known) and storage in the bed is neglected. Sometimes the ratio $K_{vu}/\Delta z_u$ (with units t^{-1}) is called the <u>leakance</u>.

If a portion of the aquifer's upper boundary is unconfined, this boundary (called the free surface or water table) may rise or fall in response to pressure changes. When the water table rises, groundwater is stored in the unsaturated soil above. Conversely, groundwater is released from the draining soil column when the water

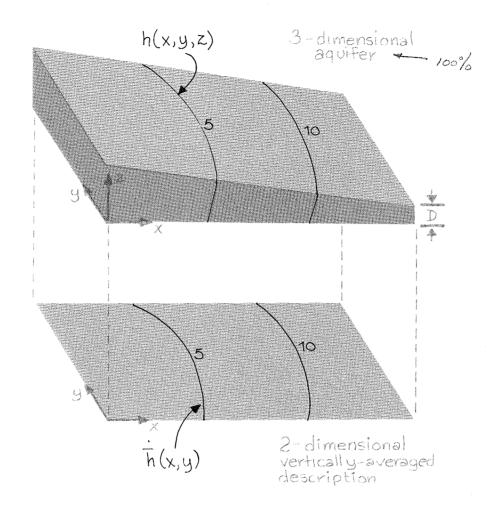


FIGURE 3-2
USING VERTICAL AVERAGING TO OBTAIN A TWO-DIMENSIONAL FLOW MODEL

table falls. The amount of water released (in mass per unit area) in response to a change in the height of the water table is given by:

$$\Delta m = -\rho(n - \theta_r) \Delta z \tag{3-12}$$

where θ_r is the specific retention, or fraction of water retained in the unsaturated soil column, and Δz is the change in water table elevation. Since the change in elevation is approximately equal to the change in the vertically averaged head, the effective flux of water due to water table fluctuations is given by:

$$q_f = -S_y \frac{\partial \overline{h}}{\partial t} \tag{3-13}$$

Here the term $(n - \theta_r)$ has been replaced by S_y , a unitless aquifer parameter called the specific yield. It should be noted that the San Andres-Glorieta aquifer

is confined everywhere except in limited areas along the southern outcrop region. Some simple methods for dealing with the unconfined conditions found in this region are described in Chapter 4.

If all the effects contributing to the flux of water across the aquifer's boundaries are included, the depth averaged flow equation may be written as:

$$(S+S_y)\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}T\frac{\partial h}{\partial x} + \frac{\partial}{\partial y}T\frac{\partial h}{\partial y} + L_u(h_u - h) + L_l(h_l - h) + q_i - Q_w$$
(3-14)

Here L_u and L_l are leakance coefficients for the upper and lower boundaries, and h_u and h_l are the heads in the upper and lower confining layers.² Note that the compressibility and water table storage change terms are combined on the left-hand side of the equation. Because storage changes due to compressibility are minor compared to those due to water table fluctuations, S is usually several orders of magnitude smaller than S_y and can be neglected where the upper boundary is unconfined. Sometimes the sum $S + S_y$ is called the storage coefficient and is written as S. This coefficient is then varied over a wide range of values to account for transitions from confined to unconfined conditions (this is one of the approaches taken in Chapter 4).

3.2.3 Integration of the Flow Equation over Space and Time

The three dimensional and vertically averaged groundwater equations discussed in the preceding sections apply at a single location (the centroid of an REV) and at a single time. Since these equations contain derivatives of the unknown head they must be integrated over an appropriate spatial region and time period before an explicit head solution can be obtained. The integration process effectively extends the range of the equation from a single point to the entire aquifer.

Whenever a differential equation is integrated one or more arbitrary constants are introduced. A simple example is the following equation giving the position x of a particle moving in a straight line at a fixed velocity V:

$$\frac{dx}{dt} = V \tag{3-15}$$

If this equation is integrated over the time period from t = 0 to t = T, the result is:

$$x(t) = Vt + C \qquad 0 \le t \le T \tag{3-16}$$

This expression is a solution to the original equation for any constant value of C—i.e., the particle can be moving with velocity V at any location x.

The non-uniqueness of the solution is eliminated by specifying an auxiliary condition or constraint which defines the location x(0) of the particle at the time t=0. Equation (3-16) may then be solved for C, giving the unique solution:

²The overbar used earlier to denote vertically averaged head has been dropped here to simplify notation.

$$x(t) = Vt + x(0)$$
 $0 \le t \le T$ (3-17)

The position x(0) is an initial condition for the governing differential equation.

Groundwater simulation models perform numerical integrations which are similar in general concept to the above example. These models can provide unique head solutions only if a sufficient number of auxiliary conditions are specified by the modeler. Some simple examples can help to explain how these conditions arise in practice. The example presented below examines the use of an initial condition in a hypothetical transient flow problem.

Example: Initial Conditions

Consider the rectangular aquifer shown in Figure 3-3. This aquifer has a constant depth and is isolated from its surroundings by flow barriers (e.g., geological faults) on all four sides and by confining layers above and below. A small leakage flux moves uniformly across the upper boundary into or out of an adjoining formation. The aquifer geometry of Figure 3-3 suggests that the head distribution can be adequately described by the two-dimensional vertically averaged groundwater equation (Equation 3-14). The transmissivity is assumed to be a constant and the pumpage and infiltration rates are assumed to be zero. The upper boundary leakage is described by Equation (3-11), with the leakance and adjoining aquifer heads assigned constant values. The initial head is assumed to have a uniform value h_0 throughout the aquifer.

Since the aquifer's geometry, physical properties, and inputs are completely uniform in space there is no reason to expect the head to vary over the x and y coordinates, although it may change over time. It is therefore reasonable to tentatively assume that the x and y derivatives in Equation (3-14) are zero. This assumption may later be checked by substituting the resulting head distribution into Equation (3-14) to insure that it is indeed a solution.

If h depends only on time, the groundwater equation becomes an ordinary differential equation:

$$S\frac{dh}{dt} = L_u(h_u - h) \tag{3-18}$$

This may be integrated using standard methods to give (see Hildebrand, 1976):

$$h(t) = Ce^{-\frac{L_u}{S}t} + h_u \left[1 - e^{-\frac{L_u}{S}t} \right]$$
 (3-19)

Here C is an unknown constant of integration.

Equation (3-19) gives a head distribution which satisfies the vertically averaged groundwater equation for any value of C (this is easily checked by direct substitu-

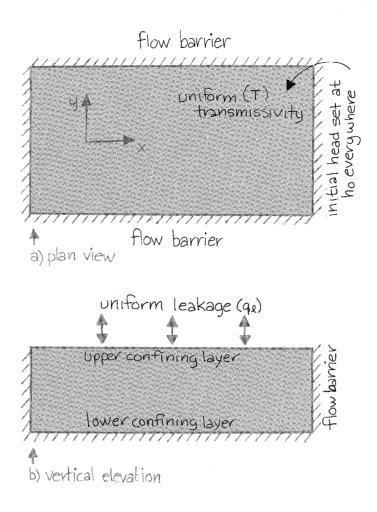


FIGURE 3-3
PLAN AND ELEVATION OF THE AQUIFER ANALYZED IN EXAMPLE 1

tion). This non-uniqueness is eliminated by imposing the initial conditions defined earlier. If the head at time t = 0 is equal to h_0 , then Equation (3-19) at this time becomes:

$$h(0) = h_o = C (3-20)$$

The integration constant is equal to the initial head. Consequently, the unique solution to the problem of Figure 3-3 is:

$$h(t) = h_o e^{-\frac{L_u}{S}t} + h_u \left[1 - e^{-\frac{L_u}{S}t} \right]$$
 (3-21)

This head applies uniformly throughout the aquifer.

The solution of Equation (3-18) implies that the head will rise, fall, or remain constant, depending on the relationship between h_0 and h_u . These three possibilities are illustrated in the plots of head vs. time presented in Figure 3-4. Note that the

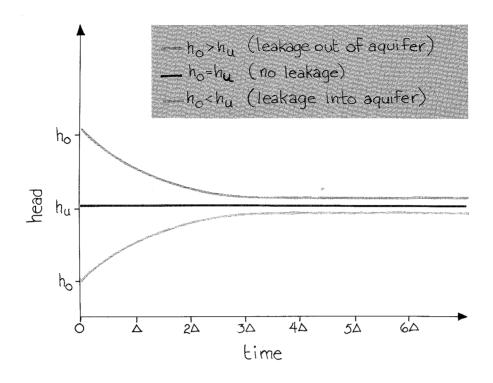


FIGURE 3-4 SIMULATED HEADS FOR EXAMPLE 1

head in every case eventually approaches the constant value h_u . This implies that water will leak either into or out of the aquifer until the two heads h(t) and h_u are equal. As this occurs, the leakage gradually diminishes and the aquifer approaches steady-state (i.e., its head stops changing). The steady-state water budget in this case is very simple—all supplies and demands are zero.

The above example illustrates several points which are of general interest. It is apparent that an initial value of head must be specified throughout the solution region if the simulation problem is dynamic (i.e., if the head changes over time). The time-varying head will approach a steady-state if all the aquifer inputs are approximately constant, as they often are in relatively undeveloped aquifers. The manner in which the head converges to steady-state depends strongly on the initial head distribution. The rate at which this convergence takes place is inversely proportional to the storage coefficient (large coefficients give slower response than small coefficients).

When the head distribution varies with location, the solution of the groundwater flow equation becomes more complicated. In this case, the auxiliary constraints needed to insure uniqueness are specified (for all times) along the boundary of the solution region. Such boundary conditions generally take the following forms:

- 1. Specified-Head Boundary Conditions In this case, the head h(x, y, t) is set equal to a known value $h_b(x, y, t)$ over some specified portion of the boundary.
- 2. Specified-Flux Boundary Conditions In this case, the normal component of the vector velocity crossing the boundary is specified. This is usually accomplished by imposing the constraint:

$$-K\frac{\partial h(x,y,t)}{\partial n} = q_b(x,y,t) \tag{3-22}$$

where n represents the distance along an outward pointing vector normal to the boundary at the location (x, y) and $q_b(x, y, t)$ is the normal component of the flux into the aquifer (in units l/t).

Either but not both of the above conditions must be imposed at every point along the boundary. In practice, the specified flux boundary condition is usually only used when the boundary is known to be a barrier to flow (i.e., when $q_b(x, y, t)$ is zero). This is because non-zero boundary flux values are difficult to estimate from available data, particularly when boundaries are located far from monitored wells.

The second example in this section considers the use of combined head and flux boundary conditions in a hypothetical problem similar to the one examined in Example 1.

Example: Boundary Conditions

Suppose that the aquifer from the preceding example is not totally isolated but is, rather, influenced by recharge from a stream that runs above the left-hand boundary. This effect may be simulated by imposing a constant head boundary condition $(h(x, y, t) = h_0)$ along the entire boundary. Suppose, in addition, that leakage and pumpage are negligible but that the infiltration rate has a uniform non-zero value of q_{i0} . As before, the transmissivity and initial heads are assumed to be equal to T and h_0 throughout the aquifer. This revised problem is illustrated in Figure 3-5.

