Exact Expression and Accurate Approximations for the Dependence of the Radii of Solution Drops on Relative Humidity

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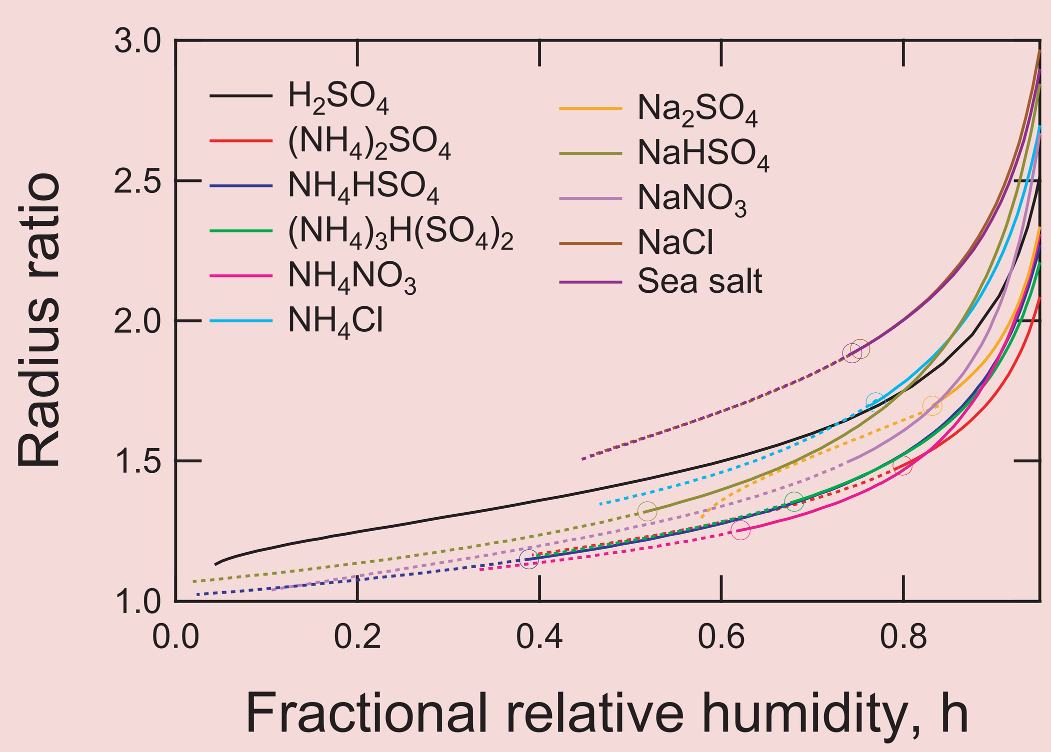
Introduction

The radius of an aqueous aerosol particle is a key property that determines its dynamics and light-scattering ability. Its dependence on relative humidity RH is of fundamental interest in quantifying and understanding aerosol properties in the atmosphere.

An exact expression and simple but accurate approximations for the radius ratio (relative to the volume-equivalent dry radius) of aqueous solution drops as a function of RH are presented for inorganic solutes of atmospheric interest.



Solid lines denote bulk solution data.
Open circles denote saturated solution.
Dotted lines denote measurements from suspended drops (mainly from Tang et al.) that are supersaturated with respect to the solute.

Exact Expressions

$$\frac{r}{r_{\text{dry}}} = \left\{ \left(\frac{\nu \rho_{\text{dry}} M_{\text{w}}}{\rho_{\text{w}} M_{\text{s}}} \right) \left[\frac{(1-h)\phi(h)}{(-\ln h)} \right] \frac{1}{(1-h)} + \left[\frac{\rho_{\text{dry}} V_{\phi}(h)}{M_{\text{s}}} \right] \right\}^{1/3}$$

where red quantities depend on h:

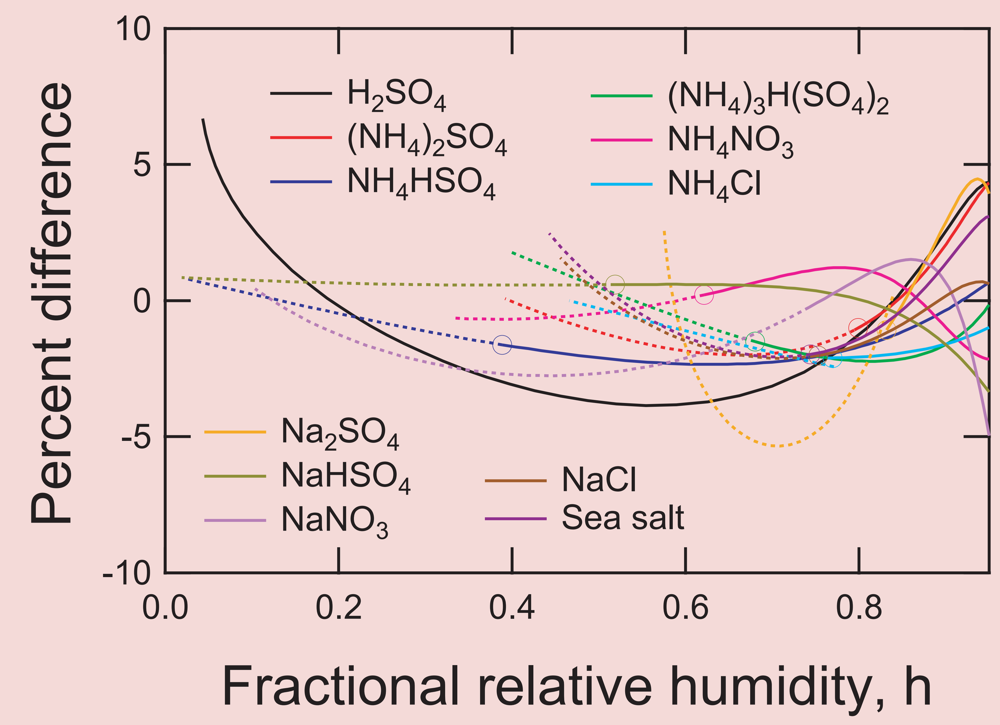
1) ϕ, related to molality m by $h = \exp(-\nu m M_{\text{w}} \phi)$

