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Two-Dimensional Co-Oscillations in a Rectangular Bay: Possible Application to Water-Level Problems

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The two-dimensional response of a rectangular bay of uniform depth to a fluctuating water-level disturbance imposed at its mouth is examined in the framework of linear shallow-water equations on a nonrotating earth. The imposed forcing is periodic in time but spacially varying in the transverse direction along the mouth of the bay. The response is presented both in terms of the amplification factor, which is the ratio of the imposed amplitude at the mouth to that at the closed end of the bay, and the structure of the height field within the bay. The two-dimensional character of the response becomes more pronounced as the wavelength of the disturbance at the mouth decreases and as the width of the bay increases. Positive and negative amphidromic systems can be generated in the bay for disturbances propagating along the mouth of the bay even though the earth's rotation is neglected. The origin of the water-level fluctuations at the mouth of the bay could be due to tides, storm surges, or tsunamis. This study indicates the importance of measuring the spacial variations in the water-level fluctuations along the mouth of the bay, instead of assuming them to be spacially uniform, when attempting to explain the water-level response within the bay.

Keywords co-oscillations, rectangular bay, two-dimensional problem, no-rotation case

All bays are subjected to forcing at their mouths due to water-level fluctuations imposed by the external water bodies to which they are connected. These water-level fluctuations in the external water bodies could be produced by any number of causes, such as astronomical tides, storm surges, or tsunamis. This imposed forcing at the mouth produces a co-oscillating response within the bay. These co-oscillations in a rectangular bay have been previously examined in the context of one-dimensional shallow-water theory for simple-analytical-shaped bottom topographies. For basins of variable shapes and bottom

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topographies, the problem has been examined in the context of the channel equations using numerical procedures (see Defant, 1961). In these one-dimensional studies, the water-level fluctuation at the mouth is periodic in time and uniform in space along the mouth of the bay. The response of the bay is generally given in terms of the amplification factor, which is the ratio of the imposed forcing elevation at the mouth to that at the closed end of the bay. This amplification factor depends on the basin geometry, bottom topography, and the frequency of the forcing at the mouth. For a rectangular bay of length L and uniform depth H, for example, the amplification factor is given by $1/\cos(\omega L/C)$, where $C \equiv \sqrt{gH}$ is the speed of long gravity waves and ω is the frequency of forcing imposed at the mouth. The amplification factor is generally ≥ 1, since $\cos(\omega L/C)$ is always ≤ 1, indicating that the amplitude at the head of the bay is larger than the forcing amplitude imposed at the mouth. The amplification becomes infinite (resonant response) in the inviscid case whenever $\omega L/C$ becomes an odd multiple of $\pi/2$, which represents the value of the frequency of a free oscillation mode of the bay.

These simple considerations are made complicated in nature due to various factors. One such factor is that all free modes of oscillation of the bay are assumed to be characterized by the existence of a nodal line—a line of zero water-level amplitude—at the mouth of the bay. Imposition of this condition makes the frequencies of free oscillations equal to odd multiples of $\pi/2$ in the one-dimensional case mentioned above. In reality, the nodal lines for different free modes of a bay connected to a larger water body occur at different locations within the domain of the entire system as shown, for example, in the study of Rao et al. (1976) on Lake Michigan. Hence, some of the free modes of a bay can have a nonzero water-level amplitude at what might normally be considered the physical location of the mouth of the bay. This feature then changes the frequency of the mode from what it would have been if a nodal line were imposed at the mouth. The resulting effect is a change in the amplification of a forced oscillation (and conditions for resonance within the bay). This problem has been addressed by Heaps (1975).

Another factor that changes the simple one-dimensional response is that the amplitude of the forcing imposed at the mouth will not normally be uniform along the mouth of the bay, even for a narrow bay, but will exhibit some spacial variation. Such a spacial variation in the forcing will result in a two-dimensional response within the bay. As a consequence, the magnification factors at the head of the bay will be not only different from what one would obtain for a spatially uniform forcing at the mouth, but would also change in the transverse direction. In the study of the oscillations of Green Bay by Heaps et al. (1982), several discrepancies were encountered between the observed and computed water levels. As speculated by them, some of these differences could indeed be related to the fact that spacial variations in the forcing imposed by the Lake Michigan water level at the mouth of the Green Bay have not been taken into account. In comments on a study of Dorrestein (1983), Murty and El-Sabh (1989) speculated that local topography and friction may be responsible for producing anamolous clockwise (negative)-rotating amphidromic systems in the co-oscillating tides in some of the marginal seas in the northern hemisphere and vice versa in the southern hemisphere. This is probably true for large marginal seas and bays, whose dynamics are influenced significantly by the earth's rotation. As will be shown in this study, for smaller water bodies or those at very low latitudes, for which the effects of the earth's rotation are negligible, disturbances propagating along the mouth can produce co-tidal oscillations that exhibit either positive or negative amphidromic systems even though the earth's rotation is ignored.

In order to illustrate the nature of the two-dimensional response of a bay to a periodic but spacially varying water-level fluctuation imposed at the mouth, we consider here a

simple case of a nonrotating rectangular bay of uniform depth. Modifications due to the possible nonexistence of a nodal line at the mouth for some of the free modes, effects of nonuniform bottom topography, and the earth's rotation will be considered later.

