ANL/DIS-11-3

GRiP—A Flexible Approach for Calculating Risk as a Function of Consequence, Vulnerability, and Threat

Decision and Information Sciences Division

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by
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January 2011

Contents

Table

Figures

Figures (Cont.)

Acknowledgments

The authors gratefully acknowledge the role of U.S. Department of Homeland Security Protective Security Advisors – Greg Gardner, Craig Freer, Ollie Gagnon, Ralph Ley, Rob Mooney, and Joe O'Keefe – in motivating the development of the Gravitational Risk Procedure (GRiP).

We also acknowledge the contributions of several Argonne colleagues. Lon Carlson, Frederic Petit, and Michael Samsa reviewed the report and made several helpful suggestions that greatly improved the report. Mark Jusko found the V3D software and developed the first application for GRiP. Andrea Manning carefully edited the report, and Vicky Skonicki prepared it for publication.

Acronyms and Abbreviations

3D	three-dimensional
C	consequence(s)
DHS	U.S. Department of Homeland Security
GRiP	Gravitational Risk Procedure
PSA	protective security advisor
R	risk
SME	subject matter expert
T	threat
V	vulnerability

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Executive Summary

Get a GRiP (Gravitational Risk Procedure) on risk by using an approach inspired by the physics of gravitational forces between body masses!

In April 2010, U.S. Department of Homeland Security Special Events staff (Protective Security Advisors [PSAs]) expressed concern about how to calculate risk given measures of consequence, vulnerability, and threat. The PSAs believed that it is not "right" to assign zero risk, as a multiplicative formula would imply, to cases in which the threat is reported to be extremely small, and perhaps could even be assigned a value of zero, but for which consequences and vulnerability are potentially high. They needed a different way to aggregate the components into an overall measure of risk.

To address these concerns, GRiP was proposed and developed. The inspiration for GRiP is Sir Isaac Newton's Universal Law of Gravitation: the attractive force between two bodies is directly proportional to the product of their masses and inversely proportional to the squares of the distance between them. The total force on one body is the sum of the forces from "other bodies" that influence that body.

In the case of risk, the "other bodies" are the components of risk (R): consequence, vulnerability, and threat (which we denote as *C*, *V*, and *T*, respectively). GRiP treats risk as if it were a body within a cube. Each vertex (corner) of the cube represents one of the eight combinations of minimum and maximum "values" for consequence, vulnerability, and threat. The risk at each of the vertices is a variable that can be set. Naturally, maximum risk occurs when consequence, vulnerability, and threat are at their maximum values; minimum risk occurs when they are at their minimum values. Analogous to gravitational forces among body masses, the GRiP formula for risk states that the risk at any interior point of the box depends on the squares of the distances[1] from that point to each of the eight vertices. The risk value at an interior (movable) point will be dominated by the value of one vertex as that point moves closer and closer to that one vertex.

GRiP is a visualization tool that helps analysts better understand risk and its relationship to consequence, vulnerability, and threat. Estimates of consequence, vulnerability, and threat are external to GRiP; however, the GRiP approach can be linked to models or data that provide estimates of consequence, vulnerability, and threat. For example, the Enhanced Critical Infrastructure Program/Infrastructure Survey Tool produces a vulnerability index (scaled from 0 to 100) that can be used for the vulnerability component of GRiP.

[1] Other exponents on distance are possible, some of which we will explore in this report.

We recognize that the values used for risk components can be point estimates and that, in fact, there is uncertainty regarding the exact values of *C*, *V*, and *T*. When we use $T = t_o$ (where t_o is a value of threat in its range), we mean that threat is believed to be in an interval around t_o. Hence, a value of $t_o = 0$ indicates a "best estimate" that the threat level is equal to zero, but still allows that it is not impossible for the threat to occur. When $t_o = 0$ but is potentially small and not exactly zero, there will be little impact on the overall risk value as long as the *C* and *V* components are not large. However, when *C* and/or *V* have large values, there can be large differences in risk given $t_o = 0$, and t_o = epsilon (where epsilon is small but greater than a value of zero). We believe this scenario explains the PSA's intuition that risk is not equal to zero when $t_o = 0$ and *C* and/or *V* have large values. (They may also be thinking that if *C* has an extremely large value, it is unlikely that *T* is equal to 0; in the terrorist context, *T* would likely be dependent on *C* when *C* is extremely large.) The PSAs are implicitly recognizing the potential that t_o = epsilon. One way to take this possible scenario into account is to replace point estimates for risk with interval values that reflect the uncertainty in the risk components. In fact, one could argue that T never equals zero for a man-made hazard.

This paper describes the thought process that led to the GRiP approach and the mathematical formula for GRiP and presents a few examples that will provide insights about how to use GRiP and interpret its results.

1 Introduction

In April 2010, U.S. Department of Homeland Security (DHS) Special Events staff (Protective Security Advisors [PSAs]) expressed concern about how to calculate risk given measures of consequence, vulnerability, and threat. The PSAs believed that additive and multiplicative formulas were not "right." That is, they understood that it is not truly accurate to assign zero risk to cases in which the level of threat is believed to be extremely small, and perhaps could even be assigned a value of zero, but for which the consequences and level of vulnerability are potentially high. They needed a different way to aggregate the components into an overall measure of risk.

Calculating risk is a hotly debated topic in many fields and disciplines. Haimes (2006; 2009) discusses the definition of vulnerability in measuring risk to infrastructures, which is the context assumed here. DHS defines risk (DHS 2009) as follows:

> "… it is important to think of risk as influenced by the nature and magnitude of a threat, the vulnerabilities to that threat, and the consequences that could result:
>
> $$R = f(C, V, T)\text{"} \qquad \text{(Equation 1)}$$

where R is risk, C is consequence, V is vulnerability, and T is threat. C, V, and T are the components of risk. Thus, risk is a *function* of C, V, and T.

It is interesting to note that the risk formula used in the DHS RAMCAP[SM] Plus process (ASME 2006; 2009) is expressed as a multiplicative formula (Risk = Consequence x Vulnerability x Threat) that may be written as follows:

$$R = C \times V \times T \times k, \qquad \text{(Equation 2)}$$

where k is a constant introduced to scale R. For example, if C, V, and T are scaled (i.e., they are indexes[2]) from 0 to 100, then $k = 1/10{,}000$ scales R from 0 to 100 (which also is an index). Willis (2007) uses the multiplicative formula to compare estimates of terrorism risks in urban areas that received Federal funding.

The multiplicative formula, however, is fraught with potential problems. For one thing, those who use it often ignore the units of measure for C, T, and V. When T and V are treated as probabilities, the unit of measure for C becomes the unit of measure for R. The measure is often expressed as dollars, number of fatalities, or dollar equivalents of a number of consequence types. Cox (2008) addresses other limitations of the multiplicative formula that may undermine effective allocation of resources to reduce risk.

