Solving an Empirical Puzzle in the Capital Asset Pricing Model[*]

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ABSTRACT

A long standing puzzle in the Capital Asset Pricing Model (CAPM) has been the inability of empirical work to validate it. Roll (1977) was the first to point out this problem, and recently, Fama and French (1992, 1993) bolstered Roll's original critique with additional empirical results. Does this mean the CAPM is dead? This paper presents a new empirical approach to estimating the CAPM. This approach takes into account the differences between observable and expected returns for risky assets and for the market portfolio of all traded assets, as well as inherent nonlinearities and the effects of excluded variables. Using this approach, we provide evidence that the CAPM is alive and well.

KEY WORDS: The capital asset pricing model; An empirical puzzle; Effect of mismeasuring the market portfolio; Omitted-variables bias; Unknown functional forms; Consistency of assumptions; Concomitants.

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[**]John Leusner died on March 5, 1996. We mourn his tragic death and will miss our good friend.

1. Introduction

The Capital Asset Pricing Model (CAPM) of Sharpe (1964), Lintner (1965), and Black (1972) in its various formulations provides predictions for equilibrium expected returns on risky assets. More specifically, one of its formulations states that the expected excess return over the risk-free interest rate of an asset (or a group of assets) equals a coefficient, denoted by β, times the (mean-variance efficient) market portfolio's expected excess return over the risk-free interest rate. This relatively straightforward relationship between various rates of return is difficult to implement empirically because expected returns and the efficient market portfolio are unobservable. Despite this formidable difficulty, a substantial number of tests have nonetheless been performed, using a variety of ex-post values and proxies for the unobservable ex-ante variables. Recognizing the seriousness of this situation quite early, Roll (1977) emphasized correctly that tests following such an approach provide no evidence about the validity of the CAPM. The obvious reason is that ex-post values and proxies are only approximations and therefore not the variables one should actually be using to test the CAPM. The primary purpose of this paper is to provide a new approach to testing the CAPM that overcomes this deficiency.

Recently, Fama and French (1992, 1993) conducted extensive tests of the CAPM and found that the relation between average stock return and β is flat, and that average firm size and the ratio of book-to-market equity do a good job capturing the cross-sectional variation in average stock returns. These findings suggest, among other things, that a formal accounting of the effects of "excluded variables" may resurrect the CAPM. This will be the central issue in this paper.

According to Fama and French (1993), some questions that need to be addressed are: (i) how are the size and book-to-market factors in returns driven by the stochastic behavior of earnings? (ii) how does profitability, or any other fundamental, produce common variation in returns associated with size and book-to-market equity that is not picked up by the market return? (iii) can specific fundamentals be identified as state variables that lead to common a variation in returns that is independent of the market and carries a different premium than general market risk? This paper attempts to answer these questions.

In an interesting article, Black (1995) gives three theoretical explanations of the measured flat line relating expected return and $\$$: (i) mismeasuring the market portfolio, (ii) restricted borrowing, and (iii) reluctance to borrow. Even if such reasoning is correct, we have found that the relation between the observed counterparts of expected return and $\$$ is nonlinear. Finally, we shall provide some answers to questions posed by Black (1995) concerning the future prospects of the CAPM: (i) will the line be flat in the future? (ii) will it be steep as the CAPM says it should be? and (iii) will it be flatter, but not completely flat?

First, the CAPM is modified to take into account the differences between expected and observable returns and between the market portfolio and its proxy. In this modified model, $\$$ is not required to be a constant, but instead is permitted to vary. Second, the effects of excluded variables and departures from a linear functional form are taken into account. Third, all the modifications are then expressed in terms of observable variables. Finally, the coefficients on the observable regressors are modeled as stochastic functions of the variables that Fama and French (1992) include in their test of the CAPM and find to have reliable power in explaining a cross-section of average stock returns. Once this has been done, the resulting model is estimated using data for 10 stock portfolios formed on the basis of both firm size and the ratio of book-to-market equity. (This procedure of forming portfolios was originated by Fama and French (1993).)

The specific model to be estimated is developed in Section 2. The issue regarding what constitutes a reasonable inference based upon this model is addressed in Section 3. A brief description of the data used to estimate the model is presented in Section 4. Section 5 discusses the empirical results and their applications. Section 6 contains the conclusions.

2. Interpreting and Extending the CAPM

2.1 A brief description of the model

The CAPM may be expressed as

$$Er_{it} - r_{ft} = \$_{it}(Er_{Mt} - r_{ft}), \qquad (1)$$

where Er_{it} is the (subjective) expected return on an asset (or a group of assets) an investor chooses to hold, Er_{Mt} is the (subjective) expected return on the mean-variance efficient market portfolio, r_{ft} is the risk-free rate, i indexes assets or groups of assets, t indexes time, and $\$_{it}$ is equal to the ratio of the covariance between r_{it} and r_{Mt}, denoted by $cov(r_{it},\ r_{Mt}),$ and the variance of r_{Mt}, denoted by F_M^2. (Alternative definitions of $\$_i$ are provided in Ingersoll (1987, pp. 92, 124, and 134) and Constantinides (1989).) The time variability of this variance and covariance implies that $\$_i$ is time varying. It is important to note that as in the case of Er_{it} and Er_{Mt}, both $cov(r_{it},\ r_{Mt})$, and F_M^2 are the moments of a subjective distribution. Homogeneous expectations or beliefs are not assumed here by allowing the subjective distributions to be different for each and every investor.

2.2 Some problems with the CAPM

A difficulty with empirically testing whether $\$_{it}$ is significantly different from zero in equation (1), from a statistical standpoint, is that it represents a statement about expected returns, which are not observable. To transform the relationship into observable variables for testing purposes, we introduce the following two equations relating observable returns to expected returns:

$$r_{it} = Er_{it} + v_{it}, \tag{2}$$

$$r_{Mt} = Er_{Mt} + v_{Mt}, \tag{3}$$

where r_{it} and r_{Mt} are the observable returns, and v_{it} and v_{Mt} are random variables. These latter variables will be distributed with zero means only if the data-generating processes and subjective processes of returns possess the same means. Substituting equations (2) and (3) into equation (1) yields

$$r_{it} - r_{ft} = \$^*_{it}(r_{Mt} - r_{ft}) + v_{it}, \tag{4}$$

where $\$^*_{it} = \$_{it}[1 - \frac{v_{Mt}}{r_{Mt} - r_{ft}}]$.