As time progresses, the infiltrating water from above will begin to accumulate and the head will rise above its uniform initial value (except along the fixed head boundary). Eventually, the internal head gradient will be sufficiently large to drive all the infiltrating water out across the left side of the aquifer. The head will then stop changing and the aquifer will be in steady-state. The steady-state head profile

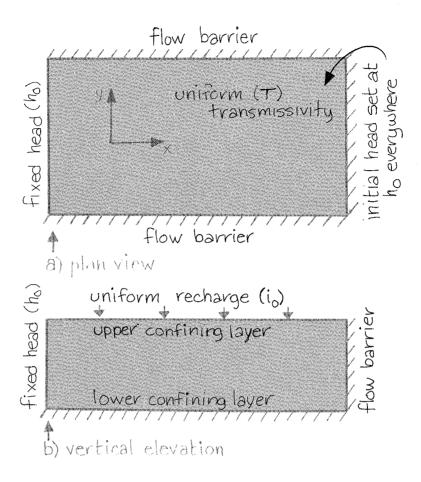


FIGURE 3-5
PLAN AND ELEVATION OF THE AQUIFER ANALYZED IN EXAMPLE 2

may be found by setting the temporal derivative in Equation (3-14) equal to zero, The resulting partial differential equation is:

$$\frac{\partial}{\partial x}T\frac{\partial h}{\partial x} + \frac{\partial}{\partial y}T\frac{\partial h}{\partial y} + q_{io} = 0$$
 (3-23)

Here h is a function of location (x and y) but not of time.

The integration of Equation (3-23) may be simplified by noting that the flow problem defined in Figure 3-5 is completely uniform in the y direction, i.e., there is no reason why the head should vary in this direction. Consequently, it is reasonable to (tentatively) assume that the head depends only on the x coordinate. This assumption may later be checked by substituting the resulting head distribution into Equation (3-23) to insure that it is, in fact, a solution.

If h depends only on x, $\partial h/\partial y$ is zero and Equation (3-14) becomes an ordinary

differential equation:

$$\frac{\partial}{\partial x}T\frac{\partial h}{\partial x} + q_{io} = 0 (3-24)$$

This may be integrated once to give:

$$T\frac{\partial h}{\partial x} = -q_{io}x + C_1 \tag{3-25}$$

Here C_1 is an unknown constant of integration. Equation (3-25) may be solved for $\partial h/\partial x$ and integrated a second time to give a general expression for the head:

$$h(x,y) = -\frac{q_{io}}{2T}x^2 + \frac{C_1x}{T} + C_2$$
 (3-26)

where C_2 is a second integration constant.

Equation (3-26) gives a head distribution which satisfies the basic groundwater equation for any values of C_1 and C_2 . This non-uniqueness is eliminated by imposing the boundary conditions defined earlier. If there is no flow across the shaded boundaries, the specific discharge across these boundaries must equal zero. But Darcy's law (Eq. 3-6) states that the specific discharge along the right-hand boundary is given by:

$$q(L,y) = -K \frac{\partial h(L,y)}{\partial x}$$
 (3-27)

Since the aquifer has constant depth the no-flow condition may be written as:

$$-\frac{T}{d}\frac{\partial h(L,y)}{\partial x} = 0 \tag{3-28}$$

where d is the depth. Equation (3-25) may be substituted into Equation (3-28) and solved for C_1 (note that x is equal to L since the no-flow condition applies at the right-hand end of the aquifer):

$$C_1 = q_{io}L \tag{3-29}$$

The head solution now has only one arbitrary constant which may be found by noting that the head along the left-hand boundary (where x = 0) is equal to h_0 . Setting x = 0 in Equation (3-21) gives:

$$h(o, y) = C_2 = h_o (3-30)$$

The unique solution obtained subject to the assumption that $\partial h/\partial y=0$ is therefore:

$$h(x,y) = -\frac{q_{io}}{2T}x^2 + \frac{q_{io}Lx}{T} + h_o$$
 (3-31)

Direct substitution reveals that this expression satisfies Equation (3-23). It clearly satisfies the left- and right-hand boundary conditions since it was constructed from them. It satisfies the upper and lower boundary conditions since the specific discharges and y derivatives across these boundaries are both zero.

The head and velocity distributions for this example are plotted vs. x in Figure 3-6. Note that the head is flat at the right end where the flux is required to be zero. The head at the left end is fixed at the value h_0 , as specified by the left-hand

boundary condition. The velocity vectors point in the x direction and increase in magnitude from right to left.

This example clearly shows that boundary conditions must be imposed to give a unique solution to the steady-state flow equation. If a dynamic solution is desired, both initial conditions and boundary conditions must be imposed. It should be noted that boundary conditions cannot be chosen arbitrarily – they must be consistent with assumptions made in the flow equation. If, for example, the aquifer of Figure 3-5 had no-flow boundaries on all four sides, there would be no way for the infiltrating water to leave. This water would accumulate indefinitely, causing the head to rise steadily throughout the aquifer. In this case, there is no steady-state solution, i.e., there is no head distribution which can satisfy both Equation (3-23) and the imposed boundary conditions.

Since auxiliary conditions must ultimately be estimated from limited quantities of field data the model's boundaries and simulation period should be selected to simplify the estimation process as much as possible. The following general guidelines are often helpful:

- 1. Boundaries should be laid out, whenever possible, along flow barriers or lines of symmetry.
- 2. Specified head boundaries should pass through regions where the head is relatively constant through time. It is best if these boundaries lie near monitored wells
- 3. In impact studies such as the San Andres-Glorieta, where the effects of pumping are of primary concern, it is wise to locate the model's outer boundaries well beyond the region likely to be affected by the pumpage. Internal boundaries maybe used to describe faults (no-flow boundaries) or surface water bodies such as lakes or perennial streams (constant head boundaries).
- 4. If possible, the simulation should be started at a time when the aquifer is known to be at steady-state. In this case, a steady-state head solution may be used to initialize subsequent dynamic simulations.
- 5. If a steady-state initialization is not feasible, the simulation should be started at a time when a reasonable number of reliable well observations are available for defining an initial condition.

If the above suggestions are followed, the model's data requirements can be reduced significantly (see Section 3.3.3).

Most groundwater flow problems cannot be solved using the analytical integration procedures outlined in the examples. These procedures require assumptions of uniformity, symmetry, and geometric regularity which rarely hold in real aquifers. Fortunately, there are many numerical methods for integrating the flow

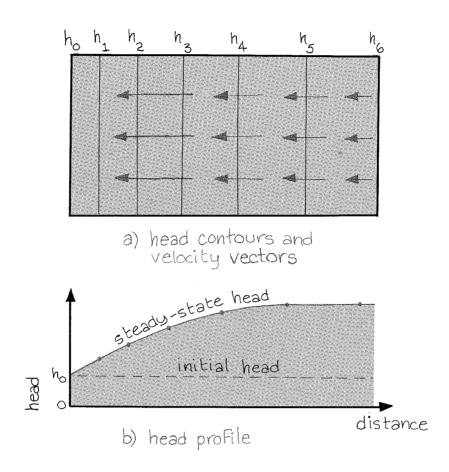
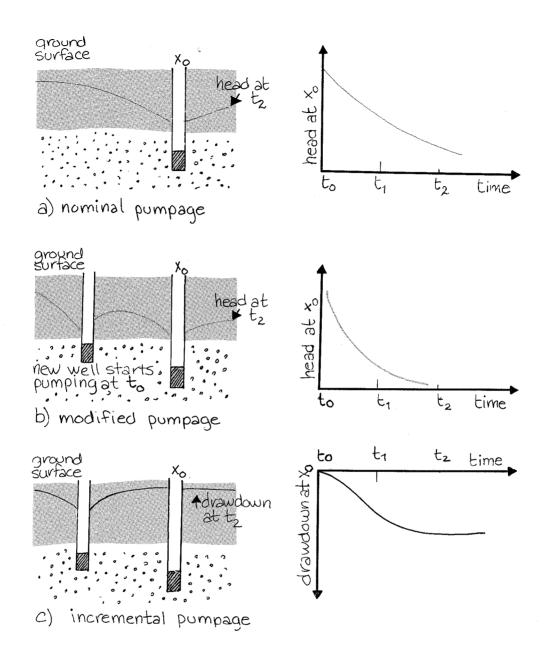


FIGURE 3-6
HEAD AND VELOCITY DISTRIBUTIONS FOR EXAMPLE 2

equations when boundary conditions and aquifer geometry are more complex. These methods involve repetitive calculations that are ideally suited for electronic computers. Section 3.3 briefly discusses some of the numerical methods and computer programs available for solving realistic groundwater problems.

3.2.4 Impact Evaluation, Drawdown, and Superposition

The procedure for simulating a vertically averaged head distribution for the San Andres-Glorieta case study should now be apparent. First, a solution region is laid out and boundary conditions are defined. If the simulation is dynamic, an initial head distribution must also be specified. Then the various coefficients and source terms in the flow equation are estimated. These include the aquifer parameters (transmissivity, storage coefficient, and leakance), infiltration and pumpage rates,



 $\mbox{FIGURE 3-7} \\ \mbox{SIMULATION RESULTS FOR A TYPICAL IMPACT EVALUATION PROBLEM}$

and adjoining aquifer heads. Finally, the flow equation is solved for the hydraulic head distribution. The hydrologic impacts of different management strategies are evaluated by comparing the head solutions obtained with appropriate pumping rates.

Hydrologic impact evaluation is concerned primarily with the change in ground-water elevation which results from a change in some specifed (or "nominal") pumping strategy. The nominal strategy may be selected in many ways—it could represent undeveloped conditions (no pumpage), it could be a continuation of current pumping levels, or it could be a "low growth" option based on a slowly increasing pumpage rate. The head associated with the nominal pumping strategy may be called the nominal head while the head associated with some modified pumping strategy (usually more pumpage) may be called the modified head. It is customary in impact studies to use the nominal head as a reference for computing drawdown, which is then defined as:

$$d(x, y, t) = h_m(x, y, t) - h_n(x, y, t)$$
(3-32)

Here the subscripts n and m represent nominal and modified heads, respectively.

These concepts are illustrated in Figure 3-7, which shows nominal and modified heads and drawdown for a simple impact evaluation problem. The nominal pumping strategy consists of pumpage from a single well. This pumpage results in a general decline in water levels, as indicated in Figure 3-7a. The modified strategy illustrated in Figure 3-7b includes a second well which further accelerates the rate of decline. The drawdown curves plotted in Figure 3-7c are obtained by differencing nominal and modified heads.

The situation pictured in Figure 3-7 may be described mathematically with two vertically averaged flow equations, one for each pumping strategy:

Nominal pumping strategy

$$S\frac{\partial h_n}{\partial t} - \frac{\partial}{\partial x}T\frac{\partial h_n}{\partial x} - \frac{\partial}{\partial y}T\frac{\partial h_n}{\partial y} = q_i + Q_n$$

$$+ L_u(h_u - h_n) + L_l(h_l - h_n)$$
(3-33)

$$h_n(x,y,o) = h_o(x,y)$$
 initial condition $h_n(x,y,t) = h_b(x,y,t)$ head boundary condition $-K \frac{\partial h_n}{\partial x}(x,y,t) = q_b(x,y,t)$ flux boundary condition

Modified pumping strategy

$$S\frac{\partial h_m}{\partial t} - \frac{\partial}{\partial x}T\frac{\partial h_m}{\partial x} - \frac{\partial}{\partial y}T\frac{\partial h_m}{\partial y} = q_i + \Delta q_i$$

$$+ Q_n + \Delta Q + L_u(h_u - h_m) + L_l(h_l - h_m)$$
(3-34)

$$h_m(x,y,o) = h_o(x,y)$$
 initial condition $h_m(x,y,t) = h_b(x,y,t)$ head boundary condition $-K \frac{\partial h_m}{\partial x}(x,y,t) = q_b(x,y,t)$ flux boundary condition

The terms preceded by Δ represent the changes in pumpage and recharge associated with the modified pumping schedule (recharge might change if some of the additional water pumped eventually infiltrates back to the aquifer). Note that the auxiliary conditions and the heads in adjoining aquifers (h_u and h_l) are assumed to be the same for the nominal and modified pumpage strategies. This implies that both strategies start with the same initial head distribution. It also implies that the boundary conditions are unaffected by pumpage.

