A United States Contribution to the International Hydrological Decade

HEC-IHD-1000

Hydrologic Engineering Methods For Water Resources Development

Volume 10
Principles of Ground-Water Hydrology

April 1972

IHD-10

REPORT DOCUMENTATION PAGE			*Form Approved OMB No. 0704-0188*
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the date needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.			
1. AGENCY USE ONLY *(Leave blank)*	**2. REPORT DATE** April 1972	**3. REPORT TYPE AND DATES COVERED** IHD Volume 10	
4. TITLE AND SUBTITLE Principles of Ground-Water Hydrology			**5. FUNDING NUMBERS**
6. AUTHOR(S) Dr. Richard L. Cooley, Dr. John F. Harsh, David C. Lewis			
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) US Army Corps of Engineers Institute for Water Resources Hydrologic Engineering Center 609 Second Street Davis, CA 95616-4687			**8. PERFORMING ORGANIZATION REPORT NUMBER** IHD-10
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES) N/A			**10. SPONSORING / MONITORING AGENCY REPORT NUMBER** N/A
11. SUPPLEMENTARY NOTES			
12a. DISTRIBUTION / AVAILABILITY STATEMENT Approved for Public Release. Distribution of this document is unlimited.			**12b. DISTRIBUTION CODE**
13. ABSTRACT *(Maximum 200 words)* This is Volume 10 of the 12 volume report prepared by the Hydrologic Engineering Center of the U.S. Army Corps of Engineers as a contribution to the International Hydrological Decade. This volume describes the fundamental concepts which govern the occurrence and movement of ground water, and presents the basic principles of hydrology, geology, hydraulics, and physics as they relate to ground water. The information presented is sufficient to guide hydrologic engineers in the identification and initial considerations of ground-water aspects of water resource development studies. A generalized computer program description, "Finite Element Solution of Steady-State Potential Flow Problems", is included as part of this volume. This program is intended for application to problems involving steady two-dimensional or axisymmetric flow through heterogeneous, anisotrapic porous media of virtually and internal or external geometry.			
14. SUBJECT TERMS International Hydrological Decade, groundwater, hydrology, aquifers			**15. NUMBER OF PAGES** 406
			16. PRICE CODE
17. SECURITY CLASSIFICATION OF REPORT UNCLASSIFIED	**18. SECURITY CLASSIFICATION OF THIS PAGE** UNCLASSIFIED	**19. SECURITY CLASSIFICATION OF ABSTRACT** UNCLASSIFIED	**20. LIMITATION OF ABSTRACT** UNLIMITED

Hydrologic Engineering Methods for Water Resources Development

Volume 10
Principles of Ground-Water Hydrology

April 1972

US Army Corps of Engineers
Institute for Water Resources
Hydrologic Engineering Center
609 Second Street
Davis, CA 95616

(530) 756-1104
(530) 756-8250 FAX
www.hec.usace.army.mil

IHD-10

FOREWORD

This volume is part of the 12-volume report entitled "Hydrologic Engineering Methods for Water Resources Development." The report is being prepared by The Hydrologic Engineering Center as part of the U. S. Army Corps of Engineers' participation in the International Hydrological Decade. The contents of this volume differ from the other volumes in that practical step-by-step procedures for determining the ground-water aspects of water resource development studies are not presented. Instead, the fundamental concepts which govern the occurrence and movement of ground water receive the major emphasis. The purpose of this volume is to present the basic principles of hydrology, geology, hydraulics, and physics as they relate to ground water. The information presented is sufficient to guide hydrologic engineers in the identification and initial considerations of ground-water aspects of water resource development studies. However, the report should not be construed to represent the official policy or criteria of the Corps of Engineers.

Contributions to this volume were made by:

Dr. Richard L. Cooley	Chapters 4, 5, 6, and 7, Appendices 1-4 and 6.
Dr. John F. Harsh	Chapters 1, 2, 3, 6, 7, 8, 10, 11, and 12, Appendices 5 and 6.
David C. Lewis	Chapters 6, 7, and 9, Appendix 6.

CONTENTS

		Page
FOREWORD		iii
CHAPTER 1.	GEOLOGY RELATED TO SURFACE AND SUBSURFACE WATER MOVEMENT	1-01
1.01.	Introduction	1-01
1.02.	Geologic Controls on the Occurrence and Movement of Surface Water	1-01
1.03.	Geologic Controls on the Occurrence and Movement of Subsurface Water	1-07
CHAPTER 2.	EFFECTS OF STRATIGRAPHY AND SEDIMENTATION ON GROUND-WATER OCCURRENCE AND MOVEMENT	2-01
2.01.	Introduction	2-01
2.02.	Lateral Changes in Lithology	2-01
2.03.	Attitude of Strata	2-02
2.04.	Hydrogeologic Significance of Unconformities	2-03
2.05.	Relation of Subsurface Water to Sedimentation and Stratigraphy	2-06
CHAPTER 3.	STRUCTURAL CONTROLS ON GROUND-WATER OCCURRENCE AND MOVEMENT	3-01
3.01.	Folds	3-01
3.02.	Joints and Veins	3-02
3.03.	Faults	3-03
3.04.	Relation of Subsurface Water to Structural Geology	3-09
CHAPTER 4.	THEORY OF GROUND-WATER MOTION	4-01
4.01.	Introduction	4-01
4.02.	Fundamental Concepts	4-01
4.03.	The Hydrologic Budget Equations	4-03
4.04.	Hydraulic Potentials and Darcy's Law	4-08
4.05.	Continuity Relationships	4-18
4.06.	Differential Equations Describing Fluid Flow	4-21
CHAPTER 5.	FLOW NETS	5-01
5.01.	Introduction	5-01
5.02.	Theory and Background	5-02
5.03.	Construction of Flow Nets	5-09
5.04.	Modified Areal Flow Net Analysis	5-19
5.05.	Computer Program for Determining Hydraulic Head Distribution	5-23

CONTENTS

		Page
CHAPTER 6.	GROUND-WATER LEVELS AND FLUCTUATIONS	6-01
6.01.	Introduction	6-01
6.02.	Ground Water and Streamflow	6-03
6.03.	Major Water Level Fluctuations in Wells	6-10
6.04.	Minor Water Level Fluctuations in Wells	6-21
CHAPTER 7.	FLOW TO WELLS	7-01
7.01.	Physics of Ground-Water Flow to Wells	7-01
7.02.	Curve-Fitting Techniques to Evaluate Aquifer Constants	7-13
7.03.	Applicability of Well Hydraulic Theory	7-20
7.04.	Aquifer Test Procedure	7-24
CHAPTER 8.	SURFACE INVESTIGATION OF GROUND WATER	8-01
8.01.	Introduction	8-01
8.02.	Gravity, Magnetic, and Seismic Methods	8-02
8.03.	Electrical Methods	8-04
CHAPTER 9.	SUBSURFACE INVESTIGATION OF GROUND WATER	9-01
9.01.	Borehole Logging	9-01
9.02.	Mechanical Methods	9-02
9.03.	Geophysical Logs	9-03
9.04.	Radioactive Logs	9-10
9.05.	Composite Logs	9-11
9.06.	Ground-Water Tracing	9-15
CHAPTER 10.	OCCURRENCE OF GROUND WATER IN IGNEOUS AND METAMORPHIC ROCKS	10-01
10.01.	Introduction	10-01
10.02.	Physical and Hydraulic Properties	10-01
10.03.	Well Yields	10-03
10.04.	Hydrochemistry	10-04
10.05.	Exploration	10-09
10.06.	Development of Ground Water	10-10
CHAPTER 11.	OCCURRENCE OF GROUND WATER IN SEDIMENTARY ROCKS	11-01
11.01.	Introduction	11-01
11.02.	Physical and Hydraulic Properties	11-01
11.03.	Well Yields	11-05
11.04.	Hydrochemistry	11-05
11.05.	Exploration	11-08
CHAPTER 12.	OCCURRENCE OF GROUND WATER IN UNCONSOLIDATED EARTH MATERIALS	12-01
12.01.	Introduction	12-01
12.02.	Alluvial Deposits in Small River Valleys	12-01

CONTENTS

		Page
12.03.	Unconsolidated Deposits in Large Valleys	12-07
12.04.	Eolian Deposits	12-11
12.05.	Glacial Deposits	12-13
12.06.	Shoreline and Coastal Plain Deposits	12-18
SELECTED BIBLIOGRAPHY		SB-1

APPENDIX 1, DERIVATION OF DARCY'S LAW

APPENDIX 2, DERIVATION OF CONTINUITY EQUATION

APPENDIX 3, DEVELOPMENT OF FLOW NET THEORY

APPENDIX 4, AQUIFER TEST DATA REDUCTION PROGRAM FOR WATER-TABLE AQUIFERS

APPENDIX 5, CHARACTERISTICS OF WATER-BEARING MATERIALS

APPENDIX 6, DEFINITIONS OF SELECTED GROUND-WATER TERMS

APPENDIX 7, FINITE ELEMENT SOLUTION OF STEADY STATE POTENTIAL FLOW PROBLEMS

Follow page SB-18

LIST OF FIGURES

1.01.	The runoff cycle	1-02
1.02.	Topographic sketches of drainage patterns	1-04
1.03.	Influence of igneous activity on surface drainage	1-04
1.04.	Development of weathering complex from parent rock	1-10
1.05.	Schematic diagram of physio-chemical processes	1-11
1.06.	Normal or zonal soil profile (podzol type)	1-12
1.07.	Types of rock interstices and relation of rock texture to porosity	1-16
2.01	Section showing shallow water conditions	2-03
2.02	Generalized stratigraphic column in the Illinois Basin . . .	2-05
3.01	Effects of folding on ground water	3-02
3.02	Names and definitions of principal kinds of faults	3-04
3.03	Possible effects of faults on positions of aquifers	3-06
3.04	Aquifers formed by erosion of fault scarps	3-07
3.05	Section showing displacement of an aquifer by a fault	3-07
4.01	Confined and unconfined ground water	4-03
4.02	The hydrologic budget	4-04
4.03	Approximate flow distribution for a homogeneous region . . .	4-05
4.04	Local flow system due to a hummocky water table	4-06
4.05	Saturated water flow through a soil column	4-11
4.06	Cross section through a flow system	4-12
5.01	Intersection of a streamtube and two equipotential	5-03
5.02	Intersections between free surfaces and other boundaries . .	5-05
5.03	Flow lines between regions of differing conductivity	5-07
5.04	Flow net under an impermeable structure	5-08

CONTENTS

LIST OF FIGURES

		Page
5.05.	Seepage with a free surface through a permeable structure . .	5-09
5.06.	Flow net for a trapezoidal channel	5-12
5.07.	Flow net for a triangular channel	5-14
5.08.	Flow net through a dam for two conductivity regions in series	5-16
5.09.	Flow net through a dam for two conductivity regions in parallel	5-16
5.10.	Flow net of ground-water flow	5-18
5.11.	Symbols used in modified areal flow net analysis	5-20
5.12.	An example of vertical leakage to a confined aquifer	5-21
6.01.	Hydrograph shape according to condition 1	6-05
6.02.	Hydrograph shape according to condition 2	6-06
6.03.	Hydrograph shape according to condition 3	6-07
6.04.	Hydrograph shape according to condition 4	6-07
6.05.	Hydrographs of August 10, 1959 at measuring stations 1 and 2	6-08
6.06.	Hydrographs of August 3-4, 1959, at measuring stations 1 and 2	6-09
6.07.	Regional flow system of the Nevada Test Site	6-11
6.08.	Ground-water fluctuations due to daily irrigation pumping cycle	6-14
6.09.	Surface-water fluctuations at Winnemucca gaging station . . .	6-16
6.10.	Ground-water fluctuations and natural discharge from a sand and gravel aquifer	6-17
6.11.	Definitive sketch for ET from ground water	6-24
6.12.	Ground-water level and its rate of change versus time	6-25
7.01a.	Cross section through a leaky aquifer system	7-02
7.01b.	Detail of an elemental volume element in the aquifer and aquitard	7-02
7.02.	Logarithmic graph of the function $W(u, \beta)$	7-15
7.03.	Logarithmic graph of the function $W(u, \frac{r}{\beta})$	7-16
7.04.	Recovery in observation well 16N3, Pixley	7-22
7.05.	Semilogarithmic graph of the function $W(u, \beta)$	7-23
8.01.	Cross section of direct current electrode configuration . . .	8-09

CONTENTS

LIST OF FIGURES

		Page
8.02.	Wenner electrode configuration	8-12
9.01.	Transient electrode configuration	9-05
9.02.	Idealized gamma ray and electric logs	9-14
12.01.	Relation between median grain size and permeability	12-03
12.02.	Relation between permeability and texture	12-04

LIST OF TABLES

1.01.	Range in porosity for selected types of earth materials . .	1-18
1.02.	Aquifer coefficients of unconsolidated sand or sand and gravel units	1-18
1.03.	Aquifer coefficients of indurated or semiconsolidated rocks	1-19
1.04.	Aquifer coefficients of carbonate and basalt rocks	1-19
4.01.	Increase in hydraulic conductivity	4-16
12.01.	Glacial deposits	12-14

Chapter 1

Geology Related to Surface And Subsurface Water Movement

CHAPTER 1. GEOLOGY RELATED TO SURFACE AND SUBSURFACE WATER MOVEMENT

Section 1.01. Introduction

Hydrogeology can be defined as the interdisciplinary study of the extensive interaction between water and the geologic framework (Maxey, 1964a, p. 1). Ground-water geology is concerned with that part of the hydrologic cycle which involves meteoric water and its behavior primarily within the first mile of the earth's crust (fig. 1.01). Fresh water is in contact with earth materials from the moment it strikes the lithosphere as precipitation. Some of this water is lost by evapotranspiration, which is governed in part by the nature and topographic expression of earth materials. The relative amounts of water that enter, are stored in, and leave the surface, soil, and ground-water reservoirs also depend upon the character of soil and rocks.

The following discussion is directed toward describing the basic geologic factors involved in the studies of surface water, soil water, and ground water reservoirs. Where relevant the importance of geologic methodology and field practice is emphasized.

Section 1.02. Geologic Controls on the Occurrence and Movement of Surface Water

Geologic factors that often act as controls on surface water phenomena may be classified into two types: (1) structural, comprising folds and faults that interrupt the continuity or uniformity of occurrence of a lithologic type or sequence of strata, and (2) lithologic, depending on rock fabric, mineralogy, and sequence of rock types. These controls in conjunction with climatic processes govern the development of soils and topography which in turn strongly affect the movement and distribution of water. The magnitude of effects resulting from these geologic controls alone or in combination with other controls may be classified as either large (regional) or small (local) scale.

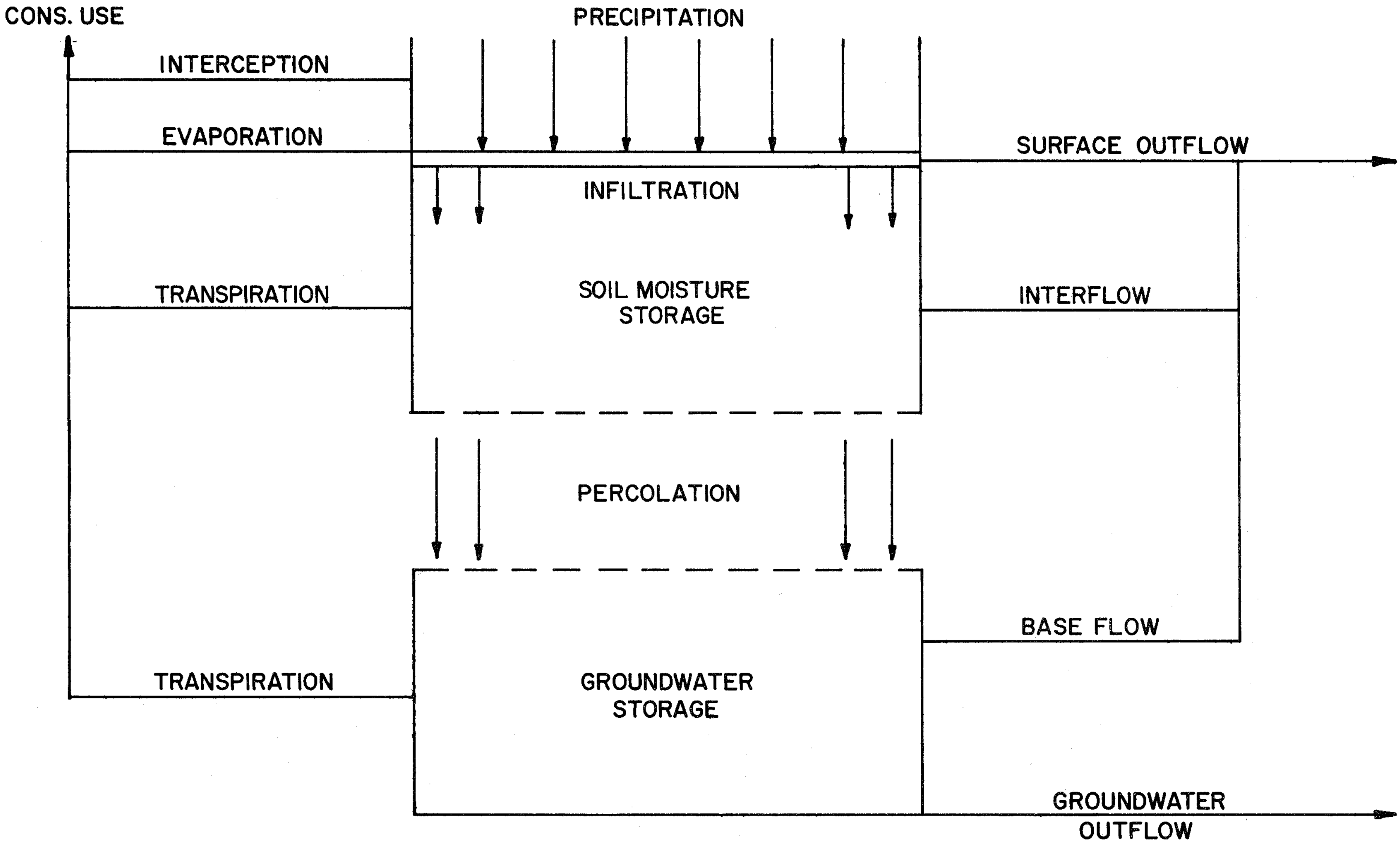


Fig. 1.01. The runoff cycle

Regional Geologic Controls

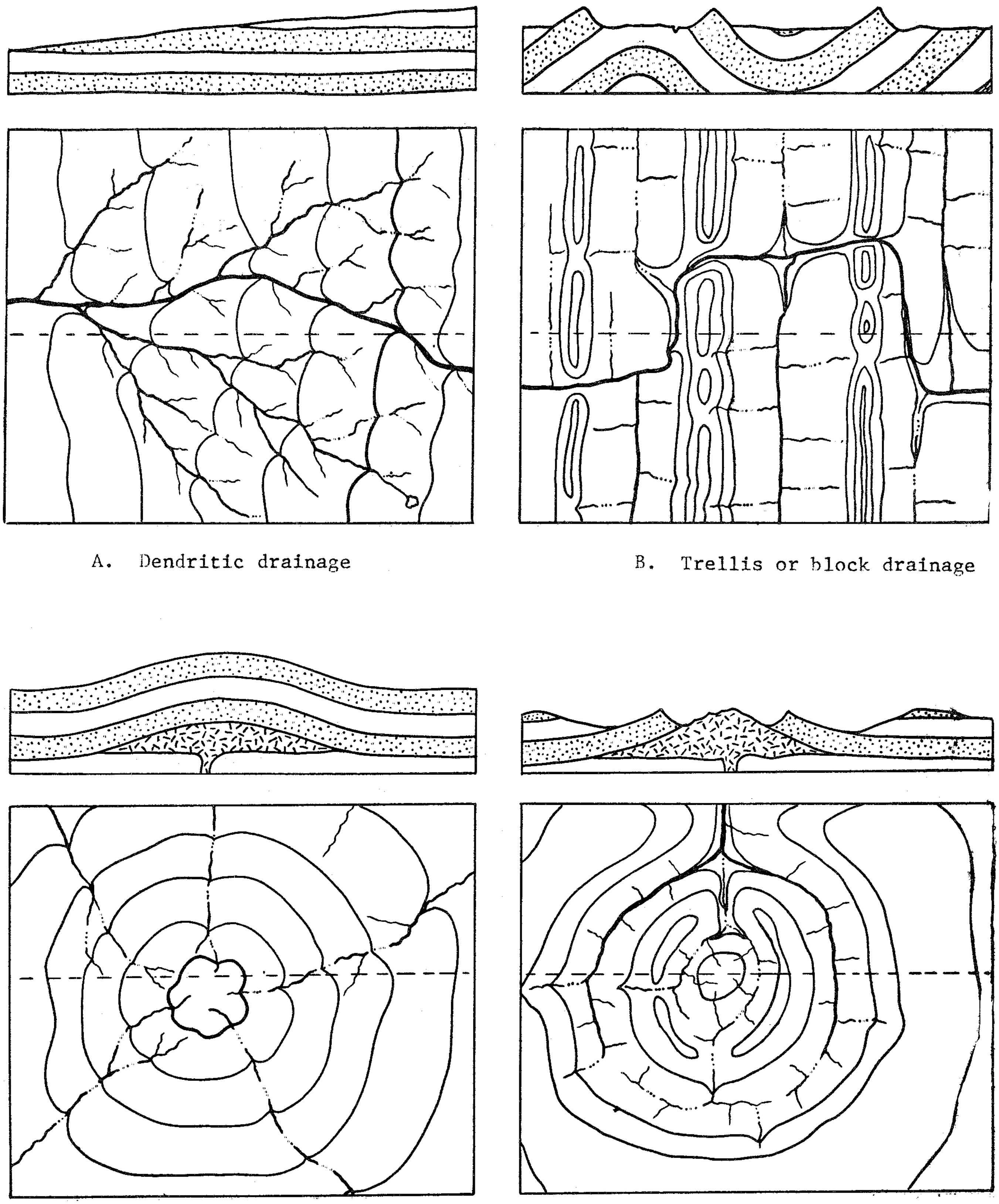
Regional controls, such as the configuration and distribution of ocean basins and continents, result from tectonic activity extending over long periods of time. This activity produces changes in erosional and depositional patterns which in turn control the placement, size, and nature of surface water bodies. The frequency and distribution of weather patterns are also influenced by large-scale tectonic changes.

Local Geologic Controls

Small-scale geologic factors often influence the type of stream drainage pattern. For example, where the strata are relatively homogenous and more or less horizontal, the drainage pattern is usually dendritic. If the strata are varied and tilted or folded, a trellis drainage pattern can result (fig. 1.02). Secondary structural controls (faults, joints) often cause streams to develop sharp bends and extraordinary straight reaches (rectangular drainage). Many other drainage patterns exist and reflect distinct geologic controls (Thornbury, 1969, pp. 117-124). In addition, geologic activities, such as tectonic movements and igneous activity, have deflected and materially altered stream courses or have blocked them to form lakes (fig. 1.03).

Lithologic and Topographic Effects

The character of the bedrock influences the behavior of streamflow. For example, the bedrock in some limestone and lava terranes is dissected by open fractures and channels. Where the soil is permeable and thin, an appreciable quantity of recharge reaches the ground-water reservoir and reappears as base flow. For example, the Deschutes River in Oregon, which drains a basaltic rock terrane, shows a relatively stable runoff pattern for its size because of the large contribution to the stream by ground water (Langbein, 1949, p. 11). On the other hand, the Yellowstone River in Montana, a stream of comparable size in a somewhat comparable climate



A. Dendritic drainage

B. Trellis or block drainage

C. Radial drainage

D. Annular drainage

Fig. 1.02. Topographic sketches of drainage patterns (Ireland 1959, p. 108

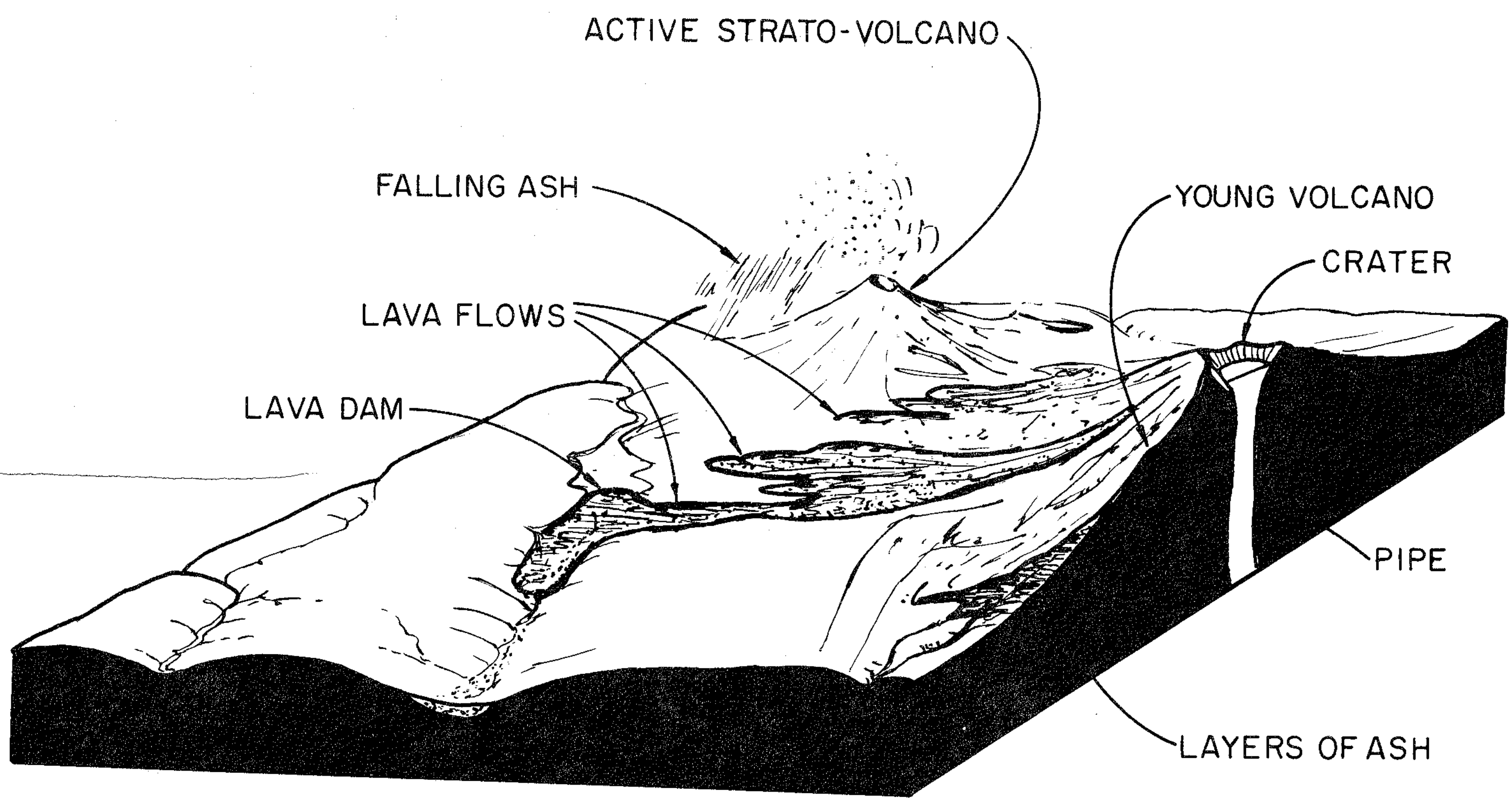


Fig. 1.03. Influence of igneous activity on surface drainage (Stahler, 1960, p. 492)

but flowing over rocks of varied types and complex structure, shows a rapidly fluctuating runoff hydrograph and low base flow.

Another geologically controlled feature of streams in limestone and lava terranes is the variation in runoff relative to recharge from one basin to another. An extreme condition occurs where streams flowing strongly in the upper reaches disappear for long stretches only to reappear again as springs. These streams are called "lost" rivers and are encountered in such areas as the Columbia Lava Plateau in Idaho, Oregon, and Washington, and in the karst regions of Kentucky and Central Europe (Maxey, 1964a, p. 5).

Anomalous distributions of runoff from basin to basin can occur because surface and ground-water divides do not coincide. Thus, low runoff in limestone and lava terranes indicates most of the discharge from these areas is ground water. In basins where ground water reaches the surface, streamflow exceeds the "normal" or expected runoff.

Topographic basins comprising large outcrop areas of confined aquifers also may show low runoff. It is again likely the cause is related to differences between surface and ground-water divides. In this case, water that recharges an aquifer is transmitted long distances away from the source of recharge. Examples are the eastward flowing streams in the Black Hills area of South Dakota (Swenson, 1968, p. 173). Streams flowing from mountains onto permeable alluvial fans in the bolsons of the Basin and Range Province demonstrate a similar, though smaller scale, process.

Runoff is often increased by the effect of highly impermeable surficial materials, particularly in mountainous terranes. Here steep slopes and low permeability surficial materials often produce erratic runoff patterns with strong flows during periods of high precipitation and negligible flow soon after precipitation ceases. Flood waters in these terranes are difficult to put to beneficial uses. However, in areas where the rocks are permeable, temporary storage of flood waters in ground-water basins allows for conjunctive use water management.

Low runoff is a common characteristic in broad, relatively flat areas of coarse textured drainage such as the High Plains of Texas, Oklahoma, New Mexico, and the Sand Hills of Nebraska (Lohman, 1953, p. 80). Shallow depressions in the relatively permeable alluvial apron collect recharge, which evapotranspires and/or infiltrates. Langbein (1949, p. 11) shows that the annual runoff from all the High Plains south of the North Platte River is less than 0.25 inches, about one-fourth of what might be expected.

Section 1.03. Geologic Controls on the Occurrence and Movement of Subsurface Water

Occurrence of Subsurface Water

The occurrence of water in the subsurface is usually divided into more or less distinct zones (Davis and DeWiest, 1966, p. 39). The following summary of these zones is modified from Davis and DeWiest (1966, pp. 39-43).

Unsaturated zone. The upper portion of this region is subject to large fluctuations in moisture content due to evaporation, plant transpiration, and precipitation. Temperature, humidity, wind speed, precipitation type and amount, and vegetation types and densities all affect the variations in moisture content.

Water below the upper portion is either percolating nearly vertically downward under the influence of gravity or is suspended due to capillarity after gravity drainage has completed. Thus, if recharge occurs, the moisture content can range from near saturation to about the value of the specific retention (Davis and DeWiest, 1966, p. 40). If little or no recharge occurs, as in a desert, then the moisture content may be very low because all precipitation that falls remains in the upper portion and is lost by evapotranspiration.

The unsaturated zone serves as a vast reservoir which, when recharged, continues to discharge water to the ground-water zone for a relatively long period after cessation of surface input. Contributions to recharge of the ground-water zone from this "in-transit storage of water" should be accounted for in water budget investigations (Wilson and DeCook, 1968, p. 1220).

Capillary Fringe. The top of the capillary fringe is the upper limit of water movement from the water table against gravity due to capillary attraction. When the surface tension on the surface of water in an interstice balances the weight of hydraulically connected water, water movement ceases. Water in the capillary fringe is under fluid pressure less than atmospheric, and moisture content can range from very low to saturated with the lower part of the capillary fringe often being saturated.

Ground-water zone. Water in this zone is called ground water and the zone is saturated. Since the lower part of the capillary fringe can also be saturated, the dividing surface, called the water table, is not necessarily the saturation surface. The water table is defined as the surface along which the fluid pressure is atmospheric (Davis and DeWiest, 1966, p. 41). Below the water table, water is under fluid pressure greater than atmospheric. The lower limit to this zone is the point below which any water present will not migrate due to lack of interconnected voids. This limit is probably measured in thousands of feet, although the limit of useful quality water is undoubtedly above this.

Geologic Controls in the Unsaturated Zone

The soil surface is the interface between the atmosphere and the lithosphere. The course water takes after falling on the surface depends primarily upon the physical, chemical and biological nature of the soil. The portion of precipitation that runs off into streams versus that which enters and percolates through the soil depends largely upon the intrinsic permeability, mineralogy, moisture content, topographic configuration and plant cover of the soil. Water is chemically modified by the soil and parent material, usually by the addition of organic acids and by the solution of various mineral salts (e.g., calcium carbonate).

Geologic factors in soil formation. Before the recognition of climatic effects, parent material was considered the most important basis for classifying soils. The reason for this can be seen from the following continuous series: bedrock ⟶ weathered rock ⟶ immature soil ⟶ mature soil (assumes soil formation in a monolithologic terrane where nearly uniform climate and vegetation exist). Immature soils and soils that develop in arid or frigid zones generally reflect the character of the parent material. Under conditions of severe weathering and leaching and in more mature soils, the effects of parent material appear to be less evident but still present. For example, in a temperate climate areas underlain by rocks of high feldspar content are characterized by mature soils with clay minerals being dominant (Gilluly et al., 1968, p. 43). The clay content of these soils varies closely with the feldspar content. In contrast, soils developed on predominantly sandy terranes contain little clay and tend to remain sandy and/or silty.

Soils formed in carbonate rock terranes are closely related to impurities in the parent rock because calcium carbonate is readily soluble and is carried away. The sand, silt, and clay fractions remain and form the parent material for the soil. Also, the presence of acid neutralizing, alkaline-earth carbonates delays normal soil development. Therefore, areas of reasonably pure limestone or dolomite are characterized by soils which are relatively thin, infertile, and immature.

Effects of soil profiles on water movement. Soil is defined here to mean the near surface cover of earth material of varying thickness (a few inches to 50 feet) that has been altered to its present form by the interaction of weather, plants, and animals. In the process of soil formation, shallow bedrock is converted into a weathering complex by decomposition, disintegration, eluviation, deposition, compaction, and alteration (fig. 1.04). The most active processes occur at the surface, and their effect is the downward movement of soluble substances (fig. 1.05). Since the effect of continuing processes is progressively downward movement, "normal" soil zones or horizons are extended consecutively deeper, forming what is called a soil profile (fig. 1.06).

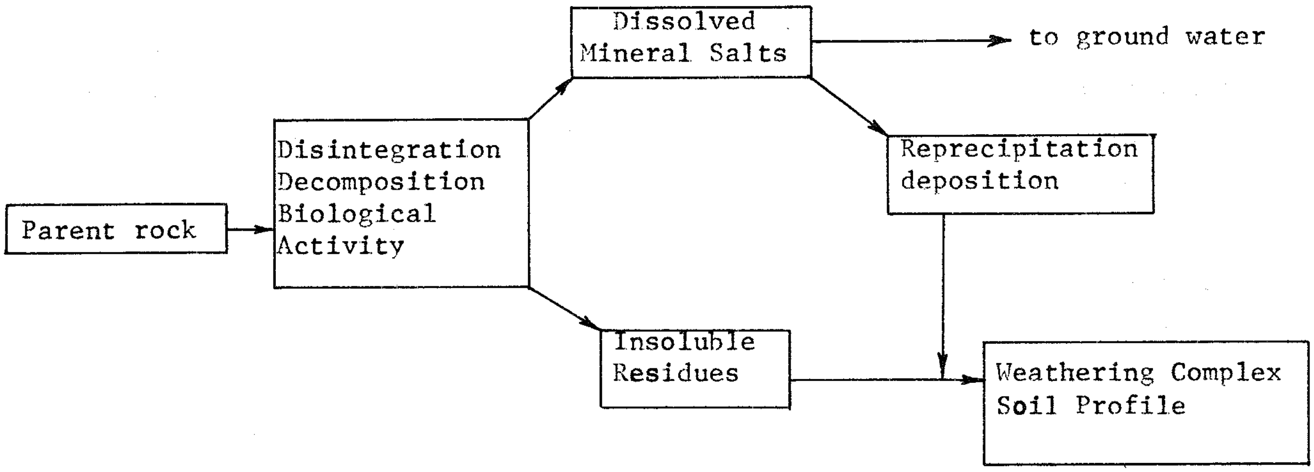


Fig. 1.04. Development of weathering complex from parent rock

The permeability distribution in a podzolic soil often mirrors the distribution of horizons. The A-horizon, being at the surface, is the most intensely weathered, with the more soluble minerals leached away and other minerals strongly altered. Structurally, the A-horizon is friable, granular, and/or platy. The A-horizon is generally more permeable than the underlying B-horizon. The B-horizon, the horizon of maximum clay accumulation, shows vertical structure in widely to closely spaced joints that may inclose columnar blocks. This horizon strongly influences the movement of water in the soil profile. When the clay is dry, vertical joints allow water to percolate downward; but when it is wet, the clay swells closing most of the openings.

The horizon of only partially weathered parent material beneath the B-horizon is called the C-horizon. In some places, carbonates and/or gypsum have accumulated in this zone and the rocks may be more or less oxidized. For the most part, this zone is more permeable than the B-horizon and does not generally impede the downward movement of water.

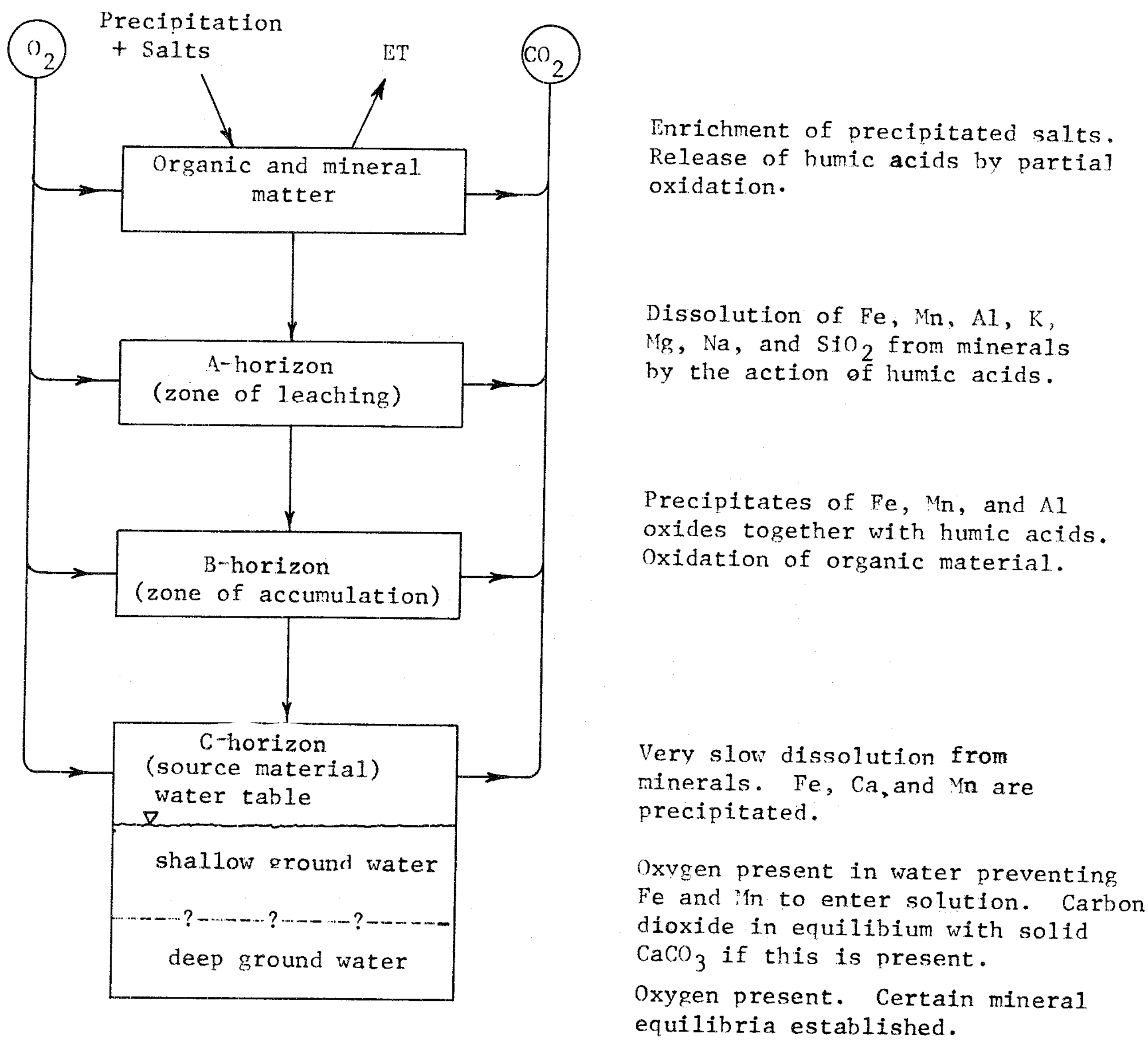


Fig. 1.05. Schematic diagram of physio-chemical processes that affect water during its passage through a podzol soil profile (Eriksson, et al., 1968, p. 100)

In addition to the zonal soils described above, there are two other categories, intrazonal soils and azonal soils. Intrazonal soils generally possess low conductivity and often form because one genetic element, for example, a high water table, a high calcium carbonate content in the parent material, or a high salt content, has altered the soil profile and prevented the development of normal horizons. Azonal soils possess no zonation and are usually thin and angular.

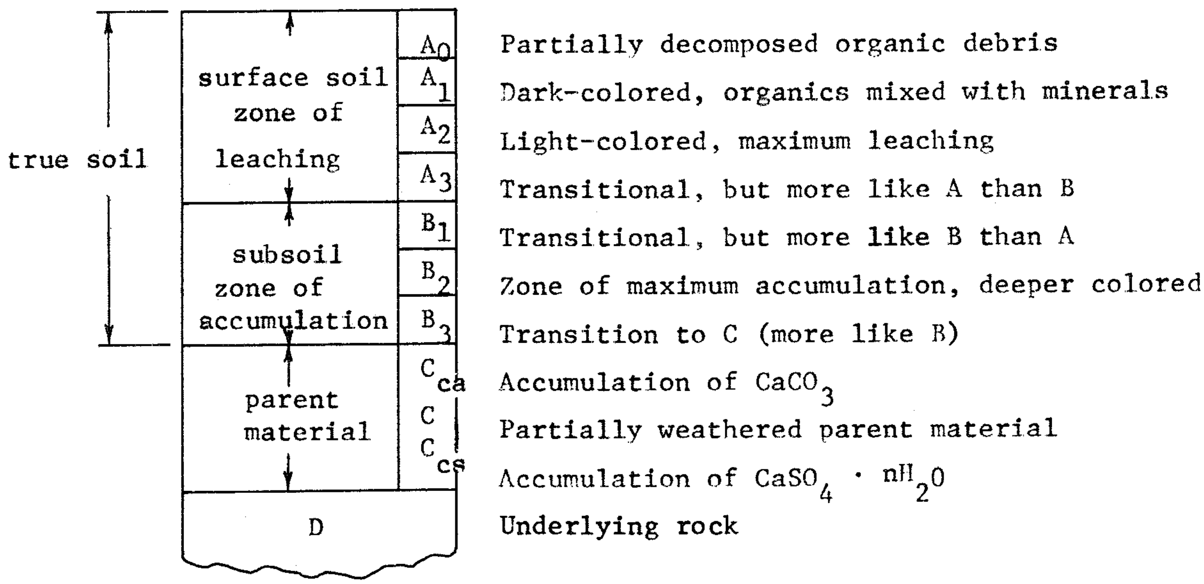


Fig. 1.06. Normal or zonal soil profile (podzol type) (modified after Rice and Alexander, 1938)

<u>Effects of secondary cementation on subsurface flow.</u> Especially in arid and semiarid regions, zones of cementation in the lower part of the B-horizon or the upper part of the C-horizon have a considerable influence on the downward movement of water. These zones have been called hardpan, caliche, mortar beds, and locally by other names. Profiles of these zones show that they range from isolated or disconnected nodules,

stringers and pipes through networks of interconnected streaks or zones (honeycomb or boxwork) to more or less uniform densely cemented horizons. The content of secondary cement generally varies from a few percent to more than 50 percent of the deposit (Maxey, 1964a, p. 8).

Genetically, the concentrations of calcium carbonate in arid and semiarid regions can be subdivided as follows (Maxey, 1964a, p. 8):

a. Water-table or ground-water cements which are formed by evaporation of ground water in shallow soil or at land surface. This process occurs in deserts where capillary attraction draws ground water toward the surface, where it evaporates, leaving a cemented deposit which impedes the movement of water.

b. Soil caliches are those calcium carbonate concentrations associated with the lower-most part of the B-horizon and the upper part of the C-horizon. These caliches, which vary from a few inches to several feet thick, occur as continuous zones, and are indurated in the upper part. A typical soil caliche is sharply defined at the top by a crenulate plane separating the zone of calcium carbonate accumulation from the leached material above. At the base, the caliche is gradational with the underlying parent material.

c. Derivatives of the first-cycle soils caliches described above. The friable A-horizon may be removed during climatic changes which are either wetter or drier than a former time. Solution and reprecipitation of the exposed calcium carbonate cause the physical character of a former soil caliche to change, usually by increasing its density and decreasing its permeability. If these caliches go through several cycles of climatic change, indurated limestones of low permeability result. These limestones may be near surface or exposed units, such as the Ogallala Formation of West Texas, or buried in sequences of alluvial deposits, such as are found in the Great Basin.

In humid climates, sufficient quantities of water are available to carry away most of the dissolved mineral matter to the ground-water reservoir. However, some pedalfer soil profiles show secondary accumulations of iron compounds in the C-horizon (called Ferrito zone, hardpan, and iron zone). These zones could locally affect the movement of subsurface water.

Geologic Controls in the Ground-Water Zone

Ground water is generally referred to as that body of water occurring below the surface of the ground under pressure greater than atmospheric (Davis and DeWiest, 1966, p. 41). This subsurface water saturates the media through which it is moving and in which it is stored and does not include very small transient bodies of water under pressure greater than atmospheric such as might result from infiltration during a period of heavy precipitation. Ground water occurs within various types of open spaces in the rocks such as interstices between grains, joints and other fractures, and solution or weathering openings. Because of unbalanced forces due mainly to pressure differentials and gravity, this water is generally in continuous motion.

The occurrence, motion, and storage of ground water are controlled by sequence, lithology, thickness, and structure of earth materials. Motion and storage capacity are chiefly controlled by hydraulic conductivity (an expression of the ability of water to move through a porous medium) and porosity (an expression of the volume of voids relative to the total volume of a porous medium).

Lithologic units which have appreciably greater transmissivity (hydraulic conductivity times aquifer thickness) than adjacent units and which store and transmit water that is recoverable in economically usable amounts are called aquifers (Maxey, 1964b, p. 10). Earth materials which have appreciably greater transmissivity than an aquifuge but less transmissivity than an aquitard are called aquicludes. Aquitards are lithologic units which have appreciably greater transmissivity than an aquiclude but considerably less transmissivity than an aquifer. Lithologic units which have very low transmissivity are called aquifuges (Todd, 1959, p. 15).

Principal aquifer types. A chief responsibility of a hydrogeologist is to locate, describe, and analyze aquifers and geohydrologic units. He must understand the character, distribution, and extent of aquifers, the stratigraphic and geographic occurrence of ground water, and the principles of ground-water motion.

Meinzer (1923, 1939) was the first hydrologist to describe the geographic occurrence of ground water and the extent of favorable aquifers in the United States. He delineated 21 ground-water provinces in a comprehensive classification based upon the character, distribution, and extent of geohydrologic units. His descriptions of geohydrologic units are arranged according to geologic age, and stratigraphic and geographic position. Thomas (1951) constructed a map of the ground-water areas in the United States on which he delineated the following types of aquifers: (1) primarily unconsolidated alluvial deposits, (2) consolidated rock aquifers, and (3) unconsolidated or semiconsolidated rock aquifers. All these types can be expected to yield 50 gpm or more of water containing less than 2000 ppm total dissolved solids. Thomas (1952) also published a series of maps delineating 10 ground-water regions by modifying Meinzer's original 21 provinces.

Upon examining the work of Meinzer and Thomas, there appear to be four distinct geologic terranes which today produce most of the ground water:

a. Unconsolidated or loosely consolidated sand and gravel deposits which are commonly interbedded with silt and clay members, but have few carbonate units.

b. Semiconsolidated and consolidated conglomerate and sandstone formations which are jointed and possess both primary (intergranular) and secondary (fracture) porosity and permeability.

c. Carbonate formations possessing both joints and other fractures which have been enlarged by solution and possess little intergranular porosity and permeability.

d. Extrusive igneous rocks, mostly jointed and fractured basalts, which have little intergranular porosity and permeability. Primary structures, such as vesicles and tubes, provide some porosity and permeability, but often these properties are secondary.

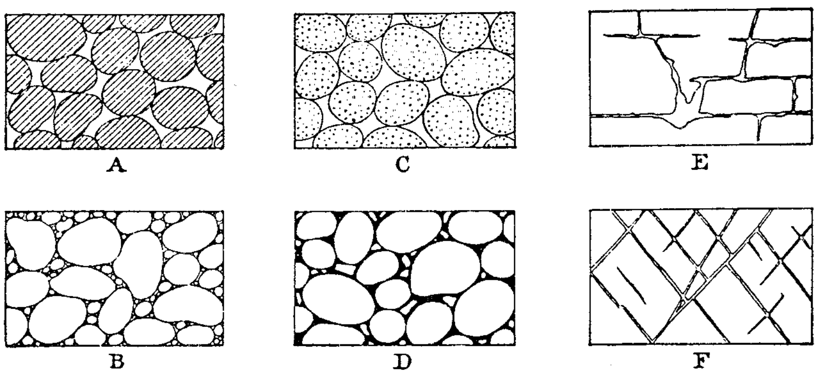
Porosity and conductivity. Open spaces, voids, or interstices in earth materials are the receptacles that store and transmit ground water. The chief factors controlling the storage capacity and conductivity of earth materials are the size, type, shape, and arrangement of the voids. Openings in earth materials are of two types (fig. 1.07):

a. Intergranular (primary) pores which are best developed in clastic sedimentary rocks both consolidated and unconsolidated.

b. Pores resulting from joints and other secondary openings which consist of two groups:

Jointed and stratified rocks that are subject to solution and therefore possess solution channels developed along fractures and bedding planes (e.g., carbonates, evaporites).

Crystalline, igneous and metamorphic rocks, some impermeable carbonate rocks, indurated clastic rocks, and some fine-grained shales, all of which are capable of transmitting and storing water in joints and other open fractures.



A, well-sorted alluvial material, high porosity; B, poorly sorted alluvial material, low porosity; C, well-sorted deposit of porous pebbles, very high porosity; D, well-sorted deposit, porosity decreased by cementation; E, rock rendered porous by solution; F, rock rendered porous by fracturing.

Fig. 1.07. Types of rock interstices and relation of rock texture to porosity (after Meinzer, 1923, p. 3)

Most aquifers with relatively high conductivities consist of clean coarse sands, mixtures of sand and gravel, and some fine-grained clastics. Aquifers which contain appreciable amounts of find sand, silt, and clay possess high porosity (table 1.01) but have low conductivity. In glacial terranes, interbedded, find-grained and coarse-grained clastics form valuable aquifers because of the high storage capacity in the highly porous fine-grained materials which leak water into the more permeable coarse-grained clastics when the hydraulic head is lowered. Table 1.02 illustrates typical values of transmissivity and storage coefficient for unconsolidated materials.

Porosity and hydraulic conductivity data that are applicable to consolidated and semiconsolidated rock aquifers are often difficult to find. However, many determinations have been made of the transmissivity and storage coefficient of such aquifers (table 1.03).

Pososity and hydraulic conductivity values of fractured and cavernous rocks are even more difficult to ascertain. The hydraulic conductivity is a function of the properties of the water and the gross nature and distribution of the interstices, the latter being extremely variable and sporadic for such rocks. Many igneous, metamorphic, and some carbonate rocks are dense and nonporous. Because the volume of fractures and caverns is very small compared to the total rock volume, their storage capacity is small. Carbonate rocks which are fractured and/or cavernous possess large solution channels which in turn are readily capable of transmitting large quantities of ground water (table 1.04). Basalts and several varieties of rhyolites form productive aquifers when jointed and dissected by other fractures. Many basalts are cavernous, vesicular or cut by tubes and thus store and transmit large amounts of ground water (table 1.04).

In some cases, fine-grained clastic rocks (shales) which have low hydraulic conductivity are sufficiently jointed and form locally important sources of ground water. Examples of such aquifers include the Brunswick Shale in New Jersey, the Brule Formation in parts of the Great Plains, and the Pennsylvanian shales in the Central States (Meinzer, 1923, p. 125).

Table 1.01. Range in porosity for selected types of earth materials (Todd, 1959, p. 16)

Rock	Porosity, percent
Gravel, clean, uniform	20 - 40
Sand, clean, uniform	20 - 40
Sand and gravel, mixed	10 - 30
Silt and clay	
As deposited	40 - 70
Compacted	0 - 40
Shale	1 - 20
Sandstone	5 - 30
Limestone	1 - 20

Table 1.02. Aquifer coefficients of unconsolidated sand or sand and gravel units (modified after Maxey, 1964a, p. 21)

Aquifer Material	Thickness, feet	Transmissivity gpd/ft (T)	Coefficient of Storage(S)
Glacio-fluviatile deposits (Hanford, Washington)	45	3,000,000	0.20
Glacio-fluviatile deposits (Hanford, Washington)	30	380,000	0.06
Ringold Conglomerate Washington	85	34,000	0.00007
Alluvial sand and gravel (Gallatin Valley, Montana)	26	100,000	0.006
Alluvial fan deposits (Gallatin Valley, Montana)	63	36,000	0.06
Sand and gravel outwash (Mattoon, Illinois)	16	25,600	0.0015
Sand and gravel outwash (Barry, Illinois)	36	119,000	0.003
Valley train sand and gravel (Fairborn, Ohio)	80	280,000	0.0008
Glacial outwash (Bristol Co., R.I.)	60	350,000	0.007
Glacial outwash (Providence, R.I.)	55	150,000	0.20

Table 1.03. Aquifer coefficients of indurated or semiconsolidated rocks (modified after Maxey, 1964a, p. 21)

Hydrologic Unit	Thickness, feet	Transmissivity gpd/ft (T)	Coefficient of Storage (S)
Cambrian-Ordovician sandstone, Wisconsin	850	23,800(ave.)[a]	0.0004(ave.)
Cambrian-Ordovician aquifer, Illinois		17,400[a]	0.00035
Carrizo sandstone, Texas	120	32,200[b]	0.00023
Aquia greensand, Maryland	20	10,000 to 20,000[b]	0.00023
Patuxent formation, Maryland	70	4,210[b]	0.00037
Patuxent formation, Maryland	14	450[b]	0.00013
Patapsco formation, Maryland	25	36,600(ave.)[b]	0.002(ave.)
Wissahickon formation (weathered material above schist), Maryland	65-100	3,000 to 10,000[b]	0.002-0.01

a = permeability and porosity result from intergranular openings and fractures
b = permeability and porosity result from intergranular openings

Table 1.04. Aquifer coefficients of carbonate and basalt rocks (modified after Maxey, 1964a, p. 22)

Hydrologic Unit	Thickness, feet	Transmissivity gpd/ft (T)	Coefficient of Storage (S)
Fort Payne chert (Limestone), Alabama	20-40 to 95-360	4,800-1,316,000	0.005(ave.)
Renault-St. Genevieve (carbonates), Kentucky	125-175	126,000(ave.)	0.0003(ave.)
Tymochtee dolomite, Ohio	225	8,000(ave.)	0.002(ave.)
Silurian dolomite, Illinois	250	61,000	0.00035(ave.)
Cockneysville marble (weathered to sand), Maryland	8	35,000	
Snake River Basalts, Idaho	100	4,000,000(ave.)	0.04(ave.)

Ground-water motion and storage. Detailed discussions of ground-water flow and storage, methods of determining directions and rates of flow, and other quantitative information are described in subsequent chapters. These discussions indicate that the common methods of determining the various quantities depend primarily upon: (1) Darcy equation and its extensions, (2) continuity equation, (3) flow net analyses based on graphical solutions and/or laboratory model studies, (4) laboratory tests of conductivity and porosity, (5) tracer studies, and (6) calculations based on water budget-type studies.

In order to apply any of the above methods successfully, a thorough understanding of geologic factors is necessary. Before rates of ground-water flow and storage can be ascertained, values of conductivity and porosity must be determined. At present, quantitative determinations of conductivity and porosity have not yet been fully developed, but useful inferences and approximations of the hydraulic properties of aquifers can be made by visual examination, surface and borehole geophysical methods, and tracers and by mechanical analyses of earth materials.

A geologic factor that is often unaccounted for in the evaluation of the transmissive properties of earth materials is the gradual lateral change in the physical nature of rocks. Such a change in lithology produces a parallel change in conductivity which affects ground-water flow patterns. In other cases, secondary structures such as joints, bedding planes, faults, and unconformities act as conduits or no-flow boundaries.

The sequency of lithologic types often determines the potential storage capacity for ground water and its direction and rate of movement. Ground water moves through aquifers, leaks through aquitards, and thus flows from one aquifer to another. Under this conditions, water not only recharges an aquifer from ground surface, but also from low permeability units beneath the aquifer. In such lithologic sequences, the highly porous aquitards considerably increase the storage capacity.

Accurate analysis of aquifer potential using equations describing ground-water motion requires knowledge of the above-mentioned geologic parameters and an understanding of their limitations. Detailed and accurate geologic data may often allow recognition of some potential problems. Several physical factors that must be considered in applying ground-water flow equations are briefly described below:

Accurate analysis of aquifer potential using equations describing ground-water motion requires knowledge of the above mentioned geologic parameters and an understanding of their limitations. Detailed and accurate geologic data may often allow recognition of some potential problems. Several physical factors that must be considered in applying ground-water flow equations are briefly described below:

a. Lack of uniformity in conductivities in all directions (anisotropy) and heterogeneity within the aquifer may be difficult to describe.

b. Turbulent flow can be anticipated to occur in strongly fractured and cavernous carbonate and basaltic rocks. Such flow may also exist in very coarse-grained aggregates and in regions where the hydraulic gradient is steep.

c. When the hydraulic head declines in a confined aquifer due to pumping, water is removed from storage from fine-grained clastics which are more compressible than the coarser-grained aquifer materials. Since these fine-grained clastics have low hydraulic conductivities and water drains slowly through them, water is not removed from storage instantaneously.

d. Withdrawals of large quantities of ground water from storage in poorly consolidated and unconsolidated deposits can result in land subsidence. Subsidence occurs because there is a decline in hydraulic head, an increase in effective stress (grain-to-grain load), and elastic compression of clay-silt units.

Chapter 2

Effects of Stratigraphy and Sedimentation on Ground-Water Occurrence And Movement

CHAPTER 2. EFFECTS OF STRATIGRAPHY AND SEDIMENTATION ON GROUND-WATER OCCURRENCE AND MOVEMENT

Section 2.01. Introduction

In a small given region, sedimentary formations lie on one another like a pile of books. The majority of the formations are themselves stratified and consist of a succession of layers or beds resting one upon another like pages of the book. Successive layers may differ in composition, texture, and degree of consolidation or may be similar but separated by films of different earth materials that produce partings between them. For example, some shale formations consist of very thin beds having the appearance of sheets of paper (fissile shale).

Stratification reflects changes in the physical and chemical conditions which occurred during deposition (Krumbein and Sloss, 1963, p. 301). Differences between successive formations are the result of relatively large and permanent changes in the geologic environment. The character, thickness, and succession of strata give the most important data on existing hydrostratigraphic units and the depths at which they will be encountered. Minor differences between stratigraphic sequences are due to local, temporal changes. For example, clay partings between similar beds of limestone may be the result of storm activity that produced turbidity in usually clear waters. Some partings in shales, however, may be caused by annual weather cycles.

Section 2.02. Lateral Changes in Lithology

Many sedimentary units are persistent and uniform through wide areas and serve as stratigraphic marker beds. However, stratified formations generally change gradually from place to place, both in thickness and in lithology. These changes commonly reflect local differences in the geologic environment. For example, a mountain stream usually deposits coarse-grained alluvium in its upper reaches and fine-grained material farther downslope. Another example is a beach consisting of coarse sand and gravel swept clean by waves and currents which are capable of removing any fine-grained materials. At other places along the shore, the waves and currents are not as strong, and fine sand, silt, and clay are deposited. Generally, there is a gradational change from thick accumulations of coarse-grained material near shore to progressively thinner, fine-grained deposits away from shore.

Lateral changes in lithology strongly influence the occurrence and movement of ground water. For example, the sediments beneath the Atlantic Coastal Plain are relatively coarse-grained and uniform where exposed and become more fine-grained away from shore. Consequently, coastal plain sediments near outcrops form excellent ground-water reservoirs, and they become less acceptable with increasing distance from the outcrops (Davis and DeWiest, 1966, p. 404).

Section 2.03. Attitude of Strata

Slight dips of rock strata may be due to deposition in a sloping position (e.g., cross bedding) or to deformation after deposition. Alluvial deposits normally have a slight original dip downstream, and lacustrine or coastal plain sediments are deposited with a gentle dip away from shore. At some distance from shore they become nearly horizontal. Lava flows have original dips away from conduits where the lava was extruded. Slight movements of the earth's crust may cause gentle tilting of the strata of only a few feet to a mile. More dramatic movements of the earth's crust may turn the strata into vertical position or overturn them. Steep dips are nearly always related to diastrophism.

Knowledge of the attitude of strata is necessary to determine the distribution of high permeability zones in them and to predict their depth with reasonable accuracy. Geologic data of this type are used in the location of productive water wells. Dipping strata may also influence the occurrence of ground water and/or the deposition of younger earth materials. For example, a durable, resistant member (e.g., quartzite, lava) in a series of tilted strata that is crossed by a stream may act as a no-flow boundary and raise ground-water levels on the upstream side (fig. 2.01). The water-yielding unit(s) under such conditions may be: (1) high permeability formation(s) belonging to the same series as the resistant member, (2) rock waste derived from weathering of less resistant rock on the upstream side, and (3) alluvium that has been deposited on the upstream side. Where the stream dissects the resistant stratum, its valley may become narrow, whereas farther upstream the valley may be rather broad. If the area of the resistant stratum is elevated with respect to the upstream area, an appreciable thickness of alluvium may accumulate.

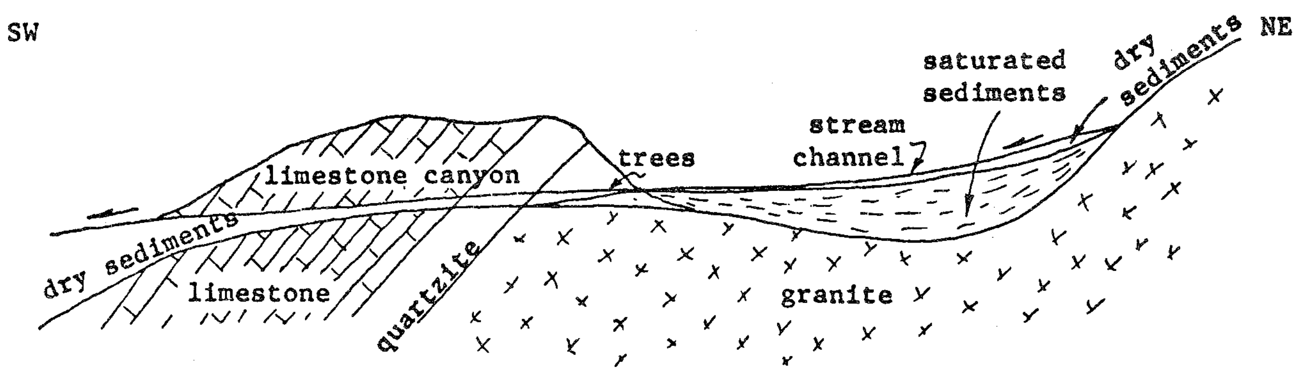


Fig. 2.01. Section showing shallow water conditions caused by resistant quartzite unit that has been tilted (Meinzer, 1923, p. 169)

Section 2.04. Hydrogeologic Significance of Unconformities

An unconformity is a surface which represents an interval of time during which either deposition was negligible or nonexistent. It normally implies that the existing rocks were eroded and/or tilted, warped, or fractured. In the geologic column unconformities represent periods when a surface stood above sea level for a considerable time. During that time, the exposed rocks are subjected to erosion and physical and chemical alteration by subsurface water.

An unconformity may be recognized by a difference in dip of two superposed series, by irregularities in the lithologic contact surface, by interruptions in the continuity of formation above and/or below the contact, and by a weathered condition in the underlying sequence at or near the contact. Field observations and detailed mapping of these features plus borehole data assist in the location of unconformities. It is difficult, however, to construct an accurate contour map of an unconformable surface because of large irregularities. An understanding of the geologic processes involved in producing an unconformity is an integral part of interpreting observations.

The eroded surface of an unconformity and the rocks at shallow depths beneath this surface have been acted upon by subsurface water. Permeable formations below the unconformity have been flushed by fresh meteoric waters which percolated to some depth. Observations of rocks directly beneath these eroded surfaces should show the modifying effects of subsurface waters, and descriptions and interpretation of these rocks should support such observations.

Maxey and Hackett (1963, p. 37) noted the importance of a weathered zone in the top of the Precambrian crystalline rocks in the northern Rocky Mountains of Wyoming. Here the weathered zone below the unconformity may reach 15 feet and is more permeable than the unweathered rock. The more permeable areas are frequently marked by springs along the outcrop of the unconformity in the Owl Creek and Big Horn Mountains.

Another example of an unconformity is highly weathered zones in the Galena and Platteville Dolomite Formations in Wisconsin and Illinois. These weathered zones are developed best where the dolomites are covered by glacial drift, indicating that they were exposed and weathered prior to glaciation.

There are many well-known aquifers containing potable water to depths of several thousand feet. This suggests that these aquifers have been flushed recently to these depths and that most of the water in the sediments at the time of deposition has been removed and replaced by meteoric water (Maxey and Hackett, 1963, p. 39).

One or more unconformities are present in the geologic column of many areas. In the Illinois Basin, for example, the stratigraphic column contains eight mappable unconformities (fig. 2.02). Geologic studies indicate that near the margin of the basin most of these surfaces are near the present land surface and have been flushed of their original water so as to contain and produce potable water.

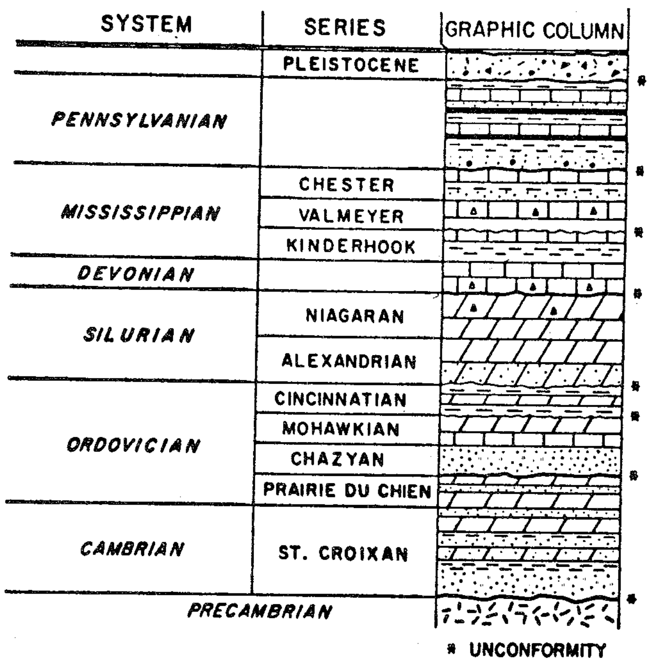


Fig. 2.02. Generalized stratigraphic column in the Illinois Basin showing the location of the most important unconformities (Maxey and Hackett, 1963, courtesy of the North-Holland Publishing Company Amsterdam, The Netherlands)

The presence of unconformities in the geologic column has a definite relationship to the concentration of brines in basins, except for basins into which brines resulting from solution of evaporites have migrated. It appears that brines in isolated basins are not entirely products of concentration of connate waters by density stratification or other processes.

Section 2.05. Relation of Subsurface Water to Sedimentation and Stratigraphy

Geohydrologic methods may be used to determine physical characteristics, continuity and extent of lithologies, and structural boundaries of stratigraphic units. They may also be used to interpret the genesis and history of the stratigraphic units.

In general, sediments laid down in terrestrial and marginal (coastal plain) environments are strongly affected by subsurface water during deposition. Glacial, alluvial, and eolian deposits are affected primarily by vadose waters, whereas lacustrine, lagoonal, and paludal deposits may be acted upon by either vadose or ground water or both. In any case, the role of subsurface water must be considered to properly interpret the genesis of sediments deposited in these environments. In eastern Utah, for example, Young (1957, p. 1760) attributes the white caps (underclays) developed on the littoral marine sandstones beneath coal to the leaching action of subsurface waters percolating downward from overlying coal swamps. Such interpretations are subject to criticism on a hydrologic basis. Usually, a swamp is an area of ground-water discharge, and the hydraulic gradient is toward the swamp with ground-water flow being upward where it is discharged by surface flow and/or evapotranspiration. It seems unlikely that paludal waters under the conditions assumed in Young's paper would percolate downward. Maxey and Hackett (1963, p. 44) believe that the formation of white caps result from the discharge of subsurface water, either during penecontemporaneous subaerial erosion, or subsequent to uplift of the area.

Chapter 3

Structural Controls of Ground-Water Occurrence And Movement

CHAPTER 3. STRUCTURAL CONTROLS ON GROUND-WATER OCCURRENCE AND MOVEMENT

Section 3.01. Folds

In many regions, strata have been warped, forming folds or irregular flexures. The common types of folds are: (1) anticlines--the strata are bowed upward, (2) synclines--the strata are bowed downward, and (3) monoclines--the strata are deformed in a step-like flexure. An anticline may be ridgelike or may form a structural dome in which the strata dip in all directions away from a common center (e.g., Black Hills uplift). A syncline may be troughlike or may form a structural basin in which the strata dip from all directions toward a common center (e.g., Michigan Basin).

Folding of rock strata introduces considerable uncertainty in locating wells and makes it necessary to obtain data at many points in an area in order to properly construct maps showing the position of highly permeable rock units and ground-water levels. Structural cross sections are used to illustrate the folds of rock strata and hence the positions and thicknesses of aquifers. Areal geologic maps, which indicate the distribution of formations at the surface, and structural contour maps, which show successive contours of the upper or lower surface of a key marker bed or formation, also are used to delineate the position of folds. Several structural cross sections and a geologic map frequently are needed to properly interpret structural conditions in the intervening areas.

The water table may traverse a series of folded strata such that a highly permeable unit may be saturated in some places and drained in other places. Folding of strata may form a structural no-flow boundary to ground-water movement. Figure 3.01 is a diagramatic section showing effects of folding and an unconformity on the occurrence and movement of ground water. The ridge of older, relatively less permeable alluvium tends to force ground water moving from the granite area through the younger permeable alluvium toward the surface. The protrusion of older alluvium may be attributed to folding.

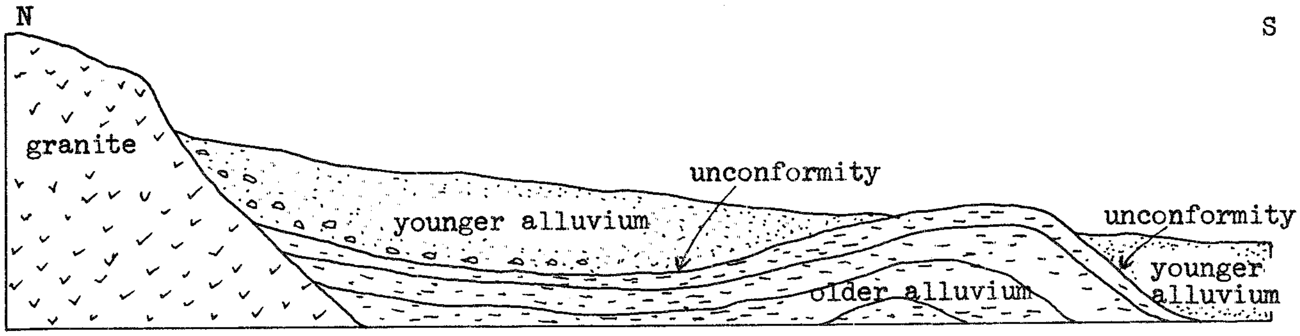


Fig. 3.01. Section showing effects of folding on the occurrence and movement of ground water (Meinzer, 1923, p. 173)

Section 3.02. Joints and Veins

Joints commonly occur in massive, hard, brittle rocks and result chiefly from compression and tension during earth movements and, in igneous rocks, from contraction due to cooling or crystallization. In massive igneous rocks, several directions of jointing usually are discernible. Two are often nearly vertical and the third is commonly horizontal. Basalts and other extensive rocks exhibit columnar jointing; that is, the rocks are fractured to form nearly vertical columns.

The spacing and continuity of joints are highly variable. Vertical joints, which are important water carriers, have little regularity of spacing, even in the same rock. Data from a large number of field observations suggest that where jointing is well developed, the spacing of vertical joints commonly ranges between 3 and 7 feet to a depth of 50 feet (Meinzer, 1923, p. 113). Although there are exceptions, joints of this type are generally continuous for some distance. The sheeted zones of close jointing and well-defined parallel joint sets may be as continuous as faults with long dimensions measured in hundreds of feet.

Horizontal joints are more regularly spaced than vertical joints (Meinzer, 1923, p. 114). They are primarily surface features and decrease in number with depth and may not exist as fractures 250 feet below the surface. The continuity of individual horizontal joints rarely exceeds

150 feet (Meinzer, 1923, p. 114). A continuous opening of several hundred feet may take the form of a curved sheet nearly parallel to land surface, each lower sheet having less curvature than the one above. Horizontal joints tend to be better developed on uplands than in valleys.

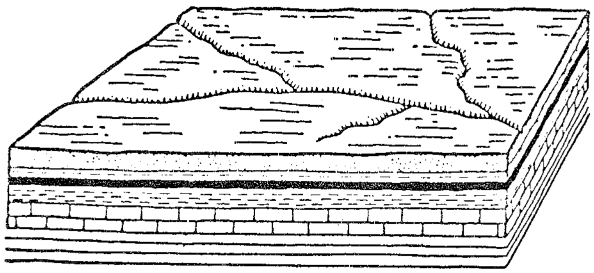
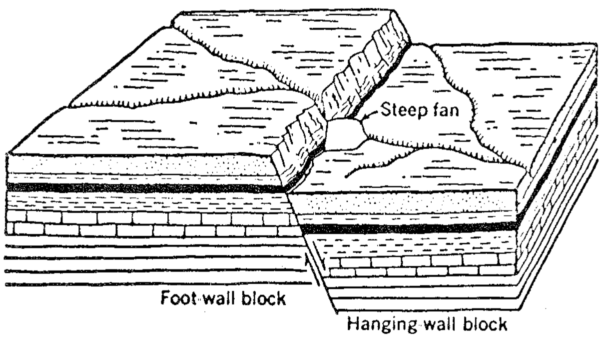
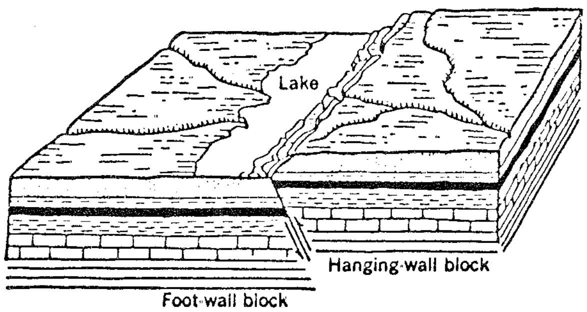
Veins are secondary openings that have been partly or completely sealed with mineral matter deposited by ground water. When these openings are filled, they no longer function as water conduits, and in some cases a vein may act as a no-flow boundary to ground-water movement. They must be considered in drilling because a resistant quartz vein is likely to deflect the drill, resulting in an inclined borehole.

Section 3.03. Faults

A fault is a fracture or fracture zone along which there has been displacement of two sides relative to one another parallel to the fracture. Faults are of three main types: (1) normal faults, (2) reverse faults, and (3) lateral faults (fig. 3.02). All faults differ considerably in amount of displacement, lateral extent, and depth. Very small faults have little effect on ground water except they may, as other open fracures, serve as small ground-water reservoirs. Large faults of considerable lateral extent, depth, and displacement affect the distribution and position of aquifers (fig. 3.03). They may also act as no-flow boundaries, impounding ground water, or serve as conduits of fluid flow.

Instead of a single sharply-defined fault, there is often a fault zone along which there are numerous parallel faults and masses of broken and pulverized rock called fault breccia. Such fault zones may represent a large aggregate displacement and may serve as passages for or barriers to ground-water movement (Tolman, 1937).

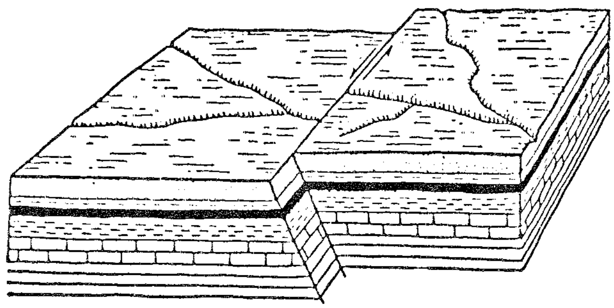
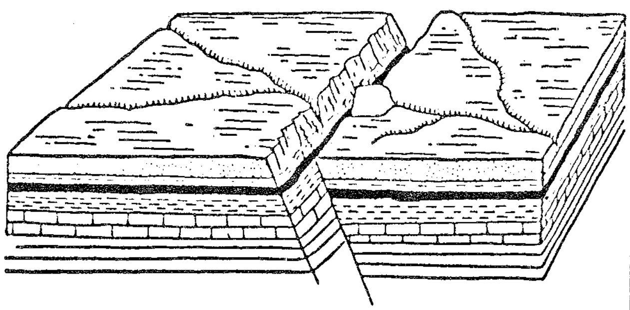
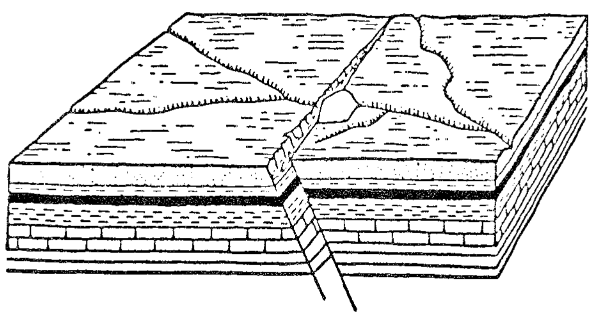
Faulting affects ground-water conditions not only by displacing high permeability rock units but also by modifying the altitude and topography of the surface on opposite sides of the fault. For example, the uplifted side of a fault may produce an escarpment as shown in fig. 3.04. Such changes may result in the deposition of highly permeable material on the downthrown side from rock debris derived by erosion of exposed source rocks on the uplifted side.

<table>
<tr><th>Block diagram</th><th>Name of fault</th><th>Definition</th></tr>
<tr><td></td><td></td><td>Reference block before faulting. Drainage is from left to right.</td></tr>
<tr><td>
</td><td>Normal fault</td><td>A fault, generally steeply inclined, along which the hanging-wall block has moved relatively downward.</td></tr>
<tr><td>
</td><td>Reverse fault</td><td>A fault, generally steeply inclined, along which the hanging-wall block has moved relatively upward.

A normal or reverse fault on which the only component of movement lies in a vertical plane normal to the strike of the fault surface is a dip-slip fault.</td></tr>
</table>

Fig. 3.02. Names and definitions of principal kinds of faults (Longwell, Flint and Sanders, 1969, pp. 414, 415)

Fig. 3.02 (continued)

Block diagram	Name of fault	Definition
	***Strike-slip fault***	A *fault on which displacement has been horizontal.* Movement of a strike-slip fault is described by looking directly across the fault and by noting which way the block on the opposite side has moved. The example shown is a *left-lateral fault* because the opposite block has moved to the left. If the opposite block has moved to the right it is a *right-lateral fault* Notice that horizontal strata show no vertical displacement.
	***Oblique-slip fault***	*A fault on which movement includes both horizontal and vertical components.*
	***Hinge fault***	*A fault on which displacement dies out (perceptibly) along strike and ends at a definite point.*

With time, erosion of the escarpment may remove any topographic evidence of faulting. Eventually the fault may be further concealed by deposition of younger sediments over the entire area.