Basic Equations and Solution

The two-dimensional linear shallow-water equations on a nonrotating earth are

$$\frac{\partial u}{\partial t} = -g\frac{\partial \eta}{\partial x} \qquad \frac{\partial v}{\partial t} = -g\frac{\partial \eta}{\partial y}$$
$$\frac{\partial \eta}{\partial t} + \left(\frac{\partial uH}{\partial x} + \frac{\partial vH}{\partial y}\right) = 0 \tag{1}$$

In these equations, u and v are the horizontal velocities in the x and y directions, t is time, η is the water-level fluctuation about the mean, H is the depth of the water in the undistributed state, and g is the gravitational acceleration. We assume that the bay occupies the domain $0 \leq x \leq L$ (length of the bay) in the longitudinal direction and $0 \leq y \leq B$ (breadth of the bay) in the transverse direction. The bay is closed at $x = L$ (head of the bay) and along $y = 0$ and B. It is open to a large external water body along its mouth at $x = 0$, where it is connected to a larger water body (see Figure 1). The appropriate boundary conditions for the free oscillations are that

$$u = 0 \text{ at } x = L \text{ (head of the bay)} \tag{2a}$$
$$v = 0 \text{ at } y = 0 \text{ and } B \tag{2b}$$
$$\eta = 0 \text{ at } x = 0 \text{ (mouth of the bay)} \tag{2c}$$

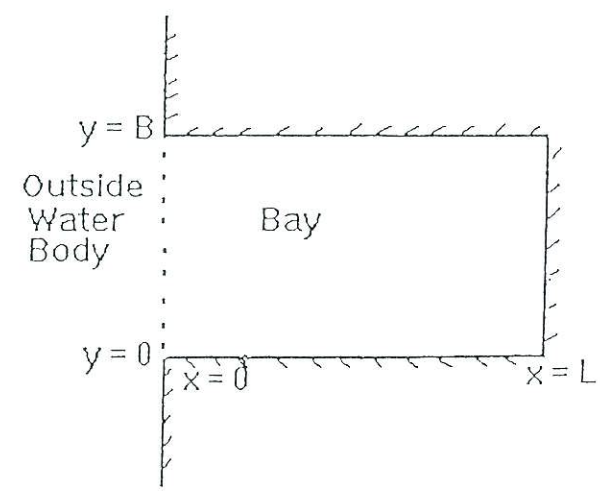


Figure 1. Geometry of the bay.

Eliminating u and v in Eq. (1) results in a single equation for η,

$$\frac{\partial^2 \eta}{\partial t^2} = \left(\frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2}\right) \tag{3}$$

In the above equation, all quantities are made nondimensional using the length of the bay L for x and y coordinates, L/C for time, and a scale amplitude η_0 for the height field. In these nondimensional units, the domain of the bay is $0 \leq x \leq 1$ in the longitudinal direction and $0 \leq y \leq \epsilon$ ($\equiv B/L$) in the transverse direction.

Assuming a periodic time dependence of the form $\cos(\sigma t)$, where σ is the nondimensional frequency of free oscillations, the solution for η that satisfies the boundary conditions given in Eq. (2) is

$$\eta = \sin\left[(2m + 1)\frac{\pi x}{2}\right]\cos\left(\frac{n\pi y}{\epsilon}\right)\cos(\sigma t) \tag{4}$$

The free oscillation frequencies are given by

$$\sigma = \pi\left[\frac{(2m + 1)^2}{4} + \frac{n^2}{\epsilon^2}\right]^{1/2} \tag{5}$$

in which m and n are integers that can take on the values 0, 1, 2, 3, ... All the free modes that correspond to $n = 0$ are obviously the one-dimensional longitudinal oscillations, and the frequencies of all these modes are equal to odd multiples of $\pi/2$. The mode that corresponds to $m = 0$ and $n = 0$ is the lowest Helmholtz mode.

For the forced oscillations, the governing equation is still the same as in Eq. (3), but the boundary condition at the mouth for the fluctuation of the water level is now given by

$$\eta(y) = f(y)\cos\omega t + g(y)\sin\omega t \qquad \text{at } x = 0 \tag{6}$$

instead of Eq. (2c), which holds for free oscillation modes. In Eq. (6), ω is the frequency of the forcing from the larger water body to which the bay is connected. The functions $f(y)$ and $g(y)$ represent the distribution of the forcing amplitude along the mouth of the bay. The solution to the forced problem is given by

$$\eta(x, y, t) = \sum_n F_n \cos\alpha_n\omega(x - 1)\cos\frac{n\pi y}{\epsilon}\cos\omega t + \sum_n G_n\cos\alpha_n\omega(x - 1)\cos\frac{n\pi y}{\epsilon}\sin\omega t \tag{7}$$

where the coefficient α_n is given by

$$\alpha_n = \left(1 - \frac{n^2\pi^2}{\epsilon^2\omega^2}\right)^{1/2} \tag{8}$$

The expansion coefficients in Eq. (7) are determined through the condition that

$$\left.\begin{aligned}\sum_n F_n\cos\alpha_n\omega\cos\frac{n\pi y}{\epsilon} &= f(y)\\ \sum_n G_n\cos\alpha_n\omega\cos\frac{n\pi y}{\epsilon} &= g(y)\end{aligned}\right\} \quad \text{at } x = 0 \tag{9}$$

Hence, the expansion coefficients are given by

$$\begin{aligned}F_n\cos\alpha_n\omega &= \frac{\gamma_n}{\epsilon}\int_0^\epsilon f(y)\cos\frac{n\pi y}{\epsilon}\,dy\\ G_n\cos\alpha_n\omega &= \frac{\gamma_n}{\epsilon}\int_0^\epsilon g(y)\cos\frac{n\pi y}{\epsilon}\,dy\end{aligned} \tag{10}$$

where

$$\gamma_n = 1 \qquad \text{for } n = 0$$
$$\gamma_n = 2 \qquad \text{for } n \neq 0$$

The coefficient α_n may be real or imaginary depending on the values of n, ϵ, and ω.

