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This volume relates the mathematical equations for flow in natural channels to the physical process. The most common method for solving these basic equations, the standard step method, is developed for complex cross sections using techniques which are suitable for both hand and computer calculations. One of the objectives of this volume is to delineate the degree of simplification that is appropriate for analysis of water surface profiles in natural channels. The procedures are illustrated by solving an example problem by manual computations; however, the more common practice in the Corps of Engineers is to utilize electronic computers to calculate water surface profiles. Programs for this purpose are readily available.

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FOREWORD

This volume is part of the 12-volume report entitled "Hydrologic Engineering Methods for Water Resources Development," prepared by The Hydrologic Engineering Center (HEC) as a part of the U.S. Army Corps of Engineers participation in the International Hydrological Decade.

Volume 6 relates the mathematical equations for flow in natural channels to the physical process. The most common method for solving these basic equations, the standard step method, is developed for complex cross sections using techniques which are suitable for both hand and computer calculations. One of the objectives of this volume is to delineate the degree of simplification that is appropriate for analysis of water surface profiles in natural channels. The procedures are illustrated by solving an example problem by manual computations; however, the more common practice in the Corps of Engineers is to utilize electronic computers to calculate water surface profiles. Programs for this purpose are readily available.

The volume was written by William A. Thomas. Editing and review comments and many helpful suggestions were furnished by Bill S. Eichert, John C. Peters, Vernon R. Bonner, Allan K. Oto, A.J. Fredrich, and Edward Hawkins.

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Chapter 1

Introduction

CHAPTER 1. INTRODUCTION

Section 1.01. Purpose and Scope

The computation of water surface profiles involves solving the one-dimensional energy equation to determine the shape of the profile between control sections where the water surface elevation is known or can be assumed. The most generally applicable procedure for steady-flow profile calculation is called the Standard Step method. In this method, the total distance is divided into reaches by cross sections at fixed locations along the channel and, starting from one control, profile calculations proceed in steps from cross section to cross section to the next control. The purpose of this volume is to relate the mathematical expressions that describe the dynamic condition of water in motion to problems involving the many complexities of natural rivers.

Many textbooks that develop the basic theory for open channel flow make use of simplified channel geometry such as a very wide or a rectangular channel because the resulting mathematical expressions are simplified. One can then turn his attention to fully understanding the basic principles. The application of basic theory to natural rivers is quite a different problem. Assumptions and simplifications must be evaluated, and those which are too extreme must be avoided.

Applications developed for the electronic computer are even more critical in that they must be carefully designed and programmed to insure that their usefulness is not impaired by simplifications which preclude "unusual" situations. For example, one cannot plan an analytical study around a computer program that is able to handle only 70 percent of the foreseeable situations. For that matter, even 90 percent effectiveness is usually not sufficient since the remaining 10 percent of the problems would require such voluminous amounts of manual calculations that the study could not be accomplished within a reasonable amount of time. A computer program must be designed so it is 95 to 100 percent effective; that is, so that it can handle 95 to 100 percent of the problems in open channel flow. This means that almost all simplifications must be avoided, and alternative algorithms must be given for every major calculation that is required.

The information presented in this volume reflects insight gained from calculating water surface profiles to satisfy requirements in planning, designing, constructing and operating water resource projects. Existing textbook material will be referenced to emphasize its usefulness and to point out its limitations for calculating water surface profiles for the general cases that occur in field problems.

The scope of this volume is restricted to rigid boundary hydraulics and steady flow. (Volume 12 of this report, "Sediment Transport," treats problems involving movable boundary hydraulics.) Procedures are presented which lend themselves to both manual and automatic computation methods.

Section 1.02. The Influence of the Electronic Computer

While methods presented in this volume lend themselves to both manual and automatic calculations, it does not necessarily follow that the two approaches are equally suitable for calculating water surface profiles. The influence of the computer has completely revolutionized the engineer's ability to understand and predict the behavior of flowing water. It frees the engineer from the time-consuming computations and permits him to focus on solving the problem.

These advantages, however, are not obtained without cost. Training is required to learn how to apply computerized techniques. The considerable expense involved in developing a computer program requires that it have a widespread application over a long period of time. The capability of computers changes so rapidly it is impossible to develop procedures that keep abreast of the computer improvements. For this reason, computer programs are constantly modified to accommodate new capabilities. Benefits obtained usually far exceed the costs.

Early applications of the computer simply duplicated the manual methods previously used. In recent years, attempts have been made to utilize computer capability more fully with primary consideration being given to improving the quality of results, expanding the capability to analyze problems and reducing computation cost. This volume presents techniques which are based upon manual methods but which attempt to minimize the computation time required when computers are utilized. For example, a technique of successive approximations for converging assumed and computed water surface elevations in two or three iterations is illustrated, the concept of calculating profiles for several discharges simultaneously is described, and techniques are presented which in most cases will reduce the number of times critical depth must be calculated.

Several computer programs are available for calculating water surface profiles. These are reviewed in reference 3. The program "Water Surface Profiles" presented in Appendix 2 of Volume 1 of this report has received widespread acceptance throughout the United States. It has been in use and under development since 1964 and provides the capability to analyze the full range of water surface profile problems normally experienced in the Corps of Engineers.

When profiles are calculated manually, major discrepancies are easily spotted. However, when computers are used, this is often not the case. It is very important, therefore, that profiles determined by electronic computers be closely reviewed when the job is completed. For example, large changes in water surface elevation, slope or velocity should be logically substantiated regardless of the method of computation.

Chapter 2

Equations for Steady Gradually Varied Flow

CHAPTER 2. EQUATIONS FOR STEADY GRADUALLY VARIED FLOW

Section 2.01. The One-Dimensional Energy Equation

Water at rest exerts a hydrostatic pressure upon the solid boundaries which contain it and, like a solid body at rest, possesses a certain potential energy due to the vertical height of that solid boundary above some datum such as mean sea level. If the pressure on the water surface is atmospheric (i.e., the water has a 'free' surface and is not confined by the roof of a pipe), the total energy at each point is the potential energy plus the pressure energy due to the weight of water above that point. Should this water begin to flow, part of the total energy would be converted to kinetic energy, and part of it used to overcome friction and other losses. Water surface profile computation is based on application of an analytical procedure to calculate quantities for each of these energy components. The principles of conservation of energy and conservation of mass are involved in the analytical procedure.

The basic relationship describing the conservation of energy of any mass being moved between two points is:

$$E_2 + \text{WORK} = E_1 + \Delta E \qquad (2\text{-}1)$$

where:

E_1 = the total energy at point 1

E_2 = the total energy at point 2

ΔE = the energy lost between points 1 and 2

and WORK = any external energy added between points 1 and 2

In fluid dynamics problems, as in the more familiar case of solid bodies in motion, the units for terms in equation 2-1 are foot-pounds (ft-lbs) in the British system and meter-kilogram force (m-kgf) in the gravitational metric system. When applied to fluid problems, the above relationship results in the Bernoulli equation for a fluid filament of infinitesimal cross section; and when integrated over the entire cross section of flow, the One-Dimensional Energy Equation is developed.

The second principle introduced above, conservation of mass, is also an important consideration in both solid and fluid dynamics, but in the case of solid bodies which display a strong resistance to deformation, conservation of mass is readily satisfied. Fluid mechanics, on the other hand, requires a mathematical expression to account for a continuously deforming body of moving fluid. The basic assumption is that a continuum of fluid exists throughout the body in both time and space. This principle is generally referred to as "continuity."

It is useful to illustrate these principles with a problem involving flow of a fluid between two points in space. To illustrate the Bernoulli equation, assume a continuous filament of flow exists in a streamtube of infinitesimal cross section and for the period of time required for a particle of fluid to pass between the two points. Furthermore, no flow crosses the boundary of this streamtube except at its end points. By definition, kinetic energy is $1/2\ MV^2$ where M is the total mass that flows and V is the flow velocity. Also, by definition, $M = W/g$ where W is weight and g is the acceleration of gravity. During the isolated time period under consideration, W is the product of flow velocity times cross sectional area of the streamtube times the time interval. The resulting kinetic energy is $W \cdot V^2/2g$.

The other energy components are the internal energy, $I \cdot W$, which is associated with the molecular forces in the fluid (where I is internal energy per unit weight); and the potential energy, $Y_o \cdot W$, where Y_o is height of the streamtube in feet or meters above a specified datum.

The principle of conservation of energy requires that total energy at every point in space be referenced vertically above the same datum. Looking then at the two points of interest, the kinetic energy, internal energy and potential energy combine to form the total energy as shown in the following equations:

$$E_1 = W \cdot V_1^2/2g + I_1 \cdot W + Y_{o_1} \cdot W \qquad (2\text{-}2)$$

and

$$E_2 = W \cdot V_2^2/2g + I_2 \cdot W + Y_{o_2} \cdot W \qquad (2\text{-}3)$$

where all terms are as previously defined.

The net work exerted between points 1 and 2 (WORK in equation 2.1) can be calculated from the following equation:

$$\text{WORK} = F_2 dl - F_1 dl \qquad (2\text{-}4)$$

where:

F = force in pounds (lbs) or kilograms (kgf)

dl = the length of the streamtube between points 1 and 2

Assuming no mechanical work is done on the system, the force (F) can result only from pressure energy due to the columns of fluid above the points in question and is equal to the product of pressure and streamtube area. Equation 2-4 can be rewritten as:

$$\text{WORK} = P_2A \cdot dl - P_1A \cdot dl$$

where:

P_1 and P_2 = pressures at points 1 and 2, respectively.

Substituting equations 2-2, 2-3 and 2-5 into equation 2-1:

$$W \cdot V_2^2/2g + I_2 \cdot W + Y_{o_2} \cdot W + (P_2A \cdot dl - P_1A \cdot dl) \qquad (2\text{-}6)$$

$$= W \cdot V_1^2/2g + I_1 \cdot W + Y_{o_1} \cdot W + \Delta E$$

Dividing both sides of equation 2-6 by W results in the following:

$$(V^2/2g)_2 + I_2 + Y_{o_2} + \left(\frac{P_2A \cdot dl - P_1A \cdot dl}{W} \right)$$

$$= (V^2/2g)_1 + I_1 + Y_{o_1} + \Delta E/W \qquad (2\text{-}7)$$

The total weight W can be expressed as the product of unit weight, γ, (in lb/ft^3 or kg/m^3) times the volume of water flowing where volume is $A \cdot dl$, (in ft^3 or m^3). A is the cross sectional area in ft^2 or m^2 and dl is the unit length in feet or meters. Replacing W in equation 2-7 with $\gamma \cdot A \cdot dl$, defining the change in energy, $\Delta E/W$, as H_L and neglecting the internal energy component, I, yields the Bernoulli equation:

$$(V^2/2g)_2 + P_2/\gamma + Y_{o_2} = (V^2/2g)_1 + P_1/\gamma + Y_{o_1} + H_L \quad (2\text{-}8)$$

The significance of the terms in equation 2-8 is illustrated in fig. 2.01 for a streamtube along a channel bottom.

The foregoing manipulations are not a rigorous development of Bernoulli's law, but rather they illustrate the relationship between the familiar form of the equation which appears to have units of feet or meters and the basic physical principles involved. Actually, the units in this equation are ft-lbs/lb (or mkgf/kg) of fluid flowing. In practice, unit total energy is usually referred to in units of feet or meters. Subsequently, it will be designated by the symbol H for total head. The kinetic energy component is referred to as velocity head.

The Bernoulli equation is equally applicable to each streamtube in a flow field. A basic assumption in developing this equation was that everywhere in the hypothetical streamtube, velocity vectors were equal in magnitude and parallel in direction. However, when integrating over all streamtubes in a flow field of finite cross sectional area for the general case, neither are velocity vectors parallel nor do they have equal magnitudes. Consequently, three energy equations result--one for each spatial direction. Each equation requires a velocity distribution correction factor, α, to produce a representative velocity head from the distribution of magnitudes. A complete solution of these equations is not necessary for fluid flow problems having a predominant velocity in one direction, and most water surface profile calculations fall into this category. As a result, the equations dealing with vertical and lateral velocity vectors are neglected yielding the so-called One-Dimensional Energy Equation upon which water surface profile calculations are based.

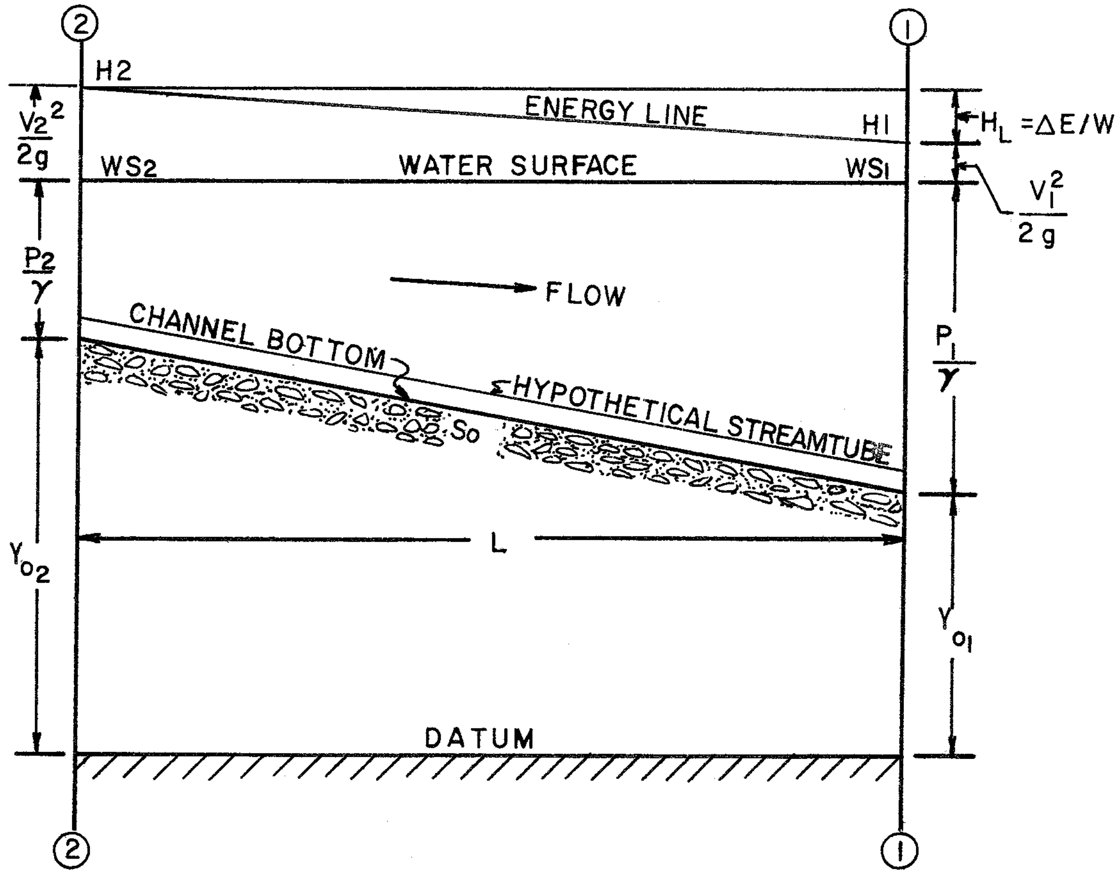
$$\frac{\alpha_2 V_2^2}{2g} + \frac{P_2}{\gamma} + Y_{o_2} = \frac{\alpha_1 V_1^2}{2g} + \frac{P_1}{\gamma} + Y_{o_1} + H_L \qquad (2\text{-}9)$$

The One-Dimensional Energy Equation has the same form as the Bernoulli equation and the same terms are present. In addition, an α term has been added to correct for velocity distribution. The terms in the above equation represent average conditions in the cross section rather than conditions for a single streamtube.

The proper application of this equation is left to the judgment of the engineer who must visualize the flow field and insure that it is, in fact, one dimensional. For example, two- and three-dimensional flow occurs at all expansions and contractions. When one-dimensional theory is being used, additional coefficients are often required to account for the increased rate of energy loss in such cases.

Section 2.02. Necessary Independent Conditions and Equations

At this point in its development, equation 2-9 applies equally well to water flowing under pressure (pipe or confined) or in an open channel (surface free to atmosphere). However, the nine unknowns require that eight more independent conditions be specified in order to obtain a solution. These are obtained from functions, constraints or initial conditions which depend upon classification of flow as either



H_L = Total change in energy (loss) between sections 1 and 2

L = Reach length

V = Velocity of flow

W = Weight of fluid

P/γ = Depth of flow

Y_o = Height of channel bottom above a datum

g = Acceleration of gravity

1 and 2 = Downstream cross section and upstream cross section, respectively

Fig. 2.01. Terms in the Bernoulli equation

steady or unsteady, pipe or open channel. Furthermore, if open channel flow exists, the relationship between pressure energy and kinetic energy is defined by a double-valued function which requires an additional boundary constraint (critical depth) to satisfy the number of independent conditions required for a unique solution. In the following discussion of required conditions and equations, the flow is assumed to be steady, open-channel flow. A detailed discussion of classification of flow is presented in Chapter 3. Unsteady flow is discussed in Chapter 7.

In open channel flow, the potential energy, Y_o, is specified as the height of the solid boundary confining the flow above some datum. If the pressure distribution is hydrostatic, the pressure energy, P/γ, is the depth of water above the solid boundary. These two energy terms can be added to obtain:

$$WS = P/\gamma + Y_o \tag{2-10}$$

where WS is the water surface elevation above the datum, as shown in fig. 2.01. Equation 2-9 can then be rewritten:

$$WS_2 + \frac{\alpha_2 V_2^2}{2g} = WS_1 + \frac{\alpha_1 V_1^2}{2g} + H_L \tag{2-11}$$

An average velocity can be obtained by dividing the flow, Q, by the cross-sectional area, A. Under steady flow conditions, continuity is satisfied because $Q_1 = Q_2 = Q$, and equation 2-11 can be rewritten as:

$$WS_2 + \frac{\alpha_2 Q^2}{2gA_2^2} = WS_1 + \frac{\alpha_1 Q^2}{2gA_1^2} + H_L \tag{2-12}$$

Equation 2-12 contains one independent variable, Q, and seven dependent variables. For a given cross section geometry, the area terms are functions of the water surface elevation; the α terms are functions of WS and hydraulic roughness across the section; and the energy loss term is calculated with the Manning Equation and equations for other losses (e.g., expansion, contraction, weir and bridge). The sixth independent equation comes from a functional relationship between WS and Q for starting conditions. The final independent condition required to solve equation 2-12 is the constraint, critical depth. Critical depth is that depth of flow that would produce the minimum total energy head, and it depends on cross section geometry and water discharge. Flow is classified as either subcritical or supercritical depending upon whether the starting water surface elevation is above or below critical depth. All subsequent calculations are confined to the same side of critical depth as the initial value. The depth-energy relationship is discussed in section 3.02 under "Effect of Gravity" on state of flow and in section 5.12 under "Critical Depth Calculations."

Chapter 3

Classification of Open Channel Flow

CHAPTER 3. CLASSIFICATION OF OPEN CHANNEL FLOW

Section 3.01. Modes of Conveyance

The extent to which boundary geometry confines flowing water is a basis for classifying hydraulic problems. If the water surface is free to the atmosphere, open channel flow exists. The other common mode of conveyance is pipe or pressure flow which exists when the surface of flowing water is confined, as by a culvert roof. Open channel flow can occur in a culvert provided the depth of water is less than the height of the culvert so that a free surface exists.

The importance of this classification is not its effect upon the conservation of energy principle. The same one-dimensional energy equation is valid for both open channel and pressure flow. However, the dynamic forces in pressure flow (assuming steady flow) are the viscous and inertial forces. When a free surface exists, the forces of gravity and surface tension must be added to these forces. Also, the position of the water surface is free to change with both time and space; and, consequently, the depth of flow, the discharge, and the slopes of the channel bottom and free surface are interrelated. Open channel flow problems are therefore considerably more difficult to analyze than pressure flow problems.

Section 3.02. State of Flow

Effect of Velocity

In both open channel and pressure flow three states of behavior can exist--laminar, transitional and turbulent. The ratio of viscous to inertial forces, called Reynolds number, indicates which state prevails and is defined by:

$$R_e = \frac{VL}{\nu} \qquad (3\text{-}1)$$

where:

R_e = Reynolds number

V = velocity of flow

L = characteristic length

ν = kinematic viscosity

In open channel flow the hydraulic radius is the characteristic length, L. For Reynolds numbers less than 500, laminar flow exists; and for Reynolds numbers greater than 2000, the flow is usually considered turbulent. A transitional state exists between Reynolds numbers of 500 and 2000. However, there is really no definite upper limit for the transitional range for all flow conditions. In pipe flow, pipe diameter is the characteristic length and the limiting values of R_e are 2000 and 8000 for the transitional zone, subject to the same qualifications as discussed above for open channel flow.

The state of flow is important because of its influence on the friction loss coefficients. Flow in natural channels is almost always highly turbulent; and, therefore, the friction loss coefficients depend only on boundary roughness. However, when the state of flow is laminar or transitional, viscosity of the fluid becomes an important factor in determining the value of the friction loss coefficient. The functional relationship between Reynolds number and friction loss coefficients is available for flow in pipes, and this relationship can be converted for use in open channels by relating the characteristic length terms in Reynolds number as follows:

$$R = A/P = \frac{\pi D^2}{4} / \pi D = D/4 \qquad (3\text{-}2)$$

where:

A = area of flow cross section

D = diameter of pipe

P = wetted perimeter

R = hydraulic radius (area/wetted perimeter)

Effect of Gravity

In pipe flow, the force of gravity is not a factor, but in open channel flow it plays a major role. It is the force of gravity that converts pressure energy into kinetic energy. This force is counteracted by the inertia of the water. The following figure illustrates the relationship between pressure head and velocity head as the depth of flow is changed by the force of gravity.

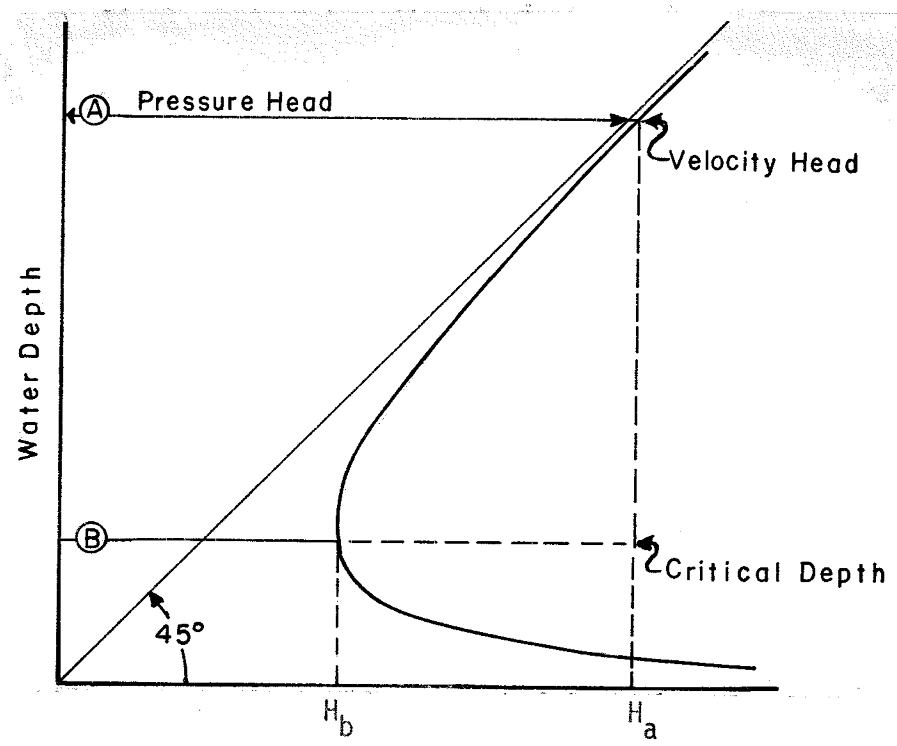


Fig. 3.01. Relationship depicting critical depth

Water flowing initially at a depth A, marked on the ordinate axis in fig. 3.01, has a total energy head marked H_a on the abscissa axis. If discharge is held constant and the water depth allowed to decrease, as in the case of water approaching a free overfall, velocity head will increase, pressure head will decrease, and total energy will decrease toward a minimum value where the rate of decrease in the pressure head is just counterbalanced by the rate of increase in velocity head. This point is critical depth, and velocity head will not continue to increase beyond the value at critical depth (i.e., critical velocity) unless additional energy is added to the flow as through a conversion of potential energy.

The ratio of inertial to gravitational forces is an important measure of the state of flow, as shown by the dimensionless number known as the Froude number:

$$F = \frac{V}{\sqrt{gD}} \qquad (3\text{-}3)$$

where:

V = velocity

D = characteristic length term defined as cross section area divided by water surface width

g = acceleration of gravity

When flow is at critical depth, the inertial and gravitational forces are equal and F = 1. If F is less than 1, the water depth is above critical and the state of flow is **subcritical.** The influence **of gravity** forces dominates the inertial forces; flow has a low velocity and it is often described as tranquil flow. If F is greater than 1, the water depth is below critical, inertial forces dominate the gravity forces, and the flow is described as rapid or shooting. The state of flow is supercritical. Therefore, "state of flow" also refers to the relationship between the flow velocity and a critical velocity.

The characteristic length used in computing the Froude number is an average depth for the cross section. This average value, then, defines an average Froude number. It is possible to have point velocities which exceed the average critical velocity and still have flow that is essentially subcritical. This is illustrated when surface waves, generated by tossing a stone into a flowing stream, are swept downstream by supercritical surface velocities even though the calculated Froude number is less than 1.

In calculating water surface profiles, calculations should begin at the downstream end and proceed in the upstream direction when flow is subcritical. When supercritical flow exists, calculations are made in the downstream direction. If the direction of profile computation is not correct for the prevailing flow condition, the calculated profile diverges from the true profile unless the starting water surface elevation is precisely correct. If the calculations proceed in the proper direction for prevailing flow conditions, the calculated water surface profile converges to the true profile even if the starting water surface is in error.

Section 3.03. Types of Flow

Possible types of open channel flow are shown in fig. 3.02.

Steady Versus Unsteady Flow

If the change in velocity with respect to time at a given location is zero, the flow is steady. Otherwise, the flow is classified as unsteady in which case the additional variable, time, must be included as discussed in Chapter 7 of this volume.