Although equation (4) is not expressed in the form of an errors-in-variables model, it reduces to such a model if the means of v_{it} and v_{Mt} are zero and $\$_{it}$ is a constant. Models possessing these properties have been extensively studied in statistics and econometrics literature (see Lehmann (1983, pp. 450-451)). As it turns out, estimation of equation (4) when it is not restricted as an errors-in-variables model is relatively straightforward, as will be shown in the next section. Furthermore, the now classic “Roll’s (1977) critique” of tests of the CAPM noted earlier does not apply to the estimation of equation (4) because v_{Mt} accounts for any differences between the “true” (unobservable) market portfolio and the particular portfolio that is chosen as a proxy. Further, v_{Mt} is permitted to have a non-zero and time-varying mean to cover situations where these differences are systematic and time varying. The presence of v_{Mt} in equation (4) makes the effects of mismeasurements noted by Black (1995) explicit, although equation (3) indicates that r_{Mt} is a good proxy for Er_{Mt} only if the mean of v_{Mt} is zero.

Even if the mismeasurement issue is resolved, equation (4) may nonetheless still be criticized insofar as important regressors are excluded. For example, no asset is perfectly liquid because all trades required to convert assets into cash involve some transaction cost. As a result, investors may choose to hold more liquid assets with lower transaction costs than otherwise. If so, an illiquidity premium should be taken into account. This can be done by allowing for trading costs to enter the right-hand side of equation (4) (see Amihud and Mendelson (1986)). Other potentially important variables, excluded from equation (4), are discussed below.

Another issue in the CAPM is whether investors face only one risk arising from uncertainty about the future values of assets. In all likelihood, investors face many sources of risk, as shown by Merton’s (1973) inter-temporal asset pricing model. In such instances, investors would supplement the market portfolio with additional positions in hedge portfolios to offset these risks. This results in separate β_{it}s and risk premiums for every significant source of risk that investors try to hedge. Equation (4), therefore, should be extended to account for the effects of extra-market hedging transactions on equilibrium rates of return. Such an expanded version of equation (4) would recognize the multidimensional nature of risk and thereby show that some important regressors are necessarily excluded from equation (4).

2.3 A generalization of the CAPM

Including previously excluded regressors in equation (4) is not trivial because the functional form of the relationship between them and the dependent variable is unknown. This difficulty is resolved in principle by modifying equation (4) as follows:

$$r_{it} - r_{ft} = \beta^{*}_{it}(r_{Mt} - r_{ft}) + v_{it} + \sum_{j=1}^{m} \gamma_{ijt} x_{ijt}, \qquad \textbf{(5)}$$

where the x_{ijt} represent excluded variables, the $\cdot_{ijt}$ denote their coefficients, and m denotes the number of excluded variables. Since m cannot be known with certainty, one may assume without restricting it to be equal to a specific number that the regressors of equation (5) form a sufficient set in the sense that they exactly determine the values of $r_{it} - r_{ft}$ in all periods.

As a general rule, by allowing all the coefficients in a linear equation to be different for each and every observation, the equation is permitted to pass through every data point and hence it coincides, for certain variations in the coefficients, with the actual process generating the data on its dependent variable. Because of this rule, equation (5) provides the only reliable way to capture unknown functional forms without relying upon strong prior information. One may assume that the coefficients of equation (5) are constants only when this equation is known with certainty to be linear. In contrast, with varying coefficients, equation (5) is truly nonlinear.

Obviously, equation (5) cannot be empirically estimated if the data on the x_{ijt} are not available. What is not so obvious is that when the x_{ijt} are not observable, one cannot prove they are uncorrelated with $(r_{Mt}-r_{ft})$ (see Pratt and Schlaifer (1984)). An approach for resolving this problem is to avoid making such uncorrelatedness assumptions and assume instead that

$$x_{ijt} = R_{0ijt} + R_{1ijt}(r_{Mt} - r_{ft}),\ j = 1,2,...,m, \qquad (6)$$

where R_{0ijt} is the portion of x_{ijt} remaining after the effect of the variable $(r_{Mt} - r_{ft})$ has been removed. Accordingly, even if the variable $(r_{Mt} - r_{ft})$ is correlated with the x_{ijt}, it can nonetheless be uncorrelated with the remainders, R_{0ijt}. Also, for certain variations in R_{0ijt} and R_{1ijt}, equation (6) exactly coincides with the true relationship between the x_{ijt} and $(r_{Mt} - r_{ft})$, if such a relationship exists. Once again, however, one cannot assume the R_{0ijt} and R_{1ijt} are constants unless equation (6) is known with certainty to be linear. Equation (6) should be recognized as an auxiliary equation, a linear form of which has been used to analyze the effects of excluded variables in the econometrics literature (see Greene (1993, pp. 245-247)). Since this equation does not impose any constraints on the coefficients of equation (5), it does not prevent the latter equation from coinciding with the true relationship between the variables. Substituting equation (6) into equation (5) yields

$$r_{it} - r_{ft} = (_{0it} + (_{1it}(r_{Mt} - r_{ft}), \qquad \textbf{(7)}$$

where $(_{0it} = (v_{it} + \sum_{j=1}^{m} \cdot_{ijt} R_{0ijt})$ and $(_{1it} = (\$^{*}_{it} + \sum_{j=1}^{m} \cdot_{ijt} R_{1ijt})$.