If the aquifer is confined or if it is unconfined but head variations are small compared to the saturated flow depth, the aquifer parameters S, T, L_u , and L_1 have the same (constant) values in each equation. In this case, the problem is linear and the principle of superposition holds (Freeze and Cherry, 1979). The modified pumpage equation may then be subtracted from the nominal pumpage equation to give the following expression for drawdown:

Drawdown

$$S\frac{\partial d}{\partial t} - \frac{\partial}{\partial x}T\frac{\partial d}{\partial x} - \frac{\partial}{\partial y}T\frac{\partial d}{\partial y} = \Delta q_i + \Delta Q - L_u d - L_l d \qquad (3-35)$$

$$d(x,y,o)=0$$
 initial condition $d(x,y,t)=0$ drawdown boundary condition $\frac{\partial d}{\partial n}(x,y,t)=0$ gradient boundary condition

This important equation reveals that drawdown depends only on the aquifer parameters and the assumed changes in pumpage and recharge. The initial and boundary conditions for the drawdown equation are identically zero (and so are known perfectly) and the nominal pumpage and infiltration drop out.

The result derived above suggests that the inputs to a vertically averaged groundwater model can be divided into two different categories -- primary inputs which affect both head and drawdown predictions and secondary inputs which affect only head. These categories are defined as follows:

Primary Inputs

- a) Time-invariant Aquifer Parameters
 Transmissivity
 Storage coefficient
 Leakance
- b) Postulated Changes in Groundwater Fluxes
 Pumpage change
 Recharge change

Secondary Inputs

- a) Auxiliary Conditions
 Initial head
 Boundary head or flux
 (Adjoining aquifer heads)
- b) Nominal Values of Groundwater Fluxes
 Nominal pumpage and discharge
 Nominal infiltration

Since drawdown is of particular interest in impact evaluations, the primary inputs deserve the most time and attention during the modeling process. Secondary inputs are important in several respects mentioned later but clearly do not have as much influence on the overall conclusions of the impact analysis.

It might seem reasonable to forget head simulation altogether and simply solve the drawdown equation directly. This is a legitimate approach which is frequently used (see Chapter 4 for an example). Drawdown simulation does, however, have some practical limitations which are worth noting.

It is important to remember that the drawdown derivation is based on the assumption of linearity. Strictly speaking, the groundwater equation is linear only if the aquifer is completely confined. If unconfined effects are significant, the transmissivity and storage coefficient both depend on head and the superposition operation needed to derive the Equation (3-35) cannot be carried out. This implies that drawdown models should not be used in areas where significant depletion or dewatering may take place.

Although drawdown is the primary measure of the hydrologic impact of development it does not tell the whole story. In some situations, it is important to

know whether the groundwater elevation will fall below a specified level, such as the top of the aquifer or the inlet of an existing well. Such questions can be answered only with a head simulation which is referred to a fixed datum. The secondary inputs required to simulate head but neglected in drawdown simulation provide the information needed to locate groundwater levels relative to such a datum.

There are clearly some advantages to the straightforward approach of simulating both nominal and modified heads. The drawdown predictions obtained by differencing these heads will be the same as those obtained by solving the drawdown equation directly and additional information will be gained on absolute water levels. Comparisons of model predictions and field observations are often easier if the head simulation approach is used. The only real disadvantage of this approach is the extra effort required to estimate secondary model inputs. If information on these inputs is very limited a simulation based on the drawdown equation may be the best alternative.

3.3 MODEL APPLICATION

3.3.1 Selection of a Computer Program

Nearly all computer programs used to simulate the hydrologic impacts of groundwater development are based on the same general concepts—the principles of mass continuity and Darcian flow discussed in Section 3.2. The primary differences between available programs are the dimensionality of the governing flow equation (either the full three-dimensional equation or the two-dimensional vertically averaged equation is usually used) and the numerical procedure used to integrate this equation.

Most groundwater modeling studies rely on two-dimensional (vertically averaged) computer models, although three-dimensional models are being used with increasing frequency. Three-dimensional analyses are generally required if groundwater is being pumped from several aquifers which are hydrologically connected. In such cases, the simple leakage relationship used in the two-dimensional vertically averaged flow equation may not be adequate. Although three-dimensional models can, in principle, provide a more realistic description of complex flow patterns they require more input data and are more expensive to run than their two-dimensional counterparts. Some three-dimensional inputs, such as the vertical component of the hydraulic conductivity, are very difficult to estimate from field observations. The actual benefits of using a three-dimensional model depend on the availability of reliable data as well as on the complexity of the groundwater system.

Several different numerical procedures are available for solving either the two or three-dimensional groundwater flow equation. The most popular are the finite difference and finite element techniques described in Section 3.3.2. Either of these will give acceptable results for most problems. The finite element method is somewhat more convenient to use with solution regions which have irregularly-shaped boundaries. The finite difference technique is often less expensive and is easier to understand and explain.

Although dimensionality and solution technique are important, there are also a number of other factors to be considered when selecting a groundwater program. The program should be well-documented, preferably with a user's manual which includes several solved examples. Data entry should be convenient and the program should be "portable" between different computers. The program's outputs should be easy to understand and should include high-resolution contour plots which are able to display both measured and simulated heads. Finally, the program should have a proven record of successful application in "real world" situations.

Perhaps the most widely used groundwater flow models available to the general public are the U.S. Geological Survey (USGS) finite difference models described in Trescott, Pinder, and Larson (1976) and in Trescott (1975). Several state agencies such as the California Department of Water Resources and the Kansas Geological Survey have also developed models which are in the public domain. The U.S. Army Corps of Engineers' Hydrologic Engineering Center has assembled a package of

model application programs designed to be used with the USGS two-dimensional finite difference model (HEC, 1983). This package includes a program for constructing model input files and a program for plotting model predictions. A somewhat out-of-date but useful review of publicly available groundwater programs is given in the American Geophysical Union's monograph on groundwater modeling (Bachmat et al., 1980).

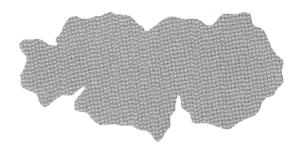
Although there are many groundwater modeling programs to choose from, the first-time or infrequent modeler would probably do best to stay with the well-documented and thoroughly-tested models of the USGS. This choice should allow the user to concentrate on problem formulation and data analysis rather than the mechanics of computer programming.

3.3.2 Spatial and Temporal Discretization

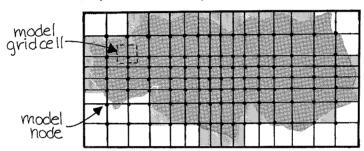
Most numerical techniques for integrating the groundwater flow equation convert the original partial differential equation into a large set of algebraic equations which may be readily solved on a digital computer. This conversion process requires the simulation problem to be discretized in both space and time. That is, the solution region is divided into a number of discrete subregions (or elements) and the simulation period is divided into a number of discrete time intervals. The partial differential equation is then used to derive one or more algebraic equations for each element and each time interval. The unknowns in these algebraic equations are "average" heads which approximate the exact solution at a given time and location. As the discretization is made more detailed, the approximate solution converges to the exact solution everywhere.

There are a number of methods for discretizing the solution regions for ground-water problems. Although these methods appear to be quite different they share a number of basic concepts. Figure 3-8a shows a typical irregularly-shaped, two-dimensional solution region. This could be a vertically averaged representation of a complete aquifer or of some portion of an aquifer. Two alternative methods of dividing up or discretizing the solution region are illustrated in Figures 3-8b and 3-8c.

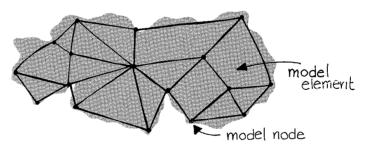
Figure 3-8b superimposes a grid constructed of unevenly spaced parallel lines on top of the solution region. The intersections of these lines are called nodes and the rectangular region surrounding each node is called a grid cell. Most computer models which use this type of discretization scheme assume that the head everywhere within each grid cell is equal to the head at the corresponding node (other assumptions are possible). If a rectangular spatial discretization technique is used, the flow equation may be converted into a set of N ordinary differential equations, where N is the number of interior nodes in the grid. The heads or head gradients at the boundary nodes are obtained from boundary conditions. Each ordinary differential equation depends only on time and may be further discretized using methods discussed below.



a) Actual aquifer outline



b) Rectangular simulation grid (e.g. finite difference)



c) Curvilinear simulation grid (e.g. finite element)

FIGURE 3–8 SOME TYPICAL SPATIAL DISCRETIZATION SCHEMES

Figure 3-8c shows an irregular discretization network which divides the solution region into elements with curved sides. In this case, the head within each element is assumed to be a weighted sum of the heads at the surrounding node points. As before, the flow equation is converted into a set of N ordinary differential equations, where N is the number of interior nodes. The rectangular grid approach is typically used with finite difference models such as the USGS finite difference model selected for the San Andres-Glorieta study. The irregular curve-sided approach is typically used with finite element models.

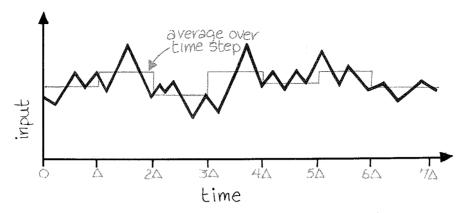
The ordinary differential equations generated by spatial discretization are generally discretized in time using a method similar to the one illustrated in Figure 3-9. The input variables and head derivatives appearing in the equation are assumed to be constant over each time interval and to jump abruptly at the end of the interval. Since the head derivative is assumed constant, the head itself is assumed to vary linearly over the time interval. When temporal discretization is applied, the each ordinary differential equation is replaced by a set of algebraic equations whose unknowns are the nodal heads at the end of each time interval. Note that the head at the beginning of the first time interval is obtained from the initial condition.

The process of laying out a discretized grid or network for a particular modeling problem depends both on the type of model selected and on the specific requirements of the problem. Program user manuals usually give guidelines for network construction. Generally speaking, the construction of finite difference grids is more mechanical and straightforward than the construction of finite element networks. The smallest cells in a finite difference grid are usually located in the area where impacts are of most concern. In most cases this is the region surrounding a well field. Grid spacing usually increases in both directions away from this region.

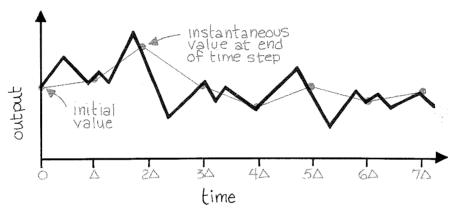
Finite element networks are usually less detailed than finite difference grids for the same aquifer. The element sides in these networks typically coincide with geological, hydrological, or institutional bour daries which are relevant to the modeling study. A given element might, for example, be defined as the area within a particular irrigation district which is devoted to grazing land and characterized by sandy soils. Such areas can be identified by overlaying transparent maps showing land uses, soil types, administrative regions and other relevant spatial features. The superimposed composite of all of these maps usually gives a good preliminary network. It is easy to either refine or simplify this network if additional information becomes available later. This flexibility is one of the most attractive features of the finite element approach.

3.3.3 Input Estimation

Input estimation is probably the most important and most neglected single task in the modeling process, particularly in groundwater applications where only a few of the relevant variables are directly observable. This is clearly illustrated in the detailed analysis of the San Andres-Glorieta modeling studies presented in Chapter 4. The need to estimate inputs throughout the solution region and over a simulation



a) input discretization (e.g. pumpage)



b) output discretization (e.g. head)

FIGURE 3-9 SOME TYPICAL TEMPORAL DISCRETIZATION SCHEMES

period extending many years into the future forces the modeler to extrapolate and generalize from the limited data available. This inevitably introduces subjectivity and uncertainty into the modeling process.

