Physics with Polarized Beams

A tutorial for experimenters,

accelerator physicists, and students

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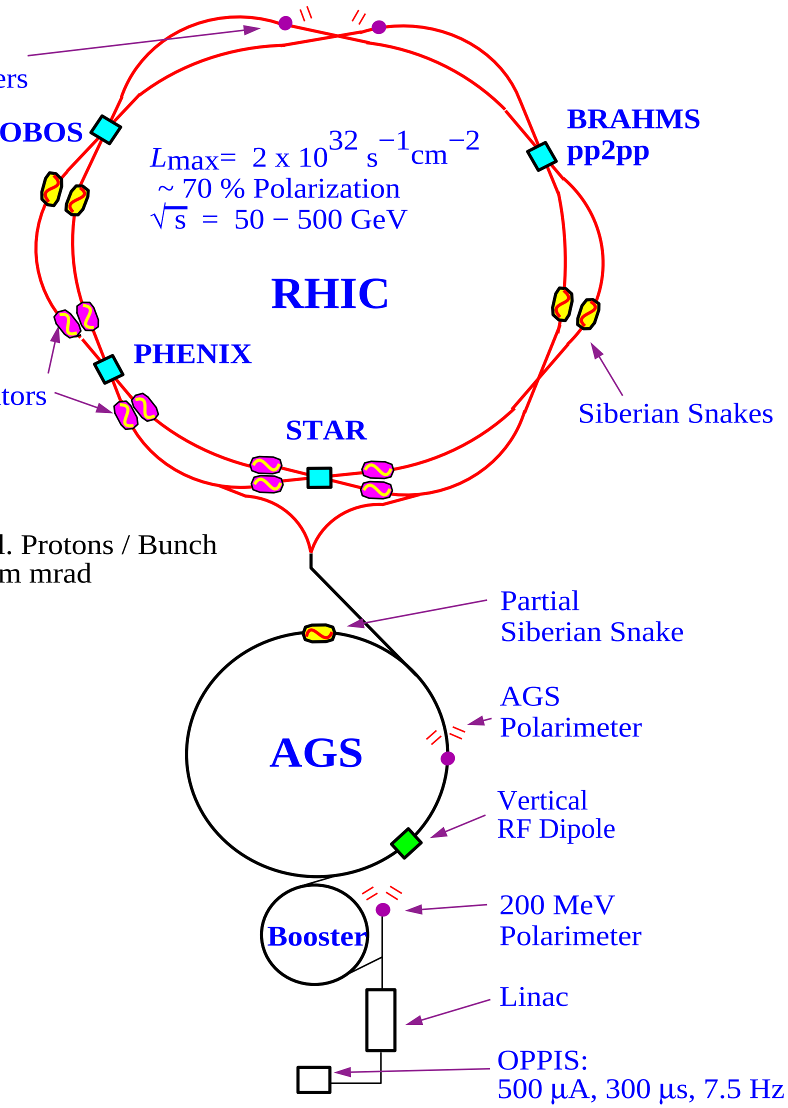
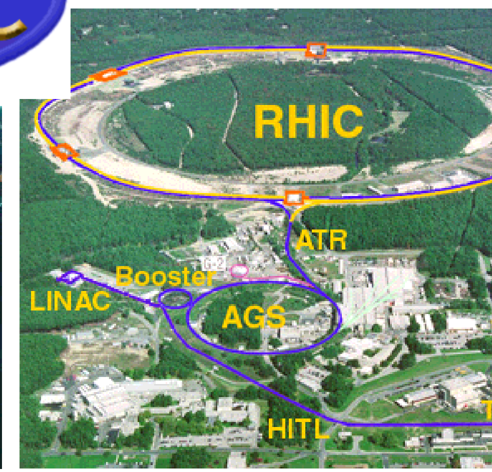
Polarized Proton Collisions at BNL






Courtesy Canadian Environmental Ministry

POLARized BEAms at RHIC

A guide to further reading :