Uniform Versus Varied Flow

If dV/dx, the change in velocity with respect to distance (where V is velocity and x is distance along the channel) is zero for any

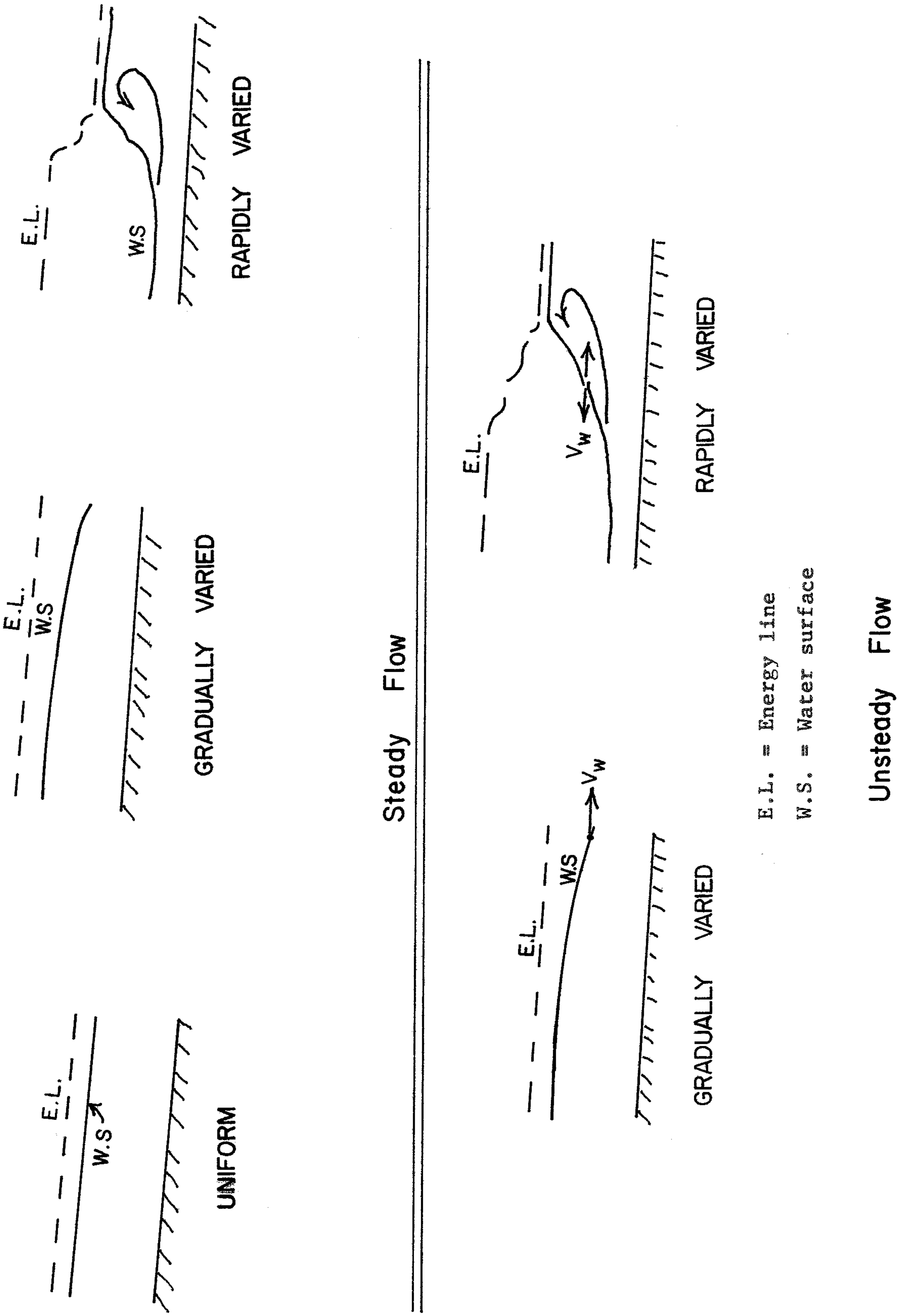


Fig. 3.02. Possible types of open channel flow

given time, the flow is uniform. Otherwise, the flow is nonuniform, and the relationship between kinetic energy and potential energy will be continuously changing. If flow is uniform, the water surface will be parallel to the channel bottom. If not, the slope of the water surface profile will differ from that of the channel bottom.

If the rate of change of water surface slope is not visible to the eye, the flow can be considered gradually varied. Rapidly varied flow exists where the change in water surface slope is apparent to the eye, such as at a hydraulic jump. The calculation of profiles under rapidly varied flow conditions requires special treatment.

Section 3.04. Classification of Steady Flow Profiles

Figure 3.03 shows the general types of steady flow profiles.

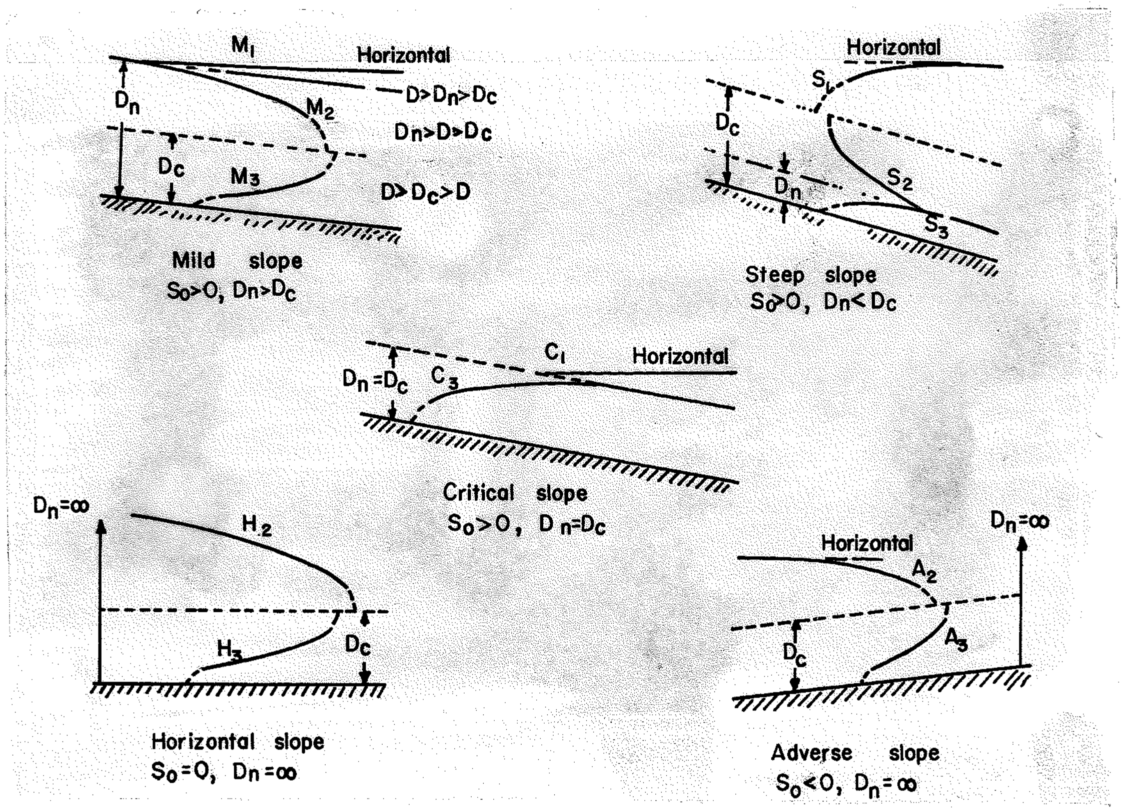


Fig. 3.03. Classification of flow profiles

The slope of the channel (S_0) is one criterion used in classification. A critical slope is one on which critical velocity is sustained by a change in potential energy rather than pressure head. A mild slope is less than critical slope, and a steep slope is greater than critical slope. When the slope is positive, the bed slope is classified as mild, steep or critical; and the corresponding flow profiles are the M, S, or C profiles, respectively. If the slope of the channel is zero, the bed is horizontal, and the profiles are called H profiles. If the bed rises in a downstream direction, the slope is negative and is called an adverse slope. Gradually varied flow profiles on an adverse slope are called A profiles.

Another parameter used in classifying gradually varied flow profiles is the magnitude of the water depth relative to normal depth, D_n, and critical depth, D_c. (The depth that would exist if the flow were uniform is called normal depth. A type 1 profile exists if the water depth is above the highest "depth" line (fig. 3.03) which may be either the critical depth line or the normal depth line depending on the channel slope.) A type 2 profile exists if the water depth is between the normal and critical depth lines, and a type 3 profile occurs when the water depth is below the lowest depth line. It is customary to identify profile type by a letter reflecting bottom slope and a number reflecting depth type (i.e., M1, A2, H3, etc.). Figure 3.04 shows a variety of profile types.

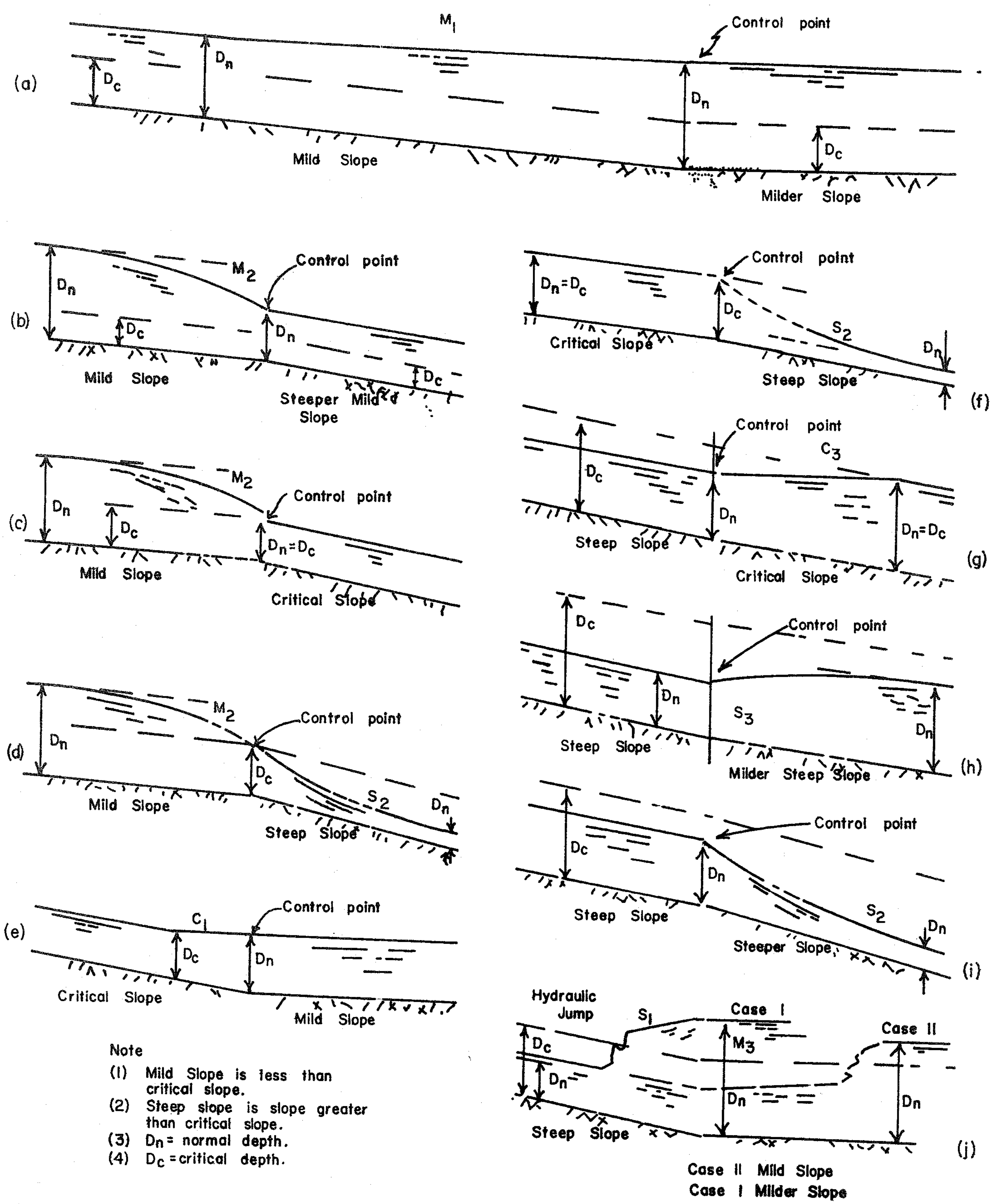


Fig. 3.04. Water surface profiles over a break in grade.

Section 3.05. Significance of Flow Type in Computerized Techniques

The fact that surface water flows naturally as open channel flow, but often is interrupted by the works of man so that pressure flow exists, is of little concern when calculations are being done manually. The engineer applies the appropriate procedures for obtaining area, wetted perimeter and losses under the prevailing flow conditions at each point. However, when a computer program is being developed for calculating water surface profiles, the capability to distinguish between pressure flow and open channel flow should be incorporated. This will permit automatic calculations to continue in the space remaining even though the mode of conveyance changes back and forth between pressure and open channel flow. Calculating area and wetted perimeter of flow, considering critical depth, and including form losses, expansion losses and other losses besides friction are primary problems in programming the automatic computations.

No special program logic is required to identify the Reynolds number in order to calculate friction loss, but one should be aware of its significance on hydraulic roughness values and be prepared to include such refinement if a problem warrants it. The classification of steady flow profiles by bed slope and profile shape is not an important feature in a computer program, but the program must distinguish between subcritical and supercritical flow. Moreover, it is important to develop program logic so computations will continue without manual intervention.

Chapter 4

Basic Data

CHAPTER 4. BASIC DATA

Hydraulic theory is well established for channels with rigid boundaries and produces consistent and accurate results when properly applied. Major sources of error when applying hydraulic theory are inaccuracies in data or improper modeling of flow conditions and current patterns.

Basic data are grouped into three categories: geometric data, hydraulic roughness values, and the water discharge and corresponding water surface elevation. The accuracy required in this data depends upon the accuracy needed in final results. At times, it is more economical to compensate for inaccurate data by safety factors such as providing liberal amounts of freeboard, and in rural areas such procedures are acceptable. However, in urban areas both property damage and loss of life can result from designs based on inaccurate data.

Section 4.01. The Geometric Model

It is customary to establish the boundary geometry for flow in natural rivers by measuring ground surface profiles perpendicular to the direction of flow at intervals along the streams and measuring the distances between these profiles. The profiles themselves are called cross sections, and the distances between cross sections are called reach lengths. Together they form a three-dimensional, digital description of the flood plain and channel through which water must flow - a digital model of the boundary geometry.

Usually a rigid boundary is assumed, and the geometric models are invariably deterministic. Cross sections are often spaced as close as 500 feet (150 meters) on major rivers such as the Missouri and Arkansas Rivers in the United States which have top widths of 2 to 6 times that distance. However, this is not a fixed rule as the type of study to be performed must also be considered. For instance, navigation-depth studies require closely spaced cross sections because local conditions are important; whereas the calculation of water surface profiles for establishing top-of-levee elevations or rating curves at various points along a stream can be made with sections spaced 1 or 2 miles (2 or 3 km) apart provided a proper formulation of the friction loss equation is used. (Friction loss is discussed in Chapter 5.) In sedimentation studies where future deposition in reservoirs is being predicted, cross sections spaced 5 or 10 miles (8 or 16 km) apart are often satisfactory. In any case, the final spacing of cross sections for the calculations depends upon the computation scheme, not the spacing selected for field measurement. It is often desirable to combine several measured sections into one average section to better define average geometry over long distances.

Some computation schemes treat each cross section as representing a reach of the river and use only one section at the midpoint of the reach to calculate losses through the entire reach. Other schemes use cross sections to define break points in the geometry, and properties of adjacent sections are averaged to calculate losses through the reach. This volume generally describes computation schemes based on the latter condition; therefore, equations are developed for cross sections located to define break points in widths and bottom slopes.

The objective of either computation scheme is to describe flow boundaries accurately enough to predict energy losses due to friction and eddy currents. The fewer the number of cross sections that are available and the further apart they are located, the greater is the amount of engineering judgment that is required to satisfactorily analyze the problem. Any deviation from a smooth profile must be explained, and in some cases it can be traced back to an inadequate description of boundary geometry. The following points will be helpful in developing the geometric model.

On the best available map locate cross sections to be surveyed such that the following information can be described for field personnel.

a. Cross sections are needed at sharp changes in bed slope.

b. Cross sections are needed at points of contraction or expansion of the channel.

c. Cross sections are needed in tributaries immediately above a confluence and in the main stream immediately below a confluence.

d. Cross sections should be spaced to provide a reasonable transition between locations that have different roughness characteristics.

e. Cross sections are required immediately above and below control sections such as weirs.

f. The maximum elevation of each end of a cross section should be higher than the anticipated maximum water surface elevation.

g. It is helpful for later reference if all sections are identified by river mile, and are located on a map. Reach lengths and n-values should be entered on the map, also.

Section 4.02. Flow Boundary Geometry at Bridges

Describing the flow boundaries in the vicinity of bridges is difficult because flow is three dimensional. It is necessary to visualize current directions for complicated flow patterns, position cross sections perpendicular to the flow and identify that portion of each cross section which conveys flow in order to model flow boundaries. The following figure illustrates the problem:

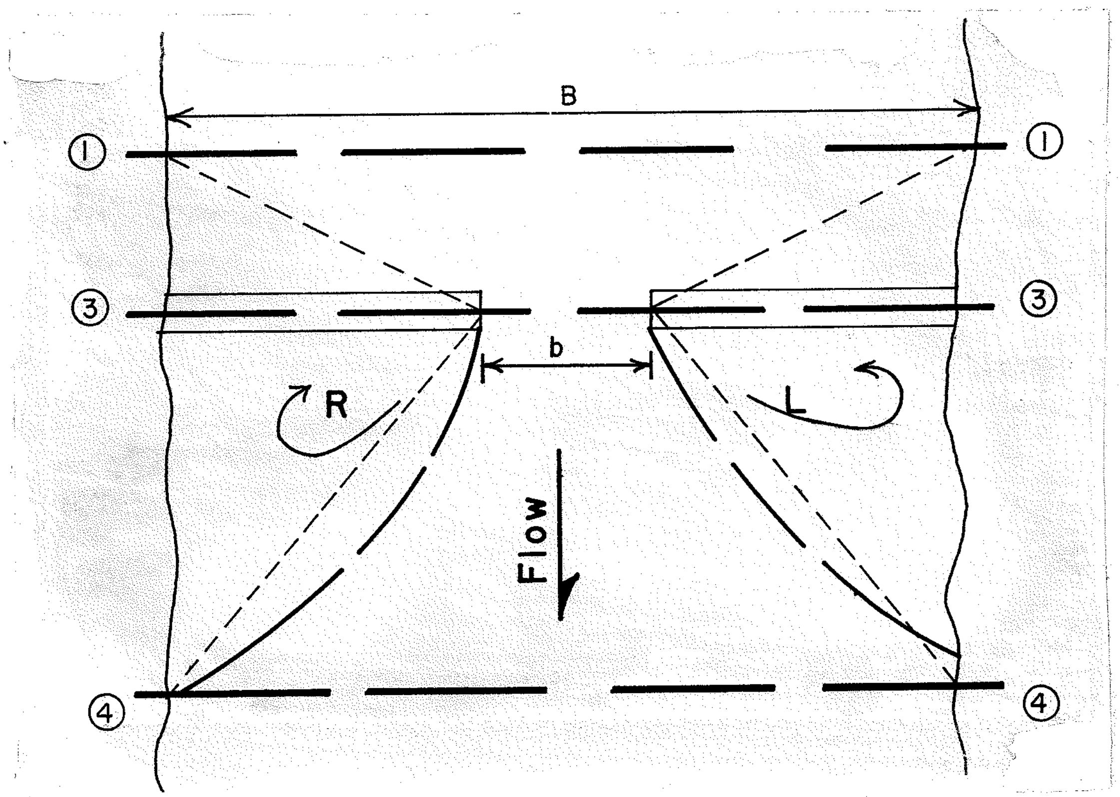


Fig. 4.01. Flow boundary geometry at bridges showing contracting and expanding flow

A stream having a top width B flows past a bridge having an opening b between abutments. Both approaches are earth fills. All flow passes through b. Cross section 3 describes the contraction by starting along the top of the road, following along the ground surface across b, and returning to the top of the road. Cross section 1 must be located far enough upstream to describe the approaching flow, and a distance equal to b is usually adequate. Cross section 4 must be located far enough downstream to describe the expanding flow. Topographic maps can be used to determine the paths that water will follow. Such paths may be difficult to locate as water may even flow down the borrow ditch beside the road. If no information is available, it may be reasonable to assume that the boundary of expanding flow downstream from the bridge is at an angle of 15° with the main flow direction.

The long dashed lines downstream from the road in fig. 4.01 represent flow boundaries between the main current and eddies (denoted as "L" and "R"). The short dashed lines show how the digitized geometric model, as described by cross sections 1, 3, and 4, would fit the estimated flow boundaries.

It is important to locate section 4 as close to the road as reasonable since the friction loss between sections 3 and 4 is calculated using the average of the conveyances of the two sections. A good test of the adequacy of the completed model is to reproduce an experienced event.

If the flow rate increases to the point of inundating the entire road way, the problem of specifying the ineffective flow area becomes important. Overbank area near the bridge, which does not convey flow until the embankment is overtopped, suddenly becomes effective as the embankment is overtopped. Since one data set is used to analyze a wide range of discharges, it should be formulated in such a way to permit this sudden change in ineffective flow area to occur.

The above discussion concentrated on only one type of bridge and flow situation. All flow passed through a rather narrow opening and then expanded to reoccupy a wide flood plain width. Many other bridge crossing configurations and flow conditions are encountered. In locating cross sections, it is important to understand what causes the energy loss and to model that aspect of the problem. For example, a bridge that spans the entire channel and flood plain and has only a few piers does not cause a significant head loss. For this type of bridge, special consideration in locating cross sections is not required. This is the opposite extreme in geometric configuration to the first case described. Bridge crossings that have the highway approaches located down on the level of the flood plain such that they cause little or no obstruction during flood flows represent another case where cross section location is not extremely critical. It is primarily the expanding flow, illustrated in the first case, which causes the energy loss at bridges. Other losses that contribute are form loss from bridge piers, the contracting flow, and friction loss through the contracted reach. Often, flow, through a single bridge crossing will exhibit all of the above types of behavior as the discharge increases from a low in-channel flow to a very high flow. In this case a model that is adequate for the case of extreme contraction will also be satisfactory for the other flow conditions.

All of the discussion on bridge crossings has pertained to use of three cross sections to model the geometry of flow. The actual number of cross sections required and their location relative to each other and to the bridge depends upon the analytical technique used to calculate the head loss. Nevertheless, all techniques which are based on solving the energy equation must accomplish the same purpose -- that is, to calculate the energy loss through a contraction. Analytical techniques may require cross sections to be located between sections 3 and 4, fig. 4.01, but if additional sections are included they must model the flow boundaries --not just the flood plain width.

Section 4.03. Hydraulic Roughness Values

Sources of n-Values

Sources for roughness values are found in publications such as references 2 and 27. Roughness values can also be obtained from hydraulically similar streams in the study area for which data is available, or from field measurements of water surface profiles and discharges. Of these sources of data, field measurements are by far the most reliable.

Published data by the U. S. Department of Agriculture and the U. S. Geological Survey provide excellent sources of information for roughness values. When information is not available from these sources the roughness value can be selected from the following table.

Table 4.01.* Coefficient of roughness, average channels

Value of "n"	Channel Condition
0.016-0.017	Smoothest natural earth channels, free from growth, with straight alignment.
0.020	Smooth natural earth channels, free from growth, little curvature.
0.0225	Average, well-constructed, moderate-sized earth channels in good condition.
0.025	Small earth channels in good condition, or large earth channels with some growth on banks or scattered cobbles in bed.
0.030	Earth channels with considerable growth. Natural streams with good alignment, fairly constant section. Large floodway channels, well maintained.
0.035	Earth channels considerably covered with small growth. Cleared but not continuously maintained floodways.
0.040-0.050	Mountain streams in clean loose cobbles. Rivers with variable section and some vegetation growing in banks. Earth channels with thick aquatic growths.
0.060-0.075	Rivers with fairly straight alignment and cross section, badly obstructed by small trees, very little underbrush or aquatic growth.
0.100	Rivers with irregular alignment and cross section, moderately obstructed by small trees and underbrush. Rivers with fairly regular alignment and cross section, heavily obstructed by small trees and underbrush.
0.125	Rivers with irregular alignment and cross section, covered with growth of virgin timber and occasional dense patches of bushes and small trees, some logs and dead fallen trees.
0.150-0.200	Rivers with very irregular alignment and cross section, many roots, trees, bushes, large logs and other drift on bottom, trees continually falling into channel due to bank caving.

* From EM 1110-2-1409 (Reference 20)

Even if the boundary geometry of a river is well defined, n-values used for the river should be considered subject to question unless discharge-stage data are available at two or more locations in the study reach to assist in verifying that the roughness values are correct.

Meaning of n-Values

If water surface elevations and the associated discharges are known, then Manning's "n" can be calculated. However, n-values computed in this manner reflect more than just friction loss even though separate evaluations are made for other losses such as contraction, expansion and bend losses. In alluvial channels, n-values from calibration studies incorporate the effects of changes in bed form and in cross section area due to scour and deposition. Both water temperature and sediment load affect bed forms. In overbanks, n-values account for energy losses due to trees and growth, where actually a form loss is involved. Finally, n-values will vary from one computation scheme to the next and should be calibrated accordingly.