[2] This treatment does not preclude the possibility that C, V, and T are real numbers (normalized between 0 and 100), or that V and T are probabilities (again normalized between 0 and 100).

Perhaps the most troublesome problem is the interpretation of risk results when one of the components of risk is equal to zero or asserted to be nearly zero. PSAs have noted this difficulty; especially sensitive are PSAs who are in charge of special events (e.g., a presidential inauguration, a Super Bowl game, a baseball World Series game), because they often identify facilities associated with events that have high vulnerability levels and potentially high consequences – but also have little or no credible information about threat. Because these PSAs "know" that it is not desirable to assign near-zero risk to the facility simply because threat is perceived to be low, they have expressed the need for a way to evaluate plausible protective measures at the facility that will result in a measurable improvement in the site's defensive posture – even if, for example, threat is perceived to be low.

The National Academy of Sciences (2010) recently examined various DHS approaches to risk analysis. With respect to the multiplicative formula, which is used in RAMCAP[SM] Plus, the organization concluded:

> "… that the multiplicative formula, Risk = T *x* V *x* C, is not an adequate calculation tool for estimating risk for the terrorism domain, within which independence of threats, vulnerabilities, and consequences does not typically hold."

Because risk is a function of C, V, and T, and because it may be desirable to have a non-zero risk when T is zero or close to it, an additive risk formula may have some merit. An additive formula at one time was used in a DHS program (National Academy of Sciences, 2010). A simple additive formula is given by:

$$R = aC + bV + cT, \qquad \text{(Equation 3)}$$

where parameters *a*, *b*, and *c* are constants. The additive formula shown in Equation 3 embodies assumptions about the way the components aggregate to risk. Because *C* and *V* are assumed to act independently on risk, the second-order derivative of *R* with respect to *C* and then *V* is equal to zero – a characteristic that may or may not be a desirable property of the risk relation. In contrast, the second cross derivative of the multiplicative formula is not equal to zero – again, a characteristic that may or may not be desirable. As will be demonstrated in Sec. 2, the GRiP (Gravitational Risk Procedure) formula, by contrast, is a complex function of *C*, *V*, and *T*.

To address concerns by the PSAs and Special Events staff about some of the limitations of calculating risk using these formulas, we developed the GRiP approach. The approach is inspired by the physics of gravitational forces among body masses (e.g., planets, moons, and satellites; see The Physics Classroom [undated]). Newton's Universal Law of Gravitation states that (1) every mass (body) attracts another mass by a force (a vector) directed along a straight line between the masses, and (2) the force is directly proportional to the product of the two masses and inversely proportional to the square of the distance between them, as follows:

$$F = G\frac{m_1 m_2}{d^2} \qquad \text{(Equation 4)}$$

where:

- *F* is the magnitude of the gravitational force between the two masses;
- *G* is the gravitational constant;
- m_1 and m_2 are the first and second masses, respectively; and
- *d* is the distance between the masses.

Therefore, the insights with respect to the GRiP approach to calculating risk include the following:

- Begin with a three-dimensional (3D) space (*C*, *V*, and *T*);

- Identify minimum and maximum values for *C*, *V*, and *T* (which define the boundaries of the space and which in turn can be visualized as a cube or box);

- Specify the magnitude of the risk at each vertex (corner) of the cube;

- Imagine a point body inside the cube subject to risk forces (analogous to gravitational forces) from bodies at each of the vertices;

- Calculate the effect of each vertex on the interior point (the magnitude of the effect is proportional to the magnitude of the risk at a vertex and inversely proportional to the square of the distance of the interior body from the vertex); and

- Sum those effects to obtain a measure of the risk at the interior point of the cube.

The analogy of risk calculation based on *C*, *V*, and *T* to the physics of gravitational forces among body masses leads to its name, GRiP – Gravitational Risk Procedure.

This paper describes the thought process that led to GRiP, the mathematical formula, and a few examples that provide insights about how to use GRiP and interpret its results. GRiP results are also compared to those yielded by a simple multiplicative formula and a simple additive formula for calculating risk.

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2 The Risk Cube

To begin, consider a three-dimensional space (a cube; Figure 1) in which the dimensions (axes) are consequence (*C*), vulnerability (*V*), and threat (*T*). *C*, *V*, and *T* are scaled from 0 to 100, where 0 represents minimal (possibly equal to zero) *C*, *V*, or *T*; and 100 represents maximal *C*, *V*, or *T* values.[3] The eight vertices are labeled with index numbers from 1 to 8.

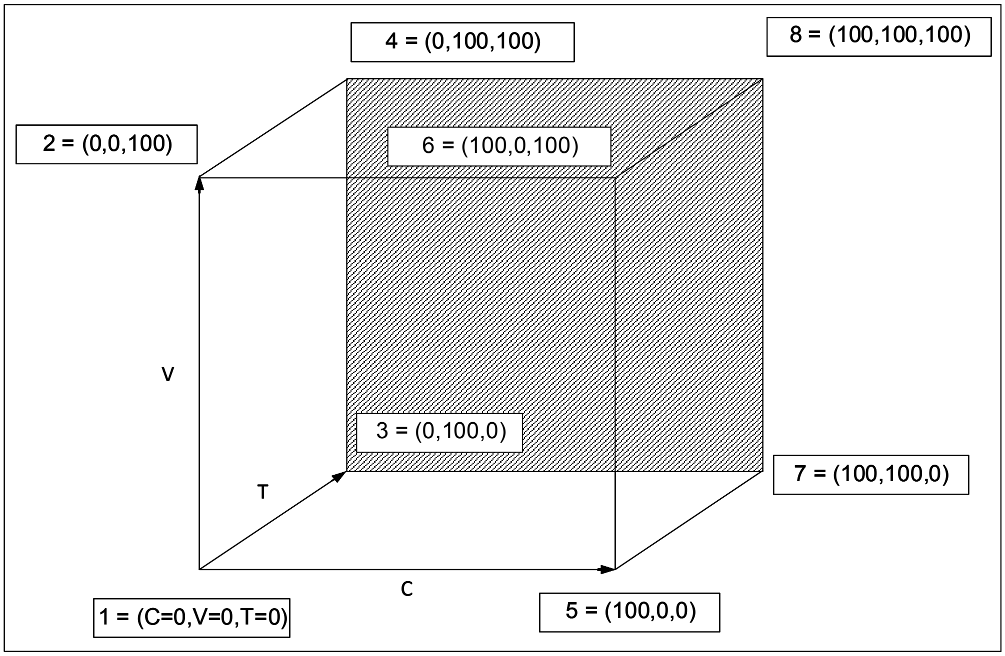

FIGURE 1 The Risk Cube Defined by Consequence, Vulnerability, and Threat

Vertex 1, labeled 1, is the origin and is associated with C = 0, V = 0, and T = 0; it can also be written (C = 0, V = 0, T = 0) or (0, 0, 0). Minimum risk is associated with this vertex. Vertex 8 is furthest from the origin. It can be written (100, 100, 100). Maximum risk is associated with this vertex (C, V, and T are maximal) For vertices 2-7, at least one of C, V, and T is at a maximum value and at least one of C, V, and T is at a minimum value.