The coefficient, $(_{1it}$, has a relatively straightforward economic interpretation. It consists of three parts, the "true" beta, $\$_{it}$, of the CAPM, a mismeasurement effect, $-\frac{\$_{it} v_{Mt}}{r_{Mt} - r_{ft}}$, and an omitted-variables bias, $\sum_{j=1}^{m} \cdot_{ijt} R_{1ijt}$. More so than the "true" beta, the omitted-variables bias changes over time because the set of excluded variables undoubtedly changes quite frequently, lending further real-world, economic plausibility to time variability of $(_{1it}$. Similarly, the connection of $(_{0it}$ with the intercepts of equations (5) and (6) clarifies its real-world origin.

The preceding discussion proposes the introduction of varying coefficients and auxiliary equations into the estimation procedure as an important approach to dealing with unknown functional forms and the effects of omitted variables. Equation (7) provides a useful formulation that does not suffer from various specification errors when testing the CAPM, and it avoids such serious errors by not relying on any definitions of $(_{0it}$ and $(_{1it}$ other than those provided by equation (7).[1]

3. Econometric Underpinnings of the Extended CAPM

Estimation of equation (7) requires specific stochastic assumptions about $(_{0it}$ and $(_{1it}$. The permissible set of assumptions is, however, restricted. For example, one cannot assume that $(_{1it}$ is a constant because doing so would contradict the assumption that v_{Mt} is a random variable, even ignoring any variations in $\$_{it}$ and any omitted-variable bias. In addition, the fact that $(_{1it}$ depends on $(r_{Mt} - r_{ft})$ via $\$^{*}_{it}$, and $(_{0it}$ and $(_{1it}$ are functions of the common set of time-varying coefficients $\cdot_{ijt}$ prohibits one from assuming that the variables $(_{0it}$, $(_{1it}$, and $(r_{Mt} - r_{ft})$ are uncorrelated with one another. (Remember that the nonlinearities involved in equation (5) cannot be captured without the time-varying $\cdot_{ijt}$.) In other words, one

[1]The main difference between this approach and the previous approaches to estimating the CAPM lies in equations (3)-(6) which do not appear in the latter approaches (see Jagannathan and Wang (1996) and the previous studies by Harvey, Shanken and others referred to therein). The linear least squares residuals introduced in these previous studies cannot represent measurement errors and the effects of omitted variables (see Pratt and Schlaifer (1984, p. 11-12)).

cannot assume that $E[(r_{it} - r_{ft})|(r_{Mt} - r_{ft})] = " + \$(r_{Mt} - r_{ft})$, where $"$ and $\$$ are constants, without contradicting the definitions of $(_{0it}$ and $(_{1it}$. This argument lies at the heart of Roll's (1977) criticism of earlier tests of the CAPM. In principle, generalizing the set of assumptions about $(_{0it}$ and $(_{1it}$ can help in this respect. The reason is that general assumptions are more likely to encompass true assumptions as special cases than more restrictive assumptions. We shall proceed therefore by weakening the assumptions about $(_{0it}$ and $(_{1it}$.

Suppose that r_{it} refers to the rate of return on the i-th group or portfolio of assets. Then $r_{it} = \sum_{j=1}^{n} w_{ijt} r_{ijt}$, where r_{ijt} is the rate of return on the j-th asset included in the i-th portfolio, one of the r_{ijt} is equal to r_{ft}, and w_{ijt} is the proportion of the investor's total budgets allocated to the j-th asset. It follows that the variance of r_{it} is

$$F_i^2 = \sum_{j=1}^{n} \sum_{\substack{k=1 \\ j \neq k}}^{n} w_{ijt} w_{ikt} cov(r_{ijt}, r_{ikt}) + \sum_{j=1}^{n} w_{ijt}^2 var(r_{ijt}), \qquad \textbf{(8)}$$

where $var(r_{ijt})$ denotes the variance of r_{ijt} and $cov(r_{ijt}, r_{ikt})$ denotes the covariance between r_{ijt} and r_{ikt}.

The variance of r_{it} can also be obtained directly from equation (7). It is

$$F_i^2 = var((_{1it}(r_{Mt} - r_{ft})) + var((_{0it}) + 2cov((_{0it}, (_{1it}(r_{Mt} - r_{ft})). \qquad \textbf{(9)}$$

Equation (9) eases the computational burden compared to equation (8) because the latter equation involves a large number of variances and covariances that may be time varying and cannot be estimated unless one knows how they vary over time even if the data on all n securities in equation (8) are available. However, it does not have the advantage of parsimony (in terms of a preference for a model with fewer parameters and in all other respects almost as good as other competing models) if these two equations yield different values for the same variance F_i^2. One should, therefore, consider the conditions under which the same values would be obtained. Suppose that the third term on the right-hand side of equation (9) is zero. Suppose also that the covariances and variances given in equation (8) are attributable to $(r_{Mt} - r_{ft})$ and other specific variables, respectively. In this case, the first and second terms on the right-hand side of equation (9) can be equal to those on the right-hand side of equation (8), respectively. If, in addition, the first and second terms on the right-hand side of equation (8) tend to nonzero and zero, respectively, as $n \to \infty$, then the first and second terms on the right-hand side of equation (9) can be defined to be the systematic (or nondiversifiable) and nonsystematic (or diversifiable) risk components of the portfolio variance, respectively (see Swamy, Lutton and Tavlas (1995)). These definitions are more comprehensive than the corresponding definitions found in the finance literature because, as has been shown above, equation (7) captures all sources of risk whereas equation (1) captures only one such source.