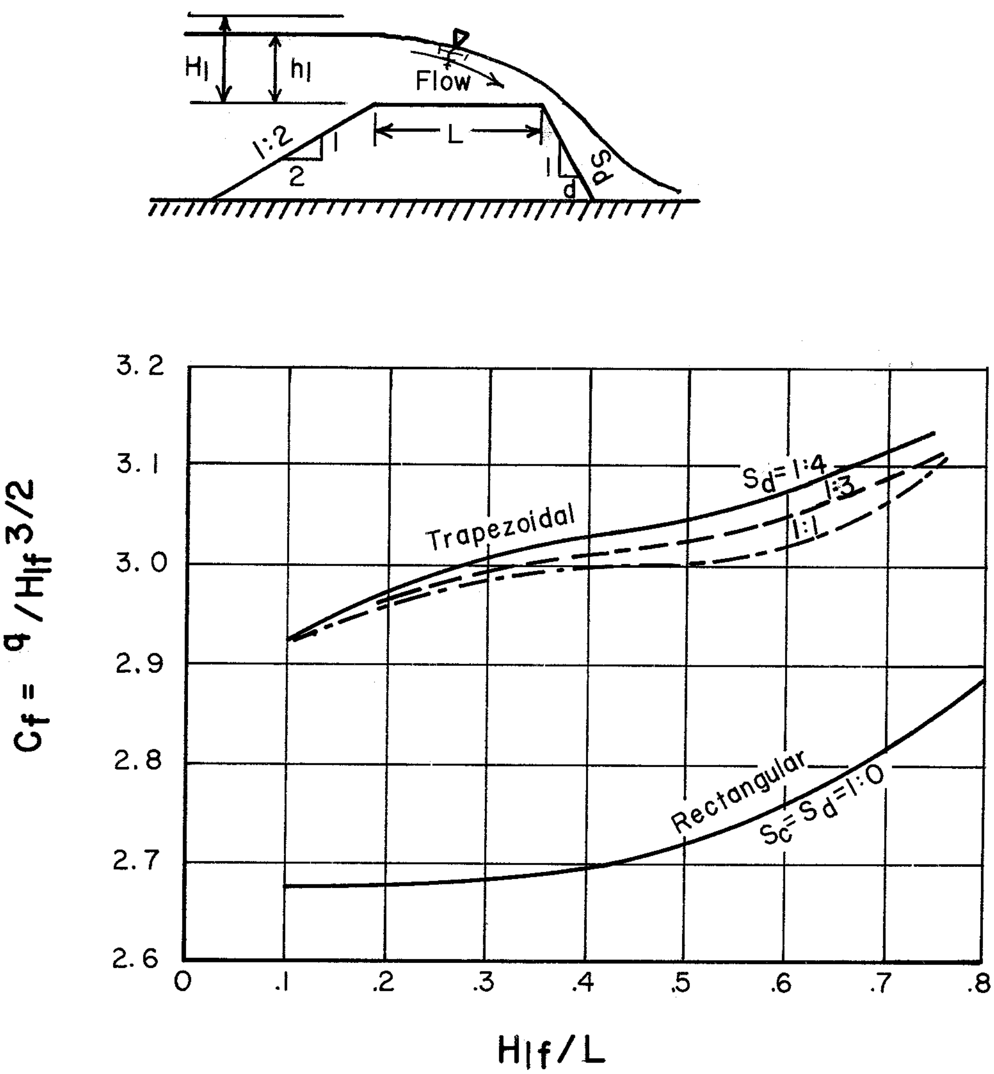
It is particularly important to appreciate the meaning of n-values when evaluating flood profiles through urban areas. In determining n-values through residential areas, it is desirable to assign n-values to streets, lawns, and sidewalks and to deduct the space occupied by buildings from available cross-sectional area. The same net effect can be accomplished by manipulating n-values alone. For example, if a cross section in a residential area is about 50 percent flow area and 50 percent obstructed by buildings, increasing n-values by a factor of 2 will produce the same friction loss as dividing the cross sectional area by 2 to account for space occupied by buildings. Increasing n values is the most convenient approach and is often used.

Section 4.04. Bridge Loss Coefficients

Basic data for bridge loss coefficients and the corresponding equations are contained in Appendix 2 of Volume 1. Since loss coefficients are empirical they must always be associated with the equation used in their determination.

Section 4.05. Weir Coefficients

Flow over weirs is divided into two types: free flow where the downstream water surface (tailwater) elevation does not influence the upstream water surface elevation, and submerged flow where the upstream water surface elevation is influenced by tailwater depth. References 7, 9, 11, 15, 16, 21 and 32 contain coefficients for free flow over sharp-crested and broad-crested weirs. References 9 and 21 contain coefficients for submerged flow over ogee weirs and highway embankment-shaped weirs, respectively. The following figures (from reference 15) give additional information on the discharge coefficients for broad-crested weirs having variable downstream slopes and crest widths.



$H_l = h_l + v_l^2/2g$

$H_{lf} = H_l$ for free flow conditions

Fig. 4.02. Effect of weir geometry on free flow discharge coefficients

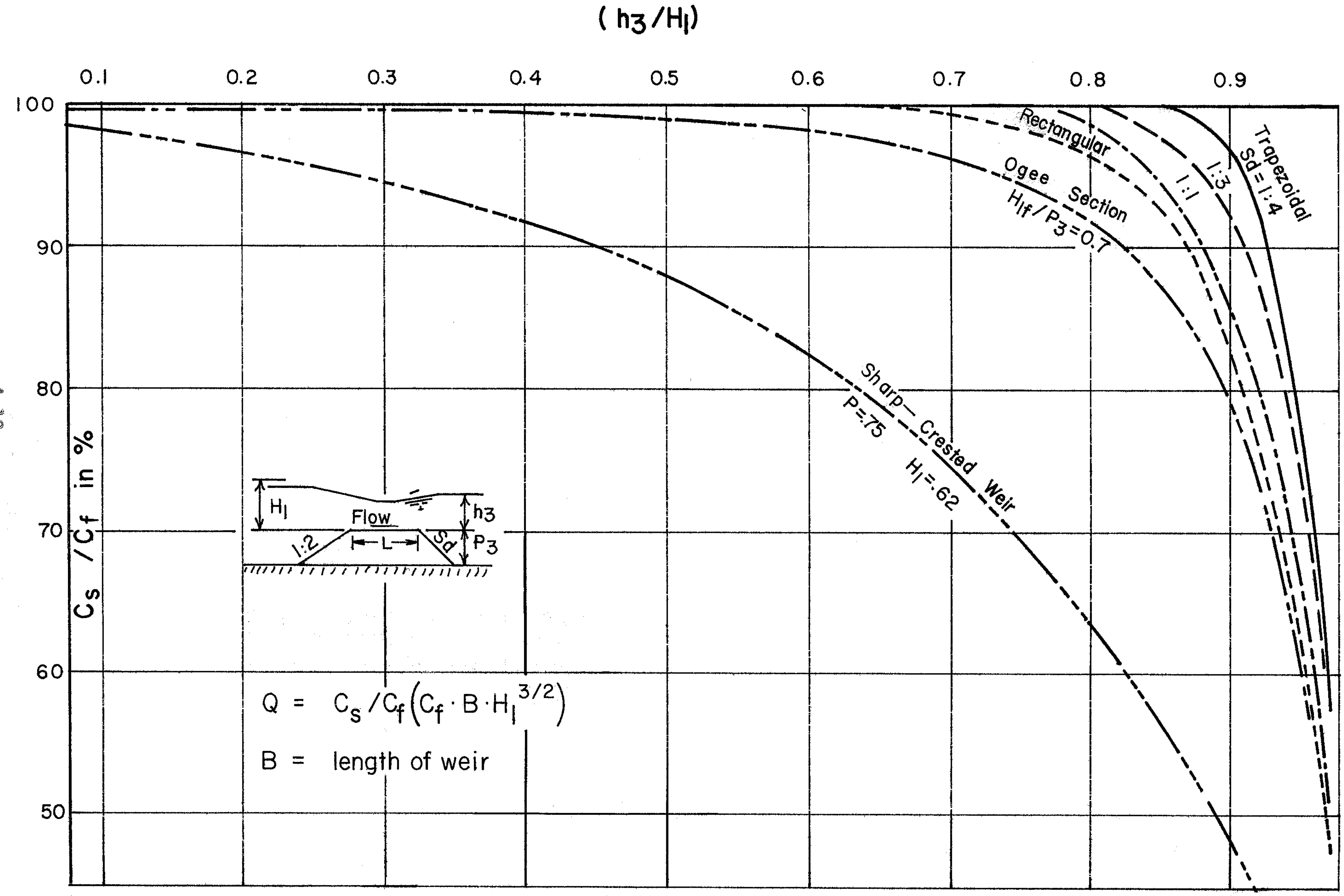
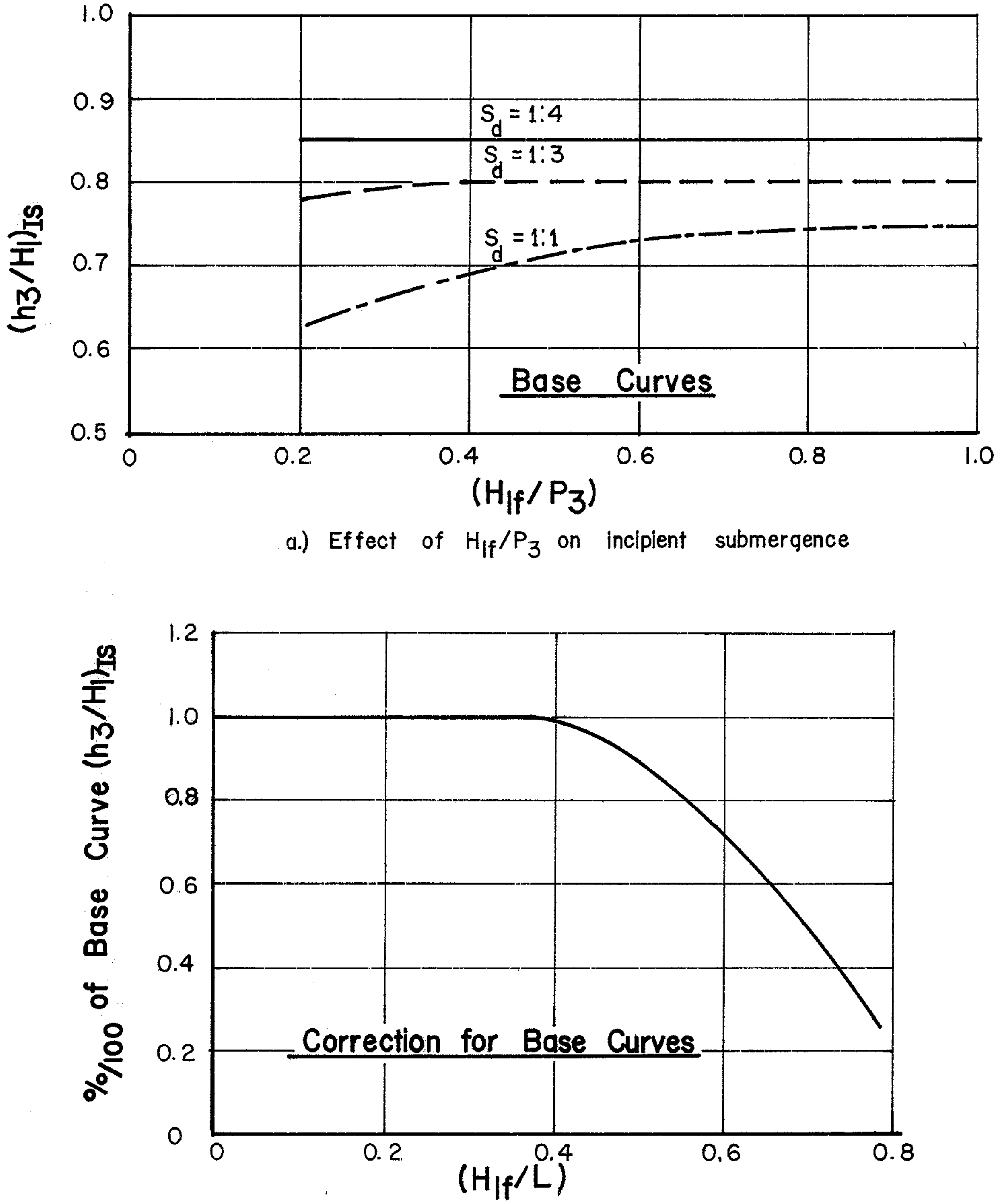


Fig. 4.03. Effect of weir geometry on submerged flow discharge coefficients



a.) Effect of H_{1f}/P_3 on incipient submergence

b.) Effect of L on incipient submergence

Fig. 4.04. Incipient submergence

The free flow discharge coefficients are shown in fig. 4.02 for four basic weir shapes. Figure 4.03 shows the effect of submergence on these four shapes as well as the effect on an ogee-shaped weir and a sharp-crested weir. Figure 4.04 shows the influence of shape and flow on the point of incipient submergence (i.e., that point below which changes in downstream water level have no influence on water elevation upstream from the weir).

The weir equation is discussed in Section 5.04. The coefficients in figures 4.02 and 4.03 were calculated from hydraulic model tests in which the upstream and downstream gages were 1.5 feet (0.5 m) and 7 feet (2.1 m) respectively from the weir. The flume was 18 inches (0.5 m) wide and could support a flow of 1.5 cfs/ft (140 liter/sec/meter). Weir crest lengths of 12, 18, and 24 inches (30.5, 45.7, and 61 cm) were tested. The weir crest was 9 inches (22.9 cm) above the approach channel and 7-3/4 inches (19.7 cm) above the exit channel.

These coefficients were not determined for the purpose of calculating discharge from measured field data, but rather for calculating head loss for a range of discharges, weirs, and flow conditions.

As shown in fig. 4.02, weir shape is as important in submerged flow as previous studies have shown it to be for free-flow conditions. It is not wise to use the coefficient for one shape of weir in analyzing head loss at a weir of another shape.

Section 4.06. Contraction and Expansion Coefficients

Fluid mechanics texts treat losses due to contractions and expansions as "minor losses" that can be evaluated by multiplying the difference in velocity heads due to the contraction or expansion by a loss coefficient. When such losses are considered in water surface profile calculations for irregular streams, coefficients typically assigned are 0.1 for contraction and 0.3 for expansion. In general, however, the values are based on the amount of contraction or expansion. In no case should these loss coefficients be less than zero or greater than 1.0. For manmade canals of uniform cross section, these coefficients are normally set to 0.

Section 4.07. Rating Curves

A rating curve defines a relationship between discharge and water surface elevation at a given location. The curve may be unique as in the case of a weir; or parametric as in the case of a stream entering another stream or a regulated reservoir. So far as the energy equation is concerned, the discharge and starting water surface elevation can be completely independent from each other, however, the physical characteristics of the river will govern the relationship that actually exists. The term "control" is used to describe a location where there is physical dependence between water surface elevation and discharge, and profile calculations generally must start at a "control cross section."

In alluvial rivers, stage-discharge measurements usually do not plot along a single line, but exhibit a wide scatter of points. This scatter does not represent measurement error but rather distinct trends in the behavior of the river. In positioning a line through the scatter of points, the line should generally be positioned high for levee grade flow lines and low for navigation depth requirements. Other requirements might require other positions of the line.

Measured data often do not extend high enough or low enough to bracket the full range of discharges, in which case an extrapolation technique is necessary. One approach is to plot log-stage versus log-discharge and extrapolate the resulting relationship beyond the experienced range since, often times, rating curves will plot as straight lines or as lines which are piecewise-straight. Another technique is to calculate a representative energy slope from the experienced data and use this slope, cross section information, and n-values, in a uniform flow equation such as the Manning equation to calculate points on the rating curve beyond the range of experienced data. This technique depends a great deal on the representative slope to give reliable results.

Section 4.08. Obtaining Field Data

Traditionally, survey parties have gone into the field to obtain the needed geometric data. Levels and taped distances were sometimes used, but the alidade, plane table and stadia measured distances were more popular since the resulting data could be plotted while being recorded in the field.

The more modern techniques utilize sonar sounding equipment for defining the geometry under the water by recording channel measurements on a digital tape and a strip chart as a boat moves across the stream; the boat is kept on course by following a laser beam. The resultant data is handled by computer. Aerial photography is being used to develop topographic maps of the flood plain on either side of the stream. Cross-section positions are established and data cards are punched automatically by digitizers operating in conjunction with plotting equipment. Aerial and ground surface photographs are used to establish characteristics of bridges and for determining roughness coefficients for the flow.

It is more desirable to obtain topography from a single source than to attempt to match up results of various surveys. With a single source, all elevations will have a common datum. Therefore, relative to each other, locations in the vertical are correct. Of course, it is most desirable to reference all vertical control to mean sea level, but sparse data often prevents it.

Another useful type of field data is observed water surface profiles and corresponding discharges. These data are needed to verify that the geometric model and n-values are correct. Without field measured events for verification, the calculated water surface profiles must be viewed with some uncertainty.

Section 4.09. Minimum Data Requirements

While it is desirable to have all of the data mentioned in the previous section, preliminary estimates can be made with a minimum amount of data. For example, cross sections may be taken by a hand level and bridge data may be collected with a measuring tape. Estimated roughness coefficients can be obtained by visually comparing the natural ground cover with calibrated photographs from standard textbooks. Topographic maps can sometimes be used to obtain cross sections where field data are not available. However, such information should be verified by spot checking a few elevations in the field.

Section 4.10. Preparing Field Data for Use

Natural channels are usually irregularly shaped, and consequently the required geometric properties of area and wetted perimeter are not easily expressed. The first step is to plot each cross section in graphic form. It is usually convenient to establish some basic policy to follow every time field data is collected. This permits the bits and pieces of one or several field surveys to be fitted together easily and without a major effort. For example, it is convenient to establish a common scale to which all cross sections will be plotted, and if cross-section sheets are to be filed in flat files, the size of paper available, and therefore the plotting scale, is determined by the length and width of the drawer. However, it is essential that the selected size be convenient to use.

It is also good practice to establish criteria for plotting data. A useful procedure is to plot all cross sections from left to right facing downstream, have the upstream cross section at the bottom of the paper, and have the cross sections plotted in sequence with the most downstream section located at the top of the paper. This gives the sensation of standing and looking in the downstream direction, and is a valuable aid in perceiving the 3-dimensional boundary geometry that must be described digitally or analytically.

Chapter 5

Application of the One-Dimensional Energy Equation to Natural Rivers

CHAPTER 5. APPLICATION OF THE ONE-DIMENSIONAL ENERGY EQUATION TO NATURAL RIVERS

Section 5.01. Introduction

The one-dimensional energy equation and supporting functional relationships and constraints were introduced in Chapter 2. They are summarized below:

$$WS_2 + \frac{\alpha_2 Q^2}{2gA_2^2} = WS_1 + \frac{\alpha_1 Q^2}{2gA_1^2} + H_L \quad \text{energy equation} \qquad (5\text{-}1)$$

A_1 = f(WS_1 and cross section geometry) (5-2)

A_2 = f(WS_2 and cross section geometry) (5-3)

H_L = friction and other losses (5-4)

Q = independent variable (usually) (5-5)

WS_1 = f(Q) (5-6)

WS_2 = dependent variable (usually); constrained by critical depth (5-7)

α_1 = f(WS_1, geometry and hydraulic roughness) (5-8a)

α_2 = f(WS_2, geometry and hydraulic roughness) (5-8b)

The variables A, K and α are non-linear, and often discontinuous functions of WS. As a result, equations (5-1) through (5-8b) are usually solved by the standard step method - a computation method which uses successive approximations. Cross sections are located to describe the boundary geometry confining the flow.

There are many ways to formulate a solution technique which employs the standard step method. The primary objective, however, is to minimize the time required to solve the set of equations subject to maintaining an acceptable accuracy in the calculated result. This requires a maximum spacing between cross sections and a minimum number of iterations for converging the trial and calculated water surface elevations. A second objective is to minimize the time required to prepare data describing the various functional relationships. It is necessary to formulate a solution technique that can analyze the entire range of possible discharges with only one set of cross sectional data. In some cases all flow will be conveyed in the main channel, while in others both channel and flood plain areas will convey flow.

The following sections in this Chapter discuss the requirements for determining representative values of area, wetted perimeter, hydraulic roughness, reach length and velocity for the above flow conditions and will present techniques to accomplish these objectives. Each of the functional relationships (5-2) through (5-8) is discussed.

Section 5.02. Averages from Distributed Parameters

The primary consideration in applying one-dimensional theory to the calculation of water surface profiles in natural rivers involves converting hydraulic parameters, which are distributed across complex sections, into averages. In applying the energy equation, 5-1, flow velocity is required. The flow velocity must be a representative velocity for each cross section. Both friction loss and the contraction and expansion losses are calculated in terms of this representative velocity. The following figure illustrates a case where the average velocity in the cross section is not sufficiently representative to serve in the energy calculations. Consider the distribution of discharge and flow area shown.

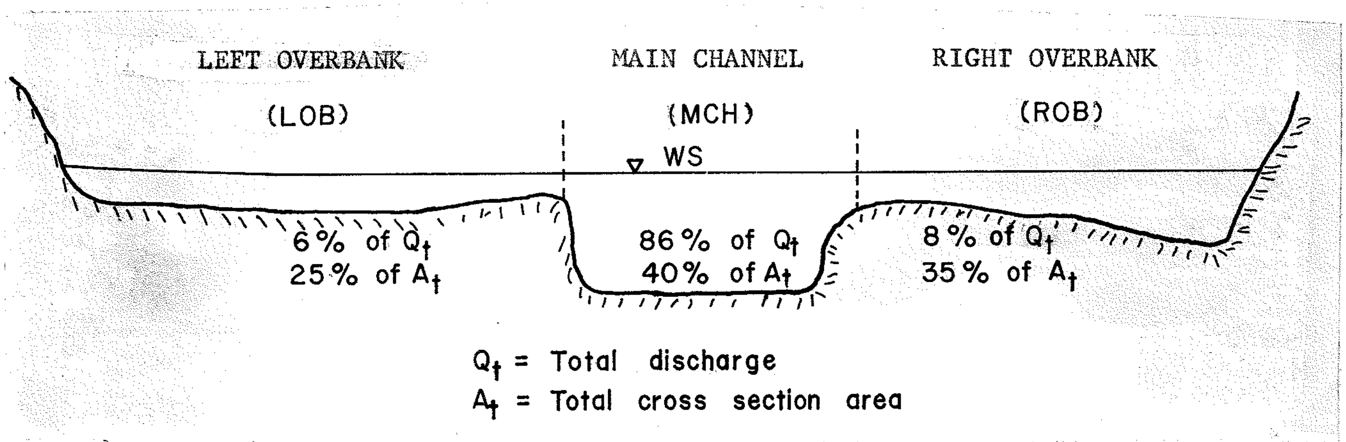


Fig. 5.01. Distribution of discharge and flow area at a cross section

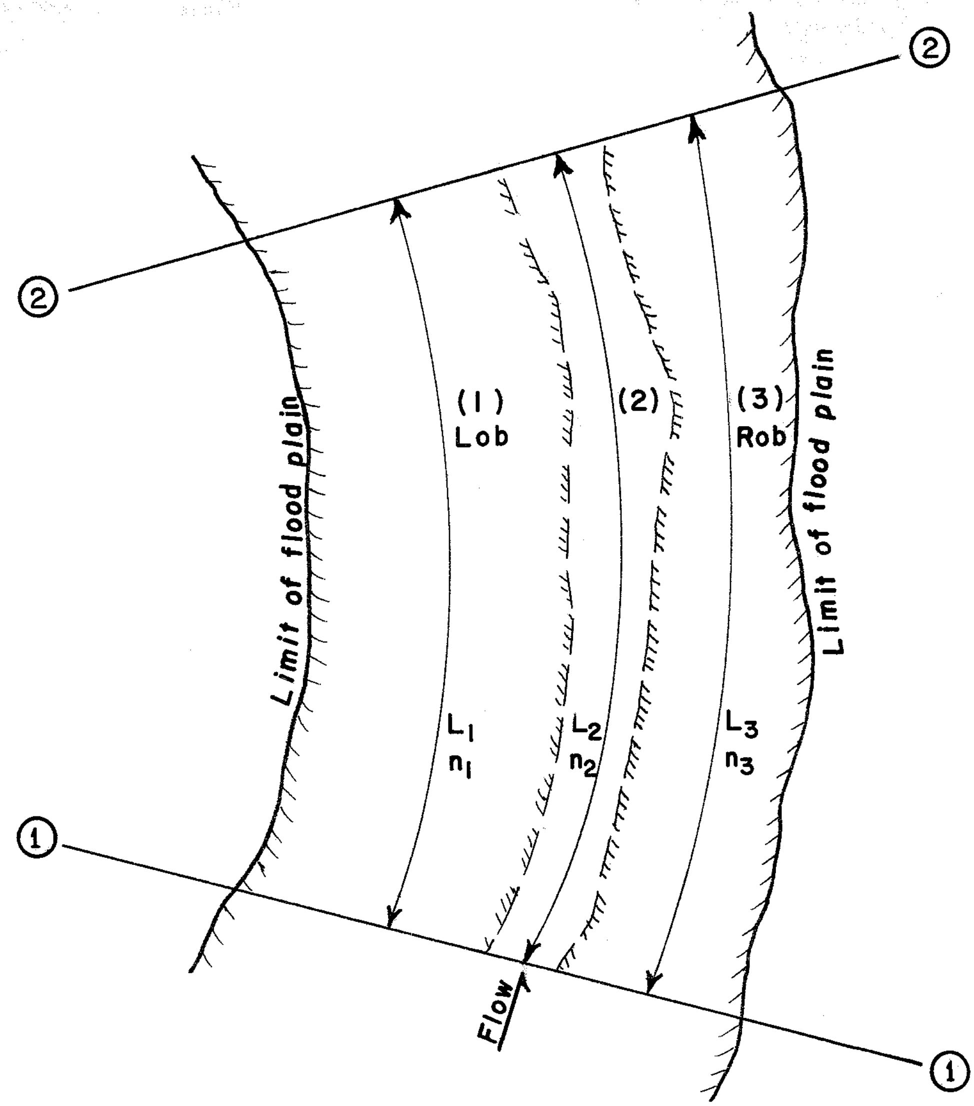
The average velocity at this cross section V_A by definition, is equal to Q_t/A_t. However, the main channel conveys 86 percent of the total discharge and therefore has an average channel velocity of $2.15 \cdot V_A$. Neither the average velocity nor the average channel velocity is sufficiently representative to be used for calculations for this case. Moreover, the percent of total flow conveyed by the channel changes with discharge. The velocity distribution factor, α, is used to overcome this problem.

Another important distributed property is the distance between cross sections. Except in the very special case of a straight channel, cross sections should not all be parallel to each other. It is important to have a solution technique that can calculate a representative reach length in order to insure that one set of cross sectional data can be used for the entire range of discharges.

Hydraulic roughness values, for example Manning's n-values, are a third important property that can vary greatly across a section. Manning's n-values also often vary in the vertical. A general solution technique for solving the energy equation must accommodate such variations.

All of these requirements can be satisfied by subdividing the flood plain into strips having similar hydraulic properties in the direction of flow. Figure 5.02 illustrates this concept with three strips: (1) = left overbank, (2) = main channel and (3) = right overbank.