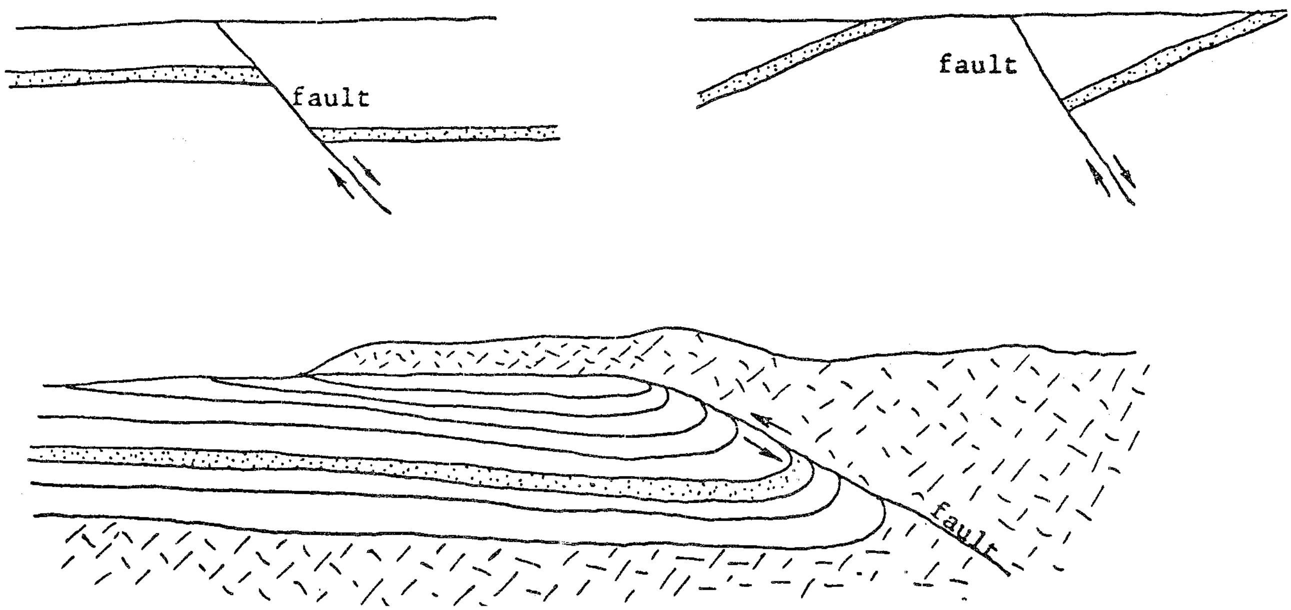


Fig. 3.03. Diagrammatic sections showing possible effects of faults on positions of aquifers (Meinzer, 1923, p. 181)

A fault may act as a no flow boundary, impounding water percolating through earth materials and forcing it to the surface along the fault and/or cause an appreciable difference in water levels along opposite sides of the fault. The impoundment may occur because of impermeable fault gouge in the fault zone or displacement of alternating aquifers and aquicludes in such a way that the aquicludes abut against the aquifers (fig. 3.05).

Mylonized rock and/or clay gouge of low permeability, may also cause impoundment of water along the fault surface. The impounding effect of faults is more prevalent in unconsolidated deposits rich in clay than in crystalline rocks (Meinzer, 1923, p. 182). A number of factors which account for the fact that faults tend to form no-flow boundaries in unconsolidated deposits are summarized from Davis and DeWiest (1966, p. 396). First, the deposition of mineral matter along the fault surface reduces

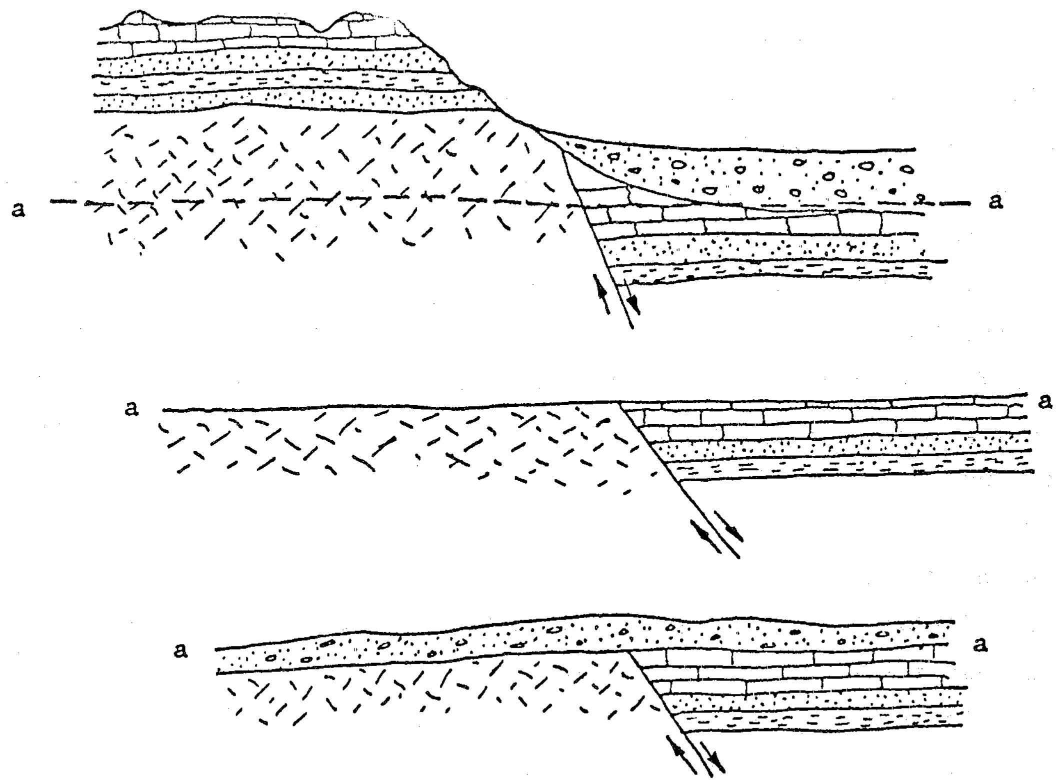


Fig. 3.04. Aquifers formed by erosion of fault scarps (Meinzer, 1923, p. 182).

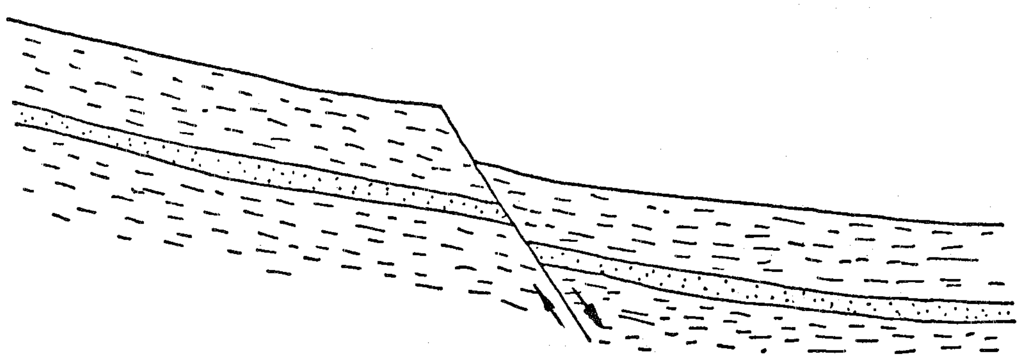


Fig. 3.05. Section showing displacement of an aquifer by a fault causing a no-flow boundary to ground-water movement

permeability. Second, elongated and flat detrital sediments tend to be rotated parallel to the fault surface also reducing the permeability perpendicular to the fault. Third, aquicludes may be offset along the fault to block aquifers. This effect is important where the number of aquifers is limited. Fourth, faulting pulverizes rocks along the fault surface. This may be effective at greater depths where lithostatic pressure will increase the friction between grains. In contrast, brecciated material along faults in crystalline rocks does not readily conpact together to reduce permeability along the edges of highly permeable rock units. Also, the opposite sides of normal faults in crystalline rocks usually are not everywhere in contact and form fissures through which ground water may flow. This is primarily due to the fact that the fracture surfaces are uneven. After displacement, rock projections on opposite sides may abut each other such that intervening openings are produced.

A fault crossing a stream channel underlain by alluvium resting on an aquifuge may expose the low permeability unit in the streambed so that ground-water levels will be raised in the alluvium upstream. Such a condition is likely to occur in regions where the erosive power of the stream has not been able to keep pace with tectonic activity.

As stated earlier, a fault zone commonly consists of pulverized, angular, and mylonized rock fragments with intervening interstices and nearly parallel fractures resulting in a zone of considerable width. Such fault zones may become important reservoirs of ground water.

Faults also act as conduits for flow of deep or shallow ground water. In regions where faults reach considerable depth, openings associated with them allow the ascent of deep-seated, thermal waters. Also, many springs owe their origin to passageways that follow faults. Examples of springs, many of which are thermal, associated with pre-Quaternary faults exist along the edges of mountain ranges in Nevada and western Utah. The ranges of this region consist primarily of tilted fault blocks, and in many places, recent fault scarps occur in the alluvial slopes at their bases. Some springs along these latter faults represent recirculated meteoric water that has percolated into the sediments of nearby alluvial slopes.

Section 3.04. Relation of Subsurface Water to Structural Geology

The chemical character, occurrence, and movement of subsurface water reflect structural controls. It has long been recognized that water is an active agent in the movements of earth masses (landslides), compaction and subsidence, gravity sliding, and other responses to gravitational forces.

Perhaps the most recent example of appreciation of the relation of fluid flow and hydraulic pressures to structural movements is the work of Hubbert and Rubey on the role of fluid pressure in the mechanics of overthrust faulting (1959, p. 116). Their results establish the need for considering the hydrologic environment, not only as it pertains to thrust faulting, but to nearly all types of tectonic activity. Their investigations showed that pressure-strain relationships within the earth's crust affect not only lithologic materials, but the fluids and gases contained within them, and in turn the response of rock materials to such forces is altered by the hydrologic environment (Maxey and Hackett, 1963, p. 43).

The physical concepts of ground-water flow in aquitards may be usefully employed in the analytical study of land subsidence caused by fluid withdrawals. Domenico and Mifflin (1965, p. 573) have reported that the amount and rate of land subsidence or ground-water recovery from aquitards is a function of their specific storage and the development of an effective-pressure area across them. The amount of subsidence at any given benchmark for a given period of time equals the product of specific storage and the effective-pressure area for that time. Effective-pressure area is shown to depend upon the aquitard thickness, the magnitude of artesian pressure decline, the manner of basin development, and time.

Chapter 4

Theory of Ground-Water Motion

CHAPTER 4. THEORY OF GROUND-WATER MOTION

Section 4.01. Introduction

The theory of ground-water motion is encompassed by several aspects: (1) a statement (or statements) of the dynamics of flow, i.e., the forces responsible for the movement and the energy transfers that take place during movement, (2) a statement of continuity, that is, that mass may be neither created nor destroyed in the system, and (3) a statement of pressure-temperature-volume relations in the system. These basic principles are used in studies ranging from a ground-water budget analysis of a basin to a particular analytical solution of a partial differential equation describing fluid flow.

After reviewing some basic concepts regarding the occurrence of ground water, this chapter formulates the principles of ground-water motion and demonstrates how these principles are applied in equations commonly used for analyses.

Section 4.02. Fundamental Concepts

The amount of water that a rock unit can store is measured by its porosity, n, which is the proportion of the total volume of material occupied by openings, or:

$$n = \frac{V_v}{V_t} \tag{4-1}$$

where:

V_v = volume of voids

V_t = the total volume

Porosity is not necessarily a measure of the amount of water that the unit will ultimately yield upon pumping or drainage. This quantity, called specific yield, S_y, is the amount of water per unit total volume which can be yielded under gravity drainage. Specific retention, S_r, is its complement or the amount of water per unit total volume which will not drain under the influence of gravity. Obviously:

$$S_y + S_r = n \qquad (4\text{-}2)$$

Capillary forces prevent all water from draining from a porous material. As water drains from an initially saturated unit, the weight of water remaining continually diminishes. When the capillary forces on the surface of water in an interstice balance the weight of hydraulically connected water, all of this water is held firmly. In homogeneous material, this residual water generally occurs as isolated or semi-isolated rings or segments between grains or fracture surfaces. Stratified material will generally hold more water than homogeneous material (see, for example, Wilson and DeCook, 1968, p. 1219).

Ground water occurs under two basic conditions: unconfined and confined. Unconfined conditions occur at the upper part of the saturated zone. The upper surface of ground water is called the water table and is defined as the atmospheric pressure surface. This definition is convenient because water under atmospheric or greater pressure will stand in and be yielded to wells. Water generally occurs above the water table under less than atmospheric pressures and will not be yielded to wells. In nearly homogeneous rocks the water table is a subdued replica of the topography.

If a lense of ground water occurs above the true water table, it is called perched ground water, and the upper surface of such a zone is a perched water table. Perched lenses often are supported by lenses of earth materials of low permeability.

Confined ground water refers to water under sufficient pressure that water levels in wells tapping a zone rise above the bottom of the confining layer of low permeability earth materials. For example, a clay layer overlying a sand layer confines the ground water in the sand if the water table is above the bottom of the clay. The surface that represents the levels at which water stands in wells penetrating the confined system is called the piezometric surface.

Figure 4.01 shows some conditions leading to both confined and unconfined ground water.

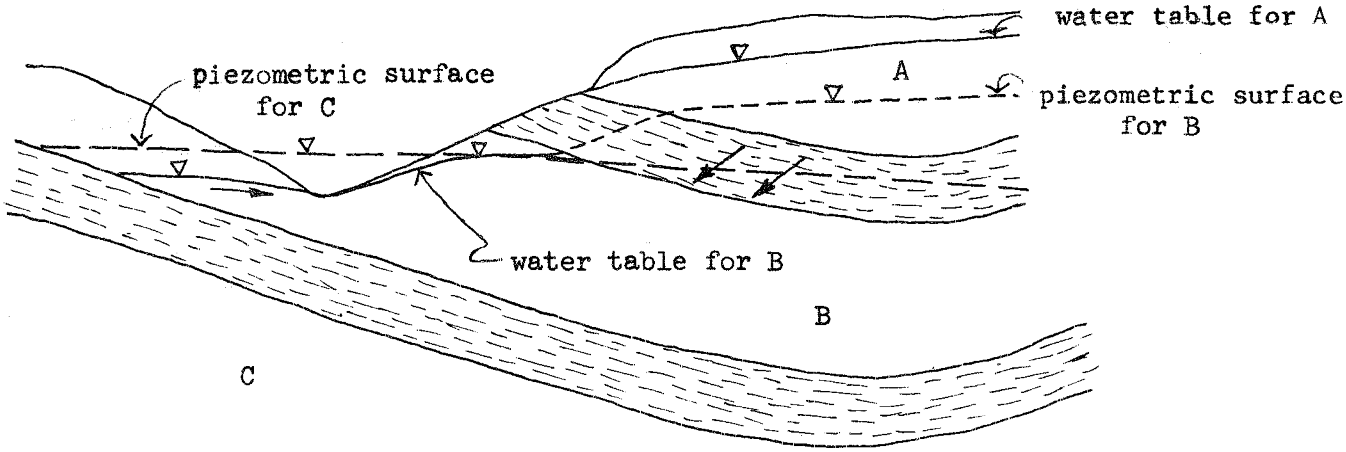
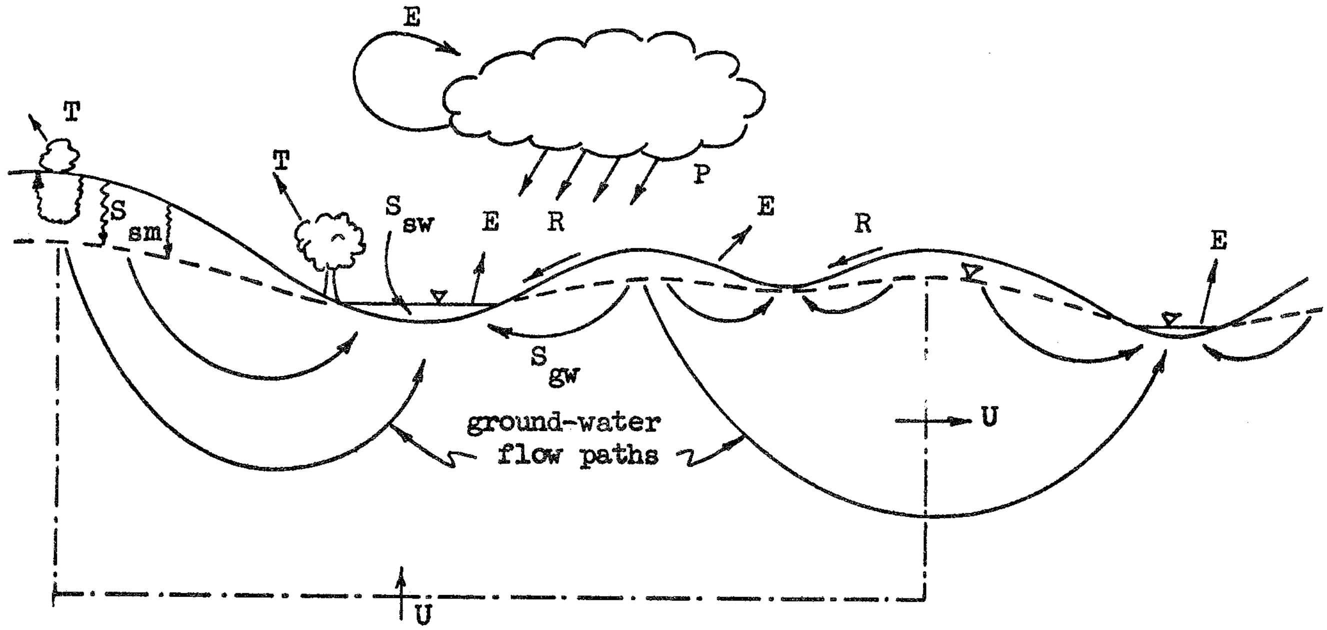


Fig. 4.01. Confined and unconfined ground water in a hypothetical sequence of high (indicated by letters) and low permeability strata

Section 4.03. The Hydrologic Budget Equations

The total hydrologic budget equation is a convenient place to begin the study of ground-water motion because a consideration of the budget demonstrates the major sources and paths of movement of water that will become ground water.

Fig. 4.02 depicts an idealized cross section through a hypothetical region together with an identification of the various sources and losses of water. Ground water occurs only below the dashed line, which is the water table. The rocks are saturated below this surface, and the system is unconfined because no low permeability units are present to restrict vertical movement of water.

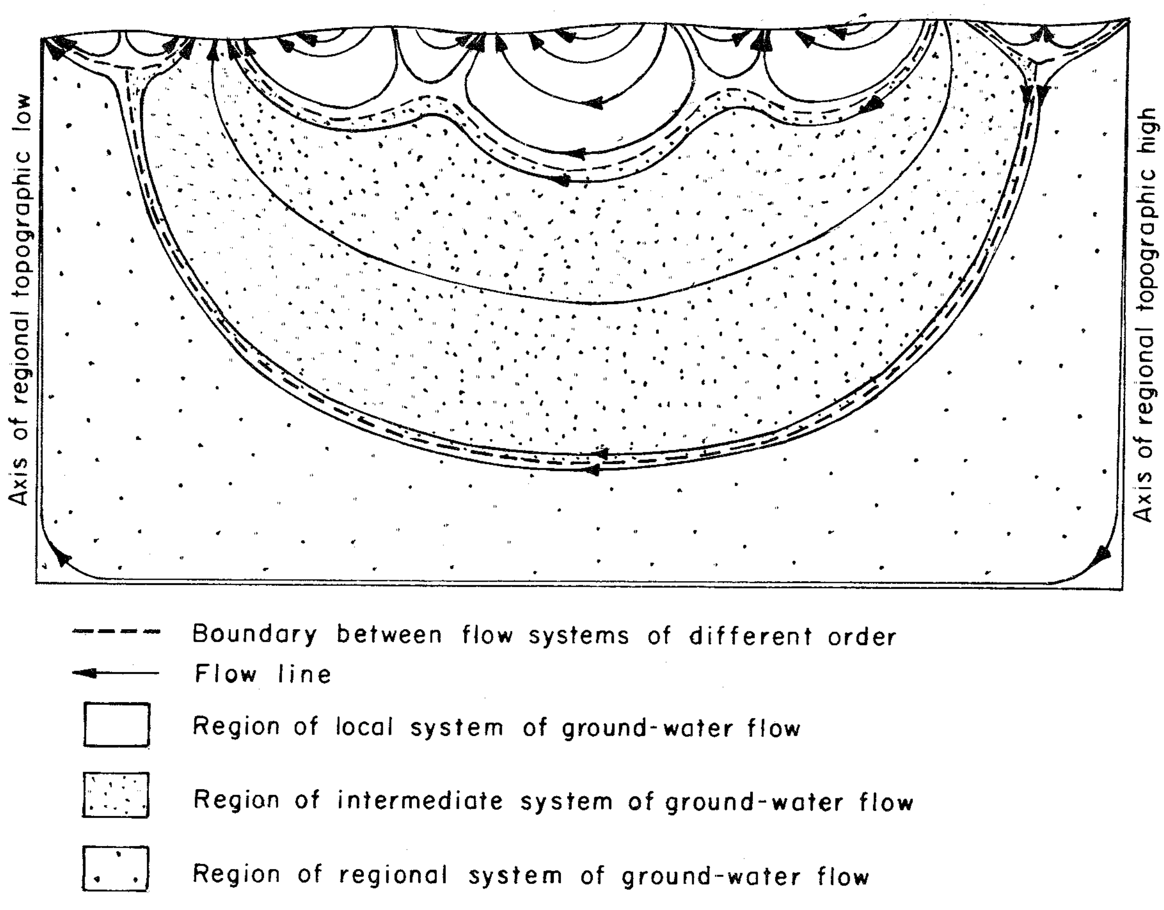


Atmospheric pressure surfaces on water are located with the inverted triangles. The subsurface atmospheric pressure surface is the water table (modified from Parizek, unpublished manuscript).

Fig. 4.02. The hydrologic budget

The basic source of water for the hydrologic budget (which is an equation expressing local parts of the hydrologic cycle) is precipitation (P). Of the water which reaches the earth, part runs off (R), part evaporates (E), and part infiltrates to become soil moisture (S_{sm}). Part of the soil moisture is drawn to the ground surface under the influence of evaporation from, and near, the ground surface; part is used by vegatation (T); and part, the ground-water recharge (P_{gw}), flows downward to become part of the ground-water body (S_{gw}). A portion of the runoff is stored in surface-water bodies (S_{sw}) and, if the bottom of the surface-water body is above the water table, some will infiltrate to become S_{sm}. Water will also seep from the stream bottom if the water table intersects but slopes away from the stream. For the latter case, the seepage is direct ground-water recharge. Ground water often moves toward surface-water bodies in areas where the water table is a subdued approximation of the topography. It may then discharge to become surface-water storage or ground-water runoff (R_g).

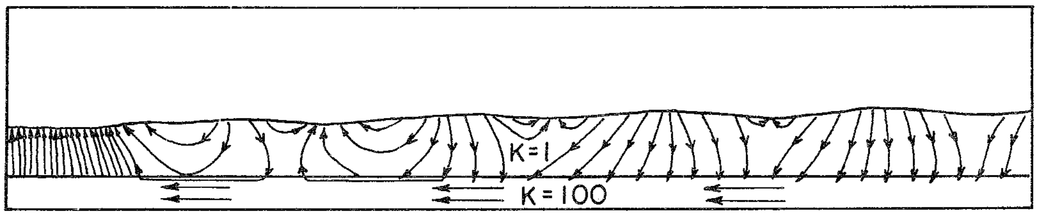
Ground water is generally in continuous motion. For nearly homogeneous rocks where the water table undulates in response to topography, flow paths, or flow directions, are arcuate, trending downward in ground-water mounds (recharge areas) and upward in ground-water troughs or valleys (discharge areas). Some flow paths extend from major highs to major lows, passing under minor irregularities. However, the minor irregularities have their local recharge and discharge areas too. These conditions are termed regional and local flow systems, respectively. Intermediate flow systems extend between intermediate highs and lows in the water table (see fig. 4.03).



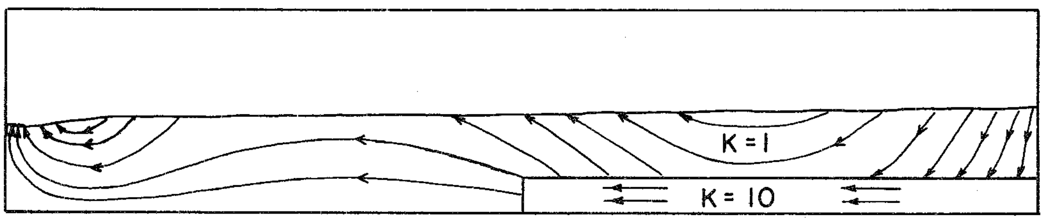
Note: Approximate flow distribution for a homogeneous region (modified from Toth, 1962b, p. 94). The region is bounded by a regional topographic low, a regional topographic high, an undulating water table, and a deep impermeable base.

Fig. 4.03. Development of flow systems of different order

Changes in permeability laterally and vertically alter these relationships considerably but seldom destroy them (see fig. 4.04). The basic ideas briefly summarized here were first introduced by Hubbert (1940, pp. 926-930). They were subsequently developed thoroughly by Toth (1962a and 1962b), Freeze and Witherspoon (1966, 1967 and 1968). Freeze (1967) has used a mathematical model of a ground-water flow system as an aid in a hydrologic budget study.



a. Local flow systems due to a hummocky water table superimposed on a regional flow system showing the influence of a stratum of high permeability underlying less permeable materials.



b. Influence of a lense of high permeability material on a regional flow system. In both cases, the high permeability material causes the induction of flow into it (modified from Freeze and Witherspoon, 1967, pp. 628-629).

Fig. 4.04. Changes in regional flow systems due to local changes in permeability.

The budget or total water balance equation can be written for a natural (unmodified by man) system as:

$$P = R + E + T \pm \Delta S_{sw} \pm \Delta S_{gw} \pm \Delta S_{sm} \pm U \qquad (4\text{-}3)$$

where:

P = total precipitation*

R = total runoff

E = total evaporation of surface and subsurface water

T = total transpiration from vegetation ($E + T$ is usually designated evapotranspiration, ET)

ΔS_{sw} = change in surface-water storage (positive for an increase in surface-water content and negative for a decrease)

ΔS_{gw} = change in ground-water storage (the signs are used in the same manner as for ΔS_{sw})

ΔS_{sm} = change in soil moisture content (the signs are used in the same manner as for ΔS_{sw})

U = ground-water underflow (positive for an excess of outflow over inflow and vice versa)

A ground-water budget equation can be separated from the total budget equation by balancing ground-water inflow and outflow components against ground-water recharge (Walton, 1962, p. 22):

$$P_{gw} = R_g \pm U \pm \Delta S_{gw} + ET \qquad (4\text{-}4)$$

where:

P_{gw} = ground-water recharge

R_g = ground-water discharge into streams

U = ground-water underflow

ΔS_{gw} = change in ground-water storage

ET = evapotranspiration loss from the ground-water reservoir.

Modifications to the natural system by man can easily be added to equations 4-3 and 4-4. For example, discharge from wells would be added as

*All of the quantities in the equation are determined over some fixed time span, usually in terms of an equivalent depth over an area.

a positive quantity to the right side of equation 4-4. The quantity added would be net use, or the total amount withdrawn in the area minus the amount returned to the ground-water system in the area.

Upon examination of equations 4-3 and 4-4, a very important point becomes obvious. These equations treat the system as a lumped system. Total inflows and outflows are balanced, but their locations within the area are not considered. Also, many local inflow-outflow relationships may not appear as transfers across the area boundaries and, thus, never appear in the budget equation. For many purposes, distributed answers are desired. For example, the locations of discharging wells and their radii of influence, discharge and recharge from streams, major recharge and discharge areas, paths which pollutants follow from some initial point, etc., can be of major importance in many studies. Consideration of the system as a distributed system generally requires the solution or approximate solution of the differential equations describing ground-water movement.

Section 4.04. Hydraulic Potentials and Darcy's Law

The Significance and Forms of Darcy's Law

Rates and quantities of flow are generally computed using the ideas that fluid flows from one point to another when a hydraulic head difference exists between the two points. The equation used to compute the flow rate in most ground-water studies is Darcy's law.

Darcy's law is the dynamic equation expressing the conditions which control flow rate under low and medium velocity, laminar flow conditions in a porous medium. The equation was originally published by Henry Darcy, a French engineer, in 1856. His derivation was empirical, based on measurements of the quantity of flow through a sand-filled apparatus under various conditions of head loss and length of flow.

A simple form of Darcy's law is:

$$Q = KA\left(\frac{h_1 - h_2}{L}\right) = k \frac{\rho g}{\mu} A \left(\frac{h_1 - h_2}{L}\right) \qquad (4\text{-}5)$$

where:

Q = discharge or volumetric flow rate [(length)^3(time)$^{-1}$]

K = hydraulic conductivity [(length)(time)$^{-1}$]

k = intrinsic permeability [length^2]

ρ = mass density [(force)(time)^2(length)$^{-4}$]

g = acceleration due to gravity [(length)(time)$^{-2}$]

μ = viscosity [(force)(time)(length)$^{-2}$]

A = cross-sectional area through which flow occurs [(length)^2]

h = hydraulic head [length] (the subscripts refer to two points along the same flow path, 1 being upstream from 2)

L = the length along the flow path between points 1 and 2

$\left(\frac{h_1 - h_2}{L}\right)$ = the hydraulic gradient

The significance of the terms in Darcy's law in relation to the forces aiding and resisting fluid flow may be developed through the idea that hydraulic head and hydraulic gradient are measures of energy and energy dissipated, respectively. M. K. Hubbert (1940) derived a force potential, ϕ, which is the total mechanical energy contained in a unit mass of fluid:

$$\phi = gz + \int_{p_o}^{p} \frac{dp}{\rho} + \frac{v^2}{2} \tag{4-6}$$

where:

z = elevation above some selected datum plane [length]

p = fluid pressure [(force)(length)$^{-2}$]

v = velocity [(length)(time)$^{-1}$]

gz = gravitational potential energy

$\int_{p_o}^{p} \frac{dp}{\rho}$ = work available due to pressure change from reference pressure, p_o (usually atmospheric pressure)

$\frac{v^2}{2}$ = kinetic energy

For liquids at low velocity

$$\phi = gz + \frac{p - p_o}{\rho} \tag{4-7}$$

because the mass density, ρ, is nearly constant and kinetic energy is negligible (Hubbert, 1940, p. 802). Both equations 4-6 and 4-7 assume that mass density is a function of pressure only. Otherwise, the integral is indeterminate.

The fluid pressure at any point is determined by the height water would stand in a manometer open to the point and is (see fig. 4.05):

$$p = \rho g\ (h - z) + p_o \tag{4-8}$$

Substituting equation 4-8 into equation 4-7 there results:

$$\phi = gz + \frac{[\rho g\ (h - z) + p_o] - p_o}{\rho} = gh \tag{4-9}$$

(Hubbert, 1940, p. 802). Thus, the hydraulic head, a potential, is simply the total mechanical energy divided by the acceleration due to gravity. Hydraulic head is, therefore, written:

$$h = \frac{p - p_o}{\rho g} + z \tag{4-10}$$

The hydraulic gradient is the hydraulic head loss per unit length in the direction where this quantity is a maximum. Therefore, it is proportional to the mechanical energy dissipated by the water in that direction. The energy is transferred to the porous medium as heat from the frictional resistance to flow as water moves along the solid surfaces. In fig. 4.05 the hydraulic gradient is:

$$\frac{h_1 - h_2}{L} = \frac{(\frac{p_1 - p_o}{\rho g} + z_1) - (\frac{p_2 - p_o}{\rho g} + z_2)}{L}$$

$$= \frac{1}{\rho g}\ (\frac{p_1 - p_2}{L} + \frac{\rho g(z_1 - z_2)}{L}$$

$$= \frac{1}{\rho g}\ (\frac{p_1 - p_2}{L} + \rho g \cos\theta) \tag{4-11}$$

The pressure p_o is considered to be constant and does not enter into the calculation of the gradient. It is convenient to let it equal zero, or atmospheric gauge pressure. The term $\rho g \cos\theta$ is the component of unit fluid weight acting in the flow direction. Because the flow direction in fig. 4.05 has a downward component, the term is positive, i.e., the fluid weight aids flow. If the flow direction were opposite, the term would have been negative and would have tended to decrease the gradient. For horizontal flow $z_1 - z_2 = 0$, thus $\cos\theta = 0$, and the unit fluid weight has no effect on the gradient.

Note that for the example in fig. 4.05 the flow direction is toward increasing pressure. This is possible because flow is toward lower hydraulic head not just toward lower pressure. Also, note that the flow is linear, parallel to the sides of the column. Therefore, L is measured parallel to the sides of the column.

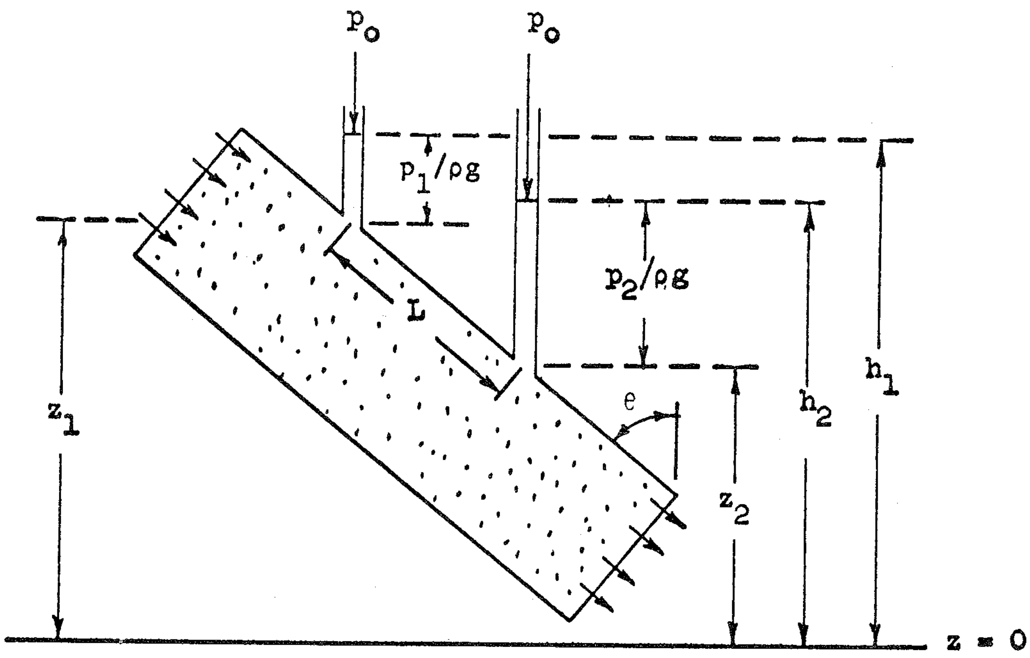
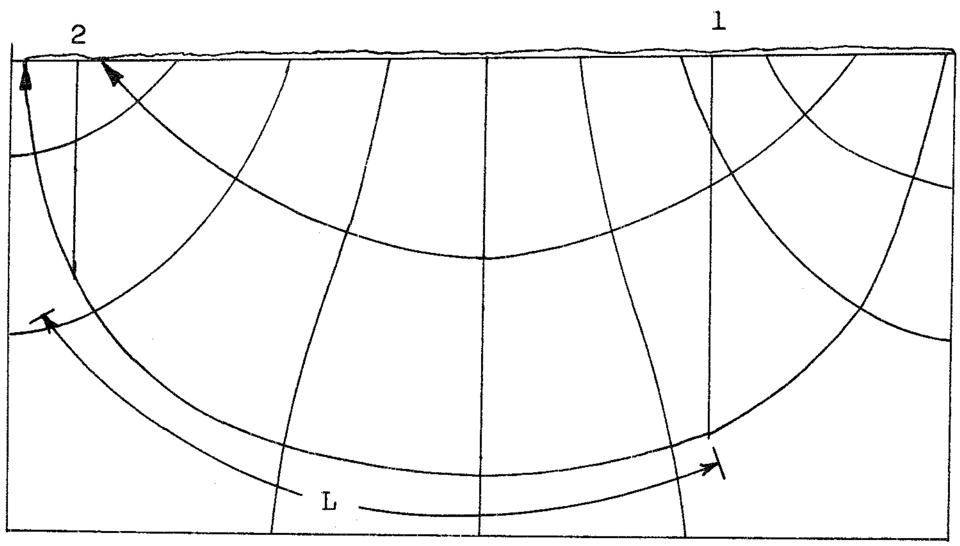


Fig. 4.05. Saturated water flow through a soil column

In fig. 4.06 two wells, open only at their bottoms, penetrate an idealized unconfined flow system with a recharge area on the right side and discharge area on the left side. Note that the length, L, must be measured along the curved flow path to calculate the true hydraulic gradient between the two wells, although little error would result from using the distance between their bottoms in this case. If well 2 were shallower than it is, the equipotential line crossed at the bottom of the well would have to be extended to cross the flow line passing through the bottom of well 1 in order to measure L. Only through an understanding of the flow system is it possible to predict whether a good estimate of the true hydraulic gradient can be computed between two wells.



Note: A recharge area is on the right (flow crosses the water table to enter the system) and a discharge area is on the left (flow crosses the water table to leave the system) (modified from Toth, 1962a, p. 4380). The lines at right angles to the flow lines are lines of equal hydraulic head. The left, right, and bottom boundaries are impermeable.

Fig. 4.06. Cross section through a flow system

By substituting equation 4-11 into equation 4-5, the following expression for Darcy's law is obtained:

$$Q = \frac{k}{\mu} A \left[\left(\frac{p_1 - p_2}{L} \right) + \rho g \cos \theta \right] \qquad (4\text{-}12)$$

In Appendix 1 it is shown that, in order for Darcy's law to be true, the force in the flow direction due to viscous drag must be in equilibrium with the forces in the flow direction due to the pressure gradient and fluid weight. The control that viscous drag has on flow rate is represented in the term k/μ, and the controls that the pressure gradient and fluid weight have on flow rate are represented by the terms in the bracket.

There are a number of commonly used equivalent forms of Darcy's law. Equations 4-5 and 4-12 are two, others are:

$$Q = KiA \tag{4-13}$$

$$Q = TiW \tag{4-14}$$

$$V = \frac{Q}{A} = K\left(\frac{h_1 - h_2}{L}\right) = Ki \tag{4-15}$$

$$V_r = \frac{K}{n_e} i \tag{4-16}$$

$$\Delta\ell = \frac{K}{n_e} i\Delta t \tag{4-17}$$

$$\vec{v} = -K\nabla h = -K \text{ grad } h = -K \frac{1}{g} \nabla\phi = -K \frac{1}{g} \text{ grad } \phi \tag{4-18}$$

where:

$i\nabla$ = $-\nabla h = -\text{ grad } h = -\frac{1}{g}\nabla\phi = -\frac{1}{g}\text{ grad } \phi$ = hydraulic gradient

W = width through which flow occurs

T = transmissivity = Kb

b = thickness through which flow occurs

V = discharge per unit area

V_r = true average velocity of flow

n_e = effective porosity

$\Delta\ell$ = distance a fluid particle moves during time interval, Δt

$\vec{v}$ = discharge per unit area written in vector notation

Several assumptions and limitations are contained in Darcy's law (see Appendix 1 for the derivation), and these are summarized as follows:

a. Inertial forces (i.e., those forces associated with the mass of the fluid times its acceleration) are assumed to be negligible so that the flow paths do not distort with increasing velocity (Hubbert, 1940, p. 811).

b. Neglecting inertial forces also assumes flow to be steady and nearly uniform so that the forces taken into account in the derivation balance (Hubbert, 1940, pp. 806-807).

Therefore, if inertial forces are not negligible, Darcy's law is not valid. Harr (1962, p. 13) cited investigations showing that the unsteady component of the total inertial forces can generally be neglected except for very wide variations of velocity in time. Todd (1959, pp. 46-49) cited studies indicating that Darcy's law is valid for Reynolds numbers less than the range between 1 and 10. Reynolds number, N_R, for these studies is defined as:

$$N_R = \frac{\rho \vec{v} d}{\mu} \qquad (4\text{-}19)$$

where d is "an average grain diameter" (Todd, 1959, p. 47) and other terms are as defined before.

Intrinsic Permeability and Hydraulic Conductivity

Intrinsic permeability is commonly written (Davis and DeWiest, 1966, p. 162, Todd, 1959, p. 51, Hubbert, 1940, p. 816).

$$k = Cd^2 \qquad (4\text{-}20)$$

In this equation:

C = a dimensionless coefficient characteristic of the internal structure

d = a representative grain diameter (Todd, 1959, p. 51) or "average pore size of the medium" (Davis and DeWiest, 1966, p. 162)

Apparently (see equation 10, Appendix 1):

$$\frac{1}{B} = C_1 d^2 \qquad (4\text{-}21)$$

where B is a constant characteristic of the pore geometry. In equation 4-21, C_1 is a dimensionless coefficient so that:

$$k = C_1 n_e d^2. \tag{4-22}$$

The coefficient C_1 may be interpreted as characteristic of the internal pore geometry including pore shapes, roughness of the pore surfaces, tortuosity of pores, frequency of constrictions, etc. It is not a constant. The porosity, n_e, used in equation 4-22 is effective porosity because only interconnecting pores can transmit water. Equation 4-22 provides a qualitative or semiqualitative aid to the understanding of the controls on intrinsic permeability.

The effective porosity is controlled by a number of factors depending on rock type. For an unconsolidated, sedimentary rock sorting and packing are extremely important. Sorting, which is a measure of the sediment grain-size distribution around an average, is important because, if smaller grains are added to a deposit of larger ones and the total volume is constant, the total pore space and, thus, porosity are reduced (Meinzer, 1923, p. 6). Sediment packing governs the volume of pore space; the tighter the packing the less the pore space and, consequently, the porosity. For other rock types, different factors, parallel in their effect to sorting and packing, exert important controls on the effective porosity. Other phenomena, such as air bubbles, which act as grains to the flowing water, and microbiota growing in the pore space can reduce porosity also.

The term d^2 is derived from dimensional considerations and so has been interpreted differently by different investigators. Todd (1959, p. 51) stated that the dimensions of k describe a unique pore area. Therefore, d^2 can be interpreted as this area, the cross sectional area of the pore size which controls intrinsic permeability. This unique pore size is independent of the number of such pore sizes per unit volume because this number is related to the porosity.

The factors discussed here are only a few of the many factors which can affect permeability, and other factors such as grain orientations, stratification, and clay swelling may often be important. Grain migrations through the pores and consolidation can also cause intrinsic permeability to vary with time.

Variations in hydraulic conductivity are caused by all the variations in intrinsic permeability plus mass density and viscosity of water variations. Often both vary mainly because of water temperature changes. As the temperature increases, both density and viscosity decrease. However, according to equation 4-5 they cause opposing effects. Also, the magnitude of these effects is vastly different, variation in viscosity due to temperature changes being far the greater of the two. Table 4.01 illustrates the effect of temperature on hydraulic conductivity. It shows percent increase in hydraulic conductivity, K, from that at 32^{o} F, K_{32}, due to a decrease in kinematic viscosity, $\frac{\mu}{\rho}$.

Table 4.01. Increase in hydraulic conductivity due to an increase in water temperature[a]

Temperature, °F	$\frac{K - K_{32}}{K_{32}} \times 100$ (percent)
32	0.0
40	13.8
50	27.0
60	37.0
70	45.1
80	51.8
90	57.2

[a]Data from Vennard (1961, p. 545)

Turbidity also affects hydraulic conductivity. In addition to clogging the pores, an increase in turbidity causes an increase in both density and viscosity.

Extensions of Darcy's Law

Darcy's law also apparently applies to unsaturated flow (Richards, 1931, Todd, 1959, p. 71). However, both conductivity and fluid pressure are functions of saturation (or water content of the porous material) and, thus, are functions of one another. Darcy's law for this case can be written:

$$v = -K(S_w)\frac{dh}{dL} \tag{4-23}$$

where $K(S_w)$ is the hydraulic conductivity (usually referred to for this case as capillary conductivity) as a function of water saturation, S_w for 100 percent water saturation $S_w = 1$).

The capillary conductivity decreases as water saturation increases. This probably occurs because water is confined to smaller and smaller openings or areas near grain contacts as saturation decreases; thus, there is a decrease in both effective d and n_e. When the water content approaches the specific retention of the material, the capillary conductivity approaches zero.

Darcy's law is also applied to anisotropic porous media. In this case, intrinsic permeability is no longer a scalar quantity as was assumed in the derivation but instead varies with direction at any place. Remembering that the discharge per unit area is a vector and thus can be resolved into components, Darcy's law with anisotropic conductivity can be written:

$$\left.\begin{aligned} v_x &= -K_{xx}\frac{\partial h}{\partial x} \\ v_y &= -K_{yy}\frac{\partial h}{\partial y} \\ v_z &= -K_{zz}\frac{\partial h}{\partial z} \end{aligned}\right\} \tag{4-24}$$

where the subscripts refer to the three orthogonal axes, x, y, and z. The velocity in the x direction, v_x, equals the product of the conductivity in the x direction, K_{xx}, and the component of the gradient in the x direction, $\partial h/\partial x$. The other two equations are interpreted similarly.

The three values of conductivity are the three principal components of the conductivity tensor. The x, y, and z axes must be aligned with the three principal directions of the conductivity tensor in order to write equations 4-24.

For flow in anisotropic porous media, the resultant hydraulic gradient (i.e., the resultant of the x, y and z components) is no longer parallel to the flow line as it was for isotropic porous media. For further discussion the reader is referred to other texts such as Harr (1962, pp. 26-35).

Section 4.05. Continuity Relationships

The ground-water budget, equation 4-4, can be rearranged and modified to yield:

$$U = \Delta S_{gw} + D - R \qquad (4\text{-}25)$$

Here the terms are expressed as flow rates and are defined as:

U = net underflow into the designated region per unit time (negative if outflow is greater than inflow)

ΔS_{gw} = net increase in ground-water storage per unit time (negative if there is a net decrease)

D = net ground-water discharge per unit time

R = net ground-water recharge per unit time

The term U can be regarded as the difference between ground-water inflow into the area and outflow from it, or:

$$U = Q_1 - Q_2 \qquad (4\text{-}26)$$

where:

Q_1 = net ground-water inflow into the region per unit time

Q_2 = net ground-water outflow from the region per unit time

The storage term, ΔS_{gw}, can be approximately evaluated as follows: For an unconfined system the water pressure at the water table is atmospheric or zero gauge pressure. Therefore, $h = z$ at the water table.

The total volume of porous materials dewatered (excluding the capillary fringe), V_t, during some interval of time can therefore be written:

$$V_t = A_\ell \Delta\bar{h} \tag{4-27}$$

where:

A_ℓ = map area of the region

$\Delta\bar{h}$ = the average change in water table elevation over the region during time interval Δt_{xx}

If water is being withdrawn from storage, V_t must be multiplied by the amount of water per unit volume that will drain under the influence of gravity. This function is often assumed to be the specific yield, S_y, which implies that voids drain instantaneously as the water table is lowered. However, water above the water table drains slowly under the influence of gravity, thus the function is actually variable with time. For the case of a rising water table, the magnitude of the term depends on the antecedent moisture content as well as time. It is convenient to denote the time variant amount of water taken into or released from ground-water storage per unit volume as $S_f(t)$. It should be noted that this term is actually dependent mainly on relationships between capillary conductivity, fluid pressure, and saturation in the unsaturated region above the water table. These terms are, in turn, variable in time. To obtain water withdrawn per unit time, the expression $V_t S_f(t)$ must be divided by the interval of time, Δt. Therefore:

$$\Delta S_{gw} = A_\ell S_f(t) \frac{\Delta\bar{h}}{\Delta t} \tag{4-28}$$

Equation 4-25 can be written using equations 4-26 and 4-28 as:

$$Q_1 - Q_2 = A_\ell S_f(t) \frac{\Delta\bar{h}}{\Delta t} + D - R \tag{4-29}$$

where the discharges Q_1 and Q_2 are calculated on the boundaries of the region using Darcy's law. For example, for isotropic conductivity:

$$Q_1 = K_1 A_1 \frac{\Delta h_1}{\Delta L_1} \tag{4-30}$$

The new symbol used is:

A_1 = the cross sectional area through which ground-water inflow occurs, measured perpendicular to the flow direction

It should be noted that K_1 is assumed to be the effective value over the area of inflow.

For application to confined conditions, the storage function, $S_f(t)$, in equations 4-28 and 4-29 is replaced with the coefficient of storage, S, defined as the amount of water in storage released from or taken into a column of aquifer with unit cross section under a unit change of hydraulic head (Davis and DeWiest, 1966, p. 182). Change in water storage does not result from changing thickness of saturated materials as it does for the unconfined case. For a decline in head, water is released from storage due to the elastic expansion of water and elastic compression of the rock unit. For an increase in head, the opposite occurs.

Equation 4-29 is best applied to single hydrologic units, i.e., units of relatively constant conductivity and storage properties. All of the terms apply to inflows and outflows from that unit. The equation can also be applied to adjacent units where some inflows to one unit are outflows from the other. As the regions over which each equation is applied become smaller, more distributed information is obtained. However, distributed information about the quantities, $S_f(t)$ (or S), K, D, and R is necessary in order to make the analysis.

Providing that estimates of these quantities are available or can be determined, the maximum amount of information can be obtained by writing the continuity equations and Darcy's law as differential equations. A general continuity equation is derived as equation 18 in Appendix 2, and the next section lists some of the various commonly used simplifications. It should be borne in mind that they express the same general principles as equations 4-29 and 4-30.

Section 4.06. Differential Equations Describing Fluid Flow

Equation for Flow with a Free-Surface Using Dupuit's Simplifying Assumptions

Boussinesq (1904) and Jacob (in Rouse, 1950, p. 383) derived an unsteady-state equation for unconfined or free-surface flow using Dupuit's simplifying assumptions: (1) The velocity of flow is proportional to the tangent of the angle between the water table and the horizontal, i.e., all flow paths are assumed to be horizontal, and (2) the velocity and hydraulic conductivity are uniform over the entire depth of flow. Additional assumptions are: Density is constant, S_w equals 1 in the flow field and zero outside, and all water is released from storage instantaneously with a decline in water table. One form of the equation, which allows for variation in the elevation of the base of an essentially horizontal aquifer is:

$$\frac{\partial}{\partial x}\left(K_{xx} b \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy} b \frac{\partial h}{\partial y}\right) = S_y \frac{\partial h}{\partial t} \qquad (4\text{-}31)$$

New symbols used are:

$b = h - (h_o - b_o)$ = saturated thickness of an aquifer at any time

h_o = initial head

b_o = initial saturated thickness of the aquifer

The directions x and y are both horizontal, and head, h, is measured vertically. In order to allow for variations in storage capacity with time, S_y could be replaced with $S_f(t)$. The use of equation 4-31 with time variant storage capacity is illustrated in Trescott, Pinder, and Jones (1970).

Equation for Confined Flow Through an Aquifer of Uniform Thickness

Jacob (in Rouse 1950, p. 333) showed that the equation for a horizontal confined aquifer of constant thickness, b, constant hydraulic conductivity, K, and constant coefficient of storage, S, is:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{S}{T} \frac{\partial h}{\partial t} \qquad (4\text{-}32)$$

where:

$$T = Kb$$

A special case of equation 4-32 is for radial flow (e.g., flow to a well). If the coordinates are transformed from orthogonal cartesian to cylindrical and flow is assumed to have radial symmetry, equation 4-32 becomes:

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} = \frac{S}{T} \frac{\partial h}{\partial t} \qquad (4\text{-}33)$$

The Laplace Equation

If flow is not time dependent (i.e., is steady), then the right hand term of equation 18, Appendix 2, becomes zero. For variable, anisotropic hydraulic conductivity, the Laplace equation with variable coefficients results:

$$\frac{\partial}{\partial x} \left(K_{xx} \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y} \left(K_{yy} \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z} \left(K_{zz} \frac{\partial h}{\partial x}\right) = 0 \qquad (4\text{-}34)$$

If hydraulic conductivity is constant and isotropic then:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0 \qquad (4\text{-}35)$$

This is the classical Laplace equation.

All of the above equations can be written to include the source or sink term, W (see equation 18, Appendix 2), and equations 4-31 through 4-33 can be written for steady-state conditions. Also, many more derivations of particular equations from equation 18, Appendix 2, are possible. Only the most commonly used ones have been summarized here.

Boundary Conditions

In addition to the equations describing the flow of ground water, conditions at the boundaries of the flow region and (for unsteady-state problems only) conditions in the field of flow at the initial instant must be specified. The flow system is uniquely defined when the equation of motion and the following conditions are given:

a. A geometrical statement of the boundaries of the region for which a solution is desired (the shape of the region being considered).

b. A specification of the velocity of flow normal to the boundaries, the hydraulic gradients at the boundaries, or the hydraulic heads at the boundaries (the inflows and outflows or hydraulic head conditions at the boundaries).

c. If the flow is unsteady, the initial hydraulic head distribution (the head distribution at $t = 0$).

Conclusions

By progressing through the ground-water budget equations, including calculations for the underflow and rate of storage change, to the differential equations, it can be seen that the differential equations are essentially distributed forms of the budget equations. The several differential equations listed are approximations for more general relationships actually controlling flow. A fairly general continuity equation is derived in Appendix 2, but even more general equations can be derived (see for example, Verruijt, in DeWiest 1969). Solution of these equations will not only provide more reliable information about nature of subsurface-water movement but will also provide a more rational basis for approximations than presently available. With continued developments in computers and sophisticated mathematical methods it will soon be possible to solve general three-dimensional flow problems. It is expected that measurement of parameters such as conductivity will remain a major obstacle to application of the new methods.

Chapter 5

Flow Nets

CHAPTER 5. FLOW NETS

Section 5.01. Introduction

Flow net analysis is a graphical method of solution for several different kinds of ground-water flow and seepage problems. For one variant of this method the hydraulic head distribution is unknown and is to be determined. To determine the solution, a family of curves parallel to the flow direction called flow lines are drawn orthogonally to another family of curves called equipotential lines. Each flow line represents the path a fluid particle would follow through a porous medium, and successive flow lines are constructed so that an equal quantity of discharge is contained between each pari. Each equipotential line represents a line of equal hydraulic head, and these lines are drawn so that there is an equal head loss between each successive pari. There is an infinite number of possible flow lines and equipotential lines, and only those needed to give sufficient detail for satisfactory accuracy are drawn.

Another variant of the method utilizes a contour map of either a piezometric surface or water table which has been constructed from known water level data. Pairs of flow lines are drawn orthogonally to the contour lines only at selected places. Then discharge differences are computed between several cross sections, each bounded by the pair of flow lines and oriented parallel to the contours. Differences between the discharges at two or more of the cross sections can be related to withdrawals by wells, recharge, changes in ground-water storage, or other phenomena. Knowing all but one of these variables, the remaining one can be computed. If all of these variables are known, it is also possible to use the discharge difference to estimate transmissivity.

Flow nets are very useful because they illustrate the principles of fluid motion and the influence of various types of boundary conditions on flow patterns. They also provide a means, which does not require specialized mathematical procedures, for obtaining approximate solutions to many flow problems. One limitation on the first variant of the method lies in the amount of time needed to obtain a solution because these flow nets require trial and error sketching of flow lines and equipotantial lines. This limitation becomes especially severe for free-surface problems having complicated boundaries and involving more than one hydraulic conductivity. Another limitation is that the method can only be applied to certain classes of problems.

Section 5.02. Theory and Background

The principles and methods for the first type of flow net analysis, the determination of the hydraulic head distribution, are discussed in this section and Section 5.03. The second variation, defined for the purposes of this discussion as the modified flow net analysis, is discussed in Section 5.04.

One of the important principles used in drawing flow nets is that flow lines and equipotential lines are orthogonal. The necessary conditions for these principles are derived in Appendix 3, and are summarized as follows:

a. The medium can be treated as isotropic (this will be explained further on).

b. Darcy's law is valid. A further condition used in drawing flow nets is either that flow is steady-state or the fluid and porous medium are incompressible. For unsteady flow, true flow lines, as unique paths fluid particles follow through the porous medium, do not exist. If flow were unsteady and a fluid particle were to start out at a particular place at a particular time, it would follow a course different from the one it would follow if it started from the same place at a different time. However, for an incompressible system a flow net may be drawn using pseudo-flow lines--flow lines the fluid particles would follow if flow were steady. The final condition used is that K, the hydraulic conductivity (hereafter called conductivity) be constant. Flow lines and equipotential lines are orthogonal for variable but isotropic conductivity; however, the geometric figures formed by the intersections of successive equipotential lines and flow lines do not have a constant length-to-width ratio. Thus, unless the conductivity is constant, drawing the flow nets is very difficult.

The relationships between the length and width of the geometric figures and conductivity can be demonstrated using Darcy's law, which can be written as:

$$\Delta q = K \, \Delta w \frac{\Delta h}{\Delta \ell} \qquad (5\text{-}1)$$

for discharge through a streamtube (see fig. 5.01). For steady flow, the discharge per unit thickness, Δq, is constant. Thus, for constant K and Δh, an increase in Δw, the streamtube width, is balanced by a proportionate increase in $\Delta \ell$, the equipotential spacing. Therefore, the length to width ratio is constant. If K was not constant, it can be seen from equation 4-1 that $\frac{\Delta w}{\Delta \ell}$ could not be constant.

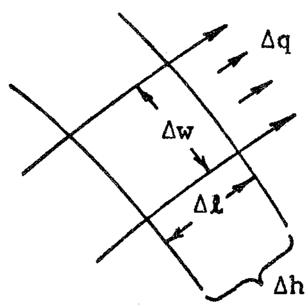


Fig. 5.01. Diagram of the intersection of a streamtube and two equipotential lines

The last condition used in constructing flow nets is that the porous medium is sectionally homogeneous (this will be explained further) and saturated.

The continuity equation for steady-state, two-dimensional flow (or two-dimensional flow of an incompressible fluid in an incompressible porous medium is:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} = 0 \qquad (5\text{-}2)$$

where:

v_x = the component of the discharge per unit area in the x direction

v_z = the component of the discharge per unit area in the z direction

Substituting in Darcy's law defined by equations 2 and 3, Appendix 3, there results for constant K:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \tag{5-3}$$

This is the Laplace equation. Flow-net analysis is, therefore, valid only when the Laplace equation is valid.

Besides satisfying the Laplace equation within the flow region, certain requirements must be satisfied on the boundaries:

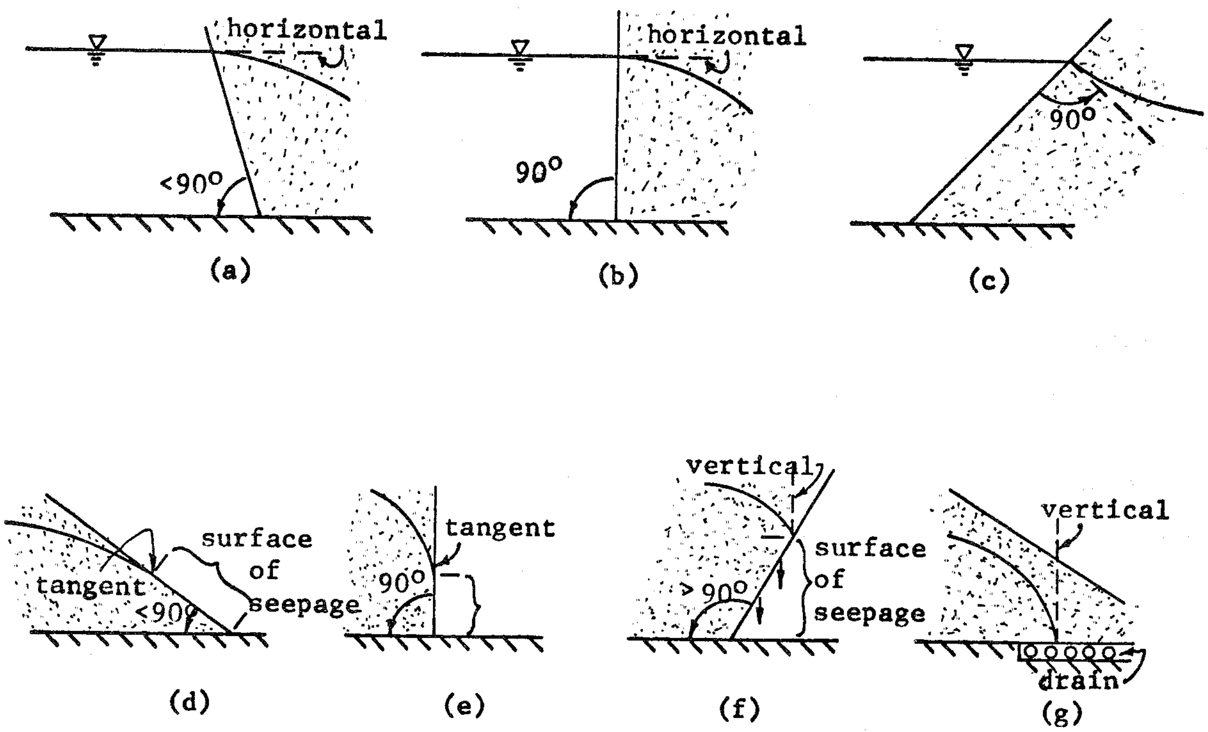
a. Impermeable boundary. For this case the component of flow normal to the boundary is zero, and the total flow is carried by the tangential component. Because flow lines are orthogonal to equipotential lines, it follows that equipotential lines terminate perpendicularly on the boundary (Hubbert, 1940, p. 847).

b. Open water boundary. Because the pressure distribution is a slowly-moving, water-filled basin or void is nearly hydrostatic, the potential, $h = \frac{p}{\gamma} + z$ (where p is the water pressure, γ is the specific weight of water, and z is the height above a reference plane), is nearly constant. The boundary is, then, an equipotential line. The orthogonality of equipotential and flow lines thus requires that flow lines terminate perpendicularly on the boundary.

c. Surface of seepage. This boundary is where water leaves the porous medium as seepage and enters the open air. The pressure at this boundary can be considered to be constant. Therefore, h - z = constant, and equipotential lines intersect the boundary at constant intervals. This boundary is neither an equipotential line nor a flow line.

d. Free surface or phreatic line. The free surface is the upper flow line in an unconfined, steady-state flow system with no recharge crossing the free surface. Thus, equipotential lines intersect it at right angles.

Also, pressure is constant along this line, and $h - z$ = constant. As for the case of the surface of seepage, equipotential lines intersect the free surface at constant intervals. Common situations where free surfaces intersect other boundary conditions are given in fig. 5.02.



Note: The free surface is tangent to the dashed lines in (a) through (c), (f), and (g) (modified from Casagrande, 1940).

Fig. 5.02. Intersections between free surfaces and other boundaries

Although it was stated that conductivity should be constant for flow-net analysis, this idea can be modified so that the flow region is divided into sections or subregions of constant but differing conductivity. The direction of flow on either side of the boundaries between regions must change (fig. 5.03) in order to conserve matter and energy. The law governing this refraction effect is derived in Appendix 3 and can be stated as:

$$\frac{K_2}{K_1} = \frac{\tan \alpha}{\tan \beta} \qquad (5\text{-}4)$$

where:

K_1 = the conductivity in region 1

K_2 = the conductivity in region 2

α = the angle between the boundary and the flow lines in region 1

β = the angle between the boundary and the flow lines in region 2

This result can also be restated in a form more convenient for flow-net analysis (Cedergren, 1967, p. 108). Between any two flow lines which cross the boundary

$$\Delta q_1 = K_1 \, \Delta w_1 \frac{\Delta h_1}{\Delta \ell_1} \tag{5-5}$$

in region 1, and

$$\Delta q_2 = K_2 \, \Delta w_2 \frac{\Delta h_2}{\Delta \ell_2} \tag{5-6}$$

in region 2 (fig. 5.03). The discharge through the streamtube is constant; therefore, $\Delta q_1 = \Delta q_2$. Also, the equipotential lines refract but are continuous across the boundary. Thus, $\Delta h_1 = \Delta h_2$. If $\Delta w_1 = \Delta \ell_1$ (i.e., equipotential lines and flow lines are made arbitrarily to form squares in region 1), then

$$K_1 = K_2 \frac{\Delta w_2}{\Delta \ell_2}$$

or

$$\frac{K_2}{K_1} = \frac{\Delta \ell_2}{\Delta w_2} \tag{5-7}$$

In other words, "squares" in region 1 change to "rectangles" in region 2 with average length to width ratios equal to K_2/K_1.

The relationships described above can be better understood when it is realized that less area and a lower hydraulic gradient are needed in the region of high conductivity to convey the same discharge as in the region of low conductivity. To accomplish this, flow lines must refract if they are to remain continuous across the boundary.

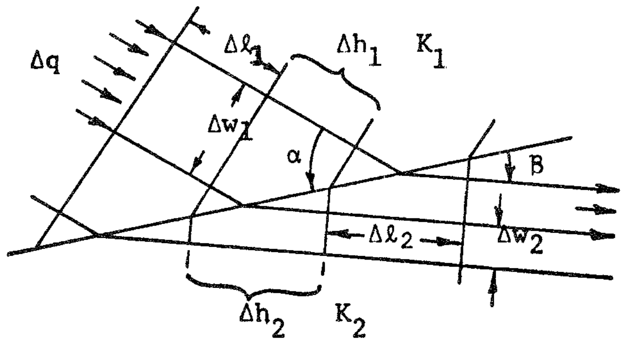


Fig. 5.03. Refraction of flow lines across a boundary between regions of differing conductivity

It was stated as one of the conditions for the validity of the Laplace equation that conductivity be isotropic. However, if there are two principal, orthogonal conductivity axes, a transformation of coordinates will yield the Laplace equation (Harr, 1962, p. 29). For two-dimensional systems having anisotropic conductivity, and where the principal components of the conductivity are aligned with the x and z directions, Darcy's law is written as:

$$v_x = -K_{xx} \frac{\partial h}{\partial x} \tag{5-8}$$

and

$$v_z = -K_{zz} \frac{\partial h}{\partial z} \tag{5-9}$$

where:

K_{xx} = principal component of conductivity in the x direction

K_{zz} = principal component of conductivity in the z direction

The flow equation then becomes:

$$K_{xx} \frac{\partial^2 h}{\partial x^2} + K_{zz} \frac{\partial^2 h}{\partial z^2} = 0 \tag{5-10}$$

The transformation

$$\bar{x} = \frac{x}{\sqrt{K_{xx}}} \tag{5-11}$$

and

$$\bar{z} = \frac{z}{\sqrt{K_{zz}}} \tag{5-12}$$

applied to equation 5-10 yields

$$\frac{\partial^2 h}{\partial \bar{x}^2} + \frac{\partial^2 h}{\partial \bar{z}^2} = 0 \tag{5-13}$$

which is the Laplace equation. Thus, a flow net can be drawn in the transformed section, then transformed back to the original section. Flow lines and equipotential lines will not intersect at right angles in the original section.

Equations to be used for the computation of seepage quantities from a flow net can now be derived. If Δq is the discharge per unit thickness between two adjacent flow lines and n_f is the number (not necessarily an integer) of streamtubes or flow channels in the flow net, then the total discharge per unit thickness is:

$$q = n_f \Delta q \tag{5-14}$$

Also, if $\Delta w = \Delta \ell$,

$$\Delta q = K \Delta w \frac{\Delta h}{\Delta \ell} = K \Delta h \tag{5-15}$$

However,

$$\Delta h = \frac{H}{n_d}$$

where:

H = the total head loss through the region

n_d = the number of potential or head drops in the flow net

Therefore,

$$\Delta q = K \frac{H}{n_d} \tag{5-16}$$

Using equation 14

$$q = KH \frac{n_f}{n_d} \tag{5-17}$$

where $\frac{n_f}{n_d}$ is called the "shape factor" (Cedergren, 1967, p. 115).

For anisotropic conductivity which can be analyzed with transformed sections,

$$q = \bar{K}H \frac{n_f}{n_d} \tag{5-18}$$

where

$$\bar{K} = \sqrt{K_{zz} K_{xx}}$$

is the effective conductivity (Harr, 1962, pp. 30-31).

The necessary background material has now been derived. The next section is concerned with the application of this material to the construction and use of flow nets.

Section 5.03. Construction of Flow Nets

Much of the material in this section has been summarized and modified from Cedergren (1967, pp. 94-116 and 148-170).

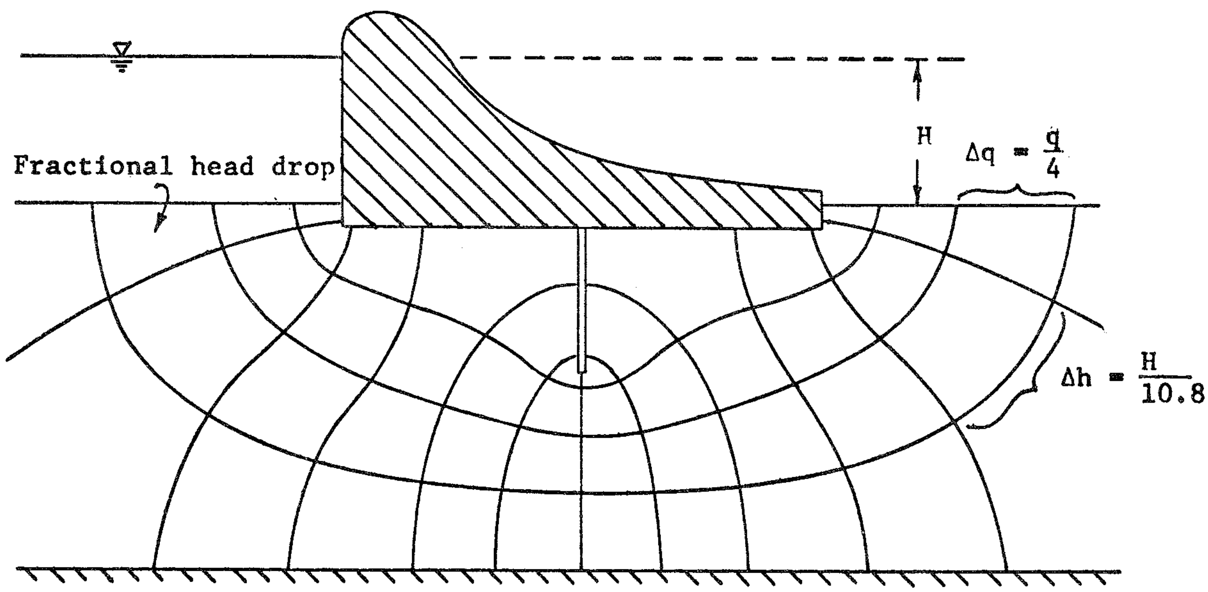
Basic Rules

These rules are a summary and some consequences of the material derived in Section 5.02.

a. Flow lines and equipotential lines should intersect at right angles and form geometric figures that are "squares." The latter requirement is not absolutely necessary; any length to width ratio could be chosen. However, the selection of squares greatly simplifies drawing of the net. A "square" is defined as a geometric figure with average width and average length equal.

b. The boundary conditions listed on page 5-04 must be satisfied.

c. Adjacent equipotential lines represent equal head losses. This rule applies to complete equipotential drops. If the flow net is constructed to contain some whole number of streamtubes, the number of equipotential drops depends on the shape of the cross section. The last drop at one or both ends of the region may, by necessity, be a fractional drop (fig. 5.04).



Note: H = total head lost through the net (modified from Davis and DeWiest, 1966, p. 192).

Fig. 5.04. Flow net under an impermeable structure with a cut-off wall

d. The same discharge flows between adjacent pairs of flow lines. Also, the number of streamtubes must remain the same throughout the net. Again, the streamtube near a boundary may be fractional; however, it must be the same fraction everywhere. If this rule is followed, the rule that the flow net be composed of squares is automatically satisfied. Along a surface of seepage, part of the flow net is cut off; thus, this rule is not satisfied (fig. 5.05).

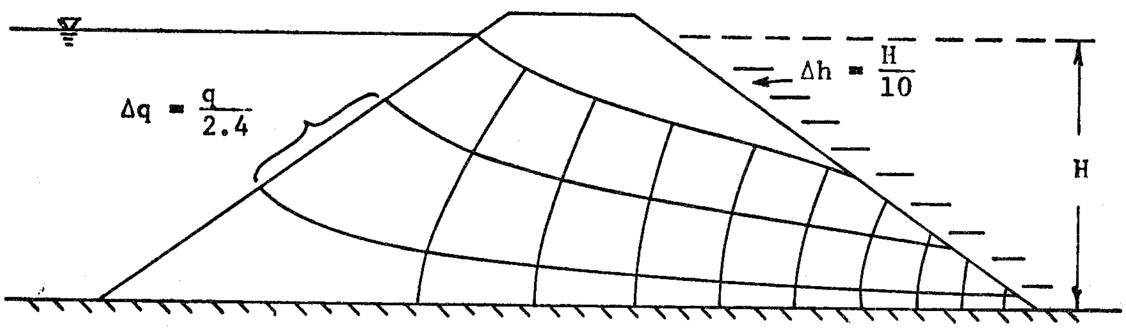


Fig. 5.05. Seepage with a free surface through a permeable structure underlain by an impermeable boundary (modified from Cedergren, 1967, p. 159)

General Methods

The following outline should be viewed as suggestions for drawing flow nets and not as hard and fast rules. Experience of each individual will determine what methods are best for him.

a. The cross section to be studied should be drawn on one side of good tracing paper, and the flow net should be drawn on the other side. It can then be traced onto the former side or onto another sheet of paper.

b. Just enough flow lines and equipotential lines to bring out the essential features should be used. If detail needs to be emphasized in some parts of the flow net, flow lines and equipotential lines in those parts can be subdivided after the flow net is completed.

c. The scale of the drawing should be just large enough to draw essential details. Often a letter-size sheet of paper is large enough.

d. The boundary conditions, especially prefixed flow lines and equipotential lines, should be studied before starting to draw the flow net.

e. Either the number of streamtubes or the number of equipotential drops should be made a whole number to simplify flow net construction. However, if necessary, both can be fractional.

f. The overall shape of the flow net should be kept well in mind while working on details. Because the shape of each section of a flow net affects the rest, a small portion should never be refined before the rest is almost completed.

Flow with a Free Surface

In a flow system which has a free surface, the free surface is unknown at first and must be determined as the flow net is developed. This is in contrast to a flow system where all boundaries are known from the start (fig. 5.04). The condition which permits the determination of the free surface is that equipotential lines with equal head losses between them must intersect the free surface at equal vertical intervals. Thus, the flow net is adjusted until the conditions discussed in Section 5.03 are met and until equipotential lines intersect the free surface at the correct elevation.

Capillary effects are generally neglected at the free surface so that the boundary condition is $p = 0$ or atmospheric gage pressure. Therefore, the head along this line is $h = z$, and equipotential lines intersect elevation lines (figs. 5.05 and 5.06). Often capillary effects are not negligible, and the free surface should be located at a pressure surface less than atmospheric. The reason for this is that an appreciable quantity of water often flow in the unsaturated region, principally in the zone of numerically small negative pressures. Also the negative pressures existing in the unsaturated region tend to expand the flow system, especially if the velocity has a large vertical component. Bouwer (1964 and 1965) presents the theory and concepts behind the procedures of locating the free

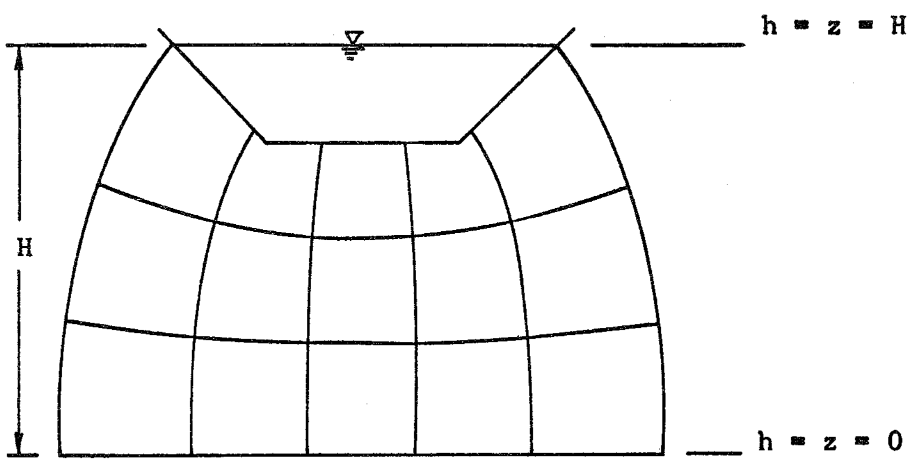


Fig. 5.06. Flow net for seepage from a trapezoidal channel to an infinitely permeable horizon at $z = 0$ (modified from Jeppson, 1968, p. 272)

surface at some pressure less than atmospheric and gives typical values of the negative pressure boundary condition for various materials. To apply the procedure given here for finding the free surface to finding the position of the negative pressure boundary, one simply adjusts the free surface until the pressure has the specified value along it. The boundary condition is $h = z + p_c$ where p_c is the negative pressure boundary.

Before beginning construction of a flow net with a free surface, the total head, H, should be divided into a convenient number of equal parts, $\Delta h = \Delta z$ should be drawn across the flow region. Each equipotential line should intersect one and only one of these elevation lines at the free surface.

A trial phreatic line (i.e., free surface) should be drawn before any other flow lines or equipotential lines are constructed. Next a plausible family of equipotential lines followed by one or more intermediate flow lines can be drawn. Correct boundary conditions, the existence of squares, a constant number of streamtubes, etc., should then be checked and the net adjusted accordingly.

Anisotropic Conductivity

To construct a flow net for anisotropic conductivity, either the dimension of the cross section in the direction of greatest conductivity can be reduced or the dimension in the direction of smallest conductivity can be increased. Thus, if $K_{zz} < K_{xx}$, either

$$\bar{x} = x\sqrt{\frac{K_{zz}}{K_{xx}}} \tag{5-19}$$

and

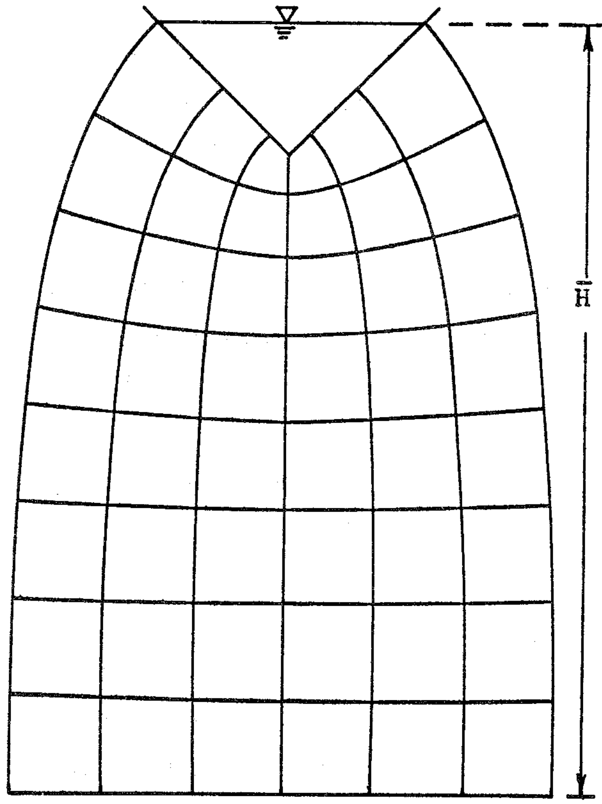
$$\bar{z} = z$$

or

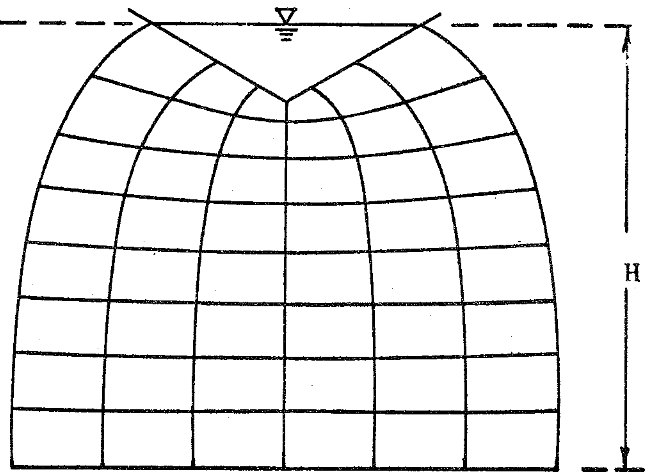
$$\bar{z} = z\sqrt{\frac{K_{xx}}{K_{zz}}} \tag{5-20}$$

and $\bar{x} = x$

The flow net is then drawn in the transformed section in the normal manner. The true flow net is obtained by transforming back to the original section. This can be accomplished by determining the location of a number of points on the flow lines and equipotential lines on the transformed section and replotting these points on the original section (fig. 5.07).