Using a specific model for testing the CAPM, following Fama and French (1993), the third term on the right-hand side of equation (9) can be zero under the following general Assumptions I and II:

Assumption I. The coefficients of equation (7) satisfy the stochastic equation

$$\gamma_{it} = A\boldsymbol{z}_{it} + \epsilon_{it}, \qquad \textbf{(10)}$$

where γ_{it} denotes the two element column vector $(\gamma_{0it}, \gamma_{1it})'$; A denotes the 2×7 matrix $[B_{kj}]$, k = 0, 1, j = 0, 1, ..., 6; $\boldsymbol{z}_{it}$ denotes the seven element column vector
$(1, z_{i1,t-1}, z_{i2,t-1}, z_{i3,t-1}, z_{i4,t-1}, z_{i5,t-1}, z_{i6,t-1})'$,
$z_{i1,t-1}$ = the log of average size over all firms in the i-th portfolio (a firm's size is equal to its market equity, ME = a stock's price times shares outstanding, for June of year t-1),
$z_{i2,t-1}$ = the average of book-to-market ratio over all firms in the i-th portfolio (a firm's book-to-market ratio is equal to its book equity, BE = book value of its common equity as measured by Fama and French (1993, p. 11), for the fiscal year ending in calendar year t-1, divided by its market equity, ME, in December of t-1),
$z_{i3,t-1}$ = the dividend price ratio (dividend/price) for the S&P 500,
$z_{i4,t-1}$ = the default premium (Moody's Baa bond rate minus Moody's Aaa bond rate),
$z_{i5,t-1}$ = the yield on the 10-year Treasury bill minus the 1-year Treasury bill rate,
$z_{i6,t-1}$ = a dummy variable that is 1 in January and 0 in other months[2], and ϵ_{it} denotes the two element column vector $(\epsilon_{0it}, \epsilon_{1it})'$ that
satisfies the stochastic difference equation

$$\epsilon_{it} = M\epsilon_{i,t-1} + \boldsymbol{a}_{it}, \qquad \textbf{(11)}$$

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[2]The variable $z_{i6,t-1}$ proxies for the January effect which is explained in Bodie, Kane and Marcus (1993, pp. 380-381).

where M denotes the 2×2 diagonal matrix $\text{diag}(N_{00}, N_{11})$ with $-1 < N_{00}, N_{11} < 1$ and $\boldsymbol{a}_{it} = (a_{0it}, a_{1it})'$ is distributed with mean zero and variance-covariance matrix $F_a^2)_a = F_a^2[*_{kja}]$, k = 0, 1, j = 0, 1.

Note that Assumption I permits the v_{it} and v_{Mt} of equations (2) and (3) to have nonzero and time-varying means. Fama and French (1992, 1993) found that the current values of $\boldsymbol{z}_{it}$ have reliable power to explain the cross-section of average returns, even though their chosen variables do not appear directly in the CAPM. They also found that stock risks are multidimensional and the elements of $\boldsymbol{z}_{it}$ proxy for different dimensions of risk. As discussed earlier, the coefficients of equation (7) capture the multidimensional nature of risk. For all these reasons, equation (10), relating the coefficients of equation (7) to $\boldsymbol{z}_{it}$ is an appropriate specification.

Assumption II. The $(r_{Mt} - r_{ft})$ are independent of the $,_{it}$ given a value of $\boldsymbol{z}_{it}$. This assumption is weaker than the assumption that the $(r_{Mt} - r_{ft})$ are independent of the $(_{0it}$ (see Dawid (1979, p. 5)) and both of Assumptions I and II are weaker than the assumptions made by Jagannathan and Wang, Harvey, and others in testing the CAPM.

The variables, denoted by $\boldsymbol{z}_{it}$, are called ‘concomitants’ in Pratt and Schlaifer (1988) and form a sufficient set of regressors for equation (10) if they completely explain all the variation in $(_{1it}$. Algebraically, this condition can be expressed as

$$|N_{11}| = 0 \text{ or } 1,\ *_{01a} = *_{10a} = *_{11a} = 0,\ B_{kj} \neq 0 \text{ for } k = 0, 1 \text{ and } j \neq 0. \quad \textbf{(12)}$$

The implication of these conditions on M and $)_a$ is that the distribution of $,_{1it}$ is degenerate. If it is degenerate and if the expectation of $,_{0it}$, given the values of $\boldsymbol{z}_{it}$ and $(r_{Mt} - r_{ft})$, is zero, as implied by Assumptions I and II, then the third term on the right-hand side of equation (9) is zero given a value of $\boldsymbol{z}_{it}$, and model (7) reduces to a regression model with first-order autoregressive errors. In this case, the usual consistency proofs apply to Swamy, Mehta and Singamsetti’s (1995) parameter estimators of model (7).[3]

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[3]The conjunction of Assumptions I and II and restrictions (12) is weaker than the assumptions given in Greene (1993, pp. 375-9) for the consistency of instrumental variable estimators (see Pratt and Schlaifer (1988, pp. 47-48)).

Under Assumptions I and II,

$$E\left[\left(\$_{it} - \$_{it}\frac{v_{Mt}}{(r_{Mt}-r_{ft})} + \sum_{j=1}^{m} \cdot_{ijt}\mathrm{R}_{1ijt}\right) | z_{it}\right] = \left[\mathrm{B}_{10} + \sum_{j=1}^{6} \mathrm{B}_{1j} z_{ij,t-1} + E(,_{1it}|z_{it})\right]. \qquad \textbf{(13)}$$

If the sum of the second and third terms on the left-hand side of this equation is equal to the sum of the second and third terms on its right-hand side and $E(\$_{it}|z_{it})$ is a constant, then $E(\$_{it}|z_{it}) = \mathrm{B}_{10}$. Thus, Assumptions I and II can aid in the estimation of the conditional mean of $\$_{it}$ in the original CAPM, given z_{it}. Even when $E(\$_{it}|z_{it}) = \mathrm{B}_{10}$,

the conditional mean $\mathrm{B}_{10} + \frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{6}\mathrm{B}_{1j}z_{ij,t-1}$ is preferable to B_{10} as a measure of the risks inherent in the i-th asset (or group of assets) if $E(\frac{1}{T}\sum_{t=1}^{T} -\$_{it}\frac{v_{Mt}}{(r_{Mt}-r_{ft})}|z_{it})$ is equal to $\frac{1}{T}\sum_{t=1}^{T}E(,_{1it}|z_{it})$, since equation (5), unlike equation (1), covers all sources of risk. For large T, the mean $\frac{1}{T}\sum_{t=1}^{T}(_{1it}$ will be equal to $\mathrm{B}_{10} + \frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{6}\mathrm{B}_{1j}z_{ij,t-1}$ if $\frac{1}{T}\sum_{t=1}^{T}(_{1it}$ converges in probability to $E(\frac{1}{T}\sum_{t=1}^{T}(_{1it}|z_{it})$ and if both $E(\frac{1}{T}\sum_{t=1}^{T} -\$_{it}\frac{v_{Mt}}{(r_{Mt}-r_{ft})}|z_{it})$ and $\frac{1}{T}\sum_{t=1}^{T}E(,_{1it}|z_{it})$ tend to zero as $T \to \infty$.