- C. Bourrely, J. Soffer, E. Leader, *Polarization Phenomena in Hadronic Reactions*, Phys. Rept. **59** (1980) 95
- N. Craigie, K. Hidaka, M. Jacob, F. Renard, *Spin Physics at Short Distances*, Phys. Rept. **99** (1983) 69
- E. Leader, *Spin in Particle Physics*, Cambridge University Press 2001.
- G. Bunce, N. Saito, J. Soffer, W. Vogelsang, *Prospects for Spin Physics at RHIC*, Annu. Rev. Nucl. Part. Sci. **50** (2000) 525.
- A. Manohar, *An introduction to spin-dependent deep inelastic scattering*, hep-ph/9204208
- B.W. Filippone, Xiangdong Ji, *The spin structure of the nucleon*, hep-ph/0101224
- Hai-Yang Cheng, *Status of the Proton Spin Puzzle* Int. J. Mod. Phys. **A11** (1996) 5109 (hep-ph/9607254)

- M. Anselmino, A. Efremov, E. Leader, *The Theory and Phenomenology of Polarized Deep-Inelastic Scattering*, Phys. Rept. **261** (1995) 1 (hep-ph/9501369)

- B. Lampe, E. Reya, *Spin Physics and Polarized Structure Functions*, Phys. Rept. **332** (2000) 1 (hep-ph/9810270)

- V. Barone, A. Drago, P.G. Ratcliffe, *Transverse Polarization of Quarks in Hadrons*, Phys. Rept. **359** (2002) 1 (hep-ph/0104283)

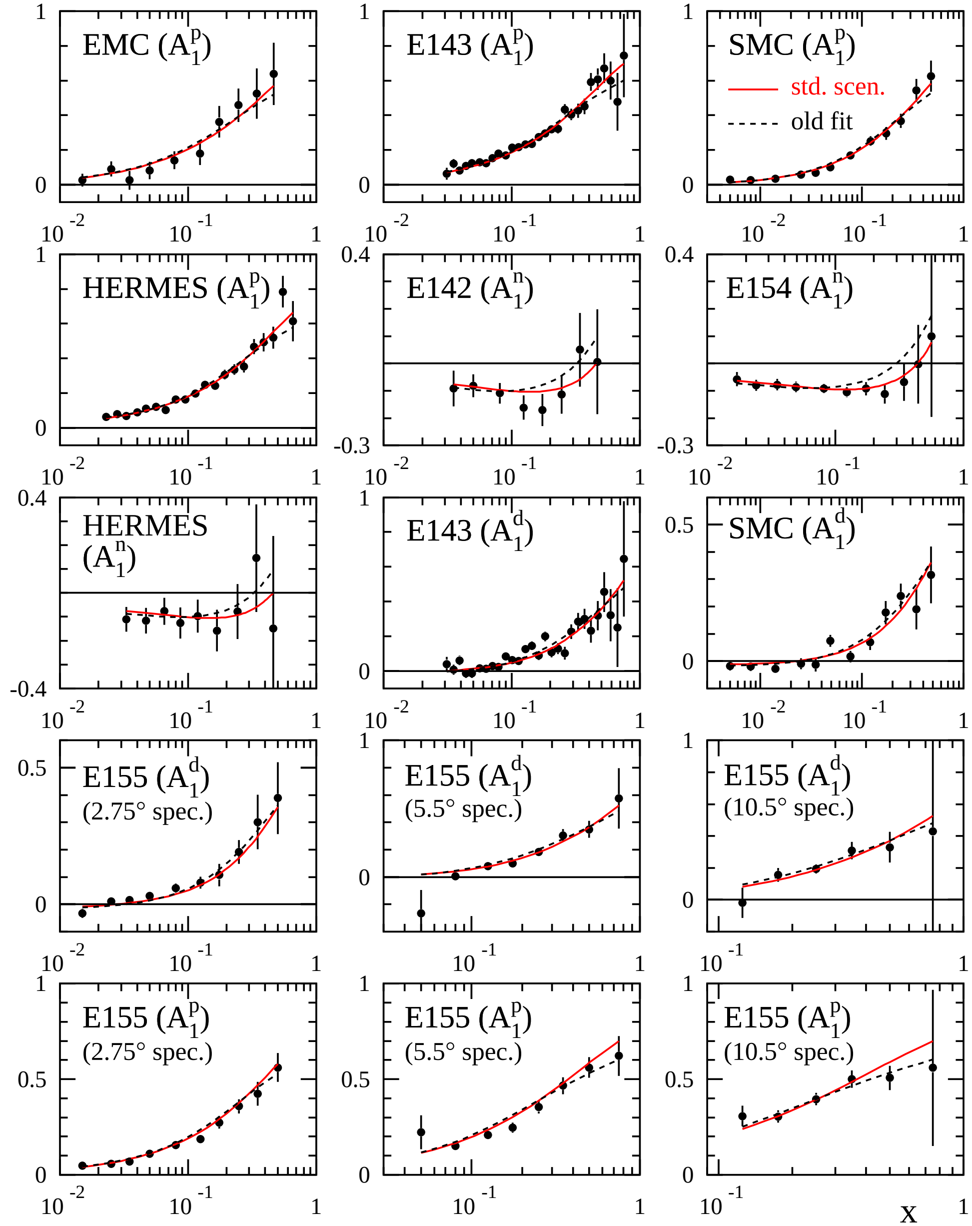
If you haven't been to Spin Caffé yet, look what you've been missing!