(a)



(b)

NOTE: H is the depth below the channel surface drawn in the transformed section to an infinitely permeable horizon (modified from Jeppson, 1968, p. 273. Here:

$K_{zz} < K_{xx}$, $\bar{x} = x$, and $\bar{z} = z\sqrt{\dfrac{K_{xx}}{K_{zz}}}$

(a) Flow net of seepage from a triangular channel to an infinitely permeable horizon

(b) The same flow net transformed back to the original scale (modified from Jeppson, 1968, p. 275)

Fig. 5.07. Flow nets under triangular channel

Seepage quantities can be calculated by counting the number of flow lines and equipotential lines, determining the shape factor, and using equation 5-18. Because the shape factor is the same on both sections, either section can be used for this determination. Hydraulic gradient determinations, however, must be made on the original section because true distances can only be measured there.

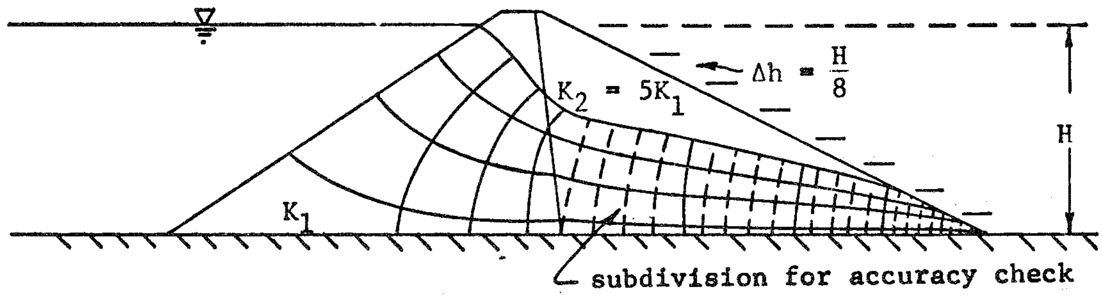
Composite Sections

Composite sections are flow nets involving more than one conductivity. Because of the refraction rule (equation 5-4) which must be followed when flow lines and equipotential lines cross boundaries between the conductivity regions, "squares" are formed in only one region. In all other regions rectangles are formed, and the basic rule governing the shape of these rectangles is equation 5-7.

Before beginning construction of the flow net, one should look for the dominating parts of the cross section and determine whether the conductivities are basically in series or parallel. Conductivities are in series when the cross section perpendicular to the flow direction is of one conductivity, and different conductivity regions occur sequentially in the flow direction (fig. 5.08). Conversely, conductivities are in parallel when more than one region occurs perpendicular to the flow direction, and most flow lines remain in the same region throughout the net (fig. 5.09).

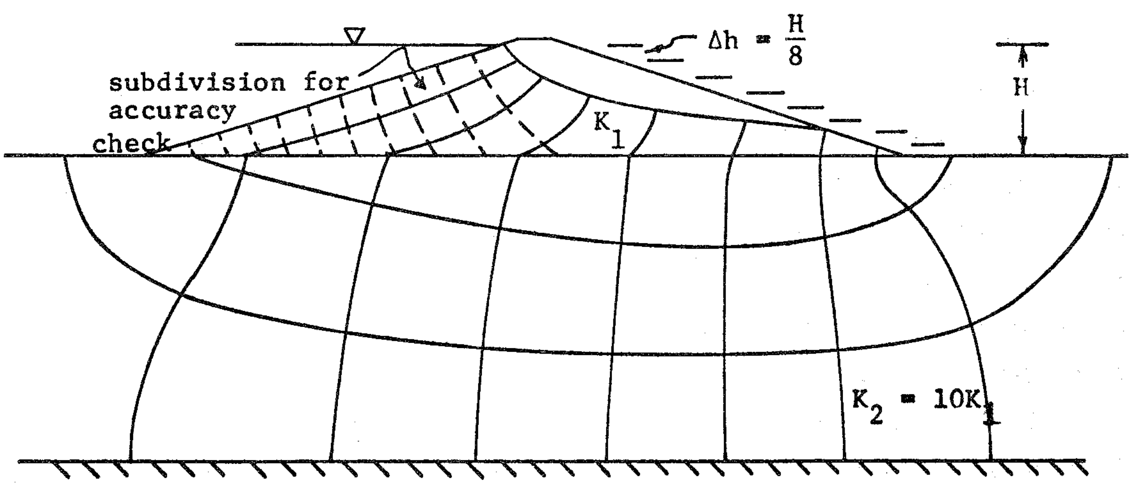
For conductivities basically in series, draw a preliminary flow net choosing one region to have "squares" and making length-to-width ratios in the other region as nearly correct as possible. If the flow net has a free surface, the slope of the free surface will be greater in the region of lower conductivity (fig. 5.08). Repeated adjustments will be necessary to yield the final flow net.

When seepage occurs through two conductivity zones that are basically in parallel, flow through the more pervious zone usually dominates the flow pattern (fig 5.09). A flow net can be constructed for the more pervious part assuming temporarily that the other part is impermeable. The equipotential lines are then extended into the less permeable zone, and, by repeated adjustments, the flow net is completed. This process works best when the conductivities are much different, say ten times different.



Note: The dam is composed of two conductivity regions in series (modified from Cedergren, 1967, p. 165). The dashed lines are explained in the text.

Fig. 5.08. Flow net through a dam



Note: The dam is composed of two conductivity regions in parallel (modified from Cedergren, 1967, p. 169). The dashed lines are explained in the text.

Fig. 5.09. Flow net through a dam

A check on accuracy for all composite sections can be made by subdividing the "rectangles" into a number of parts equal to the number of times the conductivity is higher (or lower) than the conductivity in the region where "squares" are drawn. Each subdivision should be a "square." Whether the subdivisions are of the flow tubes or the equipotential drops depends on whether the conductivity is lower or higher, respectively, than in the region of "squares" (figs. 5.08 and 5.09).

To obtain the quantity of seepage through any composite section, count the total number of equipotential drops and streamtubes and use equation 5-17. The conductivity used should be the one for the region of "squares."

Areal Flow Nets

All flow nets discussed so far have been of vertical cross sections. However, two-dimensional flow net analysis can also be conducted on water table or piezometric surfaces if it is assumed that there is no variation in hydraulic head vertically in the aquifer (Ferris, et al., 1962, pp. 139-144), or, alternatively, if it is assumed that each equipotential line represents the mean in the vertical. It is also required that the saturated thickness of the aquifer be constant because any variation in thickness from point to point causes a change in velocity from point to point. Thus, in order for discharge to remain constant along each flow tube, the flow net would be composed of rectangles of varying length-to-width ratios, something that would be very difficult to draw. In practice the aquifer thickness is never constant. However, flow nets can be drawn where the variation in saturated thickness is small compared with the total saturated thickness. For these cases an average thickness is used to compute the discharge through the flow net.

Equation 5-17 can be rewritten for areal ground-water flow analysis using the relation:

$$Q = qb \qquad (5\text{-}21)$$

where:

Q = total discharge through the flow net

q = discharge per unit thickness

b = average saturated thickness of the aquifer

The resulting relationship is:

$$Q = KbH \frac{n_f}{n_d} = TH \frac{n_f}{n_d} \tag{5-22}$$

where:

T = aquifer transmissivity

(other variables are as defined before).

If areal transmissivity is anisotropic, the flow net can be transformed as described earlier. Areally nonhomogeneous regions can also be analyzed, but for more than two regions the flow nets are difficult to draw.

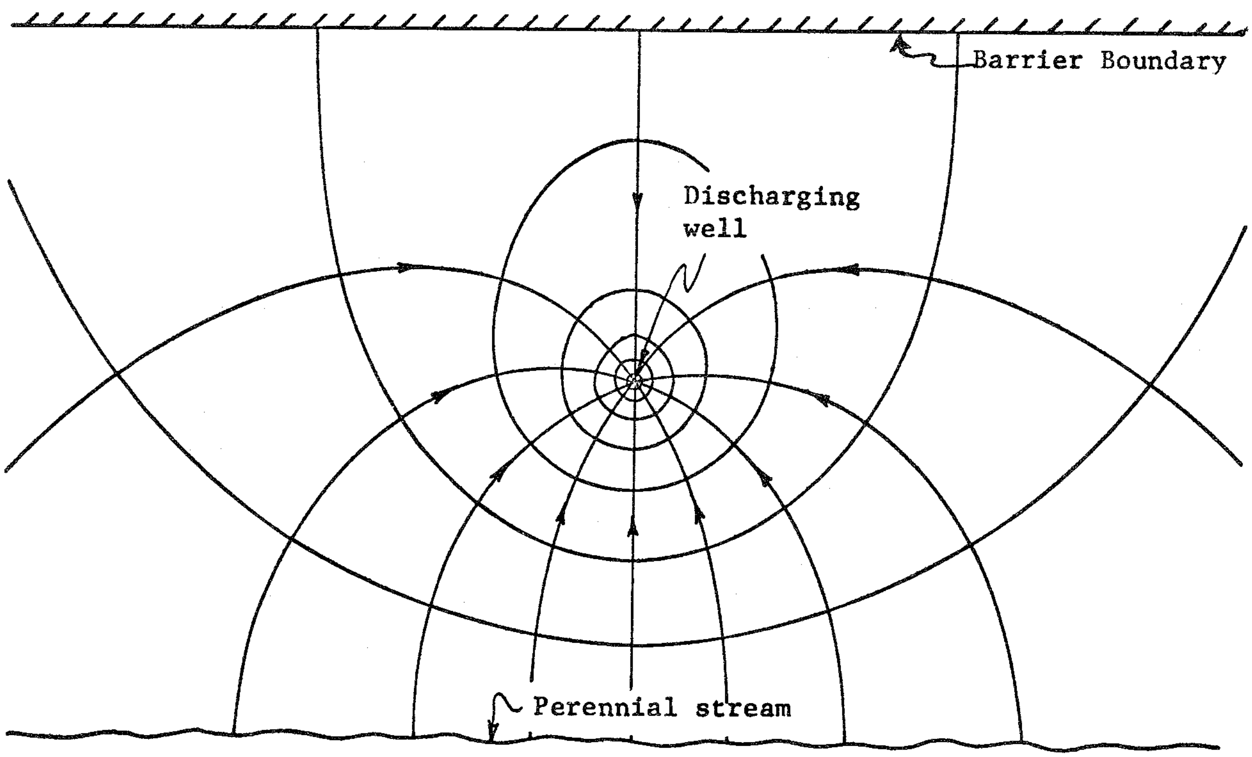


Fig. 5.10. Flow net of ground-water flow from a perennial stream to a discharging well (modified from Ferris, et al., 1962, p. 142)

An example of a flow net drawn for a discharging well located between a recharge boundary (a perennial stream) and a barrier boundary is shown in fig. 5.10. The well acts as an internal boundary of known head; therefore, the value of each equipotential drop is determined from the head difference between the perennial stream and the well.

Section 5.04. Modified Areal Flow Net Analysis

A modified method of areal flow net analysis can be developed using a map of either water table or piezometric contours. For each area being studied in a given region, two flow lines are drawn and used as limits for vertical cross sections along selected contours (fig. 5.11). The assumption that hydraulic head and conductivity do not vary with depth must be made to compute flow through the aquifer at the cross sections. An actual flow net composed of squares does not have to be drawn, so, aquifer conductivity and saturated thickness can vary areally somewhat within each area being studied. Also, unsteady-state cases can be analyzed approximately.

It is convenient to position each cross section at the average distance between two successive contours. In order to compute the flow across each cross section, a convenient form of Darcy's law was developed by Foley, Walton, and Drescher (1953) from the form Q = TiW by defining:

$$i = \frac{c}{L_a} \tag{5-23}$$

where:

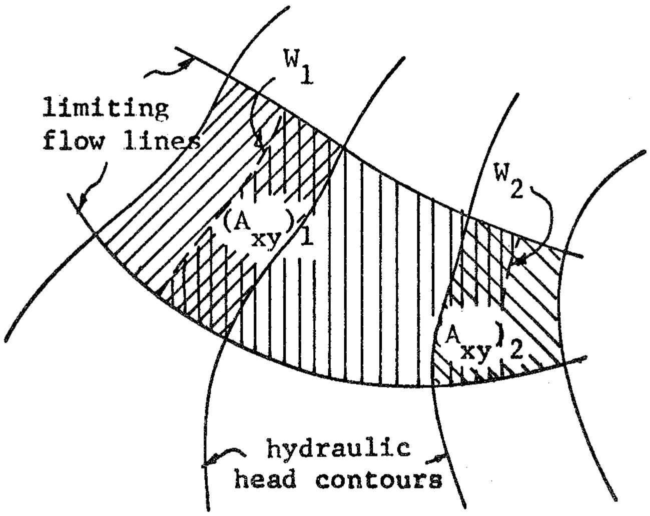
$$L_a = \frac{A_{xy}}{W}$$

c = contour interval for equipotential

A_{xy} = surface area bounded by the two successive contours and the limiting flow lines

W = average width (i.e., parallel to contours) within A_{xy}

Fig. 5.11 illustrates two cross sections bounded by two flow lines. In this case flow across both cross sections is to be computed.



Note: The surface area of the total area being studied, A_ℓ is represented by the vertical line pattern. The areas, $(A_{xy})_1$ and $(A_{xy})_2$ are represented by the diagonal line patterns. The subscripts, 1 and 2, refer to the two cross sections across which flow is being computed.

Fig. 5.11. Symbols used in modified areal flow net analysis

If there is vertical leakage into or from an aquifer bounded by a confining bed above or below, it can be computed from Walton (1962, p. 22):

$$Q_c = K'A_c \frac{\Delta h}{b'} \tag{5-24}$$

where:

Q_c = leakage through the leaky confining bed

K' = vertical hydraulic conductivity of the leaky confining bed

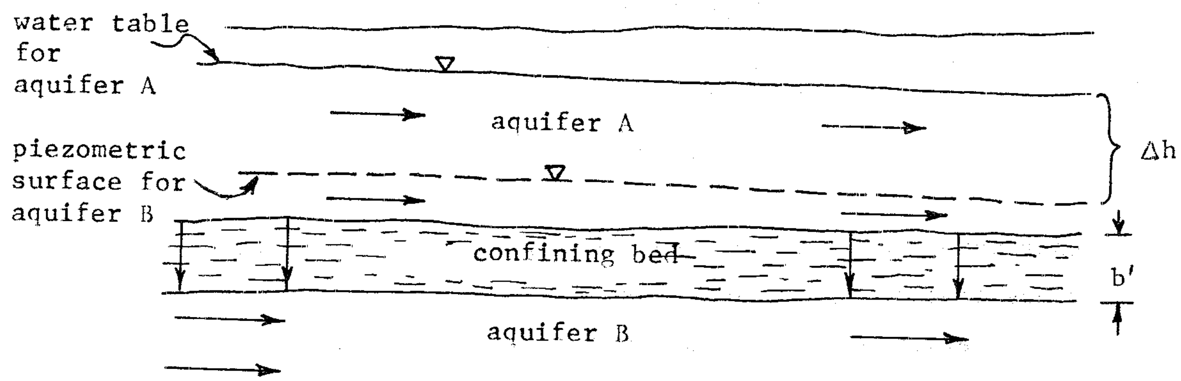
A_c = area of leaky confining bed within the study area

Δh = difference between the head in the aquifer and the head at the distal surface of the confining bed

b' = the thickness of the confining bed

It will be remembered that flow lines are refracted across a conductivity boundary. If the conductivity contrast is large enough, the 90° refraction implied by horizontal flow in the aquifer and vertical flow in the confining bed (or aquitard) is a good approximation.

An example of a flow system where vertical leakage takes place is shown in fig. 5.12. The cross section shows leakage from aquifer A to aquifer B through the confining bed. Leakage occurs because the water table is higher than the piezometric surface for aquifer B.



Note: Arrows represent assumed flow directions.

Fig. 5.12. Vertical leakage to a confined aquifer (B)

A general water balance or continuity equation can be formulated to apply to the section of aquifer between the limiting flow lines and two cross sections. This equation is derived from equation 4-29. It represents a modification of one presented in Walton (1962, p. 22) and is stated as:

$$Q_1 - Q_2 = 2.1X10^8 A_\ell S \frac{\Delta h}{\Delta t} + D - R_s A_\ell \pm Q_c \qquad (5\text{-}25)$$

where:

$Q_1 - Q_2$ = average difference in discharge in gpd between the two cross sections during the time interval Δt computed using equation 5-23. The discharge Q_1 is upstream from Q_2.

A_ℓ = the map area between the two assumed flow cross sections and the limiting flow lines in mi^2 (see fig. 5.11). If a leaky confining bed is present, then $A_\ell = A_c$.

S = coefficient of storage (dimensionless). This will equal S_y for an unconfined case.

$\frac{\Delta h}{\Delta t}$ = an average rate of water table or piezometric surface rise in ft/day calculated as: (average h in the area at time t_2 - average h in area at time t_1)/($t_2 - t_1$), $t_2 > t_1^2$). The quantity will be negative for a decline.

D = discharge rate in the section in gpd.

R_s = net recharge rate in gpd/mi^2. This variable is assumed to represent the balance between the actual average recharge rate and the average ground-water evapotranspiration rate over the interval of time, Δt.

Other terms have been defined above. The last term in equation 5-25, Q_c, is positive for leakage from the aquifer.

Equation 5-25 contains several assumptions. The most important ones are:

a. Vertical flow exists through confining beds and horizontal flow exists through the aquifers (see fig. 5.12).

b. Depuit's simplifying assumptions hold for analysis of water table aquifers. In this case, T = Kb where b = f(h) (see equation 4-31).

c. Conductivities at all elevations in the a]uifer or confining beds are equal.

d. No changes in storage in the confining bed(s) takes place during the time interval Δt.

e. Darcy's law is valid.

f. The coefficient of storage for the aquifer is constant.

These assumptions are rarely if ever met in practice, and averages are generally used for quantities assumed to be constant. Large deviations from the assumptions can produce large errors.

This method is very useful where the assumptions are approximately met and where all but one of the quantities in equation 5-25 are either known are can be shown to be negligible. Values of the unknown can be computed for a number of areas in the region covered by the water table or piezometric maps and geological data. The method also can be used to check the consistency of data when all of the quantities can be estimated using other methods. The equation should approximately balance for all areas in the region.

Section 5.05. Computer Program for Determining Hydraulic Head Distribution

Previous sections have described the theory of flow net analysis and methods for constructing and applying flow nets. For situations where porous medium geometry and properties are basically simple, flow nets can be readily constructed by hand. However, where the geometry is complex and where the medium is heterogeneous and anisotropic, the hand sketching of flow nets becomes a formidable task.

Among the many computer programs that have become available in recent years to aid in the determination of flow nets is the program, "Finite Element Solution of Steady State Potential Flow Problems," developed in The Hydrologic Engineering Center. The program is intended for application to problems involving steady, two-dimensional flow through heterogeneous, anisotropic porous media of virtually any internal or external geometry.

The program uses the finite element numerical method to solve the partial differential equation and associated boundary conditions that describe flow. The program write-up (Appendix 7) contains a description fo the theory on which the program is based, detailed descriptions of input and output, and example solutions to three problems.

Chapter 6

Ground-Water Levels And Fluctuations

CHAPTER 6. GROUND-WATER LEVELS AND FLUCTUATIONS

Section 6.01. Introduction

Ground-water levels include both elevation of the water table for unconfined systems and elevation of a piezometric surface for confined or semiconfined systems. These elevations are a measure of the hydraulic head at some level in the ground-water flow system. The principal method of determining them is the well hydrograph. A well hydrograph, like a stream hydrograph, fluctuates in both a long-and short-term sense. The fluctuations recorded tell something about recharge rates, discharge rates, and the mechanical properties of the aquifer and the material overlying it (Davis and DeWiest, 1966, p. 53).

The purpose of this chapter is to discuss these fluctuations. However, in view of the importance of well hydrographs, the common methods of obtaining them are briefly summarized first.

Well Hydrographs

Methods of measuring water levels in wells. The principal methods of obtaining a well hydrograph are (Davis and DeWiest, 1966, p. 54):

a. Chalked tape. A steel surveyor's tape is covered with chalk from its tip to several feet from its tip. When it is lowered down the well and into the standing well water, the water level shows as wetted chalk dust on the end of the tape.

b. Air line. An air line is a tube that extends from the well surface to below the elevation of the lowest seasonal water level. For a reading, air is pumped into the line until it bubbles out the bottom. This point registers as the maximum pressure on a pressure gage connected to the line on the surface. The depth of water is calculated from

$$\text{Depth of water} = \text{length of air line in well} - \frac{\text{Maximum pressure}}{\text{Specific weight of water}}$$

_c_. Electric drop line. Because electric current flows through ground water, an open circuit involving an ammeter and battery is closed when the exposed ends of the two wires are immersed in water. The ammeter and battery are usually mounted on a reel on which insulated two-wire electric cord is wound. The electric cord is lowered into the well by unwinding the reel and has distances from a weighted tip marked along its length. Small holes in a plastic shield on the tip allow air to escape when water is reached so that the two exposed wires inside the shield are immersed in the water.

_d_. Pressure transducer. A calibrated pressure transducer is permanently suspended in a well below the water level, and recorded water level fluctuations versus time are read directly from a chart at the surface.

_e_. Float. A brass cylinder which floats on the free water surface in the well is affixed to a beaded cable which loops over a pully on a recorder at the ground surface. The system is balanced by a counter weight which hangs on the free end of the cable. The recorder pully turns a drum on which a chart is attached, and a clock-driven pen traces fluctuations versus time on the chart.

Accuracy of the methods. If properly weighted, the chalked tape can be accurate to about 0.005 feet (Davis and DeWiest, 1966, p. 55); however, the chalk can be wet above the waterline by seepage into the well above the water table. The air gage generally yields values accurate to about 0.2 feet if it is functioning properly (Davis and DeWiest, 1966, p. 55). If used with a surveyor's tape, the electric drop line can yield accuracies of 0.001 foot; otherwise, it is generally only accurate to about 0.1 foot (Davis and DeWiest, 1966, p. 55). Both the pressure transducer and the float methods can measure water level changes of less than 0.001 foot (Davis and DeWiest, 1966, p. 55), but the actual accuracy depends on the initial calibration.

Both of the latter are continuous recorders yielding continuous well hydrographs; whereas, all of the others are spot measurements. To yield an accurate hydrograph, spot measurements on a well must be spaced in time according to the variability of the water level in the well--the more variable, the closer the spacing of the measurements.

Piezometers and Observation Wells

Well hydrographs are obtained from both observation wells and piezometers. Any borehole, cased or uncased, which is open or perforated in the saturated zone of an aquifer may serve as an observation well. Whenever there is a vertical flow component, the water level in an observation well will be higher or lower than the natural water table depending on whether seepage is upward or downward. The measured water level provides an estimate of the average hydraulic head over the perforated interval for the well. An observation well penetrating and open to a series of stratified aquifers provides no real information on hydraulic head in any of the aquifers.

A piezometer is a pipe, open only at the ends, extending from ground surface into a saturated formation. The piezometer provides a reading of the hydraulic head at its lower terminus. Obbink (1969, p. 434) provides a good technique for installation of piezometers by jetting. Groups of piezometers may be installed with their lower ends at varying vertical distances below the water table. Such an installation provides information about vertical changes in hydraulic head and thus about vertical seepage.

Neither piezometers nor observation wells provide data about water movement in the unsaturated zone or in the capillary fringe.

Section 6.02. Ground Water and Streamflow

Gaining and Losing Streams

A stream or reach of a stream which receives or is sustained by ground-water inflow is termed a gaining stream. If a stream contributes water to the ground-water basin between two cross sections, it is termed a losing stream in that reach. A stream which is generally gaining discharge may lose it through seepage during high flood stages when the water level in the stream is above that of the water table nearby. An intermittent (flows seasonally) or ephemeral (flows only after a large storm) stream normally loses water by seepage because neither of them contribute sufficient recharge as a yearly total to sustain a water table at the elevation of the stream. Some perennial (flow all year) streams such as the Humboldt River lose water in some reaches and gain water in others (Cohen, 1963, p. 61).

Base flow. Natural ground-water inflow to streams is termed base flow. It most commonly occurs where the water table is above the stream level. As ground-water discharge depends mainly on the physical and geological properties of the drainage basin, its magnitude and variability provide information about the permeability and storage capacity of earth materials.

Various sources of ground water contributing to base flow can be recognized (Meyboom, 1961, p. 1204):

a. Contact springs. Water which is discharged to the surface from permeable material overlying less permeable material that prevents part of the water from moving farther downward is a contact spring (Meinzer, 1923, p. 51).

b. Artesian springs. Water penetrating into bedrock becomes part of a confined system that, despite apparent inhomogeneity of the bedrock, acts as a single hydrologic unit (Meyboom, 1961, p. 1205). In this type of hydrologic system the river and its associated flood plain usually act as an area of natural ground-water discharge.

c. Bank storage. During flood periods river water enters the deposits along a stream. When the river level drops below the water table nearby, the stream gradually receives some of the water held in bank storage.

In some instances computations of average net recharge to the hydrologic system by precipitation do not consider the fact that natural ground-water discharge occurs while well water levels are rising (Williams and Lohman, 1949, p. 215). Little or no base flow leads to greater increases in well water levels and thus a greater increase in storage from recharge.

Terrains consisting of unfractured igneous rock and shale (or clay) provide relatively small contributions to ground water. Further, if two adjacent watersheds have similar climatic conditions, the watershed underlain by rock of lower permeability will be characterized by flashier floods after storms and lower flows during periods of drought (Kazmann, 1965, p. 79). As an example, Rocky River near Berea, Ohio, drains an area of glacial till, whereas Sandy Creek near Sandyville, Ohio, drains a region of sand and gravel. Flow duration curves from the two terrains indicate that the peak flows of Sandy Creek are lower than those of Rocky River (peak flows are expressed as discharge per square mile of drainage area). The low flows of Sandy Creek are far higher than those of Rocky River.

Controls on Stream Hydrograph Shapes

Four different hydrograph shapes result from four combinations of runoff parameters and runoff components (surface runoff, interflow, ground-water flow, and channel precipitation) (Davis and DeWiest, 1966, p. 23).

Condition 1. Precipitation intensity (i) < maximum infiltration rate (f_i) and volume of infiltrated water (F_i) < soil-moisture deficiency (SM_d) (fig. 6.01).

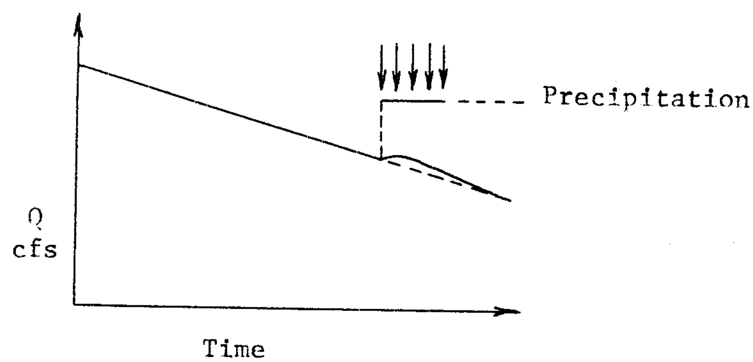


Fig. 6.01. Hydrograph shape according to condition 1 (Davis and DeWiest, 1966, p. 27)

Contributions to the increase in streamflow from interflow and ground-water flow are negligible because the volume of infiltrated water is less than the soil-moisture deficiency (defined here as the volume of water necessary to cause a measurable increase in water table elevation and a contribution to ground-water flow or to cause the formation of perched water bodies or other saturated zones contributing to interflow).

Surface runoff occurs when $i > f_i$; therefore, any addition to streamflow comes from channel precipitation resulting in a slight increase in stream discharge with time (fig. 6.01).

Condition 2. $i < f_i$ and $F_i > SM_d$ (fig. 6.02).

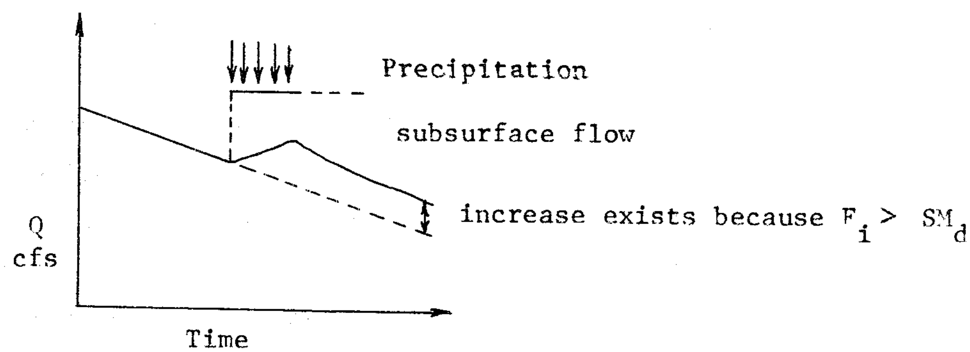


Fig. 6.02. Hydrograph shape according to condition 2. (Davis and DeWiest, 1966, p. 27)

Measurable interflow and increased ground-water flow occur after the moisture content of the earth materials reaches field capacity (defined by Todd, 1959, p. 19, as the amount of water retained in the material after gravity drainage has materially decreased with time). Contributions to streamflow from these components and channel precipitation result in an increase in stream discharge (fig. 6.02).

Condition 3. $i > f_i$ and $F_i < SM_d$ (fig. 6.03).

In this case, contributions to streamflow are derived from surface runoff and channel precipitation, and there is no additional ground-water flow (due to the storm) to increase the existing base flow (fig. 6.03).

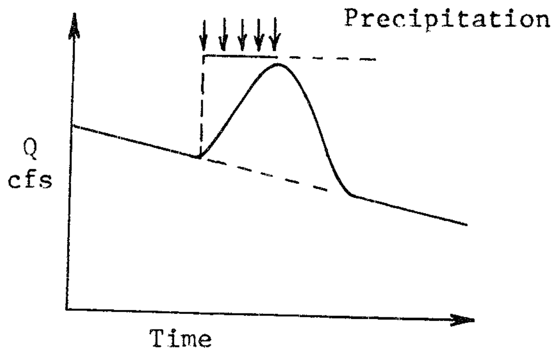


Fig. 6.03. Hydrograph shape according to condition 3 (Davis and DeWiest, 1966, p. 27)

Condition 4. $i > f_i$ and $F_i > SM_d$ (fig. 6.04).

This condition is typical of a large storm. Contributions to streamflow are due to channel precipitation, surface runoff, interflow, and ground water flow. However, the ground-water component may be negative when the stream at peak discharge loses water through the channel (fig. 6.04).

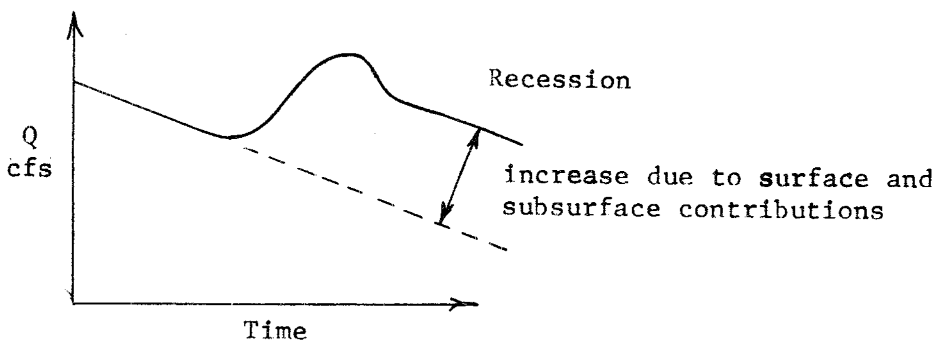


Fig. 6.04. Hydrograph shape according to condition 4 (Davis and DeWiest, 1966, p. 27)

Through-Bank Flow

When a stream intersects permeable material and the water table is below the water level in the stream, the stream loses discharge to the material. This is termed through-bank flow. Transmission losses or through-bank flow have a pronounced effect on the downstream hydrograph. Two distinctly different phenomena have been observed for an ephemeral stream in the Walnut Gulch Watershed, near Tombstone, Arizona (Keppel and Renard, 1962, p. 65).

In the first case, the hydrographs of August 10, 1959, for gaging stations 2 and 1 indicate that both the peak discharge and runoff volume decrease downstream (fig. 6.05). During low discharges, channel resistance effects appear to dominate the flow regime.

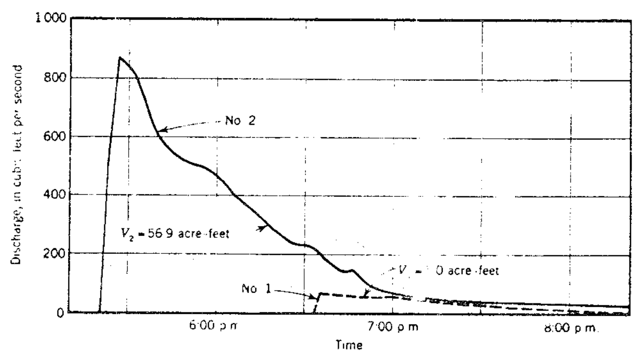


Fig. 6.05. Hydrographs of August 10, 1959, at measuring stations 1 and 2 (after Keppel and Renard, 1962)

In case two, the hydrographs from the same two stations for the event of August 3-4, 1959, show that runoff volume has been reduced downstream, but the peak discharges are nearly the same magnitude (fig. 6.06).

At high discharges, it appears that translatory waves control the flow regime. The intensity of convective thunderstorms and the steep channel

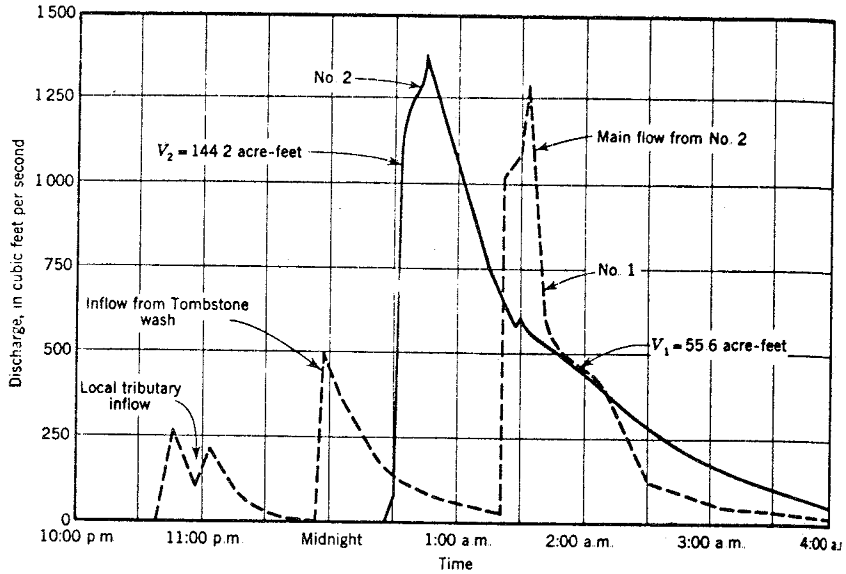


Fig. 6.06. Hydrographs of August 3-4, 1959, at measuring stations 1 and 2 (after Keppel and Renard, 1962)

gradients cause the water to move through the channel as abrupt translatory waves. A second storm, displaced only slightly in time from the first, generates waves that move faster through an already wetted channel and these waves tend to override the existing flow. This leads to a rapid rise in the peak discharge at the downstream station, and the peak discharge at the downstream station will be nearly equal to the peak discharge recorded at the upstream station. However, the runoff volume recorded at the downstream station has been substantially reduced by transmission loss.

In many instances through-bank constitutes an important source of ground-water recharge. Consumptive use removes a large portion of this water in areas where the regional water table is close to the surface.

Interbasin Ground-Water Flow

Interbasin movement of ground water has been observed by a number of investigators. Data compiled by these investigators indicate that mountain ranges are not necessarily barriers to ground-water flow. In some instances mountains may be considered as major sources of what Feth (1964, p. 14) has termed "hidden recharge," as the subsurface movement of water from basin-margin mountains directly into aquifers occupying valley basins. In areas where the concept of hidden recharge is ignored, hydrologic budget computations may show rather large imbalances. Chemical quality data often lends considerable support to this concept. Additional support comes from water level contour maps and weir records of subsurface water discharge from tunnels drilled into mountains.

In some areas, openings or conduits in consolidated rocks transmit large volumes of water through mountain ridges. A study by Hackett, et al., (1960) reports the natural diversion of snowmelt through mountains to streams in the Bridger and Gallatin Ranges, Montana. Fault zones, fractured carbonates, and shattered metamorphic rocks accept snowmelt that helps to sustain baseflows of streams, some on opposite sides of the ridges from where recharge occurs.

Hunt and Robinson (1960, p. 273) offer evidence that spring flow in Death Valley, Mesquite Flat, California, and Ash Meadow Valley, Nevada, is the product to an extensive interbasin ground-water flow system (fig. 6.07). Small drainage areas and low recharge rates imply that large spring flows cannot be readily accounted for by recharge within the basins where the springs emerge.

The inference by these investigators is that recharge to the system occurs in the Spring Mountains, Nevada, and moves from that source beneath mountain ranges and valley floors alike to discharge as springs. Hydrologic and chemical data both indicate that much of the ground-water flow between basins occurs within thick sequencs of limestone and dolomite which underlie a large part of the region (Winograd and Thordarson, 1968, p. 35 p. 35).

Section 6.03. Major Water Level Fluctuations in Wells

Water level fluctuations are the result of a change in total ground water stored. Although this section is concerned with water level fluctuations

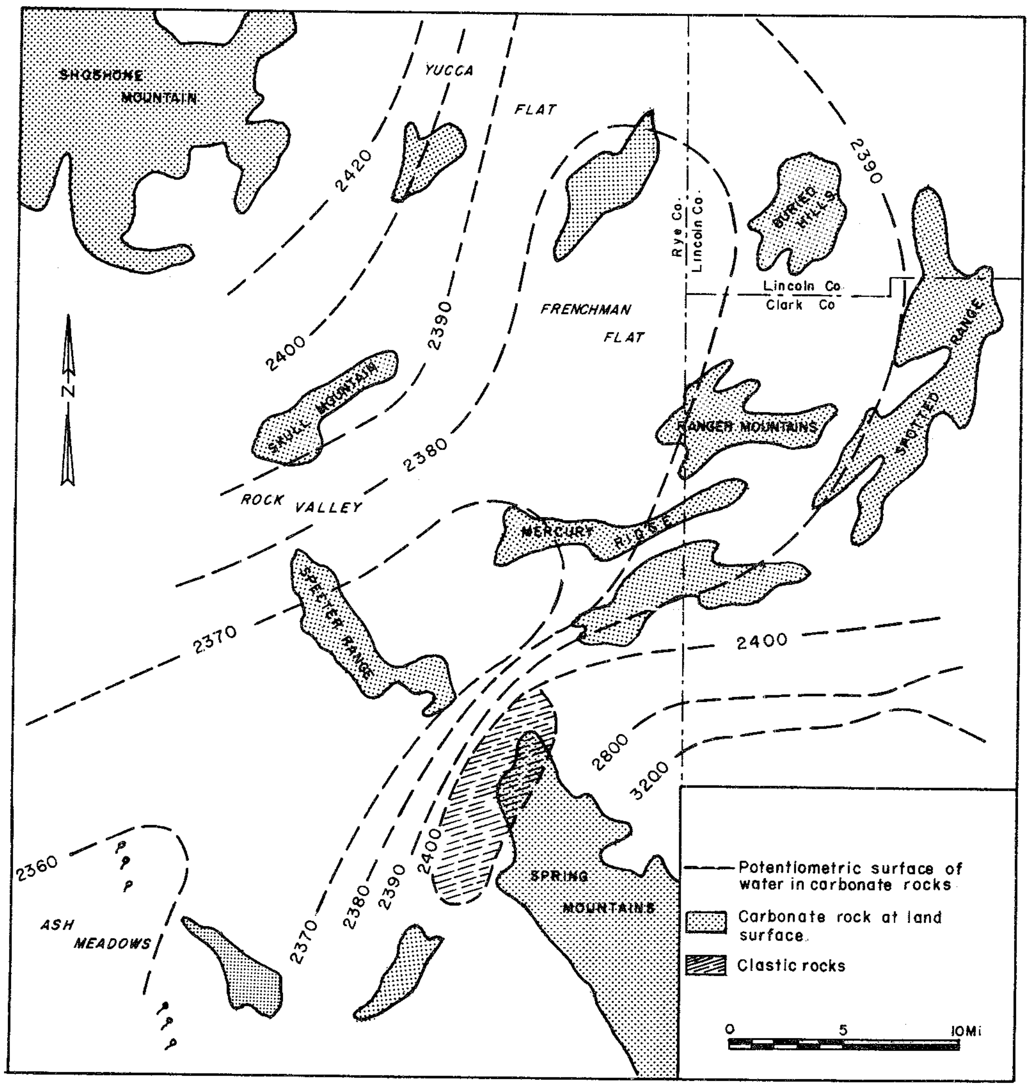


Fig. 6.07. Regional flow system of the Nevada Test Site (after Winograd, 1962)

in response to recharge and discharge, considerable quantities of water can be recharged or discharged with little or no water table movement. The ground-water system is in dynamic equilibrium with recharge balancing discharge (see figs. 4.03 and 4.04).

Recharge

Recharge is the process by which ground water is replenished. Two main trends in recharge can be noted from a well hydrograph: seasonal and secular trends. Recharge is generally a seasonal occurrence, the season(s) during which it takes place depending on the climate. This produces a roughly cyclic trend to a hydrograph with increases in water table elevation occurring during the season(s) of recharge. If, as a yearly average for some period of time, recharge exceeds discharge for a given area, then there may be a secular increase in water table elevation.

Occurrence of recharge. For any type of recharge to occur in a permeable porous medium, two conditions must be met:

a. There must be a head decrease between the source of recharge and the water table, and

b. All of the water must not be lost by evapotranspiration.

Infiltration from direct precipitation. Water from melting snow, rain, or other forms of precipitation seeps or infiltrates into the ground. Part of this water can reach the water table unless it is used by vegetation or drawn back toward the surface by capillary movement and evaporated. The amount of infiltration which is actually recharged depends on the climate, season, the geological conditions and other factors.

The unsaturated zone above the water table affects both the quantity and timing of direct recharge from precipitation. Important parameters are the rate and duration of precipitation, the subsequent conditions at land surface, the steady or background ground-water recharge or discharge rate, the antecedent soil moisture conditions, the water table depth, the allowable depth of surface-water ponding, and the soil type (Freeze, 1969, p. 169). The interaction of these parameters results in a lag and attenuation of the recharge as a function of depth (Davis and DeWiest, 1966, pp. 27, 38).

Discharge

Discharge, which is the process by which ground water is lost from an area, occurs mainly into "discharge areas" (e.g., streams, swamps and lakes) as springs, through pumping, and into drains. As with recharge, it can vary secularly, seasonally, for short terms and daily.

Seasonal, short term and daily trends in pumping are often evident. For example, heavy pumping for irrigation which occurs during the spring and summer months can produce a drop in water table elevation between about April and September (Todd, 1959, p. 150). During that season rapid fluctuations of well water levels through vertical distances of several feet are often evident. Figure 6.08 shows a well hydrograph with a daily irrigation pumping cycle superimposed on a generally downward trending curve. Seasonal industrial water demands should be expected to produce similar results. As another example, municipal water supplies are often derived from both surface and ground water. When the surface-water supplies (e.g., reservoirs) are depleted to a certain level, then ground-water supplies are used, resulting in nonsteady pumping. The pumping may be cyclic in this case in response to seasonal replenishment of the surface-water supplies.

As a final example, seasonal recharge can also increase the quantity of discharge from an area by increasing the mean ground-water stage. Ground-water mounds due to recharge in the area result in higher gradients, thus higher discharge.

Secular downward trends in water levels can result from a total pumping rate which exceeds the recharge rate in a basin, or from a series of years when the average recharge from precipitation is lower than normal. These trends represent a gradual loss of ground water stored in the basin.

Natural Ground-Water Recession

Following a seasonal high ground-water elevation related to recharge, a natural ground-water recession may be seen in the well hydrograph. The rate of decline in the water level is a function of the local values of

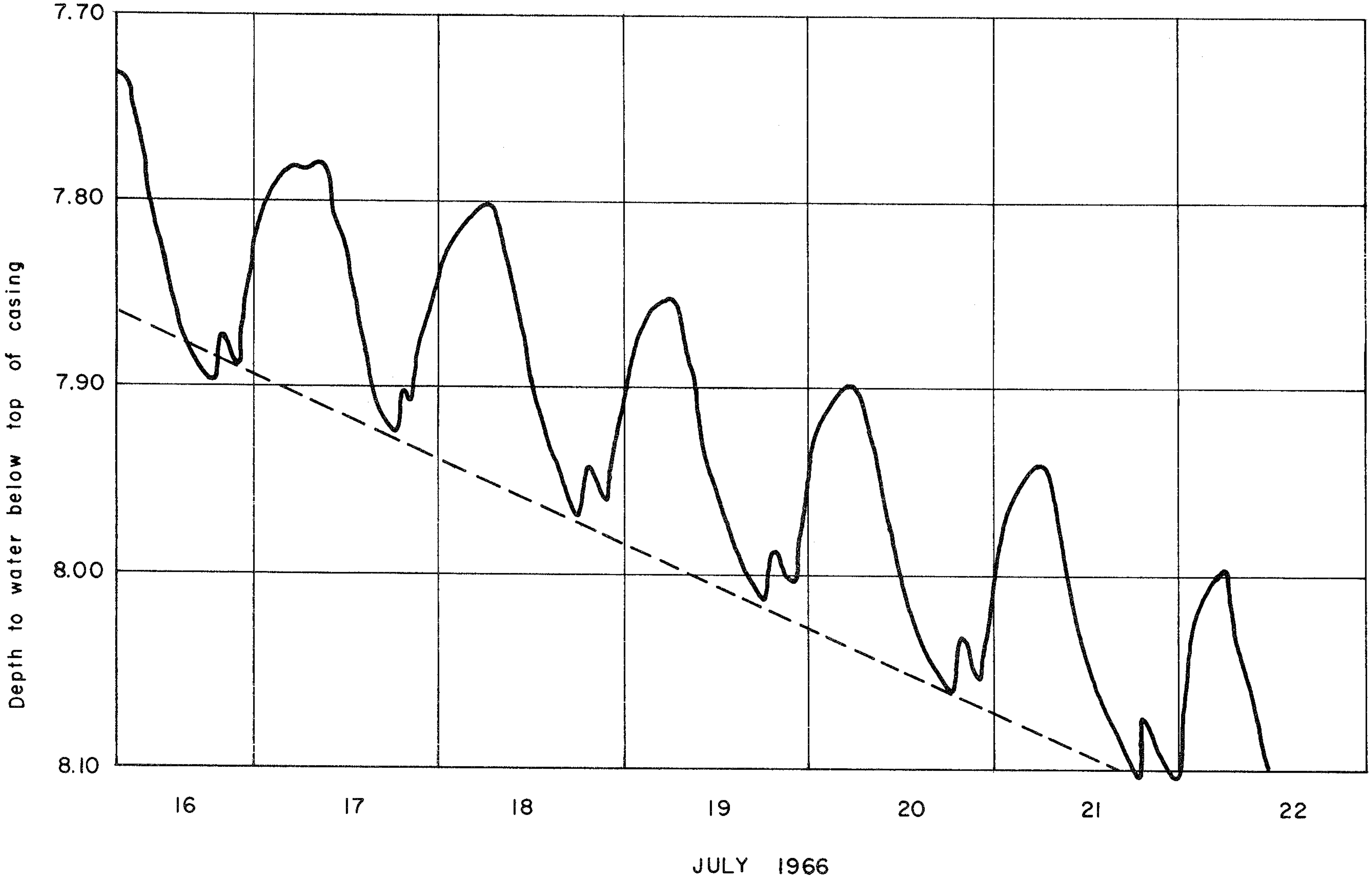


Fig. 6.08. Ground water fluctuactions due to a daily irrigation pumping cycle superimposed on natural ground water discharge

hydraulic conductivity, storage coefficient, aquifer thickness and hydraulic gradient. The rate of decline is a measure of the excess of subsurface outflow over subsurface inflow at a given point and decreases exponentially with time due to the decrease in saturated thickness and hydraulic gradient. In the absence of new recharge, the natural ground-water recession can be plotted on semilogarithmic graph paper (elevations vs. time) to obtain a straight line. The projection of the straight line can be compared with actual records to delineate periods of excessive recharge or discharge (Davis and DeWiest, 1966, p. 58).

Evapotranspiration from ground water may be superimposed on a natural recession. If a heavy frost causes cessation of transpiration, the rate of ground-water decline will decrease to the natural recession rate. By extrapolating the natural recession backwards in time, the difference between the natural recession and the actual record will be the discharge due to transpiration (Farvolden, 1963, pp. 234-235).

In areas where there is a direct recharge from precipitation, there is usually some degree of correlation between precipitation and ground-water levels. The cumulative departure of annual precipitation values from a long-term mean is often used to evaluate the secular trends of ground-water levels. Declining water tables in periods of above normal precipitation are indicative of excessive ground-water withdrawls (Todd, 1959, p. 150).

Maps of equal water level change can be prepared from well hydrograph data for periods of 1 water year or longer. The areas which show positive changes are usually recharge areas while areas with negative charges are discharge areas. This technique can be used to delineate local and regional flow systems (Davis and DeWiest, 1966, p. 53).

Aquifer Storage Changes and Stream Depletion

Change in storage, as used here, means the net addition of water to or the net withdrawal of water from an aquifer. Storage changes account for most of the large fluctuations in well water levels. Natural changes of storage such as those caused by recharge or spring flow generally give

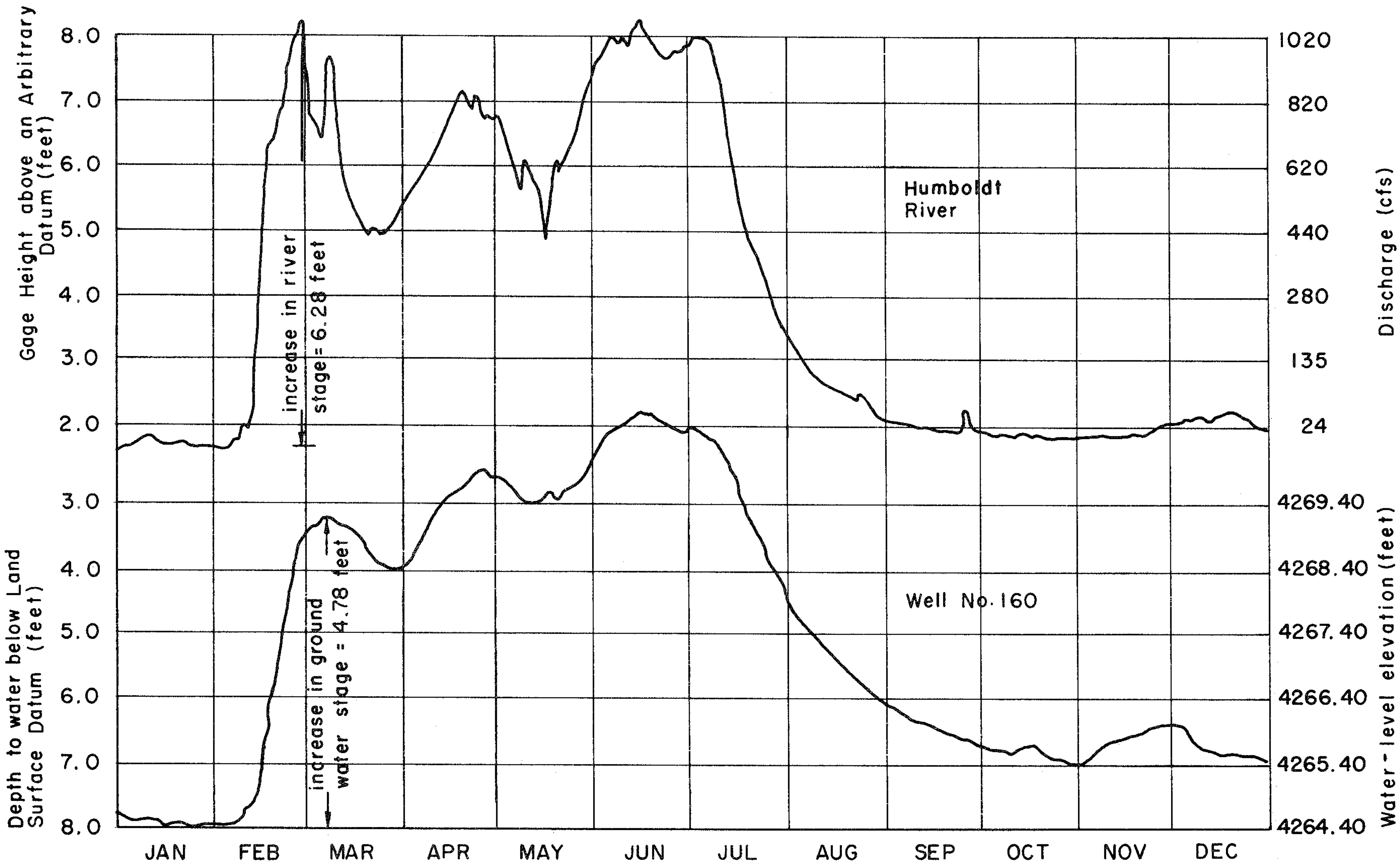


Fig. 6.09. Relationship of surface-water fluctuations at Winnemucca gaging station to ground-water fluctuations in Well No. 160, 1962, Humbolt River, Nevada

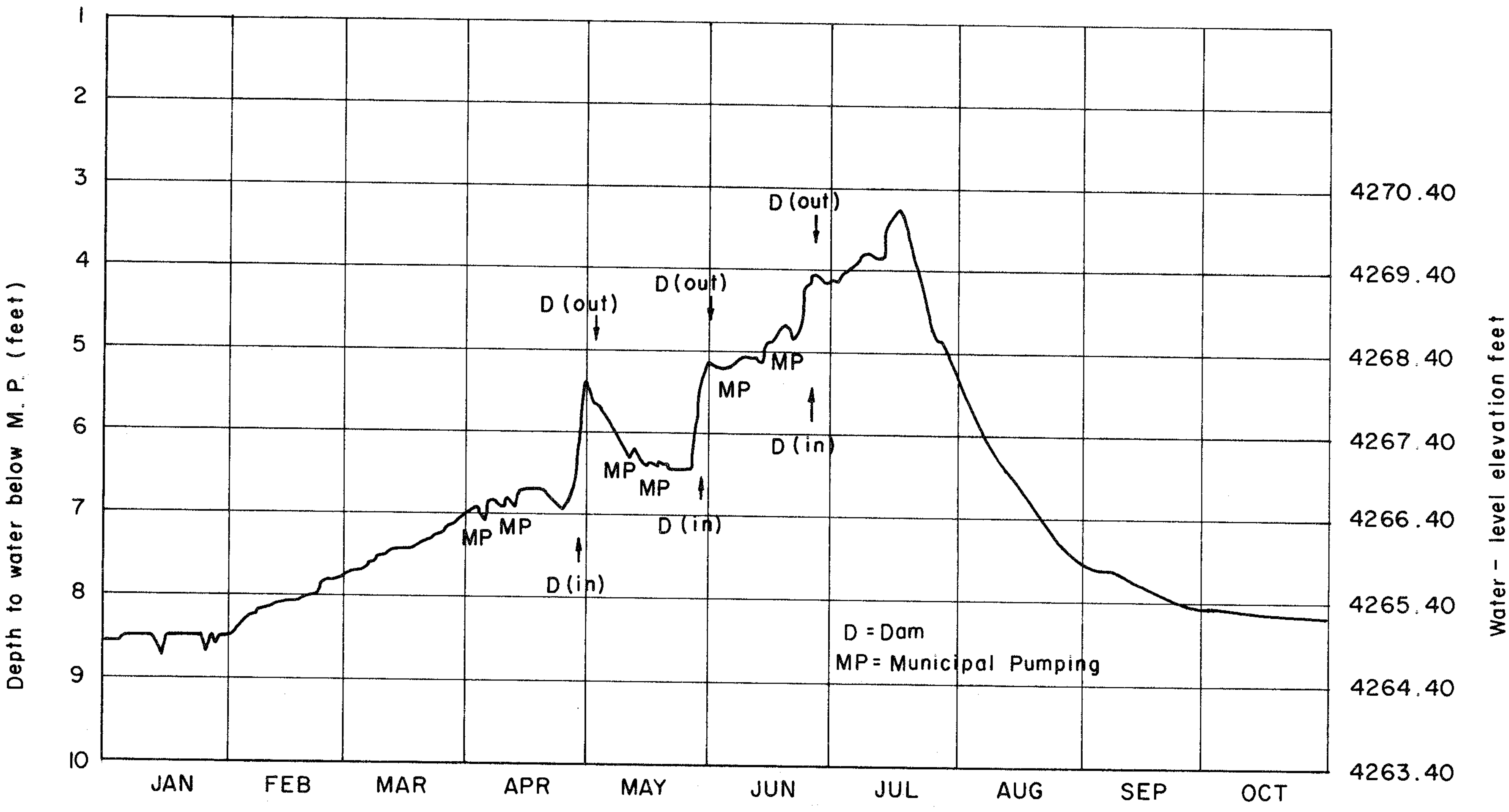


Fig. 6.10. Ground-water fluctuations and natural discharge from a sand and gravel aquifer, Humbolt River, Nevada

rise to rather gradual changes in water levels. Near river channels, canals and reservoirs, however, the increase in storage may be rather abrupt in response to rapid surface-water level changes. This will cause rapid rises in well water levels near the surface-water bodies. Figure 6.09 shows the effect of changing river stage on the fluctuations of water level in a well. Figure 6.10 shows the influence of long-term storage changes caused by natural ground-water discharge from July through October.

Due to pumping, a stream can often lose water by seepage with no resulting aquifer storage change. The water table along the stream near the well is lowered by pumping which causes water from the stream to discharge into the aquifer. This process of streamflow sustaining well production is termed induced infiltration. As an example, Rorabaugh (1956) estimated that the rate of induced infiltration along the Ohio River near Louisville varies from near zero to as much as 10 or 12 feet of water per day. Another study in Kansas has shown that after 4 years of pumping from the Wichita field, baseflow to the Little Arkansas River at various points was computed to be about 10 percent less than the calculated ground-water discharge before pumping began (Kazmann, 1965, p. 171).

A study by Walton (1967, p. 40) has shown that: (1) leakage of water through a streambed is proportional to the drawdown beneath the streambed until the ground-water level falls below the streambed, after that (2) leakage of water is proportional to the average depth of water in the stream and varies with changes in stream stage and changes in stream temperature.

Section 6.04. Minor Water Level Fluctuations in Wells

Evapotranspiration

Evapotranspiration of ground water results in secular, seasonal and short-term changes in ground-water levels. One phreatophyte, saltcedar, covers one million acres in the western states and currently transpires (comsumptively wastes) 5 million acre-feet of ground water annually (Robinson, 1965, p. A-1).

Evapotranspiration quantity is affected by the depth to ground water, vapor pressure near ground surface, the temperature of both the air and the water, wind barometric pressure, dissolved solid content of the water and vegetation types and quantities. A discussion of these factors and their effects can be found in Meinzer, (ed.) 1942, pp. 58-65 and 259-330. Tovey (1969, pp. 525-535) has reported on the relationship of alfalfa growth to water table depth. Total transpiration and yield decreased as depth to water table increased in the range of 2 to 8 feet. Fluctuating water tables adversely affected the yields as compared to static water tables. For water tables shallower than 8 feet, the standard methods for ET prediction generally gave low values.

Estimation of evapotranspiration from the hydrologic budget. In Chapter 4, the hydrologic budget equation for natural conditions was given as:

$$P = R + ET \pm \Delta S_{sw} \pm \Delta S_{gw} \pm \Delta S_{sm} \pm U \qquad (6\text{-}1)$$

This equation can be rewritten to solve for ET as follows:

$$ET = P - R + \Delta S_{gw} + \Delta S_{sw} \pm \Delta S_{sm} \pm U \qquad (6\text{-}2)$$

The solution of equation 6-2 on a short-term basis involves measurement of surface and subsurface inflows, outflows, and storage quantities. Any measurement errors are lumped into the calculated value of ET. Moreover, the detailed measurements are not usually available. Water-year estimates of ET may be made from equation 6-2 when hydrogeologic conditions are favorable. This requires that any of the unmeasured quantities must be assumed to be equal to zero with little or no error.

Estimates of ET for use in the ground-water budget equation. Very often it is advantageous to use an estimated value of ET in equation 6-1 with other known or estimated values in order to evaluate one of the unknown components.

The value of ET for use in equation 6-1 for moderately shallow water tables can be estimated most readily from panevaporation data. The evaporation from U. S. Weather Bureau Class A Land Pan exposed in an irrigated pasture environment is equal to potential ET (Pruitt, 1966, p. 58).

Potential ET is defined as the ET from a fully covered vegetated surface with transpiration not limited by soil moisture deficiency. Potential ET may be determined for pans in dry exposure by correction factor of 0.8 applied to evaporation. The actual ET is estimated by use of a series of crop coefficients for each week of the year or growing season (Hargreaves, 1968, p. 100). The ET estimate may be formulated as:

$$ET = E_{pan} C_{ex} K \tag{6-3}$$

where:

ET = evapotranspiration per period

E_{pan} = recorded panevaporation

C_{ex} = correction for pan exposure (1.0 for irrigated pasture, 0.8 for dry soil)

K = crop coefficient for period

In most areas there is an abundance of published experience to permit selection of K values for most crops and natural vegetation types.

Diurnal effects of ET on ground-water levels. If the ground water is unconfined and near surface diurnal fluctuations produced by evapotranspiration are often noted on the well hydrograph (Todd, 1959, p. 155). These diurnal effects are observed to change seasonally, being higher during the summer months, and the pattern of diurnal change is nearly the same for evaporation, transpiration or their combined effect (Todd, 1959, p. 157). Midmorning is the time of the highest water table and represents a temporary equilibrium between evapotranspiration rate and recharge rate from surrounding ground water. Then, until early evening, the evaporation rate exceeds the recharge rate from surrounding ground water, and the water table falls. There is another equilibrium point in the evening; then during the night, the recharge rate exceeds the evapotranspiration rate, and the water table rises (Todd, 1959, p. 157). Jaworski (1968, p. 730) has reported extensive observations of water-table fluctuation due to ET.

The interrelations of water-table elevation, recharge from surrounding ground water, and evapotranspiration can be illustrated by fig. 6.11. The expression:

$$Q_{in} - Q_{out} = A_{xy} S_y \Delta h/\Delta t + Q_{ET} \tag{6-4}$$

describes the relations among the variables. The total quantity of recharge can be found from a plot of dh/dt against time as shown in fig. 6.12. Van Hylckana (1968, p. 761) points out the need for applying barometric corrections to diurnal fluctuations before estimating ET.

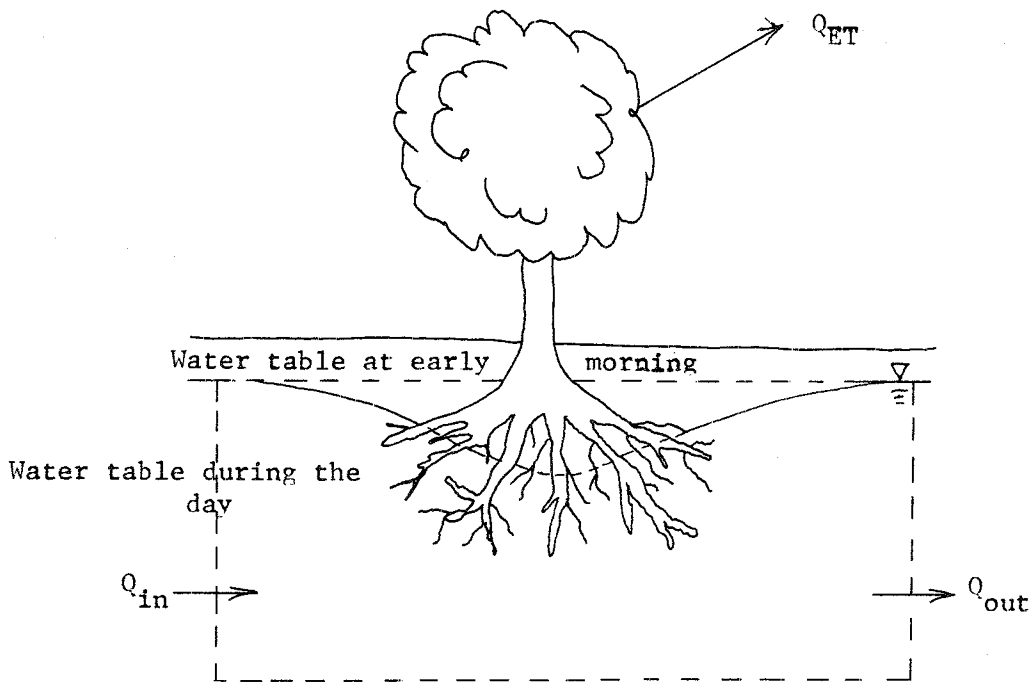


Fig. 6.11. Definitive sketch for ET from ground water

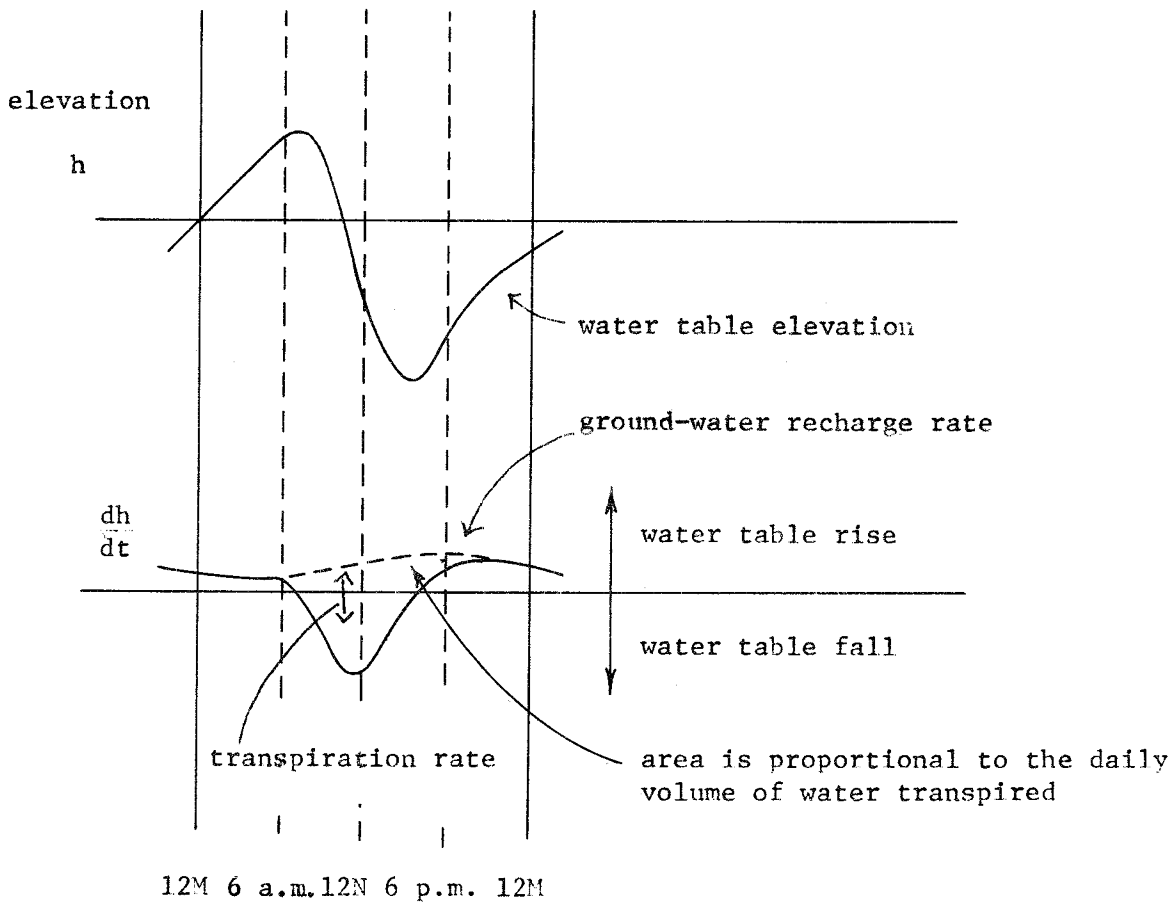


Fig. 6.12. Ground-water level and its rate of change versus time (Todd, 1959, p. 158)

Earthquakes

Earthquakes affect ground-water levels in wells because of the shock waves produced by them. As a shock wave passes, the pressure in the aquifer increases and the water level rises, then it decreases and the water level drops (Ferris, et al., 1962, p. 87). This can be elastic if no permanent rearrangement of grains takes place, or nonelastic if consolidation occurs. The elastic disturbances are noticed in wells in confined aquifers as rapid rises and falls in the water level (Ferris, et al., 1962, p. 87). In unconfined aquifers, the pressure increase produces a slight rise in the water table which is probably undetectable in a well. Thus, earthquake-induced fluctuations serve to identify confined aquifer conditions.

Davis and DeWiest (1966, p. 61) state that consolidation is common within 100 km of the epicenter of large earthquakes, and compaction of recently deposited sands can cause wells to flow. Besides disturbances in wells, permanent deformation can cause changes in spring discharges, appearance of new springs and eruptions of mud and water out of the ground (Todd, 1959, p. 170).

Barometric Fluctuations

Barometric fluctuations have been studied extensively by Jacob (1940) using elastic deformation theory. Pressure relationships in a ground-water system are analyzed considering changes in atmospheric pressure. The expression:

$$p_t = p_w + p_s \tag{6-5}$$

states the pressure and stress relationships at the top of a confined aquifer. Here:

p_t = the total pressure = p_a + overburden pressure,

p_a = atmospheric pressure,

p_w = the water pressure, and

p_s = the intergranulär pressure or stress.

The water pressure, p_w, is assumed to be supporting part of the overburden load.

At the top of the confined aquifer in a well,

$$p_w = p_a + \gamma h \tag{6-6}$$

where h is the height from the top of the confined aquifer to the water level in the well and γ is the unit weight of water.

If there is an increase in atmospheric pressure, Δp_a, there is an increase in both p_w and p_s in the aquifer (i.e., Δp_w and Δp_s) because the low permeability of the confining bed prevents the air pressure change from being transmitted directly and completely to the water in the confined aquifer. In the well, however, the total change in p_a is transmitted directly and completely to the water in the well. Thus, a pressure gradient exists across the well bore, and water flows from the well into the aquifer until equilibrium is reached (Ferris, et al., 1962, p. 84). A decrease in atmospheric pressure causes the opposite effect.

Stated mathematically, in the aquifer after the pressure increase,

$$p_t + \Delta p_a = p_w + (\Delta p_w)_1 + p_s + \Delta p_s \qquad (6\text{-}7)$$

Subtracting equations 6-5 and 6-7 (Todd, 1959, p. 159),

$$\Delta p_a = (\Delta p_w)_1 + \Delta p_s \qquad (6\text{-}8)$$

where $(\Delta p_w)_1$ is the water pressure increase in the aquifer. In the well after the pressure increase,

$$p_w + (\Delta p_w)_2 = p_a + \Delta p_a + \gamma h' \qquad (6\text{-}9)$$

where h' is the elevation of water in the well from the top of the confined aquifer after the atmospheric pressure change, and $(\Delta p_w)_2$ is the increase in water pressure in the well at the elevation of the top of the confined aquifer. Subtracting equations 6-6 and 6-9 (Todd, 1959, p. 160),

$$\begin{aligned}(\Delta p_w)_2 &= \Delta p_a + \gamma(h' - h)\\ &= \Delta p_a - \gamma(\Delta h) \qquad (6\text{-}10)\end{aligned}$$

At equilibrium (i.e., after the change in depth of water in the well),

$$(\Delta p_w)_1 = (\Delta p_w)_2$$

Dividing equation 6-10 by Δp_a,

$$\Delta p_w/\Delta p_a = 1 - \gamma(\Delta h)/\Delta p_a \qquad (6\text{-}11)$$

where

$$(\Delta h)/\Delta p_a = \text{the barometric efficiency of the well} = BE$$

$$\text{and } \Delta p_w = (\Delta p_w)_1 = (\Delta p_w)_2$$

A barometric efficiency of one indicates that the confining layer transmits none of the atmospheric pressure change to the water in the aquifer (i.e., $\Delta p_w = 0$); whereas, a barometric efficiency of zero indicates that the confining layer transmits all of the atmospheric pressure change to the water in the aquifer (i.e., $\Delta p_w = \Delta p_a$). The barometric efficiency of an artesian well is thus a measure of the ability of the upper confining layer to transmit atmospheric pressure changes to the water in the aquifer and provide a relative measure of the rigidity of the overlying or confining beds and the aquifer (see Gilliland, 1969, p. 245).

For a water table well with high overburden permeability and for a slow atmospheric pressure change, $BE = 0$ because at all times $\Delta p_w = \Delta p_a$. Each small increment in pressure is transmitted through the overburden rapidly enough that the pressure on the water table is at all times approximately equal to the pressure on the water in the well. However, if the pressure change is rapid and the overburden permeability is low, then the pressure at the water table takes some time to come to equilibrium with the atmospheric pressure change. The resulting time lag produces a pressure gradient across the well bore and some $BE > 0$ results. The BE would not be expected to be constant because the pressure gradient across the well bore at any time would depend on the rate of change of atmospheric pressure with time.

The BE of a well can be determiend as the slope of $\gamma(\Delta h)$ plotted against Δp_a. If the relationship is linear with little spread in values so that BE is constant, then a confined aquifer responds elastically.

Detailed analysis of well hydrographs, such as is required for aquifer tests should be preceded by correction of the water levels for barometric fluctuations. An initial water level and the corresponding barometric pressure are chosen as reference. Subsequent water levels are adjusted for changes in barometric pressure. The value of Δp_a in inches of mercury is multiplied by 1.12 to obtain the equivalent Δp_a in feet of water. This Δp_a is multiplied by (1 - BE) to obtain the corresponding

Δp_w in feet of water. If p_z decreases, the resulting water level rise should be reduced by the calculated value of Δp_w to obtain the water level as unaffected by barometric changes.

Tidal Fluctuations

Wells tapping confined aquifers which pass under the ocean but are effectively separated hydraulically from it show fluctuations resulting from the change in loading on the aquifer in response to ocean tides. The response is direct; as the tide rises, the pressure in the aquifer increases in response to the increased load. As a result, the water level in the well rises. The opposite effect occurs as the tide drops.

Equations 6-5 and 6-6 apply to the time before tidal rise. After rise,

$$p_t + \Delta p_h = p_w + \Delta p_w + p_s + \Delta p_s \qquad (6\text{-}12)$$

Subtracting equations 6-5 and 6-12,

$$\Delta p_h = \Delta p_w + \Delta p_s \qquad (6\text{-}13)$$

Here, Δp_h is the increase in load due to the tidal rise. Also, in the well after the rise,

$$p_w + \Delta p_w = p_a + \gamma h'' \qquad (6\text{-}14)$$

Subtracting equations 6-14 and 6-6,

$$\Delta p_w = \gamma(h'' - h) = \gamma(\Delta h_2) \qquad (6\text{-}15)$$

where h'' is the height of water in the well above the top of the confined aquifer after the tidal rise. Then,

$$\Delta p_w/\Delta p_h = \gamma(\Delta h_2)/\Delta p_h \qquad (6\text{-}16)$$

$$= \text{tidal efficiency} = TE$$

A tidal efficiency of one indicates that all of the increase in loading is transmitted to the water in the aquifer (i.e., $\Delta p_w = \Delta p_h$). A tidal efficiency of zero indicates that none of the increase in loading is transmitted to the water in the aquifer. Tidal efficiency is thus a measure of the competency of an elastic confined aquifer and the beds overlying it to resist pressure changes.

If it is assumed that any type of loading on the elastic, confined aquifer produces the same result, then

$$\Delta p_w / \Delta p_a = \Delta p_w / \Delta p_h \tag{6-17}$$

and

$$BE + TE = 1 \tag{6-18}$$

from equations 6-11 and 6-16. Equation 6-18 can be used to provide a check on measurements, on the degree of elasticity, and on the degree of confinement of an assumed elastic, confined aquifer.

In a water-table aquifer near the ocean, tidal flucturations cause actual movement into or out of the aquifer, depending on whether the tide was rising or falling (Ferris, et al., 1962, p. 85).

Other Aquifer Loading Phenomena

When a change in loading on a confined aquifer takes place (e.g., a passing train) a water level fluctuation in a well tapping it occurs. The phenomenon is similar to the tidal effect except the application of loading is confined to a smaller area in the present case. Because of the smaller area of application, the higher pressure produced by an increase in loading (or vice versa) is allowed to dissipate. Thus, the rise (or fall) in water level gradually shifts back toward the normal water level after the application of the stress. While the stress remains on the aquifer, however, the water level will always be higher (or lower) than normal because of the general increase (or decrease) in pressure due to the stress (Todd, 1959, p. 170). See Thomas, 1963, p. 77, for a case study treating a number of causes of fluctuations.

Earth Tides

Earth tides often produce small (<0.1 foot) fluctuations in artesian wells (Ferris, et al., 1962, p. 86). The hydrographs show two minima each day corresponding to the moon's upper and lower culminations and two maxima corresponding to moonrise and moonset. Theis (1939) explained these relations as follows: As the earth's crust bulges slightly in response to culmination, the aquifer dilates, which lowers the pressure. Thus, the water level drops. Because the earth is compressed at 90 degrees from the bulge, the water level in an artesian well rises at moonrise and moonset.

Bredehoeft (1967, p. 3075) points out that the same aquifers which show earthquake fluctuations will have earth tide fluctuations. The magnitude of these fluctuations is a function of the specific storage of the aquifer. Theoretically, the porosity of an aquifer can be calculated from the magnitude of earth tide fluctuations.

Miscellaneous Fluctuations

Small fluctuations in wells often occur for which none of the foregoing explanations apply. A gust of wind over an open well may create a partial vacuum in the well and cause a slight momentary rise in the water level. A small animal or object may drop into the well. Often the float for a continuous recorder sticks in the well bore momentarily. Many small fluctuations due to these and other miscellaneous causes are never identified.

Chapter 7

Flow to Wells

CHAPTER 7. FLOW TO WELLS

Section 7.01. Physics of Ground-Water Flow to Wells

Formulation of the Confined Flow Problem

A general formulation of flow to a well which penetrates one or more elastic confined aquifers separated by aquitards could be obtained using a three-dimensional generalization of equation 4-32, and the conditions expressing refraction of flow lines across the boundaries between aquifers and aquitards given by equations 11 and 12, Appendix 3, together with appropriate boundary and initial conditions. Since an analytical solution to this general problem is not available, an approximate formulation given by Hantush (1960) and further developed by Neuman and Witherspoon (1969a, b and c) must be used.

Consider the leaky aquifer system given in fig. 7.01. The well, which is assumed to have an infinitesimal radius, penetrates the horizontal sequence of source bed, aquitard, and aquifer, but is open only to the aquifer. The discharge, Q, is constant and is small enough that the water level in the well is not lowered below the bottom of the aquitard. Flow velocities are assumed to be vertical in the aquitard and horizontal in the aquifer. This latter assumption leads to errors of generally less than 5 percent in drawdown when $K/K' > 3$; therefore, it can generally be used safely (Neuman and Witherspoon, 1969c, pp. 134-135). The system is assumed to have an infinite external radius so that an infinite amount of water is available from storage.