Estimation of equation (7) under Assumptions I and II is performed with and without the restrictions that[4]

$$\mathrm{B}_{kj} = 0 \text{ for } k = 0, 1 \text{ and } j \neq 0. \qquad \textbf{(14)}$$

These restrictions, when imposed, eliminate the time-varying z's from equation (10).

Note that Assumption I does not permit the restrictions on M and $)_a$ given in (12) to be exactly satisfied. They can only be nearly satisfied if the estimates of M and $)_a$ are equal to the boundary values

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[4]Equation (7) is estimated under Assumptions I and II, using a computer program developed by I-Lok Chang and Stephen Taubman. This program uses an algorithm developed by Chang, Hallahan and Swamy (1992) and is based on a methodology introduced by Swamy and Tinsley (1980). For further discussion of this methodology, see Swamy, Mehta and Singamsetti (1996).

$|N_{11}| = .99, *_{01a} = *_{10a} = *_{11a} = .0001$. If these restrictions are nearly satisfied when the restrictions on A given in (14) are not imposed than when they are imposed, then one can conclude that the regressors included in equation (10) are appropriate in the sense that they adequately explain the variation in the coefficients of equation (7).

Equations (7) and (10) jointly describe the time-series model being estimated to explain the excess returns $r_{it} - r_{ft}$ in Section 5 below. Thus, substituting equation (10) into equation (7):

$$r_{it}-r_{ft} = [1 \ (r_{Mt}-r_{ft})]\begin{pmatrix} B_{00} & B_{01} & \cdots & B_{06} \\ B_{10} & B_{11} & \cdots & B_{16} \end{pmatrix}\begin{pmatrix} 1 \\ z_{i1,t-1} \\ \vdots \\ z_{i6,t-1} \end{pmatrix} + [1 \ (r_{Mt}-r_{ft})]\begin{pmatrix} ,_{0it} \\ ,_{1it} \end{pmatrix}. \qquad \textbf{(15)}$$

This equation is not linear and contains an error term that is both heteroscedastic and serially correlated. The explanatory variables in this equation are the excess market return $r_{Mt} - r_{ft}$, the six concomitants, z, (introduced in Assumption I that includes firm size and the ratio of book-to-market equity), and the interactions between the excess market return and each of the concomitants. Used alone without these interactions, the concomitants may not have adequate power to explain stock returns because of the multidimensional nature of stock risks. Previous tests of the CAPM neglect to consider these interactions. They also use two-pass regressions (see Bodie, Kane and Marcus (1993, Chapter 11) for a survey of these tests). In principle, applying one-pass regression to (15) is superior to two-pass regressions, even when the second-pass regression overcomes the measurement error problem created by the $ estimates. Under Assumptions I and II, the conditional mean of the dependent variable of equation (7), given the values of $\mathbf{z}_{it}$ and $(r_{Mt} - r_{it})$, is equal to the first term on the right-hand side of equation (15). For empirical estimation of equation (15), any one of three data sets (time-series data, cross-section data, and time-series-cross-section data) may be used, although Assumptions I and II are well suited only to time-series data (see Swamy and Tavlas (1995)).[5]

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[5]If time-series-cross-section data are used to estimate equation (7), then equation (10) may be changed to $(_{it} = A\mathbf{z}_{it} + \boldsymbol{\mu}_i + ,_{it}$, where $\boldsymbol{\mu}_i = (\mu_{0i}, \mu_{1i})'$ is a constant through time; it is an attribute of the i-th asset (or group of assets) which is unaccounted for by the included

Now suppose that Assumptions I and II and the restrictions given by (12) hold. It follows that: (i) equation (15) explains how firm size and the ratio of book-to-market equity influence the excess returns, $r_{it} - r_{ft}$, which, in turn, influence the stochastic behavior of earnings, (ii) the sum $B_{01}z_{i1,t-1} + B_{02}z_{i2,t-1}$ measures variation in excess returns, $r_{it} - r_{ft}$, associated with firm size and BE/ME that is not captured by the market return, and (iii) the $,_{it}$ can be identified as state variables that lead to common variation in the excess returns, $r_{it} - r_{ft}$, that is independent of the market and thus carries a different premium than general market risk. Note that (i)-(iii) are directly responsive to issues (i)-(iii) raised by Fama and French (1993) and restated in the Introduction.

It is useful here to consider variations of the model proposed above. Clearly, the conjunction of the model given by equation (7) and Assumptions I and II is false if it cannot explain and predict the underlying phenomenon better than the following inconsistent or restrictive alternatives introduced earlier,

$$B_{kj} = 0 \text{ for } k = 0, 1 \text{ and } j \neq 0,\ N_{11} = 0,\ *_{01a} = *_{10a} = *_{11a} = 0,\ *_{00a} = 1, \quad \textbf{(16)}$$

or,

$$B_{kj} = 0 \text{ for } k = 0, 1 \text{ and } j \neq 0,\ M = 0,\ *_{01a} = *_{10a} = *_{11a} = 0,\ *_{00a} = 1. \quad \textbf{(17)}$$

Restriction (16) implies that equation (7) is a fixed-coefficients model with first-order autoregressive (AR(1)) errors, while Restriction (17) implies that equation (7) is a fixed-coefficients model with white-noise errors.

