U.S. DEPARTMENT OF COMMERCE
NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION
OCEAN PRODUCTS CENTER

TECHICAL NOTE

FORECASTING WAVE CONDITIONS AFFECTED BY CURRENTS AND BOTTOM TOPOGRAPHY

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JANUARY 1987

OPC Contribution No. 10

OPC CONTRIBUTIONS

No. 1. Burroughs, L. D., 1986: Development of Forecast Guidance for Santa Ana Conditions. National Weather Digest. (in press).

No. 2. Richardson, W. S., D. J. Schwab, Y. Y. Chao, and D. M. Wright, 1986: Lake Erie Wave Height Forecasts Generated by Empirical and Dynamical Methods -- Comparison and Verification. Ocean Products Center Technical Note, 23pp.

No. 3. Auer, S. J., 1986. Determination of Errors in LFM Forecasts of Surface Lows Over the Northwest Atlantic Ocean. Ocean Products Center Technical Note/NMC Office Note No. 313, 17pp.

No. 4. Rao, D. B., S. D. Steinrod, and B. V. Sanchez, 1986: A Method of Calculating the Total Flow from a Given Sea Surface Topography. NASA Technical Memorandum. (in press).

No. 5. Feit, D. M., 1986 Compendium of Marine Meteorological and Oceanographic Products Center. NOAA Technical Memorandum NWS NMC 68, 98pp.

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No. 9. Auer, S. J., 1986: A 5-Year Climatological Survey of the Gulf Stream and Its Associated Ring Movements. Journal of Geophysical Research. (Submitted).

No. 10. Chao, Y. Y., 1987: Forecasting Wave Conditions Affected by Currents and Bottom Topography. Ocean Products Center Technical Note, 11pp.

No. 11. Esteva, D. C., 1987: The Editing and Averaging of Altimeter Wave and Wind Data. Ocean Products Center Technical Note, 4pp.

1. Introduction

A marked change in the characteristics of wind-generated surface waves occurs due to the presence of prevailing surface current and major bottom topographic features. A well known region characterized by intense and dramatic interactions of waves, currents and shoals is the Columbia River entrance on the west coast of the United States. It is recognized as one of the most dangerous coastal inlets in the world, where hundreds of search and rescue missions are conducted yearly by the Coast Guard and tragic losses of life continue to occur.

The National Oceanic and Atmospheric Administration (NOAA) is responsible for forecasting sea state conditions at the Columbia River entrance, as well as a number of other potentially dangerous sites on the United States coasts. In order to assist marine forecasters in these efforts, Gonzalez (1984) developed a one-dimensional wave-current-bathymetry interaction model under the assumptions of: monochromatic waves, straight depth contours parallel to the shoreline, and unidirectional tidal currents without lateral shear. The results of this simple model compare reasonably well with a limited set of field measurements but a major problem remains. For large incident wave angles there is a significant discrepancy between predicted values and measured data. This discrepancy may result from ignoring the two-dimensionality of the actual current and depth fields. Near a typical coastal inlet, the bottom configuration and current conditions are generally characterized by the presence of submarine shoals and tidal jet. This highly irregular bottom and nonuniform currents would cause significant difference in the wave condition over a relatively small coastal area. Thus, it is not always adequate to approximate the coastal region with parallel bottom contours and one-dimensional currents. It is also an accepted opinion that spectral consideration of ocean surface waves provides more realistic, and perhaps more accurate description of wave conditions than a monochromatic wave assumption.

Aiming toward providing forecast guidance for various coastal regions whose bottom bathymetry and current conditions vary in a general manner, a two-dimensional numerical model was developed. This paper outlines the model and presents some comparison of numerical results with analytical solutions and field data.

2. The Model

The theory of wave refraction by current and bathymetry has been comprehensively presented by Phillips (1977). The theory applies the kinematic and dynamic conservation laws, and the dispersion relation. The dynamic conservation law can be expressed in terms of an energy balance equation involving the radiation stress or a wave action conservation equation. The conservation equation of wave action is certainly preferable given its greater computational simplicity. Two numerical methods are commonly used. One is the ray tracing method and the other the finite difference method. As with bathymetric refraction, when only a few forecast points are of interest, it is computationally more feasible to used the 'backward' ray tracing method than the 'forward' ray tracing method or a finite difference scheme. In this model, we assume this is the case and thus the backward ray tracing approach is followed.