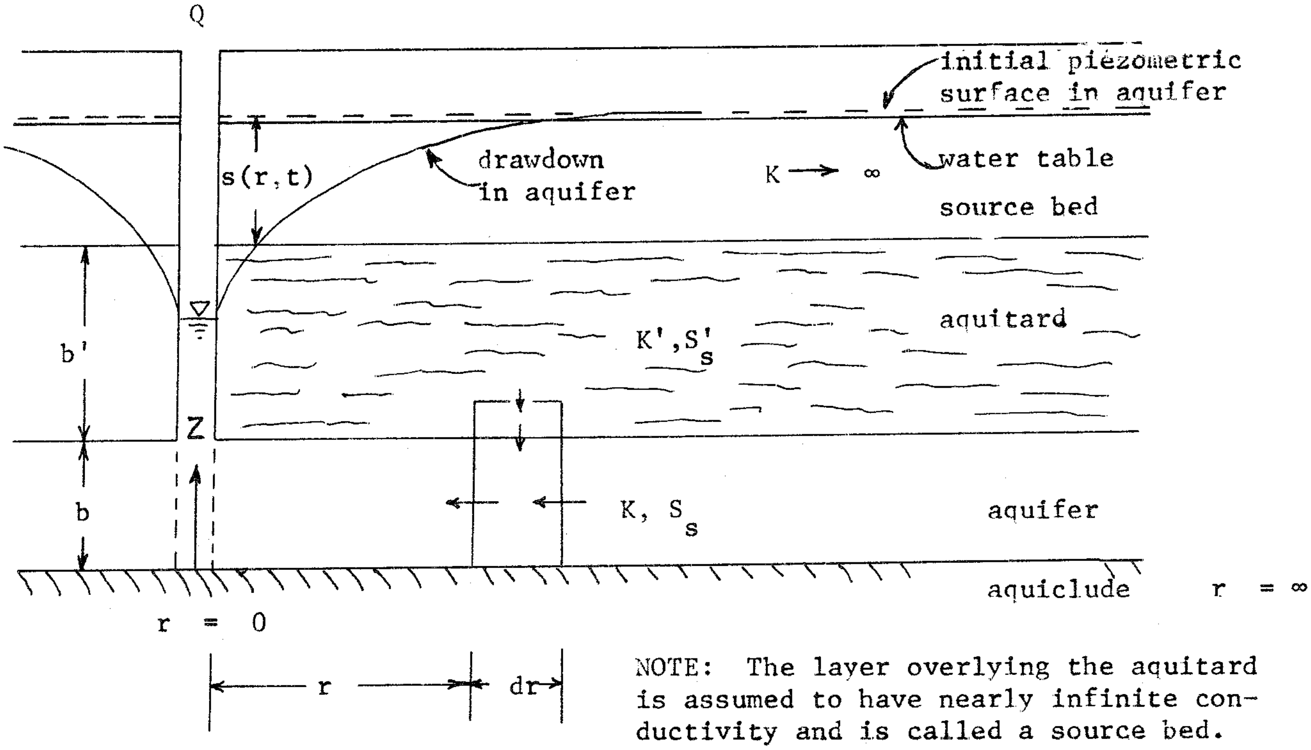
Flow equations for the system described above can be derived by balancing inflows and outflows in the elemental volume in fig. 7.01b. For flow in the aquifer the following quantities are defined:

Radial flow at inner face $= Q_r$

Radial flow at outer face $= Q_r + \frac{\partial Q_r}{\partial r}$

Vertical flow at upper face (at the surface of the aquifer) $= Q_z$

Water accumulated in storage $= Q_s$



NOTE: The layer overlying the aquitard is assumed to have nearly infinite conductivity and is called a source bed.

Fig. 7.01a. Cross section through a leaky aquifer system which is penetrated by a well open only to the aquifer

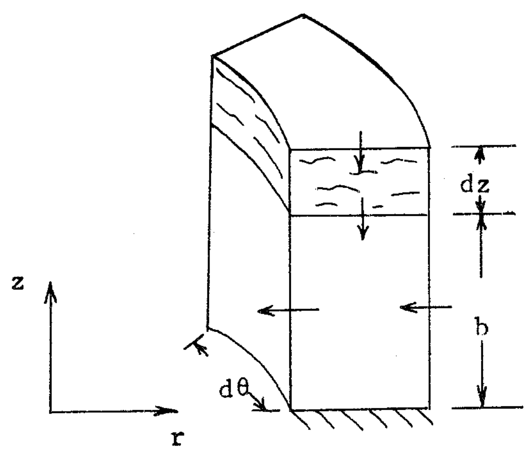


Fig. 7.01b. Detail of an elemental volume element in the aquifer and aquitard

The flow quantities are positive if flow is in the positive coordinate direction. Balancing inflows and outflows:

$$Q_r - (Q_r + \frac{\partial Q_r}{\partial r} dr) - Q_z = Q_s$$

or

$$-\frac{\partial Q_r}{\partial r} dr - Q_z = Q_s \qquad (7\text{-}1)$$

By Darcy's law:

$$Q_r = KiA$$

$$= -K \frac{\partial h}{\partial r} rd\theta b$$

or, because $s = H_o - h$

$$Q_r = \frac{\partial s}{\partial r} rd\theta b \qquad (7\text{-}2)$$

For the development leading to equation 7-2, let

K = hydraulic conductivity of the aquifer
i = hydraulic gradient
A = area across which flow occurs
h = hydraulic head
s = drawdown in the aquifer
H_o = initial elevation of the piezometric surface
b = aquifer thickness

Additional symbols are defined in fig. 7.01.

For flow entering the aquifer from the aquitard:

$$Q_z = -K' (\frac{\partial h'}{\partial z}) rd\theta dr$$

$$= K' (\frac{\partial s'}{\partial z})_{z=b} rd\theta dr \qquad (7\text{-}3)$$

where the notation for the gradient indicates that this term is to be evaluated at elevation $z = b$, the top of the aquifer and the primes indicate that the terms apply to the aquitard. Finally:

$$Q_s = S_s \frac{\partial h}{\partial t} \, r d\theta dr b$$

$$= -S_s \frac{\partial s}{\partial t} \, r d\theta dr b \qquad (7\text{-}4)$$

where:

t = time

S_s = specific storage for the aquifer, the amount of water in storage that is released from a unit volume of aquifer per unit decline of head (Davis and DeWiest, 1966, p. 182). For further explanation see 2.

Substituting equations 7-2 through 7-4 into equation 7-1 and dividing by $r d\theta dr$ there results:

$$Kb \left(\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r}\right) + K' \left(\frac{\partial s'}{\partial z}\right)_{z=b} = S_s b \frac{\partial s}{\partial z} \qquad (7\text{-}5)$$

or:

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} + \frac{K'}{T}\left(\frac{\partial s'}{\partial z}\right)_{z=b} = \frac{S}{T}\frac{\partial s}{\partial t} \qquad (7\text{-}6)$$

where:

$S = S_s b$ = storage coefficient

$T = Kb$ = transmissivity

The equation for vertical flow in the aquitard can be derived in a similar manner using the elemental volume in the aquitard (fig. 7.01b). Let:

Vertical flow at the upper surface of aquifer = Q_z

Vertical flow at the upper surface of the volume element = $Q_z + \frac{\partial Q_z}{\partial z} dz$

Water accumulated in storage = Q'_s

Balancing the inflows and outflows:

$$Q_z - (Q_z + \frac{\partial Q_z}{\partial z} dz) = Q'_s$$

or,

$$- \frac{\partial Q_z}{\partial z} dz = Q'_s \qquad (7\text{-}7)$$

Using Darcy's law:

$$Q_z = K'iA$$

$$= K' \frac{\partial s'}{\partial z} \; rd\theta dr \qquad (7\text{-}8)$$

where:

K' = hydraulic conductivity for the aquitard, and other symbols are as defined previously.

Also:

$$Q'_s = -S'_s \frac{\partial s'}{\partial t} \; rd\theta drdz \qquad (7\text{-}9)$$

In equation 7-9,

S'_s = specific storage for the aquitard

Substituting equations 7-8 and 7-9 into equation 7-7 and dividing by $rd\theta drdz$ gives:

$$K' \frac{\partial^2 s'}{\partial z^2} = S'_s \frac{\partial s'}{\partial t} \qquad (7\text{-}10)$$

The formulation of the problem is completed by a statement of the boundary and initial conditions. At the top of the aquitard:

$$s' = 0 \qquad 0 < r < \infty \qquad t > 0$$

This boundary condition is a consequence of the fact that $K = \infty$ in the source bed.

At the bottom of the aquifer:

$$\frac{\partial s}{\partial z} = 0 \qquad 0 \leq r < \infty \qquad t > 0$$

because the bottom of the aquifer is assumed to be an impermeable boundary. The external lateral boundary is at $r \to \infty$. Here:

$$s \to 0 \qquad\qquad t > 0$$

$$s' \to 0 \qquad b \geq z \geq (b + b') \qquad t > 0$$

The well is treated as a line sink (i.e., it is assumed to have an infinitesimal radius). Therefore, at the well bore where $r \to 0$:

$$Q = KiA$$

$$= -K \frac{\partial s}{\partial r} (2\pi rb)$$

or,

$$-\frac{Q}{2\pi Kb} = -\frac{Q}{2\pi T} = r \frac{\partial s}{\partial r}$$

where the minus is necessary to make the discharge, Q, positive because $\frac{\partial s}{\partial r}$ is negative. The line sink condition is expressed formally as:

$$\lim_{r\to 0} r \frac{\partial s}{\partial r} = -\frac{Q}{2\pi T} \qquad (7\text{-}11)$$

The final boundary condition is at the interface between the aquifer and the aquitard. Here:

$$s = s' \qquad 0 < r < \infty \qquad t > 0$$

Initial conditions are of zero drawdown, i.e.:

$$s = 0 \qquad 0 < r < \infty \qquad\qquad t = 0$$

$$s' = 0 \qquad 0 < r < \infty \qquad b \leq z \leq (b + b') \qquad t = 0$$

The solution for the above initial value problem was given by Hantush (1960) and, when

$$t < \frac{1077\ S_s'\ (b')^2}{K'}, \text{ is:}$$

$$s = \frac{114.6Q}{T} W(u, \beta) \quad (7\text{-}12)$$

$$u = \frac{2693\, r^2 S}{Tt} \quad (7\text{-}13)$$

$$\beta = \frac{r}{4} \lambda \quad (7\text{-}14)$$

$$\lambda = \sqrt{\frac{K' S'_s}{TS}} \quad (7\text{-}15)$$

In equations 7-12 through 7-15,

s = drawdown in feet

Q = well discharge in gpm

T = aquifer transmissivity in gpd/ft

S = aquifer storage coefficient (dimensionless)

$W(u, \beta)$ = the well function of u and β (it is actually a dimensionless drawdown)

u = the reciprocal of dimensionless time

r = radius in feet

t = time from the start of pumping in minutes

K' = vertical hydraulic conductivity of the aquitard in gpd/ft^2

S'_s = specific storage of the aquitard in ft^{-1}

When $t \geq \frac{53856\, S'_s (b')^2}{K'}$, the solution is:

$$s = \frac{114.6Q}{T} W\left(u', \frac{r}{B}\right) \quad (7\text{-}16)$$

$$u' = \frac{2693 r^2 S}{Tt} \left(1 + \frac{S'}{3S}\right) \quad (7\text{-}17)$$

$$\frac{r}{B} = \frac{r}{\sqrt{\frac{b'T}{K'}}} \quad (7\text{-}18)$$

where:

$W(u', \frac{r}{B})$ = the well function of u' and $\frac{r}{B}$ (a dimensionless drawdown)

u' = the reciprocal of dimensionless time

S' = aquitard storage coefficient (dimensionless)

b' = aquitard thickness in feet

It should be noted that there is a period of time between the two limiting times where neither solution applies.

The assumptions underlying the solutions given above have been mentioned at various points in the development. They can be summarized as follows:

a. The aquifer is confined between an aquitard and an impermeable base.

b. The pumped well is of infinitesimal diameter and is open to the entire aquifer. It is not open to an aquitard.

c. The water table and piezometric surface in the aquifer are horizontal and coincident before pumping begins.

d. The aquifer and aquitard are infinite in areal extent and each is of the same thickness everywhere.

e. The aquifer and aquitard are each homogeneous.

f. The discharge from the well, Q, is constant.

g. Darcy's law is valid everywhere.

h. Flow is vertical in the aquitard and horizontal in the aquifer.

i. There is no drawdown at the upper surface of the aquitard.

j. Water is removed from storage instantaneously with decline in head in both the aquifer and aquitard.

Special Cases of Confined Flow to a Well

The aquitard has no storage capacity. This solution, first given by Hantush and Jacob (1955), assumes that all leakage into the aquifer is derived from the source bed above the aquitard, therefore the aquitard is nearly incompressible. Flow is linear and vertical in the aquitard. To express these conditions mathematically the leakage term, Q_z, in equation 7-1 is written:

$$Q_z = -K' \frac{H_o - h}{b'} r d\theta dr = -K' \frac{s}{b'} r d\theta dr \qquad (7\text{-}19)$$

Substituting equations 7-2, 7-4 and 7-19 into equation 7-1 and dividing by $rd\theta dr$:

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} - \frac{s}{B^2} = \frac{S}{T}\frac{\partial s}{\partial t} \qquad (7\text{-}20)$$

where:

$$B^2 = \frac{b'T}{K'} \qquad (7\text{-}21)$$

The initial and boundary conditions are similar to the previous problem except that flow in the aquitard is not considered explicitly because it is controlled entirely by the head in the aquifer. The condition at the external lateral boundary is:

$$s \to 0 \qquad r \to \infty \qquad t > 0$$

Specifications for the upper and lower boundaries of the aquifer are not necessary because flow is entirely radial. The condition in the well bore is:

$$\lim_{r \to 0} r\frac{\partial s}{\partial r} = -\frac{Q}{2\pi T} \qquad (7\text{-}22)$$

and the initial condition is:

$$s = 0 \qquad 0 < r < \infty \qquad t = 0$$

The solution to this initial value problem yields:

$$s = \frac{114.6Q}{T} W\left(u, \frac{r}{B}\right) \qquad (7\text{-}23)$$

where the terms are as defined previously.

The assumptions involved in the solution are the same as those given for the first problem except for j. Instead, water is removed from storage in the aquifer instantaneously with decline in head, and this induces flow into the aquifer through the aquitard from the source bed. The aquitard is assumed to have no storage capacity (i.e., nearly incompressible).

The aquitard is impermeable. The first unsteady-state solution involving flow to a well was made by Theis (1935) and is this case. The basic differential equation can be derived by letting Q_z equal zero in equation 7-1. Following a similar development as that leading to equation 7-6, there results:

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} = \frac{S}{T}\frac{\partial s}{\partial t} \qquad (7\text{-}24)$$

Boundary and initial conditions are the same as those for the previous section.

Solving the problem:

$$s = \frac{114.6Q}{T} W(u) \qquad (7\text{-}25)$$

$$u = \frac{2693r^2S}{Tt} \qquad (7\text{-}26)$$

where:

W(u) is read as the well function of u and, as before, is actually a dimensionless drawdown.

The remaining terms are defined as they have been for the previous cases.

If it is realized that the aquitard is impermeable, and thus no flow takes place through it, the list of assumptions on page 7-08 is applicable with appropriate modifications to this problem.

Synthesis of Confined Flow Solutions

The preceding three confined flow solutions are actually only one, with the latter two being special cases of the first. Consider equations 7-12 through 7-18 and their explanations: as $S' \rightarrow 0$ the time criterion for application of the early-time solution shrinks to zero, and thus, the early-time solution cannot be applied. However, the late-time solution may be applied for all time for this case (see equations 7-16, 7-17 and 7-18). Also for this case, $u' \rightarrow u$ because $1 + S'/(3S) \rightarrow 1$. The resulting solution is the Hantush and Jacob solution presented previously. The analysis may be carried further, however. In the Hantush and Jacob solution (equations 7-13, 7-18, and 7-23), let $K' \rightarrow 0$. Then $B \rightarrow \infty$ and $r/B \rightarrow 0$. The resulting solution is that of Theis, given previously. The Theis solution may also be derived from the early-time Hantush solution (equations 7-12 through 7-15). Let $K' \rightarrow 0$. Then the time criterion is that the time during which the early-time solution can be applied need only be less than infinity. Finally, it should be noted that $\beta \rightarrow 0$, which again yields the Theis solution.

Unconfined Flow to a Well

Unconfined flow differs from the cases described previously for two major reasons. First, the water table may be considered to be a moving recharge boundary. The withdrawal of water from a well penetrating an unconfined flow system (water table aquifer) results in a lowering of the upper flow boundary coincident with the decline in head. Water stored above the water table is not removed from storage instantaneously with the decline in head. The delayed yield from storage contradicts the assumption made in the solution of the confined flow problem, rendering the above solutions inapplicable to the unconfined flow case until the delayed yield has ceased. The influence of the delayed yield may last from 2 to 10 days (Stallman, 1971, p. 11). Second, for some time after initiation of pumping, vertical flow components are very important. None of the solutions described previously take vertical flow components in the pumped aquifer into account. Because there is no solution available that treats both vertical flow and delayed yield simultaneously, existing solutions must be modified to treat unconfined flow after the effects of vertical flow and delayed yield have dissipated.

Stallman (1965, pp. 305-306) has given graphs from which have been computed times and distances from the pumped well beyond which vertical flow is negligible and a modification of the Theis equation may be applied. The criteria, in terms of dimensionless time ($1/uy = Tt/(2693\ r^2 Sy)$ and dimensionless distance ($r_D = (R/b\ \sqrt{K_{zz}/K_{rr}})$ are:

If $r_D > 0.03$,	$1/uy$ must be >	4,000
" > 0.15,	" " " >	400
" > 0.7,	" " " >	60
" > 1.5,	" " " >	12
" > 3.0,	" " " >	6
$r_D > 6.6$,	$1/uy$ must be >	Z

In order to apply these criteria, $\sqrt{K_{zz}/K_{rr}}$ must usually be estimated. A reasonable, conservative value is 0.1. These criteria may also suffice as criteria for the dissipation of delayed yield effects.

Jacob (in Bentall, 1963, pp. 253-254) derived an approximation for a flow equation which can be used for analysis beyond the critical times and distances enumerated previously where flow can be considered to be essentially horizontal and b >> s. Equation 7-2 can be written for this analysis as:

$$Q_r = K(b - s) \frac{\partial s}{\partial r} r d\theta \tag{7-27}$$

This form of equation 7-2 adjusts for decreasing cross sectional area due to drawdown of the water table. Substitution of equation 7-27 into equation 7-1 with $Q_z = 0$ yields:

$$K \frac{1}{r} \frac{\partial}{\partial r} [(b - s) r \frac{\partial s}{\partial r}] = S_y \frac{\partial s}{\partial t} \tag{7-28}$$

or

$$K [(b - s) \frac{\partial^2 s}{\partial r^2} - (\frac{\partial s}{\partial r})^2 + \frac{(b-s)}{r} \frac{\partial s}{\partial r}] = S_y \frac{\partial s}{\partial t} \tag{7-29}$$

Next s' is defined as:

$$s' = s - \frac{s^2}{2b} \tag{7-30}$$

and equation 7-30 is substituted into equation 7-29. By neglecting all $(\partial s/\partial r)^2$ terms (because, by assumption, s >> b and $\partial s/\partial r$ >> 1), then:

$$T \left(\frac{\partial^2 s'}{\partial r^2} + \frac{1}{r}\frac{\partial s'}{\partial r} \right) = S'_y \frac{\partial s'}{\partial t} \qquad (7\text{-}31)$$

where:

$$T = Kb$$

$$S'_y = \frac{b}{b-s} S_y$$

Equation 7-31 is to be applied at times and distances where the drawdown and rate of drawdown are small. Thus S'_y may be considered constant. The Theis solution of equation 7-31 suffices to describe flow to the well under these conditions, so that, after redesignating s by s' and S by S'_y, equations 7-25 and 7-26 may be applied.

Section 7.02. Curve-Fitting Techniques to Evaluate Aquifer Constants

The evaluation of aquifer constants by application of analytical solutions to time-drawdown data, while seemingly the most direct and realistic approach, is at best expensive and somewhat imprecise. In spite of economic and physical limitations, the method is superior to other methods for the determination of field values. The importance of careful planning of aquifer tests and careful analysis cannot be overemphasized. The reader is referred to three recent publications (Kruseman and DeRidder, 1970, Stallman, 1971, and Walton, 1970) for a more comprehensive treatment of the subject than is presented herein. The presentation in this volume is intended to familiarize the reader with some basic analytical solutions and the methodology of using type curves which have been developed from each of them.

Leakage, Water Derived from Storage in Aquitard

If leakage through the overlying aquitard is significant and water is derived from elastic storage in the aquitard, time-drawdown field data curves (from two or more observation wells) will be analogous to a family of β type curves (fig. 7.02).

For the conditions where $t < \frac{1077\ S_s'\ (b')^2}{K'}$, values of $W(u, \beta)$ may be plotted against values of $\frac{1}{u}$ on logarithmic paper and a family of β type curves constructed. Values of the function $W(u, \beta)$ and u are referenced in Walton (1970, p. 152-153) and Hantush (1960, p. 3719). Time-drawdown field data curves (s on the ordinate plotted against $\frac{t}{r^2}$ on the abscissa of logarithmic paper) are constructed from field values of time and drawdown plotted on logarithmic paper of the same scale as the β type curves.

The time-drawdown field data curves and the family of β type curves are superposed and matched, taking particular care to keep the two sets of axes parallel. The ratio(s) of β values should compare closely to the ratio(s) of observation well distances from the pumped well. A match point is selected and marked on the time-drawdown field data curves and match point coordinates $W(u, \beta)$, $\frac{1}{u}$, s, and $\frac{t}{r^2}$ are substituted in equations 7-12, 7-13 and 7-15 to determine T, S, and the product $K'S_s'$, respectively.

For the condition when $t \geq \frac{53856\ S_s'\ (b')^2}{K'}$, a family of $\frac{r}{B}$ type curves and equation 7-16 may be used to determine aquifer transmissivity provided K of the source bed is very large (Walton, 1970, p. 221). The curve-fitting technique is the same as for the first condition. K' may be obtained from equation 7-18 and b' may be obtained from borehole logging data. If K' can be determined, S_s' can be computed from the known value of $K'S_s'$. The aquifer storage coefficient is not readily obtainable from the late-time curves by curve fitting because time-drawdown field data curves have negligible slope and there is no unique solution to equation 7-17. Physically, subsurface flow to the pumped well has nearly reached steady state and little water is being derived from elastic storage in the aquitard.

Leakage, No Water Derived from Storage in Aquifer

If leakage through the overlying aquitard is significant, negligible water is derived from elastic storage in the aquitard and K of the source bed approaches infinity, time-drawdown field data curves will be analogous

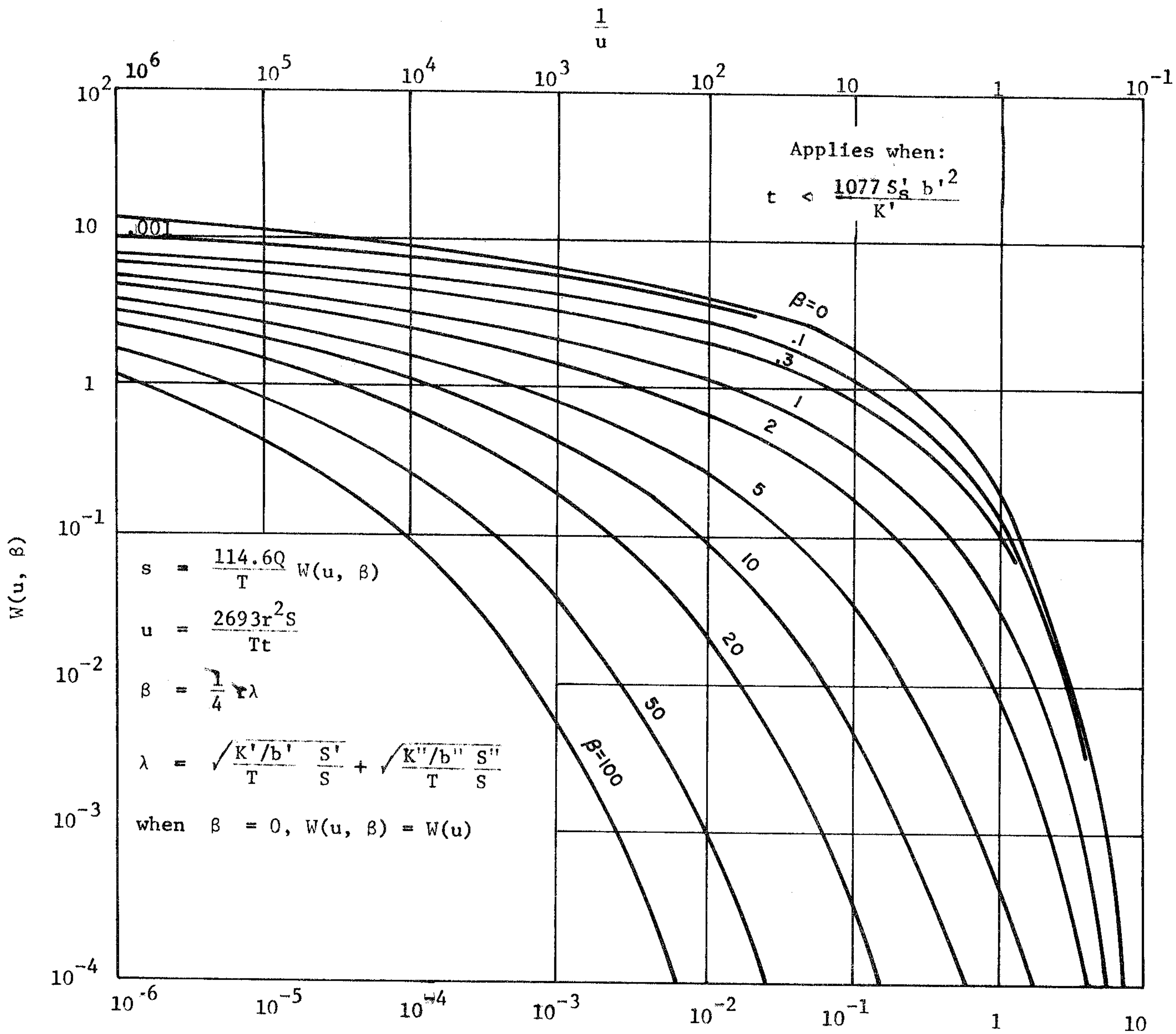


Fig. 7.02. Logarithmic graph of the function W(u, β) (Hantush, 1960, p. 3719)

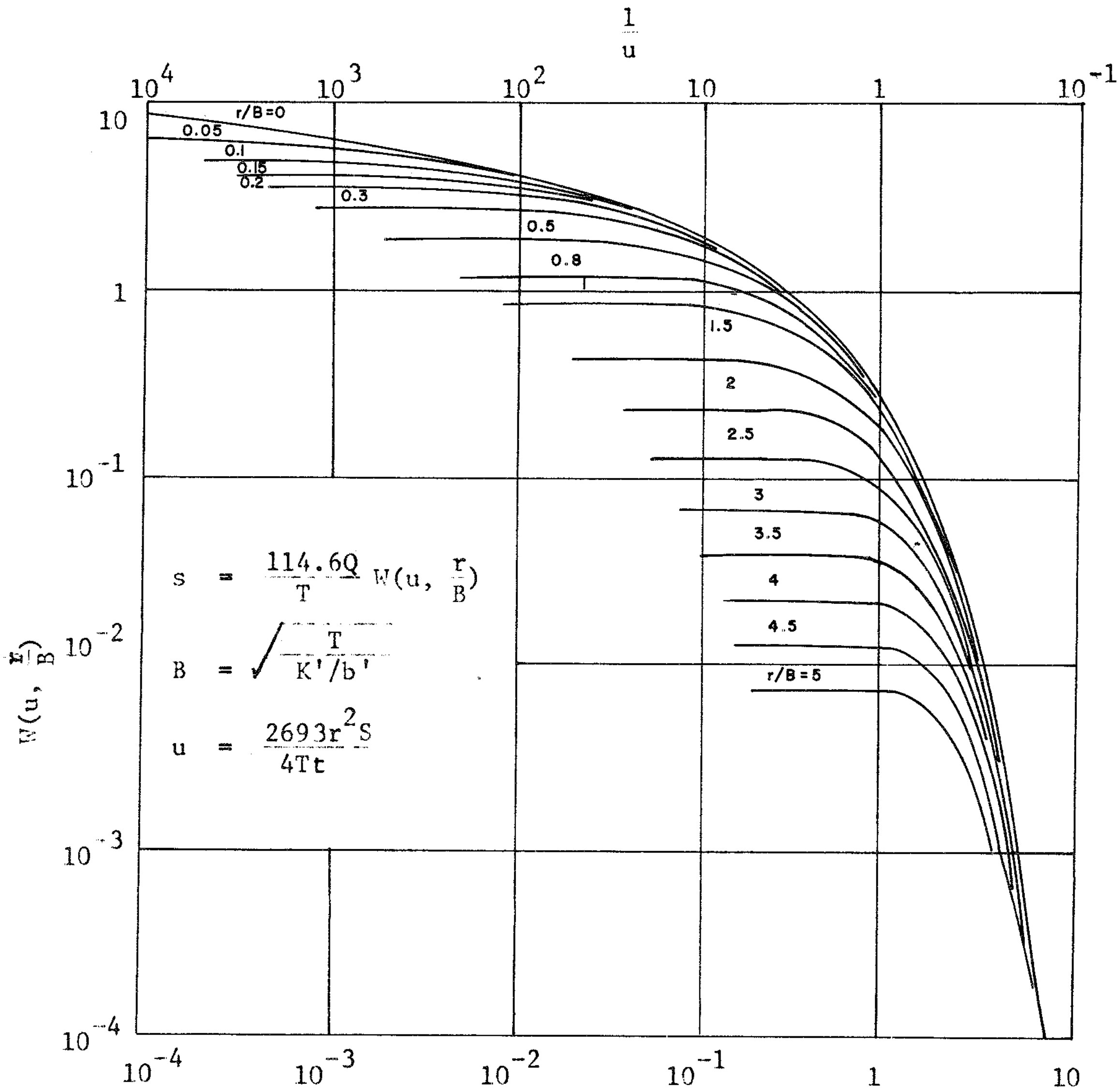


Fig. 7.03. Logarithmic graph of the function $W(u, \frac{r}{B})$ (Walton, 1970, p. 146)

to a family of $\frac{r}{B}$ type curves (fig. 7.03). Values of the function $W(u,\frac{r}{B})$ and u are referenced in Walton (1970, p. 146). Time-drawdown field data curves are constructed as described above on logarithmic paper of the same scale as the $\frac{r}{B}$ type curves.

The time-drawdown field data curves and the family of $\frac{r}{B}$ type curves are superposed and matched, taking care to keep the two sets of axes parallel. The ratio of $\frac{r}{B}$ values should compare closely to the ratio of observation well distances from the pumping well. A match point is selected and marked on the time-drawdown field data curves and match point coordinates $W(u, \frac{r}{B})$, $\frac{1}{u}$, s, and $\frac{t}{r^2}$ (or t if a single observation well is used) are substituted in equations 7-13, 7-18, and 7-23 to determine T, S, and K', respectively. The aquitard thickness, b', in equation 7-18 may be obtained from borehole geophysical data.

No Leakage

Theis method. If leakage to an aquifer from adjacent confining beds is negligible and all water is derived from elastic storage in the aquifer, the time-drawdown field data will be analogous to the nonleaky type curve (Walton, 1970, p. 210). The time-drawdown field data curve and the nonleaky type curves are superposed and matched. Match point coordinates $W(u)$, $\frac{1}{u}$, s, $\frac{t}{r^2}$ (or t for a single observation well) are substituted in Equations 7-25 and 7-26 to determine T and S. Any point can be used as a match point, but as with the previous solutions, an intersection of major axes on the type curve makes calculations easier.

The coefficients T and S can also be determined with distance-drawdown data. Values of W(u), given in Walton (1970, p. 131), may be plotted against values of u on logarithmic paper to obtain the nonleaky type curve. Values of s, measured at the same time in several wells, are plotted against distance squared from the pumped well on logarithmic paper of the same scale as the type curve to obtain a distance-drawdown field data curve. The distance-drawdown field data curve is superposed on the nonleaky type curve keeping the axes of the two graphs parallel. The two curves are matched and match point coordinates, W(u), u, s, and r^2, are substituted into equations 7-25 and 7-26 to determine T and S.

The values calculated by both the time-drawdown and distance-drawdown methods should agree approximately if the drawdown water level is reasonably symmetrical around the well, and if leakage from confining beds is negligible.

Jacob method. Jacob (in Rouse 1950, p. 368) noted that u is small for small values of r and large values of t. This means that the terms of the convergent series, $W(u) = -0.5772 - \ln u + u - \frac{u^2}{2 \cdot 2!} + \frac{u^3}{3 \cdot 3!} \ldots$ may be terminated after the first two terms. Since $0.5772 = \ln 1.78$, drawdown, s, can be expressed as:

$$s = \frac{114.6Q}{T} \left(\ln \frac{Tt}{2693\ r^2 s} - \ln 1.78\right) \qquad (7\text{-}32)$$

which reduces to:

$$s = \frac{264Q}{T} \log \frac{Tt}{4790 r^2 S} \qquad (7\text{-}33)$$

This simplification should be restricted to values of u less than about 0.01 to avoid large errors. With increasing time, u becomes small and the plot of drawdown versus the logarithm of t should form a straight line on semilogarithmic paper. From drawdown measurements in an observation well during an aquifer test, s and t are known and are plotted on semilogarithmic paper. When Δs is the drawdown difference in feet per log cycle of time, the transmissivity can be obtained from:

$$T = \frac{264Q}{\Delta s} \qquad (7\text{-}34)$$

The elastic storage coefficient can be determined from the time intercept, t_o, of the straight line on the log t axis. When s = o, equation 7-33 becomes:

$$S = \frac{Tt_o}{4790 r^2} \qquad (7\text{-}35)$$

where t_o is in minutes.

Equation 7-33 may also be applied to: (1) water level observations in different wells at the same time, and (2) to water level observations in various wells at various times.

In the first case, only the distance, r, from the pumped well varies in equation 7-33 which may rewritten as:

$$s = \frac{264Q}{T} \log \frac{Tt}{4790S} - \frac{528Q}{T} \log r \qquad (7\text{-}36)$$

This is the equation of a straight line of s versus log r on semilogarithmic paper. Thus T can be determined from the slope of the straight line and S is determined from the distance intercept, r_o, of the straight line on log r axis. When s = o, equation 7-33 becomes:

$$S = \frac{Tt}{4790 r_o^2} \qquad (7\text{-}37)$$

where t is in minutes, r_o is in feet and T is in gpd/ft.

In the second case, a composite drawdown graph may be obtained by plotting values of drawdown measured at various times in several wells against the logarithms of the respective values of r^2/t (Jacob, in Rouse, 1950, p. 368). This semilogarithmic plotting approximates a single straight line. From the slope, the transmissivity, T, can be determined using equation 7-24 and S can be determined from the intercept, $(r^2/t)_0$ of the straight line on log (r^2/t) axis using equation 7-35 or 7-37.

Water Table Conditions

No aquitard is present in an unconfined flow situation, so the upper boundary is the water table. The Theis solution provides an approximate late-time solution.

Plot corrected drawdown, s' (drawdown corrected for decrease of aquifer saturated thickness using equation 7-30), in the observation well against $\frac{t}{r^2}$ (or t if only one observation well is used) on the abscissa. The computer program given in Appendix 4 expresses time-drawdown field data in columns of measured drawdown and corrected drawdown, respectively, for easy data plotting. Superimpose the plotted graph on the Theis type curve and match as much of the late-time data as possible to the type curve taking care to keep the $\frac{t}{r^2}$ (or t) and $\frac{1}{uy}$ axes parallel. Note the coordinates s', $W(u_y)$, $\frac{1}{uy}$, and $\frac{t}{r^2}$. Compute the value of T from equation 7-25 and the value of S'_y from equation 7-26. The approximate value of Sy is calculated from:

$$Sy = \frac{b - \bar{s}}{b} S'_y \qquad (7\text{-}38)$$

where:

$\bar{s}$ = the drawdown at the geometric mean distance the observation wells are from the pumped well (Jacob, in Bentall, 1963, p. 354).

Next, calculate r_D and $\frac{1}{uy}$ for the observation well (s) (p. 7-11) and determine whether the time and distance criteria (p. 7-11) have been met. If the late-time data have fit the Theis curve well then these criteria have probably been fulfilled.

Section 7.03. Applicability of Well Hydraulic Theory

Generalization of Results for Leaky Aquifers

In multiple leaky aquifer systems, the β and $\frac{r}{B}$ solutions may be extended. Terms for the additional aquitards are additive as follows (Hantush, 1960, p. 3716):

$$\beta = \frac{r}{4} \left(\sqrt{\frac{K'/b'}{T} \frac{S'}{S}} + \sqrt{\frac{K''/b''}{T} \frac{S''}{S}} \right) \qquad (7\text{-}39)$$

$$\frac{r}{B} = r \sqrt{\frac{K'/b'}{T} + \frac{K''/b''}{T}} \qquad (7\text{-}40)$$

$$u' = \frac{2693r^2S}{Tt} \left(1 + \frac{(S' + S'')}{3S}\right) \qquad (7\text{-}41)$$

If the flow boundaries at the top of the upper aquitard and at the bottom of the lower aquitard are considered to be impermeable, a modification of the Theis equation can be used instead of the $\frac{r}{B}$ solution (Hantush, 1960, p. 3717).

$$s = \frac{114.6Q}{T} W(S_2 u) \qquad (7\text{-}42)$$

where:

$$S_2 = 1 + \frac{(S' + S'')}{S},$$

maximum $t \geq t_{cr}$, and t_c is either $\frac{53856 S'_s (b')^2}{K'}$ or $\frac{53856 S''_s (b'')^2}{K''}$.

The two conditions given by equations 7-16, 7-17, 7-18 and 7-42 yield the limiting equations for real data. If K', K'', S' and S'' are all nonzero, then these parameters cannot be individually determined. Geologic data in some instances may suggest that a good approximation is that $K' \approx K''$ and $S' \approx S''$. By assuming probable limits for one, the probable limits for the other may be determined.

The early-time solution does not contain any assumptions as to the outer boundaries because drawdowns have not yet reached them. Thus, a good solution for T and S can theoretically be obtained from early-time data no matter what the conditions are beyond the aquitards. Applying this idea, Neuman and Witherspoon (1969c, p. 153-162) have developed a method of obtaining all of the aquifer parameters (including K', etc.) if piezometers are installed in the aquitards. If a sequence of aquifers is to be investigated then each aquifer will have to be tested separately.

Limiting and Special Cases

If $\frac{S'}{S}$ or $\frac{S''}{S} \leq 0.01$ and only one of the parameters S' and S'' is present, then the limiting equation (equation 7-23) can be used with little error (Hantush, 1960, p. 3718). The Theis equation may be applied provided the upper boundary is impermeable. If the upper boundary is treated as a source bed, then the $\frac{r}{B}$ solution applies.

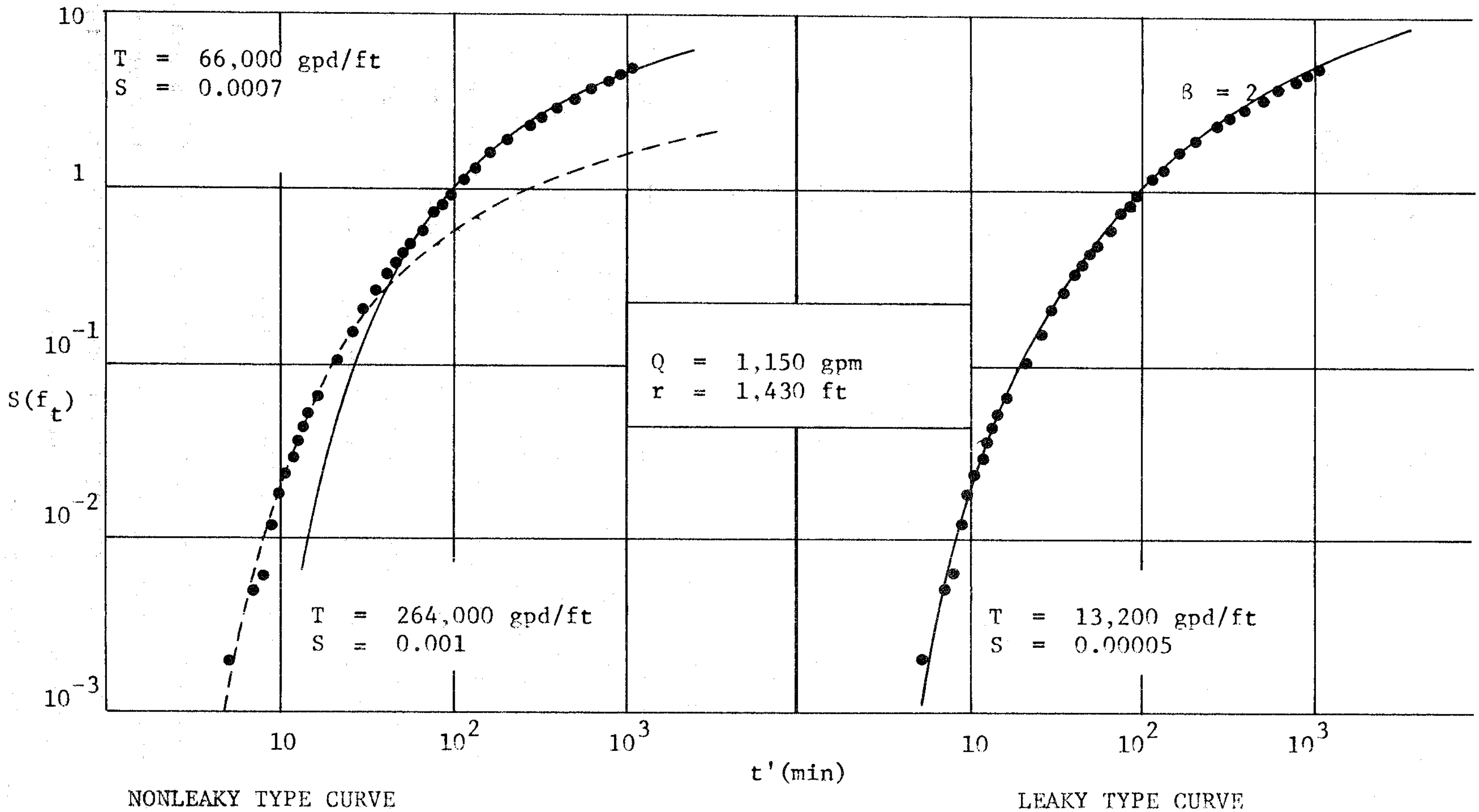


Fig. 7.04. Recovery in observation well 16N3, Pixley (Riley, personal communication)

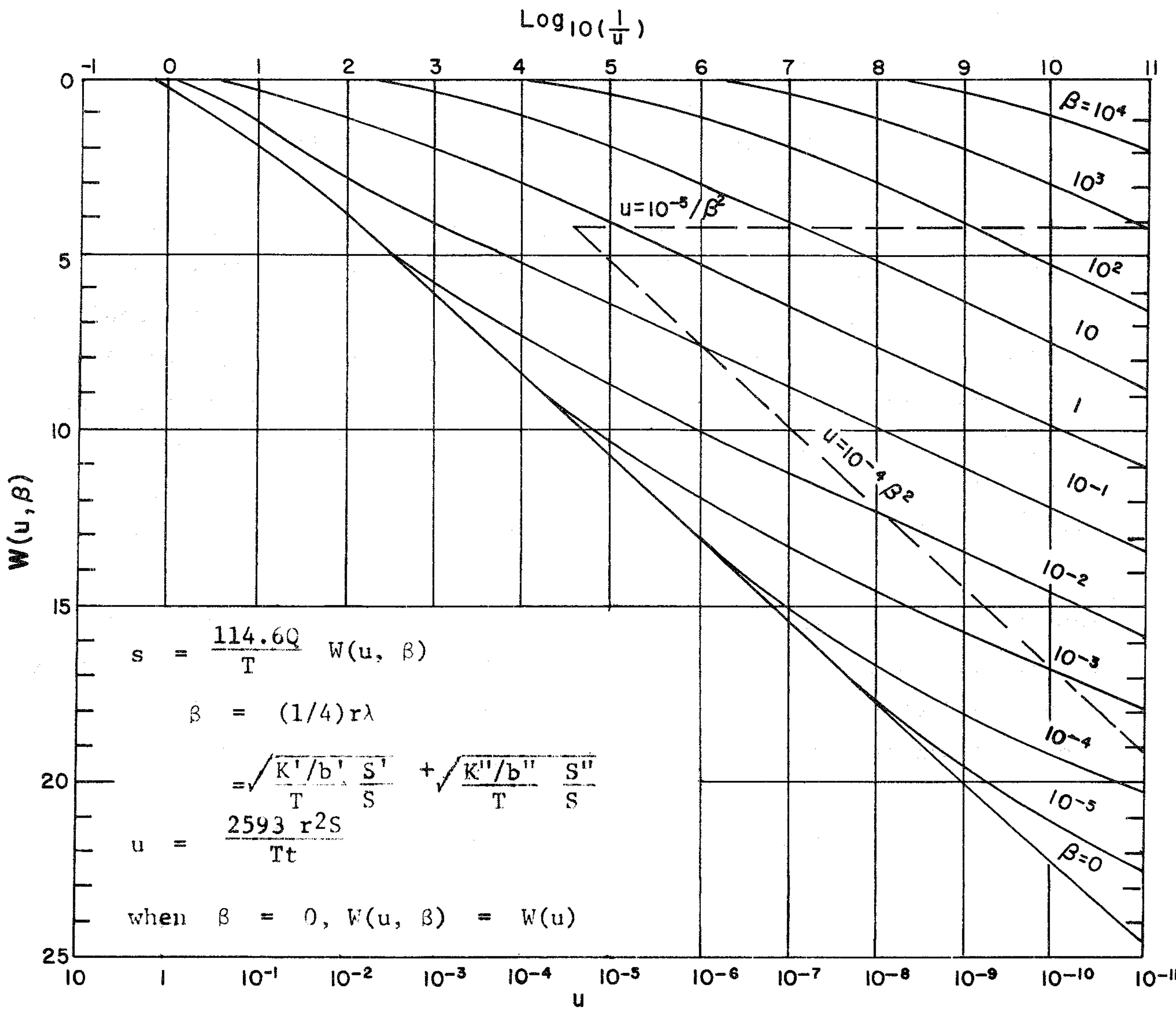


Fig. 7.05. Semilogarithmic graph of the function W(u, β) (Hantush, 1960, p. 3720)

If K' and/or K'' << K or the release of ground water in the aquitard is delayed so that little release of water occurs during the aquifer test, then the Theis equation may be applied with little error. When the release of ground water from storage is delayed, the Theis equation will not predict accurately the future drawdown water table because delayed gravity yield from storage is not taken into account.

Consequences of Violating the Assumptions

Consider the time-recovery field data for the observation well given in fig. 7.04. For the lower "fit" T is calculated as 264,000 gpd/ft. In this case abundant ground water is derived from storage in the aquitard. The Theis equation assumes that all ground water comes from storage in the aquifer, thus the calculated T is much too large. At later time the Theis curve for the lower "fit" trends below the field data curve indicating leakage is decreasing somewhat. However, the T calculated from the upper "fit" is still too large showing that leakage to the aquifer is still appreciable. A semilogarithmic plot of the function W(u, B) yields a family of straight lines for a leaky aquifer system (fig. 7.05). Any attempt to use the Jacob straight line method will yield erroneous results. Only if all of the data fit the curve or if the observation wells all plot approximately colinearly, will the Theis solution be valid.

Section 7.04. Aquifer Test Procedure

General Considerations

Good practice in conducting aquifer tests includes the following (J. L. Mogg, personal communication):

The type (confined or unconfined) and thickness of the aquifer must be known. The uncased portion of the pumped well should be equal to at least 66 percent to 75 percent of the thickness of the section of the aquifer penetrated by the well. If less than the full thickness of the aquifer is penetrated, then the data will have to be corrected for partial penetration (Hantush, in Chow, 1964, pp. 347-358, 368). The diameter of the pumped well should be large enough to hold both the pump and a water level measuring device. The test pump should have a capacity of at least half the maximum expected discharge of the well.

It is best to use as many observation wells as possible; however, two are sufficient except for the analysis of hydrologic boundary effects and multiple aquifer systems. The uncased lengths (lengths open to the aquifer) should be at least half as long as that of the pumped well and placed so that the centers of these lengths are at the same depth. In any case, the uncased lengths should not be less than 3 feet long. The diameter of the observation well should be large enough to admit a water level measuring device. A 1-1/4-inch diameter well is sufficient. The observation well closest to the pumped well should be placed at a distance equal to two times the aquifer thickness. Logarithmic spacing of wells should be used so as to provide good dispersion of the results on graph paper. To accurately reflect water level changes, the observation wells must be properly developed and cleaned of drilling mud. A good check is to fill the pipe with water and measure the time for the water to reach a nearly steady level. If the well is open, the interval of time required for the water level to change is dependent on permeability of the aquifer.

Drawdown and yield measuring equipment. Any of the devices discussed in Chapter 6 may be used to measure the water level. The choice depends on the frequency of measurement and the depth of water. The choice of yield measuring devices depends on the flow. Orifice meters are generally used for large volumes of water. Volumetric measure of volume and weight is the most accurate but is not convenient for use with large flows. Orifice buckets are generally used for small flows of water. Commercial water meters should be selected for the quantity of the flow to be measured. The same meter should not be used to measure both large and small flows.

Length of test and frequency of measurements. The test should be conducted continuously for a period of from 2 to 3 days for a water table condition and for a period of 18 to 36 hours for confined conditions. Prior to starting the test, water levels should be determined for every well. These should be measured at frequent time intervals before the test in order to determine the time trend of the water table and the barometric efficiency of the well. The pumping rate should be constant and measured continuously throughout the test. At the beginning of the test, the water level decreases rapidly and measurements should be taken as often as possible (e.g., every 10 to 15 seconds). As the water level begins to decrease less rapidly, the measurements may be taken at less frequent intervals. Watches should be synchronized prior to beginning the test to insure that water level measurements are taken simultaneously at each observation well. Log data sheets should be used to record measurements.

Corrections to measured drawdown. Prior to aquifer test (about 1 week) measurements of barometric pressure should be obtained, and these measurements should be continued through the entire aquifer test. The barometric efficiency of the aquifer should be ascertained from the data obtained before the test and used to correct drawdown (Chapter 6 of this volume and Todd, 1959, pp. 158-162). If the aquifer to be tested is near a coastline, water level measurements should be obtained prior to the test (about 1 week) and during the test. From the data obtained before the test, the tidal efficiency of the aquifer can be evaluated and used to correct drawdown (Chapter 6 of this volume and Todd, 1959, pp. 163-168). The drawdown at any time is the difference between the nonpumping water level at that time extrapolated from the time trend of the water table and the pumping level at that same time. This drawdown must then be corrected for barometric and tidal effects.

Chapter 8

Surface Investigation of Ground Water

CHAPTER 8. SURFACE INVESTIGATION OF GROUND WATER

Section 8.01. Introduction

The usual objectives of exploratory ground-water investigations re to delineate and evaluate new sources of ground water or to enlarge ithdrawals from known underground sources. Such studies require the ocation and identification of aquifers, the determinations of structural aracteristics and lithology, and the determination of the quantity and uality of water that may be obtained by wells. A knowledge of the ydraulic relationships between ground water and the water on the land urface and in soils is also required.

Kinds of Methods

The following surveys and methods are used to provide only indirect ndications of ground water, as subsurface hydrologic data must be nferred from surface information (Richter, 1967, chapter 5):

a. Aerial photographic (black and white, color, infrared and adar imagery). Photographs are required to obtain information in mapping eology, ecology, hydrology, land and water use conditions, and to provide asic data for preparation of topographic maps.

b. Topographic. Maps of suitable scale ($\frac{1}{24,000}$ or $\frac{1}{62,500}$) based n aerial photographs are required as a basis for remaining surveys and or presenting all phases of the investigation.

c. Geologic. Geological mapping, test hole drilling and sampling, quifer tests, and preparation of maps are needed to fully describe the ature and hydraulic properties of earth materials.

d. Geophysical. Specialized surface geophysical surveys magnetic, seismic, gravity, and electrical) are often required to determine epth to, and extent and thickness of hydrostratigraphic units.

e. Geochemical and geothermal. These specialized studies are equired to determine the nature of and variations in chemical character f waters as well as the source and depth of water.

f. Hydrologic. This survey would determine the nature and extent of surface waters, rainfall, runoff, infiltration and evapotranspiration, and the relation of these factors to subsurface water.

No one method or survey is commonly adequate to evaluate or predict accurately the location and delineation of aquifers. Ordinarily the demands of a given investigation dictate which method or methods will be required for meaningful results.

Section 8.02. Gravity, Magnetic, and Seismic Methods

Any conceivable property or process that could be measured or applied at ground surface and that would be affected by attitude or nature of rocks through a mantle of intervening earth materials might provide the basis for a method of geophysical prospecting. Many principles have been suggested and tried, but most geophysical exploration depends on several basic physical principles. A brief summary of these principles follows:

Gravitational Methods

Gravitational methods are based on measurement of small variations in the gravitational field at ground surface. Small differences or distortions in the gravitational field from point to point over the earth's surface are caused by lateral changes in the distribution of mass in earth's crust. Therefore, if rocks of differing density are present in any area, the resulting irregularity in mass distribution will make a corresponding change in the intensity of gravity and in the space rate of change of gravity at the surface.

Sensitive instruments are used to measure either the relative value of gravity or its space derivatives. Three common types are (Davis and DeWiest, 1966, p. 271):

a. Pendulum - This instrument is kept at constant length so that differences in period are related to differences in gravity. A rapidly vibrating rod or reed is employed as the pendulum.

_b._ Gravimeter - This instrument measures vertical intensity of pull of gravity on a mass suspended by a delicate spring.

_c._ Torsion balance - This instrument measures the gradient rather than the direct force of gravity. Two small masses are attached to the ends of a rod. This rod, which is suspended by a quartz filament, is subjected to torsion when gravitational attraction on the two masses is unequal.

The measured change in gravity is interpreted in terms of probable mass distributions below ground surface, from which inferences are made as to probable geologic conditions.

Magnetic Methods

Magnetic methods are based on measurements of small variations in the magnetic field at ground surface. Earth's magnetic field is affected by any variation in the distribution of magnetized (or polarized) rocks. Most sedimentary rocks are nearly nonmagnetic, but underlying igneous or basement rocks are slightly magnetic. Therefore, if these magnetic rocks are present, the resulting irregularities will produce corresponding changes in the intensity of magnetic field at ground surface.

Magnetometers are used to measure the relative values of magnetic intensity (usually the vertical component). The measured variation is interpreted in terms of the probable distribution of magnetic material below ground surface, from which inferences are made as to the probable geologic conditions.

Seismic Methods

Seismic methods are based on the measurement of travel times of artificial elastic waves. Such waves are produced by explosives at or near ground surface which propagate outward in all directions from the shot point. Seismic refraction is based on the principle that seismic waves travel along least-time paths (Johnson, 1966, p. 177). These waves often encounter high-velocity layers (solid crystalline rocks) along which they are refracted, eventually returning to ground surface, where their arrival times are picked up by seismometers and recorded by an oscillograph. Seismometers are placed on ground surface at various distances from shot point and are connected to the oscillograph. The refracted waves overtake the direct waves, which travel more or less horizontally through surficial material. The first arrival time at each seismometer is plotted against the distance from shot to detector, giving a travel-time curve from which wave speeds, depths and thicknesses of layers can be computed. Travel time of these waves depends on the nature (elastic properties) of rocks penetrated, corresponding speed of wave travel, and on existence of discontinuities in velocity or density which determine points of reflection or refraction below land surface.

In many cases, seismograms show also reflected waves which give information as to depth to discontinuities in the lithologic character of the rock series. Under favorable conditions, a geologic unit may be delineated accurately by reflection even though it may be several thousand feet deep.

Under favorable conditions, seismic methods may provide information that can be interpreted in terms of geologic conditions. Under less favorable conditions, detection of reflected or refracted waves or their interpretation in terms of definite geologic conditions may be uncertain and leave many questions unanswered.

Section 8.03. Electrical Methods

There are a number of ways in which electrical principles have been applied to clarify subsurface conditions. The theory of electrical methods is relatively complicated, and there is voluminous periodical literature treating this subject. Electrical prospecting methods are classified into natural and artificial current methods.

Natural Current Methods

Because natural current methods have limited application to ground water, the discussion will be brief. Natural currents may be subdivided into two types:

a. Local currents. Local currents occur because sulfide ores are electrochemically active and behave like the elements of a battery. For example, when the upper end of an ore body near ground surface becomes oxidized, it has a negative charge (cathode). The lower end of the ore body becomes positive (anode). Thus, an electric current tends to flow downward within the ore body and the return current in the bounding rock flows upward and toward the upper end of the ore body. By measuring and delineating differences in potential between observation points on ground surface, it is possible to predict directions and source of current flow. Ordinary metal electrodes in contact with the ground will sometimes generate their own self potential with respect to the ground, making it necessary to use "nonpolarizing" electrodes. These electrodes commonly consist of a porous clay pot filled with a saturated solution of copper sulfate having an excess of copper sulfate crystals. Measurements are conducted by obtaining the difference in potential between pairs of such pots located at central points along profiles or in a more or less regular grid over the area surveyed.

b. Telluric currents. Telluric currents are intimately related to diurnal magnetic changes and have corresponding diurnal and short-period variations. They are usually active during magnetic storms. Measurements are made by comparing potential differences between two pairs of electrodes positioned along perpendicular lines at a fixed station with potential differences between similar sets of electrodes at a mobile station. These measurements show that time variations of potential difference at points many miles apart are similar in form, but the amplitudes are variable with position. These variations in amplitude are controlled by changes in electrical resistance of earth's crust. By delineating the amplitude changes or the ratios of the potential differences at mobile stations to the simultaneous potential differences at the fixed station, inferences can be made as to the presence or absence of relatively conducting rocks in larger lithologic units. For example, a sedimentary basin would have less resistance to telluric currents than an area composed of shallow granodiorite. Testing of this theory has been limited to date.

Artificial Current Methods

All the various artificial current methods apply an electric current or impulse of some kind to the ground. Conductive methods apply an electric impulse directly to the ground by means ~~of electrodes~~. Inductive methods allow current to flow in the ground by electromagnetic induction from alternating currents in lines or coils at or near ground surface. Measurements are made either of the differences in potential or of the electromagnetic field produced by currents in the ground.

Electrical resistivity of a rock limits the quantity of current flowing through it when an electrical potential is applied. Resistivities or rock units are dependent upon the nature of the material, that is, its density, porosity, pore geometry, water content and quality, and temperature. Most rocks and minerals (except some clay minerals) are poor conductors. Electrical properties of a rock depend upon pore geometry and the fluids which occupy the voids. Oil and gas are poor conductors, but water is a relatively good conductor when it contains dissolved salts. Current is conducted in water by movement of ions and can, therefore, be termed electrolytic conduction. Resistivity of any material which is used to describe the ability of the material to conduct current is the resistance in ohms between opposite faces of a unit cube of material. Resistivity is defined by the following equation:

$$\rho = \frac{rA}{L} \tag{8-1}$$

where:

ρ = resistivity or specific resistance in ohm-meters or ohm-cm

A = cross-sectional area of the conductor in cm^2

L = length of the conductor in cm

r = resistance in ohms

Laboratory measurements of electrical properties require a knowledge of the dimension of the rock, the fluid saturation of the rock, the resistivity of the water in the rock, and a suitable resistivity cell in which to test the sample. The resistance of a sample is obtained from Ohm's law:

$$r = \frac{V}{I} \tag{8-2}$$

where:

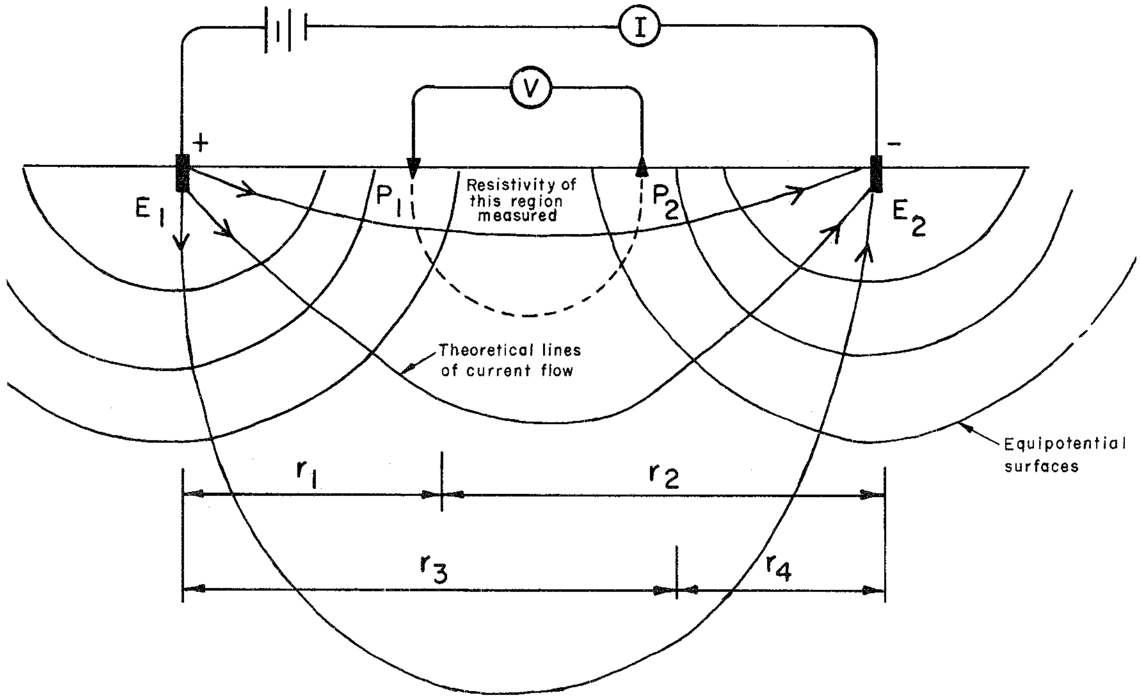
V = potential drop in volts

I = electric current in amperes

By substituting the resistance, r, into equation 8-1 and knowing A and L, the resistivity can be determined.

_a_. Conductive methods. Direct current measurements are applied to the ground by having two electrodes make electrical contact with the ground. The difference in potential is usually measured between two other electrodes. These determinations primarily give the conductivity of the ground (Maxey, 1964, p. 25).

An ideal case is shown diagrammatically by a current flow between current electrodes E_1 and E_2 (fig. 8.01). The corresponding equipotentials would be spheres centered on E_1 and E_2. The potential electrodes would be positioned at P_1 and P_2. The voltage drop between the potential electrodes (P_1 and P_2) is then measured. This arrangement (fig. 8.01) represents a very crude electrical resistivity unit. The primary function of a resistivity instrument is to measure the ratio V/I which has resistance units. Thus, the resistivity method is a field application of Ohm's law.



Potential at P_1

$$V_1 = \frac{I\rho}{2\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Potential at P_2

$$V_2 = \frac{I\rho}{2\pi}\left(\frac{1}{r_3} - \frac{1}{r_4}\right)$$

Potential between P_1 and P_2 = $V_1 - V_2$

g. 8.01. Vertical cross section showing direct current electrode configuration for a material of uniform resistivity (Diagrammatic)

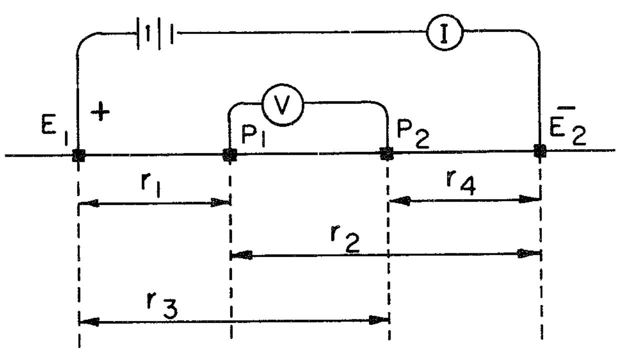


Fig. 8.02. Wenner electrode configuration

The "Wenner" electrode configuration is commonly used in field operations. This scheme consists of four equally spaced electrodes at an interval L along a straight line (fig. 8.02). For this electrode system with material of uniform resistivity, the true ground resistivity between P_1 and P_2 is given by:

$$\rho = \frac{2\pi V}{I} \quad \frac{1}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right) - \left(\frac{1}{r_3} - \frac{1}{r_4}\right)} \qquad (8\text{-}3)$$

In the case of nonuniform material, the value given by equation 8-3 is called the apparent resistivity.

The resistivity value obtained from equation 8-3 applies to a volume of ground that depends on electrode spacing. By increasing the spacing, the current penetrates more deeply into the ground. If the earth materials are uniform, homogeneous layers, the distance between the current and potential electrodes is equal to the depth of current penetration. In general, effective subsurface resistivities vary with depth; thus, apparent resistivities vary as electrode spacings are increased. Because changes of resistivity at great depths have only a slight effect on apparent resistivity compared to those at shallow depths, the method is seldom useful to determine actual resistivities below several hundred feet.

There are two types of field procedures used with electrical resistivity units. Electrical traversing employs a constant electrode separation at each station as the survey proceeds across the area. Since the effective depth of investigation is related to electrode spacing, the depth is nearly constant for all resistivity readings. As the survey proceeds along ground surface, changes in subsurface strata lying under the profile of the survey and above a depth approximately equal to the electrode separation are reflected by the readings. This survey detects only lateral variations in subsurface conditions. The readings may be presented graphically on ordinary (cartesian) graph paper as a resistivity-separation curve or entered on a map of the area and then contoured to form an equal-resistivity contour map (Soiltest, Inc., 1968, p. 7).

In electrical sounding, the center of the electrode spread is maintained at a fixed location while the electrode spacing is increased from one reading to the next. The spacing between adjacent electrodes is always equal and the spread is always symmetrical about the center position. Because the effective depth of investigation increases as the electrode spacing increases, the sounding field procedure is particularly suited for defining the sequence of rock layers, for choosing the optimum electrode spacing for profiling measurements, or for delineating rock or gravel zones within a depth range. A complete discussion of the presentation of sounding data may be found in the "Earth Resistivity Manual" published by Soiltest, Inc. (1968, pp. 31-42).

Alternating current measurements apply a low-frequency alternating current to the ground. It is difficult to distinguish theoretically between direct current and alternating current methods. This is because the current distribution at low frequency in the ground is essentially the same as that for direct current.

b. Inductive methods. Inductive methods depend upon currents induced into the ground by a primary alternating current and measurement of the resulting electromagnetic field at ground surface. The electromagnetic field around the primary current depends upon the currents that are induced in any subsurface conductors. These fields have a known form if the earth materials are homogeneous. By determining the actual phase and direction of the electromagnetic vector and comparing it with that which would be expected for a homogeneous material, it is possible to outline the nature of any changes in conductivity of subsurface material. Since these measurements are most often affected by horizontal changes in conductivity, they have been applied primarily to delineating faults.

Two different systems of induction are used. One of the systems uses a current loop or coil of relatively small dimensions so that the effect at a distance is that of an oscillating dipole at the coil. The other system employs an insulated cable laid out on the ground that measures the field around the cable by a series of short traverses perpendicular to the cable. Interpretation becomes difficult if more than two or three conducting layers are considered. Various empirical charts and schemes of interpretation are employed to facilitate the determination of changes in conductivity.

Induced polarization is based upon the fact that resistivity of earth materials is usually a function of the frequency of an applied current. This frequency-dependent resistivity is due to the development of a double layer of electrical charge at the surface of some materials. Clays show a pronounced double layer because substitutions within the crystal lattice lead to a net unbalance of electrical charge which must be balanced by an adsorbed layer of cations.

To use this system, current at selected frequencies is applied to two ground electrodes, and the resulting ground potentials are measured at two other electrodes. If the ratio of measured voltage to induced current is not the same when two different frequencies are applied, it is apparent that polarization is occurring and an induced polarization anomaly exists. Using the frequency domain technique, the induced polarization effect is determined by measuring the magnitude of ground impedance at two frequencies (e.g. 0.03 and 5 cycles). This is accomplished by applying a known current to the ground through two electrodes and measuring the potential drop across two other electrodes, the measurement being conducted at the two frequencies concurrently (Bodmer 1967, p. 15).

c. Transient method. The petroleum industry has developed a special electrical method called eltran. The transient method depends either upon the introduction into the ground of a sharp current pulse (this can be produced by suddenly closing or opening an electric circuit connected to grounded electrodes) or upon inpressing an electric current of a certain wave form on the ground. Measurements are made either of the form of the current or the form of the resulting potential. The electrode configuration consists of four electrodes in line, with two potential electrodes outside the two current electrodes (fig. 8.03). Interpretation of results depends upon obtaining in some way the time constants (rate of build-up or decline of the potential). Variations in this rate are expected to correlate with changes in the electrical properties of the subsurface medium.

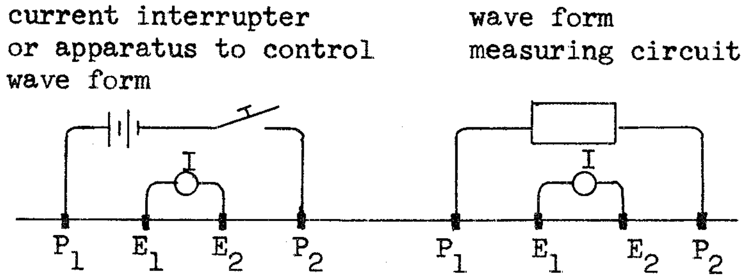


Fig. 8-03. Transient electrode configuration

Chapter 9

Subsurface Investigation of Ground Water

CHAPTER 9. SUBSURFACE INVESTIGATION OF GROUND WATER

Section 9.01. Borehole Logging

A detailed and comprehensive study of ground water and its occurrence can only be obtained from subsurface investigations. Subsurface examinations provide quantitative data concerning the location, composition, thickness, permeability, and storage capacity of aquifers and the depth, movement, and quality of ground water. However, subsurface investigations are conducted solely by personnel at ground surface who operate equipment extending into the subsurface. Test drilling provides data on subsurface strata in a vertical line from the land surface. Logging techniques within a well can provide information on properties of the formation, water quality, size of well bore, and rate of ground-water movement. Proper evaluation of these factors assist in the location, construction, and development of wells.

Kinds of Logs

- a. Driller's log
- b. Drilling time log (rate of penetration)
- c. Geologic log (identification of rock units)
- d. Geophysical logs
  - (1) SP (self-potential)
  - (2) Resistivity
    - (a) Normal (most important for ground-water applications)
    - (b) Lateral
    - (c) Wall Resistivity and Microcaliper
  - (3) Elastic wave propagation (sonic)
  - (4) Temperature
- e. Radioactive logs
  - (1) Gamma ray
  - (2) Gamma-Gamma

(3) Neutron

(a) Neutron-thermal neutron

(b) Neutron-fast neutron

(c) Neutron-gamma

Uses of Logs

a. Locate discontinuities in vertical profiles.

b. Correlate lithology from point to point.

c. Qualitatively interpret lithology.

d. Quantitatively interpret formation parameters and water quality.

All logs are useful for the first two purposes listed, and in combination most logs are useful for the latter two purposes. All developed logging techniques apply in consolidated rock but not all are adapted for use in unconsolidated rock; hence, the current limited usage in ground-water work. Most logs can only be run in uncased fluid-filled holes. The exceptions are the radioactive logs.

Section 9.02. Mechanical Methods

Driller's Logs

Economic considerations usually prevent employing well-trained geologists to log water wells. Therefore, it is common practice for the well-driller to construct the log. Despite lack of formal geologic training and the necessity of performing other duties during drilling, many drillers are able to make reasonably accurate logs. The skillful driller records the size of particles, harness, color, lithologic changes and thicknesses, and depth of water.

Drilling Time Logs

A drilling time log records the time (minutes and seconds) required to drill each foot of the test hole. This technique is most practical with hydraulic rotary drilling; however, it is used with other drilling methods. Because the texture of a lithologic unit being penetrated principally governs the drilling rate, such logs can be interpreted in terms of lithologic durabilities, boundaries, and depths (Todd, 1959, p. 237).

Geologic Logs

Logs, or samples of rock encountered in drilling, enable significant water-yielding units to be delineated. The drilling method greatly affects the accuracy and quality of the sample log. For example, cable tool drilling gives more accurate samples than hydraulic rotary drilling because a bailer is used to clean out the hole. In hydraulic rotary drilling where the rock materials are circulated out of the hole, the exact character of the lithologic units is difficult to determine, especially where the rock units are thin. However, rotary drilling is faster than cable tool and economically more feasible when other sampling or recording methods are available.

Section 9.03. Geophysical Logs

Spontaneous Potential Logs

The spontaneous, or self-potential (SP) log measures the direct current in a borehole due to different solutions which are in contact with each other. A record is made of naturally occurring potential differences between a ground surface electrode and an electrode in the column of drilling mud while this latter electrode is pulled up the borehole. The electrodes consist of a relatively stable material (e.g., lead) and any constant potential difference between the surface electrode and the one in the borehole may be balanced out by an adjustable voltage from a potentiometer circuit. Since the surface electrode is stationary, its potential is constant. Therefore, the SP log records the variations in the potential of the borehole electrode.

On the SP log, the potential differences are counted positively from left to right. It is possible to recognize on the SP log a rather well-defined base line, corresponding to shale zones, and located at the right side of the SP "track" on the recorded film. Deflection from this base line to the left (negative SP) indicates that the fluid in the formation is more saline than the fluid in the well bore.

Location of bed boundaries is always given by the inflection point of the SP curve. In correlation use, it is noted that SP curves reflect the vertical changes in lithology. Logged values of SP are measures of the salinity difference between drilling mud and formation fluid. Large negative values are recorded opposite porous permeable formations.

The SP curve generally provides the best logging approach to determine water quality, and the relationships between SP and water activity, resistivi and concentration are well established for oil field brines. However, when these oil field relationships are applied to fresh water sands, the results can be confusing. The usual relationships fail because the dissolved salts in fresh waters are no longer dominated by sodium chloride (NaCl) as they are in oil field waters. Divalent cations in dilute formation waters solutions have a much stronger effect on the SP than does sodium (Na) cation. For example, when the concentrations of calcium (Ca) and magnesium (Mg) are significant, they affect the SP as though the water were more saline than indicated by its resistivity (Alger, 1968, p. 3).

The relationships between SP and water activity, resistivity and concentration may be expressed as:

$$SP = -K \log \frac{R_{mf}}{R_w} \tag{9-1}$$

or,

$$SP = -K \log \frac{a_w}{a_{mf}} \qquad (9\text{-}2)$$

where:

R = resistivity

a = ion activity

mf = subscript indicating drilling mud

w = subscript indicating formation water, and

K = a function of formation temperature.

In oil field work, R_w is calculated using measured values of SP and R_{mf} in equation 9-1. Because of the differences in ionic composition of fresh water, equation 9-2 must be used for fresh water sands with the appropriate activity information as determined for regional or local conditions. Often, an equivalent water resistivity, R_{we} is determined from equation 9-1. Local information is used to convert R_{we} to R_w.

Resistivity Logs

The various resistivity logs are designed to measure the effect of an artificial electric current, which is transmitted to earth materials through electrodes in a sonde, at certain distances from the center line of the borehole as depicted in the fig. 9.01.

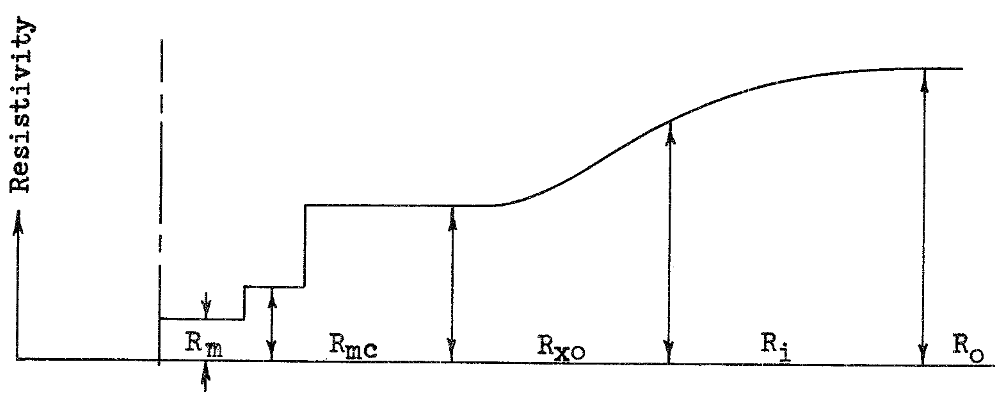


Fig. 9.01. Sketch of resistivities

In fig. 9.01, the resistivities shown refer to

R_m = drilling mud

R_{mc} = mud cake

R_{mf} = mud filtrate

R_{xo} = flushed zone = FR_{mf}

R_z = mixed resistivity

R_i = invaded zone = FR_z

R_o = clean water sand = FR_w

R_w = formation water resistivity, and

F = formation resistivity factor.

a. Formation resistivity factor. Assume that a clean lithologic unit is completely saturated with ground water of resistivity R_w, and that the resistivity of the lithologic unit is R_o. The ratio R_o/R_w is a constant referred to as the "formation resistivity factor," F, and is defined by the equation:

$$F = R_o/R_w > 1 \tag{9-3}$$

In water wells, the short normal electrode configuration usually penetrates far enough to record R_o.

The formation factor depends on the fabric of the lithologic unit. If the pore spaces inside the material consisted of parallel cylindrical channels, R_o would vary inversely with porosity, n. However, an electric current which penetrates a rock unit must flow along multiple meandering paths because of pore geometry, grain orientation, and the presence of cementing agents. The cross sections of these passageways vary rapidly from relatively large values in pores to very small values in the intervals connecting the pores to one another. In oil field work, F is related empirically to porosity by the Archie formula (Pirson, 1963, p. 23):

$$F = \frac{n}{m} \tag{9-4}$$

where:

m = cementation factor

n = porosity.

In ground-water work, F is not uniquely related to porosity but it is a function of n, R_w, and grain size. Values of R_w for water quality studies can be obtained from resistivity logs with sufficient experience in an area to permit estimates of F. Resistivity logs can be used to evaluate relative productivity of water-yielding sands, the best zones being indicated by the highest resistivities.

b. Microlog and microcaliper logs. A microlog is a resistivity log obtained with electrodes mounted a short distance from each other on an insulated rubber pad which is placed against a wall of the well bore. This system measures the average resistivity of a small volume of earth which is located directly in front of the pad and which is electrically shielded from the short-circuiting action of the mud.

The microlog has been developed primarily to detect permeable units in areas where the SP log alone does not give satisfactory results (e.g., well consolidated formations). When the lithologic units are relatively more resistive than the mud (e.g., limestones), the SP log generally detects the presence of permeable units, but does not define their boundaries. Because of its short penetration, the microlog can detect these boundaries. The microlog can be used with a high degree of success in moderately consolidated units such as sand-shale series for a detailed representation of units. The microlog (wall resistivity) can also be used to measure the formation factors of porous formations penetrated by the drill by measuring the flushed zone resistivity (R_{xo}). Charts are available in Pirson (1963, pp. 123-124) showing how micrologs are interpreted in terms of R_{xo}, F, and n.

The microcaliper is usually run simultaneously with the microlog. Borehole instruments include two pads of same shape, which are applied against opposite sides of the wall by means of a spring system. A recording of the distance in inches between the outer faces of the pads constitutes the microcaliper log and is located on the left-hand track of the film. The same track includes the SP curve and a vertical line which gives the bit size.

The microcaliper permits a precise interpretation of the microlog because it gives a detailed record of mud cakes and cavings. (Note: In an uncaved hole, the mud-cake thickness is approximately equal to one-half the difference between the recorded diameter of the borehole and bit size.)

Sonic Log

Sonic logs record the travel time in microseconds per foot for a sound wave to move through a definite length of earth material. These travel times are recorded continuously with depth as the probe is pulled up the borehole and are inversely proportional to the speed of sound in the various formations.

Speed of sound in rock depends upon its elastic properties, fluid content, and fluid pressure. Below the weathered zone, the average range of sonic velocities is from about 6,000 feet per second for shales to 26,500 feet per second for dolomites. This range is far less than the extremes of resistivity which are often encountered.

The sonic log is basically a porosity tool, since higher acoustic velocities occur in more dense materials. In consolidated rock, the engineering formula:

$$\frac{1}{V_f} = \frac{n}{V_l} + \frac{1 - n}{V_m} \qquad (9\text{-}5)$$

may be applied, where n is porosity and V is velocity. The subscripts f, l, and m refer to formation, liquid, and matrix, respectively.

Temperature Logs

It is a well-known fact that temperature in the earth increases with depth. The rate of temperature increase with depth under conditions of thermal equilibrium is known as the geothermal gradient. The geothermal gradient depends on the location and heat conductivity of the lithologic units. Because of the dependence of geothermal gradients on heat conductivity, measurements of temperature gradients at various well depths in which thermal equilibrium has been established have been used for correlation purposes. Temperature gradients are in general low in lithologic units having high heat conductivities and high in those having low heat conductivities. For example, temperature gradients are generally lower in sands than shales because the thermal conductivity of sands is greater than that of shales.