It is useful to digress for a moment to the subject of model validation based on forecast comparisons. A rationale for this type of comparison is provided by the cross-validation approach--which consists of splitting the data sample into two subsamples. The choice of a model, including any necessary estimation, is then based on one subsample and its performance is assessed by measuring its prediction against the other subsample. The premise of this approach is that the validity of statistical estimates should be judged by data different from those used to derive the estimates (see Mosteller and Tukey (1977, pp. 36-40)). Friedman and Schwartz (1991, p.

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variables but varies across i. When equation (7) is estimated separately for different i, the vector $\boldsymbol{\mu}_i$ gets absorbed into $(B_{00}, B_{10})'$. So B_{00} and B_{10} in equation (15) are implicitly allowed to vary across i.

47) also indicate that a persuasive test of a model must be based on data not used in its estimation. Furthermore, formal hypothesis tests of a model on the data that are used to choose its numerical coefficients are almost certain to overestimate performance: the use of statistical tests leads to false models with probability 1 if both the null and alternative hypotheses considered for these tests are false, as shown by Swamy and Tavlas (1995, p. 171 and footnote 7). That this is a problem in the present case follows from the lack of any guarantee that either a null or an alternative hypothesis will be true if the inconsistent restrictions (14) or (16) or (17) are necessary parts of the maintained hypothesis. Conversely, a hypothesis is true if it is broad enough to cover the true model as a special case. This is the motivation for extending the CAPM: to make it broad enough so that there is a better chance of encompassing the true model as a special case. Accordingly, in Section 5 below, model (7) and Assumptions I and II with or without restrictions (14) or (16) or (17) are evaluated based upon forecast comparisons.

Model (7), Assumptions I and II, and the three sets of restrictions can be combined as conjunctions, listed here in decreasing order of generality regarding the restrictiveness of assumptions.

Conjunction I: model (7), Assumptions I and II.

Conjunction II: model (7), Assumptions I and II, and set (14).

Conjunction III: model (7), Assumptions I and II, and set (16).

Conjunction IV: model (7), Assumptions I and II, and set (17).

The reason for considering Conjunctions II-IV--even though they are inconsistent--is to examine how they perform in explanation and prediction relative to Conjunction I. Doing this is especially useful for understanding earlier empirical work leading to the CAPM puzzle.

The accuracy of the model, or its validity, is determined as follows. Let $R_{it} = r_{it} - r_{ft}$. After estimating the models defined by Conjunctions I-IV, forecasts of the out-of-sample values of R_{it} are generated from each of the estimated models. Let these forecasts be denoted by $\hat{R}_{it}$. Then two formulas are used to measure the accuracy of $\hat{R}_{it}$,

$$(i)\ \text{root mean-square error} = RMSE = \sqrt{\frac{1}{F}\sum_{s=1}^{F}(\hat{R}_{i,T+s} - R_{i,T+s})^2} \qquad \textbf{(18)}$$

and

$$(ii)\ \text{mean absolute error} = MAE = \frac{1}{F}\sum_{s=1}^{F} |\hat{R}_{i,T+s} - R_{i,T+s}|, \qquad \textbf{(19)}$$

where F is the number of periods being forecasted and T is the terminal date of the estimation period.

4. Data

The r_{it} are the monthly value-weighted stock returns on each of 10 portfolios that are formed following Fama and French's (1993, p. 11) procedure: "Each year t from 1963 to ... [1993] NYSE quintile breakpoints for size (ME, ...), measured at the end of June, are used to allocate NYSE, Amex, and NASDAQ stocks to five size quintiles. Similarly, NYSE quintile breakpoints for BE/ME are used to allocate NYSE, Amex, and NASDAQ stocks to five book-to-market equity quintiles." The 10 portfolios are formed as the intersections of the five-firm size and the lowest- and highest-BE/ME quintiles, denoted nsibje, i = 1, 2, ..., 5 and j = 1, 5. For example, the ns1b1e portfolio contains the stocks in the smallest-ME quintile that are also in the lowest-BE/ME quintile, and the ns5b5e portfolio contains the biggest-ME stocks that also have the highest values of BE/ME.

The proxies for r_{ft} and r_{Mt} are the same as those employed by Fama and French (1993). That is, r_{ft} = the one-month Treasury bill rate, observed at the beginning of the month, and r_{Mt} = the value-weighted monthly percent return on the stocks in their 25 size-BE/ME portfolios, plus the negative-BE stocks excluded from the portfolios.

The sources of the data employed here on the $z_{i1,t-1}$ and $z_{i2,t-1}$ are explained in Fama and French (1992). The variables $z_{i3,t-1}$, $z_{i4,t-1}$, and $z_{i5,t-1}$ were obtained from the FAME data base maintained by the Board of Governors of the Federal Reserve System. The index t denotes the months that occurred in the period from July 1963 through December 1993. The subscript i of variables in equations (7), (10), and (11) should not be confused with the i of nsibje.

5. Empirical Results and Applications

Tables 1a-c show the estimates for Conjunction I. The maxima, minima, and ranges of the estimates ($\hat{\gamma}_{0it}$) of γ_{0it} in Table 1a show considerable variation over time for all 10 portfolios. By contrast, for the same portfolios, the volatilities of the estimates ($\hat{\gamma}_{1it}$) of γ_{1it} are quite low. When estimated without the nonnegativity constraint, the estimates of γ_{1it} were negative only for 5 of 341 months and for 1 (ns1b5e) of 10 portfolios.

Black (1995) provides three theoretical reasons why the line relating expected return and β is flatter than suggested by the CAPM, as stated in the Introduction. The question arises then, do the values in Tables 1a-c support such a flat line? The answer is “no,” as is shown below. Under the conditions stated below equation (13), the arithmetic means of $\hat{\gamma}_{1it}$ in Table 1a give measures of portfolio risks. These arithmetic means are positive for all 10 portfolios and are significantly different from zero for 7 of these portfolios. The arithmetic means of $\hat{\gamma}_{0it}$ are not significantly different from zero for these 7 portfolios and are significantly different from zero for the 3 highest-BE/ME portfolios (ns2b5e, ns3b5e, and ns4b5e) for which the arithmetic means of $\hat{\gamma}_{1it}$ are insignificant. The significant means of $\hat{\gamma}_{0it}$ and the insignificant means of $\hat{\gamma}_{1it}$ for the 3 portfolios cannot be interpreted as evidence of a flat expected return-β line because they arise as a direct consequence of the significant estimates of some of the coefficients on z’s and on the interactions between $(r_{Mt} - r_{ft})$ and each of the z’s that are discussed, in detail, below.