The model evaluates the refracted spectrum at the point of interest at a specific time step as given by:

$$E_{ij}(\theta_m,\omega_n,t_0) = \sum_{\ell} A_\ell^2\, E_{i+q,j+r}(\theta_{m+\ell},\omega_n,t_0-\Delta t) \tag{1}$$

where, E_{ij} is the spectral component of a band of directions $\Delta\theta$ and frequencies $\Delta\omega$ centered at θ_m and ω_n for the forecast point of the grid coordinates i,j at the time t_0, and $E_{i+q,j+r}$ is the corresponding incident spectral component. The amplification factor A_ℓ is calculated as:

$$A_\ell^2 = \frac{(C_g + u\cos\theta_{m+\ell} + v\sin\theta_{m+\ell})_{i+q,j+r}}{(C_g + u\cos\theta_m + v\sin\theta_m)_{i,j}}\;\frac{(\sigma k\,\delta\theta_m)_{ij}}{(\sigma k\,\delta\theta_{m+\ell})_{i+q,j+r}} \tag{2}$$

where u and v are current velocity components, σ is the intrinsic wave frequency, k is the wave-number and C_g is the wave group velocity. These variables are connected through the dispersion relation

$$\omega = \sigma + k(u\cos\theta + v\sin\theta) \tag{3}$$

$$\sigma^2 = gk\tanh kh \tag{4}$$

$$C_g = \frac{1}{2}\frac{\sigma}{k}\left(1 + \frac{2kh}{\sinh 2kh}\right) \tag{5}$$

where g is the gravitational acceleration and h is the water depth. The change of local wave direction θ along the ray, on a system of orthogonal Cartesian coordinates, is given by

$$\frac{d\theta}{dt} = \frac{1}{k}\frac{\partial\sigma}{\partial h}\left(\sin\theta\frac{\partial h}{\partial x} - \cos\theta\frac{\partial h}{\partial y}\right) + \cos\theta\left(\sin\theta\frac{\partial u}{\partial x} - \cos\theta\frac{\partial u}{\partial y}\right) + \sin\theta\left(\sin\theta\frac{\partial v}{\partial x} - \cos\theta\frac{\partial v}{\partial y}\right) \tag{6}$$

with

$$\frac{\partial\sigma}{\partial h} = \frac{gk^2}{2\sigma} - \frac{\sigma^3}{2g} \tag{7}$$

It is assumed that the current velocity field as a function of time can be specified from field data or from a numerical model. In performing backward ray tracing, the current directions must be reversed prior to calculations.

For a given frequency, wave rays are calculated for a number of initial directions, θ_m, from the forecast point until the rays reach the source wave region at a time step Δt where the corresponding wave directions are found to be θ_{m+l}. The source wave region is characterized by a constant depth or deep water and by a uniform current or null current. The slope of this $\theta_m \sim \theta_{m+l}$ curve is the value of $(\delta\theta_m)_{ij}/(\delta\theta_{m+l})_{i+q,j+r}$ given in the right-hand side of (2). This approach has been used by Dorrestein (1960) to evaluate refraction coefficients for bathymetric refraction problems.