Temperatures encountered in drill holes depend not only on the geothermal gradient, but also on the circulation of the mud. If the mud in the well bore is thoroughly mixed, its temperature tends to become uniform. When the drill pipe is removed and the well is allowed to stand, the mud in the borehole at each depth gradually approaches the temperature of the material around it. The mud temperature at each level changes at a rate which depends on the heat conductivity of the surrounding materials and on the quantity of mud, which is controlled by the hole diameter. Almost all termperature surveys are made during the transition period, that is, before the thermal equilibrium has been achieved.

At present, temperature measurements are used primarily for (Haun, 1958, p. 285):

a. Locating the height of cement behind casing, and possible zones of channeling.

b. Locating the depth of lost circulation.

c. Locating gas-producing horizons during actual production.

d. Correlation with electrical logs for depth control in well-casing perforation operations.

Any use of a temperature log presupposes that the temperature anomalies being measured are of a greater magnitude than 4 or 5 degrees Farenheit, these being the order of magnitude of formation thermal differences.

Section 9.04. Radioactive Logs

Radioactive logs are of two general types: the gamma ray log and the neutron log. With Schlumberger's method, the type of radiation which is detected in neutron logging is also gamma radiation.

Gamma Ray Logs

The gamma ray log measures the natural radioactivity of earth materials. Gamma rays are high energy electromagnetic waves emitted by the nuclei of atoms. A nucleus will emit gamma rays either when disturbed by collision with a particle or when naturally unstable.

Gamma rays are comparable to radio waves except that the wave length of gamma rays is considerably shorter. When an atom is disturbed by some external force, it oscillates, gives off excess energy as gamma radiation, and returns to its former stable state. Gamma rays are not discrete particles in the normal sense but may be thought of as such. A burst of gamma radiation is sometimes called a photon. Gamma ray energies are expressed in units of "million electron-volts" (Mev); most energies lie in the range from 0.1 to 10 Mev.

When penetrating matter, gamma rays do not pass a definite distance and then stop suddenly. They are absorbed gradually, each additional unit length of material reducing its intensity by a given percentage. The absorbing capacity of materials for gamma rays is expressed as the half-value thickness (HVT), which is the thickness of material needed to reduce an incident gamma ray beam to half its intensity. Two half-value thicknesses will reduce the beam to 1/4 of its original strength, three will reduce it to 1/8, etc. Absorption in heavy materials such as lead is large, but in less dense materials (e.g., limestone or water), gamma rays can pass several inches.

The natural radiation of earth materials is very low in the case of homogeneous, clay-free sediments (pure limestone and dolomite, clean sands and sandstones). Clays and shales have the highest natural radioactivity. The gamma ray log is thus a lithologic tool and low gamma records may indicate permeable formations. In combination with electric logs, gamma ray logs permit useful qualitative interpretation of aquifer potentials.

Gamma-Gamma Logs

The gamma-gamma log is a density or porosity tool. Gamma rays are emitted from a source spaced apart from a detector of gamma rays. The amount of detected backscatter is inversely related to density or directly related to porosity. The porosity may be estimated from:

$$n = \frac{D_{grain} - D_{bulk}}{D_{grain} - D_{fluid}} \tag{9-6}$$

where D is density. The recorded response of the source-detector probe is calibrated against known standards to allow bulk density measurement. Fluid and grain densities may be estimated from other information. Interpretation is affected by casing, mud cake, and cavities, and caliper logs are helpful. The probe may be mechanically forced against the well bore to obtain better penetration.

Neutron Logs

The neutron logs measure radiation reflected from or induced in the media resulting from bombardment with neutrons emitted from a source. They combine a source of fast neutrons and a detector of thermal neutrons, fast neutrons, or gamma rays.

Neutrons are electrically neutral particles, each having a mass almost the same as the mass of the hydrogen atom. During logging, neutrons are emitted at high velocities (10,000 kilometers per second) from a special source. They collide with other nuclei as they propagate from the source, and each collision results in a loss of momentum, so that they finally achieve "thermal" velocities of the surrounding medium (i.e., they approach 2 kilometers per second). At these thermal velocities, neutrons are readily absorbed by most materials. In logging, these neutrons are absorbed by atoms of H, Si, Na, Cl, etc., and in turn, high energy gamma rays are emitted.

The distance neutrons travel before being absorbed depends upon the character of the atoms they encounter. Neutrons collide and bounce off atoms in the medium, losing energy that is comparable to the gradual slowing down of a fast-moving billiard ball that collides with other balls. In this type of collision, the moving ball loses more of its energy when it hits a ball of similar mass than when it hits a larger ball. Hydrogen atoms are commonly dominant, and, because they have masses similar to that of neutrons, they are the most effective in slowing down neutrons. Other atoms, such as Ca or silicon (Si), absorb very little energy from a neutron because their mass is much greater.

When the hydrogen (H) concentration of the medium surrounding a neutron source is large, neutrons are slowed down and absorbed near the source. If the H concentration of the medium is small, neutrons travel farther from the source before being absorbed. In most neutron logging tools, the distance between source and detector is one or two feet or more, and only those gamma rays emitted near the counter are detected, whereas those coming from near the source do not reach the counter. Thus, the counting rate decreases for increasing H concentrations of the environment and vice versa. The logs quantitatively indicate the presence of hydrogen and thus the amount of saturated rock.

Neutron-gamma logging consists of placing a source of fast neutrons and a gamma detector close together and lowering them down the borehole. Schlumberger method uses a Geiger-Mueller counter sensitive to the absorption of gamma rays. The neutron-gamma log gives a direct response to cholorine and may be useful in identifying sands containing NaCl.

Section 9.05. Composite Logs

In practice, the logs previously described are used in combinations to obtain specific information or to provide complimentary interpretation capability. The most popular logging combination for water well work includes spontaneous potential, resistivity (short normal), and natural gamma ray logs. An idealized sample combination log is shown in fig. 9.02 with the corresponding geologic section.

Interpretation

The gamma ray log is independent of formation fluids and drilling mud and serves to positively identify clay or shale. Reasonably pure sandstone and limestone both have very low gamma response. Since resistivity is a function of porosity, water quality, and grain size, a similar generalization for resistivity logs is not possible. Porous formations with fresh water and large grain size usually exhibit a high resistivity, and dense formations exhibit very high resistivity. Increases in porosity or salinity result in reduced resistivity. In the SP logs, dense formations exhibit low SP response, and an increase in porosity or salinity will increase the SP response.

_a_. Discussion of sample log (fig. 9.02). In a sand-shale sequence, the best aquifer potential will be found in the intervals where the SP and resistivity curves both show high response or are farthest apart. Low responses or close curves indicate shale. The layers numbered 1 and 2 exemplify these responses. A shaley sand has a lower porosity and smaller grain size than clean sand. Layer 3 shows a lower SP and resistivity response than layer 1. The slight gamma response confirms the suspected presence of some shale. Layers 2, 4, and 8 typify the logging response previously described for clay or shale, i.e., high gamma with low potential and resistivity responses.

The dense limestone in layer 5 exhibits the typical responses of low gamma and SP with very high resistivity. The thin shale at 6 is detected

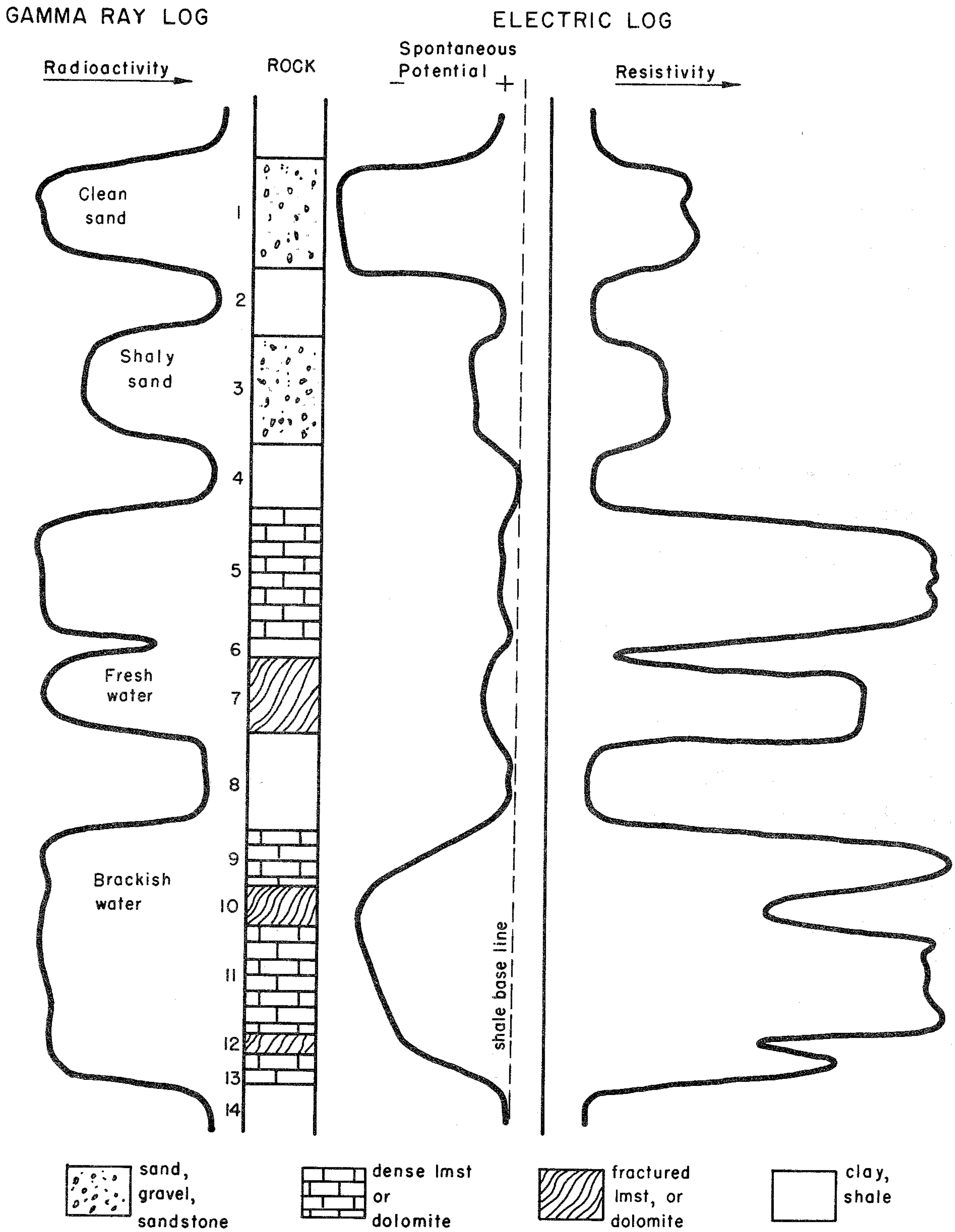


Fig. 9.02. Idealized Gamma ray and electric logs

by the moderate gamma response and the decrease in resistivity. The freshwater fractured limestone in layer 7 is detected by the high resistivity response and the slight SP response. The sequence of dense and fractured limestone in layers 9-13 contain brackish water as indicated by the high SP response. The dense layers have very high resistivity. Resistivity of layers 10 and 12 is lower than that of layer 7 due to the more saline formation water.

b. Extension of logging results. Proper use of borehole logging in a given area requires initial correlation of geologic samples and formation water samples with the measured geophysical responses. Subsequently, the geophysical measurements can yield quantitative information on water quality and water yield as well as qualitative information on changes in the lithologic thicknesses and gradual variations in formation characteristics from point to point.

Section 9.06. Ground-Water Tracing

The study of the physics of ground-water flow is difficult because of the remoteness of the flow system, the nonhomogeneity of the medium, and the extremely low ground-water velocities. Ground-water tracing is recognized as a useful tool, both in engineering and research, for solving problems of ground-water movement. An ideal ground-water tracer should: be determinable quantitatively in low concentrations, be inexpensive and readily available, be either entirely absent or present in very low concentrations in normal ground water, not be adsorbed by the porous medium, not form a precipitate with materials in the test environment, and not be influenced by the test environment so that the precision of detection is impaired. Salts, dyes, and radioisotopes have been successfully used as ground-water tracers.

Specifically Designed Tracer Experiments

The amount of time available for the performance of a ground-water tracing experiment is generally limited. In addition, the problems of tracer dilution, slow rates of ground-water flow, and protracted contact of the tracer with the aquifer materials combine to make tracers generally unsatisfactory for large-scale (regional) experiments. Exceptions to this are found in the use of environmental isotopes or bomb tritium on a regional scale. Tracers are useful for solving problems in limited areas within a ground-water basin if the tracer experiments are designed to give specific results.

Before any field work employing tracing techniques is undertaken, one or more of the following predictions, depending on the desired results, must be made: (1) the path which the tracer will take, (2) the dilution to which the tracer will be subjected, and (3) the amount of loss and retardation of the tracer by reaction with the constituents of the porous medium. A map of ground-water equipotential lines obtained from observation well soundings in the experimental area is the best method available to predict the flow path of the tracer. An estimate of the volume of water with which the tracer can mix and of the minimum detectable concentration of the tracer indicates the amount of the tracer to be injected for an experiment. Laboratory tests of tracer movement through samples of the porous medium through which the tracer will travel in the field should give accurate values of tracer recovery and retardation. After these preliminary predictions are made, the tracer may be utilized in field work.

a. Direction of flow. This type of experiment may confirm an existing notion or provide new information of the direction of ground-water flow between an injection well and a monitoring well or between wells and surface waters.

b. Source location. Information concerning the process of aquifer recharge, and the interconnection of aquifers can be obtained by tracer experiments. Bomb tritium has been used extensively for this work. Since large quantities of water are involved in recharge work, the artificial injections must be very large to produce a detectable tracer concentration. It is possible to determine approximately how much water each layer of a multilayer aquifer contributes to a well by injecting a tracer into the various layers at different times at a given distance from the well.

c. Average flow velocity through the pores. The volumetric flow rate through a porous medium is expressed by:

$$Q = n_e A V_r \tag{9-7}$$

where:

Q = the flow rate

n_e = effective porosity

A = the gross area of the flow cross section

V_r = the average pore velocity

Since a one-dimensional flow system is generally assumed in field experiments, a value for V_r results from dividing the distance between the points of injection and sampling by the travel time of the tracer.

If the injection well and the sampling well are on the same streamline, and little diffusion or dispersion takes place between the wells, a typical curve of tracer concentration versus time should be obtained at the sampling well. The only certain deduction that can be made is that the maximum pore velocity is represented by the first appearance of the tracer. Since pores vary in size, the most that can be concluded for flow in porous media that has negligible dispersion and diffusion effects is that the average pore velocity is generally less than, or at most equal to, one-half the maximum velocity corresponding to the initial appearance of the tracer. The distance between the injection and sampling wells divided by the time corresponding to the peak concentration gives an estimate of the average pore velocity. If a sizeable portion of ground-water flow takes place in the unsaturated zone, significant errors may occur in the calculations for the average pore velocity.

When there is no interaction between the tracer and the porous medium, the degree of frontal dispersion or dilution along the axis of flow may be considered as a function of several individual phenomena. These include microscopic variations in the permeability that result in channeling causing rapid travel; microscopic variations in the permeability which cause velocity differences from pore to pore and thereby increase the length of the front; nonuniform velocity distribution within a single pore space causing longitudinal dispersion and lengthening of the front; molecular diffusion in the direction of the flow, especially when the actual linear flow rate is small; and presence of nontransmitting interstitial cavities connected to water-transmitting pores by small openings that permit the interception and retention of a portion of the tracer by diffusion. In addition to longitudinal dispersion, lateral dispersion of tracer concentration exceeding that attributable to molecular diffusion alone also occurs. Lateral dispersion is the result of water continually dividing and reuniting as it flows around the individual or aggregated grains of the porous medium. The combined effects of longitudinal and lateral dispersion produce a cone of tracer material opening outward in the direction of flow.

After all of the above factors have been considered, an estimate of the average pore velocity of the tracer can be made provided the sampling well is located in the path of the tracer. If a value is assumed for the porosity, and the hydraulic gradient between the injection sampling wells is measured, a value for the hydraulic conductivity can be obtained from:

$$K = \frac{n_e V_r}{dh/dl} \tag{9-8}$$

where dh/dl is the hydraulic gradient and the other terms are as previously defined.

_d_. Bulk flow velocity through the porous medium. Three field methods for obtaining the bulk flow velocity, which is the discharge per unit area through a porous medium, are the two-well method, the dilution method, and the single-pulse method. In the two-well method discussed in the previous section, once the average pore velocity is calculated and a value is assumed for the porosity, the bulk velocity can be found by the relation:

$$\frac{Q}{A} = v = n_e V_r \tag{9-9}$$

where:

v = the discharge per unit area

n_e = the effective porosity

V_r = the average pore velocity

In the dilution method, it is assumed that potential flow governed by Darcy's law exists in the porous medium. A tracer is injected into a well, and the diminution with time of the tracer concentration due to the natural flow of water across the well is observed. If the tracer is injected at zero velocity and the flow is assumed linear, the bulk flow velocity can be obtained from:

$$\log_{10}\frac{C_o}{C} = \frac{1.106}{d}\ vt \tag{9-10}$$

where:

C_o = the concentration of the tracer at the time of the first measurement (considered as $t = 0$)

C = the concentration of the radioisotope at the time of sampling

t = the time the sample was taken

v = the discharge per unit area

d = the diameter of the well

After the apparent velocity is obtained, the hydraulic conductivity can be calculated from:

$$K = \frac{v}{dh/dl} \tag{9-11}$$

where the terms are as previously defined.

The value for the apparent velocity as determined by the single-pulse method is based on the two equations:

$$r = \frac{v}{n_e} \tau \tag{9-12}$$

and

$$V = Qt = n_e b \pi r^2 \tag{9-13}$$

where:

r = the distance the tracer traveled

v = the discharge per unit area

τ = the time interval between injection and start of pumping

V = the volume of water pumped

Q = the discharge of the well

t = the time since the start of pumping

n_e = the effective porosity

b = the aquifer thickness

Increasingly higher concentrations of a tracer are injected into the aquifer through a well, are left to move with the natural flow for increasingly longer time periods, and are then recovered by pumping the same well. The peaks of the curves of activity versus volume pumped define the various times since the start of pumping. If a sufficient number of experiments with varying τ are made and if n_e and b are known, v can be determined from the linear relationship:

$$\tau = \frac{1}{v} \sqrt{\frac{n_e Qt}{b \pi}} \tag{9-14}$$

A relatively large value of the bulk velocity in a homogeneous porous medium is needed to keep the pulse as a slug in the single-pulse method.

The influence of tracer adsorption on the results of these three field methods is reduced by injecting relatively large concentrations. Diffusion may become significant in all three methods if the bulk velocity is extremely small. The method of dilution requires a tracer which can be detected with continuous field monitoring equipment, except when the bulk velocity is small. When this occurs, a sampling technique is feasible wherever large boreholes are sampled or small quantities are withdrawn from small boreholes.

e. Effective porosity of the medium. The effective porosity of an aquifer can be determined by combining the two-well method with a pumping test. The sampling well is pumped until equilibrium conditions are obtained. At that time, the tracer is injected into an observation well located some distance from the pumped well. After two transformations are made to obtain a symmetrical output pulse, a curve of the output concentration as a function of distance between the wells versus time is obtained.

If cylindrical symmetry is assumed, the effective porosity can be calculated from:

$$n_e = \frac{V}{\pi r^2 b} \qquad (9\text{-}15)$$

where:

n_e = the effective porosity

V = the volume of water pumped

r = the distance between wells

b = the aquifer thickness

If the aquifer thickness is unknown, a value of bn_e can be obtained.

Conclusions

Oftentimes differences between tracer recovery rates and flow rates in porous media have no satisfactory explanation. Perhaps they are caused by an extreme dependence of the tracer on some gradient other than the concentration and/or hydraulic gradient, or on some chemical and/or mineralogical composition of the porous media.

Each tracer has its limitations and should not be used without full appreciation of the conditions imposed by both the carrying liquid and the medium through which it must pass. Radioisotopes are just another type of tracer whose usefulness depends on the ability of the scientist and engineer to properly design the experiments. In addition to problems encountered with other types of tracers, problems, such as health hazards, arise when radioisotopes are utilized in experiments.

Chapter 10

Occurrence of Ground Water in Igneous and Metamorphic Rocks

CHAPTER 10. OCCURRENCE OF GROUND WATER IN IGNEOUS AND METAMORPHIC ROCKS

Section 10.01. Introduction

The occurrence of ground water in usable quantities in many igneous and metamorphic rocks is governed by heterogeneously distributed zones of secondary porosity and permeability. Furthermore, zones of high primary permeability like lava tubes and buried stream deposits in many volcanic terranes are also heterogeneously distributed. These problems have made the acquisition of usable quantities of ground water in these rocks difficult. Vegetation and mantle material often conceal bedrock and thus help to complicate geological exploration.

Unfortunately, time, funds, and personnel are limitations which restrict the scope of many extensive hydrogeologic investigations in crystalline rock terranes. However, useful results can be obtained from limited hydrogeologic reconnaissance type programs.

Section 10.02. Physical and Hydraulic Properties

Primary void spaces developed in igneous rocks during crystallization comprise small angular (miarolitic) cavities, small intercrystal spaces, vesicles caused by escaping pneumatolytic gases, and cavities produced in lava flows by the motion of lava while it solidifies. The latter two types of openings may be large enough and so abundant as to give the rock a high porosity. However, many of these openings are largely isolated and are imperfectly interconnected. Fluid lavas can produce numerous caverns and tunnels during solidification (e.g., Lava Beds National Monument, California). A surface crust may form after which the more fluid lava beneath drains away leaving cavernous openings. Irregular cavities are developed when this crust is inundated or broken up by a new lava flow. Many of these primary openings are interconnected by numerous large tension joints.

Secondary openings in igneous rocks form by hydration of feldspars and other minerals in weathered rock, minor solution, jointing, and faulting. Hydrated minerals develop loose aggregates which have porosities nearly three times that of fresh fragments of unfractured igneous rock (Davis and DeWiest, 1966, p. 320). Unweathered rhyolites, welded tuffs, and certain basalts may develop relatively low secondary porosities (1 to 10 percent) because of tension joints and other fractures. The porosities of unaltered pyroclastics can be attributed to fragment size, sorting, and the degree of cementation (Davis and De Wiest, 1966, p. 337).

Solid fragments of fresh, unweathered metamorphic rock have porosities from only 1 to 3 percent and very often less (Davis and DeWiest, 1966, p. 332). Secondary void space in schists develop through jointing, faulting, and weathering. Other fractures or fissility openings parallel to the schistosity and near the surface can be expected to contain small amounts of ground water. Because of the small size and lack of continuity of these openings, they would yield ground water very slowly to wells (Meinzer, 1923, p. 113).

The specific yields of igneous and metamorphic rocks usually decrease with depth and time and are generally less than 20 percent everywhere (Stewart, 1962, p. 107). This decrease can be attributed to the combined effect of the weight of overlying bedrock, the short penetration of surface disturbances, and the filling of pores by secondary mineralization.

The hydraulic conductivities of intrusive igneous rocks are usually low because of the lack of interconnected void spaces. However, because of the uniform orientation of openings developed along highly jointed and fractured zones, these rocks as a whole are anisotropic with respect to hydraulic properties (see Appendix 5). In contrast, the conductivities of extrusive igneous rocks, especially basaltic rocks, cover a wide range of values from almost zero to more than 25 feet per day (Davis and DeWiest, 1966, p. 337) (see Appendix 5).

Many metamorphic rocks contain carbonate minerals which are readily dissolved by percolating ground water. Even though numerous solution cavities may occur in rocks such as marble, the void space of large volumes of rock is usually less than 5 percent (Davis and De Wiest, 1966, p. 322). This is because solution is localized along fractures and other secondary openings.

Ultramafic rocks and serpentinites have a loose, porous cover of soil and rock fragments. The soils are characteristically thin, giving way in a meter or less to disaggregated rock. At modest depths (order of 200 meters) the rocks become quite impermeable except along open fractures (Barnes and O'Neil, 1969, p. 1956).

Aquifer test data from wells in metasedimentary rocks of northern Michigan indicate that conductivities parallel with the strike of the strata are two to three times the average conductivity (Davis and DeWiest, 1966, p. 319).

Section 10.03. Well Yields

In general, yields of wells are low in many types of intrusive igneous and metamorphic rocks. Data from groups of wells in different regions show average yields between 10 and 25 gpm (Davis and DeWiest, 1966, p. 323). Differences in well yields usually reflect differences in topography, degree of weathering, and orientation of secondary openings. Higher yields may be obtained from wells drilled and developed on flat upland areas and along or in valleys and broad ravines developed by faulting (Davis and De Wiest, 1966, p. 328).

The yields of wells in andesitic and basaltic rocks are appreciably higher than the yields from intrusive igneous rocks. Data from groups of wells in central California which penetrate fractured, reworked tuffs and andesites show yields in excess of 500 gpm (Piper, et al., 1939, p. 210). The specific capacity of wells in some of the highly permeable basaltic rocks of the Columbia Plateau and Hawaii is limited only by the diameters of the boreholes (Davis and DeWiest, 1966, p. 342). Other data from the same regions show well yields from other basalts to be about 1 gpm for every foot of open well depth below the nonpumping water level (Newcomb, 1959, p. 14). In contrast, wells pumping from heterogeneously distributed deposits of rhyolitic ash and tuff and buried fine-grained alluvium yield only several gallons per minute (Meinzer, 1923, p. 141).

Many types of metamorphic rocks resemble crystalline igneous rocks with respect to their yield of ground water to wells. For example, data from groups of wells in Connecticut show that wells penetrating schists have an average yield of 14 gpm and wells pumping from gneiss have an average yield of 12 gpm (Meinzer, 1923, p. 147). Data from other regions show that the smallest yields come from phyllites and other soft foliated rock types in which fault and joint openings apparently close with depth (Davis and DeWiest, 1966, p. 326).

Section 10.04. Hydrochemistry

Igneous Rocks

The chemical character of ground water from intrusive igneous rocks is frequently acceptable for most domestic, agricultural, and industrial uses. Exceptions occur in arid regions where due to evapotranspiration certain mineral salts may become part of the available recharge and in places where brines have migrated into rock openings. Water of poor chemical quality also may be found in desert areas where evaporites or wind-blown materials are dissolved in downward percolating water (Davis and DeWiest, 1966, p. 331).

The ability of water to react with soil and rock rich in alkali silicates is a function of the hydrogen (H) ion concentration. The H ion concentration is regulated by the solution of carbon dioxide (CO_2) which is derived primarily from biogenic processes in the soil (Hem, 1970, p. 292). Potassium (K) and sodium (Na) ions are preferentially released from mineral lattices when the H ion concentration is high. Continued reactions between water and rock reduce the H ion concentration, and the system approaches equilibrium (further release of alkalies is reduced).

Weathering of granites and other silica-rich rocks is a hydrolysis reaction, which generally produces water of higher silica concentrations in relation to total ion content than for water associated with basalt or ultrabasic rock (Hem, 1970, p. 294). The initial mole ratios of silica to sodium (2:1) and of silica to bicarbonate (1:1) suggest the formation of montmorillonite and kaolinite or both during weathering of plagioclase feldspars (Feth, et al., 1964, p. 41). The hydrolysis reactions can be expressed as follows:

$$3 \text{ moles andesine} + 14\ H_2O + 4\ CO_2 = 2 \text{ moles kaolinite} + 2\ Na^{+} + Ca^{+2} + 4 \text{ moles silicic acid} + 4\ HCO_3^{-} \text{ and} \tag{10-1}$$

$$6 \text{ moles andesine} + 18\ H_2O + 6\ CO_2 = 3 \text{ moles montmorillonite} + 2\ Ca^{+2} + 2\ Na^{+} + 6 \text{ moles silicic acid} + 6\ HCO_3^{-}. \tag{10-2}$$

An appreciable amount of the silica released by weathering processes remains in a noncrystalline form or as residual clays. The K ion is partly immobilized by the combined processes of adsorption by clay minerals and utilization by plants. Aluminum (Al) and iron (Fe) are immobilized at the sites of weathering by forming hydrated oxides that are insoluble at the prevailing conditions of pH and Eh. Calcium (Ca) and magnesium (Mg) dissolve and are carried off in solution. When the cations are released, the Al-Si-O frameworks of the original silicate minerals are in part decomposed, in part reconstituted into the frameworks of clay minerals. Thus, only part of the silicon and very little Al go into solution (Krauskopf, 1967, p. 104).

In the Sierra Nevada and in other granitic terranes of varying mineralogy but comparable continental climate (e.g., Idaho batholith), the ground water is likely to be a mixed-cation bicarbonate type having a total dissolved solids (TDS) ranging from 25 to 200 parts per million (ppm), but averaging between 75 and 150 ppm (Feth, et al., 1964, p. 66). Where climatic conditions are different, the mineral content is likely to be two to five times larger and the percentages of sodium, sulfate (SO_4), and chloride (Cl) tend to be high. Under these conditions, part of the SO_4 and Cl available from precipitation and weathering is removed from solution. Kaolinitic clay and hydrated oxides of iron in soil have the capacity to adsorb these anions where the pH is less than 7.0 (Bear, 1955). The adsorption process will dampen the utility of chloride as an effective water tracer in granitic terranes containing acidic kaolinitic soils.

Spring waters from granitic rocks in the Mojave Desert, California, contain high ionic percentages of Cl and SO_4 (Feth, et al., 1964, p. 56). These anions concentrate in areas where ground water leaves the flow system by evaporation or via streams and by the solution of dust derived from playas and other deposits of mineral salts. Alteration of rock by subsurface water along fault zones and other secondary openings in the granites may also increase the concentrations of Cl and SO_4.

Well waters from granitic terrane in North Carolina indicate that mineralogical differences in the different aquifers sampled may account for the rather high concentration of dissolved constituents (Feth, et al., 1964, p. 39). In addition, precipitation in this area is rather evenly distributed except for greater precipitation during seasonal tropical and winter storms. This usually leads to deeper and more complete weathering of rock and the development of deep soils under heavy plant cover. The reaction of soil minerals with water under the existing climatic conditions could account for the increased concentration of dissolved constituents observed in well waters.

Magnesium bicarbonate waters constitute most of the surface water and shallow ground water discharged as springs and seeps from fresh ultramafic rocks in five California localities (Barnes and O'Neil, 1969, p. 1950). These waters result primarily from the hydrolysis of Mg rich silicates by meteoric water. Chemical analyses of water samples are strikingly similar in that the waters are low in Fe, manganese (Mn) (below detection limit of 0.05 mg/1), K, and Ca. The pH of these waters ranges between 7.8 and 8.9 and accounts for the high ratio of bicarbonate to carbonate.

Ground water from extrusive igneous rocks contains the same major chemical constituents as intrusive rocks because extrusive rocks are chemical equivalents of intrusive rocks. However, the proportion of certain ions and the total dissolved solids of waters in extrusive rocks differ from that of intrusive rocks. These differences may be attributed to the origin and evolution of the two rock types, mineral stability, and rate of weathering. Extrusive igneous rocks generally yield a single or mixed cation bicarbonate water which is comparable to the type of water associated with granitic rocks. For example, ground waters from 25 perennial springs in extrusive rocks of the Sierra Nevada and southern Cascade Mountains contain nearly 1.5 times the dissolved solids content of spring water in granitic rock and are a single or mixed cation bicarbonate type (Feth, et al., 1964, p. 64). In contrast, volcanic spring waters are reported to have higher concentrations of HCO_3, SiO_2, and Mg, and lower concentrations of SO_4 and Cl. The greater concentration of Mg in volcanic spring water is attributed to the larger proportion of Mg rich silicates composing volcanic rocks. The higher concentration of dissolved constituents in volcanic spring water results from the presence of soluble volcanic glass in the less completely crystallized lava and tuffs. In some instances, the average concentration of strontium (Sr) in volcanic spring water is greater than that found in water from granitic rocks. However, Feth, et al., (1964, p. 63) state that variations in Sr content should not be used to differentiate water from the two rock types because the absolute and percentage concentrations of Sr overlap.

The chemical character of ground water from volcanic rocks is nearly always acceptable for most domestic, agricultural, and industrial uses. Exceptions occur in areas of active thermal activity of volcanic origin where volcanic emanations contain high concentrations of boron (B), Na, sulfur (S), K, Cl and lithium (Li). These chemical constituents along with high temperature and low pH make this water unacceptable.

Metamorphic Rocks

Since rocks of any kind may be metamorphosed, all degrees of alteration of the original rocks occur, from minor chemical or physical change to a complete change and reassembly of minerals. In some instances, rocks of any kind may undergo several episodes of metamorphism of varying intensity. However, a few reliable generalizations regarding the chemical character of ground water from metamorphic rocks are possible. Gneiss and schist have silicate minerals similar to igneous rocks and produce solutions similar to those associated with igneous rock. Subsurface waters associated with dense, unweathered slates and quartzites are likely to have: (1) high ratios of K/Na, (2) low content of dissolved solids and low pH, and (3) less than 30 ppm silica (White, et al., 1963, p. 8). Limestone may be transformed to marble without much chemical change and the processes of solution are comparable to those in limestone. However, some marbles contain other minerals which are less soluble than calcite.

A study by Barnes and O'Neil (1969, p. 1950) indicates that magnesium bicarbonate waters are associated with incompletely serpentinized ultramafic rocks in five California localities. A less abundant type of water in the same areas is a calcium hydroxide water which results from hydrolysis of dunities and perdotites by meteoric water. These waters discharge as springs and seeps which are structurally controlled along fractures and faults.

Chemical analyses of the calcium hydroxide type waters representative of the five areas are low in Mg, Fe (below detection limit of 0.05 mg/l), Mn, chromium (Cr), and SiO_2. The reported high concentrations of Al, which are unusual for dilute solutions, may be due to the alteration of ortho-pyroxenes (e.g., enstatite). Some, if not most, of the Ca in solution is derived from the serpentinization of enstatite (Barnes and O'Neil, 1969, p. 1952).

Ground waters from metamorphosed shales, siltstones, and tuffs are comparable to waters from their unmetamorphosed equivalents. In general, the ratio of Ca:Na is more than unity and the dissolved solids are low in these rocks. Compositional variations can be attributed to compaction and decrease of porosity of the rocks before metamorphism, loss of interstitial water of high salt content, and reconstitution of clay minerals with high cation exchange capacity to micas and other anhydrous minerals of lower exchange capacity (White, et al., 1963, p. 8).

Section 10.05. Exploration

Existing data are inconclusive as to the extent to which geological and geophysical methods can improve on "trial and error" methods of locating ground water in igneous and metamorphic rocks. The fact that a few wells in many different regions produce more than 50 gpm indicates there is ground water which is recoverable and that geologic and geophysical techniques might be applied to improve materially the chances of locating heterogeneously distributed zones of high permeability.

Where igneous and metamorphic rock exposures are abundant, detailed geologic mapping of the extent of fracturing and the location of faults, dikes, and lithologic contacts will be extremely useful in delineating the most favorable water-yielding zones. For example, ultramafic rocks in five California localities represent topographic highs (Barnes and O'Neil, 1969, p. 1949). All yield ground water as seeps and springs at elevations above the surrounding terrane. The springs and seeps are structurally controlled along fractures and faults which are crudely linear, ranging from a few meters to a kilometer in length (Barnes and O'Neil,1969, p. 1951).

In regions of thick mantle material or vegetation cover, more indirect techniques of finding ground water must be used. For example, detailed topographic and geomorphic studies are effective in delineating thick weathered zones. Aerial photographs are commonly used to find slight differences in the tone of rock, soil, or vegetation, in drainage texture, and in the alignment of ridges. These differences may indicate the presence and particular rock types, intrusions, and faults and other lineations.

The most recent indirect techniques of hydrogeologic exploration which have great promise for the future are those in the realm of remote sensing (Sabins, 1967, p. 743).

Geophysical methods are often useful and successful in determining subsurface geologic features. Of all the geophysical techniques, seismic and electrical resistivity methods have been used most extensively in areas of igneous and metamorphic rocks. The prime application has been to delineate the thickness and distribution of weathered rock and other unconsolidated deposits, and to locate the position of lava tubes, caverns and other cavities.

Hydrogeologic studies of volcanic rock terranes depart slightly from the more conventional investigations primarily for two reasons: (1) surface drainage by streams may be nearly absent because of the high infiltration capacity of permeable rocks and soil (e.g., Oahu, Hawaii and Thousand Springs, Idaho), and (2) ground-water movement is strongly influenced by the position of ancient valleys, buried soils and ash beds, stratigraphic sequence of lava beds, and other structural controls related to volcanism.

Geophysical techniques generally have limited application in volcanic rock terranes. The reason for this is the lack of significant differences in magnetic susceptibilities, average densities, elasticity constants, and electrical resistivities. A very important exception has been in the delineation of the fresh water saline water interface in some of the extrusive rock aquifers of Hawaii (Davis and DeWiest, 1966, p. 341).

Section 10.06. Development of Ground Water

In igneous rocks most of the faults and joints are nearly vertical except for narrow fractures more or less parallel to the rock surface associated with sheeting and exfoliation (Longwell, Flint, and Sanders, 1969, p. 149). Since the spacing between these vertical joints may be from 0.5 to 10 feet, vertical wells tend to intersect only one or two joint planes. Construction of horizontal "wells" may be more successful in these areas because a greater number of fractures can be penetrated (Davis and DeWiest, 1966, p. 330).

The most favorable water-yielding zones in metamorphosed carbonates occur in fractures caused by local faulting and in other openings enlarged by solution. Next in importance are extensive fracture zones associated with major faults. Metamorphic rocks which have been deformed appreciably develop open fractures near physical discontinuities such as along resistant, brittle, quartz veins which penetrate relatively soft foliated rocks.

Aquifer tests in intrusive igneous and metamorphic rocks give rise to many different shapes of drawdown curves which can be attributed to fracture location, fracture width and spacing, and quantity of ground water in storage. Because of the low storage capacity of these rocks, it is more desirable to locate wells in weathered zones or in saturated deposits of alluvium or colluvium. Wells may also be placed near perennial streams, lakes or springs to decrease the rate of ground-water depletion.

In volcanic rock terranes, unaltered and poorly sorted unconsolidated sediments are interbedded with relatively thick accumulations of volcanic material. Under favorable conditions these porous sediments of low permeability leak water into the more permeable lava beds which have greater transmission rates.

Tunnels have been successfully used in Hawaii to recover ground water which is perched on buried soils and ash beds or is impounded behind igneous intrusions (Meinzer, 1942, p. 690). Shorter tunnels are used in other places to secure smaller amounts of ground water without drawing down water levels to induce salt water intrusion. Wells and tunnels aligned perpendicular to the strike of dipping lava beds have high yields because these tunnels penetrate several high permeability zones between successive lava flows.

Chapter 11

Occurrence of Ground Water in Sedimentary Rocks

CHAPTER 11. OCCURRENCE OF GROUND WATER IN SEDIMENTARY ROCKS

Section 11.01. Introduction

Sedimentary rocks cover more than two-thirds of the lithosphere and under favorable conditions are a major source of ground water. Scores of varieties of sedimentary rocks have been described, but more than 99 percent of the total volume consists of three: shale (including siltstone), sandstone (including graywacke), and limestone (including dolomite). Shale, claystone, siltstone, and other fine-grained clastic rocks account for nearly 50 percent of all sedimentary rocks (Gilluly, et al., 1968, p. 368). Next in abundance are sandstones (26 percent), then carbonate rocks (22 percent), and finally, several minor types (less than 2 percent) such as gypsum, conglomerate, halite, and chert (Gilluly, et al., 1968, p. 368).

Section 11.02. Physical and Hydraulic Properties

Sedimentary Interstices

Primary sedimentary interstices consist mainly of void spaces between adjacent fragments of sedimentary rock. Since most of these fragments have been reworked by wind, water, or other erosive agents, they are more or less rounded. Thus, when they were deposited, void spaces remained between them forming important reservoirs for ground water. Theoretically, material composed of uniform grains of small size is as porous as one of uniform grains or pebbles of large size (Meinzer, 1923, p. 110). Also, perfect spheres can be packed with porosities ranging from 25.95 to 47.64 percent (Graton and Fraser, 1935). However, highly angular grains tend to be held apart by irregular sharp corners (Tickell and Hiatt, 1938). In addition to angularity of grains, the overall shape of grains has a large influence on packing. Tabular grains may form box-like openings, particularly in fine-grained sediments. Plate-shaped grains, on the other hand, may be packed with porosities ranging from near zero to nearly

100 percent (Graton and Fraser, 1935). The interstices of a sedimentary rock can eventually become cemented with mineral matter precipitated from percolating ground water, and the rock becomes nearly nonporous.

The size of primary openings range from microscopic pores to openings several inches wide. Because about half of the sedimentary rocks are fine-grained, the majority of these openings are usually not more than a small fraction of an inch. Although the openings of this type are small, they are quite abundant, evenly distributed, and interconnected. The many small pore openings in fine-grained clastic rocks provide storage space for appreciable amounts of water. Slow drainage of this water into an aquifer can be induced by decreasing the hydraulic head in the aquifer. As the water moves into the aquifer, the volume of pore spaces in the aquitard is reduced by compression. Capillary and electrochemical forces prevent all water from draining out of the smallest pore openings in fine-grained sedimentary rocks.

The most important secondary openings with respect to ground water are fracture and solution openings. Where there has been extensive deformation, sedimentary rocks may have been displaced along nearly vertical fractures, producing faults and brecciated zones that are likely to store and yield significant amounts of ground water. Solution openings are produced primarily by subsurface water that penetrates preexisting openings. Openings caused by the dissolution of soluble rocks are of two kinds: (1) those formed by the removal of a soluble cement from a nearly insoluble rock, and (2) those found in soluble rocks, such as limestone, gypsum, and halite.

Fine-Grained Clastic Rocks

Most fine-grained clastic rocks have relatively high porosities, but low conductivities (see Appendix 5). Siliceous shale, some claystones, and argillites (e.g., Brule clay) may develop closely spaced joints near the surface and fracture zones at considerable depth due to deformation. In general, fine-grained rocks act as surfaces across which there is little or no ground-water flow (confining beds) or serve as leaky-type aquitards in multiaquifer systems.

Porosity of fine-grained materials usually decreases with depth and to some extent with geologic time. Recently deposited muds will have porosities between 60 and 85 percent (Davis and DeWiest, 1966, p. 349). However, compaction of fine-grained material considerably reduces the porosity.

Sandstones

Porosity of most sandstones ranges from near 5 percent to about 30 percent (Davis and DeWiest, 1966, p. 348). Pore space in a sandstone is determined by rock fabric, jointing, and degree of cementation. Of these factors, cementation is the most important. For example, silica cement in sandstone will tend to form overgrowths on the sand grains and these overgrowths will interlock with nearby sand grains having overgrowths. Advanced stages of silica cementation lead to orthoquartzites which have little or no porosity. In contrast, clay-cemented sandstones are not as firm and tend to be more porous because of the clay.

Laboratory and field determinations of hydraulic conductivities of sandstones are one to three orders of magnitude lower than hydraulic conductivities of corresponding unconsolidated sediments (Davis and DeWiest, 1966, p. 351. See Appendix 5). Most of the reduction in conductivities is due to reduction in void space by cementation and closer packing of mineral grains.

The majority of sedimentary rocks are, to a certain degree, stratified which produces some anisotropy with the vertical direction usually having lower conductivity than the horizontal direction. Conductivity data from Illinois oil sands (Piersol, et al., 1940) indicate the median ratio of horizontal to vertical conductivity was 1.5, and nearly 12 percent of the sands had a ratio of more than 3.0. Less than 6 percent had vertical conductivities greater than horizontal conductivities.

Carbonate Rocks

Limestone and dolomite can form both by inorganic and biochemical processes. These rocks also originate from different sedimentary deposits, such as talus deposits, calcite sand, and shell debris. As soon as these sediments are buried, reduction of primary porosity may occur, plus partial alteration of some original sedimentary structures.

Hydraulic conductivities of many recent (less than 3 million years) carbonate rocks are low except for limestone breccias and coquina which have primary pores not filled with cement (see Appendix 5). A dense, clay-rich, crystalline limestone may have conductivity of less than one millidarcy while the conductivity of a partly-cemented limestone breccia may exceed several thousand darcys (Davis and DeWiest, 1966, p. 353).

Fracture and secondary solution openings along bedding planes and primary pores provide major avenues of ground-water flow. Additional pore space may be created by calcite transforming to dolomite and dissolution of fossil fragments. Usually,the conductivity of dolomite is low compared to limestone because the openings between crystals are so small (Davis and DeWiest, 1966, p. 353). Carbonate rocks with extensive solution channels or fractures developed primarily in one direction are anisotrophic with respect to conductivity.

Evaporites

Gypsum, anhydrite, and halite are generally found as an impurity in shale and limestone, but in places they form relatively thick units (nearly 100 feet). These minerals are softer than limestone and are readily soluble in water. Solution openings in evaporites permit adequate drainage and the development of karst features. However, many of these solution passages become sealed with depth.

Eolian deposits of evaporites may be grouped as (1) granular evaporite sand which is initially very porous, but eventually becomes more compact through recrystallization, and (2) impure evaporite dust, which is comparable to loess and which becomes compact having clay-like properties.

Strata of halite and other saline deposits are quite soluble and thus contain appreciable quantities of water high in dissolved salts.

Section 11.03. Well Yields

Wells developed in moderately indurated sedimentary rocks usually have yields between 5 and 500 gpm. Fine-grained clastic rocks usually have yields less than 10 gpm, sandstones have yields between 10 and 250 gpm, and carbonates may have very high yields (above 2000 gpm), but usually have between 10 and 50 gpm (Davis and DeWiest, 1966, p. 355; Sonderegger and Kelley, 1970, p. 95).

Frequency distributions of specific capacities of wells in many moderately indurated sedimentary rocks are negatively skewed (Davis and DeWiest, 1966, p. 6). The skewed relationship is predominant in carbonates and fractured siliceous quartzites. Walton (1962, p. 39) plotted values of specific capacity per foot of penetration against the percentage of wells on logarithmic probability paper to determine the water yielding properties for three different hydrostratigraphic units in northern Illinois. If specific capacities per foot of penetration decrease as the depth of wells and number of units penetrated increase, the upper units are considered to be more productive than the lower units (Walton, 1970, p. 321).

Primary well yields in sedimentary rocks can be improved by: (1) acid treatment of soluble rock to enlarge openings, (2) fracturing the aquifer by fluids pumped into the well under high pressure, (3) chlorine to remove iron and bacteria, and caustic soda treatment to remove oil scum left by oil-lubricating pumps (Walton, 1970, p. 323), and (4) combination of the above techniques.

Section 11.04. Hydrochemistry

Consolidated rocks such as sandstone have voids between the mineral grains partly or completely filled with cementing material which is a major source of dissolved mineral matter for that rock. The common cementing materials are calcium carbonate, silica, ferrous carbonate, sulfates, iron oxides or hydroxides, and slightly soluble clay minerals. To recognize a type of ground water that might be characteristic of sandstone is difficult because a wide range of concentration and chemical character occurs.

It is rare for ground water to contain much silica unless the rock has a considerable proportion of silicate minerals unaltered by weathering. For example, ground water from clastic material derived primarily from a silicic igneous rock in an arid region shows a high mole ratio of silica to alkali and alkaline-earth metals (Hem, 1970, p. 294). The same ground water also may yield a 1:1 mole ratio of silica and bicarbonate. This water suggests the hydrolysis of alkali feldspars as the source of silica. Other principal products are cations and clay minerals (e.g., kaolinite and montmorillonite) as residual solids. The relatively high concentration of sodium and bicarbonate is comparable to that found in ground water associated with extrusive igneous rocks high in silica.

Analyses of ground water from the Dakota Formation in Kansas and North Dakota show definite differences in water chemistry. Ground water from the Dakota Sandstone in southeastern North Dakota has high concentrations of chloride and sulfate. This may be due to negatively charged clay minerals in aquicludes above and below the aquifer that restrict the passage of these anions, and in turn these anions hold certain cations in order to maintain electrochemical neutrality (Bredehoeft, et al., 1963). In eastern Kansas ground water from the Dakota sandstone is characterized by high concentration of calcium, magnesium, and bicarbonate. This may suggest mixing of different water types or solution of cementing material by subsurface water, or variations in flow path length and direction (Williams, 1968, p. 7).

Shales and similar rocks are very porous, but have low conductivity because the voids are extremely small. In such rocks, soluble components are likely to be retained as adsorbed ions on clay minerals or in interstitial saline water that was not completely removed by flushing because of low conductivity. An outstanding characteristic of some shales is the scarcity of water with dissolved mineral contents of less than 1000 ppm (White, et al., 1963, p. 6). Less mineralized waters have relatively low ratios of calcium to sodium, bicarbonate to chloride, and iron chloride, but the ratio of magnesium to calcium is relatively high.

Analyses of ground water from most shale formations are similar in that they are high in sodium in proportion to other cations. Studies by Foster (1950) suggest the high sodium content may be due to cation exchange of calcium with sodium from clay minerals. Water from the Chattanooga Shale has a low dissolved-solids content because of abundant supply of water from rainfall. The rather high proportion of silica in this water is possibly due to the solution of unaltered silicate minerals in the shale. Most shales also contain higher concentrations of iron, aluminum, and fluoride and are slightly acidic (pH 5.0 to 6.5) (White, et al., 1963, p. 7).

Precipitate sediments result from biochemical and chemical reactions and consist mainly of a few comparatively pure compounds. Limestone often contains some impurities, of which magnesium carbonate is usually dominant. The presence of some magnesium in a limestone water is to be expected. Ground waters associated with pure dolomitic rocks contain nearly equivalent amounts of calcium and magnesium (Hem, 1970, p. 304). Water from carbonate rocks contains bicarbonate in a much greater proportion than other anions because it is through the process of bicarbonate formation that carbonate rocks become soluble. The presence of other anions is largely due to impurities and the influence of other rock types. Evaporites are deposited from highly concentrated solutions and are readily soluble. It is common for two or more salts to crystallize simultaneously. In addition, the presence of many double salts and possible hydrates, which crystallize or recrystallize due to minor changes of temperature and composition, complicate the precipitation process.

Salt deposits in arid regions are for the most part an assemblage of chlorides, sulfates, and carbonates of sodium and calcium with smaller amounts of potassium and magnesium compounds. For example, sodium sulfate may be the principal salt in a small basin among hills composed mainly of granodiorite containing pyrite, whereas magnesium sulfate may be abundant in a similar basin inclosed by pyritic greenstones (Krauskopf, 1967, p. 322). Other salt deposits in western Nevada and Central Oregon have a high proportion of bicarbonate. When evaporation has progressed long enough so that most of the sodium chloride and calcium carbonate have crystallized, the remaining concentrated solution often contains highly soluble salts of magnesium, potassium and bromine (e.g., Dead Sea). Lithium and boron compounds occur in certain desert basins of southern California.

Oil field waters (i.e., waters associated with oil and gas pools) probably contain some interstitial water of an age that is similar to that of the inclosing rocks ("fossil" connate water). In some instances, oil field waters contain slightly more than 1 percent of the dissolved solids of ocean water and they are probably of meteoric origin because much of the original water was flushed out (White, 1957, p. 1667). Other oil field waters have 5 to 10 times the salinity of ocean water and are characterized by high concentrations of calcium chloride and sodium chloride (White, 1957, p. 1663). These high density brines are abundant and occur commonly in pre-Tertiary rocks. They are not usually associated with salt beds or unconformities and show marked similarities in chemical composition over rather large areas. The origin of these waters is not yet well understood and therefore open to debate. Even though the mechanism is not clear, it is likely that oil field waters evolve into types that are more dilute or mixed with meteoric water, and other types that are more concentrated than sea water, without contact with salt beds or dilution with meteoric water (White, 1957, p. 1670).

Section 11.05. Exploration

Many marine sedimentary rock sequences have continuous beds of limestone and dark shale which serve as marker beds. The importance of these beds to hydrogeologic studies lies in the fact that they are used to predict the position of less consistent rock units which are potential aquifers or aquicludes. Fence diagrams constructed from surface geologic maps and well logs are used to delineate irregular sandstone bodies (e.g., stream channel deposits and offshore bars) which would have been nearly impossible to predict from surface data.

Seismic refraction techniques have been used to trace indurated sedimentary units long distances and to determine the depths to these various units. In addition, it is possible to locate folds, faults, and other structural discontinuities within sedimentary sequences. Except for borehole logging techniques, little geophysical work is conducted to locate ground water in sedimentary rocks, largely because of economics. If the wells are to be drilled to great depths, the cost of a geophysical survey could be greater than the cost of a single water well. Many ground-water investigations try to make use of both geophysical and stratigraphic data compiled by the petroleum industry. But, this kind of data is not readily available.

Sandstone Aquifers

The guiding principles used to locate ground water in sandstones are similar to the practices applied to locate ground water in crystalline rocks. Well-cemented sandstones may yield appreciable quantities of water to wells along fault zones, within strongly jointed zones, and along other secondary openings. More productive wells should be located in broad valleys and on flat uplands. The construction and development of deep wells in some areas should be avoided because of the presence of saline or brine-type ground waters.

Well-cemented sandstones which appear compact and impermeable at or close to the surface due to precipitation of dissolved salts can grade into unconsolidated sand or poorly indurated sandstone aquifers at depths of less than 20 feet (Davis and DeWiest, 1966, p. 361). Therefore, some sandstones which appear compact and impermeable at the surface may be moderately permeable at depth.

Carbonate Aquifers

Geologic investigations show that the localization of solution openings in dense carbonate rocks can be attributed to minor jointing, faulting, folding, differences of composition, small units of chert or shale, and primary permeability (Davis and DeWiest, 1966, p. 361). Surface geologic indications that carbonate rocks may transmit water at depth are the presence of solution cavities, closely spaced joints, or the presence of faulting in the area of interest. The presence of numerous shale partings, chert layers, and the absence of solution openings suggests the rock units are less permeable.

Observations in quarries and other excavations in carbonate rocks show that openings along bedding planes tend to remain open and are important with respect to ground-water production. Dissolution of certain foraminifera and other types of organic matter which are composed of argonite, calcite, or magnesite may provide other openings and avenues for ground-water development. In some instances, vertical joints which become widened by solution near the surface tend to be filled by clay materials derived from weathering and soil development. Since bedding plane openings are developed near surface traces of vertical joints and faults, these secondary structures should be used to delineate favorable drilling sites for productive water wells (Lattman and Parizek, 1964, p. 78; Sonderegger, 1970, p. 25).

In regions of relatively thick carbonate rock sequences, wells located in valley bottoms and on flat uplands generally have higher production than those drilled on valley slopes. Valley bottoms usually act as sites of natural ground-water discharge. Fracture traces and solution openings are primarily concentrated along anticlinal flexures and synclinal troughs which serve as favorable sites for ground-water development (LaMoreaux and Powell, 1960).

Young limestones on coral atolls, which consist of coarse detrital organic material, generally possess favorable hydraulic properties. This coarse, blocky rubble eventually becomes cemented but tends to retain appreciable conductivity. Ground-water development is restricted in these areas because of the small thickness of the fresh ground-water body. Extensive pumping of this ground-water body induces direct intrusion of sea water.

Chapter 12

Occurrence of Ground Water in Unconsolidated Earth Materials

CHAPTER 12. OCCURRENCE OF GROUND WATER IN UNCONSOLIDATED EARTH MATERIALS

Section 12.01. Introduction

Most hydrogeologic investigations indicate that unconsolidated deposits are the most important sources of ground water. The reasons for their importance are summarized from Davis and DeWiest (1966, p. 374) as:

(1) The hydraulic conductivities of these deposits are appreciably greater than other earth materials except for some recent volcanic rocks or cavernous carbonates.

(2) Unconsolidated sediments have a greater volume of pore space than consolidated rocks.

(3) The deposits are relatively easy to drill so that exploration and development may be rapid and relatively inexpensive.

(4) Many deposits are found in valleys where ground-water levels are close to the surface.

(5) The sediments are commonly recharged during peak surface flows, by flooding, by through-bank flow, and artifically by irrigation and water spreading.

Unconsolidated deposits may be subdivided genetically into a large number of categories, such as alluvium, loess, till, dune sand, marine sand, colluvium, and lacustrine clay. From the standpoint of ground-water production the most important are alluvial, glacial, eolian, and marine deposits.

Section 12.02. Alluvial Deposits in Small River Valleys

Physical and Hydraulic Properties

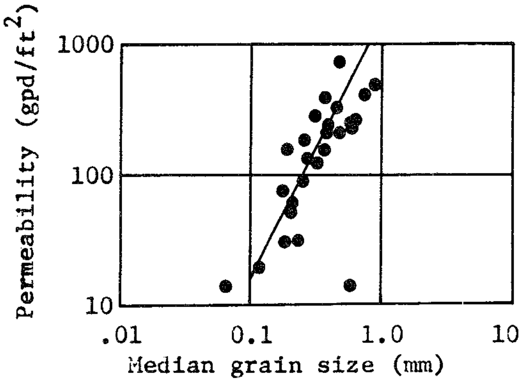
In stream valleys where the channel occupies a restricted part of the valley, the deposits may be classivied according to their position in relation to the channel and topographic form. Productive aquifers are sand and gravel deposits which are deposited directly within the channel.

Special channel deposits (e.g., point bars) which form during peak surface flows at river bends are also very permeable. As a stream meanders, point bars with intervening marshy areas remain. These depressions fill slowly with fine-grained materials during flooding. This process results in a series of arcuate strips of fine-grained sediments overlying a relatively thicker layer of coarse-grained material. Continued flooding will cause a buildup of fine sediment that grades from coarse silt at the natural levee to very fine silt and clay in the backswamp areas. This buildup and interfingering of fine and coarse-grained material result in heterogeneously distributed zones of low and high permeability (Davis and DeWiest, 1966, p. 381).

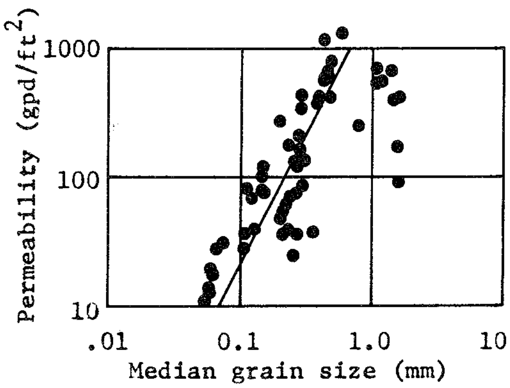
Many stream valleys do not show the general patterns and sequences of flood plain deposits described above. Streams which carry large bed loads derived from melting glaciers develop broad flood plains laced with braided channels. As another example, changes of stream regimen produced by climatic variations can increase downcutting of rivers leaving former flood plains as alluvial terrace deposits.

The vertical succession of most alluvial channel deposits is coarse to fine. The relative thickness of representative units depends on the geologic history of the river and the type of sediments carried by the river at the point of interest. Deposits typical of most Quaternary streams are 20 to 150 feet thick with 5 to 25 feet of permeable sands and gravels near their bases (Davis and DeWiest, 1966, p. 383).

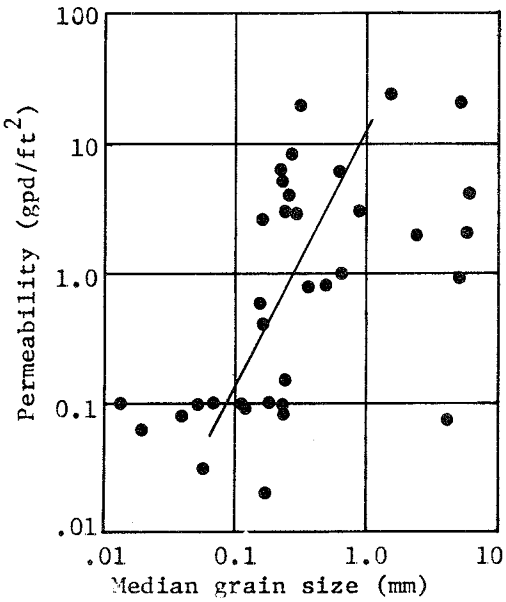
Many investigations of the hydraulic properties of alluvium have been conducted (see Appendix 5). Silts and loosely compacted clays have specific yields of less than 10 percent and conductivities of only several millidarcys (Todd, 1959, p. 53). Higher hydraulic conductivities are attributed to open structures of the original grain aggregates and to secondary structures such as root holes, desiccation cracks, and animal trails. Coarse, clean sands and gravels have specific yields greater than 25 percent and hydraulic conductivities that exceed 500 darcys (Todd 1959, p. 53). Bedinger (1961, p. 31) was able to correlate median grain size with hydraulic conductivity within single river valleys (figs. 12.01 and 12.02).



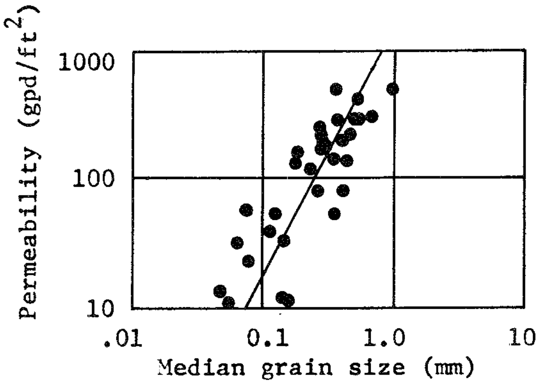
ARKANSAS RIVER VALLEY
LITTLE ROCK TO MISS. RIVER,
ARKANSAS



ARKANSAS RIVER VALLEY
LITTLE ROCK TO FORT SMITH
ARKANSAS



VERDIGRIS RIVER VALLEY
OKLAHOMA



ARKANSAS RIVER VALLEY
FORT SMITH TO MUSKOGEE,
OKLAHOMA

Fig. 12.01. Relation between median grain size and permeability of samples from the Arkansas and Verdigris River Valleys, Arkansas and Oklahoma (Bedinger, 1961, p. 31)

EXPLANATION

0.1

Contour showing average permeability in gpd/ft^2.

(11)

Numbers in parentheses indicate class numbers.

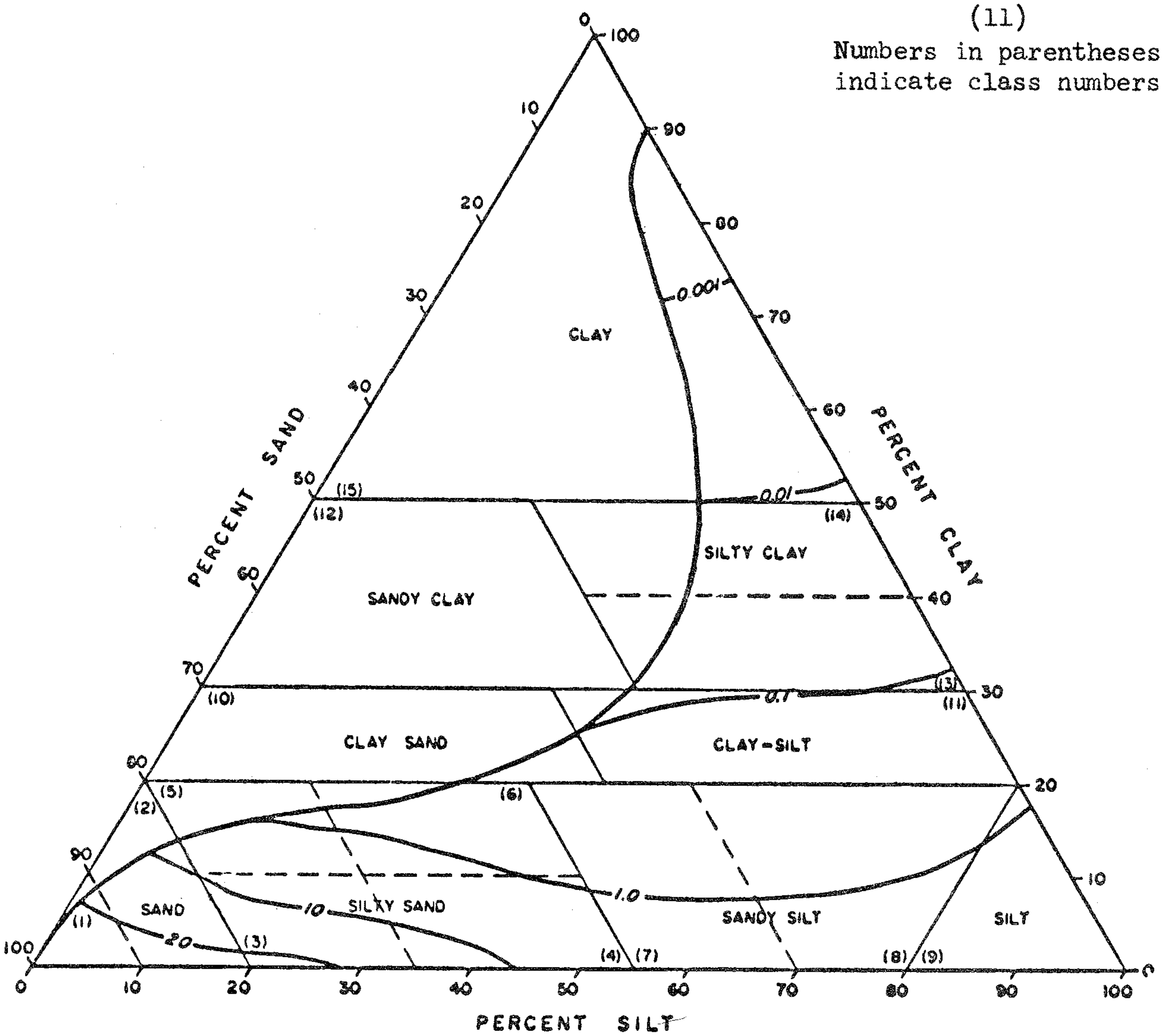


Fig. 12.02. Relation between permeability and texture of samples from the Arkansas River Valley, Arkansas and Oklahoma (Bedinger, 1961, p. 32.)

Well Yields

Wells pumping from alluvial deposits which are derived from perennial streams normally yield 10 to 50 gpm (Davis and DeWiest, 1966, p. 383). Yields up to several thousand gpm are obtained from highly permeable sand and gravel zones which underlie the axial part of many stream channels (Cohen, et al., 1965, p. 32). Logan (1964, p. 36) derived an expression which can be used to estimate well yields before drilling. However, the thickness of aquifers, the position of water levels in nearby wells, and the position of hydrologic boundaries should be known. Reasonable estimates of transmissivity can be made from graphs showing the relation between median grain size and hydraulic conductivity (figs. 12.01 and 12.02).

Hydrochemistry

Ground water in small alluvial valleys is primarily derived from local recharge on the valley floor, through-bank flow, and from lateral inflow of streams and aquifers of tributary valleys. Culture, vegetation, climate, and lithology are the important influencing factors controlling water chemistry. For example, in the central United States, ground waters contain high concentrations of calcium, magnesium, and bicarbonate because of the decomposition of carbonate rocks. In contrast, alluvial deposits along many streams in the western states contain water high in chlorides, sulfates, and carbonates of Na and Ca, with smaller amounts of Mg and K compounds derived from evaporites and regional ground-water flow.

Where aquifers are hydraulically connected with streams as an integrated hydrologic system, the chemical character of ground water is often controlled by the constituents present in surface water. Surface water in some regions is nearly identical with native ground water (Cohen, et al., 1965, p. 87). A high content of dissolved solids can be present in such interrelated systems for any of the following reasons (White, et al., 1963, p. 8): (1) brines may be contributed from connate water or from evaporite beds in the basin, (2) return flow from irrigation may contribute soluble matter leached from cultivated lands, (3) in arid climates, evapotranspiration may concentrate soluble matter in the remaining water, (4) activities of man provide salts in industrial wastes and in other forms, and (5) rivers usually act as linear discharge sites for regional flow systems. In other regions the chemistry of surface water may be influenced by relative mixtures from several sources. Two major problems of river recharge are the organic constituents and temperature changes introduced into ground water by streams containing heated industrial wastes and other pollutants. Water wells which are developed near alluvium-bedrock contacts may obtain water which is high in dissolved solids. In most places the water was present in the sediments at the time of deposition and is slowly seeping into overlying younger sediments.

Another problem of many alluvial valleys is the scattered occurrence of ground waters high in iron and manganese. Manganese in such waters is probably the result of solution of manganese from soils and rocks aided by bacteria or complexing with organic materials. The iron may become mobilized by low pH and Eh (redox potential) conditions that occur near scattered accumulations of organic matter in alluvium along the stream (Hem, 1970, p. 298).

Exploration

Hydrogeologists attempt to find subsurface water bodies near sources of highest available recharge that have a maximum saturated thickness of highly permeable material. For instance, maps are made which delineate the courses of buried stream valleys so that wells can penetrate the maximum thickness of sand and gravel. The delineation of buried stream valleys is complicated because of numerous changes in drainage patterns, introduction and interfingering of nonalluvial material with alluvium, and the lack of geologic clues at the surface.

If the valley sides consist of rocks that are resistant to erosion, steep bluffs that are now relatively distant from the stream may indicate that a former stream channel reached the valley wall. However, many recent stream channels have migrated rather rapidly and it is often difficult to relate the topography of the valley side to the position of thick accumulations of sand and gravel. The presence of small tributary streams that transport large quantities of sediment into the main stream may assist in locating the deepest accumulations of alluvium. These tributary streams tend to force the channel of the main stream to the opposite side of the valley (Davis and DeWiest, 1966, p. 386).

Geophysical methods and borehole investigations are used to ascertain the thickness and types of alluvial deposits. Interpretation of drilling data in alluvial valleys may be complicated by poor logging techniques or incorrect lithologic identifications. Clay-rich till may be mistaken for shale and boulders might be misinterpreted as bedrock. In some instances resistivity may be less reliable than seismic methods because of slight differences in resistivity values between alluvium and bedrock or because of surface inhomogeneities and severe lateral resistivity variations (Zohdy, 1965, p. 44).

Section 12.03. Unconsolidated Deposits in Large Valleys

Large valleys are primarily attributed to tectonic movements rather than stream erosion. Many of these valleys are formed by regional downwarping of the earth's crust and are bounded by extensive fault systems. Topographically these valleys may be deep and narrow or very broad with low ridges and hills.

Physical and Hydraulic Properties

Most of the debris in structurally controlled valleys is derived from the surrounding mountains. The geologic and geomorphic history control the occurrence of other types of deposits. In some areas, lacustrine, pyroclastic, and glacial material may be important sources of ground water. Unconsolidated deposits commonly exceed 2000 feet and in some cases may be more than 5000 feet thick (Davis and DeWiest, 1966, p. 391).

The Great Valley of California is a typical example of an elongate, structurally controlled basin. The valley is bounded on nearly all sides by high mountains that have contributed debris into the valley continuously for possibly as long as 100 million years. Even though the geography has changed during this time, geologists have estimated that the present valley probably has accumulated locally more than 25,000 feet of material (Hackel, 1966, p. 219). Most of these sediments are of marine origin and are saturated with water high in dissolved solids. However, about 500 to 3000 feet of nonmarine material directly underlies the surface, and it is this zone that is most productive from a ground-water standpoint (Hackel, 1966, p. 220).

The fresh water aquifers in the Great Valley consist of recently buried channel deposits and older pyroclastic materials which have been reworked by streams. Although more than 90 percent of these sediments are deposited by streams, several lacustrine clays and eolian sands are present. The lacustrine clays form aquicludes and aquitards within the southern part of the Great Valley (Davis, 1959).

The hydraulic properties of sediments in structurally controlled basins are comparable to those of small stream valleys except that larger valleys generally have a higher proportion of fine-grained material (see Appendix 5). Fine-grained sediments buried at some depth tend to have low effective porosity because of compaction and cementation. Porosities of these fine-grained sediments range from about 40 percent to 60 percent and specific yields from 3 percent to 15 percent (Richter, 1968) The hydraulic conductivities of the clay and silt zones are commonly less than one darcy. Coarse-grained and well-sorted, clean sand and gravel aquifers have hydraulic conductivities of 10 to 100 darcys, and specific yields from 20 percent to 40 percent (Conkling, et al., 1934, and Piper, et al., 1939).

A number of investigators (Cohen, et al., 1965, p. 77) have used average specific yield values in an attempt to estimate the storage capacity of large ground-water basins. Driller's logs are examined and certain values are assigned to different size ranges based on experience, field tests, and laboratory analyses. The method employed by the California Department of Water Resources and the U. S. Geological Survey is to use unit areas and unit depth zones. An average specific yield for each depth zone per unit area is generally computed. After certain favorable depth zones are established, the storage capacity for that depth zone per unit area is obtained by averaging the specific yields computed from all wells in the depth zone. Then all unit areas may be added to obtain the total storage capacity. Computations of storage capacity of ground-water basins are now being made on computers.

Hydrochemistry

The chemical character of ground water in structually controlled basins is strongly influenced by the drainage history of the valley, the mineralogy of the parent material, and climatological factors (Davis and DeWiest, 1966, p. 400). As an example of drainage-history control, ground water with very high total dissolved solids may be present in deeper aquifers or near land surface in the central parts of large bolsons (closed basins). Exterior drainage may retard the accumulation of waters high in total dissolved solids and may also permit rapid circulation and removal of poor quality ground water. In other cases, the mineralogy of different rock types can be as important as the drainage history of the valley in determining the water chemistry. Sediments that are derived from sedimentary rocks tend to have waters with high concentrations of calcium, sodium, magnesium, bicarbonate, and sulfate. Such ground waters usually contain 50 to 200 ppm calcium, 10 to 50 ppm magnesium, 100 to 250 ppm bicarbonate, and 50 to 300 ppm sulfate (Davis and DeWiest, 1966, p. 400). The chloride concentrations and dissolved solids of these waters are governed by the weathering history of sediments and the fine grain-size particles which restrict ground-water flow but provide large total surface area. In contrast, the concentration of dissolved solids in ground waters associated with sediments derived from crystalline metamorphic and igneous rocks is usually much lower. In particular, chlorides and sulfates are usually less than 55 ppm each (Davis and DeWiest, 1966, p. 400). Silica-rich water is more abundant in deposits derived from crystalline basement and volcanic rocks than from sediments of sedimentary origin. Usually the total concentration of dissolved solids in ground water obtained from volcanic rocks will lie somewhere between the two previous types.

Exploration

Surface geologic methods are successful in delineating rather broad hydrogeologic patterns such as: (1) bedrock limits of valleys, (2) stratigraphic and structural relationships, (3) lithologic units, (4) areas of maximum potential recharge, (5) tributary drainage basins, and (6) geologic features associated with faulting. The delineation of aquifers in structurally controlled valleys is complicated by the fact that the high permeability units usually consist of ancient stream channel deposits which retain sinuous and interconnected drainage patterns.

Studies of the depositional patterns in some desert basins have shown that apex areas of alluvial fans consist of older material or are dominated by mudflows and other poorly sorted material from flash floods (Davis and DeWiest, 1966, p. 396). In general the average conductivity of these sediments is considerably lower than the sediments farther down the fan slope. The sediments of the central part of some fans may be reworked by streamflow and have higher conductivities despite finger grain sizes. The more distal parts of the fan interfinger with playa sediments which are primarily fine-grained. This interfingering results in localized zones of differing conductivity. Wells are sometimes more productive in the central part of fans than in the distal parts (Davis and DeWiest, 1966, p. 397.)

Geological logs and borehole geophysical techniques are used to delineate subsurface geologic conditions. Distinctive colors or lithologies are used as stratigraphic markers to trace the continuity of various geohydrologic units. Some of the best markers are ash beds, lava flows, buried soils, and lacustrine clays. The continuity and dip of nearby beds can be determined by using these marker beds. Geophysical logs may indicate a change from relatively fresh water near the surface to more saline water at depth.

Section 12.04. Eolian Deposits

Eolian deposits are considerably less abundant than either alluvial or glacial material. The extent of eolian material encountered in subsurface exploration is difficult to evaluate because distinct textural and structural features are not easily ascertained from sample logs. There are probably many situations where silts and sands assumed to be of alluvial or lacustrine origin are actually eolian.

There are two primary types of eolian deposits, dune sand and loess. Loess is a fine, well-sorted, nearly structureless or vertically jointed silt, usually buff color. It mantles slopes and crests, and covers uplands such as those bordering the Missouri and Mississippi Rivers. The coarsest loess near source areas approaches 0.07 milimeter. At some distance from the source the loess may be as small as 0.009 milimeter (Gibbs and Holland, 1960). The second principal eolian deposit, dune sand, is commonly cross-bedded and well sorted. The diameter of most sand grains lies between 0.05 and 0.5 milimeters (Brown and Newcomb, 1963).

Hydraulic Properties

Dune sand has nearly uniform hydraulic properties (i.e., is nearly homogeneous and isotropic). The porosity and specific yield of dune sand approach 40 percent and the hydraulic conductivity is usually between 5 and 50 darcys (Brown and Newcomb, 1963) (see Appendix 5). Weathered sand and foreign matter, such as gypsum and clay pellets, will tend to reduce the hydraulic conductivity of a dune deposit.

Dune sands are not often utilized as aquifers because of their thin saturated zones and their high conductivity which allows rapid gravity drainage of saturated units. Water wells that are constructed in dune deposits may yield considerable quantities of sand because of poor design and development techniques. Despite these drawbacks, dune areas are sometimes favorable for ground-water development because of high recharge rates and high conductivities of dune sand. For example, about 25 percent of the precipitation of the Sand Hills area of Nebraska reaches the water table (Lohman, 1953, p. 85). This area is composed of sand dunes that have been stabilized by a covering of grasses and other plants. The average rate of recharge through the dunes is estimated to be five times greater than the rest of the High Plains (Lohman, 1953, p. 81).

Loess is a porous material (40 to 55 percent porosity) because the individual silt grains tend to be held in open networks by the cementing action of clay minerals (Davis and DeWiest, 1966, p. 407). Moreover, laboratory analyses show that the specific yield of coarse-grained loess is twice that of fine-grained loess (Gibbs and Holland, 1960). The presence of vertical joints, root holes, and animal burrows gives loess an anistropic character with respect to conductivity. This anisotropy has not been fully investigated, but the vertical conductivity of loess should be greater than the horizontal (see Appendix 5). Loess is considered to be a poor aquifer because of its low horizontal conductivity and lack of abundant interconnected secondary openings.

In most cases, loess makes an excellent soil because of its high water retaining capacity and small grain size. If loess overlies buried soils or consolidated rocks of low conductivity, water wells which will yield satisfactory quantities of ground water may be developed in perched bodies of ground water in the loess.

Hydrochemistry

Dune deposits are composed of clean, well-sorted, stable, sand grains that alter the chemical character of the recharge water only slightly. For example, streams fed by ground-water discharge in

Sand Hills area, Nebraska, contain less than 300 ppm dissolved solids because the recharge water dissolves very little mineral matter as it percolates through the sand dunes (Lohman, 1953, p. 87). However, weathering and the presence of impurities in dune sand may modify considerably the chemical character of percolating water.

Ground water associated with loess deposits should have higher dissolved matter than dune sand because of chemical changes in the soil profile and the greater abundance of clay minerals having high ion exchange capacity. Abundant, freshly fractured minerals and large amounts of soluble calcium carbonate below the zone of weathering will also contribute additional dissolved matter to ground water.

Section 12.05. Glacial Deposits

From a hydrogeologic standpoint, glacial deposits consist primarily of poorly sorted material deposited directly by glaciers (till), and water sorted deposits. Water-sorted material can be divided into ice contact deposits and outwash (Flint, 1970, p. 147). The more common types of glacial deposits are summarized in table 12.01.

There are two principal types of till, lodgment till and ablation till. If the till is laid down at the base of a glacier which is slowly melting, it is called lodgment till. It may be more dense and fissile than ablation till which is deposited by a melting glacier. Till is compact, unsorted, and consists of fine-grained material and scattered boulders.

Ice contact deposits are characterized by a variety of textural types ranging from fine-grained material deposited in ponds and lakes to well-sorted coarse-grained material deposited by melt water streams. A large variety of topographic forms, which alter the boundaries of both surface and underground drainage basins, are produced by accumulations of ice contact deposits. Eskers are the most distinctive of the various landforms because they are characterized by long sinuous ridges composed of coarse sand and gravel. They represent the bed load deposits of former streams that occupied subglacial ice tunnels or streams on the ice surface. Kame terraces are formed in valleys by the accumulation of debris along the margins of stagnant glacial ice. When the ice melts the debris is left as a terrace along the valley walls. The part of the terrace that previously touched the ice is characterized by collapse and slump features, which form irregular borders facing the center of the valley. Small hills of ice contact material deposited when the ice melts are called kames. Kames also may be erosional remnants of larger masses of water sorted material. Kettle holes are formed by the collapse of till and ice contact debris as isolated masses of residual ice melt. They are found on kame terraces, till plains, terminal moraines, and wide parts of eskers.