In Figure 2, point *P* is visualized inside of the cube. It has coordinates (which specify its location within the cube) of c_p, v_p, and t_p, which can be written (c_p, v_p, t_p).

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3 Where appropriate, these variables may be interpreted as probabilities. Taking threat as an example, 0 would mean a probability of zero (i.e., not possible), and 100 would mean a probability of 1 (i.e., certain).

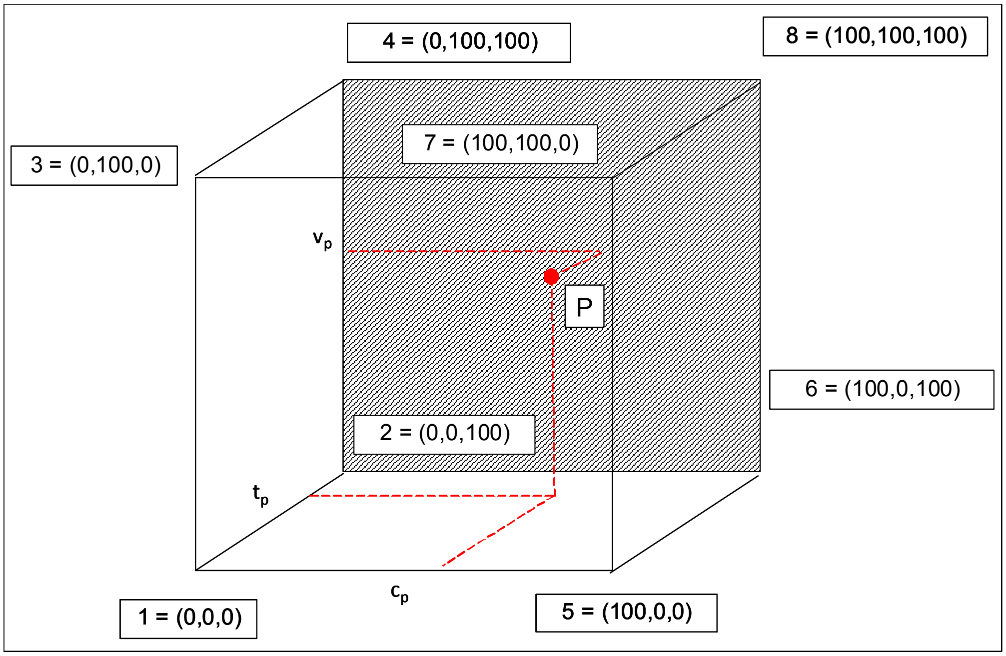

FIGURE 2 Point *P* Inside the Risk Cube Has Coordinates (c_p, v_p, t_p)

In Figure 3, a line is drawn from *P* to each of the eight vertices. The length of each line is the distance from *P* to each of the vertices.

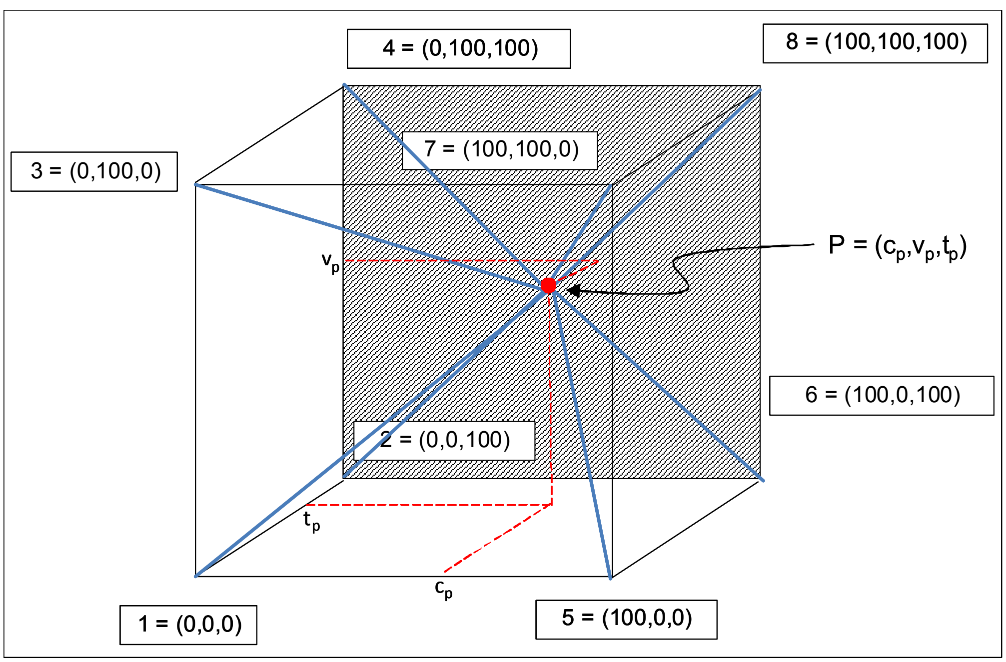

FIGURE 3 Solid Blue Lines Represent the Distance from Point *P* to Each Vertex

To make the next step in the development, additional notation is needed. As stated earlier, *P* is denoted by coordinates (c_p, v_p, t_p). Furthermore, the coordinates of vertex *i* are denoted as (c_i, v_i, t_i). The vertex 6 coordinates (C = 100, V = 0, T = 100) are denoted as (c_6, v_6, t_6). Finally, the distance from vertex *i* to *P* is denoted as $d_{i,p}$. Therefore, the distance from vertex 6 to *P* is $d_{6,p}$. Figure 4 illustrates the notation.