Results for Specific Forcing Functions

The forcing imposed at the mouth of the bay depends on its orientation with respect to the connecting water body and the dynamics of the coupled system. In the context of nonrotating dynamics, this forcing can be of the nature of a standing wave at the mouth of the bay pumping the water level in a periodic nature or it can be of the nature of a wave propagating tangentially at the entrance of the bay. In order to take care of both of these possibilities, we shall consider forcing functions as defined below:

$$f(y) = \cos ky \quad \text{and} \quad g(y) = 0 \tag{11a}$$
$$f(y) = 0 \quad \text{and} \quad g(y) = \sin ky \tag{11b}$$
$$f(y) = \cos ky \quad \text{and} \quad g(y) = \sin ky \tag{11c}$$

The forcing functions given in Eqs. (11a) and (11b) represent standing waves at the mouth of the bay, whereas the forcing from Eq. (11c) represents a disturbance propagating along the mouth of the bay with a phase speed given by k/ω. The expansion coefficients in each case are given by

$$F_n\cos\alpha_n\omega = \frac{\gamma_n k\epsilon(-)^n\sin k\epsilon}{k^2\epsilon^2 - n^2\pi^2} \tag{12a}$$

for the cosine forcing (11a), and

$$G_n\cos\alpha_n\omega = \frac{\gamma_n k\epsilon[1 - (-)^n\cos k\epsilon]}{k^2\epsilon^2 - n^2\pi^2} \tag{12b}$$

for the sine forcing (11*b*). For the propagating disturbance, the solution is

$$\eta(x, y, t) = G(x, y) \cos[\omega t - \theta(x, y)]$$

$$G(x, y) = \left[\sum_n F_n \cos \alpha_n \omega(x - 1) \cos \frac{n\pi y}{\epsilon}\right]^2 + \left[\sum_n G_n \cos \alpha_n \omega(x - 1) \cos \frac{n\pi y}{\epsilon}\right]^2 \quad (13)$$

$$\theta(x, y) = \tan^{-1} \frac{\sum_n G_n \cos \alpha_n \omega(x - 1) \cos n\pi y/\epsilon}{\sum_n F_n \cos \alpha_n \omega(x - 1) \cos n\pi y/\epsilon}$$

where θ is the phase of propagation of high water and G is its amplitude. The expansion coefficients F_n and G_n in Eq. (13) are the same as those given in Eqs. (12*a*) and (12*b*), respectively.

It is clear from Eqs. (12*a*) and (12*b*) that F_n and $G_n \to \infty$ as $\alpha_n \omega \to$ an odd multiple of $\pi/2$. Hence the solutions for the height field η in Eqs. (7) and (13) become unbounded at the head of the bay, $x = 1$. This condition is realized whenever the forcing frequency ω becomes equal to the natural frequency σ given in Eq. (5) and represents a resonant response in the conventional sense. In view of the existence of mixed longitudinal and transverse modes, whenever the integer n representing the transverse modality of a free mode differs from zero, more resonant frequencies are present in the two-dimensional case than in the one-dimensional case.

In Equations (12*a*) and (12*b*), the case of $k\epsilon = n\pi$ appears to be a singular point. This, however, is not the case. When $k\epsilon = n\pi$, the solutions (12*a*) and (12*b*) become

$$F_n \cos \alpha_n \omega = 1$$

for cosine forcing and

$$G_n \cos \alpha_n \omega = 0$$

for sinusoidal forcing. As the wave number k of the forcing function tends to zero, the response from Eqs. (12*a*), (12*b*), and (13) tends to

$$F_0 = \frac{1}{\cos \omega} \qquad \text{for } n = 0$$

$$F_n = 0 \qquad \text{for all } n \neq 0$$

$$G_n = 0 \qquad \text{for all } n$$

which is the solution to the one-dimensional case.

In one-dimensional co-oscillation cases, the response of a bay is typically given in terms of an amplification factor, defined as the ratio of the water-level amplitude at the head of the bay to that at the mouth. Following this, we will define an amplification fact A for the two-dimensional case, given by

$$A(Y) \equiv \frac{\eta(x = 1, y)}{\eta(x = 0, y)} \quad (14)$$

It should be noted that in the one-dimensional case, the larger the magnitude of the amplification factor, the closer the bay is to a resonant condition. In the two-dimensional case, however, a similar interpretation of the amplification does not necessarily apply. This is because the amplitude of the forcing at the mouth changes from $y = 0$ to $y = \epsilon$, sometimes going through one or more zero values in this range of y, depending on the form of the prescribed forcing function. The nature of the two-dimensional response always shows a finite water-level response at the head of the bay except for a truly resonant condition when ω becomes equal to σ, as mentioned earlier. Hence the amplification factor for the two-dimensional case may occasionally become infinite for certain values of y because the forcing amplitude of the water level at the mouth is equal to zero while the response at the head has a finite nonzero magnitude at this value of y.

Numerical Results

The solutions given in Eqs. (12*a*), (12*b*), and (13) for the height field and the amplification factor (14) are functions of the forcing frequency ω, wave number k, and the aspect ratio ϵ of the bay. We show results for a square bay ($\epsilon = 1$) and a rectangular bay whose width is half its length ($\epsilon = 0.5$ or aspect ratio of 2×1). To illustrate the nature of the bay's response in this simple, no-rotation case, we have chosen values of ω and k each equal to $\pi/4$ and $3\pi/4$. The first combination may be considered as a long-wavelength, long-period forcing analogous to an astronomical tidal forcing at the mouth. The latter combination is a shorter-period and shorter-wavelength forcing. This forcing may be considered to be analogous to one imposed by a free oscillation of a large water body on a smaller connected bay. In most cases the period of the Helmholtz mode of a bay is greater than the period of the lowest mode of the larger water body, as in the case, for example, of the Lake Michigan–Green Bay system (see Rao et al., 1976; Heaps et al., 1991).

Figure 2 shows the amplification factors as a function of the transverse direction (y) for both square and rectangular bays obtained with the cosine, sine, and propagating wave forcings. In this case, ω and k are both then equal to $\pi/4$. For the cosine forcing and the propagating wave forcing, the amplification factors do not change significantly as a function of y for the $\epsilon = 0.5$ case. There is a slightly larger change in the magnitude of the amplification factor for the square basin, in view of its larger transverse dimension. An example of the changes in the amplification factor for a cosine forcing is given in Table 1. For any given value of y in the range $0 \leq y \leq \epsilon$, the amplification factor is greater for smaller values of ϵ; that is, the amplification increases in elongated bays.