What's on the Menu ?

Some (of many possible) appetizers . . .

Several generations of beautiful fixed-target experiments at

SLAC, CERN, DESY



measure a spin asymmetry

in DIS :

$$A = \frac{\sigma_{++} - \sigma_{+-}}{\sigma_{++} + \sigma_{+-}}$$

- we'll see : they tell us about polarization of quarks in the nucleon
- What carries the nucleon spin ?
- simple quark model : 3 constituent quarks, $s_i = \frac{1}{2}$, $\mu_i = e_i \frac{e}{2m_i}$

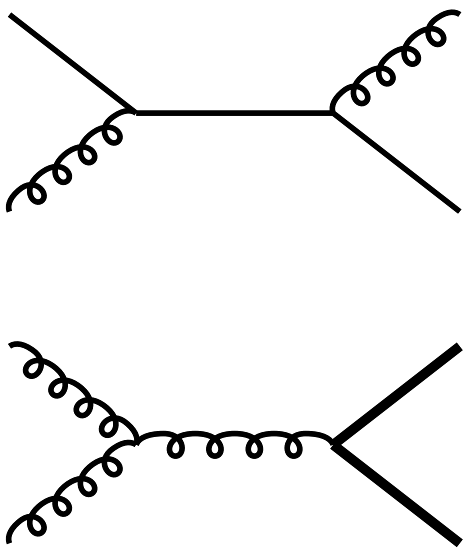
$$|P\rangle = \frac{1}{\sqrt{18}} \left[2u^{\uparrow}u^{\uparrow}d^{\downarrow} - u^{\uparrow}u^{\downarrow}d^{\uparrow} - u^{\downarrow}u^{\uparrow}d^{\uparrow} + \text{perm.} \right]$$