Calculations of the amplification factors for a simplified case were made to examine the consistency of the model and to compare the results with Gonzalez (1984) analytical solutions. The following was assumed: 1) water depth of 15 meters at the forecast point increased linearly to a depth of 60 meters at the origin of the incident waves, 2) the flow direction was perpendicular to the bathymetric contours and 3) the speeds were varied linearly from a given value at the forecast point to zero at 60 meters depth. Figure 1 shows examples of refraction parameters variability for various frequencies associated with current speeds u = 0, 3 and -3 m/sec. The negative sign indicates that the flow direction is from the forecast point toward offshore while the positive values indicate the current are from offshore to the forecast station. The ordinate labeled AP is the directions at the forecast point. The abscissa denoted as AI, KA, and KT represent, respectively, the incident wave directions at 60 meter depth, the slopes of direction curves and the amplification factors. All curves in this figure are symetric relative to AP=90 degree since it is in the direction perpendicular (parallel) to the bottom contours (current direction). Figure 2 shows examples comparing numerical and analytical results of the amplification factor for wave incidence angle of 60 degree with respect to the flow direction. For the case where the incidence angle is parallel to the flow direction, the numerical solutions are almost identical with the analytical results. For the case of large incidence angle as shown in Figure 2, the numerical results tend to deviate from the analytical solutions, particularly for strong opposing currents. The numerical results presented are calculated based on an angular increment of 7.5 degrees. Accuracy may be improved if a smaller angular increament is used.

Usually, the end point of a ray in the source region will be away from grid points on which source wave information is provided. An interpolation scheme is used to obtain required spectral information at the ray point. Furthermore, wave rays constructed from a forecast station for a given direction band usually will not stay in the same direction band in the source region and may extend to several direction bands. Thus, the spectrum of each band involved must be calculated to reflect the fact that these spectral components contribute to the spectrum of a specific band at the forecast station.

Finally, as can be seen from Eqs. (1) and (2), the simple calculation of refracted wave spectrum can produce unrealistic results. The amplification factor becomes infinite both when $(\delta\theta_{m+l})_{i+q,j+r} \rightarrow 0$ and when $(C_g + U\cos\theta_m + V\sin\theta)_{ij} \rightarrow 0$. In the first case, it means that the forecast point is lying on a 'caustic' line for waves coming from the direction θ_{m+l} . Waves of directions within a band $\Delta\theta = 7.5$ degree, say, at the forecast station are produced by deep water waves of incident angles confined in a band $\Delta\theta \to 0$. For a deep water wave direction within this narrow band, there are more than one corresponding wave directions at the forecast station. In the neighborhood of such a caustic zone, the ordinary approximations of ray method do not apply; a higher order theory involving Airy functions must be used (see e.g., Chao and Pierson, 1972). Dorrestein's (1960) method of averaging over a range of incidence angles involved in producing caustics at the forecast station provided a simpler means of estimating $(\delta\theta_m)_{ij}/(\delta\theta_{m+l})_{i+q,j+r}$. In the second case, the wave group velocity is closely matched by an opposite current velocity. For such conditions, the wavelength decreases and the wave may become so steep that it may break. Thus, the calculated spectrum must be checked to ensure the spectrum is realistic. In this model, a limiting frequency spectrum associated with a given current velocity and depth, is specified to serve as the wave breaking criterion. For those portions of the refracted spectrum which are greater than the corresponding limiting spectrum, it is assumed that the wave has broken and these portions are removed from the final spectrum. The limiting spectrum is determined in two steps. First, it is assumed that the Wallops spectrum for finite depth (Huang et al., 1983) will be a maximum at the significant slope $s = 0.505 \tanh kh$. Next, the Wallop spectrum so determined is modified by a current velocity in the manner described by Huang (1972) to obtain a current-depth limited spectrum.

3. Model Verification

Simultaneous measurement of wave and current condition, are difficult to make in areas of significant current and very little good data are available. The Pacific Marine Environmental Laboratory/NOAA has made considerable efforts to measure and collect these types of data in the vicinity of the Columbia River entrance. A field observation program was carried out during the period 10-13 September 1981. Results were presented in a comprehensive report (Gonzalez et al., 1984). In addition, the NOAA Data Buoy Center has maintained two nearby data buoys to measure offshore wave conditions routinely. Buoy 46010 is located 10 km southwest of the entrance in 60 meter of water. Buoy 46005 is about 500 km due west of the entrance. These data were used for the model verification purpose.

Three types of basic input data are required to run the model. They are offshore waves, nearshore currents and bathymetric data. From the current and bathymetric data, refraction parameters were calculated as functions of the wave frequency, the wave direction and current speed at the forecast station. The required refraction parameters include the amplification factor, the location and direction of the ray point in the source region as well as the time of wave propagation from the source to the river entrance.