In this figure the left overbank strip begins at the limits of the flood plain on the left and extends to the top of the bank on the left side of the main channel. The right overbank strip begins at the top of the right bank of the main channel and extends to the limits of flood plain on the right. All flow is conveyed within the boundary geometry defined by these three strips. The boundary between strips, however, is hypothetical, and flow is permitted to pass freely from one strip to another without a penalty for energy losses or an award for energy transfer. That is, both the water surface elevation and total energy elevation are assumed to be constant across the cross section.



5.02 Distribution of reach lengths and n-values

The strips subdivide the cross sections into portions called subsections. These subsections form the basic framework from which distributed properties are converted to averages for use in the energy equation. In the general case, three strips or subsections are not enough. For example, seven or more subsections would be required to identify a main channel, a dike field along both sides of the main channel, a strip of rather dense growth immediately adjacent to the channel on both banks and one or more strips of less dense growth or cleared fields on both flood plains.

Section 5.03. Discharge and Starting Water Surface Elevation

In open channel flow problems, any term, either in the one-dimensional energy equation or the other independent equations, can become the independent variable. However, Q is usually the independent variable. The associated starting water surface elevation, WS_1, is often obtained either from a rating curve for a "control" relationship or by some other criteria such as a rule curve specifying reservoir elevation at a dam.

If a rating curve is not available it is desirable to locate, if possible, a critical depth control section. This is a location where flows are known to pass through critical depth for the full range of discharges being analyzed--for example, a waterfall, weir or section of rapids in the river.

If a critical depth control section is not available, then select a starting point some distance away from the location where the rating curve is needed and calculate the water surface profiles for a range of starting elevations. These profiles will converge toward a common point which is the rating curve elevation for that discharge. Repeating this procedure for the range of discharges to be analyzed will permit a rating curve to be developed.

If the required location for the rating curve is in a long straight reach of the river, a procedure known as the slope-area method will give satisfactory results. The key to successful application of the slope area method is to supply the proper slope data. Before accepting the calculated rating curve, use it to calculate water surface profiles for a range of discharges and plot these profiles along with channel bed, top bank, etc. They should all be about parallel to the average slope of the channel in this location. If this test fails, the calculations and data used in slope-area method should be re-evaluated or, perhaps, a better site should be selected for establishing the curve.

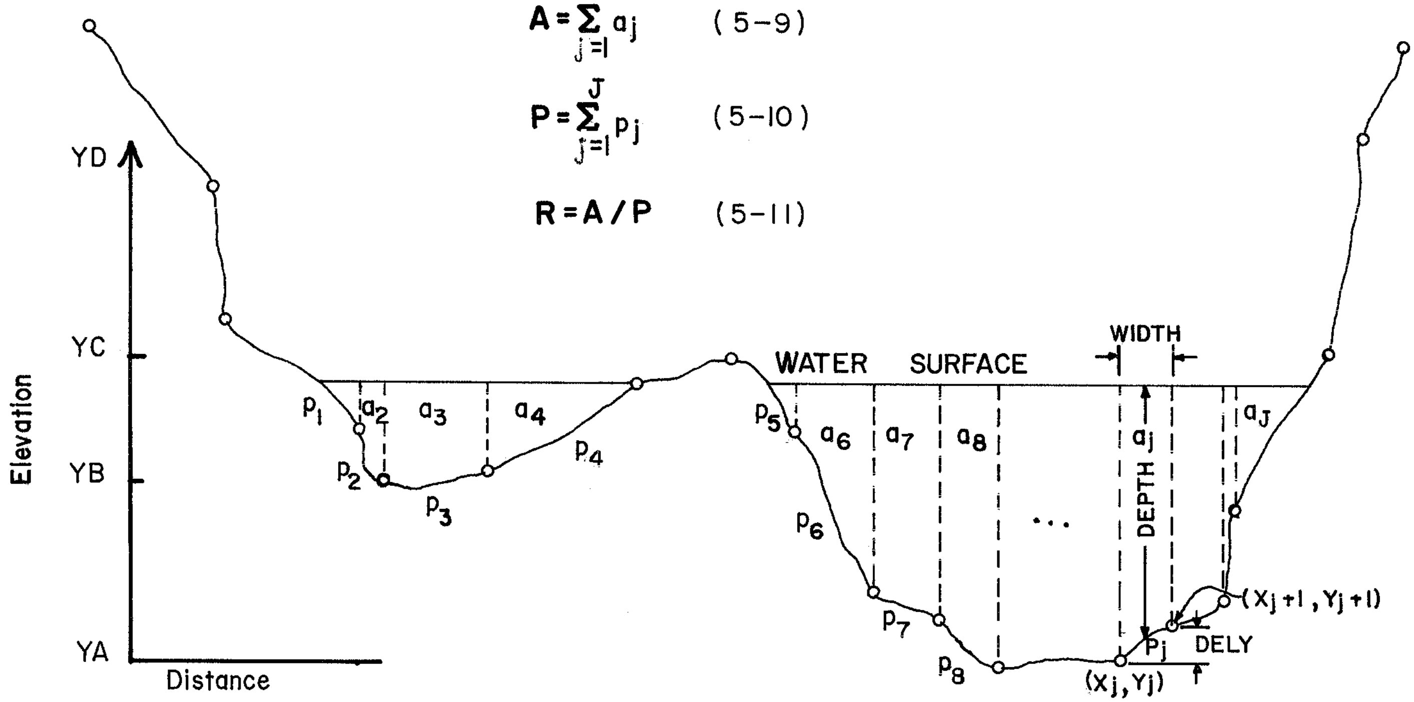
Section 5.04. Geometric Properties and Conveyance

Geometric properties required for profile calculation are cross sectional area, wetted perimeter and hydraulic radius. These properties can be determined directly from the cross section once the water surface elevation has been specified. In some cases these data are calculated for a range of elevations and tabulated or plotted in graphical form before the water surface profile calculations begin. In all cases a horizontal water surface elevation is assumed. The procedure is illustrated in figure 5.03 for one water surface elevation. Equations for area, wetted perimeter and hydraulic radius are shown on the figure. Calculations are made for each subsection in the cross section in a similar manner. Subsection values are saved for use in calculating total area and conveyance for the cross section.

$$A = \sum_{j=1}^{J} a_j \qquad (5\text{-}9)$$

$$P = \sum_{j=1}^{J} p_j \qquad (5\text{-}10)$$

$$R = A/P \qquad (5\text{-}11)$$



Legend

- A = Accumulated area of all trapezoids
- a_j = Area of j^{th} trapezoid = WIDTH · DEPTH
- DELY = $Y_{j+1} - Y_j$
- DEPTH = Average depth
- J = Total number of trapezoids
- P = Accumulated wetted perimeter of all trapezoids
- P_j = Wetted perimeter of j^{th} trapezoid = $\sqrt{\text{WIDTH}^2 + \text{DELY}^2}$
- WIDTH = $X_{j+1} - X_j$

Fig. 5.03. Area and wetted perimeter of a river channel subsection

When the water surface profile calculations are to be made with an electronic computer, it is most convenient to calculate geometric properties directly from cross section coordinates as each new trial water surface is analyzed. A less desirable approach is to interpolate values from a table because of the necessity of generating new tables for each change in cross section shape. The least desirable approach is to curve fit geometric properties versus elevation because discontinuities are common.

For example, the area-elevation curve for the subsection in fig. 5.03 would be smooth because flow is shown in areas a_1, a_2, a_3, and a_4 when the WS is lower than YC in the main channel. However, cases frequently occur where no flow can enter those areas until the WS becomes higher than YC. When this occurs, the resulting area-elevation curve would be modified as shown in fig. 5.04. From YA to YB all area is usable and a smooth curve results. Between YB and YC the amount of usable area is determined by whether water can physically enter all areas before the WS exceeds YC. If water can enter areas a_1, a_2, a_3, and a_4 the curve is smooth, but its slope changes at point YB. However, if flow cannot get into areas a_1, a_2, a_3, and a_4 until WS exceeds YC, a discontinuity results at WS = YC. This is difficult to account for in programming the computer to utilize curves.

However, when the water surface profiles are to be calculated manually, curves of geometric properties are most desirable. These are defined by selecting elevations at the major break points in the

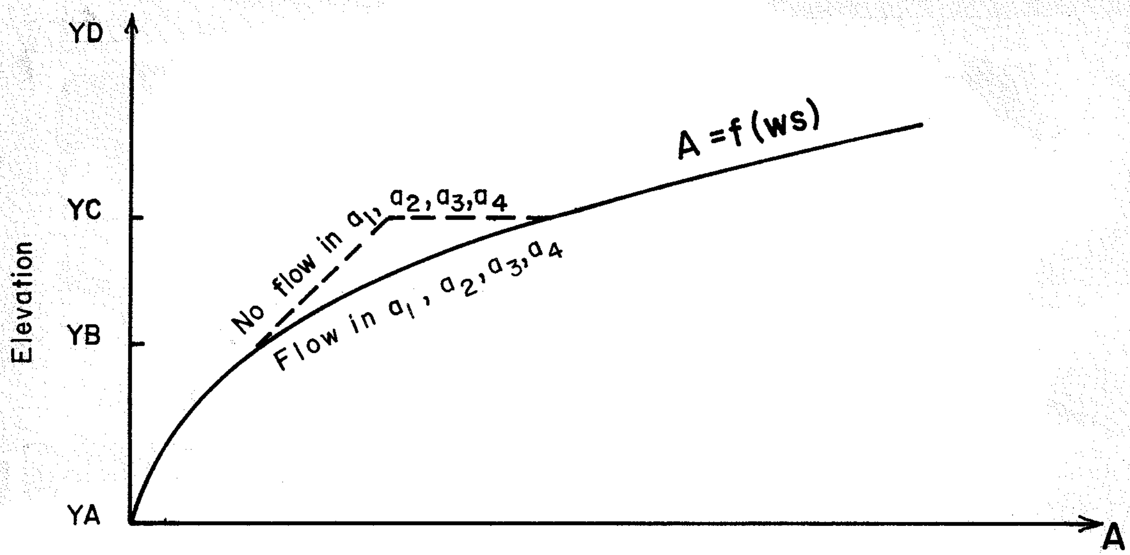


Fig. 5.04. The area-elevation relationship

cross section and then calculating the area for each subsection as illustrated above. Wetted perimeter is usually not calculated. Rather the hydraulic radius is assumed equal to the depth and a combined value of $a_j \cdot DEPTH_j^{2/3}$ is calculated for each trapezoid and the result summed over each subsection to obtain a value to be used in the place of $A \cdot R^{2/3}$ in the Manning equation.

The concept of "conveyance" is associated with the geometric and hydraulic roughness terms in the Manning Equation.

$$Q = \frac{Cm}{n} \cdot AR^{2/3} \cdot S_f^{1/2} \qquad (5\text{-}12)$$

In terms of notation in this volume:

$$Q = K_t S_f^{1/2} \qquad (5\text{-}13)$$

where

$$K_t = Cm \cdot \sum_{i=1}^{I} \frac{A_i R_i^{2/3}}{n_i} \qquad (5\text{-}14)$$

Q = total water discharge in ft^3/sec or m^3/sec

Cm = units conversion coefficient and is 1.486 for English units or 1.0 for metric units

n = Manning's hydraulic roughness value

A = cross sectional area in ft^2 or m^2

R = hydraulic radius in ft or m

S_f = total friction slope

K_t = total conveyance as defined by equation (5-14)

I = total number of subsections

i = an individual subsection such as left overbank, channel etc.

When using manual computations, curves of conveyance are often plotted for each subsection and used in the place of area and hydraulic radius curves. This decreases the amount of computations required for the water surface profile. The disadvantage of this technique is that n-values are often changed during a study which introduces a correction factor for the conveyance curves. The factor can vary from section to section, and with elevation or discharge, which makes application of the factor somewhat awkward.

Section 5.05. The Velocity Distribution Factor

Textbooks on open channel flow often associate the velocity distribution factor, α, with the shape of the velocity profile in the vertical direction. Typical values range from 1.03 to 1.36 for straight prismatic channels. This is insignificant relative to all the other uncertainties in calculating water surface profiles. However, as illustrated in fig. 5.01, the horizontal distribution of velocity in complex cross sections is quite a different problem and can result in α values greater than 2 or even 20. Analytical procedures account for this horizontal distribution by utilizing the following equation for the velocity distribution factor.

$$\alpha = \frac{\int v^2 dQ}{\bar{V}^2 Q} \qquad (5\text{-}15)$$

where

dQ = discharge associated with v

Q = total discharge

$\bar{V}$ = average velocity for cross section

v = point velocity

Applying this equation to the horizontal direction requires knowledge about the horizontal distribution of discharge which can be inferred from the horizontal distribution of conveyance as follows:

$$\alpha = \frac{\sum_{i=1}^{I} K_i (K_i/A_i)^2}{K_t (K_t/A_t)^2} \qquad (5\text{-}16)$$

where:

K_i = conveyance of the i^{th} subsection

A_i = area of the i^{th} subsection

I = total number of subsections in this cross section

t = subscript denoting sum of either K or A for this cross section

Section 5.06. Energy Loss Equations

Evaluating head loss in open channel flow requires techniques that accommodate energy converted by friction loss, form loss, change in state of flow from supercritical to subcritical through a hydraulic jump, and an abrupt change in potential energy as through a waterfall. The last case is easily accommodated since a critical depth control exists. That is, for computations to proceed in the upstream direction through a waterfall, they are restarted at the brink of the waterfall using critical depth theory.

Provided that the mode of conveyance of flow approaching the jump is free surface rather than pressure flow, the hydraulic jump may not have to be treated in detail since jump height is controlled by the downstream water surface elevation. (Note: The problem being addressed here is quite different from that confronting the hydraulic design engineer who must create a jump and stabilize its location.) The momentum equation can be used to determine the location of the jump, all that is required to continue a water surface profile calculation past a hydraulic jump is to recognize its presence and restart calculations at the next control upstream from it. It is important to recognize that the solution technique presented here for the one-dimensional energy equation, which is sometimes referred to as the standard step method, is directionally dependent upon the state of flow. That is, computations should proceed in the upstream direction for subcritical flow and in the downstream direction for supercritical flow.

If the mode of conveyance of flow approaching the jump is pressure flow, critical depth may or may not exist at the control. For example, the flow may pass through a culvert or the outlet works of a dam in which case friction and form losses must be calculated to establish the control relationship for restarting calculations. The computer program "Water Surface Profiles," which is presented in Volume 1 of this series, accomplishes this analysis automatically to insure that calculations can proceed uninterrupted.

Section 5.07. Friction Loss

The energy dissipated to overcome friction is usually calculated with the Manning equation, 5-12. It is necessary to transform the equation somewhat since in gradually varied flow the friction slope, S_f, varies in the direction of flow.

$$h_f = \int_0^L S_f dx \qquad (5\text{-}17)$$

where:

$$S_f = \left[\frac{Q}{\frac{Cm}{n} AR^{2/3}} \right]^2 \qquad (5\text{-}18)$$

A = cross sectional area

Cm = 1.486 for English units and 1.0 for metric units

h_f = head loss due to friction

L = reach length

n = Manning's n-value

Q = total discharge

R = hydraulic radius

S_f = friction slope

dx = incremental distance in direction of flow

Techniques for approximating 5-17 include:

$$h_f = 0.5 \cdot L \cdot (S_{f_1} + S_{f_2}) \qquad (5\text{-}19)$$

or $$h_f = L \cdot \sqrt{S_{f_1} \cdot S_{f_2}} \qquad (5\text{-}20)$$

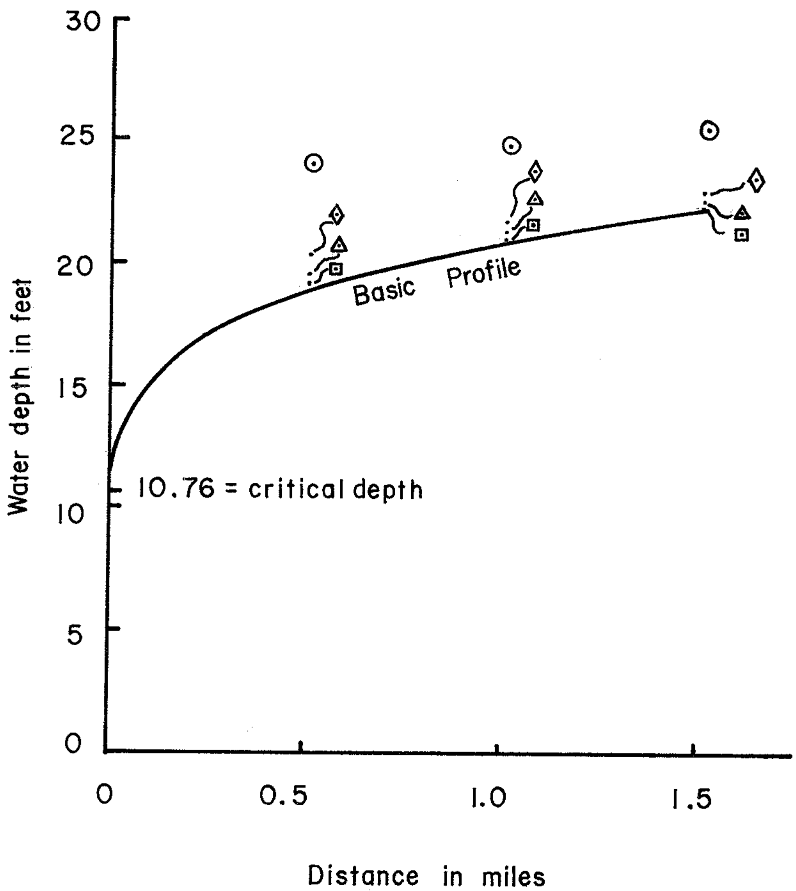
or $$h_f = L \cdot \left[\frac{Q \cdot n}{Cm\ 0.5 \cdot (A_1 + A_2)\ (R_1 + R_2)^{2/3}} \right]^2 \qquad (5\text{-}21)$$

or $$h_f = L \cdot \left[\frac{2 \cdot Q}{K_1 + K_2} \right]^2 \qquad (5\text{-}22)$$

where subscripts 1 and 2 refer to the downstream and the upstream cross sections, respectfully; K, is conveyance and L is the distance between sections. Experience indicates that equation 5-19 requires the closest spacing of cross sections and that equation 5-22 permits the longest spacing when the water surface is of the M2, H2 or A2 type. This experience is supported by the following numerical experiment.

Figure 5.05 shows a water surface profile for flow in a geometric model that was defined by repeating the cross section of fig. 5.06 at closely spaced intervals for a total distance of 1-1/2 miles and on a zero bottom slope. The profile thus obtained is essentially equivalent to one that would be obtained by integrating equation 5-17. The starting water surface elevation is at critical depth. Of particular interest is the friction slope profile which is plotted in fig. 5.07a. This friction slope profile corresponds to the basic water surface profile of fig. 5.05. According to equation 5-17, the head loss due to friction between two cross sections in a stream is the area under the friction slope profile between those two sections. For very short reach lengths equations 5-19 through 5-22 all serve equally well to approximate that area. However, in seeking a technique that will minimize the amount of computations required to solve water surface profile problems, it is desireable to use long reach lengths. Suppose equation 5-19 were applied to a reach between $x_1 = 0$ and $x_2 = 0.5$ miles (.8 km) in fig. 5.07a. It would approximate the true friction loss with an area beneath the straight line connecting those distance points on the friction slope profile. The resulting water surface profile is shown as circles in fig. 5.05. It is grossly in error because equation 5-19 overestimated the area under the friction slope curve.

Water surface elevations calculated with equations 5-20, 5-21, and 5-22 are also shown in fig. 5.05. All three of these equations



Legend

- ⊙ Eq. 5.19
- ◊ 5.20
- △ 5.21
- ⊡ 5.22

Fig. 5.05. Water surface profiles

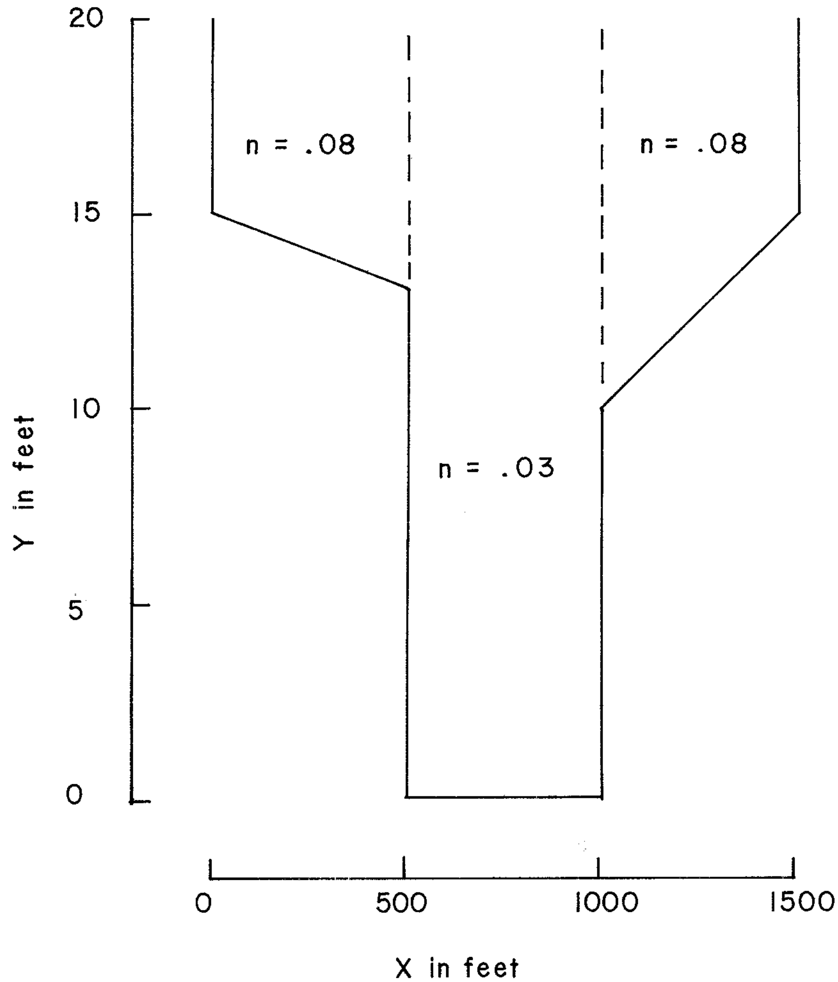
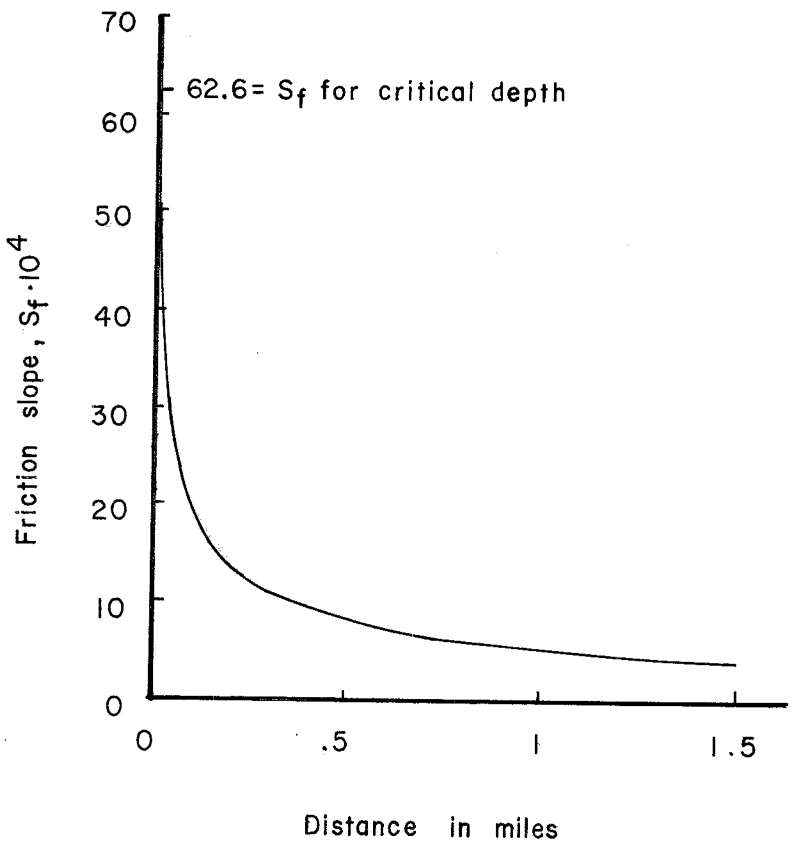
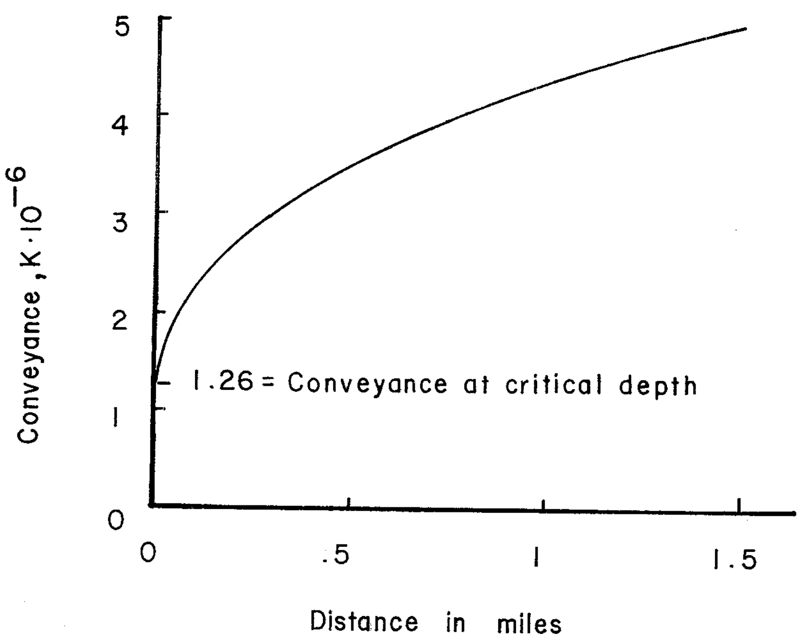


Fig. 5.06. Cross section



a. Friction slope profile



b. Conveyance profile

Fig. 5.07. Friction slope and conveyance profiles

produce elevations closer to the base profile than those calculated by equation 5-19. The best results for this case were obtained with equation 5-22 which utilizes average conveyance. Figure 5.07b shows a plot conveyance, K, versus distance along the channel. The rate of change of the S_f curve is 2 • K times the slope of the K curve because S_f is proportional to K^2. This causes more error in linear interpolation on the S_f curve than on a corresponding segment of the K curve. Equation 5-22 is not presented as "the best" technique because this example considers only one type of water surface profile and one state of flow (see fig. 3.03). In fact, equation 5-22 is less satisfactory for M1 profiles than the average-slope method of equation 5-19. However, the amount of error introduced for the M1 profile is not as sensitive to solution technique as that illustrated previously for the H2 profile. Moreover, the water surface elevation will be on the low side which is bounded by critical depth. Attention will be drawn to calculations which go below critical depth, and corrective action can be taken if deemed important.