For the sinusoidal forcing there is a large change in the amplification factor as a function of y, as shown in Figure 2. However, it should be noted that the amplitude of

Table 1

Amplification factor values for ω and k each $= \pi/4$ as a function of ϵ and y

$y =$	0.00	0.25	0.50	0.75	1.00
$\epsilon = 0.5$	1.378	1.406	1.483	—	—
$\epsilon = 1.0$	1.282	1.307	1.381	1.516	1.734

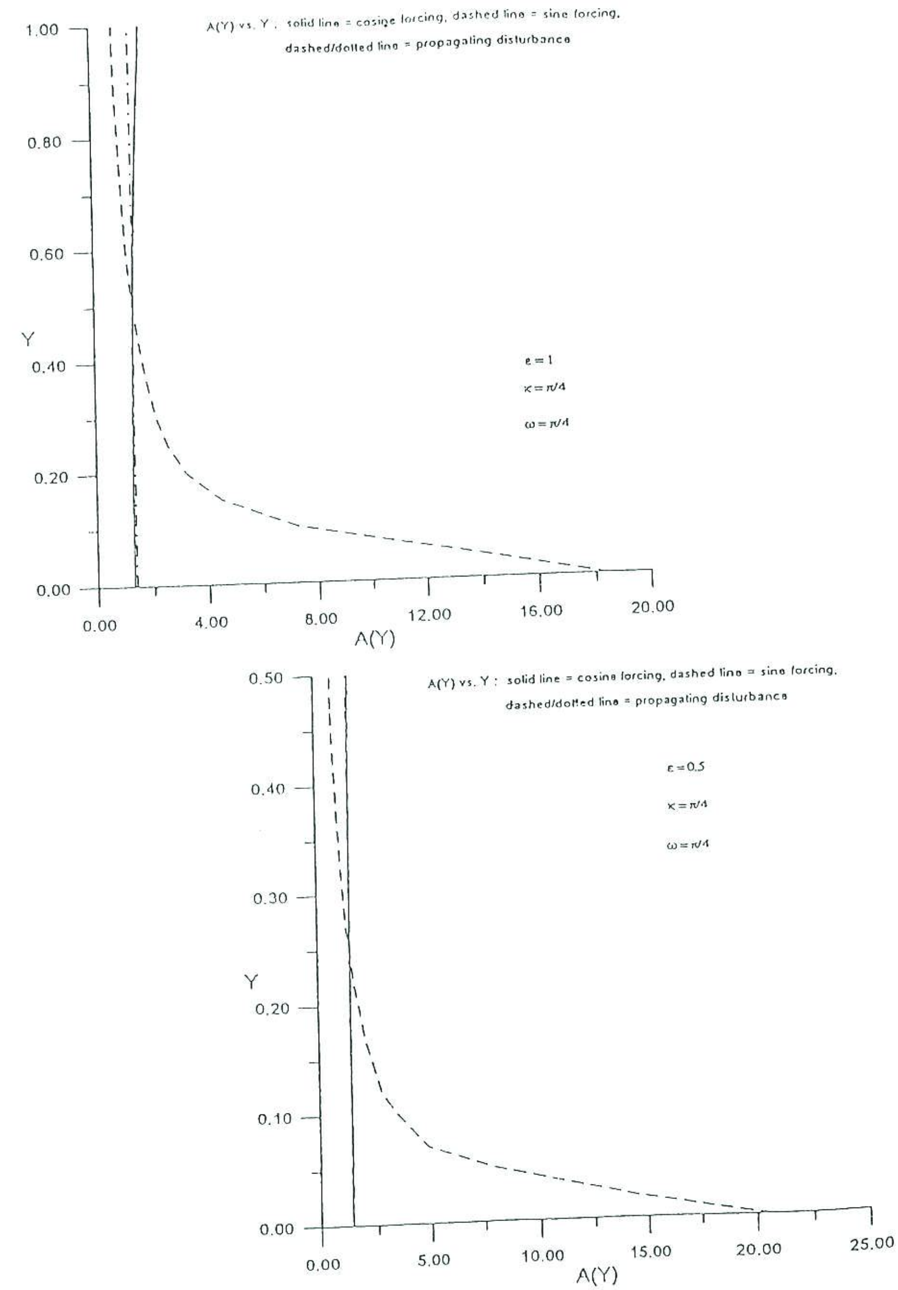


Figure 2. Changes in the amplification factor in the transverse direction of the bay for square and rectangular cases. The frequency ω and the wave number k of the forcing at the mouth are both equal to $\pi/4$.

the forcing at the mouth is zero at $y = 0$ and increases toward $y = \epsilon$. Since the response at the head of the bay is nonzero at $y = 0$, the amplification factor tends to ∞ at $y = 0$. As will be shown shortly, the two-dimensional response tends to produce an almost uniform change in the water-level response at the head of the bay for all y, even though the forcing at the mouth undergoes a greater amplitude change as a function of y. Hence, as mentioned earlier, in the two-dimensional response of a bay, a large amplification factor is not necessarily indicative of a resonant response over the basin in the usual sense, since the water-level amplitudes at the head of the bay remain finite.