The bathymetric data used for the model were digitized by the National Geophysical Data Center at 5' of angle intervals and covers an ocean region from 42° to 48° N. latitude and 124° to 126° W longitude. The location where wave conditions were predicted is at the middle of the river mouth (46° 15'N, 124° 5'W). Tidal current time series were constructed from tables published by the NOAA/National Ocean Services along with major harmonic constants for Station 695 which is about 5 km upstream from the entrance. The estimated current velocity was corrected according to a regression curve which relates the estimated speed at Station 695 to the drifter speed measured at the entrance. Insufficient data were available to specify two-dimensional variation of the current field in the region of interest. Thus, a linear decrease of flow speed from the entrance grid to a minimum at neighboring offshore grids was assumed. Figure 3 shows a sample of the calculated amplification factors. The figure is comparable to Figure 1 since both have the same frequencies and current speeds as parameters.

The wave conditions at the entrance were predicted for the following cases:

	Wave Source	Bathymetry	Wave Computed
a)	NDBC 46005	two-dimensional	spectral waves
b)	NDBC 46010	linear slope	spectral waves
c)	NDBC 46010	linear slope	significant waves

In order to calculate the refracted directional spectrum at the entrance, a cosine square angular spreading function is applied to the buoy frequency spectrum using the buoy wind direction as the mean wave direction.

The resulting significant wave heights for these three cases are compared with the observed data as shown in Figure 4. Three additional 'observed' data are given in the report cited. These data were extrapolated from upstream observations and adjusted by a breaking wave criterion for singular waves; therefore, they are questionable and are not included in the comparison. Due to sparsness of data points, it is difficult to draw a definite conclusion. Either approach seems capable of providing a reasonable prediction. However, by comparing case (b) and case (c), it seems that the significant wave approach consistently overpredicts the wave height at the entrance. And by comparing case (a) and case (b), it appears that the two-dimensional model produces results comparable to the simple linear slope consideration, even though the source waves for the two-dimensional case are further away from the river entrance.

4. Concluding Remarks

A two-dimensional wave refraction model has been developed to forecast wave conditions for a coastal location where waves coming from offshore are affected by local current and bathymetry conditions. The model uses realistic depth and current fields and therefore it can be applied to any coastal region of interest if the required input data, i.e. bathymetry and current information, as well as deep water wave spectra are prescribed.

Calculations of refracted waves were made for the Columbia River entrance to test adequacy and applicability of the model for practical forecasts. Tests of the model were done by comparing model outputs with field data and the results obtained from Gonzalez's (1984) analytical model. The analytical model is presently used as a forecast guide by the regional marine forecasters. In comparison with the analytical model, the proposed model produces the same result if identical idealized depth and current conditions are assumed. The model outputs were also generated using 5 minute depth grids and spectral wave data from the buoy NDBC 46005 during September 10-13, 1981. These forecasts are found to be consistent with field measurements available during that period.

More validation studies of the model are needed to determine if the two-dimensional effects incorporated in the model would provide improved guidance for routine forecasting of wave condtions at the Columbia River entrance. Lacking adequate field data to asses the two-dimensional effects, we have initiated a program to carry out a systematic comparison of the forecasts from the operational one-dimensional model and the present model on an experimental basis. Wave forecasts at the Columbia River entrance are being produced once a day at three hour intervals up to 72 hours taking into account time dependence of tidal currents. Required source wave data are obtained from the NOAA operational global spectral wave forecast model (Chin, 1985) at positions 45°N, 125°W and 47.5°N, 125°W. The model results are placed on OCNDAT since October 1986 and are being compared with the actual forecasts issued by Seattle WSFO.

It should be mentioned that the present model can only provide results for swell conditions propagating to the forecast point from deep water. Further extension of the model to incorporate local wind effect can, of course, be made. However, it is desirable to make as realistic a validation of the forecasts as possible with data that can be obtained from existing resources before embarking on further theoretical developments.