Section 5.08. Relationship Between Manning's n, Chezy's C and Darcy's f

Three flow resistance equations are shown below in their usual forms.

$$H_L = \frac{f}{4} \frac{L}{R} \frac{V^2}{2g} \qquad \text{(Darcy)} \qquad (5\text{-}23)$$

$$V = C\sqrt{RS} \qquad \text{(Chezy)} \qquad (5\text{-}24)$$

$$V = \frac{Cm}{n} R^{2/3} \sqrt{S} \qquad \text{(Manning)} \qquad (5\text{-}25)$$

The Darcy and Manning equations may be rearranged to yield

$$V = \sqrt{2g \cdot \frac{4}{f} \cdot \frac{H_L}{L} \cdot R} \qquad \text{(Darcy)} \qquad (5\text{-}26)$$

$$V = \sqrt{2g \cdot 4/f} \cdot \sqrt{RS} \qquad (5\text{-}27)$$

$$V = \frac{Cm}{n} R^{1/6} \cdot \sqrt{RS} \qquad \text{(Manning)} \qquad (5\text{-}28)$$

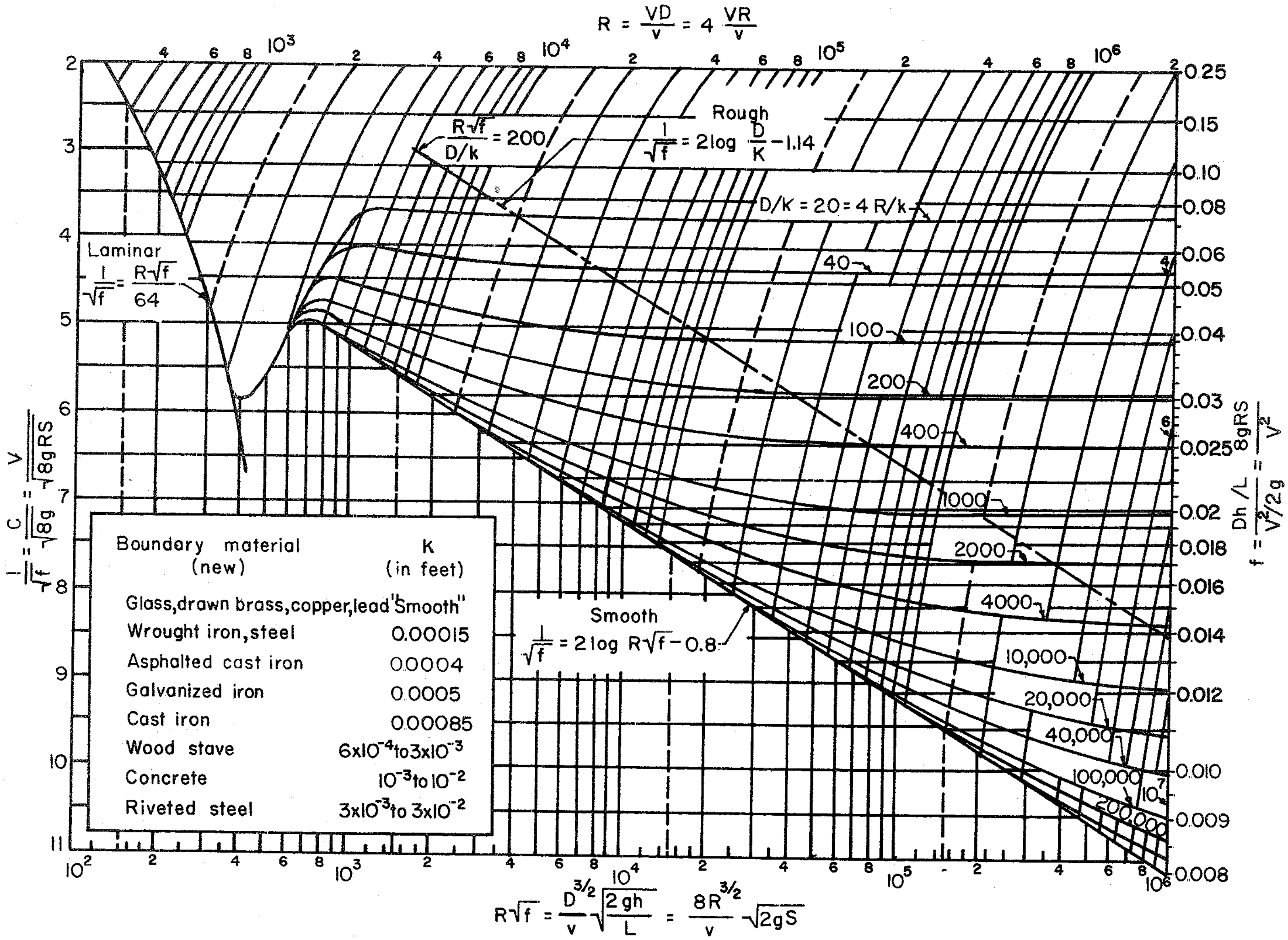
Therefore:

$$C = \sqrt{2g \cdot 4/f} = \frac{Cm}{n} \cdot R^{1/6} \qquad (5\text{-}29)$$

This relationship is useful for relating C or n to f so the influence of Reynolds number on flow resistance can be evaluted from a Moody (or similiar) diagram (fig. 5.08). In most instances, flow in natural streams is well within the fully turbulent range and Reynolds number does not affect n, but exceptions sometimes occur.

Section 5.09. Form Losses

The final energy loss equations to be discussed are those which deal with form losses (eddy losses or shock losses). The natural expansion and contraction in the width of a river causes a form loss. Other examples are bridge piers, trees, buildings, weirs or embankments that confine the flow to a smaller width than it would occupy otherwise. Form loss coefficients are introduced to permit one-dimensional theory to approximate the energy loss in these two and three-dimensional flows. For example, contraction and expansion losses are calculated



MINOR LOSS COEFFICIENTS

90° Elbow	r/D	1.0	1.5	2.0	3.0	4.0
	C_L	0.40	0.32	0.27	0.22	0.20
Abrupt Contraction	D_2/D_1	0.8	0.6	0.4	0.2	0
	C_L	0.13	0.28	0.38	0.45	0.50
Abrupt Enlargement	D_1/D_2	0.8	0.6	0.4	0.2	0
$C_L = [1-(\frac{D_1}{D_2})^2]^2$	=	0.13	0.41	0.74	0.92	1.0

Conical Diffuser	θ	4°	6°	8°	16°	60°	90°	180°
For $D_2/D_1 = 1.5$	k	0.15	0.13	0.14	0.30	1.21	1.13	1.0

Valves	Open	3/4 Open	1/2 Open	1/4 Open
Gate Valve	0.2	1.15	5.6	24.0
Globe	10.0			
Swing Check	2.5			

FLUID PROPERTIES: 60°F and Atmospheric Pressure

	γ	μ	ν
Water	γ = 62.4 lbs/ft^3	μ = 2.34×10^{-5} $lb\text{-}sec/ft^2$	ν = 1.21×10^{-5} ft^2/sec
Air	0.0763	3.74×10^{-7}	1.58×10^{-4}
Crude Oil	53.7	1.86×10^{-4}	1.10×10^{-4}

Fig. 5.08. Friction factors for pipes

by

$$h_o = C' \left| \frac{\alpha_1 V_1^2}{2g} - \frac{\alpha_2 V_2^2}{2g} \right| \qquad (5\text{-}30)$$

where subscripts 1 and 2 denote downstream and upstream cross sections respectively and

C' = coefficient of expansion if $V_1 < V_2$ or
coefficient of contraction if $V_1 > V_2$

In rivers and streams typical values for C' are 0.3 for expansion and 0.1 for contraction. However, form losses are often accounted for by increasing Manning's n-value in lieu of applying equation 5-30. Equation 5-30 should not be used in prismatic channels.

Section 5.10. The Weir Equation

The basic weir equation, $Q = CLH^{3/2}$, is applicable for determining the form loss caused by weirs provided the coefficient, C, has been calibrated for the appropriate flow magnitude and weir shape. When tailwater depth begins to exceed the weir crest elevation, some uncertainty exists as to how much the discharge or upstream water depth is being influenced. Coefficients presented in section 4.05 may be used if no better data are available. The form of the weir equation as modified to accommodate submerged flow is

$$Q = (C_s/C_f)\ C_f\ L\ H_1^{3/2} \qquad (5\text{-}31)$$

where:

(C_s/C_f) = the correction coefficient for submergence

C_f = the free flow coefficient

L = weir length (perpendicular to the flow direction)

H_1 = total energy head upstream from the weir

It is important to locate upstream and downstream cross sections at a distance from the weir equivalent to that used in the model tests from which coefficients are determined.

Incipient submergence is used to identify whether or not a correction coefficient is needed for submergence. It is the ratio of tailwater depth above weir crest to the free flow head for the discharge being analyzed. Therefore, this term does not need to appear in equation 5-31. When free flow does exist, (C_s/C_f) is 1.0.

Section 5.11. Head Loss at Contractions

The final form-loss equation to be presented is for subcritical flow through contractions and includes a friction loss term. It is applicable to bridge openings and other such contractions. Some general studies have been made to calibrate discharge coefficients for various flow conditions and boundary shapes at contractions. The general equation, related to fig. 4.01, is:

$$Q = CA_3\sqrt{2g\left(\Delta WS - h_f + \frac{\alpha_1 V_1^2}{2g}\right)} \qquad (5\text{-}32)$$

where:

ΔWS = the water surface elevation at section 1 minus the water surface elevation at section 3, with cross sections located as shown in fig. 4.01

C = discharge coefficient

A_3 = either net or gross area in the contraction depending upon whether or not "C" accounts for reduction in gross area of cross section due to piers

h_f = friction loss between sections 1 and 3

In flow through bridges, C usually ranges from about 0.6 to 1.0. The amount of contraction caused by non-overflow embankments is the primary influence on the discharge coefficient and losses take place in the expanding flow downstream from the structure. The coefficient, C, is also influenced by the Froude number in the contraction, the shape of the abutment, the angle of embankment with respect to flow, the location of the bridge opening with respect to approaching flow, the submergence of the superstructure, and the presence of piers and debris blocking the opening. When velocity through the bridge is small, little or no backwater results regardless of what discharge coefficient is used. In other cases the calculated velocity is high and several feet of head loss are calculated. It is doubtful that a sand channel could sustain more than half a foot of head loss because the sand would wash out.

Section 5.12. Critical Depth Calculations

The calculation of critical depth for complex sections of natural streams is difficult because of the complicated relationships between boundary geometry and the distribution of kinetic and potential energy. The only reliable procedure is to locate the water surface that gives the minimum total energy at the section by an indirect solution using successive approximations.

Total energy head at a cross section can be defined by:

$$H = y + \frac{\alpha Q^2}{2gA^2} + Y_o \tag{5-33}$$

where:

A = area of cross section

g = acceleration of gravity

H = total energy head in foot-pounds/pound or mkgf/kgf of fluid flowing

Y_o = height of channel bottom above the datum

y = depth of water

α = assumed to be constant for simplicity

The classic illustration for locating critical depth is to differentiate the energy equation and set the rate of change equal to zero to produce a relationship between discharge, α, cross section area and width. This is shown below to illustrate the shortcoming of applying this technique to complex sections. When specific energy is converted to total energy by including the height of channel above its datum, as in equation 5-33, the derivative of Y_o with respect to y is zero resulting in the following

$$\frac{dH}{dy} = 1 - \frac{\alpha Q^2}{2g} \cdot \frac{2}{A^3} \cdot \frac{dA}{dy} + 0 = 0 \tag{5-34}$$

$$\frac{\alpha Q^2}{g} = \frac{A^3}{B} \qquad (5\text{-}35)$$

where:

$B = dA/dy$

The value of water surface which produces the α, A and B to satisfy equation 5-35 would be critical depth. However, equation 5-35 is not satisfactory for calculating critical depth because α is not a constant with respect to y. A more realistic differentiation of equation 5-33 would include both α and A as function of y as follows:

$$\frac{dH}{dy} = 1 + \frac{\alpha Q^2}{2g} \left(\frac{-2}{A^3} \cdot \frac{dA}{dy}\right) + \frac{Q^2}{2gA^2} \cdot \frac{d\alpha}{dy} \qquad (5\text{-}36)$$

$$\frac{dH}{dy} = 0 = 1 - \frac{\alpha Q^2 B}{g\ A^3} + \frac{Q^2}{2gA^2} \cdot \frac{d\alpha}{dy} \qquad (5\text{-}37)$$

$$\frac{Q^2}{2gA^2} = \frac{1}{\left(\frac{2\alpha}{A} \cdot \frac{dA}{dy} - \frac{d\alpha}{dy}\right)} \qquad (5\text{-}38)$$

The $d\alpha/dy$ term can be evaluated over a small interval, but careful attention must be given because the relationship defined by equation 5-38 is considerably different from the well-behaved specific energy curve for which $d\alpha/dy = 0$. The derivative, dA/dy is always postive as are α and A. However, $d\alpha/dy$ can be postive, negative or zero as illustrated in fig. 5.09.

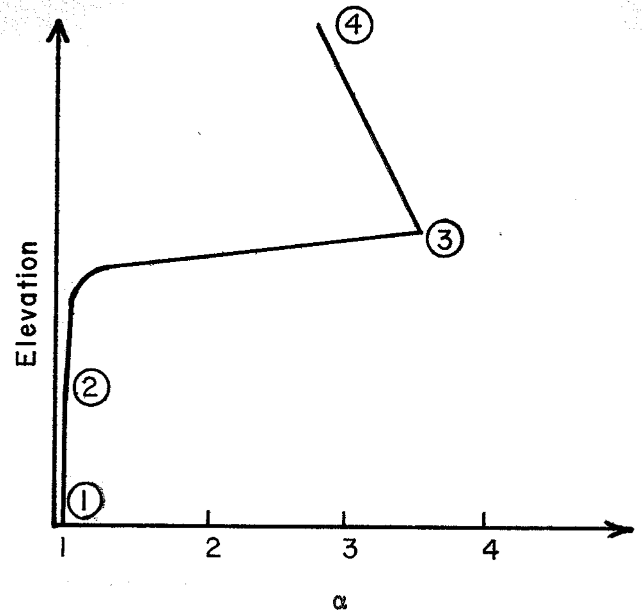


Fig. 5.09. The velocity distribution factor

Between points ① and ② , dα/dy is zero and equation 5-38 reduces to equation 5-35. From point ② to point ③ , $\frac{2\alpha}{A} \cdot \frac{dA}{dy}$ **must be** equal to or greater than dα/dy to permit a real solution, (i.e., all terms postive). The influence of the α gradient is to cause critical depth to be lower than would result if dα/dy were ignored. Between points ③ and ④ dα/dy is negative causing critical depth to be higher than would exist if the dα/dy were ignored.

Figure 5.10 shows the results of applying three methods for calculating critical depth for a discharge of 105000 cfs (2960 cms) in a complex cross section. Part (a) of fig. 5.10 shows the cross section, and α and energy curves are shown on part (b).

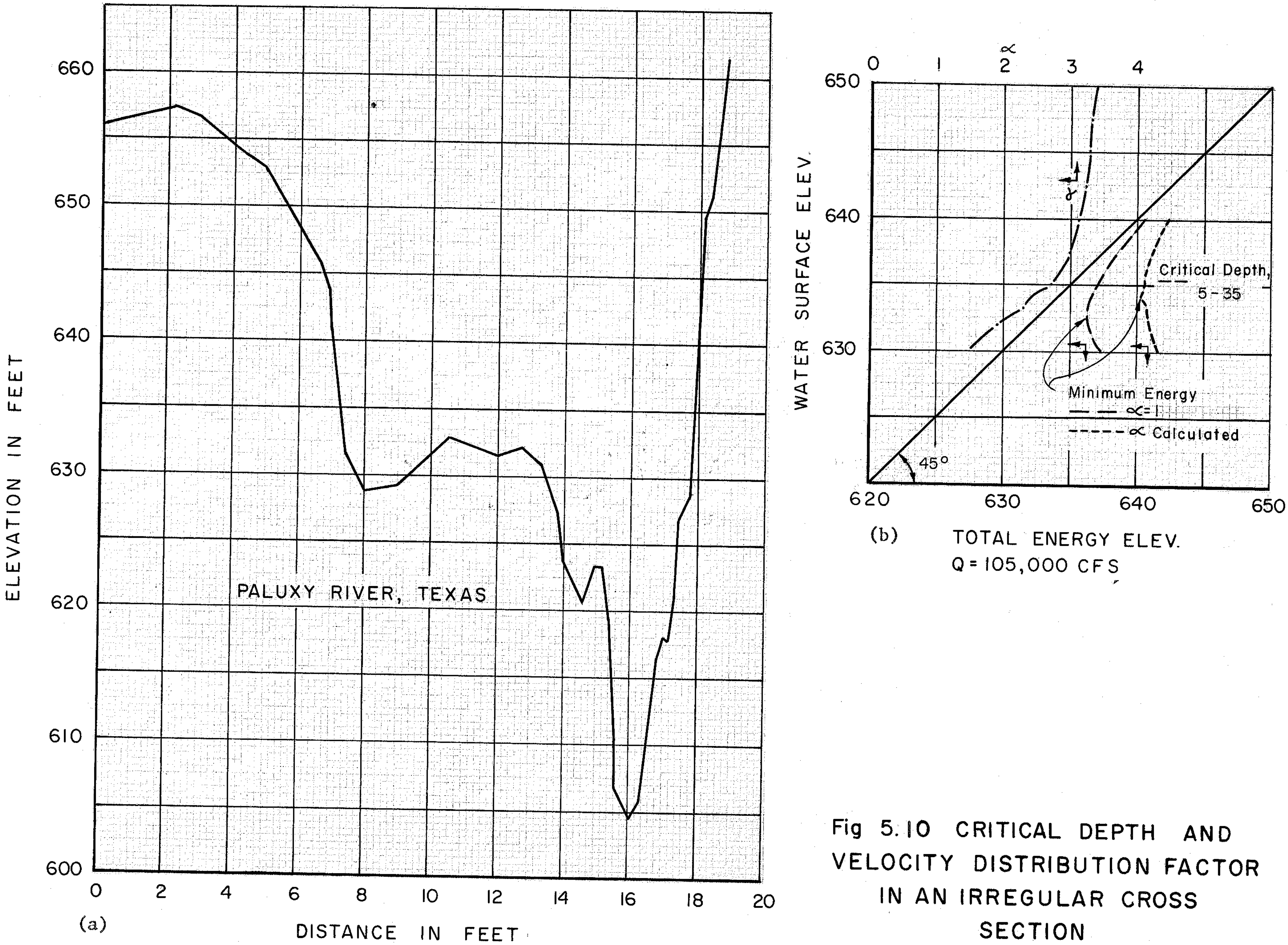


Fig 5.10 CRITICAL DEPTH AND VELOCITY DISTRIBUTION FACTOR IN AN IRREGULAR CROSS SECTION

The value for critical water surface elevation is 635.4 feet (194 m) when calculated with equation 5-35. Equation 5-33 was used to calculate two other values for critical water surface elevation. The equation itself describes the relationship between total energy and water surface elevation. By assigning an α of 1, the smooth total energy diagram, shown by long dashes on fig. 5.10b, was obtained. Tracing this diagram to the minimum total energy gives an elevation of 632.5 feet (193.1 m) for the corresponding water surface. However, α should not be constant but should vary according to equation 5-16. Values of α calculated with this equation, plotted on figure 5.10b, vary from 1.5 to 3.4 over the range of elevations analyzed. When equation 5-33 is utilized with this variable α, the calculated total energy curve, fig. 5.10b, gives a minimum total energy at elevation 634.0 feet (193.5 m). This value is accepted as the best, representative critical water surface elevation for the specified discharge because it is consistent with the manner in which the terms in the one-dimensional energy equation are evaluated for a complex cross section.

Computer programs need to test for critical depth to insure that calculations converge on the proper side. However, tracing the energy curve to its minimum value requires successive approximations and should be avoided when possible to save computer time. A satisfactory technique is to first test for critical depth by using equation 5-35. If the water surface elevation is above critical, calculations may continue. Otherwise, one cannot be certain about critical depth and the representative value must be calculated with equation 5-33 by locating the point of minimum total energy as illustrated in fig. 5.10.

This test procedure is sufficient as long as $d\alpha/dy$ is zero or positive. It is only when $d\alpha/dy$ is negative that the test is not entirely sufficient as reflected in the discussion of equations 5-36 and fig. 5.09. However, negative values of $d\alpha/dy$ are usually small relative to the other terms in equation 5-36, and the above test procedure is an acceptable simplification.

Section 5.13. Standard Step Solution

The solution of the non-linear equations 5-1 through 5-8 requires successive approximations. A trial value for WS_2 is assumed, and values for H_L and change in velocity head, are computed and summed to obtain ΔWS. This value is added to the known downstream water surface to compute WS_2. A typical convergence pattern is illustrated in figure 5.11.

The large difference between trial and computer water surface elevation is shown to converge asymptotically with successive trials. Therefore, the solution is stable.

However, the rate of convergence can be increased if an oscillation pattern is allowed to develop around the true solution. As with all non-linear equations this oscillation will often become unstable and the solution will diverge. Therefore, the solution criteria that takes advantage of the more rapid convergence characteristics afforded by oscillation must employ a damping factor for those few cases when divergence occurs. The following is such a procedure.

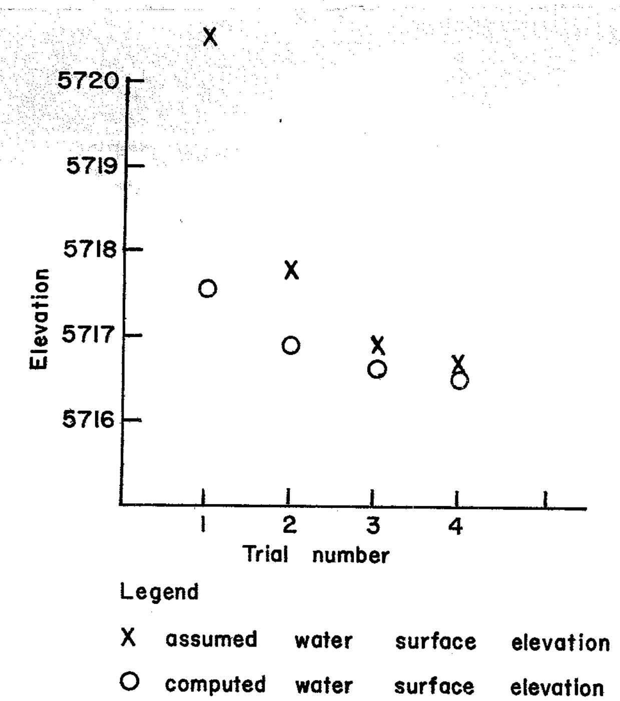


Fig. 5.11. Assumed and computed water surface elevation versus trial number

For subcritical flow the discharge is specified, and the water surface elevation is read from a rating curve for the most downstream cross section. Initially, no information is known about the next section upstream and the first trial is made by assuming the friction slope at the first cross section applies to the reach as follows:

$$TWS_{u1} = CWS_d + (Q/K_d)^2 \cdot L \qquad (5\text{-}39)$$

K = conveyance

L = distance to upstream section

Q = discharge

CWS = computed water surface elevation at downstream section

TWS = trial water surface elevation at upstream section subscripts:

u = upstream section

d = downstream section

1 = trial 1

The water surface elevation is then calculated and the error tested.

$$ERROR = |CWS_{u1} - TWS_{u1}| \leq TOLERANCE \qquad (5\text{-}40)$$

The tolerance is usually selected from the range 0.01 to 0.10 feet (3 to 30 mm). In any case the first test often fails and a second trial is needed.

The second trial can utilize information gained about the upstream section from trial 1. Tests on the Arkansas River, Arkansas USA suggested the following expression:

$$TWS_{u2} = TWS_{u1} + 0.9 \cdot (CWS_{u1} - TWS_{u1}) \qquad (5\text{-}41)$$

would result in the most rapid convergence of assumed and computed water surface elevation. Unfortunately calculations can oscillate when a value greater the 0.5 of the difference between computed and trial values is used. Therefore, a constraint criteria was developed to insure that calculations could recover if the second trial diverged from the first. If

$$|CWS_{u2} - TWS_{u2}| < |CWS_{u1} - TWS_{u1}| \qquad (5\text{-}42)$$

calculations are converging, proceed.

Otherwise, calculations are diverging in which case the present trial is ingored and a new trial 2 water surface elevation is determined according to the following equation:

$$\text{New } TWS_{u2} = 0.5 \cdot (\text{Present } TWS_{u2} + TWS_{u1}) \qquad (5\text{-}43)$$

This procedure continues until either convergence results, equation 5-42 is satisfied or the difference between "Present" TWS_{u2} and TWS_{u1} is completely damped to some fraction of the "Tolerance" in equation 5-40. The CWS_u is set equal to the final value of TWS and calculations proceed.

The third and subsequent trials are obtained by a linear extrapolation of the two previous trial and computed values as illustrated in the following figure.