2) V_ϕ, defined by $V_\phi = \frac{1}{m}\left(\frac{1 + m M_{\text{s}}}{\rho} - \frac{1}{\rho_{\text{w}}} \right)$.

Approximate Expressions

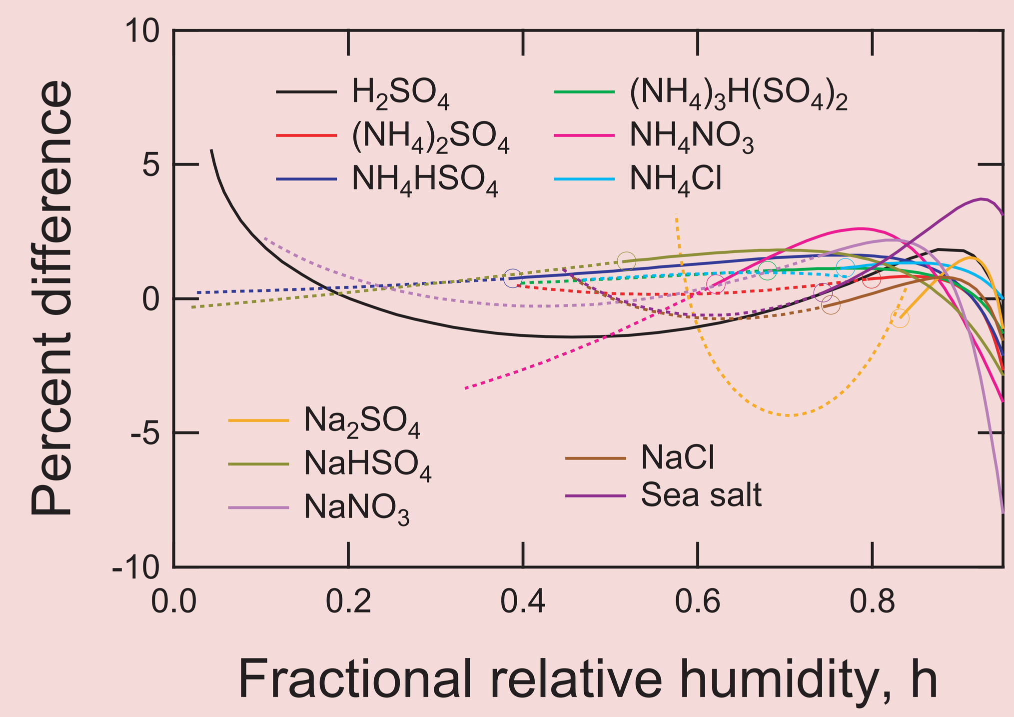
The exact expression suggests an expression of the form

$$\frac{r}{r_{\text{dry}}} = a\left[\frac{1}{(1-h)} + b \right]^{1/3}.$$



Another expression, purely empirical, is of the form

$$\frac{r}{r_{\text{dry}}} = \frac{q}{(1-h)^p}.$$



Both two-parameter expressions yield results that are accurate to within a few percent from the efflorescence RH to ~95% RH.

Table of Coefficients

Substance	a	b	q	p
Sulfuric acid	0.95	1.00	1.18	0.25
Ammonium sulfate	0.78	1.70	1.05	0.22
Ammonium bisulfate	0.83	0.90	1.02	0.26
Letovicite	0.79	1.70	1.03	0.25
Ammonium nitrate	0.82	0.95	0.96	0.28
Ammonium chloride	0.97	0.80	1.13	0.29
Sodium sulfate	0.88	1.00	1.06	0.26
Sodium bisulfate	1.01	0.20	1.06	0.32
Sodium nitrate	0.93	0.30	1.03	0.29
Sodium chloride	1.08	1.08	1.30	0.27
Sea salt	1.08	1.10	1.29	0.28

Discussion

1) The accuracy is likely to be sufficient for most purposes, but the expressions are simple enough for global models.

2) The nature of the behavior of the radius ratio with RH makes it difficult to improve the accuracy without many additional terms.

3) The first expression can be modified so that it yields results up to 100% relative humidity:

$$\frac{r}{r_{\text{dry}}} = a\left[\frac{1}{(1-h+\beta)} + b \right]^{1/3},$$

where β depends on a and r_{dry}.

4) The second expression can readily be used with a similar one for index of refraction to obtain simple expressions for dependence of light-scattering properties of particles and aerosols on RH.

Future Work

1) Similar approximations for index of refraction.

2) Organic solutes.

3) Mixing rules.