Table 1b shows that the estimates of the intercept (B_{00}) of equation (15) are insignificant for all 10 portfolios. What does this finding say about Conjunction I? The answer follows from Merton’s (1973) work, revealing that a well-specified asset-pricing model produces intercepts that are indistinguishable from 0. Fama and French (1993, p. 5) also state that “judging asset-pricing models on the basis of the intercepts in excess-return regressions imposes a stringent standard.” The insignificance of the B_{00} in Table 1b shows that, at least in the cases considered here, Conjunction I shares a property with a well-specified asset-pricing model and meets Fama and French’s stringent standard.

Table 1b also shows that the estimates of the coefficient (B_{01}) on log(size) are significant for 2 portfolios (ns2b1e and ns2b5e), the estimates of the coefficient (B_{03}) on the dividend-price ratio of the S & P 500 are significant for 4 portfolios (ns2b5e, ns3b5e, ns5b1e, and ns5b5e), and the estimates of the coefficient (B_{06}) on the January dummy variable are significant for 7 portfolios (ns1b1e, ns1b5e, ns2b5e, ns3b5e, ns4b5e, ns5b1e, and ns5b5e). This result shows, among other things, that except for 3 portfolios (ns2b1e, ns3b1e, and ns4b1e), the strong January seasonals in the returns on 7 stock portfolios are not absorbed by strong seasonals in the explanatory variables of equation (15) other than the January dummy variable ($z_{i6,t-1}$). The estimates of the coefficients on $z_{i2,t-1}$, $z_{i4,t-1}$, and $z_{i5,t-1}$ are insignificant for all 10 portfolios.

Furthermore, the estimates of the coefficient (B_{10}) on $(r_{Mt} - r_{ft})$ are significant for 7 portfolios (ns1b1e, ns1b5e, ns2b1e, ns3b1e, ns4b1e, ns5b1e, and ns5b5e)(see Table 1b). For the remaining 3 portfolios (ns2b5e, ns3b5e, and ns4b5e), the estimated coefficients on the interaction between $(r_{Mt} - r_{ft})$ and the default premium ($z_{i4,t-1}$) are significant (see Table 1b). Under the conditions stated below equation (13), the estimates of B_{10} can be viewed as the estimates of $E(\$_{it}|\mathbf{z}_{it})$ and the difference between the arithmetic mean of $\hat{(}_{1it}$ in Table 1a and $\hat{B}_{10}$ in Table 1b gives an estimate of the arithmetic mean of the sum of mismeasurement effects and omitted-variables biases. These differences do not appear to be insignificant for most of the 10 portfolios.

The interactions, the estimates of whose coefficients are significant, are $(r_{Mt} - r_{ft})*\log(\text{size})$ for 3 portfolios (ns1b1e, ns3b1e, and ns5b5e), $(r_{Mt} - r_{ft})*(\text{BE/ME})$ for the portfolio ns5b5e, $(r_{Mt} - r_{ft})*(\text{default premium})$ for 4 portfolios (ns2b5e, ns3b5e, ns4b5e, and ns5b5e), and $(r_{Mt} - r_{ft})*z_{i5,t-1}$ for the portfolio ns5b1e. The estimates of the interaction coefficients B_{13} and B_{16} are insignificant for all 10 portfolios.

All the estimates in Tables 1a and 1b unambiguously support only one conclusion: the relation between the observable counterparts of expected return and $\$$ is not a flat line but is nonlinear. This finding provides possible answers to Black's (1995) questions stated in the Introduction. The cross-section of average returns on U.S. common stocks probably shows little relation to the $B_{10} + \mu_{1i}$ defined in footnote 3 and shows significant relation to the $\mathbf{z}_{it}$, as implied by Fama and French's (1992) results, if the interactions between $(r_{Mt} - r_{ft})$ and each of the $\mathbf{z}_{it}$, and the heteroscedasticity and serial correlation of the error term in equation (15) are neglected.

Table 1c indicates the extent to which Conjunction I satisfies set (12) of restrictions. In 7 of 10 cases shown in this table, the whole set is nearly satisfied. In addition, in all 10 cases, the estimated variance of a_{1it} and the estimated covariance between a_{1it} and a_{0it} are very small in magnitude relative to the estimated variance of a_{0it}. It should be noted that exclusion of a concomitant variable from equation (10) because its estimated coefficient is insignificant is improper if the variable is needed to explain the variation in $(_{1it}$. It is better to include a concomitant that substantially explains the variation in $(_{1it}$ than to exclude it even if its inclusion means reducing the t ratios of the estimates of the coefficients of equation (10).

Tables 2ab and 2c display the results for Conjunction II. These results provide information on the effects of set (14) of restrictions on the estimates in Tables 1a-c. The values in every column of Table 2ab can be compared with those in the corresponding column of Table 1a. Plots (not included here) show that while the time profiles of $\tilde{(}_{0it}$ in Tables 2ab and 1a are the same in all 10 cases, those of $\tilde{(}_{1it}$ in these tables are the same only in 4 of 10 cases. The t ratios of $\dot{B}_{00}$ and $\dot{B}_{10}$ in Table 2ab are generally higher in magnitude than those of the arithmetic means of $\tilde{(}_{0it}$ and $\tilde{(}_{1it}$ in Table 1a. It is possible that the extra precision obtained by imposing set (14) of restrictions is spurious because several of the estimates of B in Table 1b are significant. In 4 of 10 cases, the estimates of B_{00} in Table 2ab are significant. This shows that Conjunction II does not always satisfy the property of a well-specified asset-pricing model noted by Merton (1973). In 3 of 10 cases shown in Table 2ab, the estimates of B_{00} and B_{10} are significant and insignificant, respectively, supporting the conclusion that a relation between expected return and $ is flat. However, such a conclusion is not credible because it is based on Conjunction II, which is inconsistent.