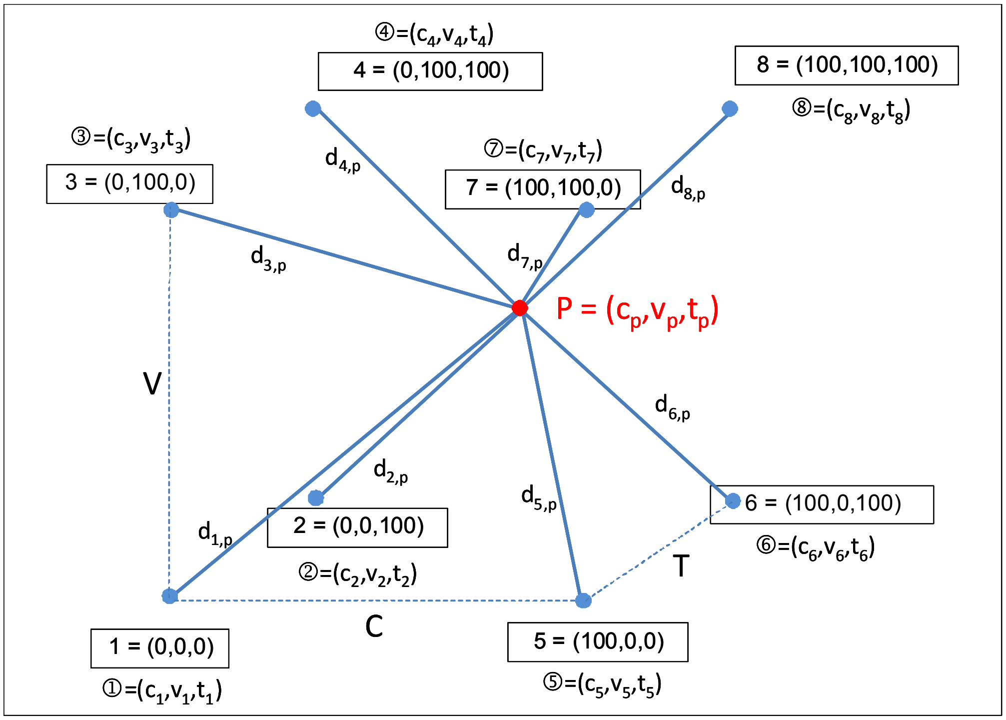

FIGURE 4 Subscript Notation to Identify Vertices and Distances from Vertices to *P*

From analytic geometry, the formula for the distance from vertex 6 to point *P* is:

$$d_{6,p} = \sqrt{(c_6 - c_p)^2 + (v_6 - v_p)^2 + (t_6 - t_p)^2}. \qquad \text{(Equation 5)}$$

Next, the risk associated with each vertex is denoted as R_i. To scale *R* from 0 to 100, R_1 must be equal to 0 and R_8 must be equal to 100. The *influence* $I_{6,p}$ of vertex 6 on *P* is defined as:[4]

[4] It is tempting to continue the analogy to gravitational forces between and among body masses and write $I_{6,p} = \frac{R_6}{d_{6,p}^2}$ and total risk as $R_p = \sum_{i=1}^{8} I_{i,p} R_i$. However, this formulation breaks down as point *P* approaches one of the vertices, say *i*, because, even in the limit, R_p will not approach R_i. Not only is this problematic when only R_8 is not equal to zero, it is especially problematic when only R_1 is equal to zero. As a result, a fractional approach must be used.

$$I_{6,p} = \frac{1}{d_{6,p}^2} \qquad \text{(Equation 6a)}$$

$$= \frac{1}{(c_6 - c_p)^2 + (v_6 - v_p)^2 + (t_6 - t_p)^2}, \qquad \text{(Equation 6b)}$$

which means that the influence of vertex 6 on *P* is inversely proportional to the square of the distance between *P* and vertex 6. Note that when $d_{6,p}$ is very small (i.e., *P* is very close to vertex 6), $I_{6,p}$ is very large.

The *fractional influence* $F_{6,p}$ of vertex 6 on point *P* is:

$$F_{6,p} = \frac{I_{6,p}}{I_{1,p} + I_{2,p} + I_{3,p} + I_{4,p} + I_{5,p} + I_{6,p} + I_{7,p} + I_{8,p}}$$

$$= \frac{I_{6,p}}{\sum_{i=1}^{8} I_{i,p}}. \qquad \text{(Equation 7)}$$

The *total influence* R_p on *P* (i.e., the risk associated with *P*) is determined from the fractional influence of each of the eight vertices and the risk value (R_i) at each of the 8 vertices:

$$R_p = \sum_{i=1}^{8} F_{i,p} R_i$$

$$= \frac{\sum_{i=1}^{8} I_{i,p} R_i}{\sum_{i=1}^{8} I_{i,p}}, \qquad \text{(Equation 8)}$$

which is called the *overall risk formula*.

Note that if, for example, *P* is very close to vertex 6, $I_{6,p}$ will be very, very large (it will approach infinity) compared to the other $I_{i,p}$ values (they will be finite and relatively small). The value of $F_{6,p}$ will approach 1.0 and the other $F_{i,p}$ will be very small and will approach 0.0. In that limit, Equation 8 reduces to:

$$R_p = \frac{F_{6,p} R_6}{F_{6,p}}$$

$$= R_6. \qquad \text{(Equation 9)}$$

3 Analysis

The properties of the overall risk formula are greatly influenced by the risk values assigned to the vertices. This section investigates several plausible settings of these values and compares results using GRiP to simple multiplicative and simple additive formulas.

3.1 One Vertex is Non-Zero

It has been commonplace to characterize risk as the product of consequence, vulnerability, and threat. With that assumption, when the value of one or more of the risk components (*C*, *V*, and/or *T*) is equal to zero, the product is equal to zero. For GRiP, this characterization means that only one vertex (100, 100, 100) has non-zero risk. The column labeled R_0 in Table 1 lists the risk values for the eight vertices based on this assumption. The data in the other column, labeled "PSA," will be discussed in Section 3.2.

Table 1 Settings of the Risk Values for the 8 Vertices of the GRiP Cube

	Components			Cases	
Vertex	C	V	T	R_0	PSA[a]
1	0	0	0	0	0
2	0	0	100	0	16
3	0	100	0	0	30
4	0	100	100	0	46
5	100	0	0	0	65
6	100	0	100	0	80
7	100	100	0	0	92
8	100	100	100	100	100

[a] These illustrative judgments were provided by a small group of PSAs.

A four-dimensional plot[5] (Figure 5) indicates the effects on risk attributable to the R_0 values at the vertices. Risk – the fourth dimension – is displayed at regular (C,V,T) intervals as a bubble, and the amount of risk is proportional to the size of the bubble. Figure 5 demonstrates that risk approaches a value of zero near seven of the vertices, and approaches a value of 100 near the (100, 100, 100) vertex. The largest bubbles – indicative of the highest risks – are predominantly in the top-back region of the cube, where *C*, *V*, and *T* are high.

The six values used for each component are 1, 20, 40, 60, 80, and 99. Therefore, there are $6^3 = 216$ points plotted in Figures 5 through 8.

[5] The four-dimensional plots in this report were generated by using V3D, "…a handy, fast, and versatile 3D/4D/5D Image Visualization & Analysis System for Bioimages & Surface Objects. It also provides many unique functions. It is also Open Source, supports a very simple and powerful plugin interface and thus can be extended & enhanced easily" (V3D undated).