Figures 3 and 4 show the height field structure for the three kinds of forcing for the square and the 2×1 rectangular bays. The two-dimensional character of the response is more pronounced for the sinusoidal forcing. For the cosine forcing, the two-dimensional character of the response appears to be confined to the region near the mouth of the bay. The response takes on the appearance of a one-dimensional nature toward the head of the bay. There does not appear to be a significant variation in the water-level amplitudes in the transverse direction. For the sine and cosine forcing, the oscillation at the mouth and at the head of the bay are in phase, since the forcing frequency is less than the lowest bay mode frequency. For the propagating disturbance, the amplitude variation of the height field shows a uniform distribution in the transverse direction, much like a one-dimensional response. However, the nature of the response in terms of time occurrence of the high water at various locations within the bay for the propagating disturbance is interesting. The high water propagates in a positive (counterclockwise) direction around the basin, indicating the existence of an amphidromic point, which is located outside the mouth of the bay in this case. Amphidromic systems, positive or negative, are normally characteristic of oscillations only in basins on a rotating earth. The example shown here appears to be the first case of an amphidromic system in a nonrotating basin. Since the disturbance is propagating from $y = 0$ toward $y = \epsilon$, the maximum value ($= 1$) of the imposed amplitude at the mouth occurs at different times for different values of y. The high water at the head of the bay, however, appears to occur almost simultaneously for all y. It takes a longer time in the case of the square bay for the establishment of this high water at the head than for the rectangular bay.

Consider now the case of ω and k each $= 3\pi/4$, which represents a frequency of forcing that is greater than the lowest Helmholtz mode of the bay. Table 2 gives amplification factors as a function of y for the cosine forcing. As before, these values are larger for the rectangular bay than for the square bay at any given value of y within the domain of the bay. The amplification factor changes significantly as a function of y when compared to the previous case. The co-oscillation amplitude at the head of the bay is even damped (amplification factor values of ≤ 1) for some values of y in the case of the square bay.

Table 2
Amplification factor values for ω and k each $= 3\pi/4$ as a function of ϵ and y

$y =$	0.00	0.25	0.50	0.75	1.00
$\epsilon = 0.5$	−1.104	−1.340	−2.627	—	—
$\epsilon = 1.0$	−0.233	−0.349	−1.126	3.011	0.953

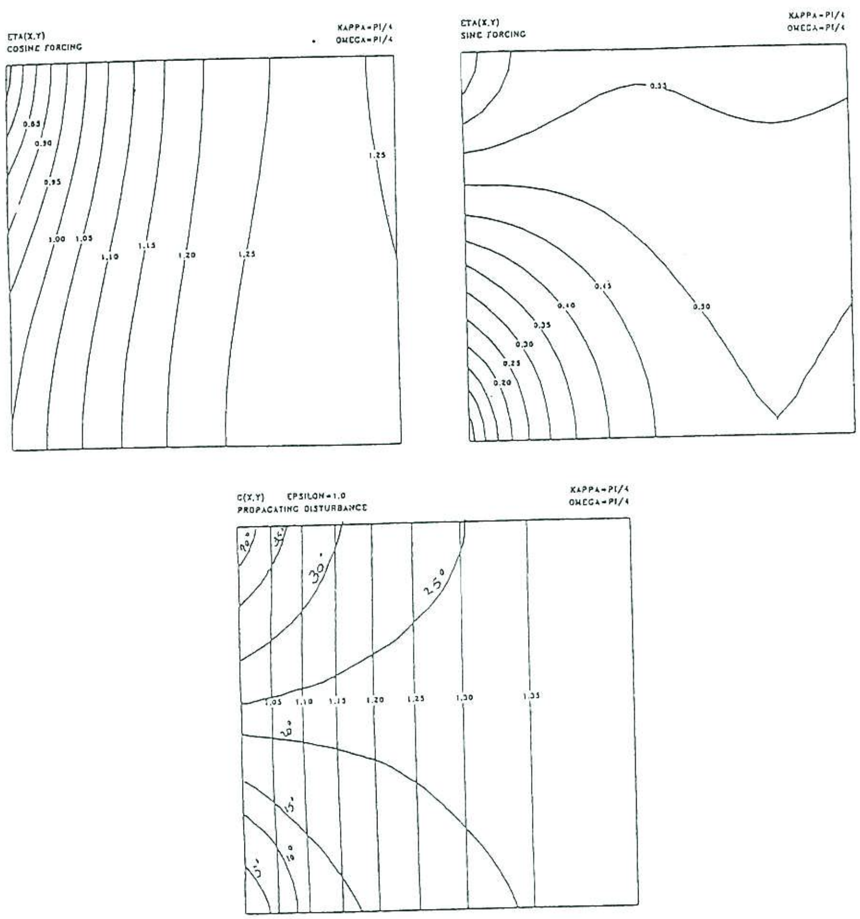


Figure 3. Structure of the water-level response in a square bay for the cosine, sine, and propagating disturbances for ω and k each equal to $\pi/4$.

Figure 5 shows the amplification factors for the square and rectangular bays for the three kinds of forcing considered earlier. In the cases of both the square and rectangular bays, the propagating wave forcing seems to exhibit less variation as a function of y than the cosine and sine forcing. The reason for this relatively small sensitivity of the amplification factor in the y direction for the propagating disturbance, as in the previous case of $\omega = k = \pi/4$, is probably related to the fact that the maximum amplitude of the forcing is the same at the mouth for all values of y, even though this maximum is reached at different times for different values of y. The sine forcing, as before, shows a large change as a function of y. For the square bay, the cosine forcing shows an extremely large amplification as y approaches ⅔ because at this value of y, the amplitude of the forcing at the mouth is zero when $k = 3\pi/4$, while the response at the head of the bay ($x = 1$) for this value of y is nonzero.