It is convenient to divide this discussion of input estimation techniques into two sections—one dealing with aquifer parameters (primary inputs) and one dealing with auxiliary conditions and nominal pumpage and recharge (secondary inputs). Although the emphasis here is on vertically averaged models of hydraulic head, most of the techniques considered are also applicable to drawdown models, provided the superposition assumption holds (see Section 3.2.4). Some practical applications of these techniques are discussed in Chapter 4.

i) aquifer parameters

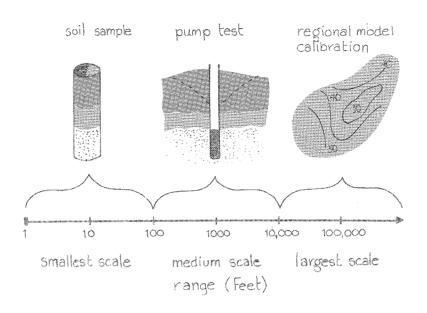
Simulations of hydraulic head and drawdown require estimates of transmissivity, storage coefficient, and leakance at each node or element in the model network (input values need not, of course, be different at every node or element). These physical parameters generally vary throughout the aquifer – variations tend to be large in regions which are geologically complex and small in regions which are homogeneous. This is illustrated in a simple way in Figure 3-10b, which shows a plot of transmissivity vs. distance along a transect in a hypothetical aquifer. Although the regional average is approximately 1000 ft²/day, local values based, for example, on soil samples may vary from 100 ft²/day to as high as 10,000 ft²/day. This type of variability is typical of the data available for the San Andres-Glorieta.

The aquifer parameter estimates produced by most estimation techniques are averages which apply over a characteristic range or length scale. This is illustrated in Figure 3-10a where the ranges for estimates derived from soil samples, pump tests, and regional model calibration are compared. The estimates produced by each of these techniques are indicated by appropriate horizontal lines in Figure 3-10b.

Soil samples and well logs obtained during well drilling give a very localized indication of the grain size distribution in a particular portion of an aquifer. Various empirical formulas are available for estimating hydraulic conductivity and transmissivity from this distribution (see Freeze and Cherry, 1979). These should be regarded with skepticism and used only in the absence of any better alternatives.

Pump tests are probably the most popular method for estimating aquifer transmissivities and storage coefficients, partly because they give a good indication of a well's producing capacity. The pump test approach is an example of an inverse method of parameter estimation. The selected well is pumped for a specified period and water levels are observed during both the drawdown and recovery process. A simplified (radially symmetric) groundwater equation is then solved for the transmissivity and storage coefficient which best reproduce the water level history observed during the test. This process has been standardized by Theis (1935), Jacob (1940), Hantush (1960) and others (see Freeze and Cherry, 1979, for a summary).

The range over which pump test results apply depends on a number of factors including the duration of the test – brief tests only produce drawdowns close to the pumped well while longer tests affect a wider area. For aquifers as large as the San Andres-Glorieta, pump test estimates should probably be interpreted as point observations (although they are admittedly less localized than soil samples).



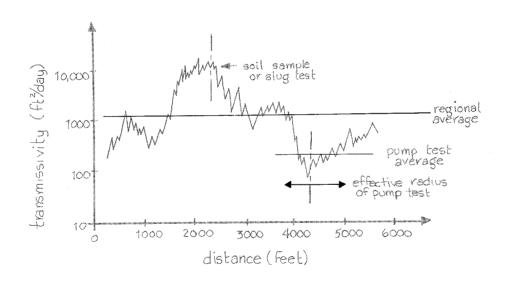


FIGURE 3-10 SPATIAL SCALES OF DIFFERENT PARAMETER ESTIMATION TECHNIQUES

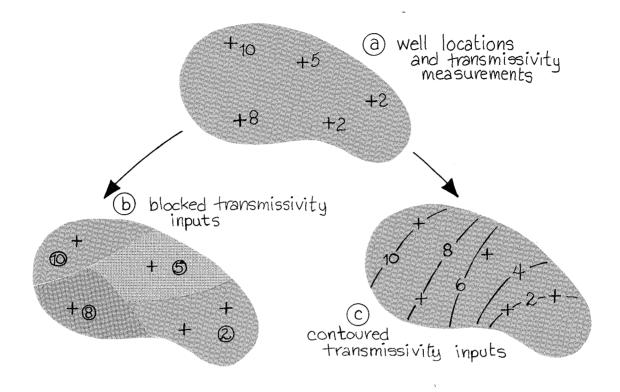


FIGURE 3-11
ALTERNATIVE WAYS TO EXTRAPOLATE LOCALIZED WELL OBSERVATIONS

If pump test results are to be used to define aquifer parameters throughout the solution region (i.e., at all nodes or elements) they must be extrapolated. Two possible extrapolation methods are illustrated in Figure 3-11. Figure 3-11a shows a hypothetical aquifer which contains five observation wells. Transmissivity estimates obtained from pump tests at these wells are indicated next to each well symbol. Figure 3-11b shows a blocked extrapolation technique which allocates each transmissivity value to a large geologically homogeneous area surrounding a particular well (or wells). Figure 3-11c shows an extrapolation technique based on a contour plot of the well values. The contours may be interpolated to give a continuously-varying distribution of transmissivity. This approach is particularly appropriate in geologically heterogeneous aquifers such as the San Andres-Glorieta.

Regional model calibration is an estimation procedure which avoids some of the disadvantages associated with extrapolation. This procedure is an inverse technique similar in concept to pump test analysis but quite different in its practical application. Regional calibration uses the complete vertically averaged flow model to simulate heads throughout the aquifer over a historical observation period. The unknown aquifer parameters are adjusted, either by trial-and-error or with an optimization algorithm, until a "best fit" is obtained between the historically observed heads and the heads simulated by the model. This parameter adjustment process is the counterpart to the type-curve fitting procedures used to obtain estimates from pump test measurements.

It is important to note that the storage coefficient can be estimated with a regional calibration approach only if heads during the historical period are varying (i.e., are not in steady-state). Otherwise, the temporal head derivative is zero and the storage coefficient has no effect on the model's predictions. In practice, it is probably best to accept this and stay with a steady-state simulation whenever possible. This is because initial condition errors introduced in a dynamic simulation could easily outweigh any information gained about the storage coefficient.

The major advantage of regional model calibration is its ability to provide estimates of the spatially discretized (regionally averaged) parameters used in the model. These estimates do not have to be extrapolated or generalized in any way—they can be input to the model as is. The major disadvantage of regional calibration is its dependence on the accuracy of the input values entered for historical pumpage, recharge, and boundary conditions. If these inputs are incorrect, the estimated aquifer parameters may be quite far from the true values.

When all the alternatives are considered, the best approach to aquifer parameter estimation seems to be a combination of the pump test and regional calibration techniques. This can be accomplished by starting a regional calibration with parameter estimates obtained from pump tests. Subsequent parameter adjustments should be constrained so that they improve upon but do not deviate too far from the initial pump test values. This approach seems to work well in practice, probably because it uses all available sources of information in an efficient way.

ii) secondary inputs

The secondary inputs required for a simulation of hydraulic head are initial heads (if the simulation is dynamic), boundary conditions (including the heads in adjoining aquifers), and nominal pumpage and recharge. The estimation problem is clearly much easier if the simulation is steady-state since boundary conditions, pumpage, and recharge are then all constants. Fixed boundary head or fluxes are usually estimated from water level observations collected in the interior of the solution region where pumpage is taking place. These must be extrapolated out to the boundaries with a technique similar to the one illustrated in Figure 3-11c.

If a dynamic simulation is required it is best to start computing at a time when the aquifer is in steady-state. In this case, the initial heads can be derived directly from a steady-state solution. Otherwise these heads must be estimated by extrapolating well observations taken at the initial time. This can be a significant source of prediction error, particularly during the beginning of the simulation period, if the model is forced to adjust to a physically unrealistic initial head distribution.

When the boundary conditions used in a dynamic head simulation are timevarying the input estimation problem becomes very difficult, if not impossible. This can be avoided if the model's boundaries are located along lines where the head or flux are reasonably constant, as suggested in Section 3.2.3. Flow barriers and lines of symmetry should, of course, be used to define boundaries whenever possible.

One might expect that historical pumping rates could be readily estimated from water use or power consumption records or indirectly derived from population and land use data. Although these information sources are all helpful, they do not necessarily provide a completely accurate record of aquifer pumping. The estimation problem is complicated considerably when major water supply wells are completed in more than one aquifer or when pumping is sporadic (depending, for example, on crop demands and weather). This is, for example, the situation encountered in the San Andres-Glorieta. It is often useful to construct a set of surface water budgets for major land uses (particularly in agricultural regions) so that pumpage estimates can be checked for consistency with other surface water data.

Although pumpage can often be estimated for a single well or a small well field, it should be remembered that the spatial resolution of a discretized model is limited by the size of its elements or grid cells. Pumpage from a point located within a given element is effectively spread over the element. If the modeler wishes to localize pumpage precisely he must make the elements in the vicinity of the well very small.

Recharge to confined aquifers such as the San Andres-Glorieta occurs primarily near outcrop areas where the aquifer is exposed to the surface or is overlain by permeable strata. The net long-term (e.g., annual) recharge may be estimated by subtracting evapotranspiration and runoff from total precipitation (including snow-

melt). Regional evapotranspiration rates may be estimated from various empirical formulas or sometimes derived from field observations. Average precipitation data are usually readily available from the National Weather Service or from local sources. Runoff may be available from stream records or, alternatively, may be estimated using a variety of empirical techniques (see Linsley et al., 1977). One of the greatest sources of uncertainty in recharge estimation is the size of the recharge area. This must be deduced from geological observations which are often not very extensive or specific.

Nominal pumpage and recharge for the future can be projected or postulated in a number of ways. Both pumpage and recharge could be assumed to remain fixed at current levels. Or pumpage could be held at current levels and recharge could be varied according to some specified climatic sequence. This approach allows the modeler to investigate the impact of prolonged droughts or wet periods. If the region being studied is growing rapidly it may be reasonable to assume that the nominal pumpage increases over time. It should be apparent that the process of estimating nominal pumpage and recharge for the future depends on assumptions made about water supply management policies, regional growth, and climatic conditions. These assumptions should, of course, be carefully spelled out when the model's results are reported.

3.3.4 Accuracy Evaluation

It should now be apparent that it is difficult to estimate the inputs of a "real-world" groundwater model without introducing significant error at one point or another. The following list identifies some of the critical stages where errors can arise in a study such as the San Andres-Glorieta:

- 1. The entire modeling process depends on the assumptions made in the governing equations. If the aquifer is assumed to be confined when it is actually unconfined, or in steady-state when its heads are actually changing, estimation and prediction errors will result.
- 2. The appropriate values for model inputs depend on the discretization method selected. Each model or elemental input represents an average over an extended region. If the model network is too coarse, these averaged inputs may not properly represent heterogeneities in the actual aquifer. If it is very fine, computational requirements may become prohibitive.
- 3. Most groundwater inputs must be estimated from a small number of measurements taken at scattered wells. The extrapolation required to extend these point measurements over a wide area can introduce significant error.
- 4. Well water level measurements can sometimes be misleading, particularly if the well is multiply completed or defective in some way. Water level measurement errors affect the validity of pump test analyses, extrapolation, and other aspects of the estimation process.

- 5. Traditional pump test analyses assume constant pumpage, homogeneous aquifer properties, negligible well loss, complete well penetration, and regular aquifer geometry. Parameter estimates based on such simplified analyses may be inaccurate in practical applications. Although pump test estimation errors can be reduced if more sophisticated (and complex) analytical methods are used, they can never be completely eliminated.
- 6. Aquifer parameter estimates obtained from a regional model calibration are highly dependent on the values used for inputs such as historical pumpage, recharge, and boundary conditions. If these inputs are incorrect, the resulting parameter estimates may be inaccurate.