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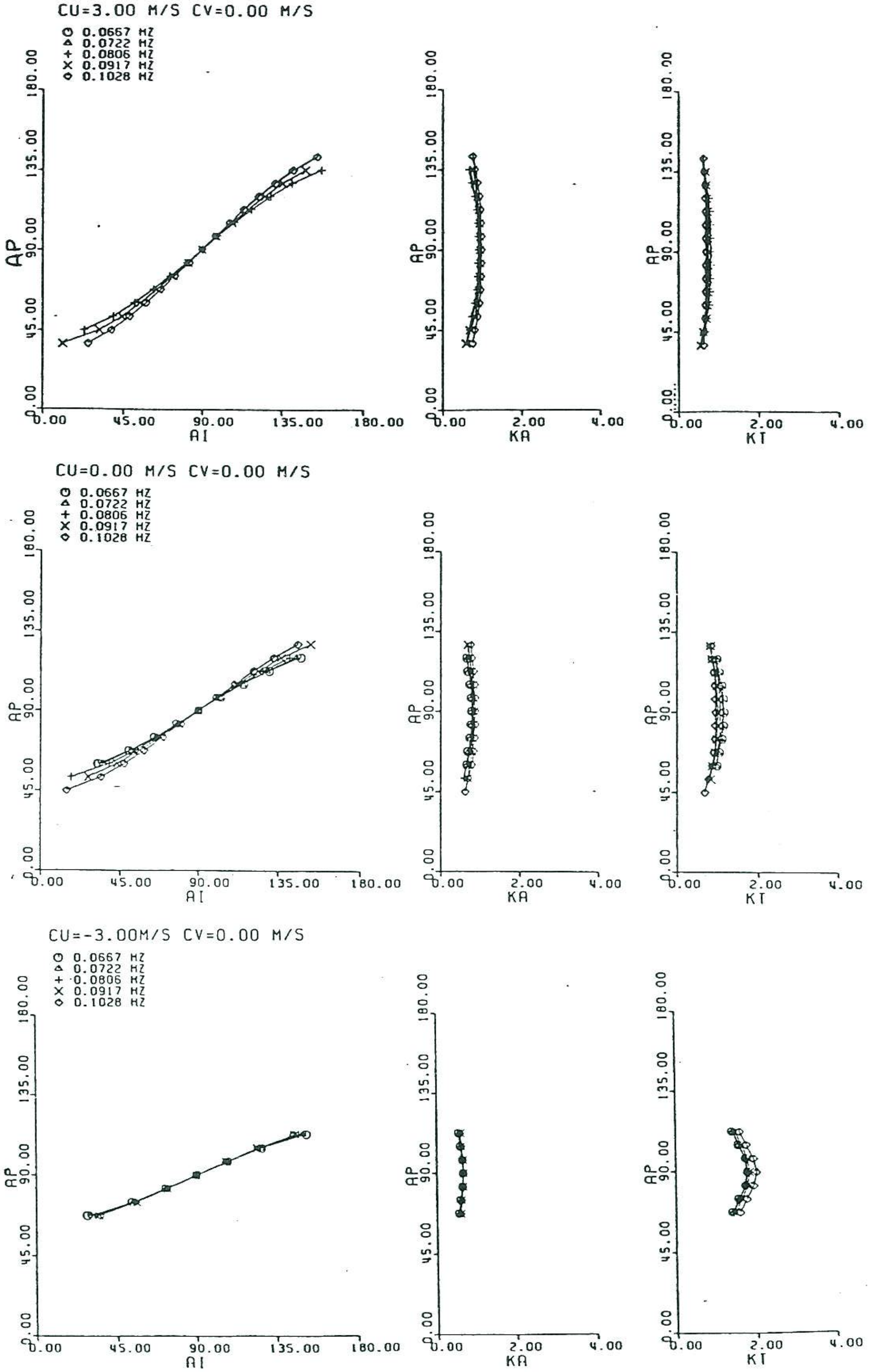

Figure 1. Examples of calculated refraction parameters as functions of the wave direction (AP), wave frequency and current velocity at the forecast point for the case of linear sloping bottom, where AI=wave direction in the source wave region, KA= slope of the AP-AI curve and KT=amplification factor.