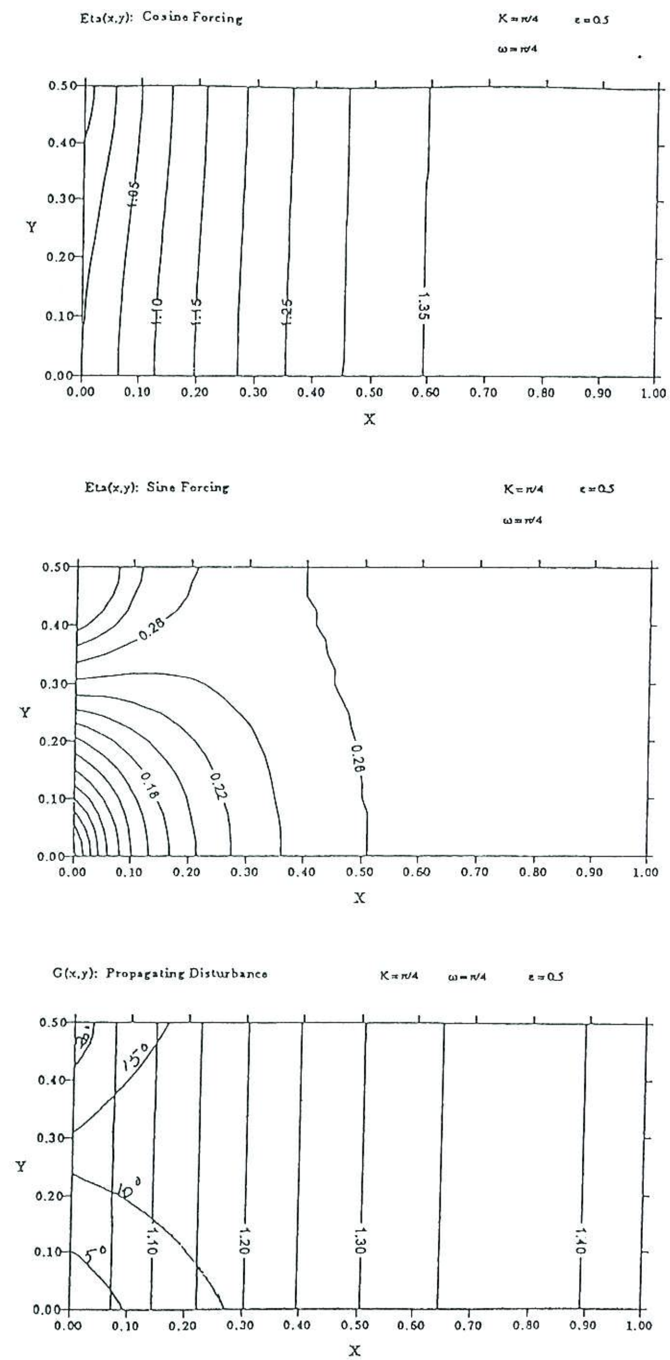
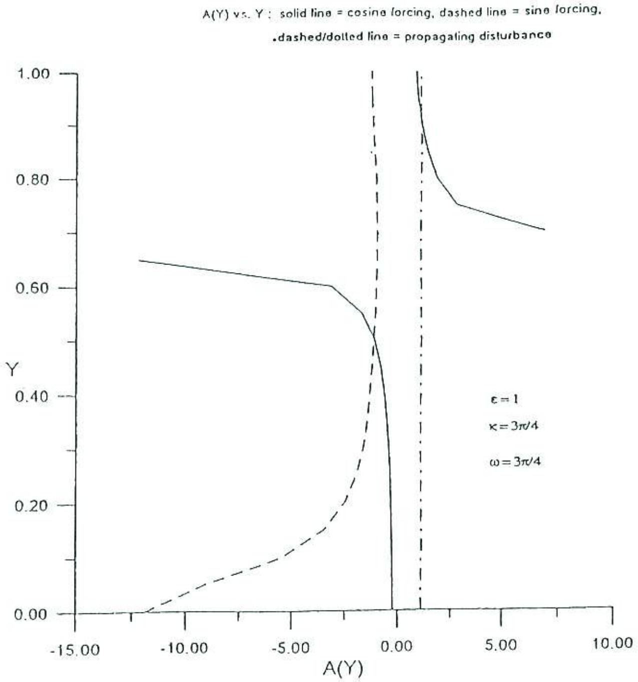


Figure 4. Same as Figure 3 but for a rectangular bay of $\epsilon = 0.5$.



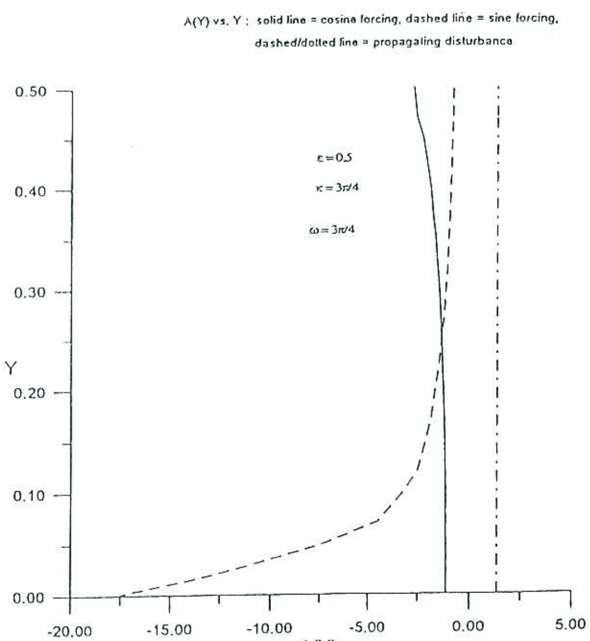


Figure 5. Same as Figure 2 but for ω and $k = 3\pi/4$.

Figures 6 and 7 shows the structure of the response in the square and rectangular bays in terms of the height field for the three forcing functions. The two-dimensional nature of the response is now very clear in the square bay. In the rectangular bay, the response to the cosine and sine forcing again appears to assume the nature of a one-dimensional response as one proceeds toward the head of the bay. This occurs even though the amplification factor changes significantly because the amplitude of the applied forcing at the mouth ranges from 1 at $y = 0$ to 0.383 at $y = \epsilon = 0.5$ for the cosine forcing and from 0 at $y = 0$ to 0.924 at $y = \epsilon = 0.5$ for the sine forcing. In the case of the rectangular bay ($\epsilon = 0.5$) for $k = 3\pi/4$, $\cos ky$ and $\sin ky$ do not pass through zero. Hence, forcing at the mouth has a positive amplitude over the range $0 \leq y \leq \epsilon$. Since the forcing frequency $\omega = 3\pi/4$ is $> \pi/2$, the lowest Helmholtz-mode frequency of the bay, the response at the head of the bay is out of phase with the forcing at the mouth.