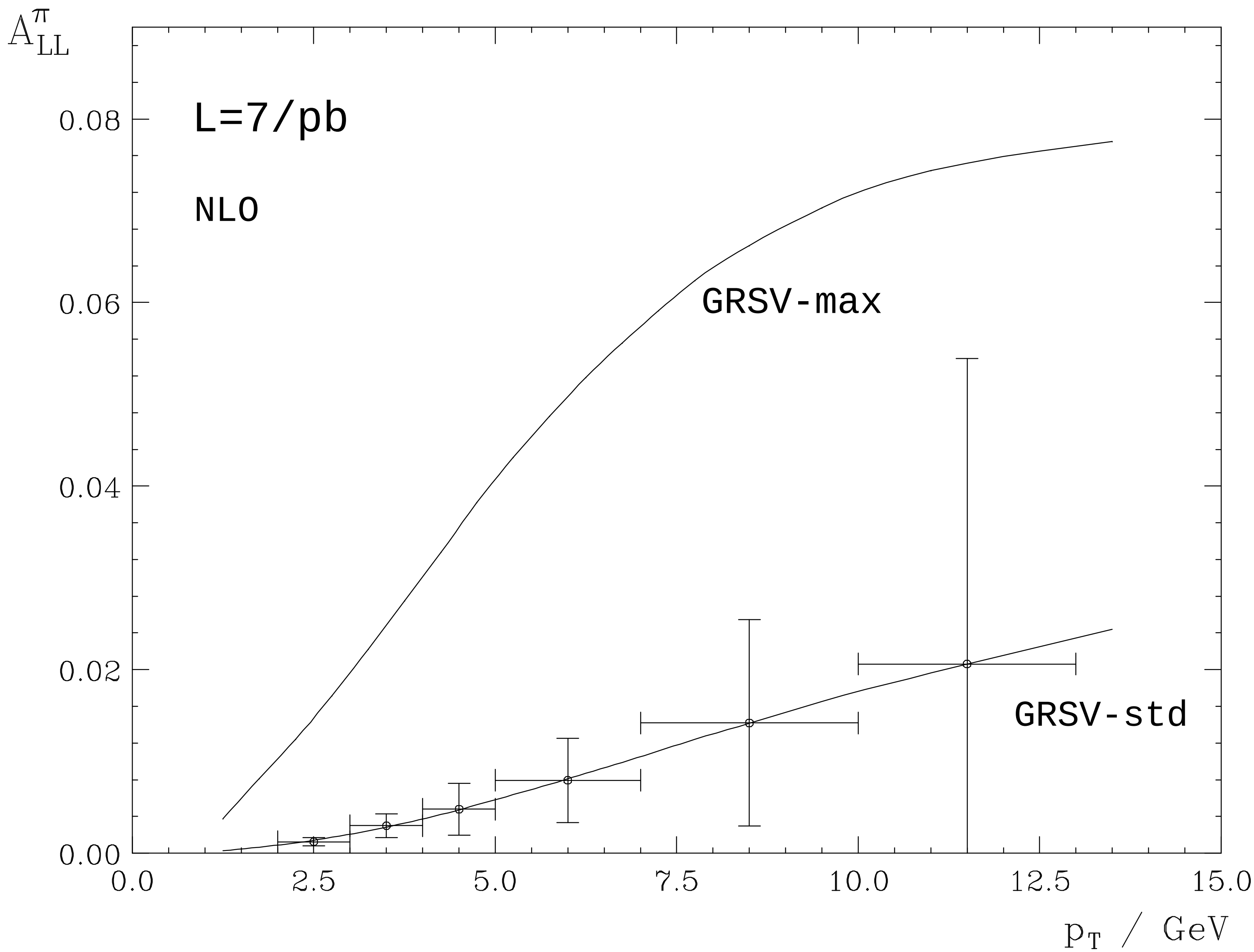
$$\Rightarrow \begin{cases} \mu_n/\mu_p = -2/3 \quad \text{vs.} \quad (\mu_n/\mu_p)_{\text{exp}} = -0.685 \qquad \checkmark \\ g_A = 5/3 \quad \text{vs.} \quad (g_A)_{\text{exp}} = 1.267 \pm 0.0035 \qquad \times \\ 2\,S_u = 4/3 \;\; , 2\,S_d = -1/3 \quad \rightsquigarrow \quad \boxed{S_q = S_u + S_d = \frac{1}{2}} \end{cases}$$

- nucleons have a rich internal structure: quarks, antiquarks and gluons
- relation between nucleon and parton spins ? How can we find out ?
- What do we learn about QCD ?