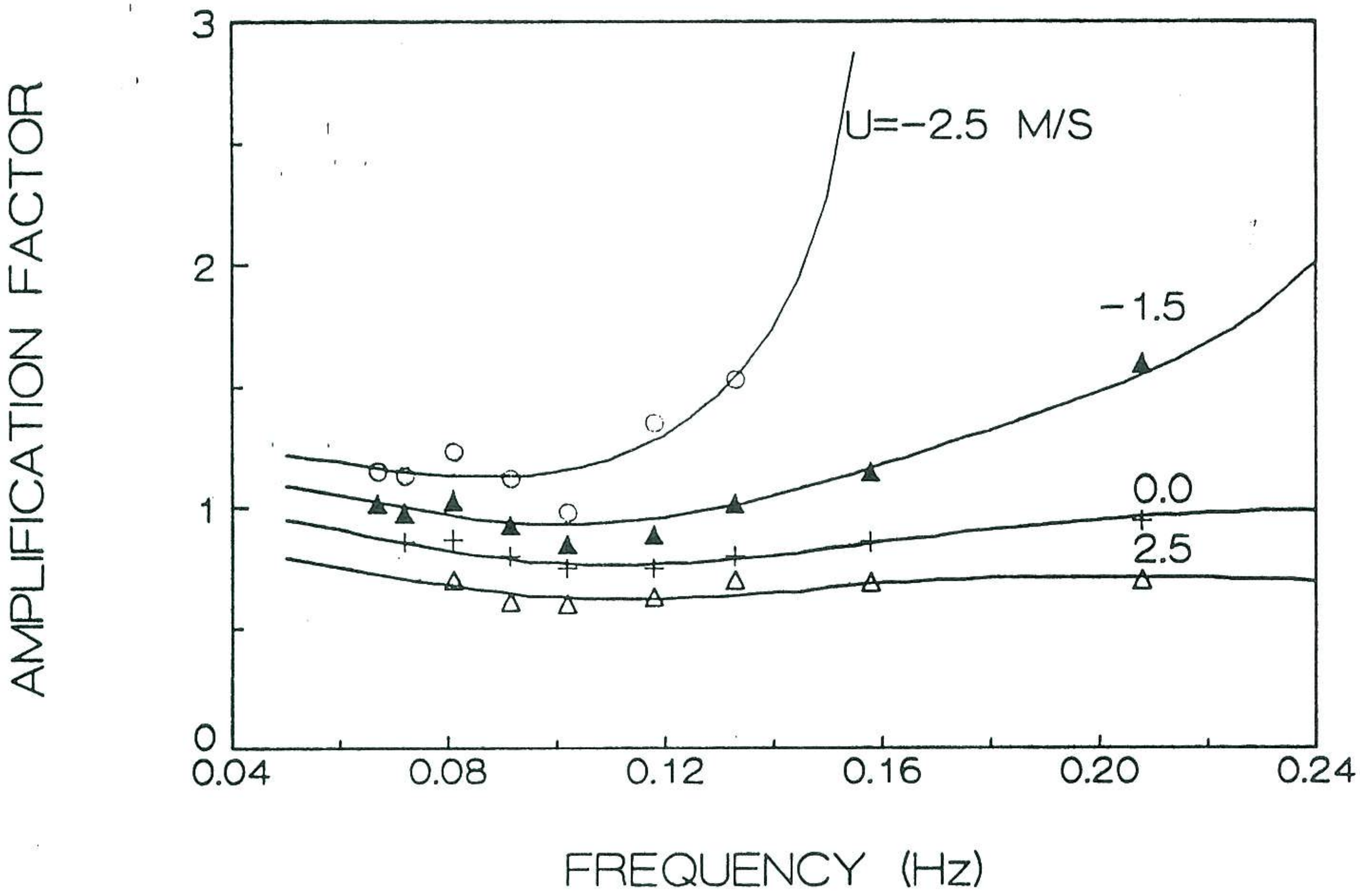

Figure 2. Comparisons of the analytical solutions (solid lines) and the numerical solutions (symbols) of the amplification factor for a linear sloping bottom. The incident wave angle is 60° from the flow direction.