Table 12.01. Glacial deposits. (Modified from Flint, 1970)

Deposits	Associated Topographic Form
Till	Till Plains
Ablation till Lodgment till	Terminal, lateral, and medial moraines. Drumlin
Water sorted	
Ice contact	Eskers, kames, kame terraces, and kettle holes
Outwash	Outwash terraces and plains

Hydraulic Properties

The unsorted character of till and the weight of the glacial ice combine to give till a low porosity and a low specific yield. The porosity of rather coarse-grained till approaches 20 percent (Meinzer, 1923). Clay rich tills may have porosities as high as 40 percent.

Hydraulic conductivities of 37 samples of clay rich tills deposited by continental glaciers ranged from 1.7×10^{-5} to 5×10^{-2} darcy with a median of 2.2×10^{-3} darcy (Norris, 1963) (see Appendix 5). Estimates of the hydraulic conductivities of till deposited by valley glaciers may be expected to be larger because of less clay and coarser-grained texture (Davis and DeWiest, 1966, p. 409).

Till covers a large part of the northern United States, yet relatively few dug wells obtain adequate water supplies for even domestic uses directly from till (Meinzer, 1923, p. 127). Drilled wells which recover adequate quantities of ground water probably penetrate perched zones and joints or small sand and gravel lenses within the till. These drilled wells generally yield from 10 gpm to as much as 100 gpm (Meinzer, 1923, p. 128). A number of wells located in mountain valleys probably recover ground water from till which is coarse-grained and less decomposed.

Water wells which penetrate ice contact deposits have yields that range from less than 1 gpm to more than 500 gpm (Meinzer, 1923, p. 117). Ground-water production is highest where the deposits are low enough topographically so that they are not subject to rapid gravity drainage. However, the limited areal extent and thickness of ice contact material restricts long-term ground-water production.

Some of the most productive wells in the United States are in outwash sand and gravel near northeastern and central Illinois (Walker and Walton, 1961, p. 19). Here specific capacities (adjusted for effects of partial penetration and well loss) from 25 gpm per foot drawdown to 80 gpm per foot drawdown have been measured. Near the cities of Tacoma and Spokane in the state of Washington, specific capacities of more than 1000 gpm per foot drawdown have been determined (Walters, 1963, p. 157). Well yields of 200 to 2000 gpm with specific capacities of from 10 to 100 gpm per foot drawdown, however, are more representative of most outwash deposits (Walker and Walton, 1961, p. 19).

The hydraulic properties of outwash are comparable to normal valley alluvium. Alluvial deposits may be distinguished from outwash deposits by the absence of anomalously large boulders, the presence of cross-bedding, and the abundance of relatively thick accumulations of fine-grained flood-plain material.

Hydrochemistry

In central North America a considerable part of the glacial material is derived from sedimentary rocks abundant in calcite, dolomite, and to a lesser extent certain evaporites. Ground water associated with such deposits may contain rather high concentrations of magnesium, calcium, bicarbonate and sulfate. Organic matter, which influences the concentrations of these ions, commonly occurs within glacial deposits. In areas of poor ground-water circulation, as below a thick till sheet, the water contains large quantities of dissolved matter. This is, in part, due to fine-grained particles which restrict ground-water movement and mineral particles whose total surface area is great. The seepage of saline ground water (high sodium:calcium ratio and magnesium) from underlying marine rocks into glacial material frequently occurs (Maclay and Winter, 1967, p. 14). The sodium ions in this mineralized water tend to exchange with the calcium ions in the clay minerals resulting in a high calcium concentration (Davis and DeWiest, 1966, p. 414).

Ground water undergoes frequent changes in chemical composition due to reduction, ion exchange, and equilibrium reactions involving changes in ionic concentration. In northwestern Minnesota, reduction of sulfate ion concentration in buried lake deposits and in lignite deposits of the underlying Cretaceous sediments, occurs by the action of anaerobic bacteria associated with organic matter (Maclay and Winter, 1967, p. 15). Ion exchange commonly softens water in the Cretaceaous deposits as indicated by a high sodium:calcium ratio and a high content of magnesium ions. Hardening of ground water by cation exchange occurs in the lake deposits.

Relatively young glacial deposits (less than 100,000 years) retain much of their original topographic expression. Conventional geological methods, aerial photographs, and topographic maps are used to delineate these relatively young glacial landforms. Relatively thick saturated zones within ice contact deposits are rare because these deposits are topographically above nearby streams and lakes, and gravity drainage is very effective. Important sources of ground water may be found within post-glacial alluvium, outwash plains, and buried glacial and pre-glacial channel deposits. Locally, broad terminal moraines may contain numerous pockets of very permeable material which may be favorable aquifers.

Thick accumulations of till often contain isolated pockets of ice contact material. Till may also contain gravel-filled channels which were formed during retreats of the glacial margin with succeeding advances of the ice leaving the material sandwiched between till sheets (Davis and DeWiest, 1966, p. 411). These channel deposits make excellent aquifers because of their high conductivity and saturated thickness, and their large areal extent. However, geophysical surveys, test wells, and several outcrops are needed to properly delineate these gravel-filled deposits.

In glaciated regions buried valleys filled with outwash and/or pre-glacial alluvium are the most important and productive type of aquifers (Horberg, 1950). In some places recent valley expressions mark the possible locations of much deeper and older valleys. In other places, recently deposited till completely covers any surface expression of buried valleys. The saturated thickness of the till deposits commonly is more than 50 feet (Walton, 1962, p. 59).

Surface geologic mapping, test drilling, and borehole geophysical techniques are used for ground-water exploration in glaciated terrains. Surface geologic mapping by itself often does not give sufficient detail to delineate high permeability units. For example, some terminal and recessional moraines are localized by the presence of preglacial bedrock highs. In some areas, glacial material forms only a thin veneer and limits further application of surface geologic mapping to locate aquifers. Seismic and resistivity surveys are used to delineate buried valley deposits. However, the data obtained from these surveys should be used in conjunction with information on the geologic and geomorphic history of the area. This enables the hydrogeologist to delineate the zones of highest conductivity in the area of interest for ground-water development. Test drilling is a more precise method to delineate the presence of aquifers, but as has already been stated, the cost of drilling is often many times that of other exploration techniques.

Section 12.06. Shoreline and Coastal Plain Deposits

Coastal plains vary in size from small isolated valley deposits that grade inland into normal alluvial deposits to extensive, almost featureless plains that fringe hundreds of miles of the coasts bordering the Artic and Atlantic Oceans (Davis and DeWiest, 1966, p. 401). The sediments of coastal plains are represented primarily by both transitional and marine environments (Krumbein and Sloss, 1963, p. 259). Three environmental conditions, deltaic, lagoonal, and littoral, are commonly classified as transitional, although other subdivisions are possible. Estuaries may range from fresh through brackish to normal sea water. Coastal swamps may be true transitional environments in that they may alternate between fresh and salt water.

Most of the well known stratigraphic units along coastal plains are mapped and grade seaward from partly alluvial deposits to typical marine units (LaMoreaux, 1960, p. 6). This gradation of sediment types is recognized by systematic changes in grain texture and orientation, distribution of fossil types, internal primary structures and depositional environment (Laporte, 1968, p. 16). Fine-grained clastics form a major part of these sediments. Along the southern Atlantic Coastal plain and the eastern Gulf Coastal plain, marls, limestones, and organic matter comprise a high proportion of the deposits.

Hydraulic Properties

The hydraulic properties of coastal plain material are comparable to unconsolidated deposits composing structurally controlled valleys.

Hydraulic conductivities of clean sands range between 1 and 100 darcys while those of clays and silts are less than 0.01 darcy (Davis and DeWiest, 1966, p. 401). Laboratory determinations of specific yield, which are representative of coastal plain sediments, are not readily available in hydrologic literature. However, the close similarity of alluvial deposits to coastal plain material suggests the specific yields of the latter should range between 18 and 29 percent for fine sand to gravelly sand and less than 18 percent for clay and sandy clay (Richter, 1968).

Ground-water yields from irrigation, industrial, and municipal wells typically are comparable to well yields of structurally controlled alluvial valleys. Lower well yields are expected in older consolidated sedimentary units and in coastal areas deposited in lagoonal and deltaic environments (Davis and DeWiest, 1966, p. 401).

Coastal plain deposits, particularly along the Gulf Coast, contain some of the most important untapped ground-water reserves of any geologic environment (LaMoreaux, 1960, p. 3). High well yields, favorable geologic conditions for recharge and storage of water underground, along with abundant rainfall (exceeds 40 inches a year) indicate that expanded ground-water development for much of the Southeast will be feasible without appreciable depletion of these reserves.

Hydrochemistry

Most coastal plain aquifers consist of four types of water based on the dominant ions and/or the concentration of dissolved solids (Davis and DeWiest, 1966, p. 403). Fresh ground water of low dissolved solids is usually characteristic of shallow wells (less than 200 feet) which penetrate high permeability units near the surface (LaMoreaux, 1960, p. 7). At depths of several hundred to more than a thousand feet the ground water is sodium bicarbonate type with relatively low concentrations of calcium, magnesium, and sulfate (LaMoreaux, 1960, p. 7). The low sulfate probably can be attributed to reduction by microorganisms in the aquifers; whereas, the calcium and magnesium are exchanged for sodium from the clay minerals (montmorillonite group). A typical brackish sodium chloride water is encountered in wells that extend to greater depths. This ground water probably represents partial mixing of both shallower fresh water zones and deeper brackish water zones. The fourth type of water is highly contaminated ground water due to sea water that has recently invaded the coastal plain sediments. This water is characterized by a relatively low silica content and a high sulfate content. The chloride:carbonate plus bicarbonate $(\frac{Cl}{CO_3 + HCO_3})$ ratio of contaminated water exceeds 15 (Revelle, 1941, p. 595).

The terrace deposits of the Gulf Coastal Plain contain ground water with low dissolved solids (Davis and DeWiest, 1966, p. 403). The high hydraulic conductivity of these deposits permits rapid circulation of recharge water which reduces the effect of rock solubility and composition. Apparently, the more soluble constituents have been removed so that recharge water flows through the deposits with only slight chemical alteration taking place.

Exploration

Electric logs, radiation logs, and stratigraphic techniques have been successful in hydrogeologic investigations to delineate high permeability zones, estimate water chemistry, and improve stratigraphic correlations (Keys, 1967, p. 478). Several of the widespread aquifers such as the Carrizo Sand of Texas extend laterally for several hundred miles (Scalapino, 1963, p. 29). Major production from this aquifer is obtained from a 5,355-foot water well that is one of the deepest in the United States. Other high permeability units commonly change character in a lateral direction and can be correlated by paleontological methods or by examination of logs from closely spaced wells.

Seismic surveys are designed to give the depth and attitude of particular reflecting or refracting horizons in order to delineate aquifers in the subsurface. At present, geologic methods and borehole geophysical techniques have been used primarily for the purposes of stratigraphic correlation. Data from oil and gas wells together with ground-water level measurements enable the hydrogeologist to successfully locate important ground-water reserves.

Selected Bibliography

SELECTED BIBLIOGRAPHY

Chapter 1

1.01. Chow, Ven T. (ed.), "Handbook of Applied Hydrology," section 4-I, Hydrogeology, by G. B. Maxey, John Wiley, New York, 1964a.

1.02. Davis, S. N., and DeWiest, R. J. M., "Hydrogeology," John Wiley New York, 1966.

1.03. Eriksson, E., et al., (ed.), "Ground Water Problems: Metals and Metalloids," v. 1, Pergammon Press, Oxford, New York, 1968.

1.04. Gilluly, J., et al., "Principles of Physical Geology," pp. 41-54, W. H. Freeman, San Francisco, 1968.

1.05. Hubbert, M. K., "The Theory of Ground-Water Motion," Jour. Geology v. 48, no. 8, pt. 1, pp. 785-944, 1940.

1.06. Ireland, H. A., "Elementary Physical Geology," Wm. C. Brown, Dubuque, 1959.

1.07. Langbein, W. B., et al., "Annual Runoff in United States," U. S. Geol. Survey Circular 52, 1949.

1.08. Lohman, S. W., "Sand Hills, Nebraska, in Subsurface Facilities of Water Management - Type Area Studies," The Physical and Economic Foundation of Natural Resources, U. S. 83rd Congress, House Committee of Interior and Insular Affairs, v. 4, chapter 5, pp. 79-91, 1953.

1.09. Maxey, G. B., "Hydrostratigraphic Units," Jour. Hydrology, v. 2, pp. 124-129, 1964b.

1.10. Meinzer, O. E., "Occurrence of Ground Water in the United States," U. S. Geol. Survey Water-Supply Paper 489, 1923.

1.11. Meinzer, O. E., "Ground Water in the United States," U. S. Geol. Survey Water-Supply Paper 836-D, pp. 157-229, 1939.

1.12. Rice, T. D., and Alexander, L. T., The Physical Nature of Soils, "Soils and Men," U. S. Department of Agriculture Year Book, 1938.

1.13. Strahler, A. N., "Physical Geography," John Wiley, New York, 1960.

1.14. Swenson, F. A., "New Theory of Recharge to the Artesian Basin of the Dakotas," Geol. Soc. America Bul., v. 79, no. 2, pp. 163-182, 1968.

1.15. Thomas, H. E., "The Conservation of Ground Water," McGraw-Hill, New York, 1951.

1.16. Thomas, H. E., "Ground-Water Regions of the United States: Their Storage Facilities," U. S. Congress House Representatives, Interior and Insular Affairs Committee, 1952.

1.17. Thornbury, W. D., "Principles of Geomorphology," John Wiley, New York, 1969.

1.18. Todd, D. K., "Ground-Water Hydrology," John Wiley, New York, 1959.

1.19. Wilson, L. G., and DeCook, K. J., "Field Observations on Changes in the Subsurface Water Regime During Influent Seepage in the Santa Cruz River," Water Resources Research, v. 4, no. 6, pp. 1219-1235, 1968.

Chapter 2

2.01. Davis, S. N., and DeWiest, R. J. M., "Hydrogeology," chapter 10, John Wiley, New York, 1966.

2.02. Krumbein, W. C., and Sloss, L. L., "Stratigraphy and Sedimentation," pp. 300-309, W. H. Freeman, San Francisco, pp. 300-309, 1963.

2.03. Maxey, G. B., and Hackett, J. E., "Applications of Geohydrologic Concepts in Geology," Jour. of Hydrology, v. 1, no. 1, pp. 35-46, 1963.

2.04. Meinzer, O. E., "Occurrence of Ground Water in the United States," U. S. Geological Survey Water-Supply Paper 489, pp. 149-178, 1923.

2.05. Young, R. G., "Late Cretaceous Cyclic Deposits of Eastern Utah," Am. Assoc. Petroleum Geologists Bul. v. 41, p. 1760, 1957.

Chapter 3

3.01. Davis, S. N., and DeWiest, R. J. M., "Hydrogeology," pp. 395-400, John Wiley, New York, 1966.

3.02. Domenico, P. A., and Mifflin, M. D., "Water from Low Permeability Sediments and Land Subsidence," Water Resources Research, v. 1, no. 4, pp. 563-576, 1965.

3.03. Hubbert, M. K., and Rubey, W. W., "Role of Fluid Pressure in Mechanics of Overthrust Faulting," Geol. Soc. America Bul. v. 70, pp. 115-206, 1959.

3.04. Longwell, C. R., Flint, R. F., and Sanders, J. E., "Physical Geology," pp. 414-416, John Wiley, New York, 1969.

3.05. Maxey, G. B., and Hackett, J. E., "Application of Geohydrologic Concepts in Geology," Jour. of Hydrology, v. 1, no. 1, pp. 35-46, 1963.

3.06. Meinzer, O. E., "Occurrence of Ground Water in the United States," U. S. Geol. Survey Water-Supply Paper.

3.07. Tolman, C. F., "Ground Water," pp. 254-255, 375-376, 534-536, McGraw-Hill, New York, 1937.

Chapter 4

4.01. Boussinesq, J., "Recherches Théoriques sur l'écoulement des Nappes d'eau Infiltrées dans le Sol et sur le Débit des Sources," Jour. de Mathématiques Pures et Appliquées, v. 10, pp. 5-78, Complement, pp. 363-364, 1904.

4.02. Davis, S. N., and DeWiest, R. J. M., "Hydrogeology," John Wiley, New York, 1966.

4.03. DeWiest, R. J. M. (ed.), "Flow through Porous Media," chapter 8, Elastic Storage of Aquifers, by A. Veruijt, pp. 311-376, Academic Press, New York, 1969.

4.04. Freeze, R. A., "Quantitative Interpretation of Regional Ground-Water Flow Patterns as an Aid to Water Balance Studies," Ground Water, General Assembly of Bern, September-October, 1967.

4.05. Freeze, R. A., and Witherspoon, P. A., "Theoretical Analysis of Regional Ground-Water Flow: 1. Analytical and Numerical Solutions to the Mathematical Model," Water Resource Res., v. 2, no. 4, pp. 641-656, 1966.

4.06. Freeze, R. A., and Witherspoon, P. A., "Theoretical Analysis of Regional Ground-Water Flow: 2. Effect of Water Table Configuration and Subsurface Permeability Variation," Water Resources Res., v. 3, no. 2, pp. 623-634, 1967.

4.07. Freeze, R. A., and Witherspoon, P. A., "Theoretical Analysis of Regional Ground-Water Flow: 3. Quantitative Interpretations," Water Resources Res., v. 4, no. 3, pp. 581-590, 1968.

4.08. Harr, M. E., "Ground Water and Seepage," McGraw-Hill, New York, 1962.

4.09. Hubbert, M. K., "The Theory of Ground-Water Motion," Jour. Geology, v. 48, no. 8, pt. 1, pp. 785-944, 1940.

4.10. Meinzer, O. E., "Outline of Ground-Water Hydrology with Definitions U. S. Geol. Survey Water-Supply Paper 494, 1923.

4.11. Richards, L. A., "Capillary Conduction of Liquids through Porous Mediums," Physics, v. 1, pp. 318-333, 1931.

4.12. Rouse, Hunter (ed.), "Engineering Hydraulics," chapter 5, Flow of Ground Water, by C. E. Jacob, pp. 321-386, John Wiley, New York 1950.

4.13. Todd, D. K., "Ground-Water Hydrology," John Wiley, New York, 1959.

4.14. Toth, J. "A Theoretical Analysis of Ground-Water Flow in Small Drainage Basins," National Res. Council of Canada, Proc. Hydrol. Symposium No. 3, Ground Water, pp. 75-96, 1962b.

4.15. Toth J., "A Theory of Ground-Water Motion in Small Drainage Basins, in Central Alberta Canada," Jour. Geophys. Res., v. 67, no. 11, pp. 4375-4387, 1962a.

4.16. Trescott, P. C., Pinder, G. F., and Jones, J. F., "Digital Model of Alluvial Aquifer," Hydraulics Div. Jour., Am. Soc. Civil Engineers, v. 96, nó. HY5, pp. 1115-1128, 1970.

4.17. Vennard, J. K., "Elementary Fluid Mechanics," John Wiley, New York, 1961.

4.18. Walton, W. C., "Selected Methods for Well and Aquifer Evaluation," Illinois State Water Survey, Bul. 49, 1962.

4.19. Wilson, L. B., and DeCook, K. J., "Field Observations on Changes in the Subsurface Water Regime During Influent Seepage in the Santa Cruz River," Water Resources Res., v. 4, no. 6, pp. 1219-1234, 1968.

Chapter 5

5.01. Bouwer, Herman, "Unsaturated Flow in Ground-Water Hydraulics," Am. Soc. Civil Engineers, Hydraulics Div. Jour., v. 90, no. HY5, pp. 121-144, 1964.

5.02. Bouwer, Herman, "Theoretical Aspects of Seepage from Open Channels," Am. Soc. Civil Engineers, Hydraulics Div. Jour., v. 91, no. HY3, pp. 37-59, 1965.

5.03. Casagrande, A., "Seepage through Dams," Contributions to Soil Mechanics 1925-1940, Boston Society of Civil Engineers, 1940.

5.04. Cedergren, H. R., "Seepage, Drainage and Flow Nets," John Wiley, New York, 1967.

5.05. Davis, S. N., and DeWiest, R. J. M., "Hydrogeology," John Wiley, New York, 1966.

5.06. Ferris, J. G., et al., "Theory of Aquifer Tests," U. S. Geol. Survey Water-Supply Paper 1536-E, 1962.

5.07. Foley, F. C., Walton, W. C., and Drescher, W. J., "Ground-Water Conditions in the Milwaukee-Waukesha Area, Wisconsin," U. S. Geol. Survey Water-Supply Paper 1229, 1953.

5.08. Harr, M. E., "Ground Water and Seepage," McGraw-Hill, New York, 1962.

5.09. Hubbert, M. K., "The Theory of Ground-Water Motion," Jour. Geology, v. 48, no. 8, pt. 1, pp. 785-944, 1940.

5.10. Jeppson, R. W., "Seepage from Ditches-Solution by Finite Differences," Am. Soc. Civil Engineers, Hydraulics Div. Jour., v. 94, no. HY1, pp. 259-283, 1968.

5.11. Walton, W. C., "Selected Analytical Methods for Well and Aquifer Evaluation," Illinois State Water Survey, Bul. 49, 1962.

Chapter 6

6.01. Bredehoeft, J. D., "Response of Well-Aquifer Systems to Earth Tides," Jour. Geophys. Research, v. 72, no. 12, pp. 3075-3088, 1967.

6.02. Cohen, P., "An Evaluation of the Water Resources of the Humboldt River Valley near Winnemucca, Nevada," Water Resources Bul. 24, Nevada Department of Conservation and Natural Resources, 1963.

6.03. Davis, S. N., and DeWiest, R. J. M., "Hydrogeology," John Wiley and Sons, New York, 1966.

6.04. Farvolden, R. N., "Geologic Controls on Ground-Water Storage and Base Flow," Jour. Hydrology, v. 1, no. 3, pp. 219-249, 1963.

6.05. Ferris, J. G., et al., "Theory of Aquifer Tests," U. S. Geol. Survey Water Supply Paper 1536-E, pp. 69-174, 1962.

6.06. Feth, J. H., "Hidden Recharge," Ground Water, v. 2, no. 4, pp. 14-17, 1964.

6.07. Freeze, R. A., "The Mechanism of Natural Ground-Water Recharge and Discharge: 1. One-Dimensional, Vertical, Unsteady, Unsaturated Flow above a Recharging or Discharging Ground-Water Flow System," Water Resources Research, v. 5, no. 1, pp. 153-171, 1969.

6.08. Gilliland, J. A., "A Rigid Plate Model of the Barometric Effect," Jour. Hydrology, v. 7, no. 3, pp. 233-245, 1969.

6.09. Hackett, J., et al., "Geology and Ground-Water Resources of Gallatin Valley, Montana," U. S. Geol. Survey Water-Supply Paper 1482, 1960.

6.10. Hargreaves, G. H., "Consumptive Use Derived from Evaporation Pan Data," Jour. Irr. Drain. Div., ASCE, v. 94, no. IR-1, pp. 97-105, 1968.

6.11. Hunt, C. B., and Robinson, T. W., "Possible Interbasin Circulation of Ground Water in the Southern Part of the Great Basin," U. S. Geol. Survey Professional Paper 400-B, p. 273, 1960.

6.12. Jacob, C. E., "On the Flow of Water in an Elastic Artesian Aquifer," Trans. Am. Geophys. Union, v. 21, pp. 574-586, 1940.

6.13. Jaworski, J., "Evapotranspiration of Plants and Fluctuation of the Ground-Water Table," Int. Assn. of Scientific Hydrology, Publication No. 83, v. II, pp. 730-739, 1968.

6.14. Kazmann, R. G., "Modern Hydrology," Harper and Row, New York, 1965.

6.15. Keppel, R. V., and Renard, K. G., "Transmission Losses in Ephemeral Streambeds," Jour. Hydraulics Div., ASCE, v. 88, no. HY-3, pp. 59-68, 1962.

6.16. Meinzer, O. E., "Outline of Ground-Water Hydrology," U. S. Geol. Survey Water-Supply Paper 494, 1923.

6.17. Meinzer, O. E. (ed.), "Hydrology," McGraw-Hill, New York, 1942.

6.18. Meyboom, P., "Estimating Ground-Water Recharge from Stream Hydrographs," Jour. Geophys. Research, v. 66, no. 4, pp. 1203-1214, 1961.

6.09. Obbink, J. G., "Construction of Piezometers and Method of Installation for Ground-Water Observations in Aquifers," Jour. Hydrology, v. 7, no. 4, pp. 434-443, 1969.

6.20. Pruitt, W. O., "Empirical Method of Estimating Evapotranspiration Using Primarily Evaporation Pans," Evapotranspiration and Its Role in Water Resources Management, ASAE, pp. 57-61, 1966.

6.21. Robinson, T. W., "Introduction, Spread and Areal Extent of a Saltcedar in the Western States," U. S. Geol. Survey Prof. Paper 491-A, 1965.

6.22. Rorbaugh, M. I., "Ground Water in Northeastern Louisville, Kentucky," U. S. Geol. Survey Water-Supply Paper 1360-B, 1956.

6.23. Theis, C. V., "Earth Tides Expressed in Fluctuations of Water Levels in Artesian Wells in New Mexico," U. S. Geol. Survey Open-File Report, 1939.

6.24. Thomas, H. E., "A Water Budget for the Artesian Aquifer in Ogden Valley, Wever County, Utah," U. S. Geol. Survey Water-Supply Paper 1544-H, pp. 63-97, 1963.

6.25. Todd, D. K., "Ground-Water Hydrology," John Wiley, New York, 1959.

6.26. Tovey, Rhys, "Alfalfa Water Table Investigations," Jour. Irr. Drain. Div., ASCE, v. 95, no. IR-4, pp. 525-535, 1969.

6.27. Van Hylckama, T. E. A., "Water Level Fluctuations, in Evapotranspirometers," Water Resources Research, v. 4, no. 4, pp. 761-768, 1968.

6.28. Walton, W. C., "Recharge from Induced Infiltration Under Varying Ground-Water Level and Stream Stage Conditions," Minnesota Water Resources Research Center, Bul. 6, 1967.

6.29. Williams, C. C., and Lohman, S. W., "Geology and Ground-Water Resources of a Part of South Central Kansas," Geol. Survey, Bul. 79, pp. 212-223, Kansas, 1949.

6.30. Winograd, I. J., "Interbasin Movement of Ground Water at the Nevada Test Site, Nevada," U. S. Geol. Survey Prof. Paper 450-C, pp. 108-111, 1962.

6.31. Winograd, I. J., and Thordarson, W., "Structural Control of Ground-Water Movement in Miogeosynclinal Rocks of South Central Nevada," Geological Soc. Amer. Memoir 110, pp. 35-49, 1968.

Chapter 7

7.01. Bentall, Roy, "Methods of Determining Permeability, Transmissibility, and Drawdown," U. S. Geol. Survey Water-Supply Paper 1536-I, 1963.

7.02. Boulton, N. S., "Unsteady Radial Flow to a Pumped Well Allowing for Delayed Yield from Storage," International Association of Scientific Hydrology, Assemblee Generale de Rome, v. II, pp. 472-477 1954.

7.03. Boulton, N. S., "Analysis of Data from Nonequilibrium Pumping Tests Allowing for Delayed Yield from Storage," Proceedings of Institution of Civil Engineers, v. 26, pp. 469-482, 1963.

7.04. Boulton, N. S., "Discussions on the Analysis of Data from Nonequilibrium Pumping Tests Allowing for Delayed Yield from Storage," Proceedings of the Institution of Civil Engineers, v. 28, pp. 603-610, 1964.

7.05. Chow, V. T., (ed.), "Advances in Hydroscience," v. 1, Academic Press, New York, 1964.

7.06. Davis, S. N., and DeWiest, R. J. M., "Hydrogeology," John Wiley, New York, 1966.

7.07. Hantush, M. S., and Jacob, C. E., "Nonsteady Radial Flow in an Infinite Leaky Aquifer," Trans. Amer. Geophys. Union, v. 36, no. 1, pp. 95-100, 1955.

7.08. Hantush, M. S., "Modification of the Theory of Leaky Aquifers," Jour. Geophys. Research, v. 65, no. 11, pp. 3713-3725, 1960.

7.09. Johnson, Edward E., (pub.), "Ground Water and Wells," UOP, Johnson Division, St. Paul, Minnesota, 1966.

7.10. Kruseman, G. P., and DeRidder, N. A., "Analysis and Evaluation of Pumping Test Data," International Institute for Land Reclamation and Improvement, Bul. 11, Wageningen, The Netherlands, 1970.

7.11. Neuman, S. P., and Witherspoon, P. A., "Applicability of Current Theories of Flow in Leaky Aquifers," Water Resources Research, v. 5, no. 4, pp. 817-829, 1969a.

7.12. Neuman, S. P., and Witherspoon, P. A., "Theory of Flow in a Two-Aquifer System," Water Resources Research, v. 5, no. 4, pp. 803-816, 1969b.

7.13. Neuman, S. P., and Witherspoon, P. A., "Transient Flow of Ground Water in Multiple-Aquifer Systems," Department of Civil Engineering, pub. no. 69-1, Univ. of Calif., Berkeley, pp. 817-818, 1969c.

7.14. Prickett, T. A., "Type-Curve Solution to Aquifer Tests Under Water Table Conditions," Ground Water, v. 3, no. 3, pp. 5-14, 1965.

7.15. Rouse, H., (ed.), Engineering Hydraulics, "Flow of Ground Water," by C. E. Jacob, John Wiley and Sons, New York, pp. 321-386, 1950.

7.16. Stallman, R. W., "Aquifer-Test Design, Observation and Data Analysis," Techniques of Water-Resources Investigations of the U. S. Geological Survey, chapter B1, book 3, 1971.

7.17. Stallman, R. W., "Effects of Water Table Conditions on Water Level Changes near Pumping Wells," Water Resources Research, v. 1, no. 2, pp. 295-312, 1965.

7.18. Theis, C. V., "The Relation Between the Lowering of Piezometric Surface and the Rate and Duration of Discharge of a Well Using Ground-Water Storage," Trans. Am. Geophys. Union, 16th Annual Meeting, part 2, 1935.

7.19. Todd, D. K., Ground Water Hydrology," John Wiley and Sons, New York, 1959.

7.20. Walton, W. C.,"Ground-Water Resource Evaluation," McGraw-Hill, New York, 1970.

Chapter 8

8.01. Antoine, J. W., and Henry, V. J., "Seismic Refraction Study of Shallow Part of Continental Shelf off Georgia Coast," Am. Assoc. Petroleum Geologists Bul., v. 49, pp. 605-609, 1965.

8.02. Bodmer, R., "Induced Electrical Polarization and Ground Water," Univ. of California, Berkeley, College of Engineering, Report no. MT 67-1, 1967.

8.03. Bodmer, R., et al., "On Induced Electrical Polarization and Ground Water," Geophysics, v. 33, no. 5, 1968.

8.04. Breusse, J. J., "Modern Geophysical Methods for Subsurface Water Exploration," Geophysics, v. 28, no. 4, pp. 633-657, 1963.

8.05. Carpenter, G. C., and Bassarab, D. R., "Case Histories of Resistivity and Seismic Ground-Water Studies," Ground Water, v. 2, no. 1, pp. 22-25, 1964.

8.06. Chow, Ven T., (ed.), Handbook of Applied Hydrology, Section 4-I, "Hydrogeology," by G. B. Maxey, John Wiley, New York, 1964.

8.07. Davis, S. N., and DeWiest, R. J. M., "Hydrogeology," John Wiley, New York, pp. 260-318, 1966.

8.08. Dobrin, M. B., "Introduction to Geophysical Prospecting," McGraw-Hill, New York, 1960.

8.09. Dugvid, J. O., "Refraction Determination of Water Table Depth and Alluvium Thickness," Geophysics, v. 33, no. 3, 1968.

8.10. Flathe, H., "Five-Layer Curves for the Hydrogeological Interpretation of Geoelectrical Resistivity Measurements Above a Two-Story Aquifer," Geophysical Prospecting, v. 11, pp. 471-508, 1963.

8.11. Geophysical Specialties Division, Engineering Seismograph Instruction Manual, Minnetech Labs, Minneapolis, 1962.

8.12. Gill, H. E., et al., "Tracing the Continuity of Pleistocene Aquifers in Northern New Jersey by Seismic Methods," Ground Water, v. 3, pp. 33-35, 1965.

8.13. Hales, F. W., "An Accurate Graphical Method for Interpreting Seismic Refraction Lines," Geophysical Prospecting, v. 6, pp. 285-294, 1958.

8.14. Hawkins, L. V., "The Reciprocal Method of Routine Shallow Seismic Refraction Investigations," Geophysics, v. 26, no. 6, pp. 806-819, 1961.

8.15. Hawkins, L. V., and Moggs, D., "Nomograms for Determining Maximum Errors and Limiting Conditions in Seismic Refraction Survey with a Blind-Zone Problem," Geophysical Prospecting, v. 9, pp. 526-532, 1961.

8.16. Keller, G. V., and Frischknecht, F. C., "Electrical Methods in Geophysical Prospecting," Pergamon Press, New York, 1966.

8.17. Mooney, H. M., and Wetzel, W. W., "Master Resistivity Curves for a Two-, Three-, and Four-Layer Earth," Univ. of Minnesota Press, Minneapolis, 1956.

8.18. Mooney, H. M., et al, "A Resistivity Computation Method for Layered Earth Models," Geophysics, v. 31, no. 1, pp. 192-203, 1966.

8.19. Neale, R. N., "The Use of Isocron Charts in Seismic Refraction Interpretation," Geophysical Prospecting, v. 12, pp. 215-224, 1964.

8.20. Orellana, E., and Mooney, H. M., "Master Tables and Curves for Vertical Electrical Sounding Over Layered Structures," Interciencia, Madrid, 1966.

8.21. Johnson, Edward E., Inc., (pub.), "Ground Water and Wells," St. Paul, pp. 157-159, 1966.

8.22. Richter, R. C., "Ground-Water Course Notes," Department of Water Resources Asilomar Conference, Pacific Grove, California, chapters 5 and 6, 1967.

8.23. Sherridan, R., et al., "Seismic Refraction-Study of Continental Margin," Am. Assoc. Petroleum Geologists Bul. v. 50, no. 9, pp. 1972-1991, 1966.

8.24. Soiltest, Inc., (pub.), "Earth Resistivity Manual," Evanston, 1968.

8.25. Steinhart, J. S., and Meyer, R. P., "Minimum Statistical Uncertainty of the Seismic Refraction Profile," Geophysics, v. 26. no. 5, pp. 574-487, 1961.

8.26. Tarrant, L. H., "A Rapid Method of Determining the Form of a Seismic Refractor from Line Profile Results," Geophysical Prospecting, v. 4, pp. 131-139, 1956.

8.27. Unz, M., "Linear Approximation of Apparent Resistivity Functions," Geophysics, v. 27, no. 1, pp. 100-108, 1962.

8.28. Zody, A. A. R., "Geoelectrical and Seismic Refraction Investigations Near San Jose, California," Ground Water, v. 3, pp. 41-48, 1965.

Chapter 9

9.01. Alger, R. P., "Interpretation of Electric Logs in Fresh Water Wells in Unconsolidated Formations," Schlumberger Well Surveying Corp., 1968.

9.02. Bennett, G. D., and Patten, E. P., Jr., "Borehole Geophysical Methods for Analyzing Specific Capacity of Multiaquifer Wells," U. S. Geol. Survey, Water-Supply Paper 1536A, 1960.

9.03. Borowczyk, M., Mairhofer, J., and Zuber, A., "Single-Well Pulse Techniques," Isotopes in Hydrology, IAEA, Vienna, pp. 507-519, 1967.

9.04. Bryan, F. L., "Application of Electric Logging to Water Well Problems," Water Well Journal, v. 4, no. 1, 1959.

9.05. Chapman, H. T., and Robinson, A. E., "A Thermal Flow Meter for Measuring Velocity of Flow in a Well," U. S. Geol. Survey Water-Supply Paper 1544E, 1962.

9.06. Chow, Ven T., (ed.) Handbook of Applied Hydrology, Section 4-I, "Hydrology," by Maxey, G. B., John Wiley, New York, 1964.

9.07. Davis, S. N., and DeWiest, R. J. M., "Hydrogeology," John Wiley, New York, pp. 260-318, 1966.

9.08. Drost, W., et al., "Point Dilution Methods of Investigating Ground-Water Flow by Means of Radioisotopes," Water Resources Res., v. 4, no. 1, pp. 125-146, 1968.

9.09. Feely, H. W., Walton, A., Barnett, C. R., and Bazan, F., "The Potential Applications of Radioisotopes Techniques to Water Resource Investigations and Utilization," AEC Research & Development Report NYO-9040, 1961.

9.10. Frimpter, M. H., "Casing Detector and Self-Potential Logger," Ground Water, v. 7, no. 6, pp. 24-27, 1969.

9.11. Halevy, E., Moser, H., Sellhofen, O., and Zuber, A., "Borehole Dilution Techniques: A Critical Revue," Isotopes in Hydrology, IAEA, Vienna, pp. 531-564, 1967.

9.12. Halevy, E., and Nir, A., "The Determination of Aquifer Parameters with the Aid of Radioactive Tracers," Jour. Geophys. Res., v. 67, pp. 2403-2409, 1962.

9.13. Hanshaw, B. B., et al., "Relation of Carbon-14 Concentrations to Saline Water Contamination of Coastal Aquifers," Water Resources Res., v. 1, no. 1, pp. 109-114, 1965.

9.14. Harpaz, Y., Mandel, S., Gat, J. R., and Nir, A., "The Place of Isotope Methods in Ground-Water Research," Radioisotopes in Hydrology, IAEA, Vienna, pp. 175-192, 1963.

9.15. Haun, J. D., and LeRoy, L. W., (ed.)"Subsurface Geology in Petroleum Exploration," Colorado School of Mines, pp. 265-427, 1958.

9.16. IAEA, "Tritium and Other Environmental Isotopes in the Hydrologic Cycle," Technical Report Series no. 73, IAEA, Vienna, 1967.

9.17. IAEA, "Guidebook on Nuclear Techniques in Hydrology," Technical Report Series no. 91, IAEA, Vienna, 1968.

9.18. Inerson, J., et al., "Electrical Investigations of Borehole Fluids," Jour. Hydrology, v. 1, pp. 204-218, 1963.

9.19. Jennings, A. R., and Schroeder, M. C., "Laboratory Evaluation of Selected Radioisotopes as Ground-Water Tracers," Water Resources Res., v. 4, no. 4, pp. 829-838, 1968.

9.20. Johnson, Edward E., Inc. (pub.), "Ground Water and Wells," St. Paul, pp. 157-179, 1966.

9.21. Kelley, J. W., and Scalf, M. R., "Aquifer Storage Determination by Radiotracer Techniques," Ground Water, v. 7, no. 1, pp. 17-22, 1969

9.22. Keys, W. S., "The Application of Radiation Logs to Ground-Water Hydrology," Isotopes in Hydrology, IAEA, Vienna, pp. 477-488, 1967.

9.23. Knutson, G., "Tracers for Ground-Water Investigations," Ground Water Problems, vol. II, Pergamon Press, pp. 123-152, 1968.

9.24. Kokesh, F. P., et al., "A New Approach to Sonic Logging and Other Acoustic Measurements," Jour. Petroleum Technology, v. 18, pp. 282-286, 1965.

9.25. Kriz, G. J., Lewis, D. C., Burgy, R. H., "Ground-Water Tracing by Tritiated Water Injection," Trans. ASAE, v. 9, no. 1, pp. 116-118.

9.26. Lewis, D. C., and Burgy, R. H., "The Relationship between Oak Tree Roots and Ground Water in Fractured Rock as Determined by Tritium Tracing," Jour. Geophys. Res., v. 69, pp. 2579-2588, 1964.

9.27. Lewis, D. C., Kriz, G. J., and Burgy, R. H., "Tracer Dilution Sampling Technique to Determine Hydraulic Conductivity of Fractured Rock," Water Resources Res., v. 2, no. 3, pp. 533-542, 1966.

9.28. Mairhofer, J., "Ground-Water Flow and Direction Measurement by Means of Radioisotopes in a Single Well," Isotope Techniques in the Hydrologic Cycle, Geophys. Monograph Series 11, Amer. Geophys. Union, pp. 119-129, 1967.

9.29. Mercado, A., and Harlevy, E., "Determining the Average Porosity and Permeability of a Stratified Aquifer with the Aid of Radioactive Tracers," Water Resources Res., v. 2, no. 3, pp. 525-531, 1966.

9.30. Moser, H., Neumaier, F., and Rauert, W., "New Experiences with the Use of Radioactive Isotopes in Hydrology," Radioisotopes in Hydrology, IAEA, Vienna, pp. 283-296, 1963.

9.31. Nir, A., "On the Interpretation of Tritium 'Age' Measurements of Ground Water," Jour. Geophys. Res., v. 69, no. 12, pp. 2589-2595, 1964.

9.32. Nir, A., "Development of Isotope Methods Applied to Ground-Water Hydrology," Isotope Techniques in the Hydrologic Cycle, Geophys. Monograph Series 11, Amer. Geophys. Union, pp. 109-116, 1967.

9.33. Payne, B. R., "Contributions of Isotope Techniques to the Study of Some Hydrological Problems," Isotope Techniques in the Hydrologic Cycle," Geophys. Monograph Series 11, Amer. Geophys. Union, pp. 62-68, 1967.

9.34. Pearson, F. J., Jr., and White, D. E., "Carbon-14 Ages and Flow Rates of Water in Carrizo Sand, Atascosa County, Texas," Water Resources Res., v. 3, no. 1, pp. 251-261, 1967.

9.35. Pirson, S. J., "Handbook of Well Log Analysis for Oil and Gas Formation Evaluation," Prentice-Hall, New Jersey, 1963.

9.36. Robinson, T. W., and Donaldson, D., "Pontracyl Brilliant Pink As a Tracer Dye in the Movement of Water in Phreatophytes," Water Resources Res., v. 3, no. 1, pp. 203-211, 1967.

9.37. Saleem, M., "An Inexpensive Method for Determining the Direction of Natural Flow of Ground Water," Jour. Hydrology, v. 9, no. 1, pp. 73-89, 1969.

9.38. Soiltest, Inc., (pub.), "Earth Resistivity Manual," Evanston, 1968.

9.39. Sternau, R., et al., "Radioisotope Tracers in Large Scale Recharge Studies of Ground Water," Isotopes in Hydrology, IAEA, Vienna, pp. 489-505, 1967.

9.40. Theis, C. V., "Hydrologic Phenomena Affecting the Use of Tracers in Timing Ground-Water Flow," Radioisotopes in Hydrology, IAEA, Vienna, pp. 193-206, 1963.

9.41. Tittman, J., et al., "The Sidewall Epithermal Neutron Porosity Log," Jour. Petroleum Technology, v. 18, pp. 1351-1362, 1966.

9.42. Tittman, J., and Wahl, J. S., "The Physical Foundations of Formation Density Logging (Gamma-Gamma)," Geophysics, v. 30, pp. 284-294, 1965.

9.43. Tixier, M. P., et al., "Sonic Logging," Jour. Petroleum Technology, v. 11, no. 5, pp. 106-114, 1959.

9.44. Todd, D. K.,"Ground-Water Hydrology," John Wiley, New York, pp. 219-251, 1959.

9.45. Toth, J., "A Theoretical Analysis of Ground-Water Flow in Small Drainage Basins," Jour. Geophys. Res., v. 68, no. 16, pp. 4795-4810, 1963.

9.46. Ubell, K., "Investigations into Ground-Water Balance by Applying Radioisotope Tracers," Isotopes in Hydrology, IAEA, Vienna, pp. 521-530, 1967.

9.47. Vogel, J. C., and Ehalt, D., "The Use of Carbon Isotopes in Ground-Water Studies," Radioisotopes in Hydrology, IAEA, Vienna, pp. 383-395, 1963.

9.48. Vonhoff, J. A., "Water Quality Determination from Spontaneous Potential Electric Log Curves," Jour. Hydrology, v. 4, pp. 341-347, 1966.

9.49. Wahl, J. S., et al., "The Dual Spacing Formation Density Log," Jour. Petroleum Technology, v. 16, pp. 1411-1416, 1964.

"Radioisotopes as Ground-Water Tracers," Jour. Geophys. Res., v. 72, no. 16, pp. 4081-4091, 1967.

9.51. Wurzel, P., and Ward, P. R. B., "A Simplified Method of Ground-Water Direction Measurement in a Single Borehole," Jour. Hydrology, v. 3, no. 2, pp. 97-105, 1965.

9.52. Youmans, A. H., et al., "Neutron Lifetime, A New Nuclear Log," Jour. Petroleum Technology, v. 16, pp. 319-328, 1964.

9.53. Zublin, C. W., "Why Electric Log Water Wells," Ground Water, v. 2, no. 2, p. 32, 1964.

Chapter 10

10.01. Barnes, I., and O'Neil, J. R., "Relationship between Fluids in Some Fresh Alpine-Type Ultramafic and Possible Modern Serpentinization, Western U.S.," Geol. Soc. America Bul., v. 80, pp. 1947-1960, 1969.

10.02. Bear, F. E. (ed.), "Chemistry of the Soil," Reinhold Pub. Company, New York, 1955.

10.03. Davis, S. N., and DeWiest, R. J. M., "Hydrogeology," John Wiley and Sons, New York, 1966.

10.04. Feth, J. H., et al., "Sources of Mineral Constituents in Water from Granitic Rocks, Sierra Nevada, California and Nevada," U. S. Geol. Survey Water-Supply Paper 1535-I, 1964.

10.05. Hem, J. D., "Study and Interpretation of the Chemical Characteristics of Natural Water, Second Edition," U. S. Geol. Survey Water-Supply Paper 1473, 1970.

10.06. Krauskopf, K., "Introduction to Geochemistry," McGraw-Hill, New York, 1967.

10.07. Longwell, C. R., Flint, R. F., and Sanders, J. E., "Physical Geology," John Wiley, New York, 1969.

10.08. Meinzer, O. E., "The Occurrence of Ground Water in the United States," U. S. Geol. Survey Water-Supply Paper 489, 1923.

10.09. Meinzer, O. E., "Hydrology," Dover Publications, New York, pp. 678-703, 1942.

10.10. Newcomb, R. C., "Some Preliminary Notes of the Ground Water of the Columbia Basalt," Northwest Science, v. 33, pp. 1-18, 1959.

10-11. Piper, A. M., and others, "Geology and Ground-Water Hydrology of the Mokelumne Area, California," U. S. Geol. Survey Water-Supply Paper 780, 1939.

10.12. Sabins, F. F., Jr., "Infrared Imagery and Geologic Aspects," Photogrammetric Engineering, v. 33, pp. 743-750, 1967.

10.13. Stewart, J. W., "Water-Yielding Potential of Weathered Crystalline Rocks at the Georgia Nuclear Laboratory," U. S. Geol. Survey Prof. Paper 450-B, pp. 106-107, 1962.

10.14. White, D. E., et al., "Data of Geochemistry," U. S. Geol. Survey Prof. Paper 440-F, pp. 1-59, 1963.

10.15. Williams, R. E., "Ground-Water Flow Systems and the Origin of Evaporite Deposits," Idaho Bureau of Mines and Geology, Pamphlet 141, 1968.

Chapter 11

11.01. Bredehoeft, J. D., et al., "Possible Mechanism for Concentration of Brines in Subsurface Formations," Amer. Assoc. Petroleum Geologists Bul., v. 47, pp. 257-269, 1963.

11.02. Davis, S. N., and DeWiest, R. J. M., "Hydrogeology," John Wiley, New York, 1966.

11.03. Foster, M. D., "The Origin of High Sodium Bicarbonate Waters in the Atlantic and Gulf Coastal Plain," Goechim. et Cosmochim., Acta, v. 1, pp. 33-48, 1950.

11.04. Gilluly, J., et al., "Principles of Geology," W. J. Freeman, San Francisco, pp. 368-900, 1935.

11.05. Graton, L. C., and Fraser, H. J., "Systematic Packing of Spheres with Particular Relation to Porosity and Permeability," Jour. Geol., v. 43, pp. 785-900, 1935.

11.06. Hem, J. D., "Study and Interpretation of the Chemical Characteristics of Natural Water," U. S. Geol. Survey Water-Supply Paper 1473, 2nd Ed., 1970.

11.07. Krauskopf, K. B., "Introduction to Geochemistry," McGraw-Hill, New York, 1967.

11.08. LaMoreaux, P. E., and Powell, W. J., "Stratigraphic and Structural Guides to the Development of Water Wells and Well Fields in Limestone Terrane," International Assoc. Sci. Hydrology, Pub. no. 52, pp. 363-375, 1960.

11.09. Lattman, L., and Parizek, R., "Relationships between Fracture Traces and the Occurrence of Ground Water in Carbonate Rocks," Jour. Hydrology, v. 2, pp. 73-91, 1964.

11.10. Meinzer, O. E., "The Occurrence of Ground Water in the United States," U. S. Geol. Survey Water-Supply Paper 489, 1923.

11.11. Meinzer, O. E., "Hydrology," Dover, New York, pp. 656-678, 1942.

11.12. Piersol, R. J., et al., "Porosity, Total Liquid Saturation, and Permeability of Illinois Oil Sands," Illinois Geol. Survey Report Investigation, no. 67, 1940.

11.13. Sonderegger, J. L., and Kelley, J. C., "Hydrology of Limestone Terranes, Geologic Investigations," Alabama Geol. Survey Bul., Part B, 1970.

11.14. Sonderegger, J. L., "Hydrology of Limestone Terranes, Photogeologic Investigations," Alabama Geol. Survey Bul. 94, part C, 1970.

11.15. Tickell, F. G., and Hiatt, W. N., "Effect of Angularity of Grain on Porosity and Permeability of Unconsolidated Sands," Am. Assoc. Petroleum Geol. Bul., v. 22, pp. 1272-1279, 1938.

11.16. Walton, W. C., "Selected Analytical Methods for Well and Aquifer Evaluation," Illinois State Water Survey Bul. 49, 1962.

11.17. Walton, W. C., "Ground-Water Resource Evaluation," McGraw-Hill, New York, pp. 311-321, 1970.

11.18. White, D. E., "Magmatic Connate and Metamorphic Waters," Geol. Soc. Amer. Bul., v. 68, no. 12, parts I and II, pp. 1637-1682, 1957.

11.19. White, D. E., et al., "Data of Geochemistry," U. S. Geol. Survey Prof. Paper 440-F, pp. 1-59, 1963.

11.20. Williams, R. E., "Ground-Water Flow Systems and the Origin of Evaporite Deposits," Idaho Bureau of Mines and Geology, Pamphlet 141, 1970.

Chapter 12

12.01. Bedinger, M. S., "Relation between Median Grain Size and Permeability in the Arkansas River Valley, Arkansas," U. S. Geol. Survey Prof. Paper 424-C, pp. 31-32, 1961.

12.02. Brown, S. B., and Newcomb, R. C., "Ground-Water Resources of the Coastal Sand-Dune Area North of Coos Bay, Oregon," U. S. Geol. Survey Water-Supply Paper 1619-D, 1963.

12.03. Cohen, P., et al., "Water Resources of the Humboldt River Valley near Winnemucca, Nevada," U. S. Geol. Survey Water-Supply Paper 1795, 1965.

12.04. Conkling, H., et al., "Ventura County Investigation," California Div. of Water Resources Bul. 6, 1934.

12.05. Davis, G. H., et al., "Ground-Water Conditions and Storage Capacity in San Joaquin Valley, California," U. S. Geol. Survey Water-Supply Paper 1469, 1959.

12.06. Davis, S. N., and DeWiest, R. J. M., "Hydrogeology," John Wiley, New York, 1966.

12.07. Flint, R. F., "Glacial and Quaternary Geology," John Wiley, New York, 1970.

12.08. Gibbs, H. J., and Holland, W. Y., "Petrographic and Engineering Properties of Loess," U. S. Bureau of Reclamation Engineering Monographs no. 28, 1960.

12.09. Hackel, O., "Summary of the Geology of the Great Valley," California Div. of Mines and Geology, Bul. 190, pp. 218-222, 1966.

12.10. Hem, J. D., "Study and Interpretation of the Chemical Characteristics of Natural Waters, Second Edition," U. S. Geol. Survey Water-Supply Paper 1473, 1970.

12.11. Horberg, L., "Bedrock Topography of Illinois," Illinois Geol. Survey Bul. 73, 1950.

12.12. Keys, W. S., "The Application of Radiation Logs to Ground-Water Hydrology," Isotopes in Hydrology, Proc. Symp. Vienna, 1966, IAEA, Vienna, pp. 477-487, 1967.

12.13. Krumbein, W. C., and Sloss, L. L., "Stratigraphy and Sedimentation," W. H. Freeman, San Francisco, pp. 257-262, 1963.

12.14. La Moreaux, P. E., "Ground-Water Resources of the South--A Frontier of the Nation's Water Supply," U. S. Geol. Survey Circ. 441, 1960.

12.15. LaPorte, L. F., "Ancient Environments," Prentice-Hall, Englewood Cliffs, pp. 7-17, 1968.

12.16. Logan, J., "Estimating Transmissivity from Routine Production Tests of Waterwells," Ground Water, v. 2, no. 2, pp. 35-38, 1964.

12.17. Maclay, R. W., and Winter, T. C., "Geochemistry and Ground-Water Movement in Northwestern Minnesota," Ground Water, v. 5, no. 1, pp. 11-19, 1967.

U. S. Geol. Survey Water-Supply Paper 489, 1923.

12.19. Norris, S. E., "Hydrogeology of a Spring in a Glacial Terrane near Ashland, Ohio," U. S. Geol. Survey Water-Supply Paper 1619-A, 1961.

12.20. Piper, A. M., et al., "Geology and Ground-Water Hydrology of the Mokelumne Area, California," U. S. Geol. Survey Water-Supply Paper 780, 1939.

12.21. Revelle, Roger, "Criteria for Recognition of Sea Water in Ground Water," Amer. Geophys. Union Trans., v. 22, pp. 593-597, 1941.

12.22. Richter, R. C., "Ground-Water Geology," Course Notes, State of California Department of Water Resources, 1968.

12.23. Scalapino, R. A., "Ground-Water Conditions in the Carrizo Sand in Texas," Ground Water, v. 1, no. 4, pp. 26-32, 1963.

12.24. Todd, D. K., "Ground-Water Hydrology," John Wiley, New York, 1959.

12.25. U. S. 83rd Congress, House Comm. Interior and Insular Affairs, The Physical and Economic Foundation of Natural Resources, Subsurface Facilities of Water Management--Type Area Studies, "Sand Hills, Nebraska," by S. W. Lohman, v. 4, chapter 5, pp. 79-91, 1953.

12.26. Walker, W. H., and Walton, W. C., "Ground-Water Development in Three Areas of Central Illinois," Illinois State Water Survey, Report of Investigation 41, p. 19, 1961.

12.27. Walters, K. L., "Highly Productive Aquifers in the Tacoma Area, Washington," U. S. Geol. Survey Prof. Paper 450-E, pp. 157-158, 1963.

12.28. Walton, W. C., "Selected Analytical Methods for Well and Aquifer Evaluation," Illinois State Water Survey, Bul. 49, 1962.

12.29. White, D. E., et al., "Data of Geochemistry," U. S. Geol. Survey Prof. Paper 440-F, pp. 1-59, 1963.

12.30. Zohdy, A. A. R., "Geoelectrical and Seismic Refraction Investigations near San Jose, California," Ground-Water, v. 3, no. 3, pp. 41-48, 1965.

Appendix 1

Derivation of Darcy's Law

APPENDIX 1. DERIVATION OF DARCY'S LAW

M. K. Hubbert (1940) showed that Darcy's law can be derived theoretically. The theoretical development used here is a modification of Hubbert's original derivation and follows, again with some modifications, one by Collins (1961). The derivation is based on a balance of the several forces influencing flow and clearly illustrates the conditions for validity of the law.

Force must be exerted on a fluid to change its velocity. When a fluid element winds it way through a porous medium, the velocity changes from point to point, thus, the forces which produce these velocity variations also vary from point to point. If the microscopic variations in velocity are random and velocity is nearly uniform, then for steady flow the only macroscopic force exerted on a fluid by the solid is that associated with viscous flow. Because of the nearly uniform distribution of velocities and the random microscopic velocity variations, the algebraic sum of the lateral forces associated with the velocity variations is nearly zero over a macroscopic volume. For unsteady flow the inertial forces (i.e., those forces associated with the mass of fluid times its acceleration) in the direction of flow will not average to zero; however, for low-flow rates they are negligible (Collins, 1961).

For steady, laminar flow the force associated with viscous resistance to flow must be in equilibrium with the external and body forces on the fluid, i.e.:

$$F_\mu + F_p + F_g = 0 \tag{I-1}$$

where:

F_μ viscous drag force

F_p = external force associated with pressure

F_g = body force or weight of fluid

For the derivation consider a differential volume element oriented in the flow field so that the length, δL, is parallel to the flow direction and the area of each end of the element, δA, is perpendicular to δL. The length, δL, is measured positive in the flow direction.

Because the exact pore geometry cannot be stated mathematically, the magnitude of the viscous drag force cannot be stated directly. However, certain facts about it can be related:

a. The general equation describing the magnitude and direction of the viscouse shear force due to fluid flowing over a solid is:

$$F_{\mu} = \mu A \frac{dv}{ds} \tag{I-2}$$

where:

F_{μ} = the viscous shear force
A = the surface area over which F_{μ} acts
μ = viscosity
s = distance from a solid surface into the fluid

b. For laminar flow the relative distribution of discharge per unit area, v, (i.e., the position of the flowlines) is independent of the magnitude of v, and hence, dv/ds. This kinematic similarity for all liminar velocities implies that:

$$\frac{dv}{ds} \propto \frac{\delta Q}{\delta A}$$

where δQ is the discharge through the differential volume element.

c. The total internal surface area of the differential porous medium element is proportional to its bulk volume, $\delta A \delta L$.

Therefore, from equation I-2:

$$F_{\mu} \propto \frac{\delta Q}{\delta A} \mu (\delta A \delta L)$$

or

$$F_{\mu} = -B\mu \delta Q \delta L \tag{I-3}$$

where

B = a constant with dimensions $(\text{length})^{-2}$ characteristic of the pore geometry.

The minus sign in equation I-3 is necessary because the force resists flow.

The external force acting on the fluid contained in the porous medium can be expressed in terms of the pressure gradient, $\partial p/\partial L$, over the length of flow, δL. The pore area on which this pressure gradient acts is given by $n_e \delta A$ where n_e is the effective porosity, or the fraction of the bulk volume made up of the interconnecting pores (Collins, 1961, p. 3). Therefore:

$$F_p = -\frac{\partial p}{\partial L} \delta L n_e \delta A \qquad \text{(I-4)}$$

The minus sign indicates that the force resists flow when flow is toward increasing pressure and vice versa.

The body force acting on the fluid is its weight. Consequently, the component in the direction of flow is expressed by:

$$\begin{aligned} F_g &= -\rho g (n_e \delta A) \delta L \cos \theta \\ &= -\rho g (n_e \delta A) \delta L \frac{\partial z}{\partial L} \qquad \text{(I-5)} \end{aligned}$$

where:

ρ = the mass density

g = acceleration due to gravity

θ = the angle between the vertical or z axis and the flow direction

The minus sign indicates that this force resists flow when flow is upward ($\partial z/\partial L$ is positive) and aids it when it is downward ($\partial z/\partial L$ is negative).

Balancing the forces:

$$-B\mu \delta Q \delta L - \frac{\partial p}{\partial L} \delta L n_e \delta A - \rho g (n_e \delta A \delta L) \frac{\partial z}{\partial L} = 0$$

or

$$B\mu \delta Q \delta L = -\frac{\partial p}{\partial L} \delta L n_e \delta A - \rho g (n_e \delta A \delta L) \frac{\partial z}{\partial L}$$

or

$$\frac{\partial Q}{\partial A} = v = -\frac{n_e}{B\mu} \left(\frac{\partial p}{\partial L} + \rho g \frac{\partial z}{\partial L}\right) \qquad \text{(I-6)}$$

Hydraulic head, h, is defined as the sum of the pressure and elevation heads, or:

$$h = \frac{p}{\rho g} + z$$

and

$$\rho g h = p + \rho g z$$

Differentiating with respect to L

$$\rho g \frac{\partial h}{\partial L} = \frac{\partial p}{\partial L} + \rho g \frac{\partial z}{\partial L} \qquad \text{(I-7)}$$

Therefore, combining equations I-6 and I-7 is obtained as:

$$v = -\frac{n_e \rho g}{\mu B} \frac{\partial h}{\partial L} \qquad \text{(I-8)}$$

If the hydraulic conductivity is defined as:

$$K = \frac{n_e \rho g}{\mu B} \qquad \text{(I-9)}$$

and the intrinsic permeability is defined as:

$$k = \frac{n_e}{B} \qquad \text{(I-10)}$$

then:

$$v = -K \frac{\partial h}{\partial L} = -k \frac{\rho g}{\mu} \frac{\partial h}{\partial L} \qquad \text{(I-11)}$$

Equation I-11 is Darcy's law.

REFERENCES

1. Collins, R. E., "Flow of Fluids through Porous Materials," Reinhold, New York, 1961.

2. Hubbert, M. K., "The Theory of Ground-Water Motion," Jour. Geology, v. 48, no. 8, pt. 1, pp. 785-944, 1940.

Appendix 2

Derivation of Continuity Equation

APPENDIX 2. DERIVATION OF CONTINUITY EQUATION

The continuity equation can be derived by considering a small volume element, $\Delta x \Delta y \Delta z$, in the field of flow:

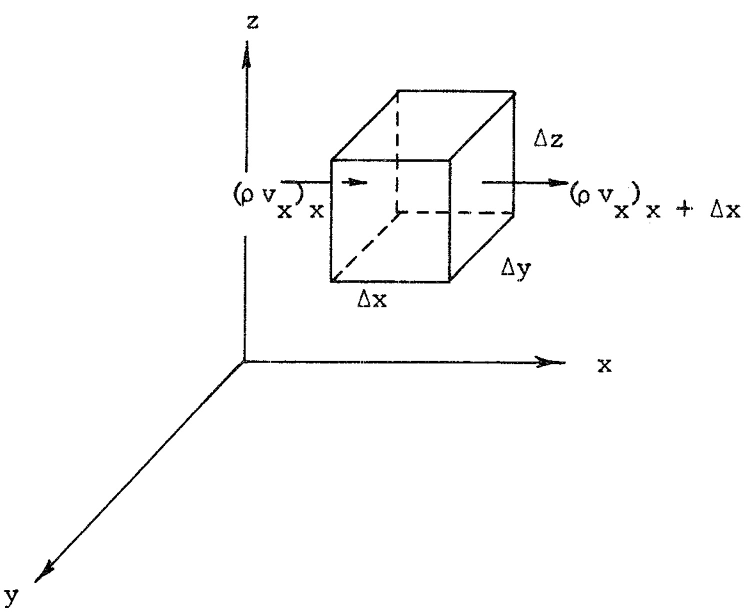


Fig. II.01. Definitive sketch of differential volume element

The rate at which mass enters the volume element through face x is $(\rho v_x)_x \Delta y \Delta z$, and the rate at which mass leaves the volume element through face $x + \Delta x$ is $(\rho v_x)_{x + \Delta x} \Delta y \Delta z$. The symbols v_x and ρ refer to discharge per unit area in the x direction and density, respectively. Similar expressions can be written for the other pairs of faces. The rate of mass accumulation is:

$$\frac{\partial \ (\rho n \Delta x \Delta y \Delta z S_w)}{\partial t}$$

where:

- n = the porosity
- S_w = the water saturation (100 percent saturation is equivalent to $S_w = 1$)
- t = time

This latter expression assumed that the dimensions of the volume element deform in response to the change in porosity. The Δx, Δy and Δz in the former expressions involving ρv_x must thus be interpreted as means in the small time interval ∂t and the discharge per unit areas are relative to the deforming coordinates. The mass balance is written:

$$[(\rho v_x)_x - (\rho v_x)_{x+\Delta x}]\ \Delta y \Delta z + [(\rho v_y)_y - (\rho v_y)_{y+\Delta y})\Delta x \Delta z +$$

$$[(\rho v_z)_z - (\rho v_z)_{z+\Delta z}]\ \Delta x \Delta y = \frac{\partial(\rho n \Delta x \Delta y \Delta z S_w}{\partial t} \qquad \text{(II-1)}$$

Dividing through by $\Delta x\Delta y\Delta z$, taking the limit as Δx, Δy, and Δz each approach zero, and defining $\Delta x\Delta y\Delta z = \Delta V$, then:

$$-\left[\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} + \frac{\partial(\rho v_z)}{\partial z}\right] = \frac{1}{\Delta V}\frac{\partial(\rho n \Delta V S_w)}{\partial t} \qquad \text{(II-2)}$$

If a source (or sink) is present, then equation II-2 becomes:

$$-\left[\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} + \frac{\partial(\rho v_z)}{\partial z}\right] + W = \frac{1}{\Delta V}\frac{\partial(\rho n \Delta V S_w)}{\partial t} =$$

$$\rho n \frac{\partial S_w}{\partial t} + S_w \left[n \frac{\partial \rho}{\partial t} + \rho \frac{\partial n}{\partial t} + \rho n \left(\frac{1}{\Delta V}\frac{\partial(\Delta V)}{\partial t}\right)\right] \qquad \text{(II-3)}$$

where W is the fluid injected into the field of flow at the rate of mass per unit time per unit volume. The sign of W is negative if fluid is withdrawn.

The quantity in brackets in equation II-3 can be evaluated a term at a time. The first term involves water volume and density changes, and thus involves water compressibility. Compressibility of water can be defined as:

$$c = -\frac{1}{Vw}\ \frac{dVw}{dp} \qquad \text{(II-4)}$$

where:

c = the volume of water released (or absorbed) per unit pressure change per unit volume

Vw = the volume of water

p = fluid pressure

Using $\rho = m/Vw$ (where ρ is the water density and m is the water mass), equation II-4 can be rewritten:

$$c = \frac{1}{\rho} \frac{d\rho}{dp} \qquad \text{(II-5)}$$

Taking the time derivative:

$$c\rho \frac{\partial p}{\partial t} = \frac{\partial \rho}{\partial t} \qquad \text{(II-6)}$$

The second two terms involve porous medium volume changes. To evaluate these terms, deformation and continuity relationships for the porous medium must be considered. It may be shown (Aris 1962, pp. 83-84) that:

$$\frac{\partial v_{sx}}{\partial x} + \frac{\partial v_{sy}}{\partial y} + \frac{\partial v_{sz}}{\partial z} = \frac{1}{\Delta V} \frac{\partial (\Delta V)}{\partial t} \qquad \text{(II-7)}$$

where:

v_{sx}, v_{sy}, v_{sz} = components of the velocity of the deforming granular skeleton of the porous medium

The volume of solids in the volume element, ΔV_s, is $(1 - n)\Delta V$. Therefore, if it is assumed that all deformation is due to porosity change so that $d(\Delta V_s) = 0$, then:

$$\frac{\partial v_{sx}}{\partial x} + \frac{\partial v_{sy}}{\partial y} + \frac{\partial v_{sz}}{\partial z} = - \frac{1}{(1 - n)} \frac{\partial (1 - n)}{\partial t} \qquad \text{(II-8)}$$

or

$$\left(\frac{\partial v_{sx}}{\partial x} + \frac{\partial v_{sy}}{\partial y} + \frac{\partial v_{sz}}{\partial z}\right) (1 - n) = \frac{\partial n}{\partial t} \qquad \text{(II-9)}$$

The components of displacement vector, u, can be used to evaluate v_s:

$$\left.\begin{aligned} v_{sx} &= \frac{\partial u_x}{\partial t} \\ v_{sy} &= \frac{\partial u_y}{\partial t} \\ v_{sz} &= \frac{\partial u_z}{\partial t} \end{aligned}\right\} \tag{II-10}$$

so that:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_{sy}}{\partial y} + \frac{\partial v_{sz}}{\partial z} = \frac{\partial}{\partial t}\frac{\partial u_x}{\partial x} + \frac{\partial}{\partial t}\frac{\partial u_x}{\partial y} + \frac{\partial}{\partial t}\frac{\partial u_z}{\partial z}$$

The volume strain, e, is defined by (Verruijt, in DeWiest, 1969, pp. 341-342):

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} = e \tag{II-11}$$

Therefore:

$$\frac{\partial v_{sx}}{\partial x} + \frac{\partial v_{sy}}{\partial y} + \frac{\partial v_{sz}}{\partial z} = \frac{\partial e}{\partial t} \tag{II-12}$$

and

$$\frac{\partial n}{\partial t} = (1 - n)\frac{\partial e}{\partial t} \tag{II-13}$$

Finally:

$$\frac{1}{\Delta V}\frac{\partial(\Delta V)}{\partial t} = \frac{\partial e}{\partial t} \tag{II-14}$$

Equations II-6, II-13, and II-14 may be substituted into equation II-3 so that the quantity in brackets is:

$$n\frac{\partial \rho}{\partial t} + \rho\frac{\partial n}{\partial t} + \rho\, n\left(\frac{1}{\Delta V}\frac{\partial(\Delta V)}{\partial t}\right) = n\rho\left(c\frac{\partial p}{\partial t} + \frac{1}{n}\frac{\partial e}{\partial t}\right) \tag{II-15}$$

If e can be considered to be the variable function $e = e(p)$ then:

$$c_f = \frac{de}{dp} \tag{II-16}$$

where c_f is a formation compressibility; c_f is only constant for special cases.

Equation II-3 can be written with the aid of equation II-15 and II-16 as:

$$\begin{aligned}\left(\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} + \frac{\partial(\rho v_z)}{\partial z}\right) + W &= \rho n \frac{\partial S_w}{\partial t} + S_w n\rho\left(c + \frac{c_f}{n}\right)\frac{\partial p}{\partial t} \\ &= \rho n \frac{\partial S_w}{\partial t} + S_w n\rho^2 g\left(c + \frac{c_f}{n}\right)\frac{\partial h}{\partial t}\end{aligned} \tag{II-17}$$

For the special case where only the z coordinate deforms:

$$\frac{\partial e}{\partial t} = \frac{1}{\Delta z}\frac{\partial(\Delta z)}{\partial t} \tag{II-18}$$

Jacob (1950, p. 329) defined

$$\alpha = -\frac{1}{\Delta z}\frac{d(\Delta z)}{d\sigma_z} \tag{II-19}$$

where σ_z = vertical component of compressive stress.

If there are no total stress changes, then

$$d\sigma_z = -dp$$

and the last term in equation II-16 becomes:

$$S_w n\rho^2 g \left(c + \frac{\alpha}{n}\right)\frac{\partial h}{\partial t} = S_w \rho S_s \frac{\partial h}{\partial t} \tag{II-20}$$

With $c = \beta$, this definition of S_s (specific storage) is identical with that given by Cooper (1966, p. 4786).

Usually the changes in water density are much smaller than the changes in other quantities, so it can be taken as constant. Therefore:

$$-\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right) + W_s = n\,\frac{\partial S_w}{\partial t} + S_w S_s\,\frac{\partial h}{\partial t} \qquad \text{(II-21)}$$

where $W_s = W/\rho$.

REFERENCES

1. Aris, Rutherford, "Vectors, Tensors, and the Basic Equations of Fluid Mechanics," Prentice-Hall, New Jersey, 1962.

2. Cooper, H. H., Jr., "The Equation of Ground-Water Flow in Fixed and Deforming Coordinates," Jour. Geophys. Res., v. 71, no. 20, pp. 4785-4790, 1966.

3. DeWiest, R. J. M., (ed.), "Flow Through Porous Media," Academic Press, New York, 1969.

4. Rouse, Hunder (ed.), "Engineering Hydraulics," Chapter 5, Flow of Ground Water, by C. E. Jacob, John Wiley and Sons, New York, pp. 321-386, 1950.

Appendix 3

Development of Flow Net Theory

APPENDIX 3. DEVELOPMENT OF FLOW NET THEORY

Derivation of Orthogonality of Flow Lines and Equipotential Lines

The conditions for the orthogonality of flow lines and equipotential lines can be derived by examining a few of the properties of these curves. The total differential along an equipotential line for steady-state, two-dimensional flow is:

$$dh = \frac{\partial h}{\partial x} dx + \frac{\partial h}{\partial z} dz = 0 \qquad \text{(III-1)}$$

Using Darcy's law, i.e.:

$$v_x = -K \frac{\partial h}{\partial x} \qquad \text{(III-2)}$$

and

$$v_z = -K \frac{\partial h}{\partial z} \qquad \text{(III-3)}$$

Equation 1 becomes:

$$\frac{v_x}{K} dx + \frac{v_z}{K} dz = 0 \qquad \text{(III-4)}$$

or, because K is a scalar:

$$v_x dx + v_z dz = 0 \qquad \text{(III-5)}$$

Therefore:

$$\frac{dz}{dx} = \frac{-v_x}{v_z} \qquad \text{(III-6)}$$

along an equipotential line. For a similar relationship along a flow line consider the differential streamtube in fig. III.01.

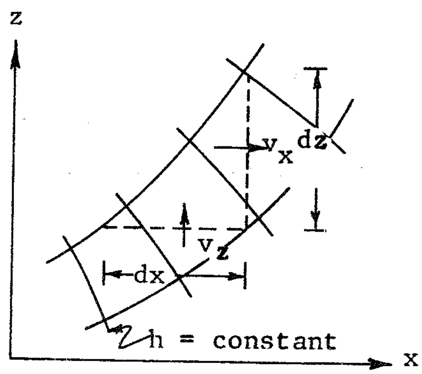


Fig. III-01. Definitive sketch of differential streamtube element

For steady-state flow with no sources or sinks, the discharge per unit thickness through the streamtube is a constant, dq. Therefore:

$$dq = v_z dx = v_x dz \tag{III-7}$$

or

$$v_z dx - v_x dz = 0 \tag{III-8}$$

or

$$\frac{dz}{dx} = \frac{v_z}{v_x} \tag{III-9}$$

along a flow line.

Equations III-6 and III-9 indicate that at any point, under the conditions considered, the slopes of the equipotential and flow lines are at right angles to one another.

Derivation of Refraction Law for Flow Across a Conductivity Boundary

Consider flow across the boundary depicted by fig. 5.03. Conservation of matter requires that the quantity of fluid entering one region must be equal to the quantity of fluid leaving the other, or:

$$V_{1n} = V_{2n} \tag{III-10}$$

where:

V_{1n} = the normal discharge per unit area leaving region 1

V_{2n} = the normal discharge per unit area entering region 2.

Also, in order to avoid a discontinuity in flow on either side of the boundary, the tangential hydraulic gradients in both regions must be equal:

$$\left(\frac{\partial h}{\partial s}\right)_1 = \left(\frac{\partial h}{\partial s}\right)_2 \tag{III-11}$$

To yield the final refraction law, equations III-10 and III-11 are modified and combined. Equation III-10 can be expressed as:

$$K_1 \left(\frac{\partial h}{\partial n}\right)_1 = K_2 \left(\frac{\partial h}{\partial n}\right)_2 \tag{III-12}$$

where:

K_1 and K_2 = the conductivities in regions 1 and 2, respectively

$\left(\frac{\partial h}{\partial n}\right)_1$ and $\left(\frac{\partial h}{\partial n}\right)_2$ = the normal hydraulic gradients in regions 1 and 2, respectively.

Also:

$$\left(\frac{\partial h}{\partial n}\right)_1 = \left(\frac{\partial h}{\partial \ell}\right)_1 \sin \alpha \tag{III-13}$$

and

$$\left(\frac{\partial h}{\partial n}\right)_2 = \left(\frac{\partial h}{\partial \ell}\right)_2 \sin \beta \qquad \text{(III-14)}$$

where:

α = the angle between the boundary and the flow lines in region 1

β = the angle between the boundary and the flow lines in region 2

ℓ = the length along a flow line

Equation III-12 can then be written:

$$K_1 \left(\frac{\partial h}{\partial \ell}\right)_1 \sin \alpha = K_2 \left(\frac{\partial h}{\partial \ell}\right)_2 \sin \beta \qquad \text{(III-15)}$$

Equation III-11 can be modified similarly to yield:

$$\left(\frac{\partial h}{\partial \ell}\right)_1 \cos \alpha = \left(\frac{\partial h}{\partial \ell}\right)_2 \cos \beta \qquad \text{(III-16)}$$

Dividing equation III-15 by equation III-16 yields:

$$K_1 \frac{\sin \alpha}{\cos \alpha} = K_2 \frac{\sin \beta}{\cos \beta} \qquad \text{(III-17)}$$

or

$$\frac{K_2}{K_1} = \frac{\tan \alpha}{\tan \beta} \qquad \text{(III-18)}$$

which is the basic "refraction law" for flow from a region of one conductivity into another.