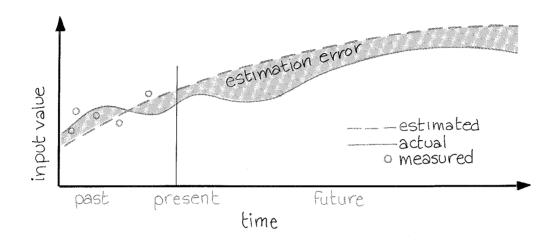
In a realistic groundwater study it is likely that one or more of these errors will be significant. It is important to have some idea of the effects these errors will have on model predictions, particularly if the model's results will be used to guide policy decisions.

A view of input estimation and subsequent predictive simulations which is particularly relevant to the San Andres-Glorieta application is illustrated in Figure 3-12. Figure 3-12a shows a typical time-varying input (e.g., recharge) plotted over the past, present, and future. Measured values available at discrete times in the past are used to estimate the actual historical recharge (solid line) and to project recharge trends for the future. Note that both the measured values and the smoothed estimates differ from the actual recharge, which is unknown to the modeler. The shaded area represents the input estimation error.

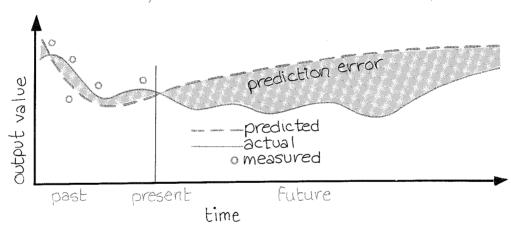
Figure 3-12b shows a time-varying observable output (e.g., head) generated by a model which uses the estimated input record plotted in Figure 3-12a. Here again, observations are available at discrete times during the historical period and the shaded area represents the model's prediction error. As can be seen, the historical prediction errors do not necessarily reveal anything about the model's prediction accuracy in the future. It should be recognized that measurements of the input and output variables do not necessarily coincide with the unknown actual values. Accuracy analyses which assume that all measurements are perfect are unrealistic and potentially misleading.

There are two approaches to the evaluation of model accuracy, one which relies on "goodness of fit" comparisons between model predictions and field observations and one which relies on sensitivity analysis. The objective in either case is to obtain a quantitative measure of the model's prediction error. The prediction error is unknown by definition (otherwise it would not be an error) but it can still be bounded or measured in a probabilistic sense. Bounds or confidence limits on prediction errors provide an intuitive and helpful guide for decision making. In the San Andres-Glorieta case study, for example, "best", "worst", and "most likely" predictions could be used to assess the risks associated with increased development.

In order for the goodness-of-fit approach to be applied, measurements and predictions must be available at the same times and locations. If model predictions



a) measured, estimated, and actual model inputs



b) measured, predicted, and actual model outputs

FIGURE 3-12 INPUT AND OUTPUT ERRORS IN A TYPICAL MODELING STUDY

are actually averages over extended spatial regions, well observations should be averaged so that compatible regional measurements can be obtained. This may be accomplished with a blocking scheme, although it is usually better to smooth out the measured head values with a contour plot (see Figure 3-11). The contoured heads may then be integrated over appropriate model elements to provide average measured heads.

Once a direct comparison of observations and predictions is possible, the goodness-of-fit errors can be computed. Statistical error measures such as the mean-squared-deviation or the value of the 95^{th} percentile error may then be derived. In fact, the entire theoretical framework of classical regression analysis may be used to analyze the "goodness-of-fit" between model and data.

The goodness-of-fit approach can be useful, particularly if it is based on a careful regression analysis, but it has some important practical limitations. Most of these are related to the fact that a goodness-of-fit analysis of model performance over a brief historical period does not necessarily convey any information about the model's long-term prediction capabilities. The extrapolation of past performance into the future presumes that past and future errors are statistically similar (i.e., are drawn from the same population). If the historical observation period is brief, as is usually the case in groundwater studies, it could very well be statistically anomalous. If, for example, recharge and pumpage during the historical period are unusually low, a model with incorrect transmissivities may give a good fit to measured heads, provided that its initial conditions and boundary conditions are adjusted appropriately. The same model inputs may give much poorer results at a later time when pumpage and recharge increase.

In order to provide a reliable assessment of accuracy the goodness-of-fit approach requires a large number of well observations. There are rarely enough observations available in a typical groundwater study to support a thorough statistical analysis of model fit. This is certainly the case in the San Andres-Glorieta application. Those measurements that are available are usually used to estimate aquifer parameters with a regional model calibration. If the same measurements are also used to evaluate accuracy, they will give a misleadingly optimistic assessment of the model's predictive ability. As an extreme example, it might be noted that three well observations taken in different years can be fit perfectly with a quadratic equation which depends only on time. The quality of this fit clearly does not guarantee that a quadratic equation will predict future water levels without error. A much better measure of the model's performance would be obtained if its prediction were compared with fifty measurements. As a general rule, the number of observations used in goodness-of-fit test should be large compared to the number of unknown parameters in the model.

A promising alternative to the goodness-of-fit approach is one based on sensitivity analysis. Sensitivity analysis does not require any field data but is, rather, based on an investigation of the model's response to specified input errors. The basic idea may be illustrated by considering a dynamic simulation run on a vertically averaged model with one transmissivity T and one storage coefficient S. Suppose

that the predicted head at a given time and location is written as h(x, y, t, T, S) to acknowledge the model's dependence on the inputs T and S. If each of these inputs were changed the resulting change in the head prediction would be approximately:

$$\Delta h(x, y, t, T, S) = \frac{\partial h}{\partial T} \Delta T + \frac{\partial h}{\partial S} \Delta S$$
 (3-36)

Here the partial derivatives $\partial h/\partial T$ and $\partial h/\partial S$ are the changes in head prediction obtained if either the transmissivity or storage coefficient (but not both) is changed infinitesimally. The perturbations ΔT and ΔS are changes in transmissivity and storage coefficient (not necessarily small) which shift the head prediction by Δh .

Equation (3-36) is a linear (or first-order) Taylor series approximation to the model's head solution, written as a function of the estimated parameters T and S. This series may be generalized to include all model inputs as follows:

$$\Delta h(x, y, t) = \sum_{i=1}^{m} \frac{\partial h}{\partial u_i} \Delta u_i$$
 (3-37)

Here u_i represents the *i*th model input (out of a total of m inputs) and $\partial h/\partial u_i$ represents the sensitivity of the prediction at location (x, y) and time t to a small change in u_i . If the input pertubations (Δu_i) 's are interpreted as errors (i.e., differences between estimated and true values) then $\Delta h(x, y, t)$ is the model's prediction error.

Equation (3-37) may not seem particularly useful for accuracy analysis since the input errors are not known. This equation may, however, be used to establish an upper bound on the prediction error if absolute values are taken throughout:

$$|\Delta h(x,y,t)| \le \sum_{i=1}^{m} \left| \frac{\partial h}{\partial u_i} \right| |e_i|$$
 (3-38)

This equation states that the magnitude of the prediction error does not exceed the summation on the right-hand side, with e_i selected as an upper bound on the error associated with input u_i .

Equation (3-38) may be used to analyze model accuracy in the following way. The model is set up and run over the desired prediction period as usual. Then its various sensitivity derivatives are computed by perturbing each input in turn. An error bound is then postulated for each model input. These bounds should be based on the modeler's best estimate of the accuracy of the input estimate, considering data availability, the degree of extrapolation, and all the other factors mentioned at the beginning of this section. Equation (3-38) is then used to compute the prediction error bound. The entire process may, in principle, be automated and a printout of $\Delta h(x, y, t)$ included in the model's output.

The sensitivity approach has several advantages which make it worth serious consideration. First, it is not data-dependent and may, in fact, be used to investigate accuracy early in the modeling study before data are compiled and inputs are estimated. Second, it provides a bound on prediction error which varies with location and time. This allows the modeler to account for the growing errors which may occur when current trends are projected far into the future. Finally, sensitivity

analysis allows the modeler to identify the largest contributions to prediction error (aquifer parameters, boundary conditions, recharge, etc.) so that he knows where to focus his efforts to improve accuracy.

The major disadvantage of sensitivity analysis is its expense. The sensitivity derivatives can be quite expensive to compute if the model has many inputs and the prediction period is long. The expense increases greatly if higher-order terms are included in the Taylor series expansion since, in this case, higher-order sensitivity derivatives must also be derived.

Limited sensitivity analyses are beginning to appear in groundwater modeling studies (Hearne, 1980; HEC, 1982) and there is a growing recognition of the connection between sensitivity and prediction error. But systematic accuracy evaluations of the type outlined here are still rare. At the moment, the goodness-of-fit approach remains the one most accepted.

3.3.5 Presentation of Model Results

The last phase of an aquifer water supply study is the culmination of all the analysis, interpretation, and manipulation discussed in the preceding sections. Here the model's predictions are generated and reported. If everything up to this point has been done properly, the model should yield a plausible set of results which can be defended from available field data and generally accepted theory. The modeling study is, however, incomplete if the modeler presents his results in a way which implies that they are perfect. It is equally incomplete if the modeler admits that his results are uncertain but does not provide any further guidance. A good modeling study acknowledges and quantifies the uncertainty of its predictions. This enables potential critics to make informed decisions about the model's credibility instead of being forced to either wholeheartedly accept or completely reject its predictions. Such an informed approach is ultimately in the best interest of everyone involved.

4. A COMPARISON OF THREE MODELING APPROACHES

The San Andres-Glorieta management problem outlined at the beginning of Chapter 2 appears to be a natural candidate for a groundwater modeling study. The impact evaluation question at issue is clear but difficult to answer without an analysis of groundwater flow. Available aquifer information is not abundant but it may be adequate for a regional modeling study. Given this, it may seem that any experienced modeler should be able to start with the problem definition and data base reviewed in Chapter 2, apply the general principles of Chapter 3, and end up with a set of reasonably credible predictions. There are, however, many judgments and decisions to be made along the way and the predictions which emerge depend nearly as much on the modeler's own abilities and biases as on objective fact. This is clearly illustrated by three modeling studies carried out to investigate the effects of pumpage in the San Andres-Glorieta aquifer. These studies all relied on the same general principles, data base, and simulation model but yielded very different predictions; so different that they virtually cancelled each other out.

The simulation approaches used in the three San Andres-Glorieta studies are distinguished primarily by differences in model formulation and input estimation. A brief summary of each approach is provided below:

Modeling Study 1 (Geohydrology, 1982)

This study solved the drawdown equation directly. No-flow boundaries were assumed on all sides of the solution region. The aquifer parameters were either postulated or estimated from pump test data. Incremental pumpage was obtained from the Plains Electric pumping schedule (see Figure 2-4) and incremental recharge was assumed to be zero.

Modeling Study 2 (Geotrans, 1982)

Drawdown was computed in this study from a simulation initialized with steady-state heads based on 1968 pumpage and recharge values. Using the terminology of Section 3.2.5, the nominal pumping strategy was defined by 1968 conditions and the modified strategy by 1968 conditions with Plains Electric pumpage added. No-flow boundaries were assumed on all sides of the solution region. Aquifer parameters were estimated from pump test data and a steady-state regional calibration.

Modeling Study 3 (HEC,1982)

This study used the same differencing approach as Modeling Study 2 to derive drawdown predictions. Specified head boundaries were used on three sides of the solution region and a no-flow boundary on the fourth side. Aquifer parameters and boundary heads were either postulated or derived from a

steady-state calibration. This study differed from the other two in that the emphasis was on the impact of pumpage on water levels at Fort Wingate Army Depot (located approximately 10 miles west of the Ciniza well field). This had some influence on the way the model was formulated and on the selection of model input values.

The maximum drawdowns predicted by each model are plotted in Figure 4-1. Note that the area pictured in this figure covers the south-central portion of the San Andres-Glorieta region. Each of the three models predicted that maximum drawdown will occur after about 18 years of plant operation. It is apparent that Model 2 predicts the greatest impact (the 1000 foot drawdown indicated at the well field locally dewaters the aquifer). Model 3 predicts relatively modest and localized drawdowns while Model 1 gives results somewhere in between. The maximum difference between these predictions in the vicinity of the well field is over 600 feet.