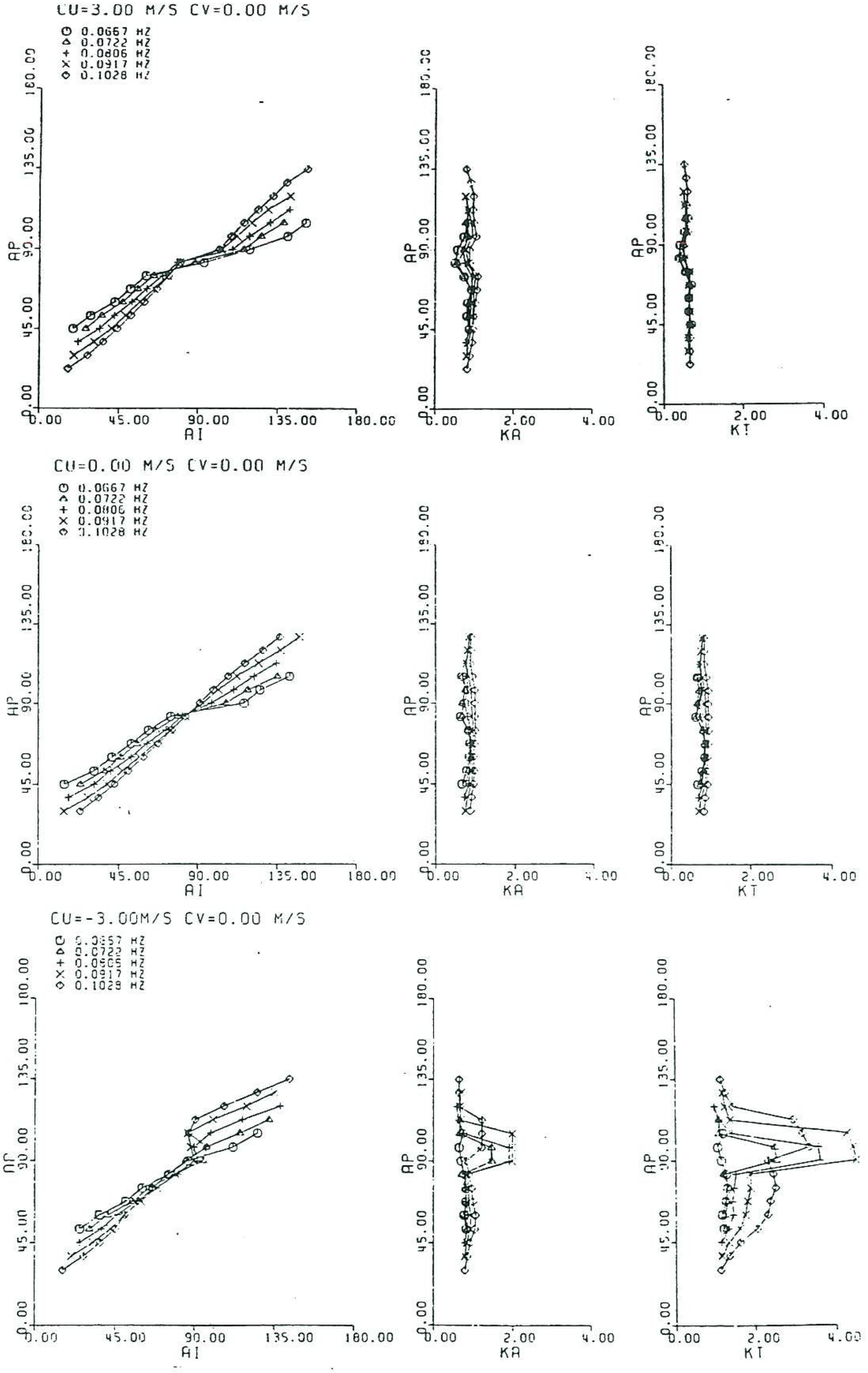

Figure 3. Same as Figure 1 except for the bottom bathymetry of the Columbia River entrance region.