Appendix 4

Aquifer Test Data Reduction Program

```
                    ===========================
                    =  A P P E N D I X    4   =
                    ===========================
```

AQUIFER TEST DATA REDUCTION PROGRAM FOR WATER-TABLE AQUIFERS

= = = = PROGRAM LISTING = = = =

```
      DIMENSION T(100),S(100),FACT(100),SC(100)
   10 FORMAT (2X,I6,I8,8F8.0)
   20 FORMAT (2X,F6.0,9F8.0)
   30 FORMAT (1H1,17X,21HOBSERVATION WELL NO. ,I3/18X,2HR=,F8.2,6X,2HB=,
     1F8.2/1H0,4X,7HTIME, T,6X,11HDRAWDOWN, S,6X,6HR**2/T,7X,11HCORRECTE
     2D S)
   40 FORMAT (1H ,1X,E12.5,3(3X,E12.5))
      READ 10,NW                                                    CARD A
      DO 100 K=1,NW
      READ 10,NOBSW,NPTS,R,B                                        CARD B
      READ 20,(T(I),I=1,NPTS)                                       CARD C
      READ 20,(S(I),I=1,NPTS)                                       CARD D
      RSQ=R**2
      TEMP=2.*B
      DO 50 I=1,NPTS
      IF (T(I).LE.0.) GO TO 50
      FACT(I)=RSQ/T(I)
   50 SC(I)=S(I)-S(I)**2/TEMP
      PRINT 30,NOBSW,R,B
      DO 60 I=1,NPTS
      PRINT 40,T(I),S(I),FACT(I),SC(I)
   60 CONTINUE
  100 CONTINUE
      STOP
      END
```

= = = = SAMPLE INPUT = = = =

```
    1
    2       10     50.3    59.4
    0        2        5       8      11      14      16      24      29      34
    0     .047     .087    .112    .162    .202    .212    .222    .242    .262
```

= = = = SAMPLE OUTPUT = = = =

```
              OBSERVATION WELL NO.   2
              R=    50.30        B=    59.40

   TIME, T       DRAWDOWN, S       R**2/T         CORRECTED S
   .00000          .00000          .00000          -.00000
   .20000+01       .47000-01       .12650+04        .46981-01
   .50000+01       .87000-01       .50602+03        .86936-01
   .80000+01       .11200+00       .31626+03        .11189+00
   .11000+02       .16200-00       .23001+03        .16178-00
   .14000+02       .20200-00       .18072+03        .20166-00
   .16000+02       .21200-00       .15813+03        .21162-00
   .24000+02       .22200-00       .10542+03        .22159-00
   .29000+02       .24200-00       .87244+02        .24151-00
   .34000+02       .26200-00       .74414+02        .26142-00
```

Appendix 5

Characteristics of Water-Bearing Materials

APPENDIX 5. CHARACTERISTICS OF WATER-BEARING MATERIALS

Table V.01. Typical Hydraulic Conductivity
(California Department of Water Resources, 1968)

Material	Hydraulic Conductivity (gpd per sq. ft. @ 60° F)		
Granite	0.0000009	-	0.000005
Slate	0.000001	-	0.000003
Dolomite	0.00009	-	0.0002
Hematite	0.000002	-	0.009
Limestone	0.00001	-	0.002
Gneiss	0.0005	-	0.05
Basalt	0.00004	-	1
Tuff	0.0003	-	10
Sandstone	0.003	-	30
Till	0.003	-	0.5
Loess	1	-	30
Beach sand	100	-	400
Dune sand	200	-	600
Clay	0.001	-	1
Silt	1	-	10
Very Fine Sand	10	-	100
Fine Sand	100	-	1,000
Medium Sand	1,000	-	4,500
Coarse Sand	4,500	-	6,500
Very Coarse Sand	6,500	-	8,000
Very Fine Gravel	8,000	-	11,000
Fine Gravel	11,000	-	16,000
Medium Gravel	16,000	-	22,000
Coarse Gravel	22,000	-	30,000
Very Coarse Gravel	30,000	-	40,000
Cobbles	Over 40,000		

Table V.02. Specific Yields of Various Materials
[Rounded to Nearest Whole Percentage]
(California Department of Water Resources, 1968)

Material	Number of Determinations	Specific Yield		
		Maximum	Minimum	Average
Clay	15	5	0	2
Silt	16	19	3	8
Sandy Clay	12	12	3	7
Fine Sand	17	28	10	21
Medium Sand	17	32	15	26
Coarse Sand	17	35	20	27
Gravelly Sand	15	35	20	25
Fine Gravel	17	35	21	25
Medium Gravel	14	26	13	23
Coarse Gravel	14	26	12	22

Table V.03. Compilation of Specific Yield for Various Materials

[All values rounded off to nearest whole percentage]

(California Dept. of Water Resources, 1968)

Material	Valley fill, Calif. (Eckis, 1934)	Mokelumne Area, Calif. (Piper and others, 1939)	Santa Ynez River Basin, Calif. (Upson and Thomasson, 1951)	Sacramento Valley, Calif. (Poland and others, 1949)	Smith River Plain, Calif. (Back, 1957)	Ventura County, Calif. (Calif. Water Resources Board, 1956)	Santa Margarita Valley, Calif. (Calif. Dept. Public Works, 1956)	Tia Juana Basin, Calif. (Calif. Water Rights Board, 1957)	San Luis Obispo County, Calif. (Calif. Water Resources Board, 1958)	San Joaquin Valley, Calif. (Davis and others, 1959)	Eureka area, Calif. (Evenson, 1959)	Santa Ynez Basin, Calif. (Wilson, 1959)	Rechna Doab, Pakistan (Kazmi, 1961)	Napa-Sonoma Valleys, Calif. (Kunkel and Upson, 1960)	Humboldt River Valley, Nev. (Cohen, 1963)	Unconsolidated Alluvium (Preuss and Todd, 1963)	Little Bighorn River Valley, Mont. (Moulder and others, 1960)	Average specific yield
Clay	1	4	2	3	1	0	1	1	3	3	3	5	3	3	0.5	--	--	2
Silt	10	4	12	3	--	3	10	10	5	5	10	5	5	5	19	4	17	8
Sandy clay	10	4	12	3	5	5	5	5	5	5	10	--	--	10	----	--	--	7
Fine sand	21	26	12	10	10	25	28	25	25	10	20	20	27	20	26	23	32	21
Medium sand	31	26	30	20	15	25	28	30	25	25	20	30	28	20	28	28	32	26
Coarse sand	31	35	35	20	25	25	28	32	25	25	20	30	23	20	27	28	32	27
Gravelly sand	31	35	35	20	25	21	22	28	21	25	20	--	23	20	----	22	32	25
Fine gravel	27	35	35	25	25	21	22	26	21	25	25	25	26	25	19	17	25	25
Medium gravel	21	--	--	25	25	21	22	23	21	25	25	25	26	25	----	13	25	23
Coarse gravel	14	--	--	25	25	21	22	18	21	25	25	25	26	25	----	12	--	22

Appendix 6

Definitions of Selected Ground-Water Terms

APPENDIX 6

DEFINITIONS OF SELECTED GROUND-WATER TERMS

Anistropy	Directional variation of a physical property at a point in the porous medium.
Aquiclude	A lithologic unit which has appreciably greater transmissivity than an aquifuge but less transmissivity than an aquitard.
Aquifer	A lithologic unit (or combination of such units) which has appreciably greater transmissivity than adjacent units and which has capability to store and transmit water recoverable in economically usable quantities (reference 1.08, p. 10).
Aquifuge	A lithologic unit which has virtually no transmissivity.
Aquitard	A lithologic unit which has appreciably greater transmissivity than an aquiclude but considerably less transmissivity than an aquifer.
Barometric efficiency	Ratio of the atmospheric pressure-induced change in water level in a well to the corresponding change in atmospheric pressure, both expressed in the same units.
Barrier boundary	Surface across which there is little or no flow. Folds, faults, ground-water divides, aquicludes and aquifuges often form barrier boundaries.
Base flow	Portion of streamflow derived from ground-water discharge.
Capillary water	Portion of soil water held by surface tension as a continuous film around particles and in capillary voids. Water in the capillary zone (reference 1.02, p. 39).
Capillary fringe	Zone above the water table, and in hydraulic continuity with it, where water occurs under negative pressure. Its thickness is usually determined as the upper limit of the capillary rise of water.

Coefficient of vertical permeability	Rate of vertical flow of water in gallons per day through a horizontal cross-sectional area of one square foot of confining bed under hydraulic gradient of one foot per foot at prevailing water temperature.
Confined aquifer	Aquifer which contains water under sufficient pressure that water levels in wells tapping it rise above the bottom of the confining bed.
Connate water	Water that has been out of contact with the atmosphere for a relatively long period of time.
Dewatering	Process of lowering the water table.
Discharge area	Area in which ground-water flow lines converge and are directed toward water table.
Drawdown	The difference between the nonpumping water level at some time and the pumping level at that time.
Effective porosity	Fraction of the bulk volume made up of interconnecting pores.
Elevation head	Elevation above some selected datum plane.
Equipotential line	Contour line which represents equal hydraulic head.
Evapotranspiration	Portion of the precipitation returned to the air through direct evaporation and/or by transpiration.
Field capacity	Amount of water held in a soil by capillary forces after gravity drainage, expressed as the ratio of water retained to the weight of dry soil.
Flow line	Line which represents the path a fluid particle would follow through a porous medium.
Flow net	A graphical representation comprising a family of flow lines and equipotential lines within a flow region.
Flow system	A set of flow lines in which any two flow lines adjacent at one point in the flow region remain adjacent throughout the entire flow region, and that can be intersected anywhere by an uninterrupted surface across which flow occurs only in one direction (reference 4.14, p. 81).

APPENDIX 6

DEFINITIONS OF SELECTED GROUND-WATER TERMS

Anistropy	Directional variation of a physical property at a point in the porous medium.
Aquiclude	A lithologic unit which has appreciably greater transmissivity than an aquifuge but less transmissivity than an aquitard.
Aquifer	A lithologic unit (or combination of such units) which has appreciably greater transmissivity than adjacent units and which has capability to store and transmit water recoverable in economically usable quantities (reference 1.08, p. 10).
Aquifuge	A lithologic unit which has virtually no transmissivity.
Aquitard	A lithologic unit which has appreciably greater transmissivity than an aquiclude but considerably less transmissivity than an aquifer.
Barometric efficiency	Ratio of the atmospheric pressure-induced change in water level in a well to the corresponding change in atmospheric pressure, both expressed in the same units.
Barrier boundary	Surface across which there is little or no flow. Folds, faults, ground-water divides, aquicludes and aquifuges often form barrier boundaries.
Base flow	Portion of streamflow derived from ground-water discharge.
Capillary water	Portion of soil water held by surface tension as a continuous film around particles and in capillary voids. Water in the capillary zone (reference 1.02, p. 39).
Capillary fringe	Zone above the water table, and in hydraulic continuity with it, where water occurs under negative pressure. Its thickness is usually determined as the upper limit of the capillary rise of water.

Coefficient of vertical permeability	Rate of vertical flow of water in gallons per day through a horizontal cross-sectional area of one square foot of confining bed under hydraulic gradient of one foot per foot at prevailing water temperature.
Confined aquifer	Aquifer which contains water under sufficient pressure that water levels in wells tapping it rise above the bottom of the confining bed.
Connate water	Water that has been out of contact with the atmosphere for a relatively long period of time.
Dewatering	Process of lowering the water table.
Discharge area	Area in which ground-water flow lines converge and are directed toward water table.
Drawdown	The difference between the nonpumping water level at some time and the pumping level at that time.
Effective porosity	Fraction of the bulk volume made up of interconnecting pores.
Elevation head	Elevation above some selected datum plane.
Equipotential line	Contour line which represents equal hydraulic head.
Evapotranspiration	Portion of the precipitation returned to the air through direct evaporation and/or by transpiration.
Field capacity	Amount of water held in a soil by capillary forces after gravity drainage, expressed as the ratio of water retained to the weight of dry soil.
Flow line	Line which represents the path a fluid particle would follow through a porous medium.
Flow net	A graphical representation comprising a family of flow lines and equipotential lines within a flow region.
Flow system	A set of flow lines in which any two flow lines adjacent at one point in the flow region remain adjacent throughout the entire flow region, and that can be intersected anywhere by an uninterrupted surface across which flow occurs only in one direction (reference 4.14, p. 81).

Fluid pressure	Pressure at a specific point determined by height a static liquid would rise above that point plus atmospheric pressure.
Force potential	The approximate amount of mechanical energy contained at a point in a system; equal to hydraulic head at that point multiplied by acceleration due to gravity.
Geohydrologic unit	An aquifer, an aquiclude, or a combination of aquifers and aquicludes that compose a framework for a reasonably distinct hydraulic system (reference 1.08 p. 26).
Gravity drainage	Free water in earth materials which flows downward under the influence of gravity.
Ground water	A body of subsurface water in which fluid pressure is greater than atmospheric.
Ground water basin	A three-dimensional closed system that contains the entire flow paths followed by all the water recharging the basin (reference 4.06, p. 624).
Homogeneity	The physical properties of the porous medium do not vary from point to point in the medium.
Hydraulic conductivity	Ratio of flow velocity to driving force (hydraulic gradient) for viscous flow under saturated conditions of a specified liquid in a porous medium.
Hydraulic gradient	Maximum increase in hydraulic head per unit length of flow path.
Hydraulic head	The sum of the pressure and elevation heads.
Hydrologic boundary	An abrupt change in hydraulic properties in or at the margin of a ground-water basin. Recharge and barrier boundaries are types of hydrologic boundaries.
Hydrostatic pressure	Pressure exerted by water at any given point in a body of water at rest.
Hydrostratigraphic unit	Earth materials with considerable lateral extent that compose a geologic framework for a reasonably distinct hydrologic system (reference 1.08 p. 126).
Hygroscopic water	Water so tightly attached to soil particles that it will not evaporate except at temperatures above the boiling point.

Infiltration	The inflow of water into earth materials.
Intrinsic permeability	Ability of a porous medium to transmit fluid. It is a function of the properties of the porous medium only.
Perched ground water	A lense of ground water separated from an underlying body of ground water by unsaturated earth material.
Piezometric surface	Imaginary surface defined by the level to which water will rise in wells tapping a confined aquifer.
Porosity	Proportion of the total volume of porous medium occupied by voids.
Potentiometric surface	General term which refers either to the water table or the piezometric surface.
Pressure head	Fluid pressure divided by unit weight of water.
Recharge area	Area in which ground-water flow lines diverge and are directed away from water table.
Recharge boundary	Surface across which there is nearly constant hydraulic head. Rivers, lakes, and other bodies of surface water often form recharge boundaries.
Reynolds number	A dimensionless ratio of inertial to viscous (or resistive) forces.
Semiconfined aquifer	Aquifer bounded by rock units or aquitards (aquicludes) of low permeability.
Sink	Refers to the discharge of water from a flow system.
Source	Refers to the addition of water to the flow system.
Specific capacity	Yield in gallons per minute per foot of drawdown for a well at a selected time after pumping is started.
Specific retention	Quantity of water per unit total volume which will not drain under the influence of gravity.
Specific storage	Quantity of water in storage that is released from (or taken into) a unit volume of aquifer per unit change in hydraulic head.
Specific yield	Amount of water yielded per unit drawdown per unit of horizontal area dewatered.

Spring	Natural surface discharge of ground water having a concentrated flow.
Static water level	Level at which water stands in a nonpumping well.
Steady-state flow	Fluid movement that is not time dependent.
Storage coefficient	Quantity of water released from (or taken into) storage in a column of aquifer with unit cross section and length equal to thickness of aquifer per unit change in hydraulic head.
Subsurface water	All water that exists in the interstices of porous earth materials.
Tidal efficiency	Ratio of the load-induced change in water level in a well to the corresponding change in the stage of the tide both expressed in the same units.
Transmissivity	Rate of horizontal water flow in gallons per day through a vertical strip of aquifer 1 foot wide and extending full saturated thickness under hydraulic gradient of 1 foot per foot at prevailing water temperature.
Unconfined aquifer	Aquifer within which the water table is located.
Unsteady-state flow	Fluid movement that is time dependent.
Vadose water	Water in the unsaturated zone.
Water table	Surface along which the fluid pressure is atmospheric and below which the fluid pressure is greater than atmospheric.
Well efficiency	Ratio of the actual specific capacity after 24 hours of pumping to the theoretical specific capacity at that time.

Appendix 7

Finite Element Solution of Steady State Potential Flow Problems

FINITE ELEMENT SOLUTION OF STEADY STATE POTENTIAL FLOW PROBLEMS

THE HYDROLOGIC ENGINEERING CENTER
GENERALIZED COMPUTER PROGRAM
723-G2-L2440

NOVEMBER 1970

WATER RESOURCES SUPPORT CENTER
THE HYDROLOGIC ENGINEERING CENTER
609 SECOND STREET
DAVIS, CALIFORNIA 95616

(916) 551-1748

FINITE ELEMENT SOLUTION OF
STEADY STATE POTENTIAL FLOW PROBLEMS

TABLE OF CONTENTS

	Page
I. INTRODUCTION	
1. Origin of Program	1
2. Purpose of Program	1
II. SCOPE OF PROGRAM	
1. Theoretical Assumptions	1
a. Darcy's Law	1
b. The Continuity Equation	2
c. Basic Flow Equations	3
2. General Areas of Application	4
a. Foundations and Structures Problems	4
b. Ground-Water Flow System Analysis	5
c. Well Flow Problems	5
d. Hydraulics Problems	5
3. Special Areas of Application	5
a. Determination of Hydraulic Conductivity	5
b. Determination of the Free Surface	6
III. PROGRAM DESCRIPTION	
1. Theoretical Basis	6
2. Program Organization	7
IV. PROGRAM USAGE	
1. Equipment Requirements	7
2. Input Preparation	7
a. Organization of Input Data	7
b. Numbering of Nodes and Elements	9
c. Boundary Conditions	10
d. Axisymmetric Cylindrical Coordinates	10
e. Units	10

TABLE OF CONTENTS - CONTINUED

3. Program Output

4. Example Problems
   a. Example 1. Regional Ground-Water Problem 11
   b. Example 2. Axisymmetric Radial Flow to a Well in a Semiconfined Aquifer 26
   c. Example 3. Steady State Seepage from a Ditch Considering the Effects of Capillarity 31

V. REFERENCES 47

ADDENDUM 1. THE FINITE ELEMENT METHOD

1. Finite Element Formulation
2. Quadrilateral Elements
3. Anisotropic Media
4. Axisymmetric Cylindrical Coordinates

ADDENDUM 2. SOLUTION OF MATRIX EQUATION

ADDENDUM 3. DEFINITIONS OF VARIABLES

ADDENDUM 4. SOURCE LISTING

ADDENDUM 5. INPUT REQUIREMENTS

1. Card Format
2. Multiple Jobs
3. Card Contents

SUMMARY OF INPUT CARDS

LIST OF FIGURES

1.	General Flow Diagram	8
2.	Potentiometric Surface and Saturated Thickness of the Alluvial Aquifer in 1950 for the Kansas River Valley near Topeka, Kansas	12
3.	Element Configuration for Example 1	13
4.	Input for Example 1	18
5.	Printout for Example 1	20
6.	Contours of Hydraulic Head for Run 1, Example 1	24
7.	Contours of Hydraulic Head for Run 2, Example 1	25
8.	Axisymmetric Radial Flow to a Well in a Semiconfined Aquifer (Example 2)	26
9.	Element Configuration for Example 2	27
10.	Input for Example 2	28
11.	Program for Generating C and D Cards for Example 2	30
12.	Printout for Example 2	32
13.	Geometry and Boundary Conditions for Example 3	35
14.	Initial Element Configuration for Example 3	37
15.	Example of Interpolation Scheme Used to Obtain Trial Positions for the Free Surface (Example 3)	38
16.	Values of Total Head and Pressure Head Obtained from Iteration 7, Example 3	39
17.	Contours of Total Head for Iteration 7, Example 3	40
18.	Contours of Pressure Head for Iteration 7, Example 3	41
19.	Input for Iteration 7, Example 3	42
20.	Printout for Iteration 7, Example 3	44
A1-1	Triangular Element	Addendum 1 4
A1-2	Quadrilateral Element	Addendum 1 7

LIST OF TABLES

		Page
1.	Data for Regional Ground Water Problem (Example 1)	14
2.	Hydraulic Conductivities for Regional Ground Water Problem (Example 1)	17
3.	Comparison of Analytic and Numerical Solutions for Example 2	31

FINITE ELEMENT SOLUTION OF STEADY STATE POTENTIAL FLOW PROBLEMS

THE HYDROLOGIC ENGINEERING CENTER GENERALIZED COMPUTER PROGRAM 723-G2-L2440

I. INTRODUCTION

1. Origin of Program

This program, with the exception of subroutine BSOLVE*, was developed in The Hydrologic Engineering Center by Richard Cooley and John Peters. Up-to-date information and copies of source statement cards for the program, which is written in Fortran IV, can be obtained from the Center upon request by Government and cooperating agencies.

In addition to the version described herein, two additional versions of the computer program are available. One version has a PLOT subroutine that provides for a printer plot of contours of the potential function. The second version permits the use of quadrilateral and triangular elements in any combination.

2. Purpose of Program

This program is intended for application to problems involving steady, two-dimensional or axisymmetric flow through heterogeneous, anisotropic porous media of virtually any internal or external geometry. Although the program was developed primarily with ground water and seepage problems in mind, it is equally applicable to potential flow problems of any type, such as those involving heat conduction or electric potential distribution.

II. SCOPE OF PROGRAM

1. Theoretical Assumptions

The basic equations that are solved numerically by the finite element method are derived on the basis of a number of assumptions as to the physics of motion and achievement of continuity of mass. These assumptions must at least be approximately fulfilled in order to apply the method. The following brief discussion outlines the basic equations and associated assumptions.

a. Darcy's law. Darcy's law is the dynamic equation expressing the conditions which control the flow rate under low and medium velocity, laminar flow conditions in a porous medium where inertial forces (i.e., forces associated with the mass of the fluid times its acceleration) can be neglected. It can be conveniently stated in vector component form for two dimensions as

---

* Subroutine BSOLVE, which solves matrix equations by a specialized Gauss elimination procedure, is described in (1).

$$\left.\begin{aligned} v_x &= -K_{xx}\frac{\partial h}{\partial x} \\ v_y &= -K_{yy}\frac{\partial h}{\partial y} \end{aligned}\right\} \qquad (1)$$

where

v_x = the component of the bulk velocity or discharge per unit area in the x direction,

v_y = the component of the bulk velocity in the y direction,

K_{xx} = a principal component of the conductivity ellipse which corresponds in direction to the x direction,

K_{yy} = a principal component of the conductivity ellipse which corresponds in direction to the y direction,

$\frac{\partial h}{\partial x}$ = the component of hydraulic gradient in the x direction,

$\frac{\partial h}{\partial y}$ = the component of hydraulic gradient in the y direction,

h = hydraulic head (a potential function) $= \frac{p}{\gamma} + z$,

p = fluid pressure,

γ = unit weight of the fluid, and

z = elevation above a datum ($z = y$ if a vertical flow cross section is being used).

The form of Darcy's law given by equation 1 assumes (a) that the porous medium is fully saturated, (b) that it is anisotropic with two principal values of hydraulic conductivity K_{xx} and K_{yy}, oriented in the x and y directions (or isotropic if $K_{xx} = K_{yy}$), and (c) that these and intermediate values of hydraulic conductivity are the same for flow in opposite directions (2, p. 137).

b. The continuity equation. This equation expresses the idea that fluid may be neither created nor destroyed. Flow is assumed to be steady state (i.e., the components of velocity do not vary with time). One form of the continuity equation for steady state flow in two dimensions is

$$- \left[\frac{\partial (v_x m)}{\partial x} + \frac{\partial (v_y m)}{\partial y} \right] = W(x,y) \tag{2}$$

where

m = the local thickness of the flow region, and

$W(x,y)$ = a source or sink term in units of discharge per unit area; it is positive for a sink and negative for a source.

In addition to steady state conditions, equation 2 assumes (a) that variations in fluid density throughout the porous medium are negligible, (b) that variations in thickness of the flow region (measured normal to the x,y plane) are small enough that they produce negligible flow components in the direction normal to the x,y plane, and (c) that the flow system is in dynamic equilibrium or is only slowly unsteady. An example of a system in dynamic equilibrium would be a ground-water basin in which fluctuations in the water table position are cyclic with seasons of the year, are small enough that they constitute a small percentage of the total flow region, and do not change the relative configuration of the water table (9, p. 583). A mean potential distribution could then be estimated using the program.

c. Basic flow equations. The equations to be solved result from combining equations 1 and 2:

$$\frac{\partial}{\partial x} \left(K_{xx} m \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y} \left(K_{yy} m \frac{\partial h}{\partial y}\right) = W(x,y) \tag{3}$$

If a cross section of a three dimensional flow system is being studied then $m = 1$, and the equation is

$$\frac{\partial}{\partial x}\left(K_{xx} \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy} \frac{\partial h}{\partial y}\right) = W'(x,y) \tag{4}$$

where

$W'(x,y)$ = source or sink in units of discharge per unit area per unit thickness.

For constant, isotropic conductivity or for flow of an ideal, incompressible fluid

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{W'(x,y)}{K} \qquad (5)$$

where K is the constant conductivity (K = 1 for flow of an ideal, incompressible fluid).

The program will also solve the equation describing flow which is symmetric around a central axis. For this case the Cartesian coordinates (x,y) are transformed to axisymmetric cylindrical coordinates (r,z), and the appropriate flow equation is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(K_{rr} r \frac{\partial h}{\partial r}\right) + \frac{\partial}{\partial z}\left(K_{zz} \frac{\partial h}{\partial z}\right) = 0 \qquad (6)$$

It should be noted that the directions of the x and y axes do not have to be constant throughout the flow region. As will be shown further on, the directions need be constant only within each element (although they must always be normal to one another). This allows for analysis of porous media with varying directions of anisotropy. In applying this idea to the (r,z) coordinate system, it must be remembered that the properties of the porous medium must be axisymmetric. For all practical purposes this limits analysis of axisymmetric ground-water flow problems to vertically stratified porous media so that the symmetry axis is vertical.

2. General Areas of Application

a. Foundations and structures problems. Problems which require steady state potential distribution, fluid pressure estimates (such as uplift pressures) at various points, or an estimate of the quantity of discharge through a foundation or structure can often be analyzed using two dimensional methods such as those incorporated in the program described here. Structurally very complicated foundations such as folded and faulted rocks can theoretically be analyzed provided that the conductivity components can be estimated. Conversely, the influence of heterogeneity or varying degrees or directions of assumed anisotropy on the potential distribution or discharge could be determined for a flow region. The use of flow nets (which can be obtained from finite element solutions) for seepage and drainage problems is covered in (3).

Often applications to seepage analysis require the determination of a free surface. The program will not do this internally, but a manual trial and error method is described under part 3 of this section.

b. Ground-water flow system analysis. The ground-water flow system concept was introduced by Hubbert (4, pp. 926-930) and was further developed by Toth (5, 6) and Freeze and Witherspoon (7, 8, 9). Freeze (10) has used the numerical solutions of ground-water flow systems to determine recharge and discharge profiles as an aid in making hydrologic budget studies in Canada. In general, the concept involves the controls which geology and topography exercise upon the water table and resulting potential distribution; three orders of flow systems - local, intermediate, and regional - are recognized. The amount and distribution of recharge and discharge, the total flow through the flow system, the age of the ground-water at any point, and the chemistry of ground water are dependent on the overall potential distribution and the distribution of the three orders of flow systems.

The program described in this report can be used to obtain the potential distribution for cross sections through flow systems. If the position of the water table is one of the unknowns, it can usually be determined by the method described in part 3.

c. Well flow problems. Steady state flow to a well in a confined multiple aquifer system can be analyzed using the finite element method. In this way information regarding such items as the ultimate drawdowns in various aquifers at various distances from the well or the ultimate percentage of well discharge conducted through various aquifers can be obtained. The application of the finite element method (mainly unsteady state) to problems involving flow to wells in multiple aquifer systems is detailed in references 11 and 12.

d. Hydraulics problems. Hydraulic design problems such as those involving fluid flow through a conduit contraction or in a reservoir in the vicinity of an intake structure or spillway are sometimes amenable to solution by application of potential flow theory. As with problems involving flow through porous media, it is necessary to describe the geometry of flow boundaries. For free surface problems for which a flow net in the vertical plane is desired, the position of the free surface must be known, or assumed and adjusted by trial and error.

3. Special Areas of Application

a. Determination of hydraulic conductivity. If piezometer, water table, and subsurface geologic data are available for a flow region, the conductivity distribution may often be estimated by trial and error matching of calculated and measured potentials at various points (9, 10). Whenever estimates of recharge and discharge are available for any time period during which the flow system is steady or slowly unsteady, the position of the water table may be estimated and compared with the known position for further verification of the calculated conductivities. These data can be used as part of the verification process in constructing an unsteady state model of a ground-water basin (13).

b. Determination of the free surface. The free surface, which is generally taken as the atmospheric pressure surface, can be located by a trial and error adjustment somewhat similar to that used when constructing a graphical flow net (see 3, pp. 158-169). The general method is to (a) provide a first estimate for the position of the free surface, (b) set up the boundary conditions and solve the problem, (c) draw an approximate free surface along the line where the pressure is zero (i.e., atmospheric), (d) use the assumed free surface as a new boundary for the flow system and solve the problem again, (e) adjust the free surface in the direction indicated by the values of pressure at node points on the free surface, and (f) repeat step 'e' until the desired accuracy is achieved.

Often the free surface should be located at a pressure surface less than atmospheric. The reason for this is that an appreciable quantity of water often flows in the unsaturated region, principally in the zone of numerically small negative pressures. Also, the negative pressures existing in the unsaturated region tend to expand the flow system if the velocity has a large vertical component. Bouwer (14, 15) found that a negative pressure boundary condition for the free surface provided a good approximation to the actual variably saturated conditions and gave typical values of the negative pressure boundary condition for various materials. To apply the procedure given here for finding the free surface to finding the position of the negative pressure boundary, one simply adjusts the free surface until the pressure has the specified value along it.

Additional details for applying the method with the finite element program are given further on.

III. PROGRAM DESCRIPTION

1. Theoretical Basis

This program uses the finite element numerical method to solve the second order, linear partial differential equation (equation 3) and associated boundary conditions. The theoretical basis for the method is outlined in Addendum 1. In applying the finite element method, the flow region is divided up into polygonal subareas. For this program quadrilateral - shaped subareas, called elements, are used.* The corners of the elements are called nodes. A solution to a particular problem consists of values for the potential function (e.g. hydraulic head) at each node. Application of the finite element method results in a series of equations with nodal values of the potential function as unknowns in the equations. The method used to solve the series of simultaneous equations is discussed in Addendum 2.

*Note that although the word 'element' is used to denote a quadrilateral subarea, the program uses triangular elements. A center node is generated internally in the program for each quadrilateral subarea and the subarea is treated as four triangles (see Addendum 1).

2. Program Organization

The program consists of a main program and two subroutines, GEES and BSOLVE, as shown in the functional flow chart, figure 1. In the main program, input data are read and printed, the size of the matrix equation to be solved in subroutine BSOLVE is determined, the subroutines for setting up and solving the matrix equation are called, and the computed values of the potential function are printed. Subroutine GEES is used to determine the elements of the matrices that constitute the final matrix equation, and subroutine BSOLVE is used to solve the final matrix equation for values of the potential function. Addendum 4 contains a source listing of the program.

IV. PROGRAM USAGE

1. Equipment Requirements

The program as dimensioned in the source listing in Addendum 4 can handle up to 400 nodes, 400 elements (equivalent to 1600 triangular elements), 20 conductivity zones, 100 specified nodal values of potential, 200 sources or sinks and 100 known-flow boundaries. It occupies about 31,000 words of core on a CDC 6600 computer. No special library functions or routines are used, and the only peripheral equipment required is a card reader. The program dimensions can be easily modified to accommodate other sizes of problems and/or computer capacity requirements.

2. Input Preparation

a. Organization of input data. Information required to describe a particular problem can be categorized as follows:

(1) job specification
(2) nodal coordinates
(3) element data
(4) media property data
(5) boundary conditions
(6) sources and sinks

Job specification consists of information that describes the size of the problem and controls use of available options. Items to be specified include the total number of (a) elements, (b) nodes, (c) media zones, (d) known nodal values of the potential function, (e) specified sources or sinks, and (f) specified normal gradient boundary conditions. In addition, indices can be set to (a) specify use of axisymmetric cylindrical coordinates, (b) convert the units of input nodal coordinates from miles to feet, (c) request a printout of the coefficient matrix that is developed in subroutine GEES, or (d) request calculation and printing of pressure heads.

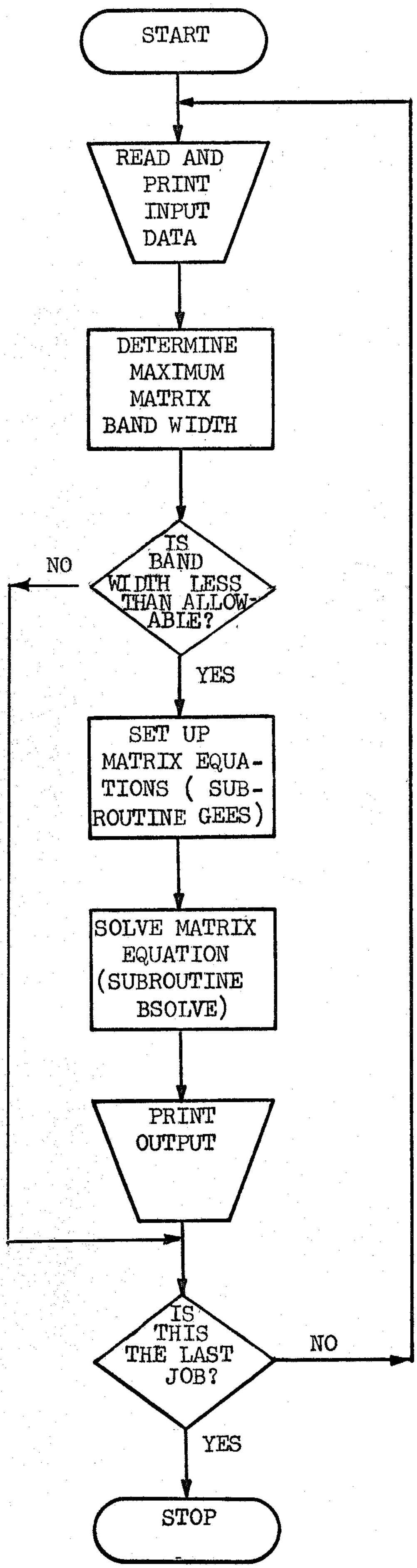


Figure 1. General flow diagram

Nodal coordinate information consists of coordinates of all nodes.

Element data consist of specification, for each quadrilateral element, of the four nodes of the element, the media zone that contains the element, and the element thickness.

Media property data consist of specification, for each media zone, of the major and minor conductivities and the angle of rotation of the conductivity axes from the axes of the global coordinate system.

Boundary conditions consist of specification of (a) nodal values of the potential function, and (b) potential flow of a given magnitude across element boundaries.

Sources and sinks are represented by designated elements for which magnitude of discharge is specified.

A detailed description of input data is given in Addendum 5. The last page of this document contains a "Summary of Input Cards".

b. Numbering of nodes and elements. An initial step in the preparation of input data is to prepare a sketch of the flow region and divide the region into elements. Each node is assigned a number in sequence beginning with 1 for the first node. Likewise each element is assigned a number in sequence beginning with 1 for the first element. Figure 3 shows a flow region subdivided into elements. It is essential that the following two rules be followed when numbering the nodes.

Rule A. Nodes for which the potential function is known (e.g. on a fixed head boundary) must be numbered last.

Rule B. Nodes should be numbered to keep the maximum difference between the smallest and largest node numbers on a single element as small as possible. Nodes for which the potential function is known are not considered when applying this rule.

In figure 3, the potential function for nodes 173 through 201 is known (i.e., input), hence these nodes are numbered last. The sequence for numbering nodes for which the potential function is known is arbitrary.

Rule B is meant to minimize computer storage requirements. The version of the program included in this document is dimensioned to permit a maximum difference of 24 between largest and smallest node numbers on a single element (see section 3 on the following page). In figure 3, the maximum difference between node numbers on a single element is 14 and occurs for nodes 91 and 105 on element 92.

The choice of a numbering system for the elements is arbitrary, with the exception that a slight savings in computer execution time can be achieved for large problems if elements that are designated as sources or sinks are numbered first.

c. Boundary conditions. A boundary of known potential is handled by specifying in the input data the known potential function for all nodes on that boundary. A boundary for which flow normal to the boundary is known is handled by specifying the magnitude of the discharge per unit length of boundary that occurs between pairs of adjacent nodes. If neither of these two types of boundary condition are specified along an external boundary, the boundary is automatically treated as a no-flow (impermeable) boundary.

d. Axisymmetric cylindrical coordinates. Input for problems using axisymmetric cylindrical coordinates is essentially the same as input for problems set up with Cartesian coordinates. The transformation from Cartesian to axisymmetric coordinates is accomplished internally in the program. An example showing input requirements for an axisymmetric flow problem is included in section 4 on page 26.

e. Units. Any consistent set of units may be used for input data. The only conversion option that is built into the program provides for conversion of the units of nodal coordinates from miles to feet.

3. Program Output

Examples of program output are shown in figures 5, 12, and 20. First, all input data are printed, followed by a statement indicating the magnitude of variable MAXBW and the node for which MAXBW was determined. MAXBW is the number of columns of the coefficient matrix that is set up in subroutine GEES. It is a function of the maximum difference between node numbers referred to in Rule B on page 9. The relationship is

$$\text{MAXBW} = (\text{Maximum Difference}) \times 2 + 1$$

For example, MAXBW for the problem described by figure 3 is 14x2+1 or 29. The version of the program included in this document is dimensioned to permit a maximum value for MAXBW of 50. If the maximum value is exceeded, a note to this effect will be printed and the program will stop. In order to overcome this problem, it is necessary to either increase the dimensions of the program or renumber the nodes so that MAXBW is reduced to a value less than the maximum allowable.

After MAXBW is printed, the elements of the coefficient matrix developed in subroutine GEES are printed, if requested. Normally a printout of the elements of the coefficient matrix would be called for only if required for tracing an error in a solution. Finally, the computed and known values of the potential function are printed. Pressure heads will be printed alongside the computed values of potential function, if requested. This latter option is useful for problems involving determination of constant pressure surfaces by trial and error or for situations where pore pressure distributions are required.

4. Example Problems

In this section three example problems are described to illustrate use of the program. Most of the options and techniques described above for using the program are applied in these examples.

a. Example 1. Regional Ground-Water Problem. This problem is intended to illustrate the versatility of the program for modeling regions of complex geometry and for handling a variety of boundary conditions. It is taken from a report by Winslow and Nuzman (20) describing an electrical analog simulation of the near-surface ground-water hydrology of a 45 square mile area in the Kansas River Valley near Topeka, Kansas. Figure 2 is a map of the area, and figure 3 shows the configuration of elements used to model the region. Dashed lines in the latter figure locate Soldier Creek and the Kansas River. Two runs were made with the model: the first without pumping, and the second with pumping from a well field in element 110. Boundary conditions, which include values used for areal recharge and flow from the influent and effluent streams, were taken from Winslow and Nuzman (20, pp. 17-19), and are shown in table 1. Each element was placed in one of three conductivity zones, depending on the aquifer thickness, as indicated in table 2.*

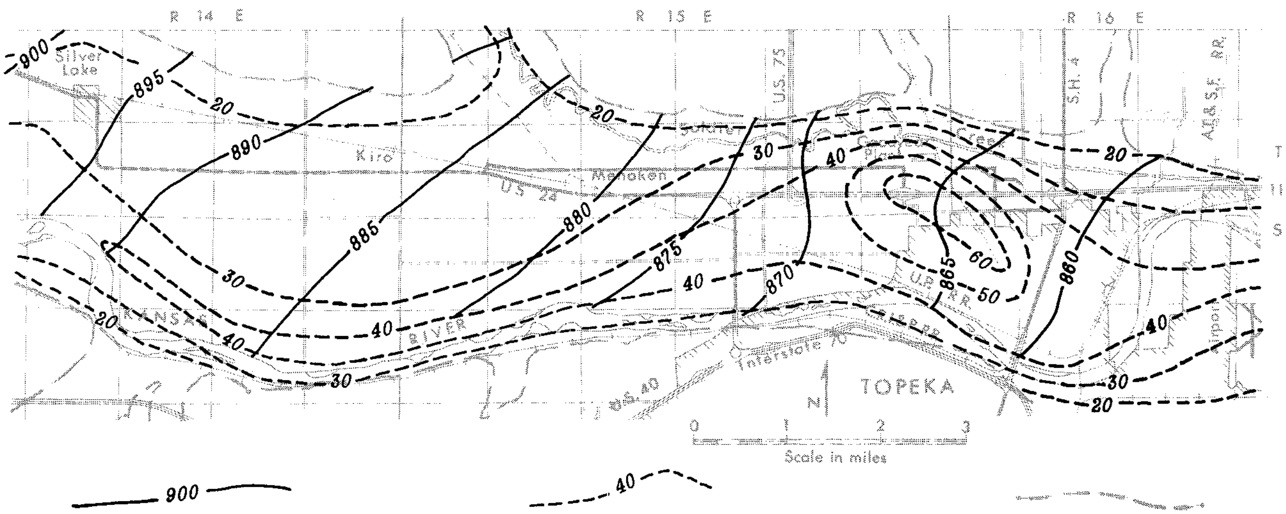
Input for the first run is shown in figure 4, and a printout is shown in figure 5. Input is listed on the first three pages and output on the fourth page of the latter figure. Contours of hydraulic head for the first run, plotted in figure 6, lie very close to those shown in figure 2. Two hundred and one nodes were used for the finite element solution compared to over 600 used by Winslow and Nuzman in their electrical analog.

Input for the second run (not shown) is similar to input for the first run, except that values in the "Run 2" columns of table 1 were used. Contours of hydraulic head for the second run, plotted in figure 7 clearly show the effect of pumping.

Central processor time for a run on a CDC 6600 computer was 6.1 sec., which includes time for program compilation.

---

*Winslow and Nuzman, (20, p. 15), used only two conductivity zones: 10,000 g.p.d./ft^2 for areas of saturated thickness greater than 40 ft. and 5000 g.p.d./ft^2 for areas less than 40 ft. thick. However, for the present simulation wherever elements encompass both zones about equally they were assigned to a third zone with a conductivity of 7500 g.p.d./ft^2.



900 — Contour lines connecting points of equal water level above mean sea level. Contour interval is 5 feet.

40 — Lines connecting points of equal saturated thickness of the alluvial aquifer. Interval is 10 feet.

Boundary of alluvial aquifer along valley wall.

Figure 2. Potentiometric surface and saturated thickness of the alluvial aquifer in 1950 for the Kansas River Valley near Topeka, Kansas. From Winslow and Nuzman (20, p. 5).

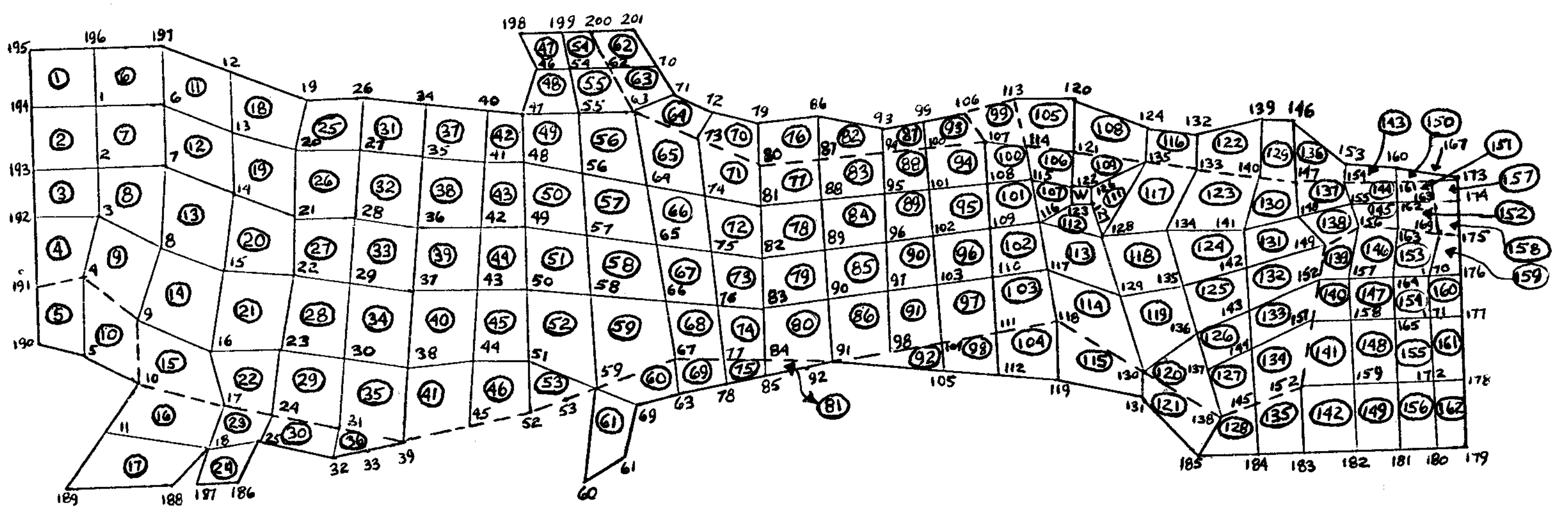


Figure 3. Element Configuration for Example 1

TABLE 1

Data for Regional Ground Water Problem (Example 1) from Winslow and Nuzman (20, p. 5, p. 8, and p. 19).

Known Head Boundary Conditions

Node	Run 1 Head (ft.)	Run 2 Head (ft.)	Node	Run 1 Head (ft.)	Run 2 Head (ft.)
173	856.0	855.2	188	887.0	882.0
174	855.7	855.1	189	887.0	882.0
175	855.5	855.0	190	893.0	888.0
176	855.2	854.8	191	894.8	889.8
177	855.0	854.6	192	896.6	891.6
178	854.5	854.4	193	898.4	893.4
179	854.0	854.0	194	900.2	895.2
180	854.5	854.2	195	902.0	897.0
181	855.0	854.4	196	900.0	895.0
182	856.0	854.6	197	897.0	892.0
183	857.0	854.8	198	892.0	885.0
184	858.2	855.0	199	892.0	885.0
185	860.0	855.2	200	892.0	885.0
186	887.0	882.0	201	892.0	885.0
187	887.0	882.0			

Known Flow Boundary Conditions

	Pair of Nodes Defining Boundary		Run 1 Discharge per Unit Length of Boundary (gal./day/ft.)	Run 2 Discharge per Unit Length of Boundary (gal./day/ft.)
External Boundaries	197	12	39.3	33.5
	12	19	39.3	33.5
	19	26	39.3	33.5
	26	34	39.3	33.5
	34	40	39.3	33.5
	40	47	39.3	33.5
	47	46	39.3	33.5
	46	198	39.3	33.5
	201	70	39.3	33.5

(continued)

Known Flow Boundary Conditions, Continued

	Pair of Nodes Defining Boundary		Run 1 Discharge per Unit Length of Boundary (gal./day/ft.)	Run 2 Discharge per Unit Length of Boundary (gal./day/ft.)
External Boundaries	70	71	39.3	33.5
	71	72	39.3	33.5
	72	79	39.3	33.5
	79	86	39.3	33.5
	86	93	39.3	33.5
	93	99	39.3	33.5
	106	99	39.3	33.5
	113	106	39.3	33.5
	120	113	39.3	33.5
	124	120	39.3	33.5
	132	124	39.3	33.5
	139	132	39.3	33.5
	146	139	39.3	33.5
	153	146	39.3	33.5
	160	153	39.3	33.5
	173	160	39.3	33.5
Influent Stream (Soldier Creek)	200	62	29.9	22.5
	62	63	29.9	22.5
	63	73	29.9	22.5
	73	80	29.9	22.5
	80	87	29.9	22.5
	87	94	29.9	22.5
	94	100	29.9	22.5
	100	107	29.9	22.5
	107	106	29.9	22.5
	106	113	29.9	22.5
	113	114	29.9	22.5
	114	121	29.9	22.5
	121	125	29.9	22.5
	125	133	29.9	22.5
	133	140	29.9	22.5
	140	147	29.9	22.5
	147	154	29.9	22.5
	154	161	29.9	22.5
	161	167	29.9	22.5
	167	168	29.9	22.5
	168	169	29.9	22.5

(continued)

Known Flow Boundary Conditions, Continued

	Pair of Nodes Defining Boundary		Run 1 Discharge per Unit Length of Boundary (gal./day/ft.)	Run 2 Discharge per Unit Length of Boundary (gal./day/ft.)
Effluent Stream (Kansas River)	191	4	-141.	-76.9
	4	9	-141.	-76.9
	9	10	-141.	-76.9
	10	17	-141.	-76.9
	17	24	-141.	-76.9
	24	31	-141.	-76.9
	31	39	-141.	-76.9
	39	45	-141.	-76.9
	45	52	-141.	-76.9
	52	53	-141.	-76.9
	53	59	-141.	-76.9
	59	67	-141.	-76.9
	67	77	-141.	-76.9
	77	84	-141.	-76.9
	84	91	-141.	-76.9
	91	98	-141.	-76.9
	98	104	-141.	-76.9
	104	111	-141.	-76.9
	111	118	-141.	-76.9
	118	130	-141.	-76.9
	130	138	-141.	-76.9
	138	145	-141.	-76.9
	145	152	-141.	-76.9
	152	151	-141.	-76.9
	151	150	-141.	-76.9
	150	149	-141.	-76.9
	149	146	-141.	-76.9
	156	163	-141.	-76.9
	163	169	-141.	-76.9
	169	175	-141.	-76.9

Areal Recharge and Pumping

A recharge rate of 0.0054 gal./day/ft.^{2} from precipitation occurs over all elements for both runs, with the exception that a discharge rate of -4.59 gal./day/ft.^{2}, due to pumping, is specified for element 110 for the second run. For this example only, the program was modified so that the input for source terms was in gal/day/ft^2 (discharge per unit area) instead of discharge units.

TABLE 2

Hydraulic Conductivities for Regional Ground Water Problem (Example 1).

Zone	Element 'Thickness' (ft.)	Hydraulic Conductivity (gal./day/ft.^{2})
1	About 40	7500
2	Greater than 40	10000
3	Less than 40	5000

GENERAL PURPOSE DATA FORM
(8 COLUMN FIELDS)

PROGRAM 723 - G2 - L2440 DATE

REQUESTED BY PREPARED BY J. C. P. CHECKED BY PAGE 1 OF 2

	1	2	3	4	5	6	7	8	9	10
A	SIMULATION OF GROUND-WATER HYDROLOGY IN THE KANSAS RIVER VALLEY									
A				NEAR TOPEKA KANSAS						
A			CONDITIONS IN 1950 (WITHOUT PUMPING)							
B	162	201	3	29	162	76			1	
C	1	.625	3.69							
C	2	.625	3.08							
C	3	.625	2.63							
C	4	.438	2.05							
				C CARDS FOR NODES 5 THROUGH 198 NOT SHOWN						
C	199	5.10	4.25							
C	200	5.39	4.25							
C	201	5.81	4.25							
D	1	195	194	1	196	1	20			
D	2	194	193	2	1	1	27			
D	3	193	192	3	2	1	31			
								D CARDS FOR ELEMENTS 4 THROUGH 159 NOT SHOWN		
D	160	170	171	177	176	3	40			
D	161	171	172	178	177	1	31			
D	162	172	180	179	178	1	22			
E	1	5000	5000							
E	2	10000	10000							
E	3	7500	7500							
F	173	856								
F	174	855.7								
F	175	855.5								
			F CARDS FOR NODES 176 THROUGH 198 NOT SHOWN							
F	199	892								

SPK Form 321
1 Jul 66

(Previous editions are obsolete)

Figure 4. Input for example 1

GENERAL PURPOSE DATA FORM

(8 COLUMN FIELDS)

PROGRAM 723-G2-L2440 DATE

REQUESTED BY PREPARED BY J.C.P. CHECKED BY PAGE 2 OF 2

	1	2	3	4	5	6	7	8	9	10
F	200	892								
F	201	892								
G	1	.0054								
G	2	.0054								
G	3	.0054								
			G CARDS FOR ELEMENTS 4 THROUGH 159 NOT SHOWN							
G	160	.0054								
G	161	.0054								
G	162	.0054								
H	197	12	39.3							
H	12	19	39.3							
H	19	26	39.3							
				70 H CARDS FOR KNOWN FLOW BOUNDARY CONDITIONS NOT SHOWN (SEE TABLE 1)						
H	156	163	-141							
H	163	169	-141							
H	169	175	-141							

SPK Form 321
1 Jul 66

(Previous editions are obsolete)

Figure 4. Input for example 1

```
SIMULATION OF GROUND-WATER HYDROLOGY IN THE KANSAS RIVER VALLEY
                      NEAR TOPEKA, KANSAS
           CONDITIONS IN 1950    (WITHOUT PUMPING)

INPUT DATA

NO. OF ELEMENTS (NELS) =     162
NO. OF NODES (NNDS) =     201
NO. OF MATERIAL PROPERTY ZONES (NZNS) =        3
NO. OF SPECIFIED BOUNDARY HEADS (NHDS) =       29
NO. OF SOURCES OR SINKS (NWELS) =     162
NO. OF BOUNDARY ELEMENT SIDES (NOBND) FOR WHICH DISCHARGE IS SPECIFIED =      76
```

NODE	XCORD	YCORD
1	.63	3.69
2	.63	3.08
3	.63	2.63
4	.44	2.05
5	.44	1.25
6	1.29	3.66

Values for nodes 7 through 196 not shown.

NODE	XCORD	YCORD
197	1.29	4.24
198	4.70	4.25
199	5.10	4.25
200	5.39	4.25
201	5.81	4.25

VALUES FOR COORDINATES CONVERTED FROM MILES TO FEET BEFORE MAKING CALCULATIONS

ELEMENT	NODE 1	NODE 2	NODE 3	NODE 4	ZONE	THICKNESS
1	195	194	1	196	1	20.00
2	194	193	2	1	1	27.00
3	193	192	3	2	1	31.00
4	192	191	4	3	1	32.00
5	191	190	5	4	1	25.00

Figure 5. Printout for example 1

Values for elements 6 through 157 not shown.

158	168	169	175	174	1	30.00
159	169	170	176	175	1	35.00
160	170	171	177	176	3	40.00
161	171	172	178	177	1	31.00
162	172	180	179	178	1	22.00

ZONE	XPERM	YPERM	ALPHA
1	5.0000E+03	5.0000E+03	-0.
2	1.0000E+04	1.0000E+04	-0.
3	7.5000E+03	7.5000E+03	-0.

NODE	SPECIFIED BOUNDARY HEAD
173	856.00
174	855.70
175	855.50
176	855.20
177	855.00
178	854.50

Values for nodes 179 through 194 not shown.

195	902.00
196	900.00
197	897.00
198	892.00
199	892.00
200	892.00
201	892.00

SOURCES OR SINKS (+ INDICATES RECHARGE, - INDICATES DISCHARGE)

ELEMENT	DISCHARGE
1	5.400E-03
2	5.400E-03
3	5.400E-03
4	5.400E-03
5	5.400E-03
6	5.400E-03

Figure 5. Printout for example 1

Values for elements 7 through 156 not shown.

157	5.400E-03
158	5.400E-03
159	5.400E-03
160	5.400E-03
161	5.400E-03
162	5.400E-03

SPECIFIED BOUNDARY DISCHARGES

UNIT DISCHARGE	BOUNDARY NODE A	BOUNDARY NODE B
3.930E+01	197	12
3.930E+01	12	19
3.930E+01	19	26
3.930E+01	26	34

Values not shown.

2.990E+01	140	147
2.990E+01	147	154
2.990E+01	154	161
2.990E+01	161	167
2.990E+01	167	168
2.990E+01	168	169
-1.410E+02	191	4
-1.410E+02	4	9
-1.410E+02	9	10
-1.410E+02	10	17
-1.410E+02	17	24
-1.410E+02	24	31

Values not shown.

-1.410E+02	149	156
-1.410E+02	156	163
-1.410E+02	163	169
-1.410E+02	169	175

Figure 5. Printout for example 1

```
OUTPUT

MAXIMUM MATRIX BAND WIDTH (MAXBW) OF   29 CORRESPONDS TO NODE   91

COMPUTED VALUES OF HYDRAULIC HEAD
      NODE              HEAD
        1           8.9820E+02
        2           8.9618E+02
        3           8.9441E+02
        4           8.9227E+02
        5           8.9037E+02
        6           8.9610E+02
        7           8.9428E+02
```

Values for nodes 8 through 167 not shown.

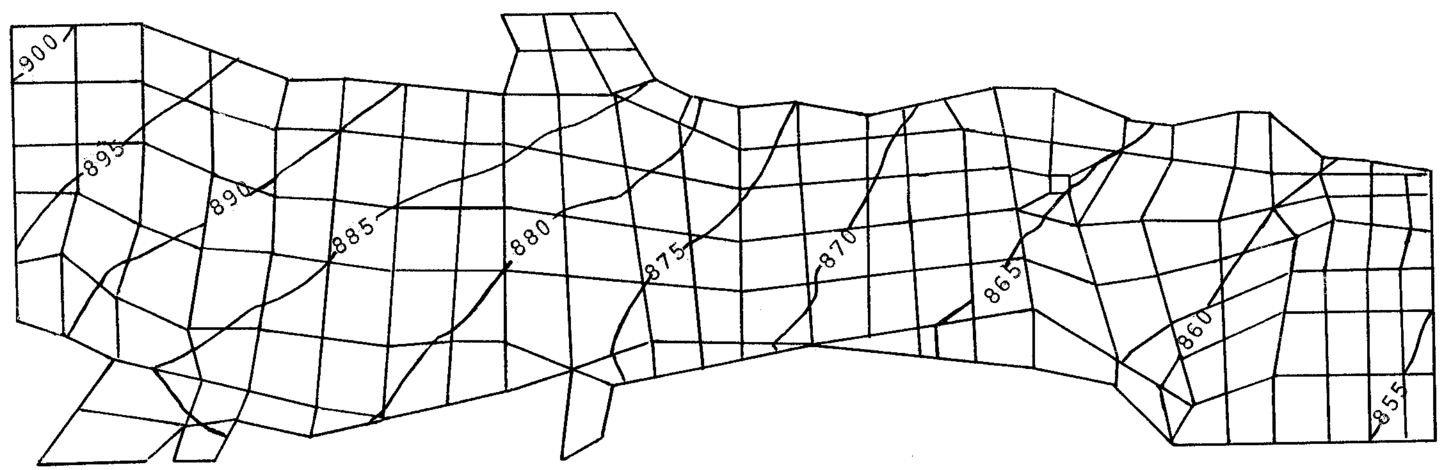
```
      168           8.5636E+02
      169           8.5552E+02
      170           8.5580E+02
      171           8.5570E+02
      172           8.5523E+02

KNOWN VALUES OF HYDRAULIC HEAD AT BOUNDARIES
      NODE              HEAD
      173           8.5600E+02
      174           8.5570E+02
      175           8.5550E+02
```

Values for nodes 176 through 196 not shown.

```
      197           8.9700E+02
      198           8.9200E+02
      199           8.9200E+02
      200           8.9200E+02
      201           8.9200E+02
```

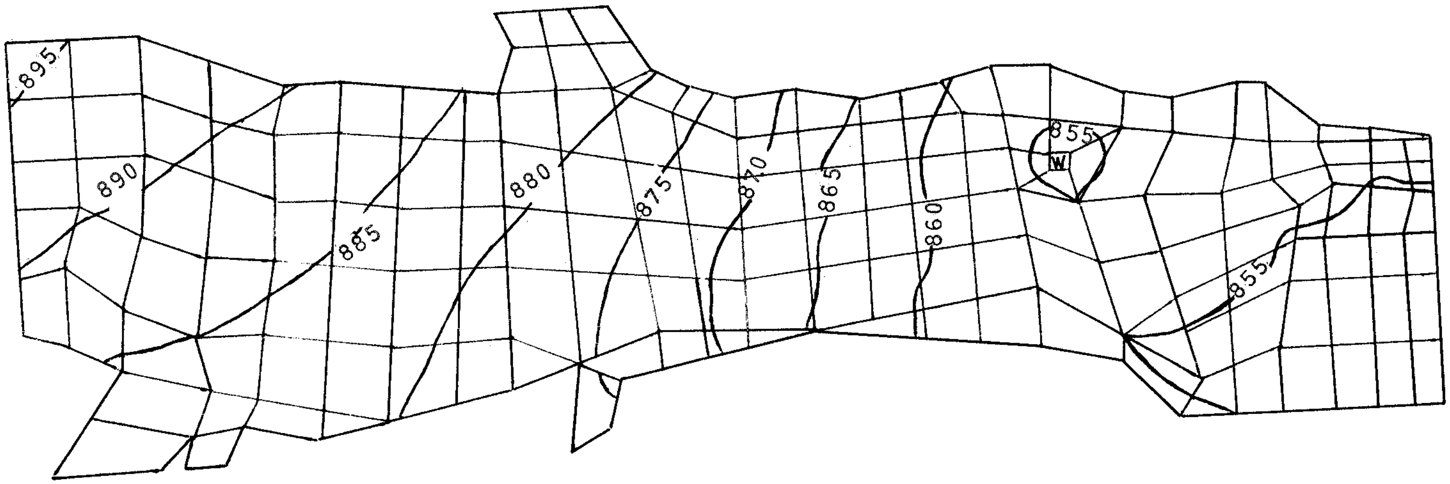
Figure 5. Printout for example 1



Explanation

890 Elevation of water level above mean sea level.

Figure 6. Contours of hydraulic head for run 1, example 1.



Explanation

890 Elevation of water level
above mean sea level

Figure 7. Contours of hyd

Figure 7. Contours of hydraulic head for run 2, example 1.

Explanation

890 Elevation of water level above mean sea level.

W Well field.

b. <u>Example 2. Axisymmetric Radial Flow to a Well in a Semiconfined Aquifer.</u> This problem is intended to illustrate the methods used for handling axisymmetric flow in nonhomogeneous media. The problem is illustrated in figure 8. A 55 ft. thick aquitard overlays a 50 ft. thick aquifer. The lower boundary of the aquifer is impervious and the well is open only to the aquifer. The hydraulic conductivities of the aquifer and aquitard are 1000 gal/day/ft^2 and 10 gal/day/ft^2, respectively. Hydraulic heads at nodes on the upper boundary of the aquitard and at nodes on the assumed radius of influence at 4000 ft. were set equal to zero so that the calculated values of hydraulic head will represent drawdown. A discharge of 50 gal/min is specified across the well bore. This discharge is assumed to be divided equally into five increments among the first five elements. Because of the small vertical variation in hydraulic gradient across the well bore due to the fact that flow is not strictly radial, this boundary condition is only an approximation. However, the drawdowns calculated at the nodes along the well bore varied only slightly indicating that for this problem the approximation is valid.

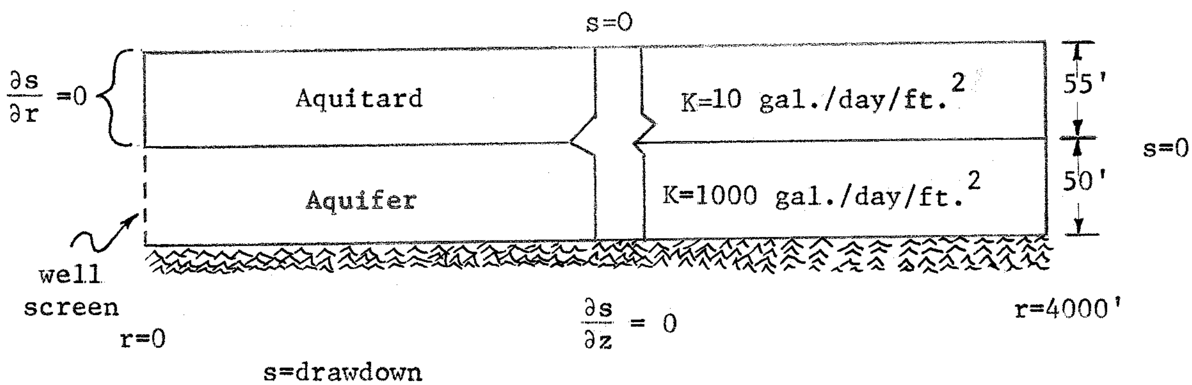


Figure 8. Axisymmetric radial flow to a well in a semiconfined aquifer (example 2).

The element configuration illustrated in figure 9 was used to model the problem. Columns of nodes in addition to those shown in figure 9 were located at 24, 41, 70, 119, 202, 343, 583, 990, 1287, 1673, 2175, 2827, 3375, and 4000 ft. from the centerline of the well. Altogether there are 320 elements and 357 nodes. Input is shown in figure 10. In order to facilitate input preparation, the program shown in figure 11 was used to generate C cards for nodes 1 through 336 and D cards for elements 1 through 300.

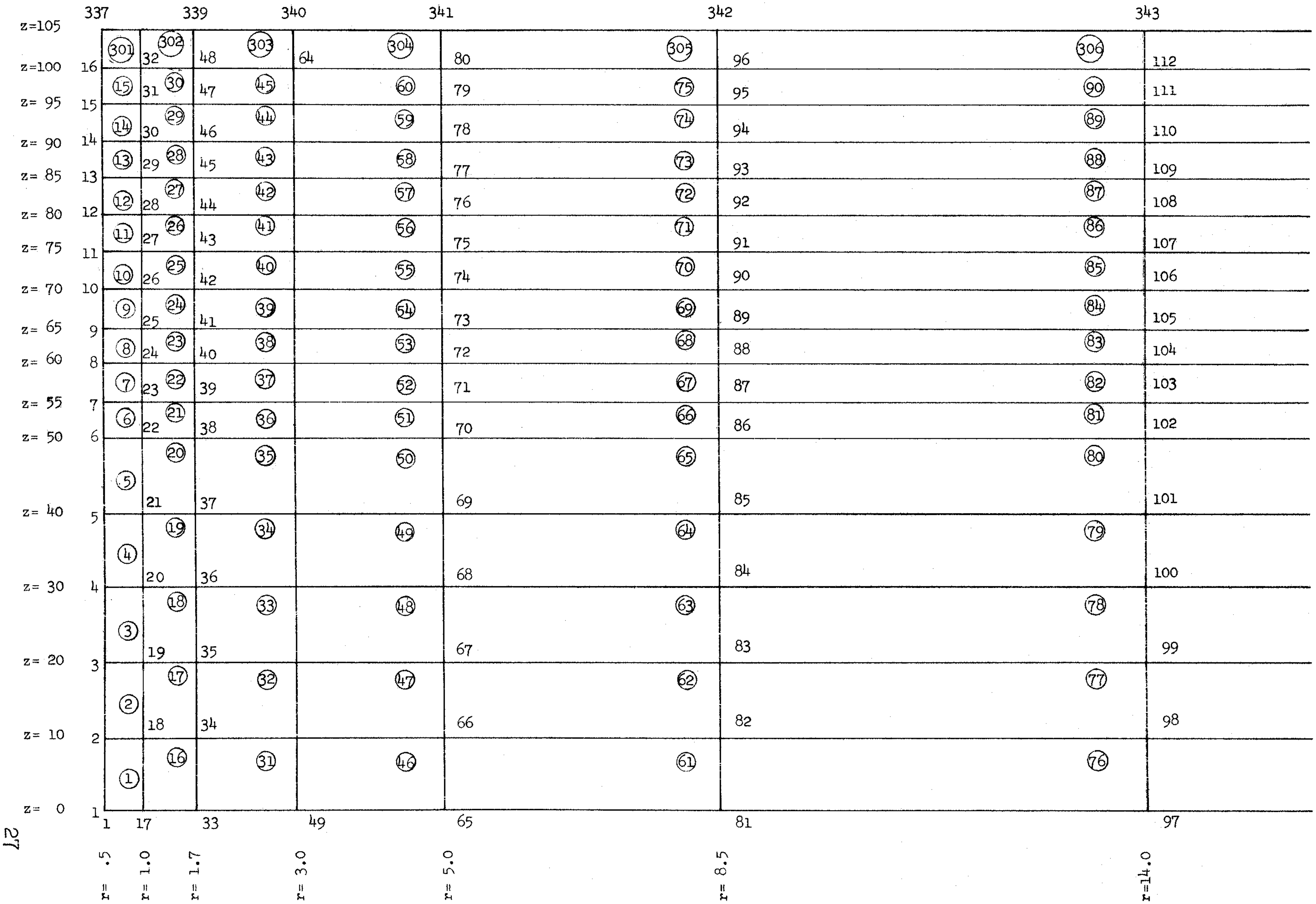


Figure 9. Element configuration for example 2

GENERAL PURPOSE DATA FORM

(8 COLUMN FIELDS)

PROGRAM 723-G2-L2440 DATE

REQUESTED BY PREPARED BY J.C.P CHECKED BY PAGE 1 OF 2

	1	2	3	4	5	6	7	8	9	10
A			STEADY-STATE FLOW TO A WELL							
A			IN A LEAKY TWO-LAYERED							
A			AQUIFER SYSTEM							
B	320	357	2	37		5	1			
C	1	.5								
C	2	.5	10							
C	3	.5	20							
C	4	.5	30							
					C CARDS FOR NODES 5 THROUGH 352 NOT SHOWN					
C	353	1673	105							
C	354	2175	105							
C	355	2827	105							
C	356	3375	105							
C	357	4000	105							
D	1	2	1	17	18	1				
D	2	3	2	18	19	1				
D	3	4	3	19	20	1				
D	4	5	4	20	21	1				
								D CARDS FOR ELEMENTS 5 THROUGH 316 NOT SHOWN		
D	317	353	272	288	354	2				
D	318	354	288	304	355	2				
D	319	355	304	320	356	2				
D	320	356	320	336	357	2				
E	1	1000	1000							
E	2	10	10							
F	321									
F	322									
F	323									

SPK Form 321
1 Jul 66

(Previous editions are obsolete)

Figure 10. Input for example 2

GENERAL PURPOSE DATA FORM

(8 COLUMN FIELDS)

PROGRAM 723-G2-L2440 DATE

REQUESTED BY PREPARED BY J. C. P. CHECKED BY PAGE 2 OF 2

1	2	3	4	5	6	7	8	9	10
F 324									
		F CARDS FOR NODES 325 THROUGH 353 NOT SHOWN							
F 354									
F 355									
F 356									
F 357									
H 1	2	-1440							
H 2	3	-1440							
H 3	4	-1440							
H 4	5	-1440							
H 5	6	-1440							

SPK Form 321
1 Jul 66

(Previous editions are obsolete)

Figure 10. Input for example 2

```
      PROGRAM DATA (INPUT,OUTPUT,PUNCH)
      DIMENSION X(30),Y(30)
   10 FORMAT (1X,I7,I8)
   20 FORMAT (1X,F7.0,9F8.0)
   30 FORMAT (1HC,I7,2F8.2)
   35 FORMAT(1H1)
   40 FORMAT (1HD,I7,4I8)
      READ 10,NR,NC
      READ 20,(X(I),I=1,NC)
      READ 20,(Y(I),I=1,NR)
      I=0
      DO 60 K=1,NC
      DO 50 L=1,NR
      I=I+1
      PRINT 30,I,X(K),Y(L)
   50 PUNCH 30,I,X(K),Y(L)
   60 CONTINUE
      IEL=0
      ND1=0
      NRM1=NR-1
      NCM1=NC-1
      ND4=NR
      PRINT 35
      DO 80 K=1,NCM1
      ND4=ND4+1
      ND1=ND1+1
      DO 70 L=1,NRM1
      ND2=ND1
      IEL=IEL+1
      ND3=ND4
      ND4=ND3+1
      ND1=ND2+1
      PRINT 40,IEL,ND1,ND2,ND3,ND4
   70 PUNCH 40,IEL,ND1,ND2,ND3,ND4
   80 CONTINUE
      STOP
      END
```

Figure 11. Program for generating C and D cards for example 2.

A printout for example 2 is shown in figure 12. Input is listed on the first two pages and output on the third page of the figure. An analytic solution for this problem has been made (see Walton, 21, pp. 145-147), under the assumptions that water seeps vertically through the aquitard and is refracted 90° so that flow is horizontal in the aquifer. A comparison of analytic and finite element solutions is made in table 3, where drawdowns in the aquifer by the two methods are given. The two solutions are very close.

TABLE 3

Comparison of Analytic and Numerical Solutions for Example 2

Dist. From Well (ft.)	Drawdown by Analytic Solution (ft.)	Drawdown by Finite Element Solution (ft.)	Dist. From Well (ft.)	Drawdown by Analytic Solution (ft.)	Drawdown by Finite Element Solution (ft.)
.5	1.61	1.59	202.0	.26	.25
1.0	1.46	1.43	343.0	.16	.16
1.7	1.34	1.31	583.0	.086	.078
3.0	1.21	1.18	990.0	.031	.029
5.0	1.09	1.07	1287.0	.015	.014
8.5	.97	.95	1673.0	.0066	.0059
14.0	.86	.83	2175.0	.0022	.0019
24.0	.73	.71	2827.0	.00062	.00048
41.0	.61	.59	3375.0	.00020	.00014
70.0	.50	.47			
119.0	.38	.36			

The numerical solution to this problem took 9.9 seconds of central processor time on a CDC 6600 computer, which includes time for program compilation.

c. Example 3. Steady state Seepage from a Ditch Considering the Effects of Capillarity. This example illustrates a method of solving problems involving flow with a free surface, in particular flow with a negative pressure specified on the free surface. The problem solved here was originally solved in 1940 by V. V. Vedernokov (see Polubarinova-Kochina, 22, pp. 160-162) using complex variable theory. Geometry and boundary conditions for the problem were taken from Polubarinova-Kochina, p. 161, table 9, and are illustrated in figure 13.

```
          STEADY-STATE FLOW TO A WELL
          IN A LEAKY TWO-LAYERED
          AQUIFER SYSTEM

INPUT DATA

NO. OF ELEMENTS (NELS) =     320
NO. OF NODES (NNDS) =     357
NO. OF MATERIAL PROPERTY ZONES (NZNS) =        2
NO. OF SPECIFIED BOUNDARY HEADS (NHDS) =       37
NO. OF SOURCES OR SINKS (NWELS) =      -0
NO. OF BOUNDARY ELEMENT SIDES (NQBND) FOR WHICH DISCHARGE IS SPECIFIED =        5

*** AXI-SYMMETRIC RADIAL COORDINATES ***

NODE          XCORD          YCORD
   1            .50            0.
   2            .50          10.00
   3            .50          20.00
   4            .50          30.00
   5            .50          40.00
   6            .50          50.00

        Values for nodes 7 through 352 not shown.

 353        1673.00         105.00
 354        2175.00         105.00
 355        2827.00         105.00
 356        3375.00         105.00
 357        4000.00         105.00

ELEMENT  NODE 1  NODE 2  NODE 3  NODE 4    ZONE    THICKNESS
      1       2       1      17      18       1       -0.
      2       3       2      18      19       1       -0.
      3       4       3      19      20       1       -0.
      4       5       4      20      21       1       -0.
      5       6       5      21      22       1       -0.
```

Figure 12. Printout for example 2

Values for elements 6 through 315 not shown.

316	352	256	272	353	2	-0.
317	353	272	288	354	2	-0.
318	354	288	304	355	2	-0.
319	355	304	320	356	2	-0.
320	356	320	336	357	2	-0.

ZONE	XPERM	YPERM	ALPHA
1	1.0000E+03	1.0000E+03	-0.
2	1.0000E+01	1.0000E+01	-0.

NODE	SPECIFIED BOUNDARY HEAD
321	-0.
322	-0.
323	-0.
324	-0.
325	-0.

Values for nodes 326 through 353 not shown.

354	-0.
355	-0.
356	-0.
357	-0.

SPECIFIED BOUNDARY DISCHARGES

UNIT DISCHARGE	BOUNDARY NODE A	BOUNDARY NODE B
-1.440E+03	1	2
-1.440E+03	2	3
-1.440E+03	3	4
-1.440E+03	4	5
-1.440E+03	5	6

Figure 12. Printout for example 2

OUTPUT

MAXIMUM MATRIX BAND WIDTH (MAXBW) OF 35 CORRESPONDS TO NODE 1

COMPUTED VALUES OF HYDRAULIC HEAD AND FLUID PRESSURE HEAD

NODE	HYD. HD.	PRESS. HD.
1	-1.5895E+00	-1.5895E+00
2	-1.5893E+00	-1.1589E+01
3	-1.5888E+00	-2.1589E+01
4	-1.5877E+00	-3.1588E+01
5	-1.5864E+00	-4.1586E+01
6	-1.5793E+00	-5.1579E+01
7	-8.2861E-01	-5.5829E+01

Values for nodes 8 through 315 not shown.

NODE	HYD. HD.	PRESS. HD.
316	-6.2299E-05	-8.0000E+01
317	-4.9846E-05	-8.5000E+01
318	-3.7389E-05	-9.0000E+01
319	-2.4928E-05	-9.5000E+01
320	-1.2464E-05	-1.0000E+02

KNOWN VALUES OF HYDRAULIC HEAD AND PRESSURE HEAD AT BOUNDARIES

NODE	HYD. HD.	PRESS. HD.
321	-0.	0.
322	-0.	-1.0000E+01
323	-0.	-2.0000E+01
324	-0.	-3.0000E+01

Values for nodes 325 through 352 not shown.

NODE	HYD. HD.	PRESS. HD.
353	-0.	-1.0500E+02
354	-0.	-1.0500E+02
355	-0.	-1.0500E+02
356	-0.	-1.0500E+02
357	-0.	-1.0500E+02

Figure 12. Printout for example 2

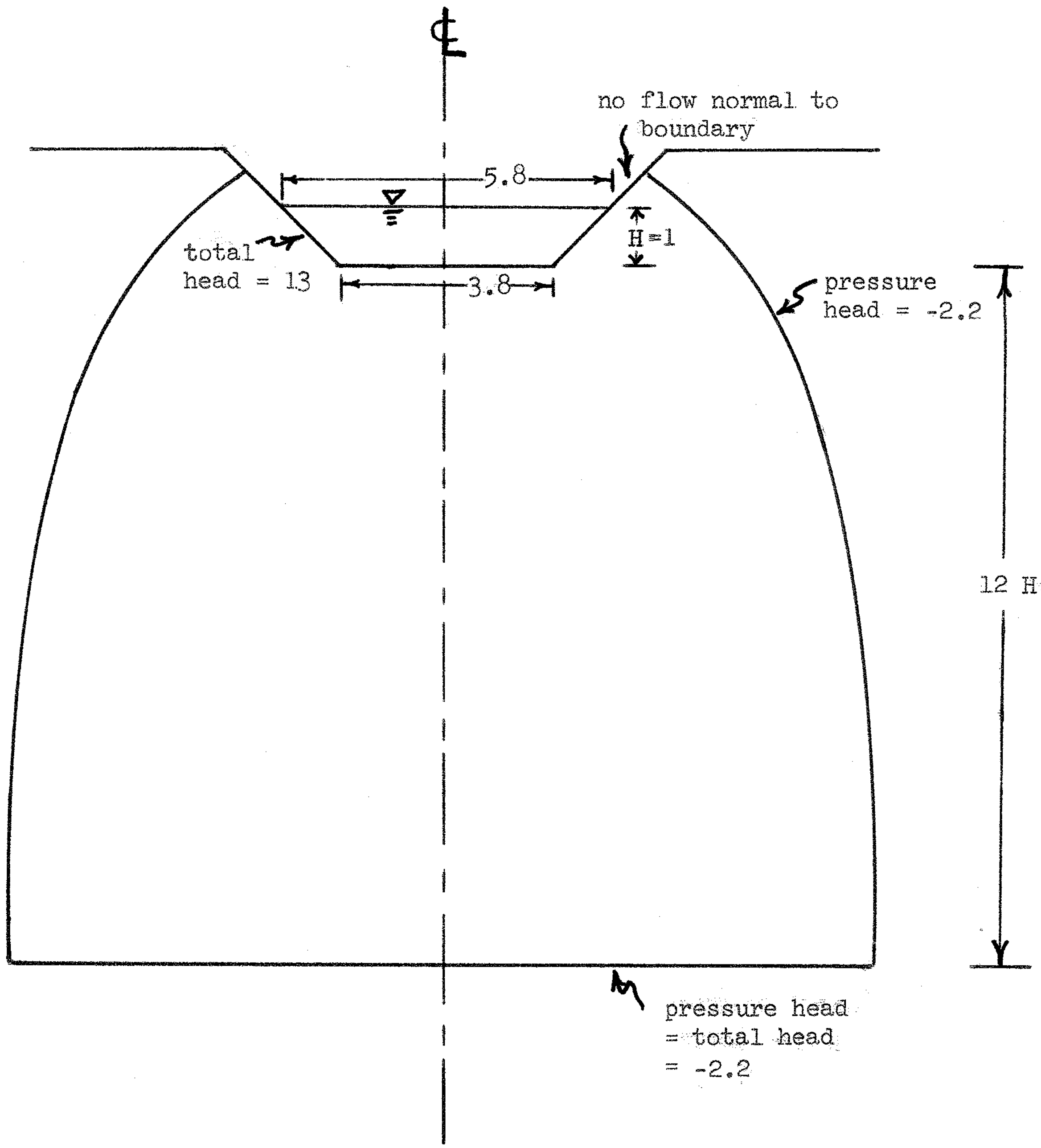


Figure 13. Geometry and boundary conditions for example 3.

The lower boundary condition used by Vedernihov was one of an infinitely permeable drain at an infinite distance below the channel bottom. This boundary condition can be simulated by specifying a constant pressure head equal to the free surface boundary condition at the depth at which flow is essentially vertical. This depth is set equal to zero, and all points above it have positive elevation. Comparison of figure 13 with figure 16 (shallow canal) in Bouwer (15, p. 56) indicates that the distance to the bottom boundary can be taken as about three times the bottom width of the canal for the problem illustrated here. This distance is about 12H (H equals the depth of water in the canal).

Calculation of the free surface was accomplished by a manual iteration procedure. Solution using an initial guess for the position of the free surface (assumed to be an impermeable boundary) provided an initial pressure head distribution. Next a contour was drawn through points equal to the pressure head specified for the boundary. This contour was used as the next trial position for the free surface. After the first two trials all subsequent trial free surfaces were obtained by linear interpolation or extrapolation between node points at the same elevation on the previous two trial free surfaces. Since this interpolation or extrapolation was accomplished manually, only three points were used, and the trial free surface was sketched from these. The finite element net was constructed so that after the first trial only node points near the free surface needed to be moved for successive iterations. Input preparation for each iteration was thus kept to a minimum. Also note that, because the flow system is symmetric about the vertical center line, only half of the flow net need be solved. The center line is treated as an impermeable boundary because it is a streamline.

The initial guess for the free surface, which was purposely chosen to be very poor, and the initial element configuration are shown in figure 14. Interpolation between iterations 2 and 3 to yield a trial surface for iteration 4 is shown in figure 15. The solution obtained after seven iterations, which is close to the correct solution, is illustrated in figures 16 through 18. As can be seen, the iteration process converged very rapidly, and the solution for iteration 7 would have probably been even closer to correct had all of the points on the boundary been used in the interpolation or extrapolation process. Finally, input and a partial printout of input and output for iteration 7 are illustrated in figures 19 and 20, respectively.

Vedernihov obtained a height of capillary rise along the canal bank of about .67H. The value obtained here was about .8H. The correct width of the flow system at infinity is about 15.3H; whereas, the finite element solution yielded a value of 15.2H. The finite element solution took about 3.9 seconds of central processor time for each iteration on a CDC 6600 computer.