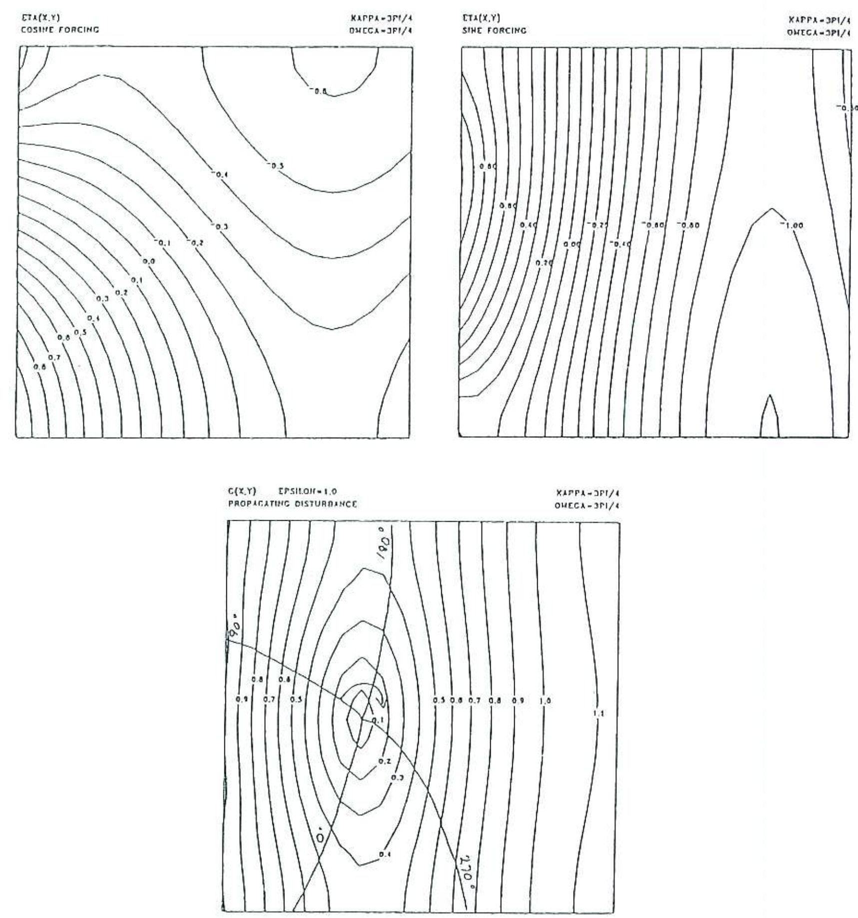


Figure 6. Same as Figure 3 but for ω and $k = 3\pi/4$.

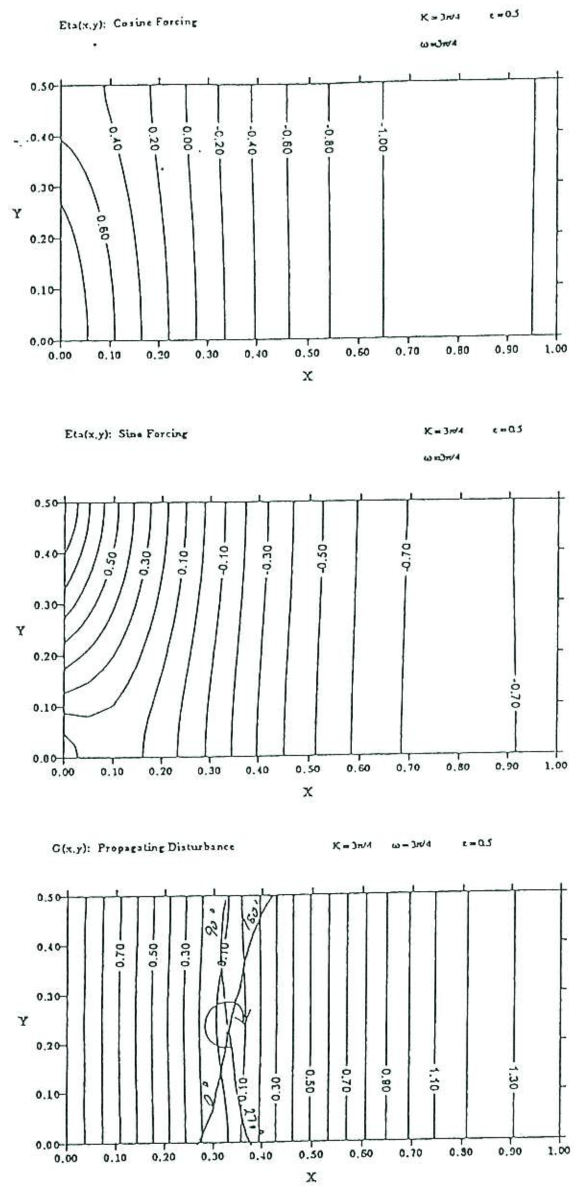


Figure 7. Same as Figure 4 but for ω and $k = 3\pi/4$.