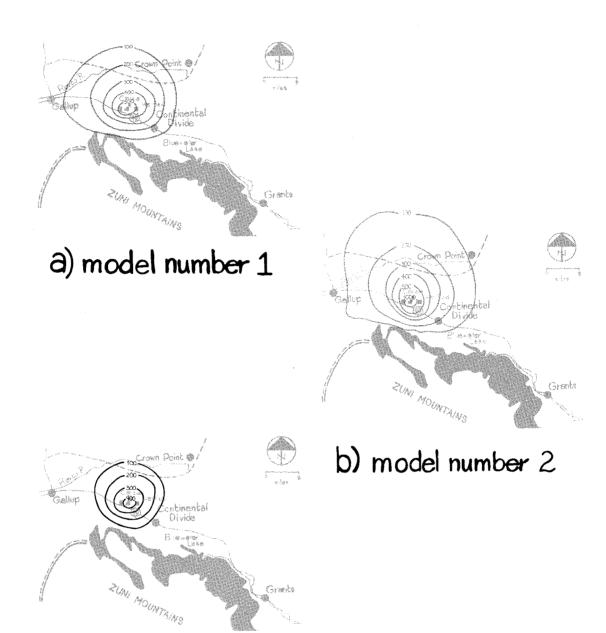
The analysis presented in this chapter attempts to discover how three modeling studies using such similar methodologies can generate such different results. The case study comparison is interesting for its own sake but it also reveals something about the practice of groundwater modeling in general. The data ambiguities and opportunities for subjective interpretation encountered in the San Andres-Glorieta play an equally important role in other water supply investigations. The differences examined here are dramatic but not unusual.

The comparison begins with a description of the general specifications for each model—the simulation approach used, the boundaries of the solution region, etc. It then considers the important application issues addressed in Section 3-3—discretization, input estimation, and accuracy evaluation. The chapter concludes with a review of model similarities and differences and an overall assessment of the case study.

4.1 MODEL FORMULATION

All three of the studies reviewed here adopted the two-dimensional vertically averaged modeling approach described in Section 3-2 In each case, the flow equation was solved with the USGS two-dimensional finite difference program documented in Trescott, Pinder, and Larson (1976). This program can handle both confined and unconfined conditions and provides reasonable flexibility for laying out boundaries and assigning boundary conditions.

The solution regions for the three models represent somewhat different approaches to aquifer simulation (see Figure 4-2). The network for Model 1 is essentially rectangular with some modification along the lower (southwestern) boundary to account for the irregular nature of the aquifer outcrop. The areal extent of this network covers much of the aquifer but does not actually reach generally acknowledged geological boundaries except along the southern edge. The limited coverage of the model network can be justified if the fluxes crossing the right (eastern), left



c) model number 3

(western), or upper (northern) boundaries are reasonably constant and are unaffected by pumpage at the Ciniza well field. Model 1 assumes that this is the case and, consequently, imposes zero gradient drawdown conditions on all four sides of the network.

The nearly rectangular network used in Modeling Study 2 extends north to the edge of the aquifer but approximates the lower boundary somewhat differently than Model 1—Model 2 appears to stop at the northern edge of the outcrop while Model 1 stops at the southern edge. Model 2 assumes that there is no flow across any of the network boundaries.

The perfectly rectangular Model 3 network makes no attempt to cover the entire aquifer but includes only the region where drawdowns are expected to be significant. The influence of the surrounding flow field is accounted for by specified head boundary conditions on the upper, lower, and left sides of the network. The right side is assumed to be a no-flow boundary. The boundary heads of Model 3 are adjusted so that groundwater enters the network from the outcrop area, moves north, and gradually curves to the west. This assumed flow pattern is based on water level data reported by Shomaker (1971) and plotted in Figure 2-5. Since the boundary heads are assigned constant values, they are implicitly assumed to be unaffected by pumpage at Ciniza.

The zero-gradient boundary condition for Model 1 is a straightforward consequence of the drawdown formulation outlined in Section 3.2.5. The boundary conditions for Models 2 and 3 are somewhat more subjective and have broader implications. The zero-flux approach taken in Model 2 treats the aquifer as a closed system which interacts with the outside world only through the horizontal boundary flux (called q_h in Section 3.2.3). In this case the aquifer can be in steady-state only if the pumpage, recharge, and leakage components of the horizontal flux balance exactly. Since pumpage and recharge are specified independently, the leakage term must adjust to give a steady-state solution. The open (specified head) boundaries used in Model 3 provide a different way to achieve a steady-state balance. In this case the head gradients along each of the specified head boundaries may adjust to allow groundwater to either enter or leave the network. The difference between pumpage and recharge does not have to be absorbed by leakage across the horizontal boundary. The leakage term of Model 2 and the open boundaries of Model 3 are both reasonable ways to account for interactions between the aquifer and its environment. They are, however, clearly not equivalent.

A comparison of the three model formulations suggests that Model 2 takes the safest (most conservative) approach since it makes the fewest assumptions. Both Model 1 and Model 3 require the network boundaries to be unaffected by pumpage at the Ciniza well field. Model 3 also requires estimation of head boundary values along three sides of the solution region. Although such requirements may not necessarily pose problems, it is best to avoid them if possible.

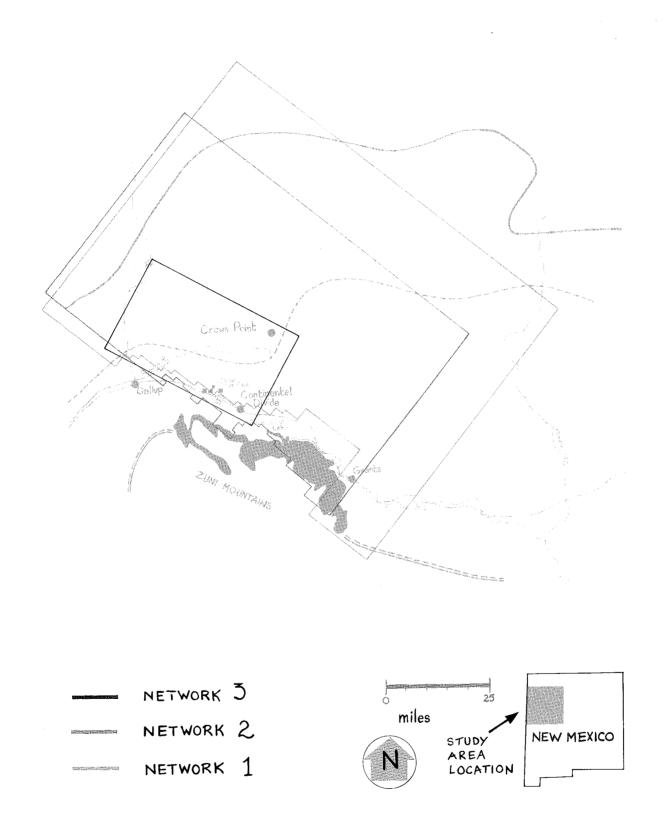


FIGURE 4-2 COMPARISON OF SIMULATION NETWORKS FOR MODELS 1,2, AND 3

4.2 MODEL APPLICATION

4.2.1 Spatial and Temporal Discretization

Since the three case study models were all run with the same finite difference computer program, they all used the same discretization approach. Each solution region was divided into a grid of parallel lines spaced close together near the middle of the region and progressively further apart toward the edges (see Figure 3-8b for an illustration of this approach). The smallest and largest grid intervals for each model are listed below:

MODEL	SMALLEST INTERVAL	LARGEST INTERVAL	
	(miles)	(miles)	
1	0.25	10.0	
2	1.00	10.0	
3	0.25	2.0	

The smallest interval defines the model's spatial resolution near the Ciniza well field. Temporal discretization for each model followed the approach illustrated in Figure 3-9. Model inputs are assumed constant over each time step and model predictions are assumed to vary linearly. The model time steps were all one year or less. Since the time horizon of interest in the impact evaluation is forty years, annual time steps probably provide adequate temporal resolution.

The discretization approach used in each of the three models is reasonable and adequate for impact evaluation purposes. It seems safe to say that discretization had no significant effect on the differences in model predictions which are of particular interest here.

4.2.2 Input Estimation

Given the general similarity of the model formulations and discretization procedures outlined above, it seems obvious that the divergent predictions generated by the three case study models were caused by differences in model inputs, i.e. in the values used for aquifer parameters, boundary conditions, and pumpage/recharge rates. What is less obvious is why qualified hydrologists should derive such different input estimates from the same data base. This issue is clearly worth further investigation.

It is helpful to summarize the inputs that need to be estimated for each model since differences in formulation lead to differing input requirements. These requirements are indicated by closed circles in the following table:

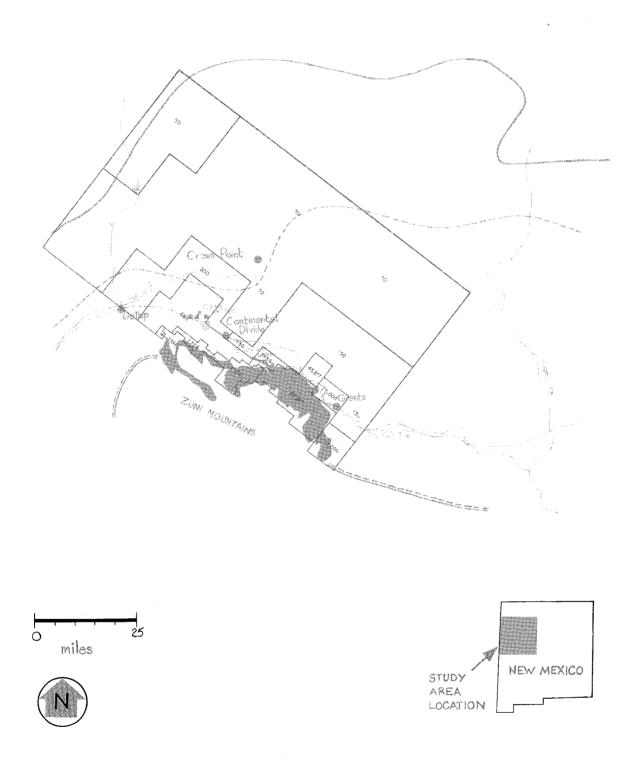
INPUT	MODEL			
	1	2	3	
Aquifer parameters	•	•	•	
Initial heads		•	•	
Adjoining aquifer head		•		
Boundary heads			•	
Nominal pumpage		•	•	
Nominal recharge		•		

In order to keep the differences between the three modeling studies clear, it is convenient to examine their input estimation methods separately. Each model is covered in one of the subsections which follow.

Model 1

Aquifer parameter estimates for Model 1 were obtained from a variety of primary and secondary sources which are not very well documented. Transmissivities were assumed to be uniform over large blocks as shown in Figure 4-3. The transmissivity estimates used vary dramatically from a low of 70 ft²/day to over 77,000 ft²/day. These estimates appear to have been extrapolated from individual pump test or soil sample values by means of a blocking scheme similar to the one illustrated in Figure 3-11b.

The procedure used to estimate the transmissivity value for the Ciniza area is worth considering in detail since this value has a significant effect on the model's drawdown predictions. Since 1956 Shell Oil has pumped about 600 acre-feet/year from a well field near Ciniza. Water level observations collected at several nearby wells over the 1956-1982 period indicate a gradual decline in head which is presumably due to the Shell pumpage. If the 1956 water level is taken as a reference, drawdowns for the historical period may be computed and plotted vs. time. A conventional pump test analysis of this plot carried out in Modeling Study 1 gave a transmissivity estimate of about 450 ft²/day. This was apparently confirmed by a limited regional calibration study.