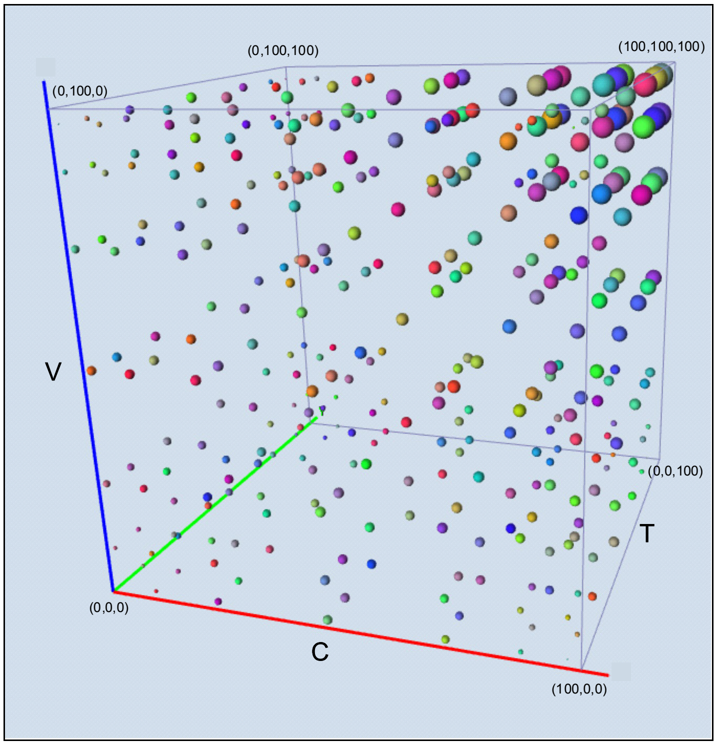

FIGURE 5 Four-Dimensional Plot of Risk at Regular Intervals in CVT-Space for the GRiP Formula; Risk, the Fourth Dimension, Is Proportional to the Size of the Bubble at the (C,V,T) Values Shown (only one vertex has a risk value that is not equal to zero)

3.2 Only One Vertex is Zero

As stated at the beginning of this report, PSAs expressed concern about how to calculate risk given measures of consequence, vulnerability, and threat. They felt that additive and multiplicative formulas are not "right." They also believed that it is not right to assign zero risk to cases in which threat is asserted to be zero (or very small) but consequences and vulnerability are potentially high. They needed a different way to "calculate" risk.

These thoughts were the catalyst for developing GRiP. Several months after the initial efforts to develop GRiP, the PSAs came together and, as a group, provided illustrative judgments about appropriate values for the vertices of the GRiP cube. The averages of their judgments at each vertex are listed in Table 1 in the column labeled "PSA." Their judgments are that the potential for high consequences is of greater concern (indicated by a higher assigned risk index of 65) than high vulnerability, and the potential for high vulnerability is of greater concern (indicated by a higher assigned risk index of 30) than high threat (assigned a risk index of 16). Statistical analysis shows that these effects are highly additive, with a slightly negative interaction between C and V.

A plot of risk versus *C*, *V*, and *T* for the PSA vertex values is shown in Figure 6. Note that there are considerably more large bubbles, which indicates higher levels of risk for more (C,V,T) combinations, as compared to Figure 5.

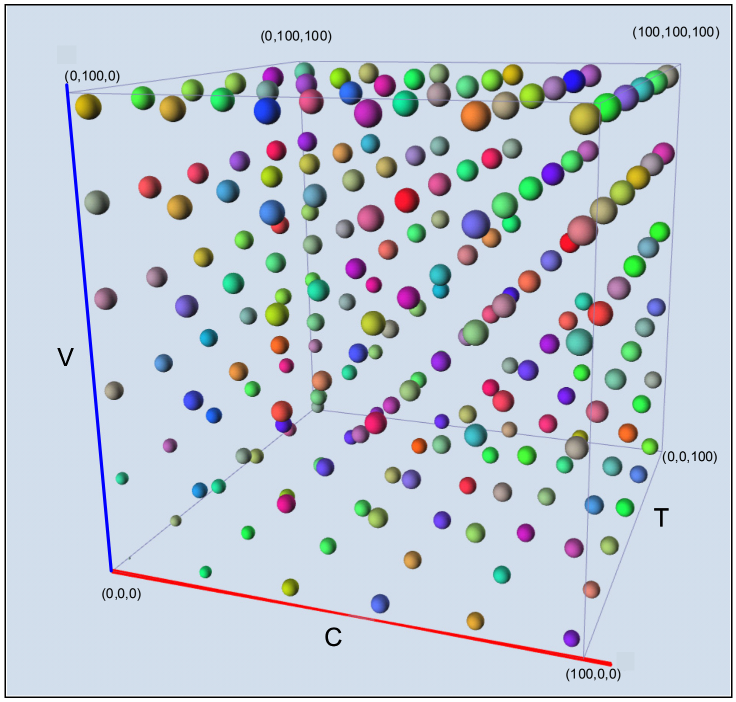

FIGURE 6 Four-Dimensional Plot of Risk at Regular Intervals in CVT-Space; Risk, the Fourth Dimension, Is Proportional to the Size of the Bubble at the (C,V,T) Values Shown (only one vertex has risk equal to zero, and the results are based on the judgments of PSAs and the GRiP formula)

3.3 Simple Multiplicative Formula

To gain insights into the characteristics of GRiP-produced risk results, we consider a simple multiplicative formula (and a simple additive formula in the next section).

Given that *C*, *V*, and *T* are indexes from 0 to 100, a multiplicative risk formula (from Equation 2) that is simply the product of these factors is:

$$R_m = C \times V \times T / 100 / 100, \qquad \text{(Equation 10)}$$

and the range of R_m in this equation is from 0 to 100.

A plot of risk versus *C*, *V*, and *T* for the multiplicative formula (Equation 10) is shown in Figure 7. Comparing Figure 7 to the GRiP-generated results for the single non-zero vertex case

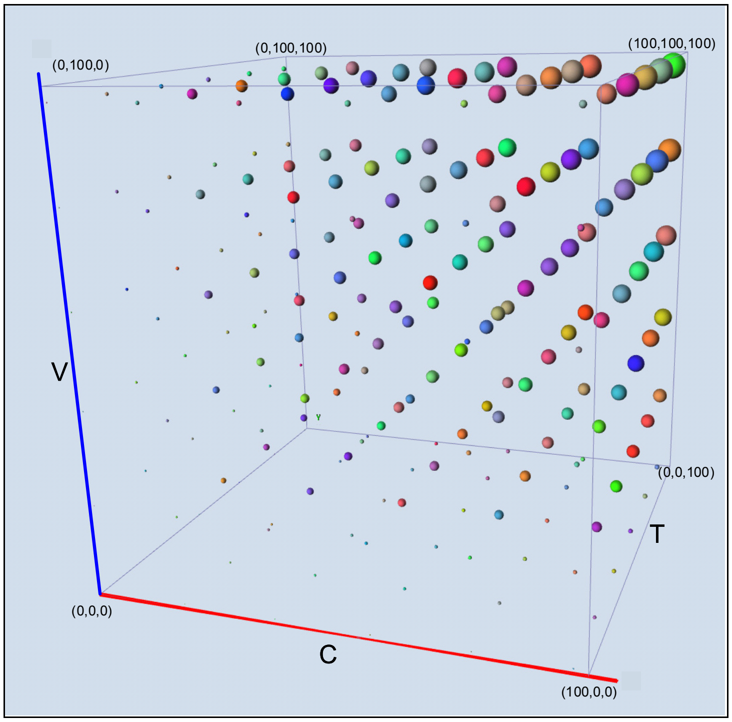

FIGURE 7 Four-Dimensional Plot of Risk at Regular Intervals in CVT-Space for a Simple Multiplicative Formula

in Figure 5, the multiplicative formula appears to yield lower risk values than the GRiP results at lower values of *C*, *V*, and *T*. The converse appears to be true at higher values of *C*, *V*, and *T*.