For the square basin, however, cos ky passes through zero at $y = 2/3$ as mentioned earlier and becomes negative for $y > 2/3$. In the case of a square bay, the response at the head of the bay is out of phase with that at the mouth over the range $0 \leq y \leq 2/3$ and is in phase over the range $2/3 \leq y \leq \epsilon\ (= 1)$ for the cosine forcing. The response to the propagating disturbance once again exhibits a uniform amplitude distribution in the trans-

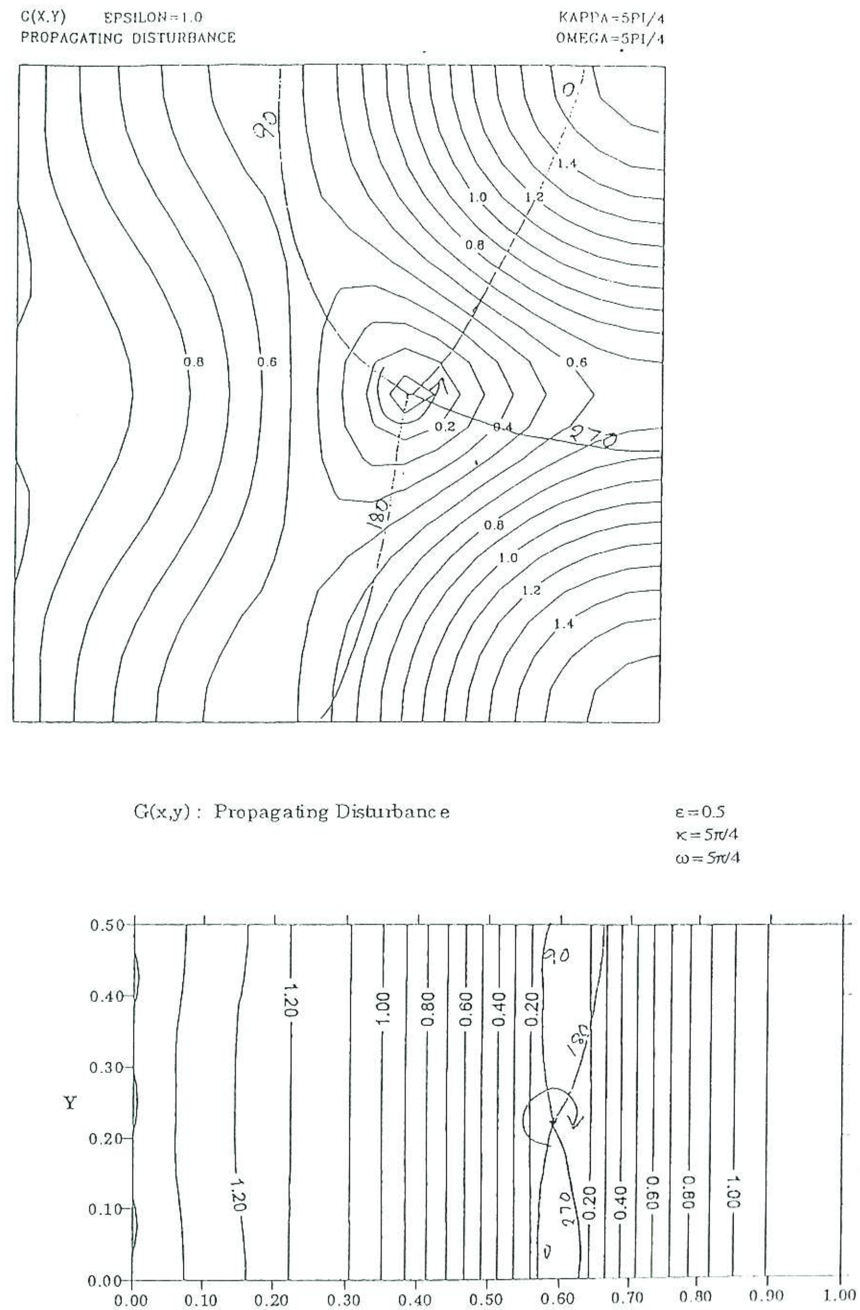


Figure 8. Response of a square and a rectangular bay for a propagating disturbance whose frequency ω and wave number k are both $= 5\pi/4$.

verse direction, but now it also shows that the propagation of the high water has a clockwise (negative) amphidromic character for this combination of ω and k in both the square and rectangular bays. The amphidromic point is now located within the domain of the bay. To illustrate further the complicated nature of the two-dimensional response of a bay as a function of the frequency and wavelength of the imposed forcing and the aspect ratio of the bay, we show in Figure 8 the response of a bay for a propagating disturbance with $\omega = k = 5\pi/4$. The square-bay response shows a positive amphidromic system inside the bay, while the rectangular-bay response shows a negative amphidromic system. Hence the sense of rotation of high water is strongly influenced by the aspect ratio of the bay and the frequency and wavelength of the forcing function.

Conclusions

The two-dimensional response of a nonrotating rectangular bay of uniform depth to periodic but spacially varying disturbances imposed at its mouth has been considered. For very long wavelength of the imposed disturbance, the response tends to be almost like that of the one-dimensional case, particularly near the head of the bay. As the wavelength of the disturbance at the mouth decreases, the response becomes more complicated, with significant two-dimensional variations in the amplitudes along the boundary of the bay in both the longitudinal and transverse directions, especially near the mouth of the bay. In particular, when a traveling disturbance is imposed at the mouth, the propagation of high water within the bay can assume an amphidromic nature, even though the earth's rotation has been ignored.

This amphidromic character of the response in the bay could be either positive or negative depending on the aspect ratio of the bay and the frequency and wave number of the imposed forcing. In real nature, the dynamics of motions in a bay are influenced both by the wind fields acting on the bay as well as by the water level forcing imposed at its mouth by the larger water body. From this simple example presented here, it appears that it may be necessary to take into account the variations of water level across the mouth of the bay imposed by the external water body (in addition to the wind) to explain the observed dynamics. The development of anomalous negative co-oscillating amphidromic systems in some marginal seas in the northern hemisphere and positive ones in the southern hemisphere, as discussed by Murty and El-Sabh (1989), may also be related to the propagating nature of the forcing at the mouths of these bays and marginal seas in addition to the influence of friction and topography.

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