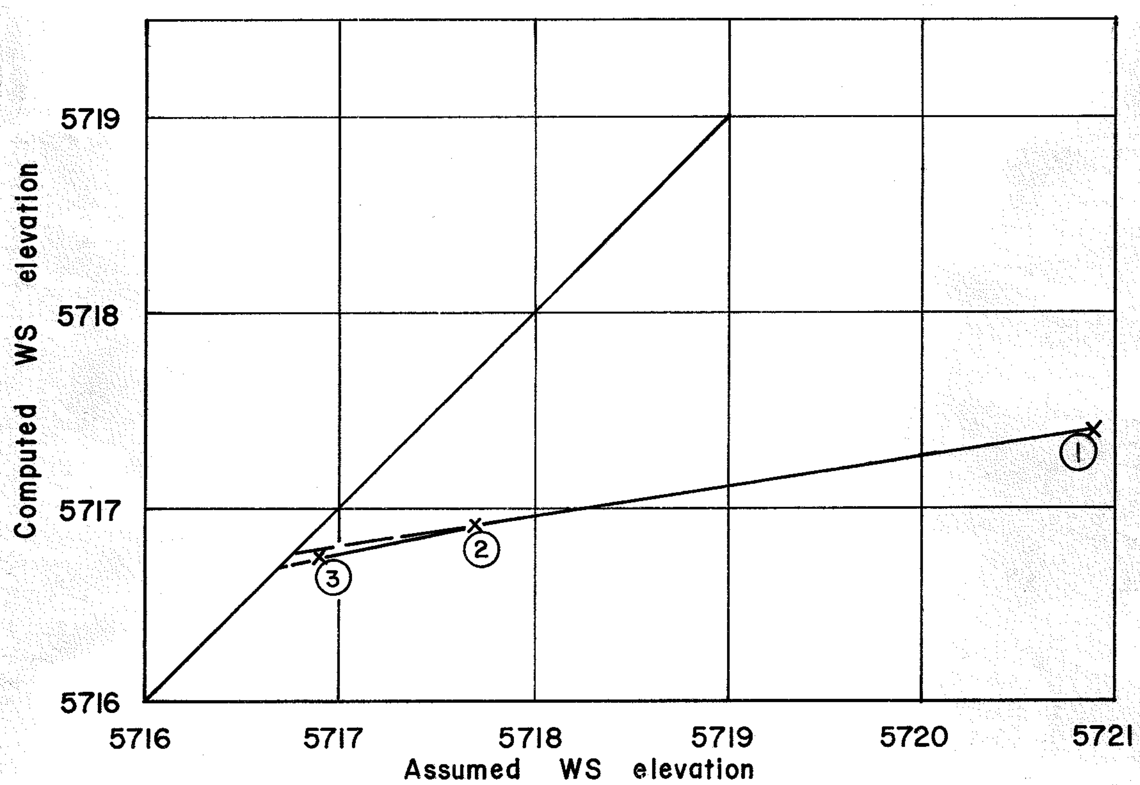


Fig. 5.12. Graphic technique for converging trial to computed water surface elevation

The algebraic equation which represents this graphical procedure for the computer is:

$$TWS_{u3} = (TWS_{u1} - XM \cdot CWS_{u1}) / (1. - XM) \tag{5-44}$$

where:

$$XM = (TWS_{u2} - TWS_{u1}) / (CWS_{u2} - CWS_{u1})$$

The oscillation and dampening criteria apply to all calculations after trial 1.

The procedure is repeated for all cross sections after the second except that previously computed energy slopes are used in calculating the first trial, rather than using only the friction slope at the downstream section of the reach.

Chapter 6

Calculating Steady Flow Profiles

CHAPTER 6. CALCULATING STEADY FLOW PROFILES

The computation of steady flow profiles by either manual or computer techniques requires similar steps as shown in the following comparison.

Table 6.01. Computation of steady flow profiles

Modeling the Study Area

Manual Calculations	Computer Calculations
1. On the best available map, locate cross sections so that best geometric description for the study area can be determined by averaging flow areas and hydraulic radii (or conveyances) of the cross sections at the ends of each reach.	1. Same as manual.
2. Subdivide the river valley into strips having similar hydraulic properties in the direction of flow (e.g., main channel, trees, fields, etc.)	2. Same as manual.
3. Specify hydraulic roughness values to be used in each strip of each reach. If these vary with elevation, specify the values.	3. Same as manual.

Manual Calculations	Computer Calculations
4. Determine the reach length for each strip. This will be the distance between cross sections unless a strip ends before reaching the next cross section.	4. Same as manual.
5. Calculate areas, hydraulic radii and conveyances for each cross section and prepare curves.	5. Computer calculates from cross section coordinates during water surface profile calculations.

Verifying the Model

6. Reproduce known water surface profiles and/or rating curves to ascertain the model accuracy.	6. Same as manual.

Perform Desired Study

7. Establish the starting water surface elevation.	7. Same as manual.
8. Calculate required profiles.	8. Calculations done with computer program.
9. Critically review each calculated profile and confirm the reason for any peculiar shapes. Plotting profiles is a valuable aid and should always be done. (Besides displaying errors, plots assist in making the first approximation of the water surface elevation for the next cross section.	9. Same as manual; however, plotting of profiles may be performed automatically by computer.

Perform Sensitivity Studies

Manual Calculations	Computer Calculations
10. The large volume of computations virtually prohibits an extensive analysis and requires considerable use of engineering judgment.	10. The computer is ideal for sensitivity studies. Once the basic model is coded it is easily modified for additional runs.
11. Prepare tables or curves showing results of study.	11. Can be done automatically by computer.

The computer can perform four of the eleven steps, and these are the repetitious and time-consuming steps. Such assistance enables the engineer to concentrate on understanding the problem better and formulating better solutions. Steps 1 through 4, and step 6 require careful study for proper definition of the problem. Thereafter, computations proceed automatically when using the computer.

Section 6.01. The Computation Form

A convenient form for use in calculating water surface profiles is shown in Table 6.02. A general discussion about the table will illustrate how conditions discussed in Chapter 5 are accommodated. This is followed by a sample problem.

In summary, columns 2 and 4 through 12 are devoted to solving Manning's equation to obtain the energy loss due to friction, columns 13 and 14 contain calculations for the velocity distribution across the section, columns 15 through 17 contain the average kinetic energy, column 18 contains calculations for "other losses" (expansion and

Table 6.02. WATER-SURFACE PROFILE CALCULATIONS

PROJECT: Red Fox River

Q = 5000 cfs

C_e = 0.3 C_c = 0.1

CROSS SECTION NO.	WATER SURFACE ELEVATION ASSUMED	COM-PUTED	AREA A	HYDRAULIC RADIUS R	$R^{2/3}$	n	K	$\overline{K}_t$	$\overline{S}_f$	L	h_f	K^3/A^2	α	V	$\alpha \frac{V^2}{2g}$	$\Delta(\alpha \frac{V^2}{2g})$	h_o	Δ(WATER SURFACE ELEVATION)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
									X 10^{-3}			X 10^6						
1		5709.0	355	6.1	3.35	0.03	58,900	--					1.0	14.1	3.08			
2	5712.6		360	6.1	3.3	0.03	58,800					1568						
			80	1.6	1.4	0.05	3,300					6						
		5713.0	440				62,100	60,500	6.83	500	3.42	1574	1.27	11.4	2.54	+.54	.05	4.01
	5713.0	5713.0	380	6.3	3.4	0.03	64,000					1815						
			100	1.8	1.5	0.05	4,460					9						
			480				68,460	63,700	6.16	500	3.08	1824	1.31	10.4	2.21	+.87	.09	4.04
3	5716.2	5715.9	820	5.4	3.1	0.03	126,600	97,500	2.63	400	1.06	--	1.0	6.1	0.58	+1.63	.16	2.85
	5715.9	5715.95	780	5.1	2.9	0.03	112,000	90,200	3.07	400	1.22	--	1.0	6.4	0.64	+1.57	.16	2.95
4	5720.9	5716.7	780	6.4	3.4	.036	109,000	110,500	2.05	400	0.82	--	1.0	6.4	0.64	0	0	0.82
	5716.9	5716.15	380	4.5	2.7	.036	42,400	77,200	4.19	400	1.68	--	1.0	13.2	2.69	-2.05	0.62	0.25
	5716.1	5715.49	315	4.1	2.56	.036	33,300	72,650	4.74	400	1.89	--	1.0	15.9	3.92	-3.28	.98	-0.41
	NOTE:	At 5716.1 V = 15.9 fps, celerity = 12.0 fps, Froude No. = V/c = 1.32. Flow is supercritical.																

(8) $K = C_m AR^{2/3}/n$

C_m = 1.486 for English units
1.0 for Metric units

(9) $\overline{K}_t = .5(K_{upstream} + K_{downstream})$

(10) $\overline{S}_f = (Q/\overline{K}_t)^2$

(12) $h_f = L\ \overline{S}_f$

(14) $\alpha = \dfrac{(A_t)^2\ \Sigma\ (K_i^3/A_i^2)}{(K_t)^3}$

where: i = incremental value

t = total value

(15) $V = Q/A_t$

(17) $\Delta(\alpha \frac{V^2}{2g}) = (\alpha \frac{V^2}{2g})_{downstream} - (\alpha \frac{V^2}{2g})_{upstream}$

(18a) $h_o = C_e \mid \Delta(\alpha \frac{V^2}{2g}) \mid$ for $\Delta(\alpha \frac{V^2}{2g}) < 0$

(18b) $h_o = C_c \mid \Delta(\alpha \frac{V^2}{2g}) \mid$ for $\Delta(\alpha \frac{V^2}{2g}) > 0$

(19) Δ (water surface elevation) $= \Delta(\alpha \frac{V^2}{2g}) + h_f + h_o$

contraction losses due to interchanges between kinetic and potential energies as the water flows), and column 19 contains the computed change in water surface elevation. Conservation of energy is accounted for by proceeding from section to section down the computation form.

Column 1, RIVER MILE, is the cross-section identification number. Miles (or kilometers) upstream from the mouth are recommended.

Column 2, ASSUMED, is the assumed water surface elevation which must agree to within ±.05 feet, or some allowable tolerance, with the resulting computed water surface elevation for trial calculations to be successful.

Column 3, COMPUTED, is the rating curve value for the first section but, thereafter, is the value calculated by adding ΔWS to the computed water surface elevation for the previous cross section (Section 5.13).

Column 4, A, is the cross section area. If the section is complex and has been subdivided into several parts (e.g., left overbank, channel and right overbank) use one line of the form for each subsection and sum to get A_t, the total area of cross section (Section 5.04).

Column 5, R, is the hydraulic radius. Use the same procedure as for column 4 if section is complex, but do not sum subsection values (Section 5.04).

Column 8, K, is conveyance and is defined as $C_m AR^{2/3}/n$ where C_m is 1.0 for metric units or 1.486 for English units. If the cross section is complex, sum subsection K values to get K_t (Section 5.04).

Column 9, $\bar{K}_t$, is average conveyance for the reach, and is calculated by 0.5 (K_{td} + K_{tu}) where subscripts d and u refer to downstream and upstream ends of the reach, respectively (Section 5.07).

Column 10, $\bar{S}_f$, is the average friction slope through the reach determined by $(Q/\bar{K}_t)^2$ (Section 5.07).

Column 11, L, is the distance between cross sections; different values may be used in each strip (Section 5.02).

Column 12, h_f, is energy loss due to friction through the reach and is calculated by $h_f = (Q/\bar{K})^2 L$ (Section 5.07).

Column 13, $K (K/A)^2$, is part of the expression relating distributed flow velocity to an average value. If the section is complex, calculate one of these values for each subsection and sum all subsection values to get a total. If one subsection is used, Column 13 is not needed and α (Column 14) equals one (Section 5.05).

Column 14, α, is the velocity distribution coefficient and is calculated by $\Sigma K(K/A)^2/K_t(K_t/A_t)^2$ where the numerator is the sum of values in column 13 and the denominator is calculated from K_t and A_t (Section 5.05).

Column 15, V, is average velocity and is calculated by Q/A_t.

Column 16, $\alpha V^2/2g$, is the average velocity head corrected for flow distribution (Section 5.05).

Column 17, $(\Delta\alpha V^2/2g)$, is the difference between velocity heads at the downstream and upstream sections. A positive value indicates velocity is increasing; therefore, use a contraction coefficient for "other losses". A negative value indicates the expansion coefficient should be used in calculating "other losses" (Section 5.01).

Column 18, h_o, is "other losses," and calculated with either C_e or C_c (Section 5.09).

Column 19, ΔWS, is the change in water surface elevation from the previous cross section. It is the algebraic sum of columns, 12, 17 and 18.

Section 6.02. Example Problem

An example of this procedure is illustrated in an application to the Red Fox River, Colorado. Figure 6.01 is a plan view showing the river, contours on the flood plain and location and alignment of cross sections. The stream flows from west to east. Cross sections are plotted in fig. 6.02. Figures 6.04 through 6.07 are area and hydraulic radius curves which, when combined with reach length, will describe the geometric model. The rating curve for section 1, a control section, is shown in fig. 6.03.

The location and alignment of cross sections are very important because they describe the geometric model which is the basis for the entire series of computations. Contour lines are used in orienting sections perpendicular to the expected current directions, and the results often require angle points to model both channel and overbank flow. In this example, no cross sections intersect. In cases where cross sections do tend to cross, the cross section alignments should run parallel to each other to high ground and some small, postive value should be assigned for each reach length. Zero reach lengths should be avoided so that dividing by zero will not occur in subsequent

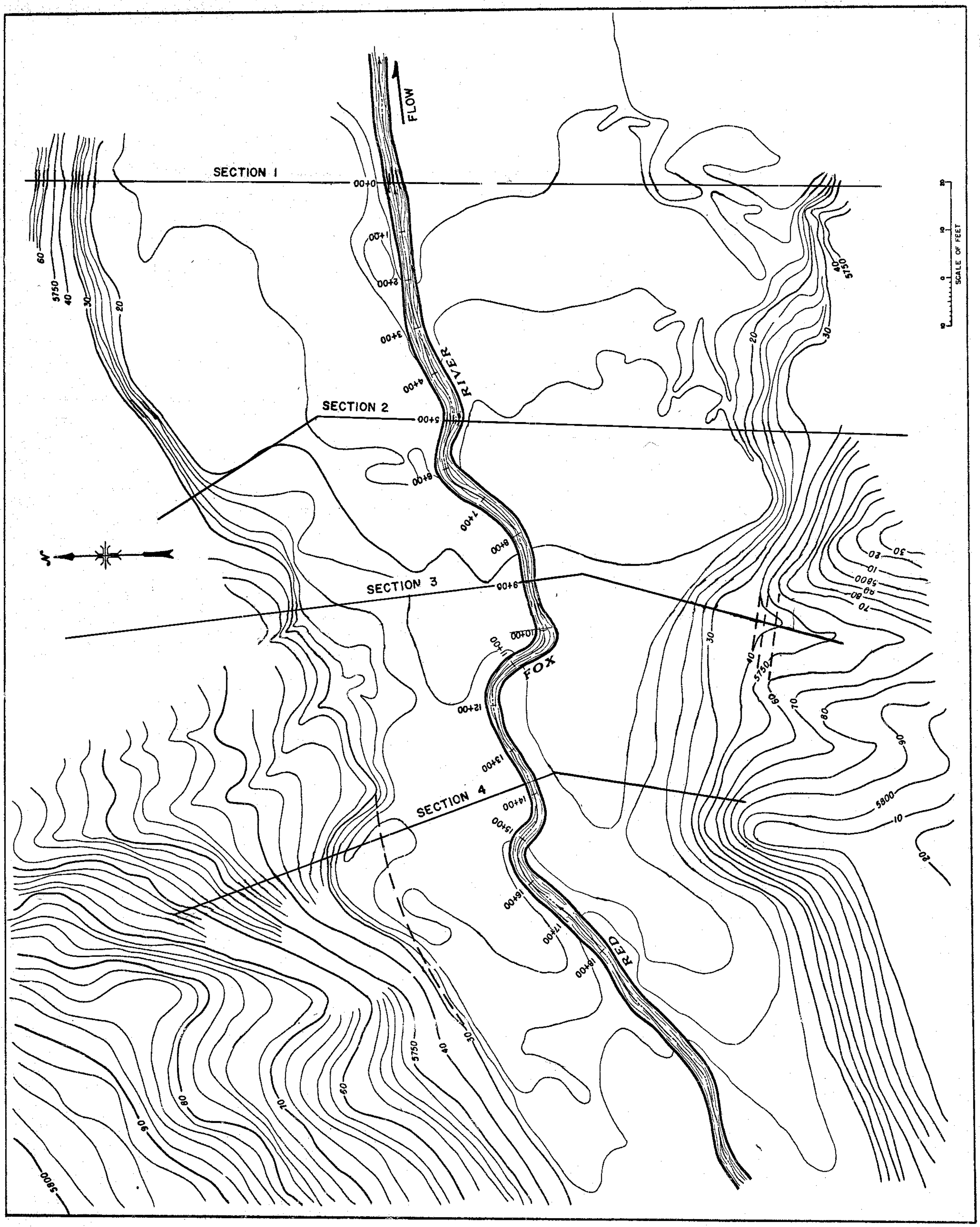


Fig. 6.01. Plan view of the Red Fox River, Colorado

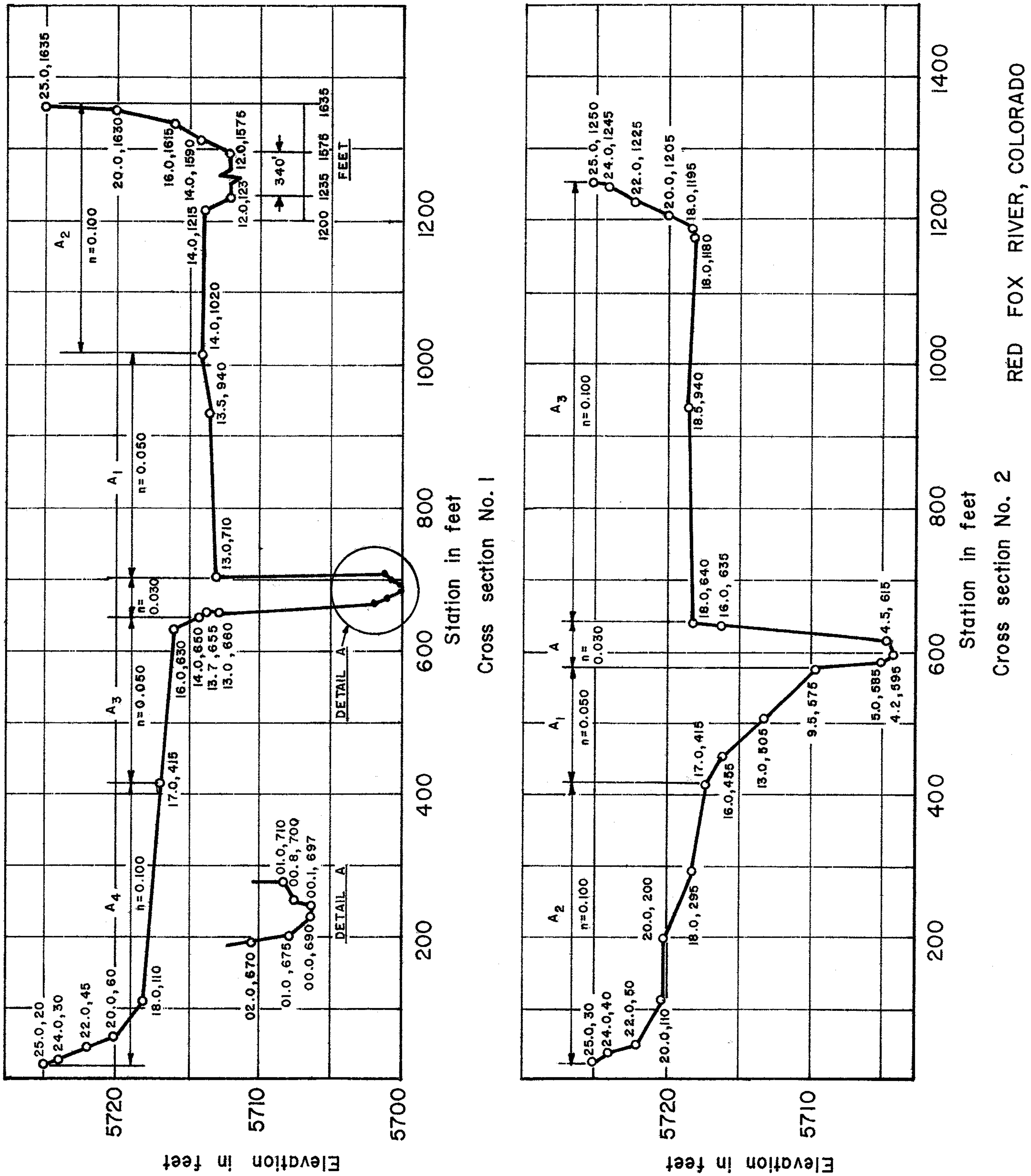
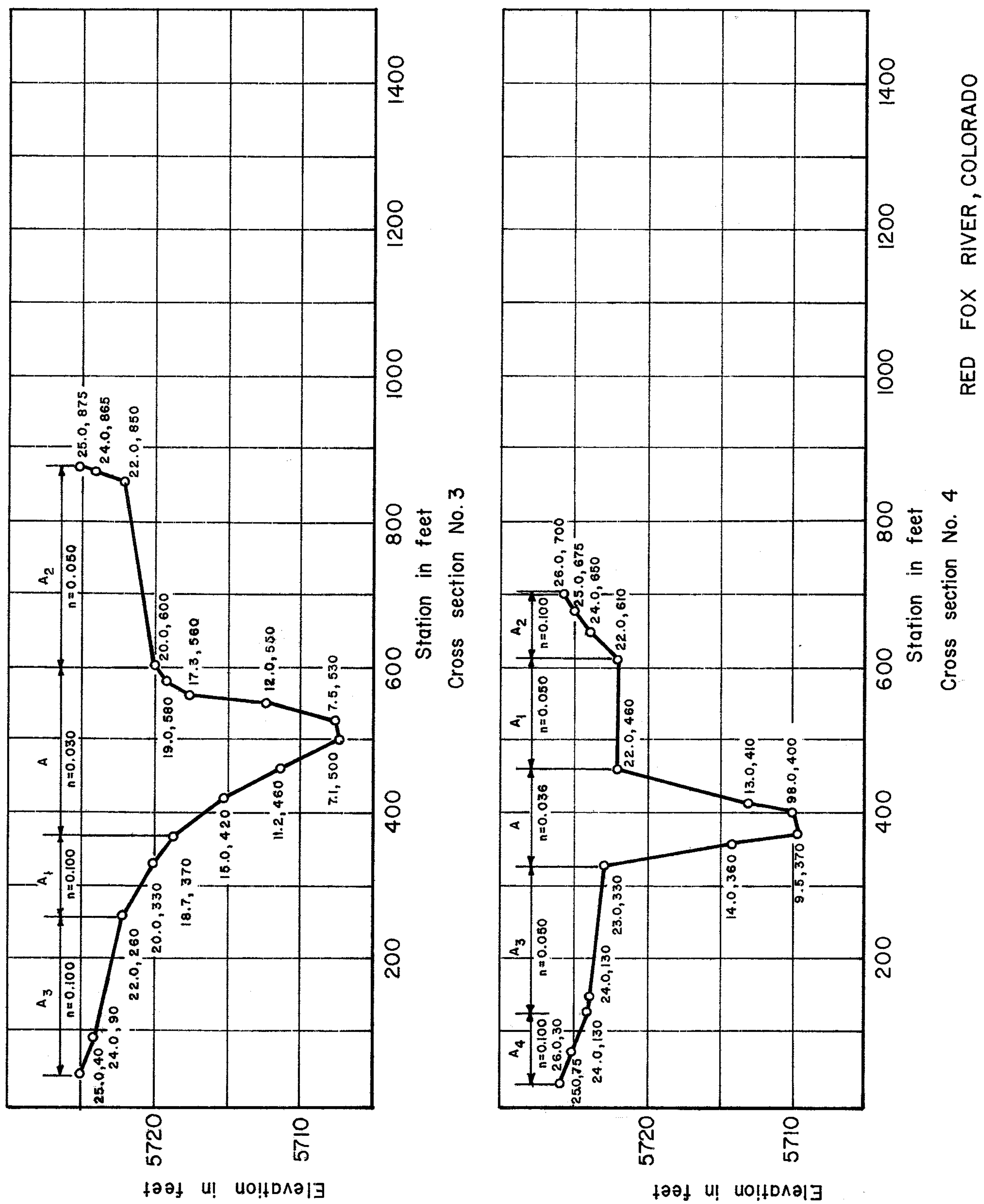


Fig. 6.02. Cross sections of Red Fox River, Colorado

Cross sections of Red Fox River, Colorado (cont)



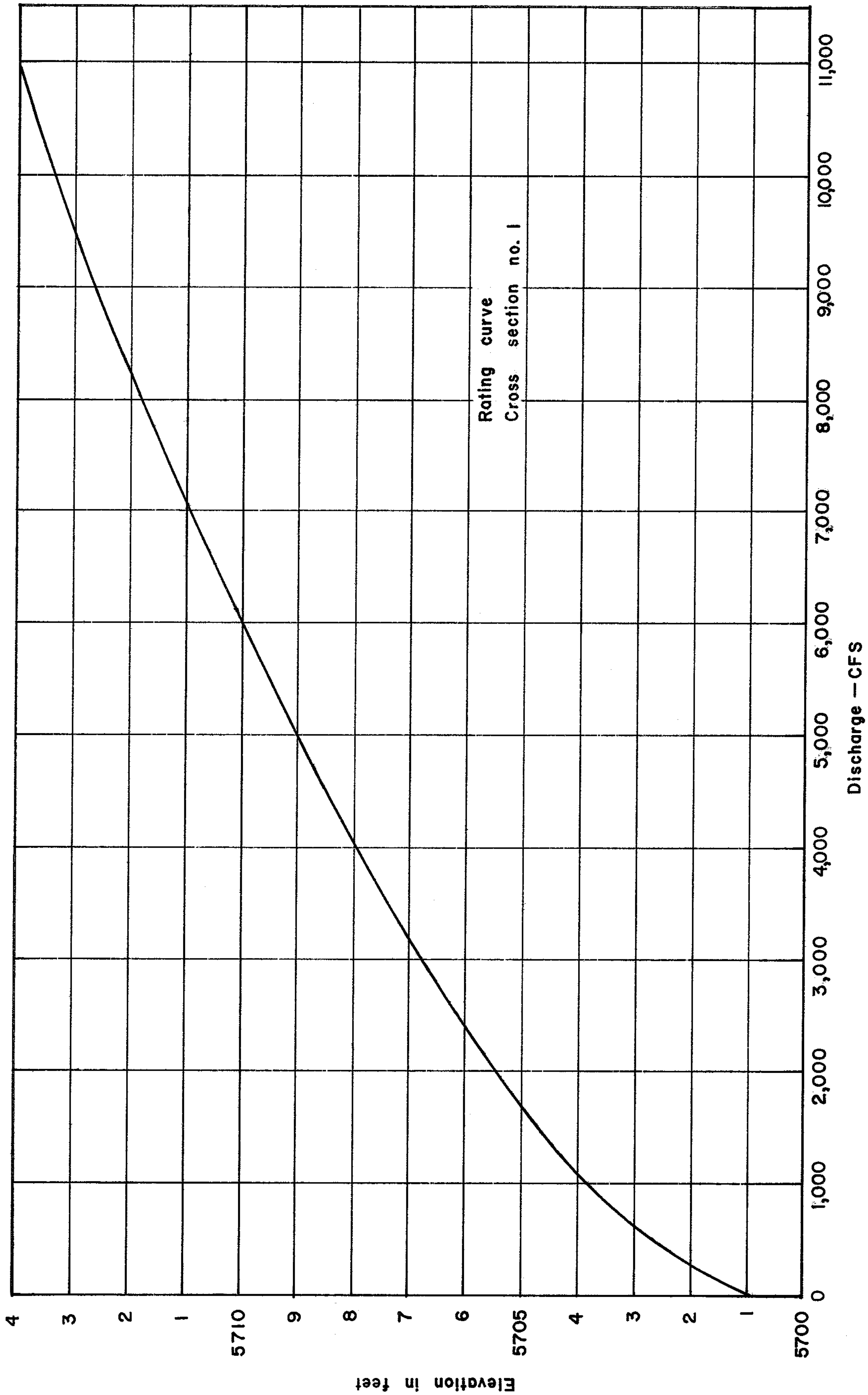


Fig. 6.03. Rating curve for cross section no. 1.