It should be noted that other pump test analyses of the Ciniza drawdown data produced transmissivity estimates which varied from less than 80 ft²/day to over 3000 ft²/day (Dames and Moore, 1982). The wide range of uncertainty observed in these estimates reflects different assumptions made in type curve computations. The average difference in computed transmissivity (about 1500 ft²/day) could be taken as an upper bound on the transmissivity estimation error for the Ciniza area.

A confined storage coefficient value of 5×10^{-4} (unitless) for the Ciniza area was derived from the Shell Oil pump test analysis mentioned earlier. This value was used throughout the confined portion of the aquifer but was increased to 0.05 in the unconfined outcrop area.³ This increase accounts for the impact which a rising or falling water table has on storage. It has the effect of making local heads less responsive to changes in pumpage. The 0.05 estimate for the outcrop storage coefficient is plausible, considering that specific yields for unconfined sandstone and limestone formations usually fall in the range from 0.02 to 0.08 (Linsley, et al.,1982).

The Model 1 leakance value was computed by assuming that the confining layer lying between the San Andres-Glorieta and the adjoining aquifer (presumably the Sonsela formation) has a vertical hydraulic conductivity of $1 \times 10^{-12} \mathrm{ft/sec.}$ and is 600 feet thick. This gives a leakance estimate of $1.7 \times 10^{-15} \mathrm{sec^{-1}}$ or about $1.4 \times 10^{-10} \mathrm{~days^{-1}}$ (see Equation 3-11). The computed leakance was assumed to apply uniformly throughout the Model 1 network.

It is instructive to see what this leakance value implies about the role of leakage in the Model 1 water budget. The incremental decrease in leakage out of the aquifer due to the Plains Electric pumpage may be calculated as follows:

$$\Delta q_l = LA\bar{d} \tag{4-1}$$

where L is the leakance and \overline{d} is the average drawdown over an area A. The maximum Model 1 drawdown prediction plotted in Figure 4-1 can be roughly approximated as 250 feet over a circle with a radius of 12 miles. Given this approximation and a leakance value of 1.4×10^{-10} days⁻¹, Equation (4-1) yields a decrease in leakage of 3.7 acre-feet/year. When compared to an incremental pumpage value of 4000 acre-feet/year this is negligible. Leakage clearly has little effect on the predictions of Model 1.

³The Model 1 outcrop area covers the two transmissivity blocks at the bottom of Figure 4-3 labeled 7754 and 43,817.

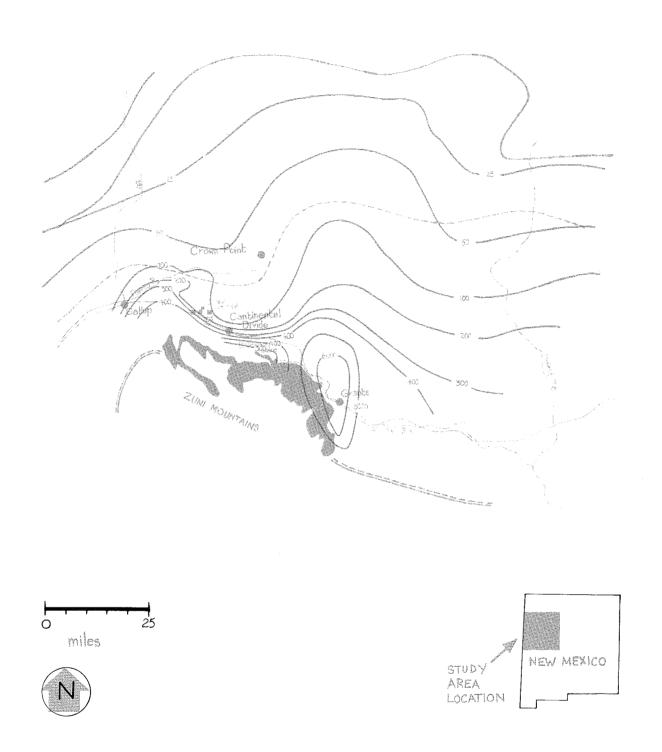


FIGURE 4-4 TRANSMISSIVITY CONTOURS FOR MODEL 2

Model 2

Aquifer parameters for Model 2 were estimated primarily from pump test results and qualitative geological information. In some cases, these estimates were refined with a regional model calibration based on steady-state head observations. The Model 2 aquifer parameters were extrapolated with a contouring procedure which allows the parameters to vary continuously throughout the aquifer, as is illustrated by the transmissivity map of Figure 4-4.

A comparison of Figures 4-3 and 4-4 reveals how different modelers can interpret and extrapolate the same set of pump test results. The two transmissivity maps are qualitatively similar—in each case transmissivities around the top and two sides of the aquifer are low and transmissivities near the eastern end of the outcrop are very high. The most important interpretive difference is in the Ciniza area, where the Model 2 contours bend in such a way as to lower the transmissivity north of the well field from the Model 1 value of 450 ft²/day to less than 100 ft²/day. The low transmissivities used in Model 2 were obtained from an independent analysis of pump test data from the Ciniza well field.

Confined storage coefficients for Model 2 were computed by multiplying a specific storage of 4×10^{-7} ft⁻¹ by the estimated aquifer thickness (in ft.). This gives values which vary throughout the aquifer but are generally less than 1×10^{-4} near the Ciniza well field. The 4×10^{-7} ft⁻¹ value used for specific storage seems rather low. A rough check on this value can be obtained by computing the specific storage from Equation (2-5). The range for the bulk compressibility of a jointed rock aquifer given by Freeze and Cherry (1979) is 10^{-8} to 10^{-10} m²/Newton (about 10^{-12} to 10^{-14} psi⁻¹). If the smaller value is used with a porosity of 0.10, the resulting specific storage is 1.5×10^{-6} ft⁻¹. This compares well with the value of 1.0×10^{-6} ft⁻¹ given by Lohman (1972). If 1.5×10^{-6} ft⁻¹ is used, the computed storage coefficient for the Ciniza area quadruples to 4×10^{-4} . This is close to the value used in Model 1. The unconfined storage coefficient for Model 2 was assumed to have a uniform value of 0.10 wherever unconfined conditions exist. This is consistent with the range of reasonable values cited in the Model 1 discussion.

The leakance value used for most of the Model 2 network was rather arbitrarily set equal to $1.0 \times 10^{-14} {\rm sec^{-1}}$. Sensitivity runs showed that this small leakance value has little effect on the model's predictions. The leakance must be increased to roughly $1.0 \times 10^{-13} {\rm sec^{-1}}$ before the resulting decrease in leakage flux becomes significant compared to the Plains Electric pumping rate. Model 2 used a much higher leakance value $(1.0 \times 10^{-10} {\rm sec^{-1}})$ along the Nutria monocline (a structural feature located about four miles east of Gallup) but the resulting effect on the predicted head is very localized. For all practical purposes, leakage plays a negligible role in the Model 2 analysis.

Since Model 2 assumes no-flow boundaries on all four sides the only inputs required for the steady-state head simulation (other than aquifer parameters) are the nominal recharge and pumpage and the head in the adjoining aquifer (the Sonsela formation). As was pointed out in Section 4.1, an aquifer surrounded by

no-flow boundaries can be in steady-state only if leakage is able to account for the difference between pumpage and recharge. A simple water balance (see Table 2-2) reveals that San Andres-Glorieta recharge exceeded pumpage in 1968 by several thousand acre-feet. Since the Model 2 leakance values are small, the head in the Sonsela would have to be at least several hundred feet lower than the head in the San Andres-Glorieta in order for this much water to leave by leakage. This is unrealistic considering that observed head differences between the Sonsela and San Andres-Glorieta are only about 150 feet. This dilemma was resolved by inserting an ad hoc "line of discharge" north of the outcrop area. Specified discharge values along this line force enough water out of the San Andres-Glorieta to achieve a steady-state consistent with the 150 foot observed head difference mentioned above. Additional lines of discharge were also apparently placed below eastern sections of Rio San Jose.

The nominal recharge inputs used in Model 2 were based on Shomaker's (1971) estimates of average infiltration in the outcrop area. A fixed infiltration rate of 0.75 inches/year was assumed to apply over an outcrop area covering approximately 250 square miles. This gives a total recharge flux of about 10,000 acre-feet/year, as compared to the value of 8040 acre-feet/year reported in Table 2-2. Nominal pumpage rates assumed in the Model 2 steady-state simulation were obtained from generally available records and are consistent with the values presented in Table 2-1.

The steady-state simulation results obtained with the discharge lines, recharge rates, and pumpages assumed in Model 2 are shown in Figure 4-5. The Model 2 simulation implies that groundwater moves from the western end of the outcrop northward (as suggested by Shomaker, 1971) and then swings southward toward Rio San Jose. This flow pattern is credible in the area where head observations are available but it is difficult to accept in the east where the flow reverses direction. It appears that the convenient but speculative lines of discharge dominate the regional flow field, forcing the suprising reversal. Although the steady-state head distribution simulated by Model 2 may be correct, it depends on some rather arbitrary assumptions. The "line of discharge" approach needs to be justified by geological and hydrological data since it could clearly be abused if applied indiscriminately. In this case the "line of discharge" seems to be primarily a method for resolving the problems which result from the imposition of universal no-flux boundary conditions on a steady-state model. This is clearly not an adequate justification, since these problems can be best handled by adopting more realistic (and more flexible) boundary conditions.

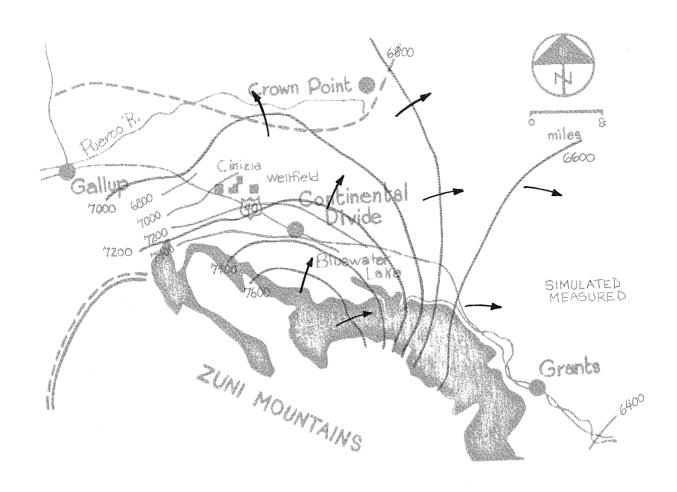


FIGURE 4-5
STEADY-STATE SIMULATION RESULTS FOR MODEL 2

Model 3

Aquifer parameters for Model 3 were initially obtained from an early version of Model 1 and then were refined with a regional model calibration based on the 1968 head observations discussed earlier. The Model 3 transmissivities were blocked in a manner similar to those of Model 1 (see Figure 4-6). The most notable feature of the Model 3 transmissivity distribution is the higher value used in the Ciniza vicinity (1500 ft²/day). This is at the upper end of the range of estimates derived from the Ciniza pump test data.