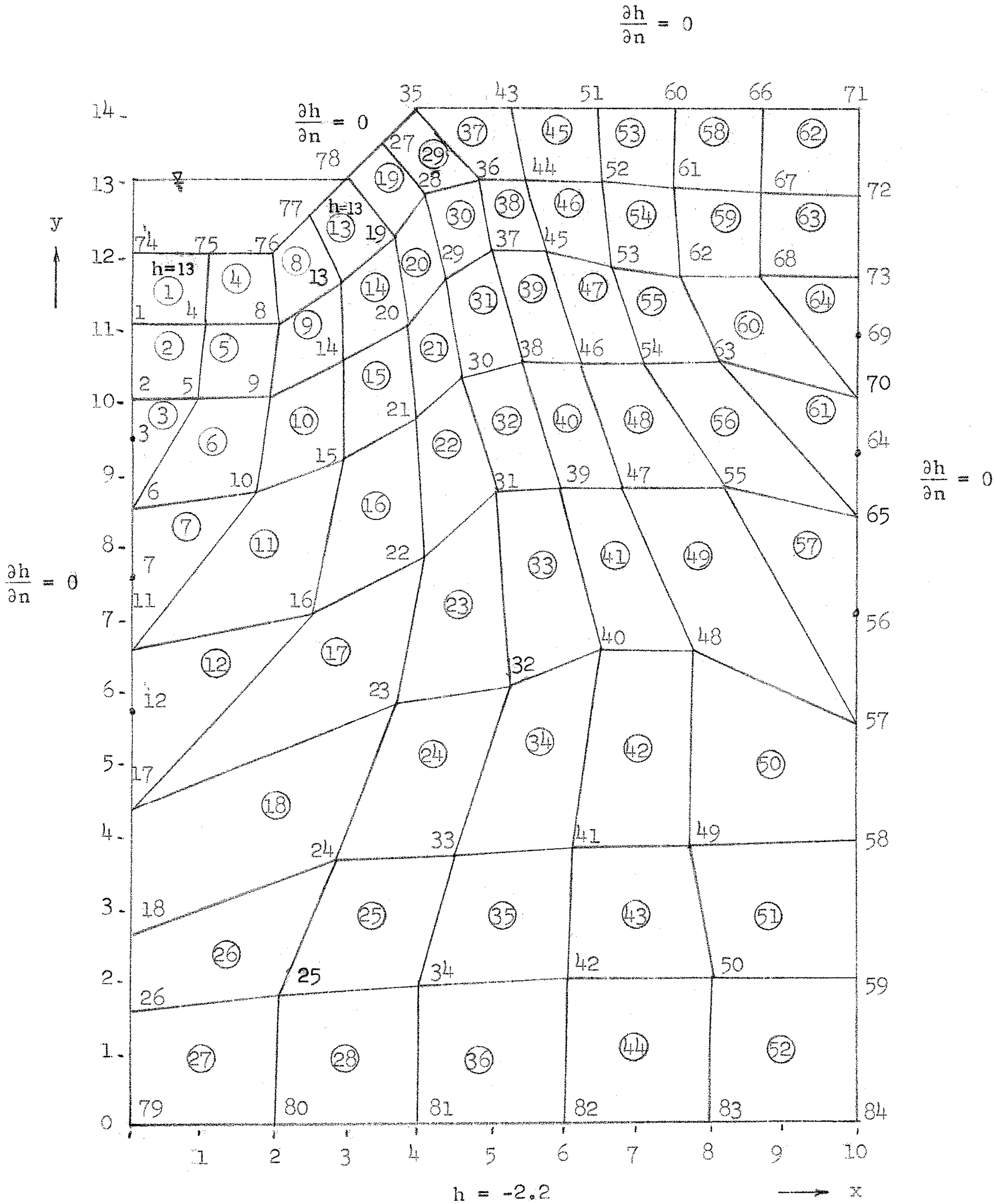


Figure 14. Initial element configuration for example 3.

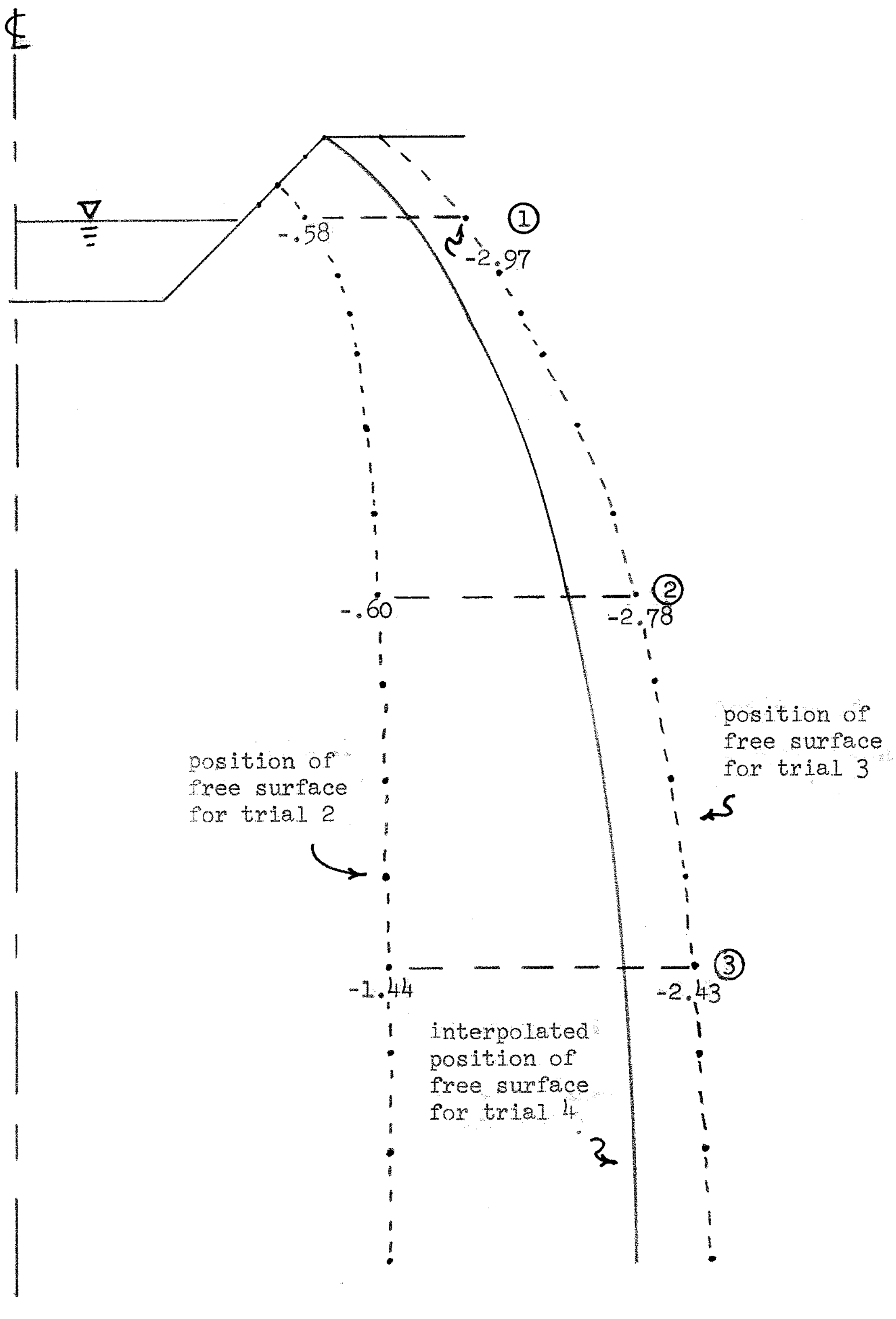


Figure 15. Example of interpolation scheme used to obtain trial positions for the free surface (example 3).

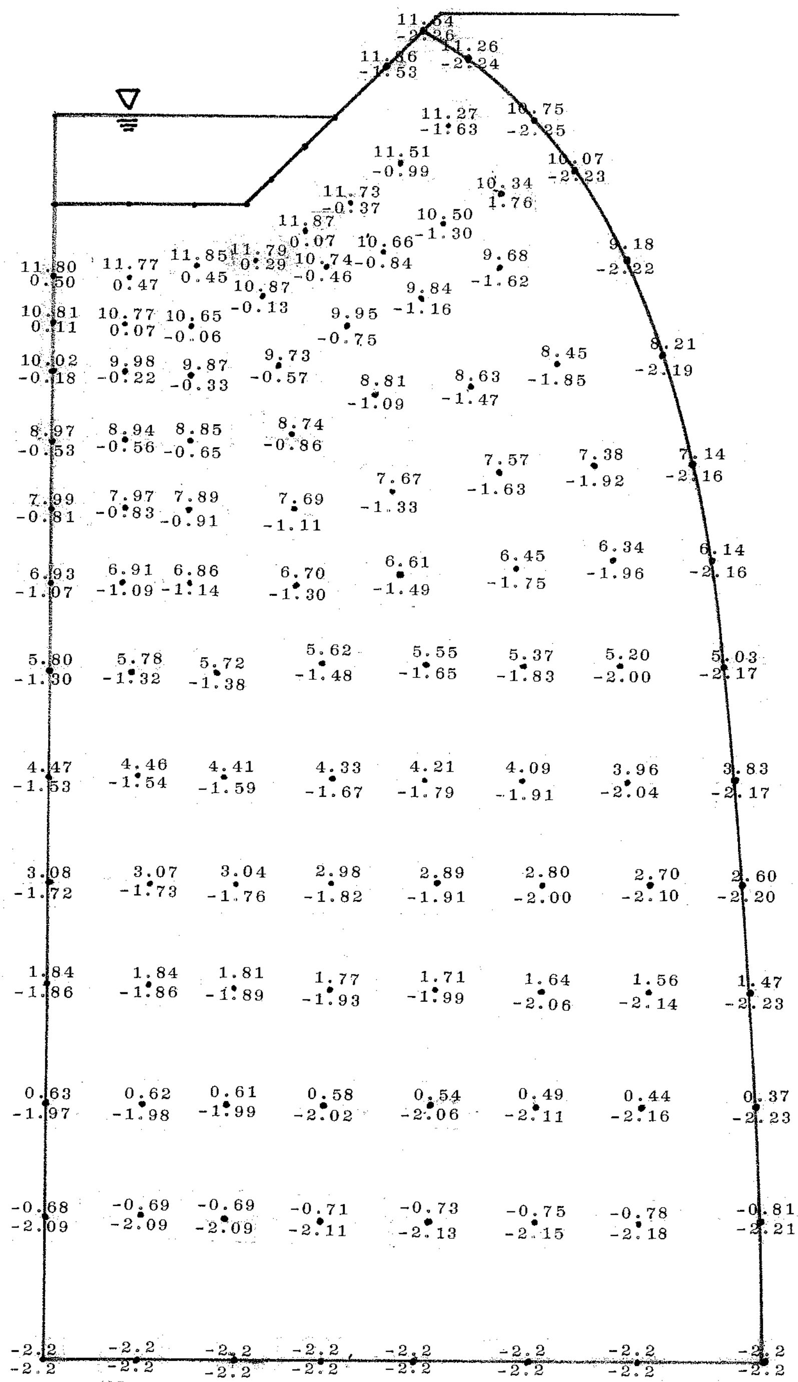


Figure 16. Values of total head and pressure head obtained from iteration 7, example 3. Total head is above each node point and pressure head is below it.

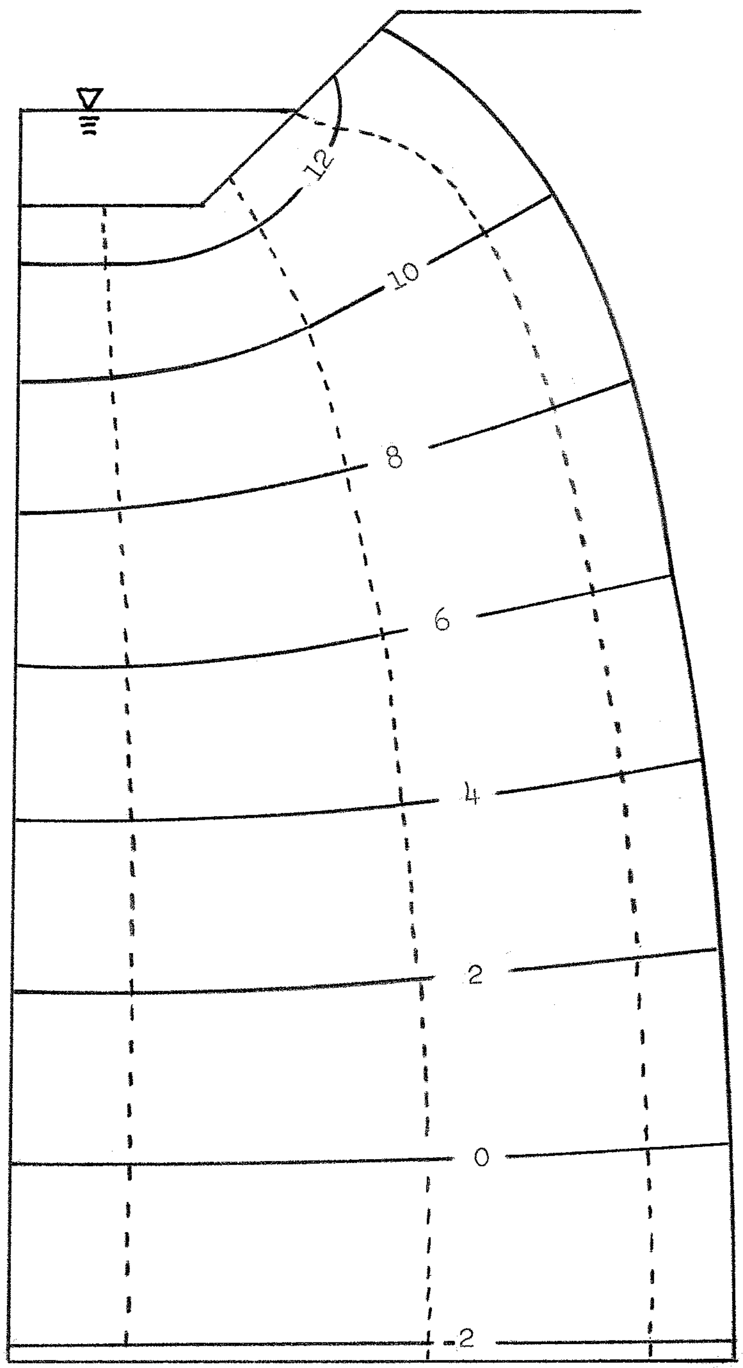


Figure 17. Contours of total head for iteration 7, example 3.

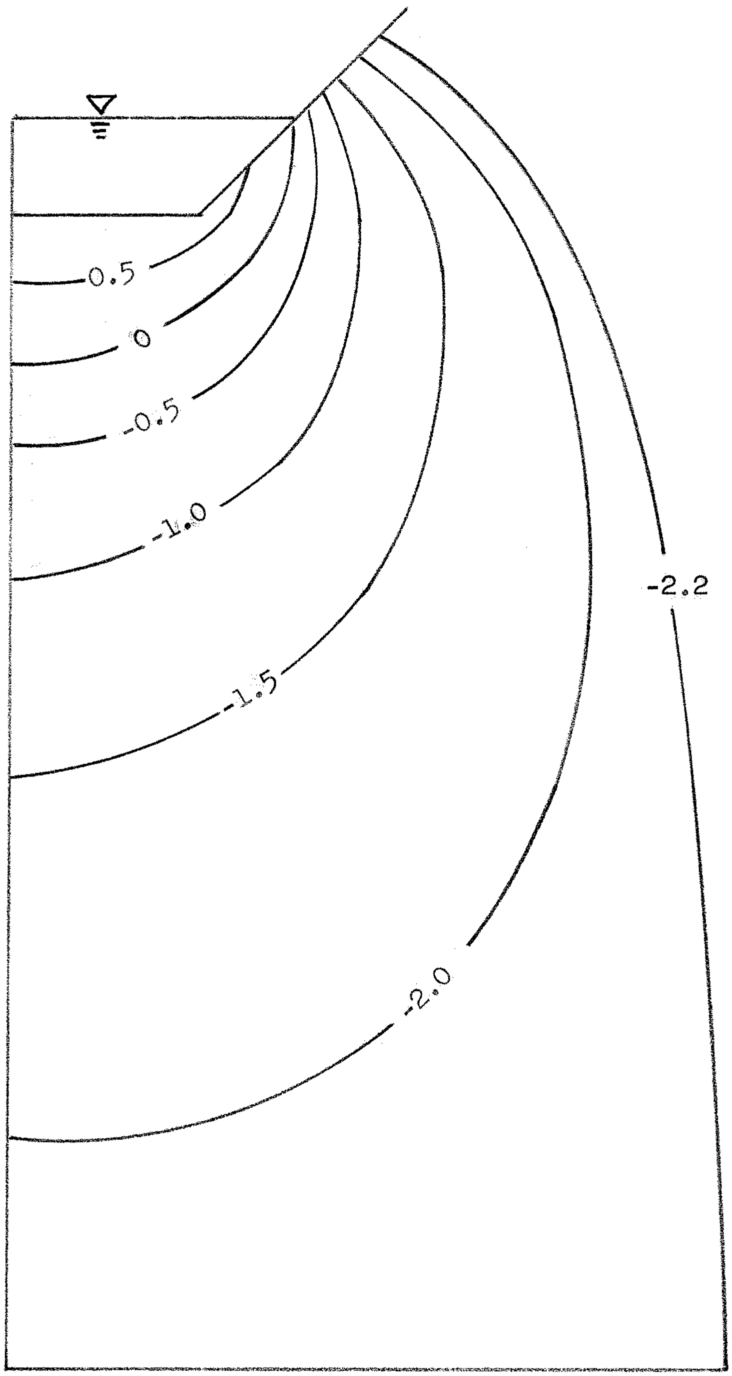


Figure 18. Contours of pressure head for iteration 7, example 3.

GENERAL PURPOSE DATA FORM

(8 COLUMN FIELDS)

PROGRAM: 723-G2-L2440 DATE:

REQUESTED BY: J.C.P PREPARED BY: CHECKED BY: PAGE 1 OF 2

	1	2	3	4	5	6	7	8	9	10
A		SEEPAGE FROM A TRAPEZOIDAL CANAL								
A		DETERMINATION OF FREE SURFACE - ITERATION 7								
A		116 NODES 94 ELEMENTS								
B	94	116	1	15						1
C	1	3.3	13.4							
C	2	3.7	13.8							
C	3	4.3	13.5							
C	4	4.0	12.9							
					C CARDS FOR NODES 5 THROUGH 112 NOT SHOWN					
C	113	1.9	12.0							
C	114	2.2	12.3							
C	115	2.5	12.6							
C	116	2.9	13.0							
D	1	1	4	3	2					
D	2	116	5	4	1					
D	3	115	6	5	116					
D	4	114	7	6	115					
						D CARDS FOR ELEMENTS 5 THROUGH 90 NOT SHOWN				
D	91	98	106	105	97					
D	92	99	107	106	98					
D	93	100	108	107	99					
D	94	101	109	108	100					
E	1	1	1							
F	102	-2.2								
F	103	-2.2								
F	104	-2.2								
F	105	-2.2								
F	106	-2.2								

SPK Form 321
1 Jul 66

(Previous editions are obsolete)

Figure 19. Input for iteration 7, example 3

GENERAL PURPOSE DATA FORM

(8 COLUMN FIELDS)

PROGRAM 723-G2-L2440 DATE

REQUESTED BY J. C. P. PREPARED BY CHECKED BY PAGE 2 OF 2

1	2	3	4	5	6	7	8	9	10
F 107	-2.2								
F 108	-2.2								
F 109	-2.2								
F 110	13								
F 111	13								
F 112	13								
F 113	13								
F 114	13								
F 115	13								
F 116	13								

SPK Form 321
1 Jul 66

(Previous editions are obsolete)

Figure 19. Input for iteration 7, example 3

SEEPAGE FROM A TRAPEZOIDAL CANAL
DETERMINATION OF FREE SURFACE-SEVENTH ITERATION
116 NODES 94 ELEMENTS

INPUT DATA

NO. OF ELEMENTS (NELS) = 94
NO. OF NODES (NNDS) = 116
NO. OF MATERIAL PROPERTY ZONES (NZNS) = 1
NO. OF SPECIFIED BOUNDARY HEADS (NHDS) = 15
NO. OF SOURCES OR SINKS (NWELS) = -0
NO. OF BOUNDARY ELEMENT SIDES (NQBND) FOR WHICH DISCHARGE IS SPECIFIED = -0

NODE	XCORD	YCORD
1	3.30	13.40
2	3.70	13.80
3	4.30	13.50
4	4.00	12.90
5	3.50	12.50
6	3.00	12.10

Values for nodes 7 through 111 not shown

112	1.40	12.00
113	1.90	12.00
114	2.20	12.30
115	2.50	12.60
116	2.90	13.00

ELEMENT	NODE 1	NODE 2	NODE 3	NODE 4	ZONE	THICKNESS
1	1	4	3	2	-0	-0.
2	116	5	4	1	-0	-0.
3	115	6	5	116	-0	-0.
4	114	7	6	115	-0	-0.
5	113	8	7	114	-0	-0.

Figure 20. Printout for interation 7, example 3

Values for elements 6 through 89 not shown

90	97	105	104	96	-0	-0.
91	98	106	105	97	-0	-0.
92	99	107	106	98	-0	-0.
93	100	108	107	99	-0	-0.
94	101	109	108	100	-0	-0.

ZONE	XPERM	YPERM	ALPHA
1	1.0000E+00	1.0000E+00	-0.

NODE	SPECIFIED BOUNDARY HEAD
102	-2.20
103	-2.20
104	-2.20
105	-2.20
106	-2.20
107	-2.20
108	-2.20
109	-2.20
110	13.00
111	13.00
112	13.00
113	13.00
114	13.00
115	13.00
116	13.00

Figure 20. Printout for interation 7, example 3

OUTPUT

MAXIMUM MATRIX BAND WIDTH (MAXBW) OF 21 CORRESPONDS TO NODE 3

COMPUTED VALUES OF HYDRAULIC HEAD AND FLUID PRESSURE HEAD

NODE	HYD. HD.	PRESS. HD.
1	1.1869E+01	-1.5307E+00
2	1.1537E+01	-2.2629E+00
3	1.1264E+01	-2.2361E+00
4	1.1265E+01	-1.6346E+00
5	1.1511E+01	-9.8866E-01
6	1.1734E+01	-3.6552E-01
7	1.1873E+01	7.3464E-02

Values for nodes 8 through 96 not shown

97	-7.2681E-01	-2.1268E+00
98	-7.0644E-01	-2.1064E+00
99	-6.9278E-01	-2.0928E+00
100	-6.8507E-01	-2.0851E+00
101	-6.8206E-01	-2.0821E+00

KNOWN VALUES OF HYDRAULIC HEAD AND PRESSURE HEAD AT BOUNDARIES

NODE	HYD. HD.	PRESS. HD.
102	-2.2000E+00	-2.2000E+00
103	-2.2000E+00	-2.2000E+00
104	-2.2000E+00	-2.2000E+00
105	-2.2000E+00	-2.2000E+00
106	-2.2000E+00	-2.2000E+00

Values for nodes 107 through 116 not shown

Figure 20. Printout for interation 7, example 3

V. REFERENCES

1. Thurnau, D. H., 1963, Algorithm 195, BANDSOLVE: Communications of the ACM, v. 6, no. 8, p. 441.

2. Liakopoulos, A. C., 1965, Variation of the Permeability Tensor Ellipsoid in Homogeneous Anisotropic Soils: Water Resources Res., v. 1, no. 1, pp. 135-141.

3. Cedergren, H. R., 1967, Seepage, Drainage, and Flow Nets: New York, John Wiley and Sons, 489 pp.

4. Hubbert, M. K., 1940, The Theory of Ground-Water Motion: Jour. Geol., v. 48, no. 8, pt. 1, pp. 785-944.

5. Tóth, J., 1962, A Theory of Groundwater Motion in Small Drainage Areas in Central Alberta, Canada: Jour. Geophys. Res., v. 67, no. 11, pp. 4375-4387.

6. Tóth, J., 1962, A Theoretical Analysis of Groundwater Flow in Small Drainage Basins: National Res. Council of Canada, Proc. Hydrol. Symp. No. 3 - Groundwater, pp. 75-96.

7. Freeze, R. A., and P. A. Witherspoon, 1966, Theoretical Analysis of Regional Groundwater Flow. 1. Analytical and Numerical Solutions to the Mathematical Model: Water Resources Res., v. 2, no. 4, pp. 641-656.

8. Freeze, R. A., and P. A. Witherspoon, 1967, Theoretical Analysis of Regional Groundwater Flow. 2. Effect of Water-table Configuration and Subsurface Permeability Variation: Water Resources Res., v. 3, no. 2, pp. 623-634.

9. Freeze, R. A., and P. A. Witherspoon, 1968, Theoretical Analysis of Regional Groundwater Flow. 3. Quantitative Interpretations: Water Resources Res., v. 4, no. 3, pp. 581-590.

10. Freeze, R. A., 1967, Quantitative Interpretation of Regional Groundwater Flow Patterns as an Aid to Water Balance Studies: Int'l. Assoc. Sci. Hydrol., General Symposium, Berne, Sept. 25-Oct. 7, 1967, pp. 154-173.

11. Javandel, Iraj, and P. A. Witherspoon, 1968, Analysis of Transient Fluid Flow in Multi-layered Systems: Univ. of California, Berkeley, Dept. of Civil Engineering, Contribution No. 124, 119 pp.

12. Neuman, S. P., and P. A. Witherspoon, 1969, Transient Flow of Ground Water to Wells in Multiple-aquifer Systems: Univ. of California, Berkeley, Dept. of Civil Engineering, Pub No. 69-1, 182 pp.

13. Blair, A. H., 1968, Simplifications of Ground-water Data Used for an Analogue of a Coastal Aquifer: Bulletin of the I.A.S.H., XIII, Annee No. 3, pp. 59-65.

14. Bouwer, Herman, 1964, Unsaturated Flow in Ground-water Hydraulics: Am. Soc. Civil Engineers, Hydraulics Div. Jour., v. 90, no. HY5, pp. 121-144.

15. Bouwer, Herman, 1965, Theoretical Aspects of Seepage from Open Channels: Am. Soc. Civil Engineers, Hydraulics Div. Jour., v. 91, no. HY3, pp. 37-59.

16. McCracken, D. D., and W. S. Dorn, 1964, Numerical Methods and Fortran Programming: New York, John Wiley and Sons, 457 pp.

17. Zienkiewicz O. C., and Y. K. Cheung, 1965, Finite Elements in the Solution of Field Problems: The Engineer, v. 220, no. 5722, pp. 507-510.

18. Zienkiewicz, O. C., P. Mayer, and Y. K. Cheung, 1966, Solution of Anisotropic Seepage by Finite Elements: Am. Soc. Civil Engineers, Eng. Mech. Div. Jour., v. 92, no. EM1, pp. 111-120.

19. U. S. Army Corps of Engineers, 1970, Preparation of Hydrologic Engineering Computer Programs: Methods Systemization Manual, The Hydrologic Engineering Center.

20. Winslow, J. D., and C. E. Nuzman, 1966, Electronic Simulation of Ground-water Hydrology in the Kansas River Valley Near Topeka, Kansas: State Geological Survey of Kansas, The University of Kansas, Special Distribution Publication 29.

21. Walton, W. C., 1970, Groundwater Resource Evaluation: New York, McGraw-Hill, 664 pp.

22. Polubarinova-Kochina, P. Y. A., 1962, Theory of Ground-water Movement (Translated from the Russian by R. J. M. DeWiest): Princeton, Princeton University Press, 613 pp.

ADDENDUM 1

THE FINITE ELEMENT METHOD

1. Finite Element Formulation

The equation to be solved is

$$\frac{\partial}{\partial x}(K_{xx} m \frac{\partial h}{\partial x}) + \frac{\partial}{\partial y}(K_{yy} m \frac{\partial h}{\partial y}) = W(x,y) \qquad \text{(A1-1)}$$

where

K_{xx} = A principal component of the conductivity ellipse which corresponds in direction to the x direction,

K_{yy} = a principal component of the conductivity ellipse which corresponds in direction to the y direction,

m = the local thickness of the flow region,

h = hydraulic head (a potential function) = $\frac{p}{\gamma} + z$,

$\frac{\partial h}{\partial x}$ = the component of hydraulic gradient in the x direction,

$\frac{\partial h}{\partial y}$ = the component of hydraulic gradient in the y direction,

$W(x,y)$ = a source or sink term in units of discharge per unit area; it is positive for a sink and negative for a source.

A problem equivalent to the problem described by equation A1-1 together with appropriate boundary conditions can be formulated using the calculus of variations (17). To employ this method the coefficients a_i for a function $h=\sum_{i=1}^{N} a_i\ f(x,y)$, where $f(x,y)$ is some assumed function of x and y, are found such that the function of h minimizes the functional, I, described by

$$I = \frac{1}{2} \iint_A \left[K_{xx} m (\frac{\partial h}{\partial x})^2 + K_{yy} m (\frac{\partial h}{\partial y})^2\right] dx\ dy$$
$$+ \iint_A W\ h\ dx\ dy - \int_S v_b\ m\ h\ ds \qquad \text{(A1-2)}$$

where

I = functional,

v_b = discharge per unit area across and normal to an element boundary,

A = area of region,

S = a line along which v_b is specified.

A single relationship to assume for h which provides an accurate solution to a given boundary value problem over the entire region is usually not available however. Therefore, the region is divided into subareas called finite elements for each of which an approximate function for h can be assumed.

In order to make the minimization tractable for numerical solution, the region is divided into triangular elements, and the variation of potential function over each element is taken to be planar, i.e.,

$$h = A + Bx + Cy \tag{A1-3}$$

Hence the potential "surface" over an element can be expressed in terms of the three nodal values of potential function and the coordinates of the nodes. Figure A1-1 shows a triangular element defined by nodes r, s, and t. The distribution of h over this element is defined by

$$h = \left[N_r, N_s, N_t,\right] \begin{Bmatrix} h_r \\ h_s \\ h_t \end{Bmatrix} \tag{A1-4}$$

where the elements of the coefficient matrix are

$$N_r = (a_r + b_r x + c_r y)/2\Delta$$

$$N_s = (a_s + b_s x + c_s y)/2\Delta$$

$$N_t = (a_t + b_t x + c_t y)/2\Delta$$

and

$$a_r = x_s y_t - x_t y_s$$

$$b_r = y_s - y_t$$

$$c_r = x_t - x_s$$

$$a_s = x_t y_r - x_r y_t$$

$$b_s = y_t - y_r$$

$$c_s = x_r - x_t$$

$$a_t = x_r y_s - x_s y_r$$

$$b_t = y_r - y_s$$

$$c_t = x_s - x_r$$

x_i = x coordinate of node i,

y_i = y coordinate of node i,

Δ = area of element e provided that nodes r, s, and t are arranged in counterclockwise order around the element,

$\begin{Bmatrix} h_r \\ h_s \\ h_t \end{Bmatrix} = \{h^e\}$ = column vector of nodal values of the potential function for element e.

Equation A1-4 was obtained by treating the three nodal values of potential as knowns in equation A1-3 and solving the resulting three equations for the three unknown values A, B, and C. Substitution of the expressions obtained for A, B, and C into equation A1-3 and rearranging the results yields equation A1-4.

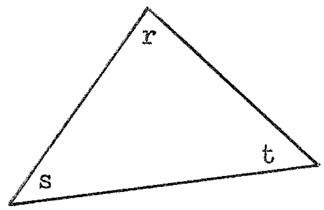


Figure A1-1 Triangular Element

Minimization of the functional described by equation A1-2 over the region is approximated by establishing equations that minimize the functional with respect to each unknown nodal value of the potential function. The resulting equations are solved simultaneously to yield desired nodal values of potential function.

The functional for the region can be considered as the sum of the functionals for each element. That is,

$$I_{region} = \sum_{1}^{N} I^e \qquad (A1\text{-}5)$$

where

N = number of elements,

I^e = functional for element e.

Because the potential over an element is defined completely by the element's three nodal values of potential, a minimization equation for a node involves only the elements that contain that node.

When setting up the minimization equations, it is convenient to first develop expressions for minimizing the functional over each element. The final minimization equations can then be assembled by adding the contributions of each element. In matrix notation the minimization equation for the typical element of figure A1-1 is

$$\left\{\frac{\partial I^e}{\partial h^e}\right\} = \left\{\begin{matrix} \dfrac{\partial I^e}{\partial h_r} \\ \\ \dfrac{\partial I^e}{\partial h_s} \\ \\ \dfrac{\partial I^e}{\partial h_t} \end{matrix}\right\} = \left\{\begin{matrix} 0 \\ \\ 0 \\ \\ 0 \end{matrix}\right\} \tag{A1-6}$$

An element of the column vector in equation A1-6 is:

$$\frac{\partial I^e}{\partial h_r} = \iint_A \left[K_{xx}\, m \frac{\partial h}{\partial x} \frac{\partial}{\partial h_r}\left(\frac{\partial h}{\partial x}\right) + K_{yy}\, m \frac{\partial h}{\partial y} \frac{\partial}{\partial h_r}\left(\frac{\partial h}{\partial y}\right)\right] dx\ dy$$

$$+ \iint_A W \frac{\partial h}{\partial h_r}\, dx\ dy - \int_S v_b\, m \frac{\partial h}{\partial h_r}\, ds \tag{A1-7}$$

where integrations in equation A1-7 apply only to the element e. When equation A1-4 is substituted into A1-7, we have:

$$\frac{\partial I^e}{\partial h_r} = \iint_A \left\{K_{xx}\, m \left[\frac{\partial N_r}{\partial x}, \frac{\partial N_s}{\partial x}, \frac{\partial N_t}{\partial x}\right] \{h^e\} \left(\frac{\partial N_r}{\partial x}\right)\right.$$

$$\left. + K_{yy}\, m \left[\frac{\partial N_r}{\partial y}, \frac{\partial N_s}{\partial y}, \frac{\partial N_t}{\partial y}\right] \{h^e\} \left(\frac{\partial N_r}{\partial y}\right)\right\} dx\ dy$$

$$+ \iint_A W N_r\, dx\ dy - \int_S v_b\, m\, N_r\, ds \tag{A1-8}$$

or,

$$\frac{\partial I^e}{\partial h_r} = \frac{K_{xx} m}{4\Delta} \left[\, b_r b_r,\ b_s b_r,\ b_t b_r \,\right] \{h^e\}$$

$$+ \frac{K_{yy} m}{4\Delta} \left[c_r c_r,\ c_s c_r,\ c_t c_r\right] \{h^e\}$$

$$+ \frac{W\Delta}{3} - \frac{v_b mL}{2} \tag{A1-9}$$

where L = length of element side (or sides) adjacent to node r for which v_b is specified (17).

The quantities K_{xx}, K_{yy}, m, and W are taken to be constant over the element in obtaining equation A1-9 from equation A1-8.

Equation A1-6 may now be written as:

$$\left\{\frac{\partial I^e}{\partial h^e}\right\} = \left[G^e\right]\{h^e\} - \{F^e\} \tag{A1-10}$$

where $\left[G^e\right]$ is a square matrix having a typical element

$$G_{rs} = \frac{m}{4\Delta}\left[K_{xx}\, b_r\, b_s + K_{yy}\, c_r c_s\right] \tag{A1-11}$$

and

$$F^e = \begin{Bmatrix} \dfrac{v_b mL}{2} - \dfrac{W\Delta}{3} \\ \dfrac{v_b mL}{2} - \dfrac{W\Delta}{3} \\ \dfrac{v_b mL}{2} - \dfrac{W\Delta}{3} \end{Bmatrix} \tag{A1-12}$$

Note that the terms $\frac{v_b mL}{2}$ in equation A1-12 are nonzero only if v_b is specified for an element side adjacent to a node.

The final equations can now be assembled by combining contributions from each element. The equation for a typical node r is

$$\frac{\partial I}{\partial h_r} = \Sigma\, \frac{\partial I^e}{\partial h_r} = 0 \tag{A1-13}$$

The summation in equation A1-13 is carried out over all elements that contain node r. Equation A1-13 is developed for all nodes, and the resulting series of equations is described by

$$\left[G\right]\{h\} = \{F\} \tag{A1-14}$$

The elements of [G] and {F} are obtained by substitution of equation A1-10 into equation A1-13. The column vector {h} is a vector of all the nodal values of the potential function. Solution of equation A1-14 is discussed in Addendum 2.

2. Quadrilateral Elements

A useful extension of the finite element method permits division of a region into quadrilateral subareas composed of triangular elements. Internally in the computer program, quadrilateral subareas are divided into four triangles by generating a center node which has coordinates that are obtained by averaging the coordinates of the four corner nodes (figure A1-2). Advantages of working with quadrilateral "elements" rather than triangular elements are that input preparation is simplified, computer storage requirements are reduced, and the computer time required for solution of a problem is reduced. When the functional, I, is minimized with respect to a center node, such as the node labled A in figure A1-2, the only unknowns that appear in the equation other than the potential at node A are the potential functions for the surrounding nodes, which in figure A1-2 are labled 1, 2, 3, and 4. Therefore an equation can be written for h_a in terms of h_1, h_2, h_3, h_4:

$$h_A = \frac{1}{G_{AA}} \left[F_A - G_{A1}\, h_1 - G_{A2}\, h_2 - G_{A3}\, h_3 - G_{A4}\, h_4\right] \qquad \text{(A1-15)}$$

where

$$G_{ij} = \Sigma \frac{m}{4\Delta} \left[K_{xx}\, b_i\, b_j + K_{yy}\, c_i\, c_j\right] \qquad \text{(A1-16)}$$

$$F_A = \Sigma \left(\frac{v_b mL}{2} - \frac{W\Delta}{3}\right) \qquad \text{(A1-17)}$$

The summations in equations A1-16 and A1-17 sum contributions from the four triangular elements.

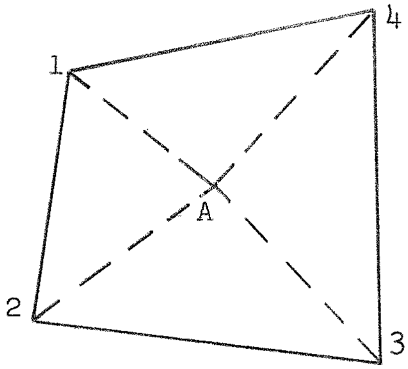


Figure A1-2 Quadrilateral Element

The right side of equation A1-15 is substituted for h_a where it appears in the minimization equations for nodes 1, 2, 3, and 4. For example, for node 1 the resulting equation is

$$\left(G_{11} - \frac{G_{1A}}{G_{AA}} G_{A1}\right) h_1 + \left(G_{12} - \frac{G_{1A}}{G_{AA}} G_{A2}\right) h_2 - \frac{G_{1A}}{G_{AA}} G_{A3} h_3$$

$$+ \left(G_{14} - \frac{G_{1A}}{G_{AA}} G_{A4}\right) h_4 = F_1 - \frac{G_{1A}}{G_{AA}} F_4 \qquad \text{(A1-18)}$$

In this manner the unknown potential functions for all center nodes are removed from the final matrix equation (equation A1-14), which can reduce the number of unknowns in that equation by as much as 50%.

3. Anisotropic Media

The terms K_{xx} and K_{yy} in equation A1-1 are principal conductivity components. If a finite element solution is desired for a region of anisotropic conductivity, the coordinates of the nodes in a global coordinate system (i.e., the coordinate system used for all nodes) must be rotated to a coordinate system in which the axes are aligned with the principal conductivity axes (18). Such a transformation can vary from element to element and is accomplished internally in the program for elements that lie in zones for which an angle of anisotropy is specified.

4. Axisymmetric Cylindrical Coordinates

If equation A1-1 is rewritten in terms of axisymmetric cylindrical coordinates, the resulting equation is

$$\frac{1}{r} \frac{\partial}{\partial r} (K_{rr}\, r \frac{\partial h}{\partial r}) + \frac{\partial}{\partial z} (K_{zz} \frac{\partial h}{\partial z}) = 0 \qquad \text{(A1-19)}$$

The source-sink term has been left out of this equation because it is of little practical value for most applications. The functional corresponding to equation A1-19 is

$$I = \pi \iint_A \left[K_{rr} (\frac{\partial h}{\partial r})^2 + K_{zz} (\frac{\partial h}{\partial z})^2\right] r\, dr\, dz$$

$$- 2\pi \int_{S_z} v_b\, h\, r\, dz \qquad \text{(A1-20)}$$

The second integral on the right side of equation A1-20 accounts for a boundary condition involving specified lateral flow across a boundary that is parallel to the z axis. Such a boundary condition would exist, for example, if flow across a well bore were specified.

When the functional in equation A1-20 is minimized, the equation corresponding to equation A1-9 is

$$\frac{\partial I^e}{\partial h_r} = \frac{K_r}{4\Delta}\left[b_r,\ b_s,\ b_t\right]\{h^e\}\ b_r$$

$$+\ \frac{K_z}{4\Delta}\left[c_r,\ c_s,\ c_t\right]\{h^e\}\quad c_j\quad \frac{1}{3}(r_r + r_s + r_t)$$

$$-\ \frac{r_b v_b L}{2} = 0 \qquad \text{(A1-21)}$$

where

r_b = radial coordinate of element side (parallel to z axis) for which discharge per unit area, v_b, is specified,

L = length of element side for which v_b is specified.

Equation A1-21 is identical to A1-9 if m in the first two terms on the right side of equation A1-9 is defined by

$$m = \frac{r_r + r_s + r_t}{3} \qquad \text{(A1-22)}$$

and if m in the fourth term on the right side of equation A1-9 is defined by

$$m = r_b \qquad \text{(A1-23)}$$

When use of axisymmetric cylindrical coordinates is called for in the computer program, the substitution described by equation A1-22 is made internally in the program. The substitution described by equation A1-23 is not required because of the manner in which boundary conditions are specified. The product $v_b m$ is specified as input for problems using Cartesian coordinates and the product $2\pi r_b v_b$ is specified for problems using axisymmetric cylindrical coordinates. The latter product is divided by 2π in the program to yield a term equivalent to $v_b m$.

ADDENDUM 2

SOLUTION OF MATRIX EQUATION

The basic matrix equation resulting from the finite element formulation is (Addendum 1, equation A1-14).

$$[G]\ \{h\} = \{F\} \tag{A2-1}$$

where the terms were defined in Addendum 1. Some of the elements of the vector {h} are known boundary potentials, thus should be treated as known values. This can be accomplished by numbering the node points on the boundaries where the potentials are known last. Equation A2-1 can then be written in partitioned form as

$$\left[\begin{array}{cccccc|ccc}
G_{11} & G_{12} & G_{13} & \cdots & & G_{1m} & \cdots & & G_{1n} \\
G_{21} & G_{22} & G_{23} & \cdots & & G_{2m} & \cdots & & G_{2n} \\
\vdots & & & & & \vdots & & & \vdots \\
G_{m1} & & & & & G_{mm} & \cdots & & G_{mn} \\
\hline
\vdots & & & & & \vdots & & & \vdots \\
G_{n1} & & & & & G_{nm} & \cdots & & G_{nn}
\end{array}\right]
\left\{\begin{array}{c} h_1 \\ h_2 \\ \vdots \\ h_m \\ \hline \vdots \\ h_n \end{array}\right\}
=
\left\{\begin{array}{c} F_1 \\ F_2 \\ \vdots \\ F_m \\ \hline \vdots \\ F_n \end{array}\right\} \tag{A2-2}$$

where

m = the number of unknown hydraulic heads, and

n = the total number of node points.

The hydraulic heads h_{m+1} through h_n are all known boundary heads, thus all the equations forming rows m+1 through n are redundant. Also, all of the terms forming columns m+1 through n belong on the right sides of the equations. After performing the operations indicated by these facts, the resulting matrix equation is

$$[G]\ \{h\} = \{b\} \tag{A2-3}$$

where $\{b\}$ is the vector composed of the known boundary, source, and sink terms and $[G]$ is the coefficient matrix of order m.

Because m is the number of nodes where the value of potential is to be determined, the matrix $[G]$ can be very large - for a 400 node problem it could be of order 400 x 400. However, most of the entries are zero, the only nonzero entries being clustered in a narrow band around the main diagonal. This fact makes it possible to solve equation A2-3 by a specialized form of the Gauss elimination technique which uses a coefficient matrix whose entries are shifted so that the main diagonal is vertical and all entries except a band of width M (an odd number) centered around the main diagonal are eliminated. The band width, M, is defined as

$$M = 2L + 1, \tag{A2-4}$$

L = the maximum spread in node point numbers in a quadrilateral element, and matrix entry (i,j) is shifted to (i,j-i+(M+1)/2).

Thus,

$$[G] = \begin{bmatrix} G_{11} & \cdot & \cdot & \cdot & \diagdown\; 0 \\ \cdot & G_{22} & & & \cdot \\ \vdots & & \cdot & & \vdots \\ \cdot & & & \cdot & \cdot \\ 0 \;\diagdown & & \cdot & \cdot & G_{mm} \end{bmatrix} \tag{A2-5}$$

where the number of entries in any row between the diagonal lines is equal to M, becomes

$$[G_s] = \begin{bmatrix} 0 & \cdots & 0 & G_{1,(M+1)/2} & \cdots & G_{1M} \\ \vdots & & & \vdots & & \vdots \\ G_{mM} & \cdots & & G_{m,(M+1)/2} & 0 \cdots & 0 \end{bmatrix} \tag{A2-6}$$

Computer storage and unnecessary arithmetic operations on zero terms are both considerably reduced using this method. A (400x400) coefficient matrix (160,000 entries) could conceivably be reduced to a (400x41) matrix (16,400 entries).

ADDENDUM 3

DEFINITIONS OF VARIABLES

Variables	Definitions
ALPHA*	Angle of rotation of conductivity axes from global coordinate axes, in radians.
ANG	Temporary variable for ALPHA.
ANGLE	Temporary variable for ALPHA.
ATRI*	Temporary array for areas of triangular components of a quadrilateral element.
B*	Known vector composed of source, sink, and boundary condition terms.
BCN	Temporary variable composed of known boundary, source, and sink terms for the center node of a quadrilateral element.
BJ	Temporary variable for y coordinate differences.
BK	Temporary variable for y coordinate differences.
BL	Temporary variable for y coordinate differences.
C*	A variable, used only in subroutine BSOLVE, that is equivalent to the variable G.
CJ	Temporary variable for x coordinate differences.
CK	Temporary variable for x coordinate differences.
CL	Temporary variable for x coordinate differences.
CONST	Temporary variable for conversion of miles to feet.
CSALP	Cosine of angle of anisotropy (ALPHA).
DIST	Length of a side of a quadrilateral element.
G*	Array of elements of coefficient matrix developed in subroutine GEES.
GC	Temporary variable that represents G for the center node of a quadrilateral element.
GT*	Temporary array used in calculating elements of coefficient matrix.
GTEMP	Temporary variable used in computing G elements.
HD*	Temporary array of known nodal values of the potential function for a quadrilateral element.

*Subscripted variable

Variables	Definitions
HEAD*	Array of known (input) values of the potential function.
I	Temporary counter used for a subscript.
II	Temporary counter used for a subscript.
IM	Temporary variable.
IPRES	Indicator calling for the calculation and printing of pressure heads.
IPRGS	Indicator calling for the printing of the elements of the coefficient (G) matrix developed in subroutine GEES.
IRADL	Indicator specifying use of axisymmetric cylindrical coordinates.
ITMP	Temporary variable.
IUNIT	Indicator calling for the conversion of the units of the input coordinates from miles to feet.
IWLEL*	Array of elements that are designated as sources or sinks.
IZONE*	Array indicating material zone that a given element is in.
J	Temporary counter used for a subscript.
JJ	Temporary counter used for a subscript.
JTMP	Temporary variable.
JM	Temporary variable.
K	Temporary counter used for a subscript.
KA	Temporary node point variable.
KB	Temporary node point variable.
KC	Temporary node point variable.
KEYND	Node for which MAXBW is determined.
KK	Temporary counter used for a subscript.
KM	Temporary subscript variable.
KN	Temporary subscript variable.
KOUNT	Counter used in subroutine GEES to count the number of source/sink elements that have been used in determining the coefficient matrix.
L	Temporary counter used for a subscript.
LR	Variable used in transformation of subscripts of coefficient matrix.

*Subscripted variable

Variables	Definitions
LS	Temporary variable.
M	Variable used only in subroutine BSOLVE to represent MAXBW.
MAXBW	The number of columns of the coefficient matrix developed in subroutine GEES.
MAXND	Temporary variable used for finding MAXBW.
MINND	Temporary variable used for finding MAXBW.
N	Temporary counter used for a subscript in main program; also, variable used in subroutine BSOLVE that is equivalent to NRED.
NA	Temporary variable representing a corner node of a quadrilateral element.
NAT	Temporary variable used in transformation of subscripts of coefficient matrix.
NB	Temporary variable representing a corner node of a quadrilateral element.
NBT	Temporary variable used in transformation of subscripts of coefficient matrix.
NCOL	Temporary variable used for a subscript.
ND*	Temporary array of node numbers for the corner nodes of a quadrilateral element.
NDA	Node number of a node on a known-flow boundary.
NDB	Node number of a node on a known-flow boundary.
NDHD	Node number of a node on a boundary of known potential function.
NELS	Number of elements.
NHDS	Number of nodes for which the potential function in known.
NNDS	Number of nodes.
NODE*	Array of node numbers that associates elements with corner nodes that delineate the elements.
NPIV	Temporary variable used for row interchange.
NQBND	Number of element sides on known-flow boundaries.
NRED	Number of nodes for which the potential function is not known (i.e. NNDS - NHDS).
NROW	Temporary variable used for a subscript.

*Subscripted variable

Variables	Definitions
NTEMP	Temporary variable.
NUDE	Temporary variable.
NWELS	Number of elements that represent sources or sinks.
NZNS	Number of material property zones.
PRES	Pressure head, which is equal to the computed value of the potential function minus YCORD for a node.
QBND*	Array of flows specified for known-flow boundaries.
QTEMP	Temporary variable used in adding boundary flows to the B vector.
QWELL*	Array of flows associated with sources and sinks.
SNALP	Sine of angle of anisotropy (ALPHA).
TEMP	Temporary variable.
THICK*	Array of element thicknesses.
TITLE*	Array used for job title.
TMP	Temporary variable.
TPXJ	Temporary variable.
TPXL	Temporary variable.
TPYJ	Temporary variable.
TPYL	Temporary variable.
V	A variable used, only in subroutine BSOLVE, that is equivalent to the variable B.
XCN	Temporary variable for x-coordinate of a generated center node of a quadrilateral element.
XCORD*	Array of x-coordinates of nodes.
XK	Temporary variable representing XPERM.
XKT	Temporary variable.
XNA	Temporary variable representing x-coordinate of a corner node of a quadrilateral element.
XNB	Temporary variable representing x-coordinate of a corner node of a quadrilateral element.
XPERM*	Array of major conductivities.
XX*	Temporary array representing x-coordinates of four corner nodes of a quadrilateral element.

*Subscripted variable

Variables	Definitions
YCN	Temporary variable for y-coordinate of a generated center node of a quadrilateral element.
YCORD*	Array of y-coordinates of nodes.
YK	Temporary variable representing YPERM.
YKT	Temporary variable.
YNA	Temporary variable representing y-coordinate of a corner node of a quadrilateral element.
YNB	Temporary variable representing y-coordinate of a corner node of a quadrilateral element.
YPERM*	Array of minor conductivities.
YY*	Temporary array representing y-coordinates of four corner nodes of a quadrilateral element.

*Subscripted variable

```
C        FINITE ELEMENT SOLUTION OF STEADY STATE POTENTIAL FLOW PROBLEMS
C        QUADRILATERAL ELEMENTS
C        PROGRAM 723-G2-L2440
C        NOVEMBER 1970  (PROGRAM NUMBERED MAY 1971)
C        SUBROUTINE GEES IS CALLED FROM STATEMENT 205.03
C        SUBROUTINE BSOLVE IS CALLED FROM STATEMENT 265
      DIMENSION TITLE(21),XCORD(400),YCORD(400),NODE(400,4),IZONE(400)
     1,XPERM(20),YPERM(20),ALPHA(20),NDHD(100),HEAD(100),G(400,50)
     2,B(400),QWELL(200),IWLEL(200),QBND(100),NDA(100),NDB(100)
     3,THICK(400)
      COMMON B,G,NRED,NELS,NNDS,NHDS,XCORD,YCORD,NODE,IZONE,XPERM,YPERM,
     1ALPHA,HEAD,NDHD,MAXBW,QWELL,IWLEL,QBND,NDA,NDB,THICK,NWELS,NQBND,
     2IRADL,LR
    1 FORMAT (1H ,I4,5X,E10.3)
    2 FORMAT (1X,I7,9I8)
    3 FORMAT (1X,1A1,2A3,18A4)
    4 FORMAT (1X,I7,9F8.0)
    6 FORMAT (1H0,10HINPUT DATA)
    7 FORMAT(1H ,I4,9(3X,F10.2))
    8 FORMAT (1H1)
    9 FORMAT (1H0,6HOUTPUT)
   10 FORMAT (1H0,5X,3HROW,8X,6HCOLUMN,16X,1HG)
   11 FORMAT (1H ,5X,I3,9X,I3,13X,E10.3)
   12 FORMAT (1H0,5X,3HROW,17X,1HB)
   13 FORMAT (1H ,5X,I3,2(11X,E11.4))
   14 FORMAT (34H0COMPUTED VALUES OF HYDRAULIC HEAD/1H ,5X,4HNODE,14X,4H
     1HEAD)
   15 FORMAT (45H0KNOWN VALUES OF HYDRAULIC HEAD AT BOUNDARIES/1H ,5X,
     14HNODE,14X,4HHEAD)
   16 FORMAT (10H0MAXBW OF I5,24H, CORRESPONDING TO NODE ,I5,68H, IS GRE
     1ATER THAN ORDER OF MATRIX (NRED) OR EXCEEDS DIMENSION LIMITS)
   17 FORMAT (26H0NO. OF ELEMENTS (NELS) = ,I6/23H NO. OF NODES (NNDS) =
     1 ,I6/41H NO. OF MATERIAL PROPERTY ZONES (NZNS) = ,I6/42H NO. OF SP
     2ECIFIED BOUNDARY HEADS (NHDS) = ,I6/35H NO. OF SOURCES OR SINKS (N
     3WELS) = ,I6/74H NO. OF BOUNDARY ELEMENT SIDES (NQBND) FOR WHICH DI
     4SCHARGE IS SPECIFIED = ,I6)
   18 FORMAT (58H0COMPUTED VALUES OF HYDRAULIC HEAD AND FLUID PRESSURE H
     1EAD/1H ,5X,4HNODE,12X,8HHYD. HD.,13X,10HPRESS. HD.)
   19 FORMAT  (5H0NODE,8X,5HXCORD,8X,5HYCORD)
   20 FORMAT (9H0 ELEMENT,2X,6HNODE 1,2X,6HNODE 2,2X,6HNODE 3,2X,6HNODE
     14,4X,4HZONE,4X,9HTHICKNESS)
   21 FORMAT (5H0ZONE,8X,5HXPERM,8X,5HYPERM,8X,5HALPHA)
   22 FORMAT (1H0,9X,9HSPECIFIED/1H ,4HNODE,6X,8HBOUNDARY/1H ,12X,4HHEAD
     1)
   23 FORMAT (1X,I7,5I8,4F8.0)
   24 FORMAT (2X,I6,5I8,4X,F10.2)
   25 FORMAT (63H0SOURCES OR SINKS (+ INDICATES RECHARGE, - INDICATE2 DI
     1SCHARGE)/1H ,19HELEMENT   DISCHARGE)
   26 FORMAT (1H0,29HSPECIFIED BOUNDARY DISCHARGES/1H ,6X,28HUNIT      B
     1OUNDARY  BOUNDARY/1H ,33H    DISCHARGE    NODE A    NODE B)
   27 FORMAT (1X,I7,I8,8F8.0)
   28 FORMAT (1H ,3X,E10.3,4X,I4,6X,I4)
   29 FORMAT (63H0KNOWN VALUES OF HYDRAULIC HEAD AND PRESSURE HEAD AT BO
     1UNDARIES/1H ,5X,4HNODE,12X,8HHYD. HD.,13X,10HPRESS. HD.)
   31 FORMAT (1H0,40H*** AXI-SYMMETRIC RADIAL COORDINATES ***)
   33 FORMAT (1H ,I4,5X,2(E11.4,2X),F8.2)
   34 FORMAT (1H1,27H***************************   /
     11X,20HPROGRAM 723-G2-L2440 /
     21X,27HVERSION DATED NOVEMBER 1970 /
     31X,27H*************************** //)
   35 FORMAT (1H0,37HMAXIMUM MATRIX BAND WIDTH (MAXBW) OF ,I4,21H CORRES
     1PONDS TO NODE ,I4)
   36 FORMAT (1H0,78HVALUES FOR COORDINATES CONVERTED FROM MILES TO FEET
     1 BEFORE MAKING CALCULATIONS)
   37 PRINT 34
C
C         READ 3 TITLE CARDS
C
      DO 40 I=1,3
      READ 3, (TITLE(N),N=1,21)
      PRINT 3, (TITLE(N),N=1,21)
   40 CONTINUE
```

```
C
C        READ JOB SPECIFICATION
C
      READ 2,NELS,NNDS,NZNS,NHDS,NWELS,NQBND,IRADL,IPRGS,IUNIT,IPRES
      IF (NELS.LE.0) STOP
      PRINT 6
      PRINT 17,NELS,NNDS,NZNS,NHDS,NWELS,NQBND
      IF(IRADL.GT.0) PRINT 31
C
C        READ NODAL COORDINATES
C
      PRINT 19
      DO 41  J=1,NNDS
      READ 4, I,XCORD(I),YCORD(I)
      PRINT 7, I,XCORD(I),YCORD(I)
   41 CONTINUE
      IF (IUNIT.LE.0) GO TO 43
      PRINT 36
      CONST = 5280.
      DO 42 I=1,NNDS
      XCORD(I)=XCORD(I)*CONST
      YCORD(I)=YCORD(I)*CONST
   42 CONTINUE
C
C        READ ELEMENT DATA
C
   43 PRINT 20
      DO 45  K=1,NELS
      READ 23, I,(NODE(I,J),J=1,4),IZONE(I),THICK(I)
      PRINT 24, I,(NODE(I,J),J=1,4),IZONE(I),THICK(I)
      IF(IZONE(I).LE.0)IZONE(I)=1
   45 CONTINUE
C
C        READ MEDIA PROPERTIES
C
      PRINT 21
      DO 50  J=1,NZNS
      READ 4, I,XPERM(I),YPERM(I),ALPHA(I)
      PRINT 33, I,XPERM(I),YPERM(I),ALPHA(I)
   50 CONTINUE
C
C        READ BOUNDARY CONDITIONS
C
      IF (NHDS.LE.0) GO TO 65
      PRINT 22
      DO 60  I=1,NHDS
      READ 4,NDHD(I),HEAD(I)
      PRINT 7,NDHD(I),HEAD(I)
   60 CONTINUE
   65 IF(NWELS.LE.0) GO TO 75
      PRINT 25
      DO 70 I=1,NWELS
      READ 4, IWLEL(I),QWELL(I)
      PRINT 1, IWLEL(I),QWELL(I)
   70 CONTINUE
   75 IF(NQBND.LE.0) GO TO 81
      PRINT 26
      DO 80 I=1,NQBND
      READ 27, NDA(I),NDB(I),QBND(I)
      PRINT 28, QBND(I),NDA(I),NDB(I)
   80 CONTINUE
C
C       QBND FOR AXI-SYMMETRIC RADIAL COORDINATES
C
   81 IF(IRADL.LE.0.OR.NQBND.LE.0) GO TO 85
      DO 83 I=1,NQBND
      QBND(I)=QBND(I)/6.283
   83 CONTINUE
   85 CONTINUE
C
C        DETERMINE MAXIMUM BAND WIDTH (MAXBW)
C
```

```
      NRED=NNDS-NHDS
      MAXBW=0
      DO 200 I=1,NRED
      MINND=NRED
      MAXND=0
      DO 180 J=1,NELS
      DO 110 K=1,4
      IF (NODE(J,K).EQ.I) GO TO 120
  110 CONTINUE
      GO TO 180
  120 DO 150 K=1,4
      NUDE=NODE(J,K)
      IF(NUDE.GT.NRED) GO TO 150
      IF (NUDE.GT.MAXND) MAXND=NUDE
      IF (NUDE.LT.MINND) MINND=NUDE
  150 CONTINUE
  180 CONTINUE
      NTEMP=I-MINND
      IF ((MAXND-I).GT.NTEMP) NTEMP=MAXND-I
      IF(NTEMP.LE.MAXBW) GO TO 200
      MAXBW=NTEMP
      KEYND=I
  200 CONTINUE
      MAXBW=MAXBW*2+1
      IF (MAXBW.LE.50.AND.MAXBW.LE.NRED) GO TO 205
      PRINT 16,MAXBW,KEYND
      GO TO 37
  205 PRINT 8
      PRINT 9
      PRINT 35,MAXBW,KEYND
C
C         CALL SUBROUTINE GEES TO SET UP MATRIX EQUATION
C
      CALL GEES
      IF(IPRGS.LE.0) GO TO 265
      PRINT 10
      DO 250 I=1,NRED
      DO 210 J=1,MAXBW
      NROW=I
      NCOL=J-LR+I
      IF (G(I,J).EQ.0.) GO TO 210
      PRINT 11, NROW,NCOL,G(I,J)
  210 CONTINUE
  250 CONTINUE
      PRINT 12
      DO 260 I=1,NRED
      PRINT 13,I,B(I)
  260 CONTINUE
C
C         SOLVE MATRIX EQUATION
C
  265 CALL BSOLVE(G,B,NRED,MAXBW)
C
C       PRINT HYDRAULIC HEADS OR HYDRAULIC HEADS AND PRESSURE HEADS
C
      IF(IPRES.GT.0) GO TO 290
      PRINT 14
      DO 270 I=1,NRED
      PRINT 13,I,B(I)
  270 CONTINUE
      IF (NHDS.LE.0) GO TO 37
      PRINT 15
      DO 280 I=1,NHDS
      PRINT 13,NDHD(I),HEAD(I)
  280 CONTINUE
      GO TO 37
  290 PRINT 18
      DO 300 I=1,NRED
      PRESS=B(I)-YCORD(I)
      PRINT 13,I,B(I),PRESS
  300 CONTINUE
      IF (NHDS.LE.0) GO TO 37
```

```
      PRINT 29
      DO 310 I=1,NHDS
      J=NDHD(I)
      PRESS=HEAD(I)-YCORD(J)
      PRINT 13,NDHD(I),HEAD(I),PRESS
  310 CONTINUE
      GO TO 37
      END
```

```
      SUBROUTINE GEES
C         SUBROUTINE TO CALCULATE MATRIX ELEMENTS
      DIMENSION TITLE(21),XCORD(400),YCORD(400),NODE(400,4),IZONE(400)
     1,XPERM(20),YPERM(20),ALPHA(20),NDHD(100),HEAD(100),G(400,50)
     2,B(400),QWELL(200),IWLEL(200),QBND(100),NDA(100),NDB(100)
     3,THICK(400)
      DIMENSION XX(5),YY(5),HD(5),GT(5),ND(5),ATRI(4)
      COMMON B,G,NRED,NELS,NNDS,NHDS,XCORD,YCORD,NODE,IZONE,XPERM,YPERM,
     1ALPHA,HEAD,NDHD,MAXBW,QWELL,IWLEL,QBND,NDA,NDB,THICK,NWELS,NQBND,
     2IRADL,LR
      LR=(MAXBW+1)/2
      DO  2 I=1,NNDS
      B(I)=0.
      DO  2 J=1,MAXBW
      G(I,J)=0.
    2 CONTINUE
C
C         ADD BOUNDARY DISCHARGES TO B VECTOR
C
      IF(NQBND.LE.0) GO TO 4
      DO  3 I=1,NQBND
      K=NDA(I)
      L=NDB(I)
      DIST=((XCORD(K)-XCORD(L))**2+(YCORD(K)-YCORD(L))**2)**.5
      QTEMP=QBND(I)*DIST/2.
      B(K)=B(K)+QTEMP
      B(L)=B(L)+QTEMP
    3 CONTINUE
    4 KOUNT =0
C
C   *** BEGIN QUAD ELEMENT LOOP
C
      DO 200 I=1,NELS
      GC=0.
      BCN=0.
      DO 5 K=1,5
    5 GT(K)=0.
      J=IZONE(I)
      ANGLE=ALPHA(J)
      XK =XPERM(J)
      YK =YPERM(J)
C
C       SET HD ARRAY
C
      DO 9 JJ=1,4
      ITMP=NODE(I,JJ)
      IF(ITMP.LE.NRED) GO TO 9
      DO 8 KK=1,NHDS
      JTMP=NDHD(KK)
      IF(ITMP.NE.JTMP) GO TO 8
      HD(JJ)=HEAD(KK)
      GO TO 9
    8 CONTINUE
    9 CONTINUE
      HD(5)=HD(1)
C
C         DETERMINE LOCAL COORDINATES
C
      DO 10 II=1,4
      KA=NODE(I,II)
      ND(II)=KA
      XX(II)=XCORD(KA)
   10 YY(II)=YCORD(KA)
      ANG=ABS(ANGLE)
      IF(ANG.LE.0.) GO TO 12
      SNALP=SIN(ANGLE)
      CSALP=COS(ANGLE)
      DO 11 II=1,4
      TEMP = XX(II)
      TMP=YY(II)
      XX(II)=TEMP*CSALP+TMP*SNALP
   11 YY(II)=-TEMP*SNALP+TMP*CSALP
```

```
   12 XX(5)=XX(1)
      YY(5)=YY(1)
      ND(5)=ND(1)
      XCN=(XX(1)+XX(2)+XX(3)+XX(4))/4.
      YCN=(YY(1)+YY(2)+YY(3)+YY(4))/4.
C
C    *** BEGIN TRIANGULAR ELEMENT LOOP
C
      DO 100 II=1,4
      NA=ND(II)
      NB=ND(II+1)
      XNA=XX(II)
      YNA=YY(II)
      XNB=XX(II+1)
      YNB=YY(II+1)
      XKT=XK
      YKT=YK
C
C           DETERMINE TRANSMISSTVITY TERMS
C
      IF(IRADL.LE.0) GO TO 19
      TMP=(XNA+XNB+XCN)/3.
      GO TO 20
   19 IF(THICK(1).LE.0.) GO TO 23
      TMP=THICK(I)
   20 XKT=XKT*TMP
      YKT=YKT*TMP
C
C          THE MAZE
C
   23 BJ=YNB-YCN
      BK=YCN-YNA
      BL=YNA-YNB
      CJ=XCN-XNB
      CK=XNA-XCN
      CL=XNB-XNA
      TEMP=2.*(XNA*BJ+XNB*BK+XCN*BL)
      ATRI(II)=TEMP
      XKT=XKT/TEMP
      YKT=YKT/TEMP
      TPXL=XKT*BL
      TPYL=YKT*CL
      TPXJ=XKT*BJ
      TPYJ=YKT*CJ
      GC=GC+TPXL*BL+TPYL*CL
      IF(NA.LE.NRED) GO TO 35
      BCN=BCN-(TPXL*BJ+TPYL*CJ)*HD(II)
      IF(NB.LE.NRED) GO TO 32
   31 BCN=BCN-(TPXL*BK+TPYL*CK)*HD(II+1)
      GO TO 100
   32 B(NB)=B(NB)-(TPXJ*BK+TPYJ*CK)*HD(II)
   33 GT(II+1)=GT(II+1)+TPXL*BK+TPYL*CK
      G(NB,LR)=G(NB,LR)+XKT*BK*BK+YKT*CK*CK
      GO TO 100
   35 GT(II)=GT(IT)+TPXJ*BL+TPYJ*CL
      G(NA,LR)=G(NA,LR)+TPXJ*BJ+TPYJ*CJ
      IF(NB.LE.NRED) GO TO 36
      B(NA)=B(NA)-(TPXJ*BK+TPYJ*CK)*HD(II+1)
      GO TO 31
   36 NAT=NA-NB+LR
      NBT=NB-NA+LR
      G(NA,NBT)=G(NA,NBT)+TPXJ*BK+TPYJ*CK
      G(NB,NAT)=G(NA,NBT)
      GO TO 33
  100 CONTINUE
      GT(1)=GT(1)+GT(5)
C
C           ADD SOURCES OR SINKS TO B VECTOR
C
      IF (NWELS.LE.0.OR.KOUNT.GE.NWELS) GO TO 50
      DO 45 JJ=1,NWELS
      IF (IWLEL(JJ).NE.I) GO TO 45
```

```
      KOUNT=KOUNT+1
      TEMP=QWELL(JJ)/((ATRI(1)+ATRI(2)+ATRI(3)+ATRI(4))*3.)
      DO 40 KK=1,4
      KA=ND(KK)
      KB=ND(KK+1)
      TMP= ATRI(KK)*TEMP
      B(KA)=B(KA)+TMP
      B(KB)=B(KB)+TMP
   40 BCN=BCN+TMP
      GO TO 50
   45 CONTINUE
C
C         ELIMINATE CENTRAL NODE POINT OF EACH QUADRILATERAL ELEMENT
C         FROM MATRIX EQUATION
C
   50 DO 61 JJ=1,4
      KA=ND(JJ)
      IF(KA.GT.NRED) GO TO 61
      GTEMP=GT(JJ)/GC
      B(KA)=B(KA)-BCN*GTEMP
      DO 60 KK=1,4
      KB=ND(KK)
      IF(KB.GT.NRED) GO TO 60
      KC=KB-KA+LR
      G(KA,KC)=G(KA,KC)-GT(KK)*GTEMP
   60 CONTINUE
   61 CONTINUE
  200 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE RSOLVE(C,V,N,M)
C         SOLVE MATRIX EQUATION BY GAUSS ELIMINATION
      DIMENSION C(400,50),V(400)
      LR=(M-1)/2
      DO 20 L=1,LR
      IM=LR-L+1
      DO 20 I=1,IM
      DO 10 J=2,M
   10 C(L,J-1)=C(I,J)
      KN=N-L
      KM=M-I
      C(L,M)=0.
   20 C(KN+1,KM+1)=0.
      LR=LR+1
      IM=N-1
      DO 90 I=1,IM
      NPIV=I
      LS=I+1
      DO 30 L=LS,LR
      IF(ABS(C(L,1)).GT.ABS(C(NPIV,1))) NPIV=L
   30 CONTINUE
      IF(NPIV.LE.I) GO TO 50
      DO 40 J=1,M
      TEMP=C(I,J)
      C(I,J)=C(NPIV,J)
   40 C(NPIV,J)=TEMP
      TEMP=V(I)
      V(I)=V(NPIV)
      V(NPIV)=TEMP
   50 V(I)=V(I)/C(I,1)
      DO 60 J=2,M
   60 C(I,J)=C(I,J)/C(I,1)
      DO 80 L=LS,LR
      TEMP=C(L,1)
      V(L)=V(L)-TEMP*V(I)
      DO 70 J=2,M
   70 C(L,J-1)=C(L,J)-TEMP*C(I,J)
   80 C(L,M)=0.
      IF(LR.LT.N) LR=LR+1
   90 CONTINUE
      V(N)=V(N)/C(N,1)
      JM=2
      DO 110 I=1,IM
      L=N-I
      DO 100 J=2,JM
      KM=L+J
  100 V(L)=V(L)-C(L,J)*V(KM-1)
      IF(JM.LT.M) JM=JM+1
  110 CONTINUE
      RETURN
      END
```

ADDENDUM 5

INPUT REQUIREMENTS

1. Card Format

Each input card is described in detail in section 3 below. The last page of this document contains a "Summary of Input Cards". Variable locations on each card are identified by field numbers. As in other HEC programs (19), each card is divided into ten fields of eight columns each except field 1. Variables occuring in field 1 may only occupy card columns 2-8 since card column 1 is reserved for an identification character. Identification characters in the first column of each card are for identification only and are not read by the computer. When the decimal is provided, the input format permits reading correctly any figure that is within the proper field. When there is no need for a decimal point, the number must be right justified in its field. Where the value of a variable is zero, the field may be left blank since a blank field is read as zero. It is convenient to use data forms and cards that are ruled in ten fields of eight columns for ease of preparing and checking the input.

2. Multiple Jobs

When several jobs are to be computed during the same run (stacked jobs), the data cards for the last job only are to be followed by four blank cards. If only a single job is to be run, four blank cards must follow the data cards for that run.

3. Card Contents

a. A-cards. Three title cards are required at the beginning of each job. Alphabetical characters and numbers may be used in any of the fields of all three cards. The contents of the cards will be printed at the beginning of the program output.

b. B-card. This is a job specification card that controls the number of subsequent cards to be read and also sets indices that call for use of available options. Variables should be specified by integers that are right justified in the fields.

Field	Variable	Description
1	NELS	Number of elements.
2	NNDS	Number of nodes.

Field	Variable	Description
3	NZNS	Number of material property zones.
4	NHDS	Number of nodes for which the potential function is known.
5	NWELS	Number of elements to be specified as sources or sinks.
6	NQBND	Number of element sides for which flow (other than zero) is to be specified.
7	IRADL	A positive integer (e.g. one) in this field indicates that axisymmetric radial coordinates are being used.
8	IPRGS	A positive integer (e.g. one) in this field calls for a printout of the elements of the coefficient matrix that is set up in subroutine GEES.
9	IUNIT	A positive integer (e.g. one) in this field calls for conversion of the units of the input nodal coordinates from miles to feet. That is, values read in for XCORD and YCORD from the C-cards are multiplied by 5280 before being used in calculations.
10	IPRES	A positive integer (e.g. one) in this field calls for calculation and printing of nodal values of pressure head.

c. C-cards. These are nodal coordinate cards. One C-card is required for each node. The choice of origin for an orthogonal coordinate system is arbitrary. If the medium is homogeneous and anisotropic, it is advantageous to align the coordinate axes with the major and minor permeability axes. If pressure heads are to be calculated internally (by making IPRES positive), the y-axis must be vertical.

Field	Variable	Description
1	I	Node number.
2	XCORD	The x-coordinate of the node.
3	YCORD	The y-coordinate of the node.

d. D-cards. These are element data cards. One D-card is required for each element.

Field	Variable	Description
1	I	Element number.
2 3 4 5	Node 1 Node 2 Node 3 Node 4	The variables in these fields are the node numbers of the element's four nodes. Node 1 may be any of the four nodes, but the remaining nodes must then be given in counter-clockwise order around the element (see example problems).
6	IZONE	The zone number of the material property zone that contains the element (see E-cards).
7	THICK	The element thickness. Element thickness is not required if it is the same for all elements or if axisymmetric radial coordinates are used.

e. E-cards. These are material-property cards. One E-card is required for each material zone (e.g. for each region of differing conductivity characteristics).

Field	Variable	Description
1	I	Zone number. Zones are numbered sequentially, beginning with one for the first zone.
2	XPERM	The major conductivity of the medium in units of length per unit time.
3	YPERM	The minor conductivity of the medium in units of length per unit time.
4	ALPHA	The angle of rotation in radians (counter-clockwise positive) of the conductivity axes from the axes of global coordinate system.

f. F-cards. These are potential function cards. One F-card is required for each node for which a value for potential function (e.g. hydraulic head) is specified.

Field	Variable	Description
1	I	Node number.
2	HEAD	Specified value of the potential function.

g. G-cards. These cards designate sources and sinks. One G-card is required for each source or sink.

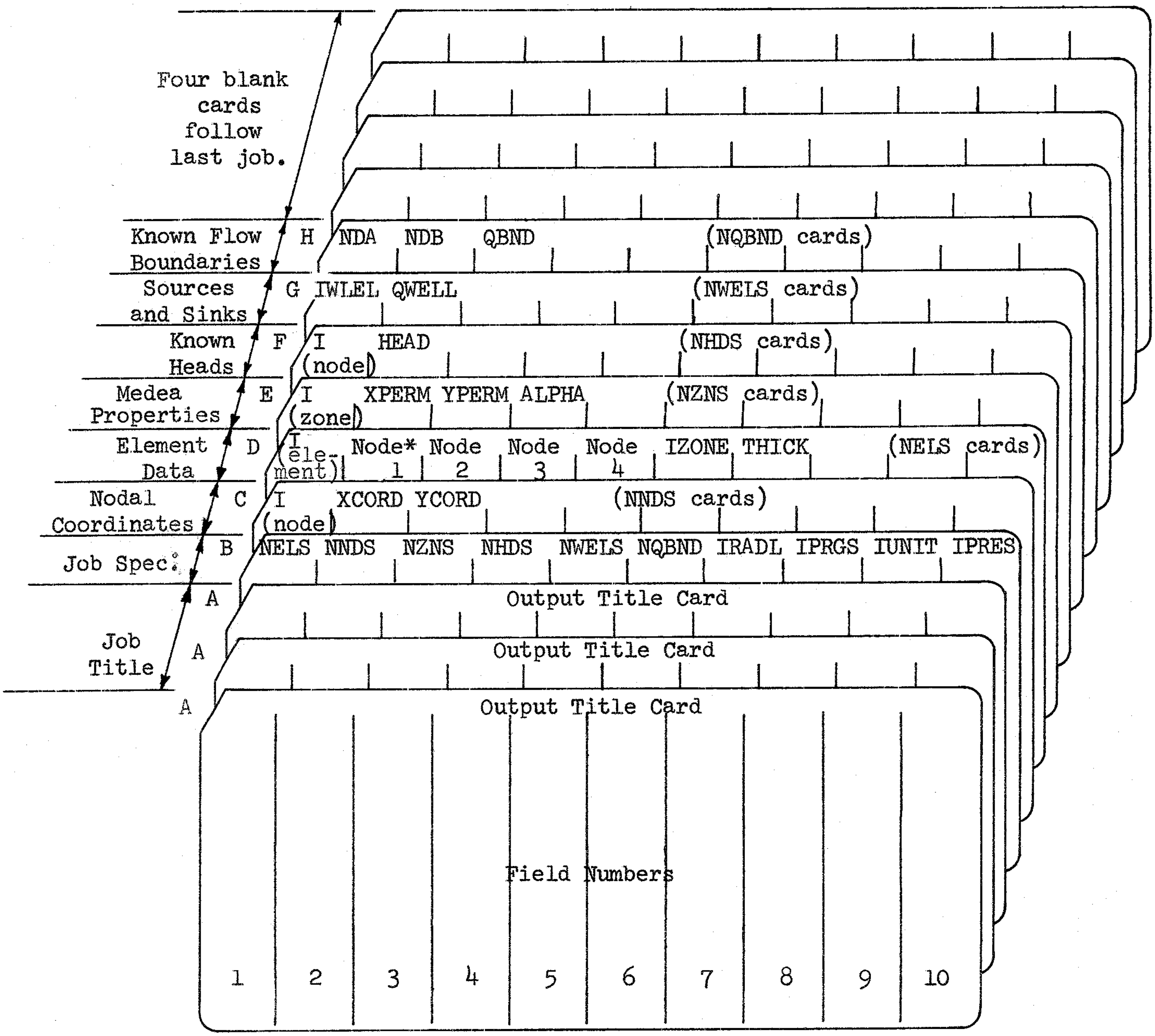
Field	Variable	Description
1	IWLEL	Element number.
2	QWELL	Magnitude of flow in length units cubed per unit time. A negative sign before the flow magnitude indicates that the element acts as a source; otherwise the element is treated as a sink.

h. H-cards. These cards indicate known-flow boundaries and the normal flow across them. One H-card is required for each element side for which flow (other than zero) is specified.

Field	Variable	Description
1 2	NDA NDB	The variables in these fields are the node numbers of the pair of adjacent nodes between which the unit flow specified in field 3 occurs. The order in which the nodes are given is arbitrary.
3	QBND	The magnitude of flow per unit length normal to the element side between NDA and NDB. A positive sign before the flow magnitude indicates that flow enters the flow region. A negative sign indicates that flow leaves the flow region. Note that units for QBND are discharge per unit thickness per unit length if thickness is not specified, and are discharge per unit length if thickness is specified. (Thickness is specified in field 7 of the D-cards.)

Field	Variable	Description
		QBND can be specified only on a side parallel to the z direction in an axisymmetric coordinate system.

i. Blank cards. Four blank cards after last job will cause computer to stop.



*Designate nodes on element data cards in counter-clockwise order.

SUMMARY OF INPUT CARDS

Hydrologic Engineering Methods
for Water Resources Development

Volume 1	Requirements and General Procedures, 1971
Volume 2	Hydrologic Data Management, 1972
Volume 3	Hydrologic Frequency Analysis, 1975
Volume 4	Hydrograph Analysis, 1973
Volume 5	Hypothetical Floods, 1975
Volume 6	Water Surface Profiles, 1975
Volume 7	Flood Control by Reservoir, 1976
Volume 8	Reservoir Yield, 1975
Volume 9	Reservoir System Analysis for Conservation, 1977
Volume 10	Principles of Groundwater Hydrology, 1972
Volume 11	Water Quality Determinations, 1972
Volume 12	Sediment Transport, 1977