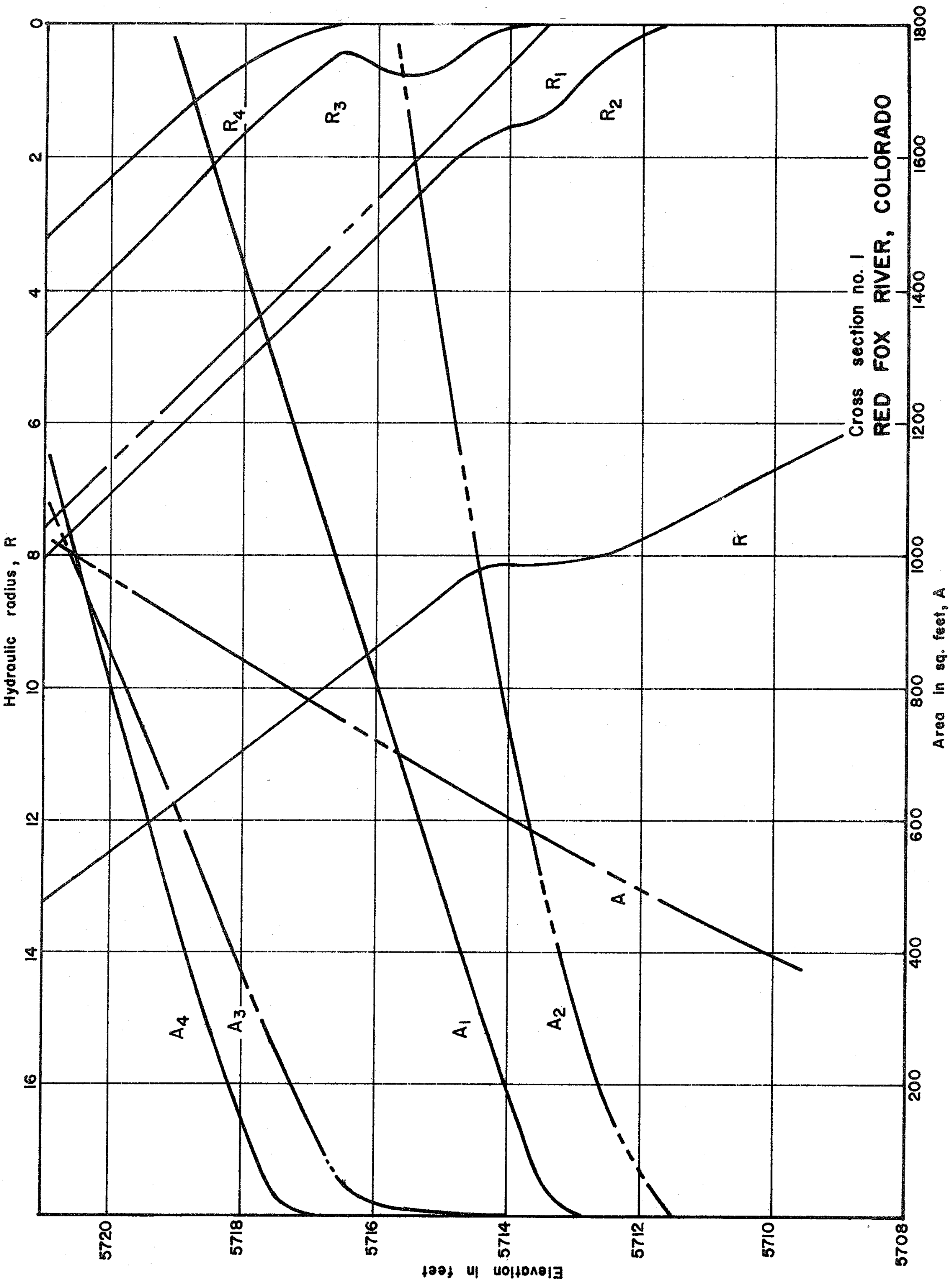


Fig. 6.04. Area and hydraulic radius curves for cross section no. 1.

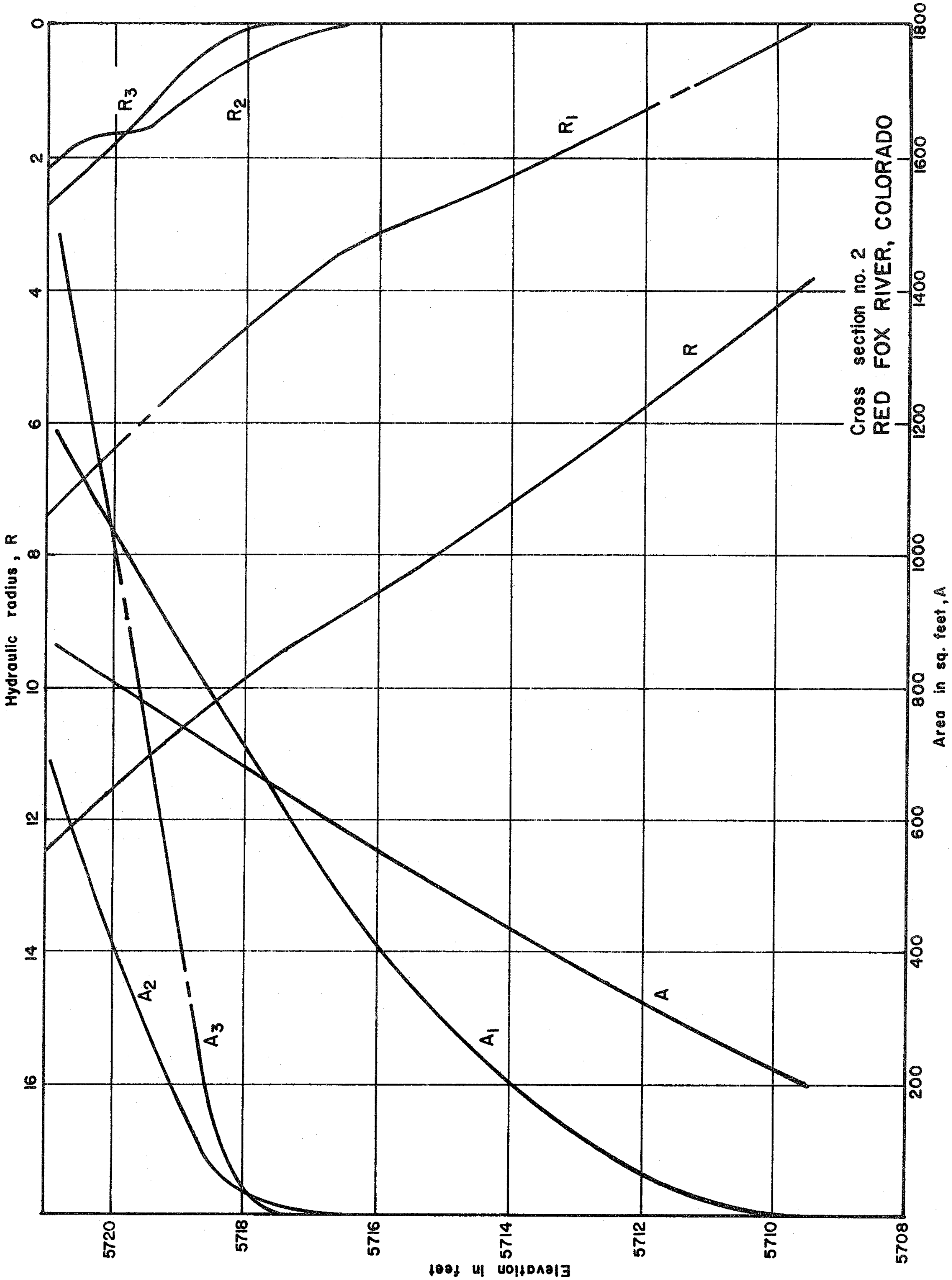


Fig. 6.05. Area and hydralic radius curves for cross section no. 2.

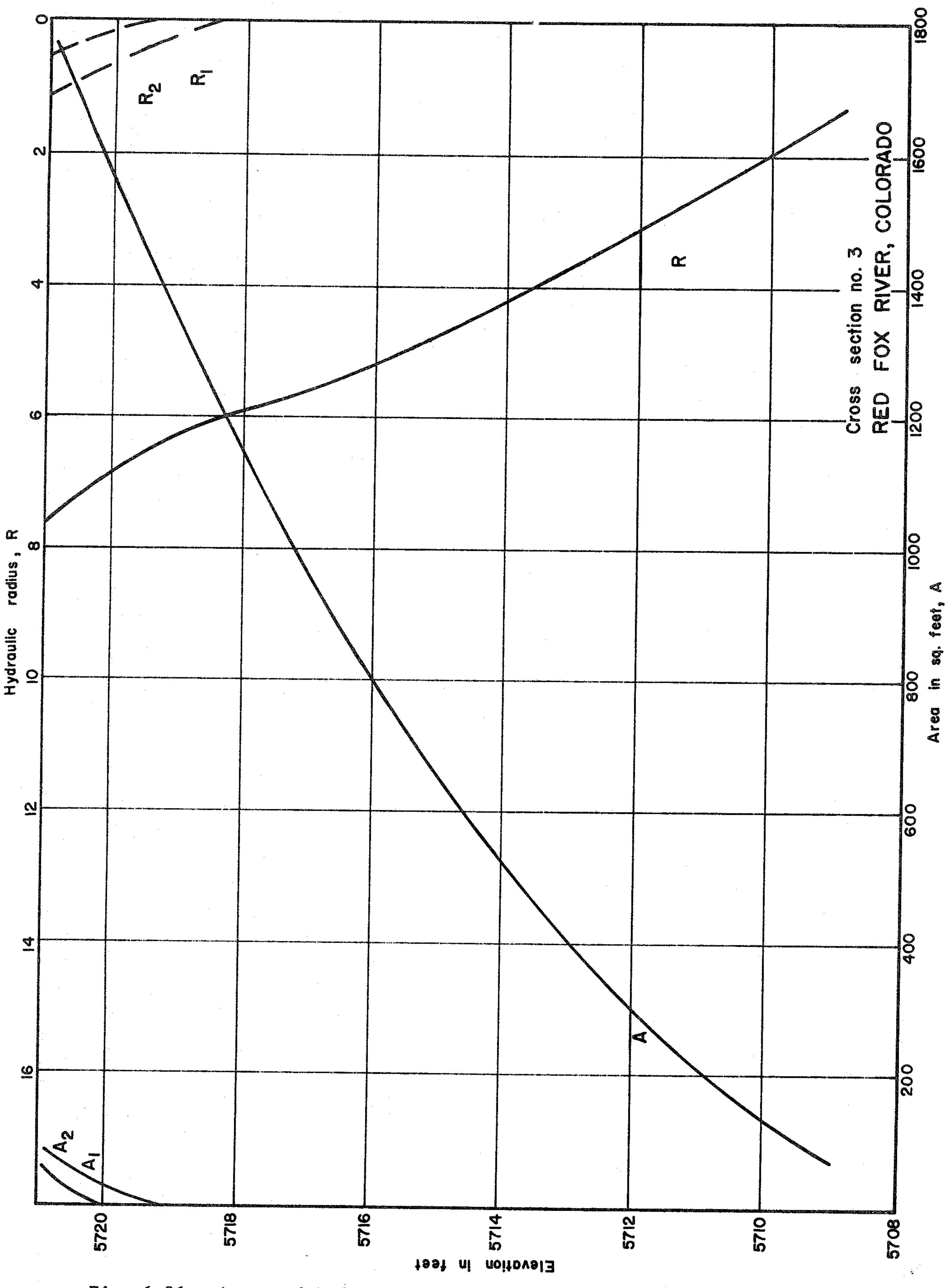


Fig. 6.06. Area and hydraulic radius curves for cross section no. 3.

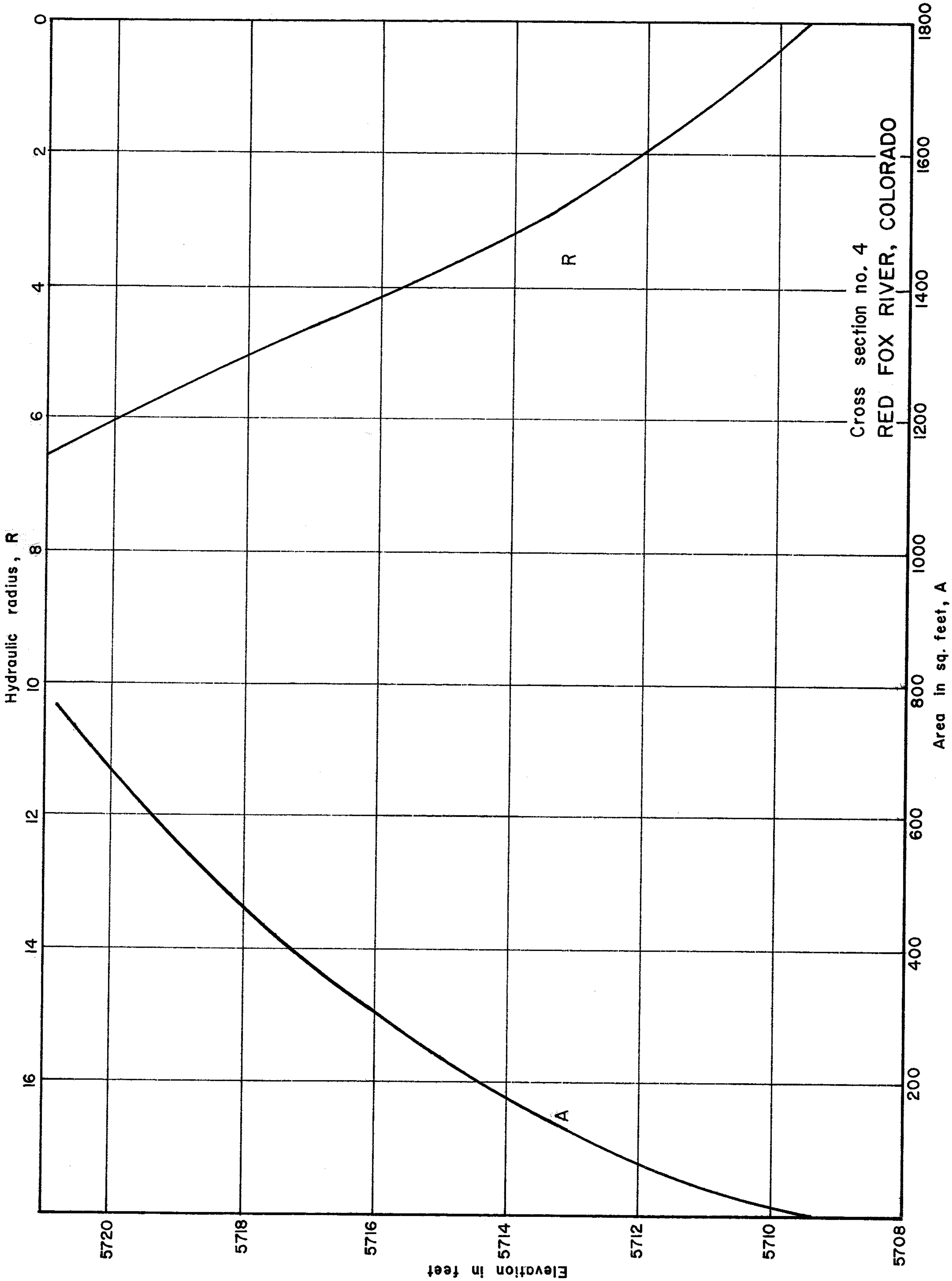


Fig. 6.07. Area and hydraulic radius curves for cross section no. 4.

computations.

Hydraulic roughness values are obtained from the field--not the contour map. The roughness values should be shown on each cross section as they are helpful in locating where a cross section should be subdivided to determine distributed properties. Values of .3 and .1 are assumed for the expansion and contraction coefficients, respectively.

At this point, all basic data have been specified except the independent variable, discharge, and the associated starting water surface elevation. Moreover, the same basic data is valid for a range of different independent variables. In this example, a water surface profile is required for 5,000 cfs and the stage-discharge relationship defined for the control (fig. 6.03) indicates a starting water surface elevation (WS_1) of 5709.0 feet (WS, in equation 5-3). Using WS_1, values of area and hydraulic radius are determined from fig. 6.04 and entered on the computation form, table 6.02, on line with RIVER MILE 1.0. (All column numbers shown in this section refer to table 6.02.) Since all flow is contained in one subsection, the main channel, calculating a value for α with equation 5-16 yields 1.0. Velocity and kinetic energy ($\alpha \frac{V^2}{2g}$) are calculated, thus completing calculations for cross section 1. Total energy is not shown on the form, but its components, water surface elevation and velocity head, are tabulated in columns 3 and 16 respectively.

The next step is to calculate total energy at section 2 of fig. 6.01, which requires the following calculations. An estimate of the water surface elevation is obtained at section 2 by equation 5-39 as follows:

$$\Delta WS = (Q/K)^2 \cdot L$$

$$= (5000/58900)^2 \cdot 500$$

$$= 3.6 \text{ feet}$$

$$WS_2 = 5709 + 3.6$$

$$= 5712.6 \text{ feet assumed for trial 1.}$$

The corresponding area and hydraulic radius are obtained from fig. 6.05, satisfying equation 5-7, and are entered in columns 4 and 5, respectively. Total area (A_t) is calculated by summing the subsectional values (column 4), and conveyance (K) is calculated for each subsection and summed to obtain K_t. The K_t of section 2 is averaged with K_t of section 1 to get $\bar{K}_t$ (column 9), and friction loss (column 12) is calculated by equation 5-22.

Column 13 contains the numerator of equation 5-16 for calculating α. A value is calculated for each subsection, entered in the column, and the subsection values are summed. The denominator of equation 5-16 is calculated from A_t of column 4 and K_t of column 9 and divided into the summed value in column 13 to obtain α, tabulated in column 14. The average velocity (Q/A_t) is entered in column 15 and the weighted velocity head calculated in column 16. The change in weighted velocity head (column 17) is the value at the downstream section minus the value at the upstream section. Relative to equation 5-1 this is:

$$\frac{\alpha_1 Q^2}{2gA_1^{\,2}} - \frac{\alpha_2 Q^2}{2gA_2^{\,2}} = 3.08 - 2.54 = +0.54$$

The sign of the value in column 17 is used to select either the contraction coefficient (positive) or expansion coefficient (negative) for calculating "other losses" in accordance with equation 5-30. The "other losses" are entered into column 18. Column 19 represents the change in water surface elevation from section 1 to section 2 and relates to equation 5-1 as follows.

$$\Delta WS = WS_2 - WS_1 = \frac{\alpha_1 Q^2}{2gA_1^{\;2}} - \frac{\alpha_2 Q^2}{2gA_2^{\;2}} + h_f + h_o$$

$$\Delta WS = \text{column } (11) + (16) + (17) = 4.01$$

Column 3 is calculated by adding ΔWS to the calculated water surface of the previous cross section as follows.

$$CMPT = CMPT_1 + \Delta WS$$

$$= 5709.00 + 4.01$$

$$CMPT = 5713.01$$

The specified tolerance between assumed and calculated values is ±.05 feet. The result from this trial is

$$ERROR = 5713.01 - 5712.00$$

$$= +.41$$

Therefore, the ERROR is not within the specified tolerance, and a second trial must be made. Equation 5-41 is used.

$$ASMD_{u2} = ASMD_{u1} + 0.9 \cdot (ERROR)$$

$$= 5712.6 + 0.9\ (0.41)$$

$$ASMD_{u2} = 5712.97$$

The second trial produced acceptable results at section 2; calculations are advanced to section 3.

Geometric properties for section 3 are shown on fig. 6.06. Calculations proceed as described for section 2.

Geometric properties for section 4 are shown on fig. 6.07. When three trials failed to produce a solution and the velocity became quite high, critical depth was calculated with equation 5-35 as follows.

$$\alpha Q^2/g = A^3/B$$

$$\alpha = 1.0;\ Q^2 = (5000)^2 = 25(10)^6;\ g = 32.3$$

$$\therefore \alpha Q^2/g = 25(10)^6/32.3$$

$$= 0.777(10)^6$$

The water surface elevation that produces a value of A^3/B equal to $0.777(10)^6$ is determined by successive approximations.

Table 6.03. Critical Depth at Section 4

Water Surface Elevation	Area (A)	Width (B)	A^3/B (10^6)
5716.9	380	82	.67
5717.7	440	98	.87
5717.2	415	91	.778

The resulting critical water surface is elevation 5717.2 feet. Since this is a subcritical flow profile, the calculated water surface elevation at section 4 must remain above the critical depth constraint.

The assumption of critical depth at section 4 is satisfactory for calculations to proceed in the upstream direction. However, to determine the true water surface elevation at section 4 would require extending calculations upstream until subcritical flow is again encountered, starting at critical depth at that point, and calculating a supercritical flow profile in the downstream direction. This example problem ends with the calculation of critical depth at section 4, however.

A closer analysis of the calculations between sections 3 and 4 revealed an interesting problem. It appears that an energy balance can not be achieved when approaching section 4 from the downstream direction. That is, the energy and energy-loss equations cannot be solved to yield a water surface elevation for that cross section using the analytical procedure developed in this volume. Consider fig. 6.08 which presents curves of water surface elevation versus total energy head for cross sections 3 and 4. Conditions at cross section 3 are represented by the water surface elevation of 5715.95 feet (1742.67 meters) and corresponding total energy head of 5716.6 feet (1742.42 meters). The total energy head for cross section 4 is equal to the total energy head for cross section 3 plus h_e, the energy loss between the two cross sections.

Given the constraint that calculations have to converge at or above critical depth, the maximum losses that can be calculated occur when the water surface elevation at section 4 is at critical depth. The corresponding velocity head and conveyance are:

$$\alpha\frac{V^2}{2g} = 2.62 \qquad K = 44{,}100$$

The resulting energy loss between cross sections 3 and 4 is:

$$h_o = 0.3\ |0.64 - 2.62| = 0.6 \text{ (18.29 centimeters)}$$

$$h_f = 400 \frac{2(5000)}{112,000 + 44,100} = 1.6 \text{ (48.77 centimeters)}$$

$$H_L = h_o + h_f = 2.2 \text{ (67.06 centimeters)}$$

Hence, the total calculated energy head at cross section 4 would be 5718.8 feet (1743.09 meters). When plotted on fig. 6.08, this value does not intersect the curve for cross section 4. This indicates that it is impossible to calculate a water surface elevation for cross section 4 on the basis of given information because more energy losses occur than are described by the energy-loss equation.

It is possible, for example, that the flow is supercritical at cross section 4 and that a hydraulic jump occurs between cross sections 3 and 4. If this were the case, the computed energy loss between cross sections 3 and 4 would have to include the energy consumed by the jump in addition to h_o and h_f.

Alternatives for continuing water surface profile calculations past section 3 are (1) to restart at section 4 using critical depth; (2) to insert additional sections between 3 and 4; or (3) to reformulate the energy loss equation. The first alternative is most expedient to program for computers.

One other alternative to continuing calculations to section 4 is to reformulate the finite difference scheme used to calculate friction loss (see equation 5-17 through 5-22). For example, equation 5-19, based on average friction slope, would predict a water surface elevation of 5718.0 feet (1742.85 meters) at section 4 starting with

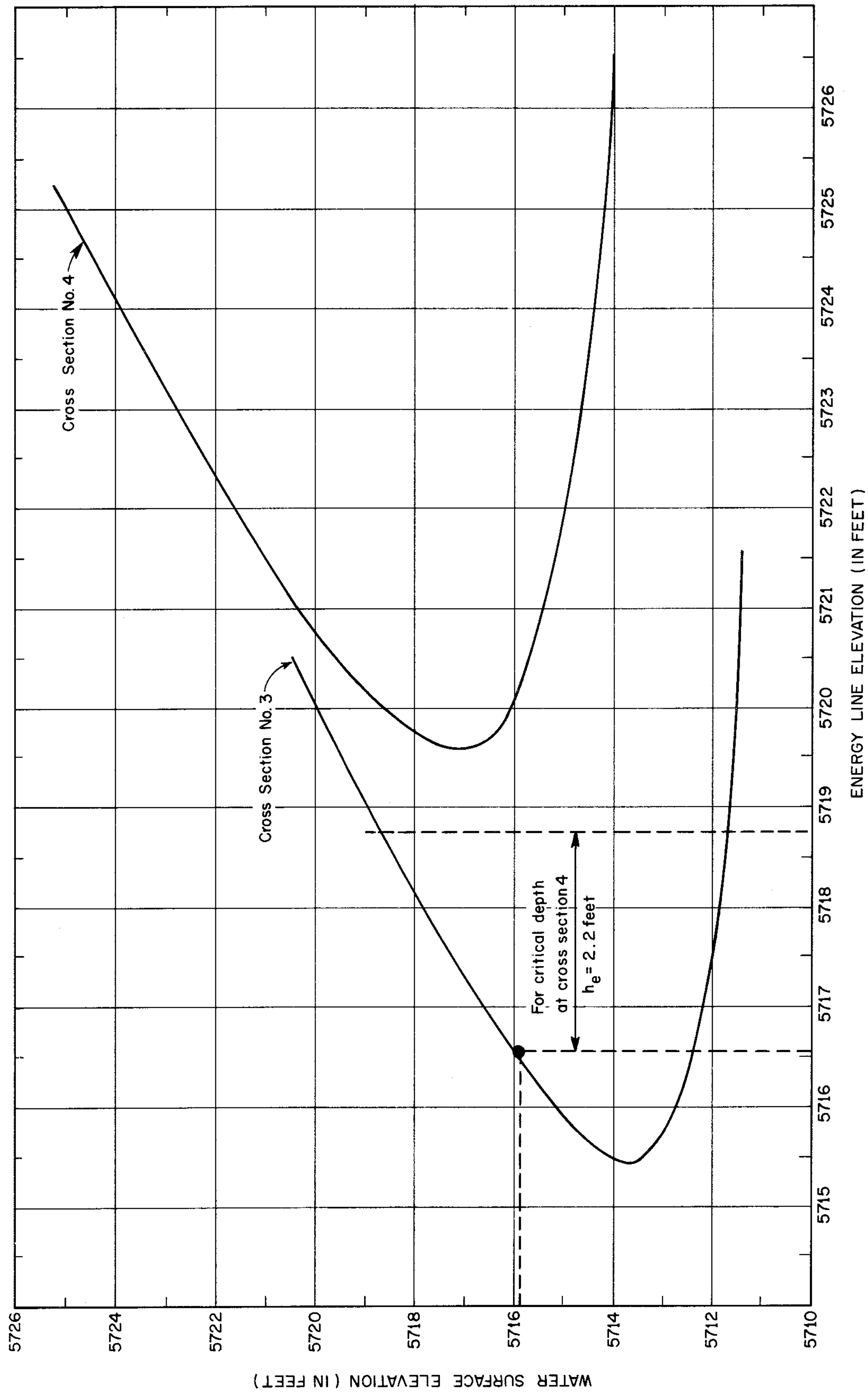


Fig. 6.08 Total energy curves for cross sections 3 and 4

the same conditions at section 3 that were presented above. In general, the average slope equation seems to predict a higher water surface elevation than the average conveyance equation. This has an advantage for this particular problem, but its disadvantages for calculating M2 profiles offset the apparent advantage on this M1 profile. In manual calculations both friction loss equations could be used; however, a computer program would require extremely detailed code to prevent instabilities and endless looping from occurring if optional formulations of the friction loss equations were permitted in a single run, Therefore, additional cross sections are recommended when this type of problem occurs.