Model 3 assumes a value of 5×10^{-4} for the confined storage coefficient (the same as Model 1). An unconfined value is not required since the Model 3 network

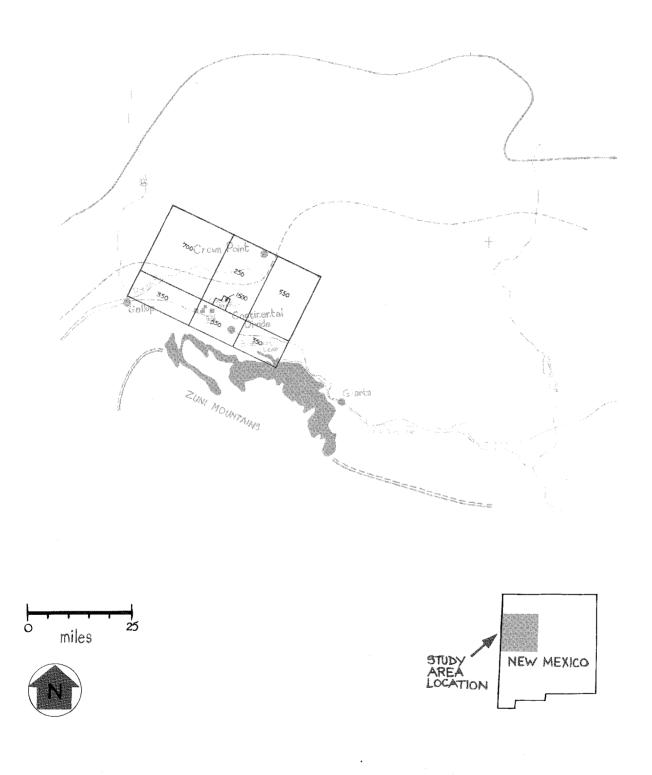


FIGURE 4-6
TRANSMISSIVITY REGIONS FOR MODEL 3

does not extend into the outcrop area. It should be noted that Model 3 uses an open (specified head) southwestern boundary to account for inflows from the outcrop region. These inflows automatically adjust to balance outflows across other boundaries and withdrawals due to pumpage. Since this approach is intended to account for both leakage and recharge, the leakance coefficient and nominal recharge rate for Model 3 were assumed to be zero. Overall, the Model 3 aquifer parameters are as reasonable as those of the other models, with the notable exception of the Ciniza transmissivity, which may be as much as an order of magnitude too high. The implications of this assumption are considered in Section 4.3.

4.2.3 Accuracy Evaluation

The approach to accuracy evaluation taken in Modeling Studies 1 and 2 represents what might be called the "model validation" point-of-view. This approach is concerned primarily with proving that the model is an acceptable description of reality. Once the proof of validity is accepted the predictions are presumed to be sufficiently accurate for their intended purpose. This all-or-nothing approach provides no way to objectively compare different models. All validated models are, by implication, equally accurate.

The goodness-of-fit and sensitivity analysis methods described in Section 3.3.4 represent a more realistic and informative approach to the accuracy issue. The goal of these methods is to derive upper bounds on the model's prediction errors. When expressed as functions of time and space these bounds clearly reveal the model's strengths and weaknesses.

Since there are not enough head measurements in the San Andres-Glorieta aquifer to support an adequate goodness-of-fit investigation, sensitivity analysis must be used to evaluate model accuracy. The "decision tree" search conducted in Modeling Study 3 is a type of sensitivity analysis which is particularly useful in impact assessments. The decision tree approach attempts to identify the set of plausible inputs which gives the worst case impact. In Modeling Study 3 transmissivities, storage coefficients, and boundary conditions were progressively varied above and below their original values and at each stage the value giving the greatest drawdown was retained. The difference between the model's original drawdown prediction and the worst case result (evaluated at Fort Wingate) was about 160 feet. This can be viewed as an upper bound on the prediction error. It should be pointed out that the choice of nominal, upper, and lower values for each parameter in a sensitivity analysis is usually rather subjective. One investigator's worst case may be another's nominal or best case. The worst case values of the Model 3 sensitivity analysis (a transmissivity of 250 ft²/day and a storage coefficient of 5×10^{-5}) are, for example, close to the nominal values used in Model 2. The Model 2 investigators presumably feel these values are reasonable ("most likely") estimates rather than worst case extremes. Such differences in interpretation do not discredit sensitivity analysis as a modeling technique but they do suggest that sensitivity results (like all modeling results) should be carefully examined before they are used to guide policy decisions.

4.3 SUMMARY AND OVERALL ASSESSMENT

When all of the assumptions, interpretations, and results of the San Andres-Glorieta case study are reviewed, it becomes apparent that many of the most noticeable differences between the three models are not very important. shape, resolution, and extent of the model network have little impact since the drawdown cone is contained well within the boundaries of all three alternatives. The analysis presented in Section 3.2.4 indicates that only the type of boundary condition (drawdown vs. drawdown gradient) influences drawdown predictions. Specific boundary values do have an effect on the simulated head which may, in turn, influence the transmissivity estimates generated in a regional model calibration. But regional calibration was not relied upon extensively in the modeling studies reviewed here. It was not used at all in Study 1 and had relatively little impact on estimated well field transmissivities in Studies 2 and 3. All three studies were designed, intentionally, to minimize the impact of boundary conditions on the drawdown results of most interest from a policy point-of-view. Similar comments apply to the nominal pumpage and recharge values used to simulate existing hydrologic conditions. Here again, these inputs have an effect on head but not on drawdown.

It follows from the brief review presented above that the dominant inputs in the San Andres-Glorieta case study must be the aquifer parameters, particularly the transmissivity and storage coefficient. This conclusion should come as no suprise since it was clearly anticipated in Section 3.2.4. It is possible, however, to go still further and state that only the transmissivity and storage coefficient values in the vicinity of the Ciniza well field have a significant impact on drawdown. This assertion is reinforced by the simple "desk top" analysis summarized in the following paragraphs.

The importance of the aquifer parameters in the Ciniza area is related to the nature of the pumping strategy investigated in the case study. The pumping wells to be used by Plains Electric are clustered in a small region located well inside the boundaries of the aquifer in a relatively geologically homogeneous region. It is interesting to note that the predicted drawdown contours plotted in Figure 4-1 are nearly concentric circles centered on the Plains well field. This suggests that the impact evaluation problem can be solved with the classical well drawdown radial flow equation (Freeze and Cherry, 1979). Various solutions to this equation are available but it is most instructive to begin with the simple approximation proposed by Jacob(1940):

$$d = \frac{2.3\Delta Q}{4\pi T} \log \frac{2.25Tt}{r^2 S} \tag{4-2}$$

Here T and S are the transmissivity and storage coefficient near a well pumping at a volumetric rate ΔQ . The drawdown d is evaluated at time t at a distance r from the well. All variables are assumed to be in consistent units.

Jacob's solution clearly reveals how drawdown depends on the aquifer parameters T and S. It is also consistent with the results of the case study—the

TABLE 4-1 $\begin{tabular}{l} \hline COMPARISON OF SIMULATED DRAWDOWNS WITH VALUES OBTAINED \\ \hline FROM A THEIS SOLUTION \\ \hline \end{tabular}$

All drawdowns computed at $t=18~{\rm years}$ Incremental pumpage assumed to be 5000 acre-feet/year

				Drawdowns	
Model	T	S	Radius	Simulated	Theis
No.	$({\rm ft^2/day})$	(unitless)	(miles)	(ft.)	(ft.)
1	400	$5 imes 10^{-4}$	4	375	380
			8	250	230
			12	150	145
2	350	$2 imes 10^{-4}$	4	500	530
-			8	350	350
			12	230	250
3	750	$5 imes 10^{-4}$	4	280	240
•		- • • = -	8	160	160
			12	50	110

predicted drawdown was greater when T and S were assumed to be small (Model 2) and less when T and S were assumed to be large (Model 3). This is a physically plausible result since head gradients must become steeper as conductivity and compressibility decrease if a given pumping rate is to be sustained.

Jacob's approximation requires the argument of the log function to be small and so is not accurate if Tt is large compared to r^2S . A more general but less descriptive well drawdown solution has been proposed by Theis (1935). The Theis solution may be compared directly with the model predictions plotted in Figure 4-1 if t is set equal to the time of maximum drawdown (18 years) and r is varied over a range of appropriate distances (e.g. 4, 8, and 12 miles). Each of the three modeling alternatives may be characterized by the combination of spatially averaged T and S values listed in Table 4-1. The simulated and Theis drawdown predictions displayed in this table are suprisingly close, confirming the hypothesis that the Plains Electric problem is a simple example of drawdown from a pumping well.

It might be argued that the only way to know that a radial flow solution is appropriate for this problem is to perform a complete two-dimensional simulation first. That may be, but it seems likely that a Theis solution would occur to many groundwater hydrologists presented with the problem description summarized in Chapter 2. It is probably more accurate to say that a desk-top analysis based on the Theis solution simply isn't as impressive as a full-blown computer simulation. It is sometimes difficult to resist the belief that a computer model is better just because it is more complicated.

Certainly there are many situations where computer models are the only reasonable way to analyze a complex problem. One case study does not prove that modeling is unnecessary or redundant. Nevertheless, the San Andres-Glorieta experience illustrates the value of carefully thinking about a problem before undertaking an expensive modeling project. A few simple hand calculations based on readily available data can help to focus attention on the truly crucial aspects of a problem and perhaps save months of work. In the San Andres-Glorieta case study, the dramatic differences in predicted drawdowns were ultimately the result of different interpretations of a handful of pump tests. The accuracy of these predictions would probably have improved significantly if more effort had been spent on field studies in the Ciniza area.

In the last analysis, the critical issue in this case study was not model formulation but data collection and interpretation. Modeling has made groundwater hydrology more technical and sophisticated but it has not eliminated the need for carefully designed and executed field studies. Ideally, the modeler and field investigator should work together so that the strengths of their two disciplines can be combined. Groundwater models can be used to help design field sampling programs by showing where additional data will be most beneficial. Conversely, sampling programs tailored to the needs and assumptions of discretized regional models can provide more reliable information for estimating model inputs. Such a cooperative approach is ultimately the most reasonable and cost-effective way to obtain credible impact predictions.

5. GUIDELINES FOR GROUNDWATER MODELING

The preceding chapters include a number of recommendations and guideline which relate to the practical details of model application. These cover topics ranging from the specification of boundary conditions to the selection of an extrapolation technique. This chapter attempts to step back from technical details and look at some of the major decisions to be made in a groundwater modeling study. The guidelines which follow are inspired by the San Andres-Glorieta case study but are meant to be generally applicable. Each modeler must, of course, adapt these general guidelines to the unique requirements of his particular problem. This could, in fact, be viewed as the "first guideline".

The major recommendations of this report can be summarized as follows:

- 1. Clearly define the objectives of the modeling study at the outset. If the study results are to be used for hydrologic impact evaluation determine what type of predictions are needed (head, drawdown, storage changes, etc.)
- 2. Consult closely with a groundwater geologist familiar with the region of interest. Local geological experience can help make data interpretation and input estimation more informed and realistic.
- 3. Always carry out a thorough desk-top analysis before starting a modeling project. In particular:
 - Construct a rough aquifer-scale water budget
 - Plot head contours from available well observations
 - Sketch in flow lines
 - Use simple radial well solutions (Theis, Jacob, etc.) to look at drawdown near well fields

The desk-top analysis should be used to determine first whether a modeling study is necessary and then how it should be conducted.

- 4. Identify the inputs which will have the most effect on the model's predictions. If drawdown is of primary interest these will be the aquifer parameters and the postulated incremental pumpage and recharge rates. Special effort should be devoted to the collection and analysis of data needed to estimate these parameters.
- 5. Carry out a steady-state regional calibration whenever possible. Do not rely solely on localized pump test analyses when estimating spatially-averaged model inputs. Instead, use pump test results to constrain a regional calibration.

- 6. Make a serious attempt to quantify the accuracy of the model's predictions throughout the entire prediction period. The accuracy evaluation may be based on a goodness-of-fit analysis, a sensitivity analysis, or both. Avoid the traditional "model validation" approach.
- 7. When presenting model results, carefully consider the needs of the reader. Are the results presented in a way which will make decisions more informed? Are prediction errors acknowledged or is the accuracy issue avoided? Proper presentation is crucial if the model's results are ever to be used.

The above list is somewhat biased and probably not complete but it does reflect the major lessons learned in the San Andres-Glorieta case study. If each of these guidelines is carefully followed the resulting modeling study is much more likely to be technically defensible and convincing. The most important factors influencing the success of any modeling effort are, of course, the modeler's competence and judgment. There are still no computerized substitutes for these.

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