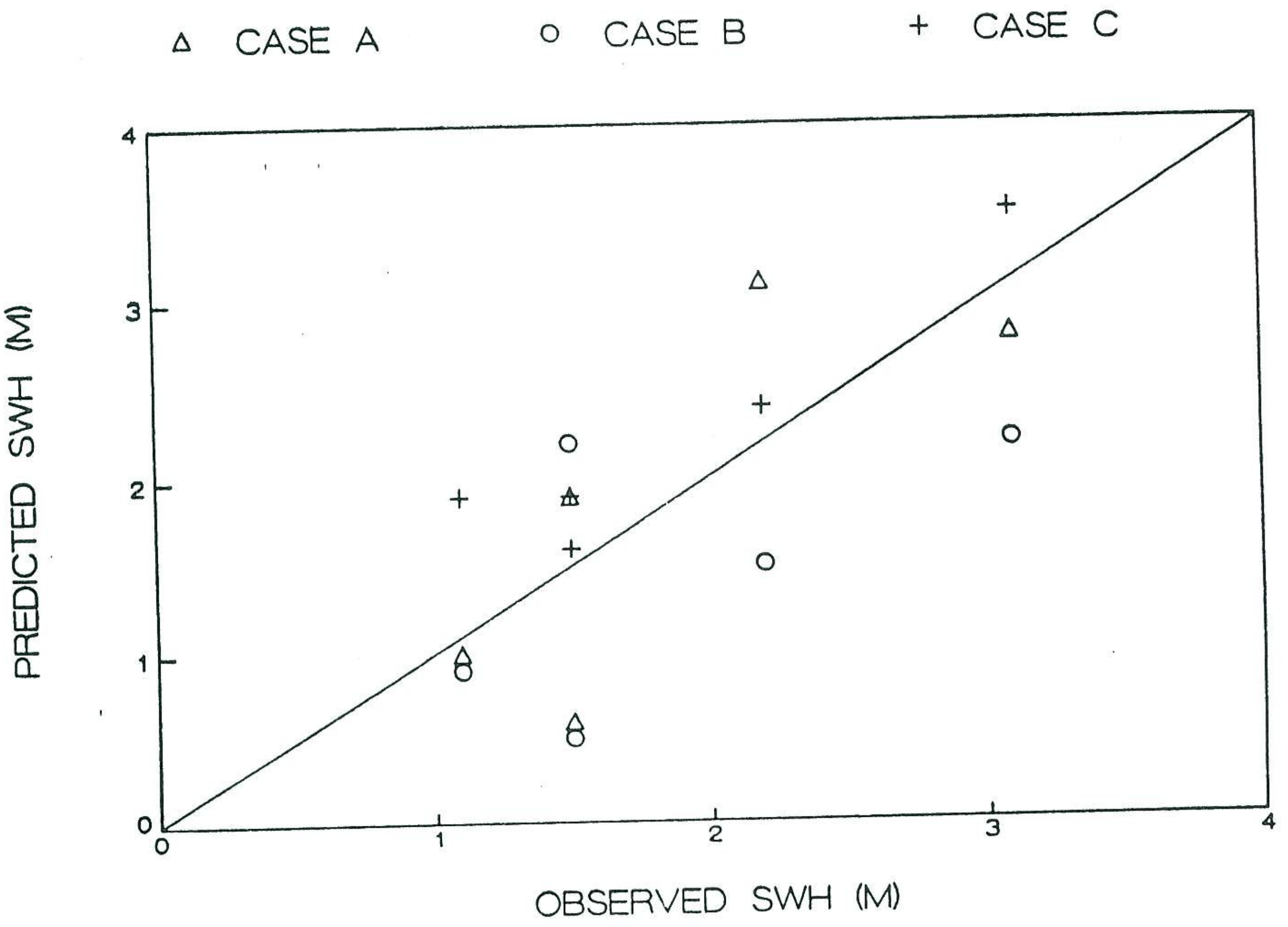

Figure 4. Comparisons of predicted and observed significant wave heights. See text for explanations of tested cases.