The water surface and energy line profiles plotted in fig. 6.09 indicate critical depth at section 4. The results from all water surface profile studies should be plotted to aid in locating errors.

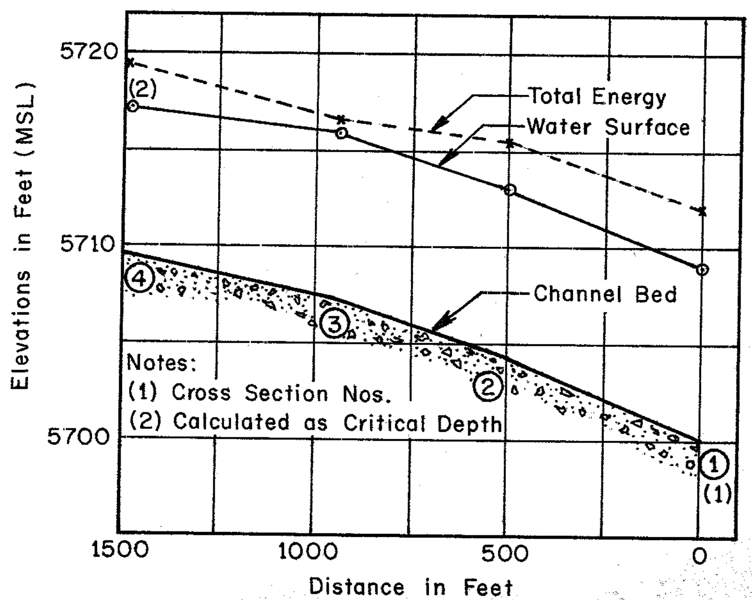


Fig. 6.09. Bed, water surface, and energy line profiles

Supercritical Flow Profiles

Even though the actual depth was below critical depth at section 4, it cannot be assumed that supercritical flow will exist upstream from section 4. The best approach is to continue computations to the upstream boundary of the model utilizing critical depth each time calculated values are less than critical. Then, if supercritical flow exists at several sections, computations can start at the upstream limit of the model and proceed downstream through the same set of cross sections. In this case, the rating curve for the upstream boundary must be available to obtain starting water surface elevations.

The calculation procedure is identical to that for subcritical flow and table 6.02 is generally applicable. However, all references to "upstream" and downstream" locations must be reversed. A more general reference would be "present section" for the cross section currently being analyzed and "previous section" for the section at the other end of the reach. Also, ΔWS will be subtracted rather than added to the previous section's value to obtain that for the "present section."

Divided Flow

Where an island or other obstruction in the river separates flow into two or more channels over a substantial length (several cross sections), the quantity of water passing on each side of the island must be determined since total energy loss, past the island, must be the same for both sides.

The example in fig. 6.10 illustrates how to solve the divided flow problem graphically. The total discharge is proportioned between the north and south channels, arbitrarily. The water surface elevation for the total flow is determined for river mile 10.0, and a water surface profile is calculated for each assumed discharge through the north channel and through the south channel. The resulting potential water surface elevations at river mile 10.8 are plotted in fig. 6.11. A "total" discharge curve is obtained at river mile 10.8 by summing north and south channel discharges for common water surface elevations. This total flow curve intersects the total river discharge, 5000 cfs, at elevation 104.32, thereby defining the upstream water surface elevation. By intersecting the north and south channel curves at elevation 104.32, their respective discharges can be read from the figure.

PROBLEM:

Using the data for the discharge of 5,000 units tabulated below, determine the water-surface elevation at river mile 10.8 when the water-surface elevation at river mile 10.0 is 100.0. What is the discharge in the channel north of the island?

WATER-SURFACE PROFILES FOR VARIOUS BACKWATER PROFILES

CROSS SECTION RIVER MILE	PROFILE 1 NORTH CHANNEL Q = 1000	PROFILE 1 SOUTH CHANNEL Q = 4000	PROFILE 2 NORTH CHANNEL Q = 1500	PROFILE 2 SOUTH CHANNEL Q = 3500	PROFILE 3 NORTH CHANNEL Q = 2000	PROFILE 3 SOUTH CHANNEL Q = 3000	PROFILE 4 NORTH CHANNEL Q = 2500	PROFILE 4 SOUTH CHANNEL Q = 2500
10.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
10.2	100.6	101.4	100.8	101.2	101.0	101.1	101.2	101.0
10.4	101.2	102.8	101.6	102.5	102.0	102.2	102.4	102.0
10.6	101.8	104.2	102.4	103.7	103.0	103.3	103.6	103.0
10.8	102.4	105.5	103.3	105.0	104.1	104.5	104.8	104.0

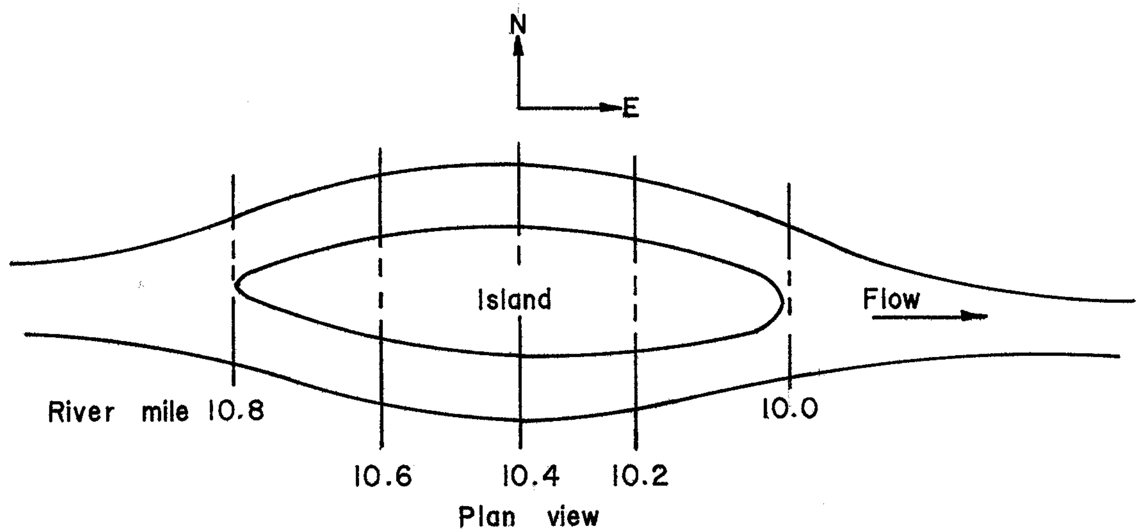


Fig. 6.10. Example problem for subcritical island type flow computations

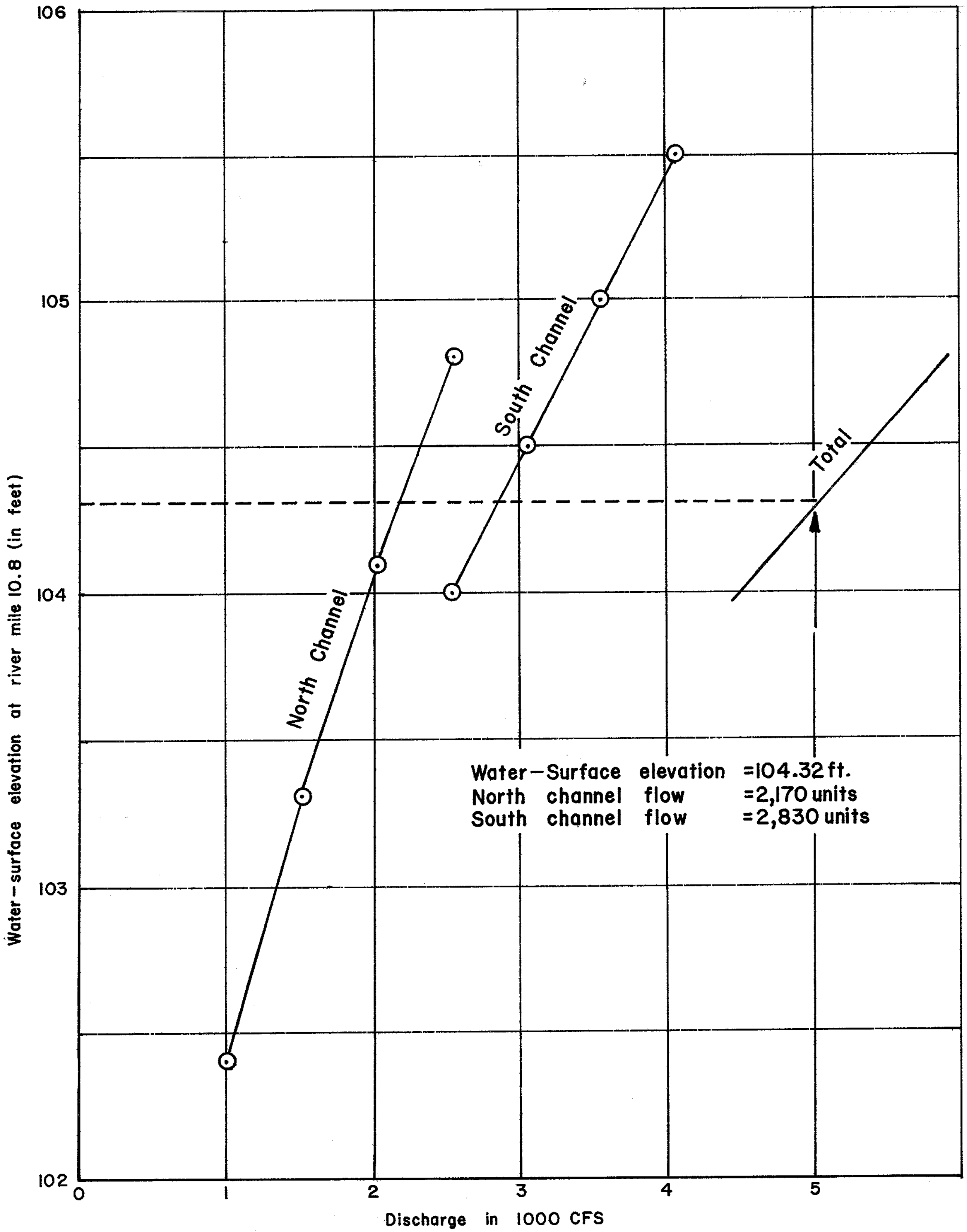


Fig. 6.11. Graphical interpolation for divided flow past an island

Section 6.03. Head Loss at Bridges and Weirs

There are three methods for evaluating the head loss at bridges: (1) continue water surface profile calculations through the bridge opening using Manning's equation and contraction and expansion coefficients; (2) use momentum equations to account for energy losses due to bridge piers and use contraction and expansion coefficients to account for the other losses; and (3) use the energy equation between sections located in zones of uniform flow on either side of the bridge and account for friction and form losses with empirical criteria from hydraulic model and prototype studies.

The advantage of method 1 is its simplicity and its ability to give reasonable answers where the bridge offers little obstruction to the flow. Energy losses at a bridge are primarily a form loss. That is, flow must contract to pass through a small area then expand to fill the entire flood plain region. Piers in the contracted opening contribute an additional form loss. The Manning equation can predict a friction loss (often very minor for bridges), and contraction and expansion coefficients can predict the form loss. However, the major problem in using this technique occurs because most data on these coefficients are calculated from profiles that do not include bridges. In some conditions these derived coefficients may not be too representive for bridges.

Method 2 can be used in supercritical as well as subcritical flow problems. The major disadvantage is that available data is limited to pier losses and, in subcritical flow problems, these are often a minor part of the total loss. The contraction and expansion losses that result from changes in flow width as water passes through the contracted opening of the bridge are usually major losses and, at the present time, contraction and expansion coefficients must account for these losses. The disadvantage is as stated for method 1.

The advantage of method 3 is that it accounts for energy losses at a contraction with coefficients developed in hydraulic model and prototype studies. The disadvantages are that tests did not include supercritical flow conditions and depths which submerge the bridge deck were not used. References 23 and 28 present more detailed information on this approach.

All three methods are used in practice and have been developed into computer solutions. Unfortunately, they do not always give comparable answers for the same fixed set of conditions. The "Water Surface Profiles" computer program presented in Appendix 2 of Volume 1 of this report utilizes methods 1 and 2 plus an energy loss equation formulated similar to method 3. The procedures also include the use of weir flow and combinations of various types of flow.

None of these methods account for local scour. However, in designing bridge piers, scour must be considered, and Volume 12 of this report gives some design criteria for scour. Also, debris often reduces the flow area of bridge openings so that large energy losses result. In one case, the head loss through a railroad trestle was reduced from 2.5 feet (.076m) to 0.5 foot (0.15m) just by removing the debris from the upstream side. The analyst must be aware of these possibilities.

A common problem in open channel flow is to evaluate the head loss at bridges where flow submerges the bridge structure and sometimes even the approach embankments. One-dimensional flow theory is very limited for this application and most procedures that account for losses do not extend to submerged bridge decks. When these problems occur, one approach is to separate flow passing through the opening from that over the top of the bridge. Energy losses for flow over the top of the bridge deck are calculated by the weir equation. Flow underneath the bridge deck is analyzed for losses by using the orifice equation. The computations are repeated for several flow distributions until a common energy elevation upstream from the bridge is determined. The results of this technique are shown in fig. 6.12 where "low flow control" means that the water surface does not contact the bridge deck of substructure. Another approach is to account for the cross section area occupied by the bridge deck and structure by using discharge coefficients to reduce the gross area of the cross section to the desired net area. Again, experimental data are not available to substantiate how much overreduction, if any, should be made to account for the three dimensional flow conditions.

The weir equation is satisfactory for calculating losses over the embankments, but corrections must be made for submerged conditions as discussed in Chapter 4. Moreover, the flow that passes over the embankment will undergo considerable friction loss passing over the flood plain to and from the embankment. Such losses must be accounted for. In these situations, it is often desirable to divide the flood plain into strips in the direction of flow and calculate the flow distribution between

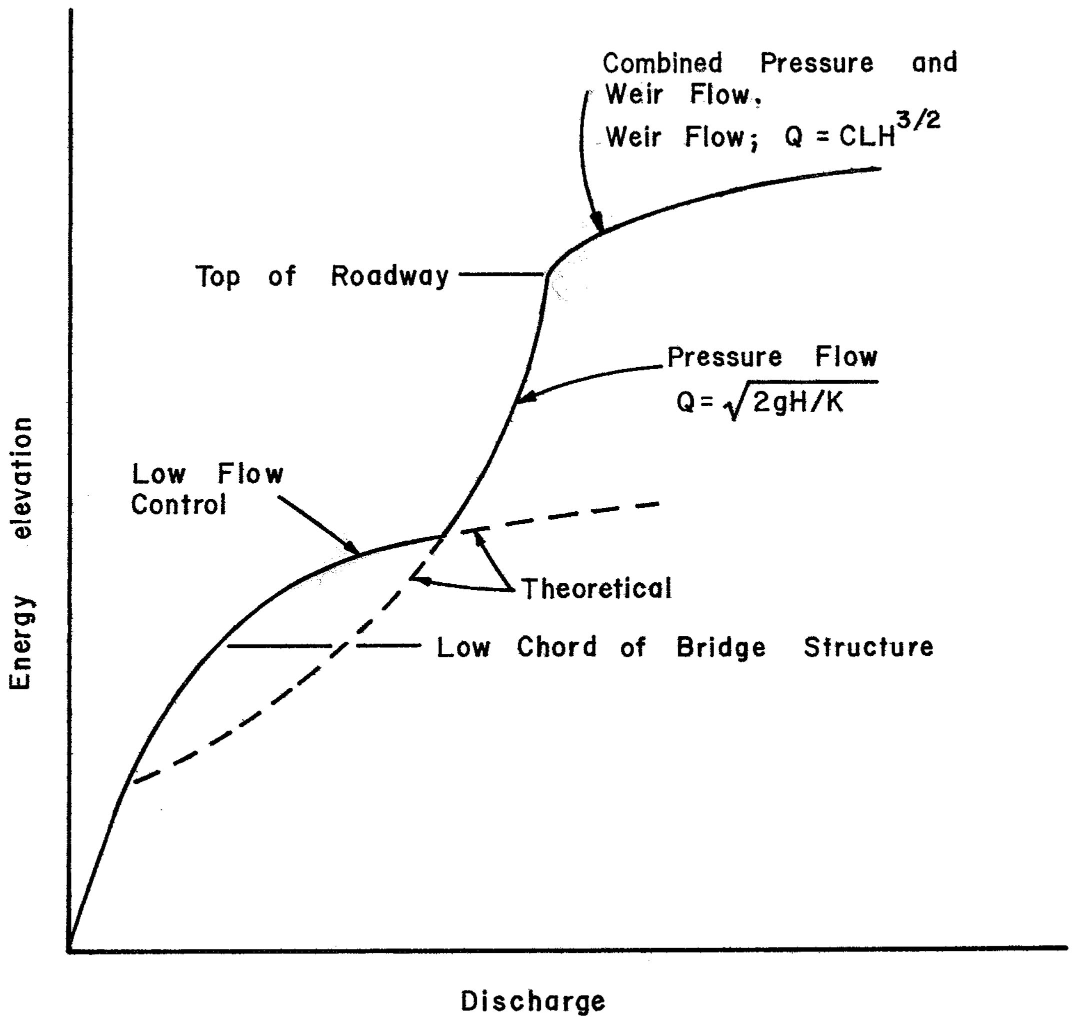


Fig. 6.12. Energy-discharge curve for bridges

bridge opening and approach embankments separately. This can best be done by beginning at some point downstream and ending at some point upstream where a common water surface in all strips can logically be expected to occur. The method was discussed earlier for water flowing past an island.

Approximating a two-dimensional flow pattern with this one-dimensional computation procedure requires the basic assumption that there is no interchange of flow between strips. As a result the water surface elevations are often quite different in adjacent strips at the same cross section. This indicates a transfer of flow between strips and although the rate of flows being transferred is not known, these differences in water surface elevation do indicate where and in which direction the transfer should occur.

Section 6.04. Energy Losses at Culverts

The distinction between flow through bridges and flow through culverts is important because of the different hydraulic conditions at these structures. Culverts produce the most complicated flow conditions and require a careful analysis to determine the proper energy loss. A structure should probably be treated as a culvert when its length, in the direction of flow, exceeds about 20 times its height. In that distance, a hydrostatic pressure distribution has developed in the barrel and the boundary layer of flow has grown to the point where friction loss becomes important.

Section 6.05. Bank Stabilization Structures

The technique of approximating a two-dimensional flow problem with a one-dimensional computation model has also been utilized to design bank stabilization structures. The spacing and height of stone-fill dikes set perpendicular to the current was established for a specified discharge such that water was attracted into the dike field by a slight gradient in that direction. In these studies, the difference in water surface between a strip through the dike field and one in the main channel was adjusted by raising or lowering individual structures until water surfaces just upstream of the structure were equal in both strips. This caused the difference in water surface downstream from the dike to be lower in the dike field strip than in the main channel resulting in a postive pressure to deflect sediment material into the dike field. At the same time, the potential to draw flow out of the main channel was arrested by controlling the head loss over the structures. The Manning's equation was used to calculate friction loss and the weir equation, corrected for submergence, was used to calculate head loss at the dikes.

Section 6.06. Flood Plain Encroachments

Often it is desirable to develop a portion of the flood plain for urban or other uses. A common criteria for determining an acceptable amount of encroachment is related to a permissible increase in the water surface elevation. To use this type of criteria it is necessary to determine how much the flood plain width must be decreased in order to increase the water surface elevation by some specified amount for the same discharge. Conveyance has proved to be the most reliable parameter to test. For example, to increase a water surface profile by 1.0 foot (0.3 m) for the same discharge means the sides can be moved toward the center line of the channel until the remaining conveyance at the 1-foot increase in elevation equals the conveyance of the original cross section at the original water surface elevation. This test seems to work better when water surface slopes are smaller than when large slopes exist. However, it is used for both.

Section 6.07. Wide Flood Plains

In the case of infinitely wide flood plains, where flows exceed the channel capacity and spill into the flood plains, the effective cross section is not easily defined. One way to solve this problem is to shape the cross-section alignment so that the overbank portions curve upstream. This curvature will contain the water in a reasonable width since the cross section elevations on the edge of the flow will be higher than in the case of the straight cross section alignment. The degree of curvature is a matter of judgment.

Chapter 7

Unsteady Flow In Open Channels

CHAPTER 7. UNSTEADY FLOW IN OPEN CHANNELS

Section 7.01. Introduction

The classification for unsteady flow is discussed in Chapter 3. Interest in unsteady flow stems from the need to calculate the movement of flood waves. Two basic techniques are being used: (1) methods which approximate in some general sense a solution to the basic equations of unsteady flow, and (2) methods which solve the basic equations themselves. The first type of methods is sometimes referred to as "hydrologic" routing methods and the second type "hydraulic" routing methods. The following discussion pertains to the second type of methods as they are presently being used. Their value lies in the fact that surges, backwater resulting from channel junctions or reservoirs, effects of tidal fluctuations, normal flood waves and other unsteady flow problems can be analyzed. Such studies may be described as the calculation of gradually varied unsteady flow profiles, that is, the determination of water surface, discharge and velocity profiles in both the time and space domains.

Interest in flood routing techniques is not new, but capability to perform routing with the complete equations came with the modern, high-speed computer and resulted in numerous methods and a great deal of discussion about "unsteady flow." The required basic data includes that required for calculating steady flow profiles and as well as inflow hydrographs of either elevation or discharge.

Section 7.02. Basic Equations

The Saint-Venant equations form the basis for unsteady flow analysis techniques:

$$\frac{\partial (AV)}{\partial x} + T \cdot \frac{\partial (WS)}{\partial t} - q = 0 \qquad \text{(continuity)} \qquad (7\text{-}01)$$

$$\frac{1}{g}\frac{\partial V}{\partial t} + \frac{\partial (WS)}{\partial x} + \frac{V}{g}\frac{\partial V}{\partial x} + S_f + \frac{q}{A}\frac{V}{g} = 0 \qquad \text{(energy)} \qquad (7\text{-}02)$$

where:

A = area of cross section

T = width at water surface

g = acceleration of gravity

q = lateral flow

S_f = friction slope from Manning equation

t = time

WS = water surface elevation

x = distance along channel in direction of flow

There are six basic solution techniques:

1. Explicit solution of basic equations, fixed mesh
2. Implicit solution of basic equations, fixed mesh
3. Explicit solution of characteristic formulation from basic equations, fixed mesh
4. Implicit solution of characteristic formulation from basic equations, fixed mesh

5. Explicit solution of characteristic equations using characteristic network

6. Implicit solution of characteristic equations using characteristic network

The anticipated advantage of the characteristic formulations is that solutions proceed along characteristic lines that are best for high velocities (near-critical and above). The disadvantages have been the inability to maintain continuity as well as the problem of calculating celerity in complex natural channel sections. Characteristic solutions have not been entirely satisfactory for the general case.

The solution techniques that have proved to be the most useful are explicit and implicit techniques for solving the basic equations. Theoretically these are valid only for subcritical, gradually varied flow, but they have been successfully used to route even the dambreak flood. The major disadvantage of explicit methods is the small time increment required in this routing technique. Two minutes is a long interval, while 30 seconds is more likely the value to expect. However, in routing surges, changes occur so rapidly that short times are required to model the event, and the short computation interval is not a major consideration.

Section 7.03. Verifying the Model

Three steps are important for verifying that the unsteady flow model reproduces historical events. First, the storage volume contained in the geometric model must accurately reproduce the actual storage. Second, the calculated water surface profiles must reproduce historical, steady flow profiles. Finally, reproducing historical flood hydrographs will verify that the model can route hydrographs through the reach of interest. Once the model is verified, a variety of problems can be analyzed by changing basic parameters. The only coefficient required is Manning's n-value, and this is calibrated in step 2 above.

Verification is a trial and error process. It can be very time-consuming. However, it is the essential first step in performing simulation studies and must be done carefully to insure a successful study. If sufficient data is not available to verify the model, consideration should be given to using the more approximate flood routing techniques.

Section 7.04. Conveyance vs. Volume

In these one-dimensional models, it is important to determine what portion of the cross section conveys flow and what portion stores water. One concept is to assume all conveyance occurs in or near the main channel, and the overbanks only store water. Since 80 to 85 percent of the flood flow often is carried in the main channel, this procedure appears reasonable. Another approach involves attempting to account for a lateral outflow/inflow interchange between main channel and overbank. This has not been entirely successful. A third method assumes that the entire cross section conveys flow. While this is the case for very large floods, there will be some difficulty in verification of the smaller events.

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