Tables 1c and 2c might be compared to determine whether the regressors of equation (10) are appropriate and sufficient. It can be seen from these tables that for 8 of the 10 portfolios, set (12) of restrictions is better satisfied when set (14) of restrictions is not imposed than when it is imposed. This result supports the conclusion that the regressors of equation (10) are the appropriate explanatory variables for $(_{1it}$ but some additional concomitants are needed to completely explain all the variation in $(_{1it}$ for all portfolios. Further work is needed to find such additional concomitants.

Tables 3 and 4 show parameter estimates for Conjunctions III and IV, respectively. The t ratios of $\dot{B}_{00}$ and $\dot{B}_{10}$ in these tables are generally higher than those of $\dot{B}_{00}$ and $\dot{B}_{10}$ in Table 2ab in absolute value. The spuriousness of the extra precision obtained by imposing inconsistent sets (16) and (17) of restrictions is more pronounced than than that of the extra precision obtained by imposing inconsistent set (14) of restrictions. Since the explanatory variables of equation (15) are not orthogonal to one another, and its error covariance matrix is not equal to a scalar times an identity matrix under all Conjunctions I-IV, the estimates $\hat{B}_{00}$ in Tables 2ab, 3, and 4 are not comparable to the estimates $\hat{B}_{00}$ in Table 1b. The estimates of N_{00} in Tables 1c, 2c, and 3 offer little support for the presence of serial correlation among the $,_{0it}$. The estimates ($\hat{F}^{2}_{a}\hat{*}_{00a}$) of the variance of a_{0it} in Table 1c are generally smaller than the estimates $\hat{F}^{2}_{a}\hat{*}_{00a}$ in Table 2c and the estimates $\hat{F}^{2}_{a}$ in Tables 3 and 4, indicating that in most cases the variance of a_{0it} is reduced as the z's are added to equation (10). This reduction in variance helps to weaken the correlations between $(_{0it}$ and $(r_{Mt} - r_{ft})$.

The values of log likelihood in Tables 1a, 2ab, 3, and 4 might be compared to determine whether one of the Conjunctions I-IV has greater support of the time-series data used for estimation than other Conjunctions. With one exception, these values in Table 1a are higher than those in Tables 2ab, 3, and 4. The exception corresponds to the portfolio ns2b1e in Table 2ab. Even in this case, the value of the log likelihood in Table 1a is only slightly smaller than that in Table 2ab. This shows that the support of the data to Conjunction I is either greater or only slightly less than the support to Conjunctions II-IV.

Since the t ratios in Tables 1a and 1b are based on a consistent set of general assumptions that are not used by earlier tests of the CAPM in the literature, they do not fall into Black's (1995, p. 2) category of "the simplest kind of data mining." Still, it is appropriate to seek RMSE and MAE measures for each portfolio of a conjunction's success in predicting the out-of-sample values of the dependent variable of equation (7). Tables 1c, 2c, 3, and 4 report for each portfolio the values of such measures in the RMSE and MAE columns. The RMSEs for Conjunction I are smaller for 7 portfolios and slightly higher for 3 portfolios than those for Conjunctions II-IV. For a conjunction which has at least 10 more unknown parameters and hence uses up at least 10 more degrees of freedom than any of Conjunctions II-IV, this is not a bad performance. Perhaps Conjunction I would have produced lower RMSEs than Conjunctions II-IV for all 10 portfolios if all the parameters of equations (11) and (15) were known. Based on the RMSEs, Conjunctions II-IV cannot be preferred to Conjunction I. Even though the MAEs for Conjunction I are smaller than those for Conjunctions II-IV in 5 of 10 cases, they are much bigger than those for Conjunctions II-IV in 2 of the remaining 5 cases. Two reasons for this result are: (i) the predictor used to generate forecasts of the dependent variable of equation (7) is optimal relative to a quadratic loss function but is not optimal relative to an absolute error loss function, and (ii) sometimes inconsistent models appear to predict better than consistent models if inappropriate formulas are used to measure the accuracy of forecasts. In the long run, only consistent models are able to tell the truth.

Any empirical result that holds for all 4 Conjunctions can be considered robust. The arithmetic means of the estimates of $(_{1it}$ under Conjunction I and the estimates of B_{10} under Conjunctions II-IV are the estimates of the conditional and unconditional expectations, $E((_{1it}|z_{it})$ and $E((_{1it})$, respectively. It can be seen from Tables 1a, 2ab, 3, and 4 that these estimates are close to one another and hence are robust.

Fama and French (1993, p. 53) list four applications--(a) selecting portfolios, (b) evaluating portfolio performance, (c) measuring abnormal returns in event studies, and (d) estimating the cost of capital--that require estimates of risk-adjusted stock returns. The estimates in Table 1b can be substituted into the first term on the right-hand side of equation (15) to obtain the estimates of risk-adjusted stock returns because $(_{1it}$ is a comprehensive descriptor of stock risk. The preceding discussion shows that these estimates do a better job in all four applications than those of previous studies. The methodology used to obtain the estimates in Tables 1a-c can also be used to obtain accurate predictions about as yet unobserved values of the dependent variable of equation (15). The discussion in this section and in Sections 2 and 3 shows that the measures of market or “systematic” risks of portfolios given by the arithmetic means of $\hat{(}_{1it}$ in Table 1a are theoretically and empirically superior to estimates of $\$$ presented so far in the literature.

6. Conclusions

This paper has extended the CAPM to account for the effects of differences between unobservable and observable stock and market portfolio returns, of excluded variables, and of departures from a linear relationship between the observable returns on individual stock and market portfolios. The extended CAPM is tested using a stochastic-coefficients methodology. For purposes of comparison, both consistent and inconsistent sets of assumptions are made in these tests. Tests based on a consistent set of assumptions show that the relation between the observable returns on stock and market portfolios is nonlinear.

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