It is difficult to see what is occurring in the four-dimensional plots. In Section 3.5, we focus on the main diagonal in each cube to gain insights about the properties of all four formulas (including the additive formula described in the next section).

3.4 Simple Additive Formula

Given that *C*, *V*, and *T* are indexes from 0 to 100, an additive risk formula (from Equation 3) that is simply the weighted sum of these factors is:

$$R_a = 0.586\ C + 0.27\ V + 0.144\ T, \qquad \text{(Equation 11)}$$

where the constants in the formula are based on the ratios judged by PSAs for the vertices having one component at 100, as listed in Table 1. For example, the coefficient on C is the ratio of 65 (the risk value at the vertex with only C at 100) to the sum of 65, 30 (the risk value at the vertex with only V at 100), and 16 (the risk value at the vertex with only T at 100). The coefficients for the additive formula in Eq. 11 must be normalized and add to 1.0 or the risk value at the (100,100,100) vertex will be greater than 100. Thus, the coefficient on C is 65/(65+30+16), which equals 0.586, and the range of R_a in this formula is from 0 to 100.

A plot of risk versus *C*, *V*, and *T* for the additive formula (Equation 11) is shown in Figure 8. When comparing Figure 8 to the GRiP-generated results for the PSA case (Table 1) in Figure 6, the results are quite similar. The differences between the additive formula and the GRiP/PSA case are examined further in the next section.

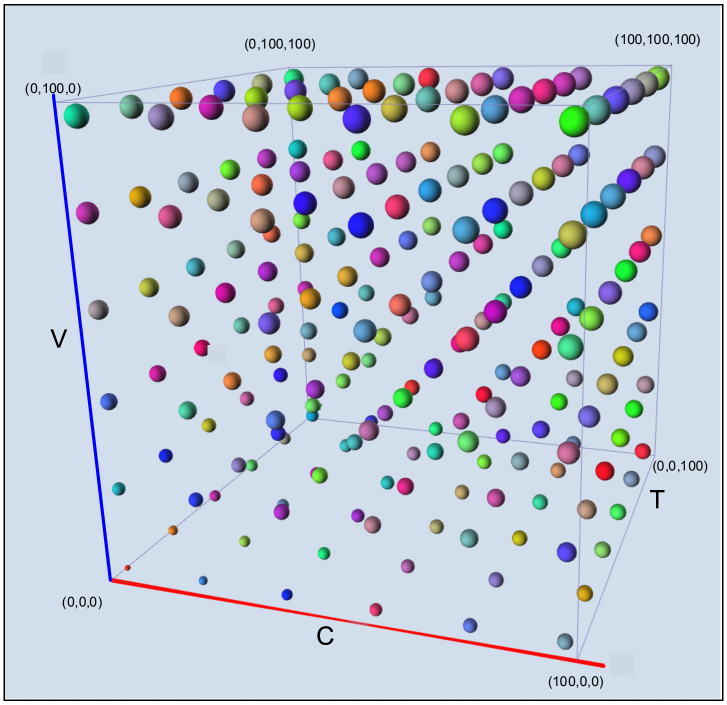

FIGURE 8 Four-Dimensional Plot of Risk at Regular Intervals in CVT-Space for a Simple Additive Formula

3.5 Comparison of the Cube's Main Diagonal Risk Values for Different Formulas

If there were a universally agreed-upon formula for risk, exploration of alternative formulas for risk as a function of consequence, vulnerability, and threat would not be necessary. Given this situation, several alternative formulas can be examined to determine the advantages and disadvantages of each. GRiP offers the convenience of representing some very complex relationships between risk and *C*, *V*, and *T* by simply specifying the risk associated with the eight vertices and the exponent of the distances from any point in the cube to each of the eight vertices.

As will be shown, several distance formulas for the GRiP approach are plausible and potentially useful. Four basic formulas are explored in Section 3.5.1, four variations of the GRiP formula are explored in Section 3.5.2, and a family of distance formulas known as L^p functions is explored in Section 3.5.3. Furthermore, a single shape parameter (the exponent on the distance) for all but the additive and multiplicative functions determines their properties. "The best" shape parameter can be determined by specifying (i.e., judging) the risk associated with one interior point in the risk cube. A process for determining the best shape parameter is presented in Section 3.5.4.

3.5.1 Four Basic Formulas

Because it is difficult to comprehend the intricacies of these formulas by viewing four-dimensional graphs, we turn attention to the main diagonal of the CVT cube – that is, the diagonal from (0, 0, 0) to (100, 100, 100). Along the main diagonal, *C* equals *V* equals *T*.

We investigate a multiplicative function, an additive function, and two GRiP functions (one with one non-zero-risk vertex and one with seven non-zero-risk vertices. Results are shown in Figure 9. Results for the multiplicative (dashed line with triangular markers) and the GRiP formula with seven zero-risk vertices (solid line with diamond markers) are similar. The most notable difference is that the graph for GRiP results has two inflection points: values are above, then below, and finally above the multiplicative results as the main diagonal value increases from 0 to 100.

The additive formula (dotted line with circle markers) and the GRiP formula with 7 non-zero-risk vertices (based on PSA judgments; solid line with square markers) produce similar results. The most notable difference is that the graph for GRiP results has several (4) inflection points: values are below, then above, then below, and finally above the additive results as the main diagonal value increases from 0 to 100.

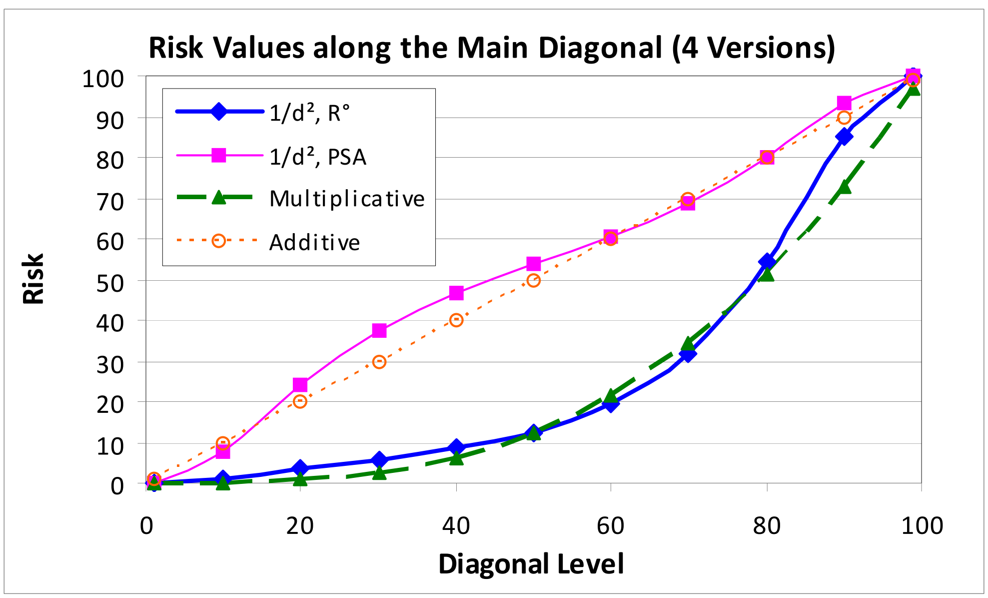

FIGURE 9 Risk Index Values along the Main Diagonal for Four Risk Formulas (2 GRiP formulas, multiplicative, additive)

3.5.2 Four Additional Formulas

The following modification presents an additional "twist": instead of using the inverse of the distance squared, use the inverse of the distance or the inverse of the distance to the fourth power in the GRiP formulas for one non-zero-risk vertex and seven non-zero-risk vertices (based on the PSA judgments). Thus, we rewrite Equation 5 as:

$$d_{6,p} = [(c_6 - c_p)^2 + (v_6 - v_p)^2 + (t_6 - t_p)^2]^{s/2}, \qquad \text{(Equation 12)}$$

rewrite Equation 6a more generally as:

$$I_{6,p} = \frac{1}{d_{6,p}}, \qquad \text{(Equation 13)}$$

and calculate risk as before using Equations 7 and 8. Therefore, in Equation 12, s = 1 yields the 1/d case, s = 2 yields the $1/d^2$ case, and s = 4 yields the $1/d^4$ case.

This modification introduces some interesting possibilities, which are illustrated in Figure 10. Perhaps the most interesting is the case for seven non-zero-risk vertices and $1/d^4$: the plot has a distinct sigmoidal (*S*) shape that is low risk at first (up to a diagonal level of about 20), rises sharply with a relatively constant slope up to a diagonal level of about 80, and levels off thereafter. This effectively defines three regions: (1) one in which the risk value is very low – with a value near zero with main diagonal values less than about 20; (2) another in which the risk value is very high – nearly 100 with main diagonal values greater than about 80; and (3) a third with main diagonal values between 20 and 80 in which the risk value is proportional to the main diagonal value and varies from about 0 to about 100.

The risk index value in the center of the risk cube is a function of the risk index values assigned to the eight vertices. When all vertices except (100, 100, 100) are assigned zero risk values, the GRiP-calculated risk index for (50, 50, 50) is 12.5 for three "R^o" cases; 12.5 is also the risk index for the multiplicative formula. For the PSA judgments, the GRiP-calculated risk index for (50, 50, 50) is 53.6; for the additive formula, the risk index is 50.

Finally, this modification leads to discussion of another family of distance formulas known as L^p functions.

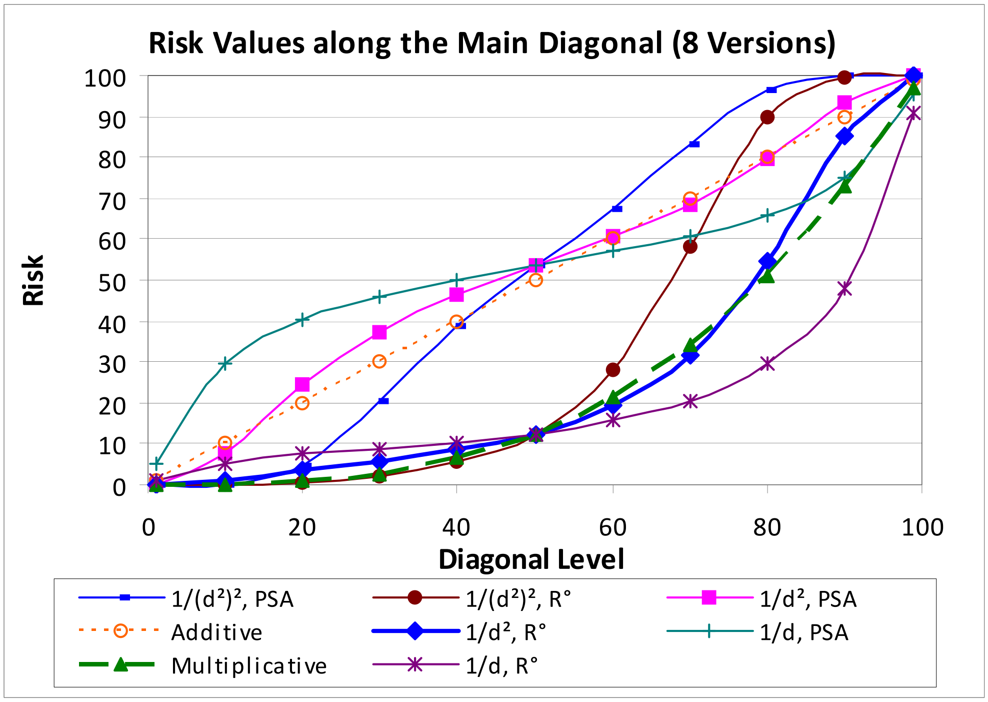

FIGURE 10 Risk Index Values along the Main Diagonal for Eight Risk Formulas (6 GRiP formulas, multiplicative, additive)

3.5.3 L^p Functions

If we interpret the parameter *s* in Equation 12 as a shape parameter, we have thus far investigated values of 1, 2, and 4 for *s*. The s = 1 case is special and an instance of a family of functions known in mathematics as L^p functions.[6] If we view the line connecting *P* in the GRiP cube to vertex 6 (as in Equation 5) as a vector, $L_{6,p}$, the general L^p formula for the length of the vector (in three-dimensional space) is as follows:

$$\|L_{6,p}\|_{s'} = (|c_6 - c_p|^{s'} + |v_6 - v_p|^{s'} + |t_6 - t_p|^{s'})^{1/s'} \qquad \text{(Equation 14)}$$

where:

- $\|L_{6,p}\|_{s'}$ denotes the magnitude (length) of $L_{6,p}$ for a given value of s', and
- $|c_6 – c_p|$ denotes the absolute value of the difference between c_6 and c_p.

Note that we use s' instead of the traditional p in the L^p formula to avoid confusion with the p-notation associated with point *P* in the GRiP cube.

[6] For information on L^p functions, see Taylor (1973).

Clearly, for s' = 2, the formula is equivalent to that of the Euclidean distance (namely, Equation 5) between point *P* and vertex 6. Note that versions of Equation 12 for s = 2 and s = 4 *are not* L^p functions.

Next, we generalize Equation 6a for L^p functions to be:

$$I_{i,p} = \frac{1}{\left\| L_{i,p} \right\|_{s'}}, \qquad \text{(Equation 15)}$$

which allows Equations 7 and 8 for calculating risk to remain unchanged.

It happens that risk results along the main diagonal for L^p distance functions are not much different from those for the 1/d formulas for main diagonal values less than about 70. Because the results for L^p functions are so similar to those for the GRiP formulas with the 1/d relationship, it is logical to conclude that raising the L^p distance to a power will again yield similar results. The primary usefulness of the L^p distance functions is that they allow for additional "fine tuning" of the basic distance calculations should that be desirable. A process for doing fine tuning is discussed in the next section.

4 A Process for Determining the Value of the Shape Parameter in the Risk Formula

With one additional judgment, the shape parameter in Equation 12 (the distance formula) can be determined. Without this additional judgment, the value of s used in analysis is in turn a judgment with little or no foundation. A process for determining the value of s that is consistent with the additional judgment is described below.

Given the risk index values of the eight vertices, the shape parameter for the distance formula (Equation 12) can be determined by specifying the risk index value for an interior point in the GRiP cube other than (C=50, V=50, T=50). As mentioned earlier, the risk index value in the center of the cube is determined by the values assigned to the vertices. A four-step process for determining the shape parameter is as follows:

1. Determine the risk index values of the eight vertices. These are value judgments that should be made by SMEs. Except for vertices (0, 0, 0) and (100, 100, 100), which must be 0 and 100, respectively, the others can be any value between 0 and 100.

2. For a point of reference, calculate the risk index value for the center of the GRiP cube. It will be greater than 0.125 if more than one vertex has a risk index value greater than zero.

3. Determine (i.e., judge) the risk index value of a point along the main diagonal. From Figure 10, it appears that a diagonal value of about 80 should give more reliable results because there is greater separation among the functions in this region. For consistency, the judged value must be greater than the value in the center of the cube because all of these functions are monotonically increasing (i.e., always increasing along the main diagonal).

4. Use a search algorithm to determine the value of s that satisfies the judged value of the point along the main diagonal.[7] This is a straightforward process using a computer software program that has a goal seek or solver tool.

Note that the shape parameter is important in the distance formula – Equation 12 – for non-L^p functions. The distance formula is then used in Equations 6, 7, and, finally, 8, to calculate risk. In the same manner, the exponent (s') for L^p distance (Equation 13) can be determined and then used in conjunction with Equations 7 through 9 to calculate a risk index value.

Risk values can be assigned to more than one interior point. Each will lead to a different s or s' value if the assignments are not perfectly consistent. Inconsistencies should be resolved before proceeding. Regression techniques can be used to resolve inconsistencies or the s and s' values can simply be averaged.

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[7] Note: Although it is possible to use a point that is not on the main diagonal, this approach may require more difficult and less reliable judgments. Whether or not these judgments are realistic can only be determined through experience.

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5 Conclusions

This paper describes a new, flexible approach and formula for calculating risk, which is a function of consequence, vulnerability, and threat. The inspiration for the formula is the physics of gravitational forces among body masses. We call this approach GRiP (Gravitational Risk Procedure).

The potential need for GRiP stemmed from an April 2010 meeting with the PSAs, who were not comfortable about how to calculate risk based on measures of consequence, vulnerability, and threat. They felt that it was not right to assign "zero risk", as a multiplicative formula would imply, to cases in which the threat level is reported to be extremely low, perhaps even equal to a value of zero, when consequences and vulnerability are potentially high. These PSAs needed a different approach for aggregating the components into an overall measure of risk.

It is recognized that the values used for risk components can be point estimates and that, in fact, there is uncertainty regarding the exact values of *C*, *T*, and *V*. When we use $T = t_o$ (where t_o is a value of threat in its range), we mean that the threat level is thought to be in an interval around t_o. Hence, a value of $t_o = 0$ indicates a "best estimate" that the threat level is equal to zero, but still allows that it is not impossible. When $t_o = 0$, but is potentially small and not exactly zero, there will be little impact on the overall risk value as long as the *C* and *V* components are not large. However, when *C* and/or *V* are large, there can be large differences in risk, given $t_o = 0$ and t_o = epsilon (where epsilon is small but greater than a zero value). We believe this scenario is the reason for the PSAs' intuition that the risk level is not zero when $t_o = 0$ and *C* and/or *V* are large. The PSAs are implicitly recognizing the potential that t_o = epsilon. They also may have been thinking that if *C* has an extremely large value, it is unlikely that *T* is equal to 0 (*T* is to some degree dependent on *C* when *C* is extremely large).

Another way to express the concerns of the PSAs introduces the concept of "exposure" from the business world. Perhaps their discomfort is closely related to an aversion to the *possibility* of large, adverse consequences even though the *likelihood* is near zero.

The GRiP functionality addresses these concerns and is flexible for four primary reasons. First, the risk associated with combinations of *C*, *V*, and *T* having one or two zero values can be judged to be non-zero. Traditionally, a risk of zero is assigned to such combinations–usually because a multiplicative function is used to calculate risk. Second, although the primary formula uses inverse-distance-squared variables, other exponents for distance are feasible (e.g., $1/d$ and $1/d^4$). Third, in the general case of L^p functions (the GRiP formula is an L^p function in the case of inverse-distance-squared variables), the feasible range of the shape variable – 1 to infinity – allows for a wide range of functions that may have desirable properties for calculating risk. Fourth, *C*, *V*, and *T* values can be real numbers or indexes.

Results, which depend on key illustrative judgments provided by the PSAs specifically for the GRiP approach, are presented for a number of combinations of exponents on distance and risk values assigned to the vertices of the GRiP cube. These are compared to a simple multiplicative formula and a simple additive formula. The additive formula necessarily uses a subset of the judgments of the DHS PSAs about the risk values of the vertices of the GRiP cube.

In considering GRiP results along the main diagonal of the GRiP cube for the $1/d^2$ case, seven vertices with the R = 0 assignment are quite similar to those for the additive formula. Likewise, in the case of $1/d^2$ with PSA judgments about the vertices assignment, GRiP results are quite similar to those for the multiplicative formula, for values along the main diagonal that are less than about 70 (see Figure 10). Thus, the differences in risk values (e.g., those from a GRiP formula vs. a multiplicative formula) at higher values of *C*, *V*, and *T* may be important.

Finally, the process for determining a shape parameter (an exponent applied to the distance) is described in Section 4. This parameter greatly increases possibilities for the overall risk formula – regardless of how distance is calculated (i.e., GRiP, L^p, additive, or multiplicative functions).

We are hopeful that risk analysts will find GRiP useful and that it will help them obtain additional insight into some of the complexities of addressing risk.

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