In coming run : $\vec{p}\vec{p} \to \pi^0 X$

PHENIX





$pp \longrightarrow \pi^0 X$ **by**

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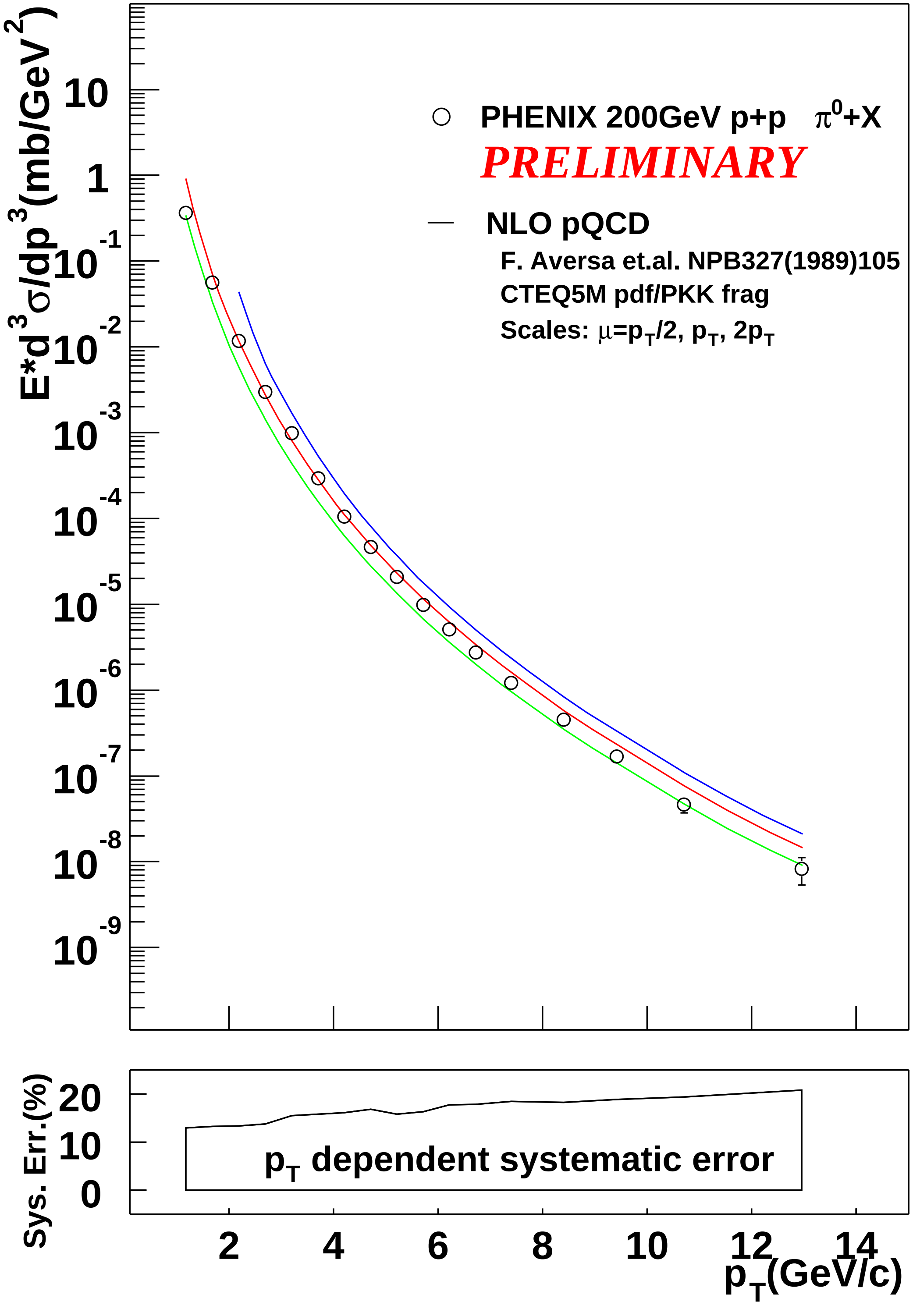
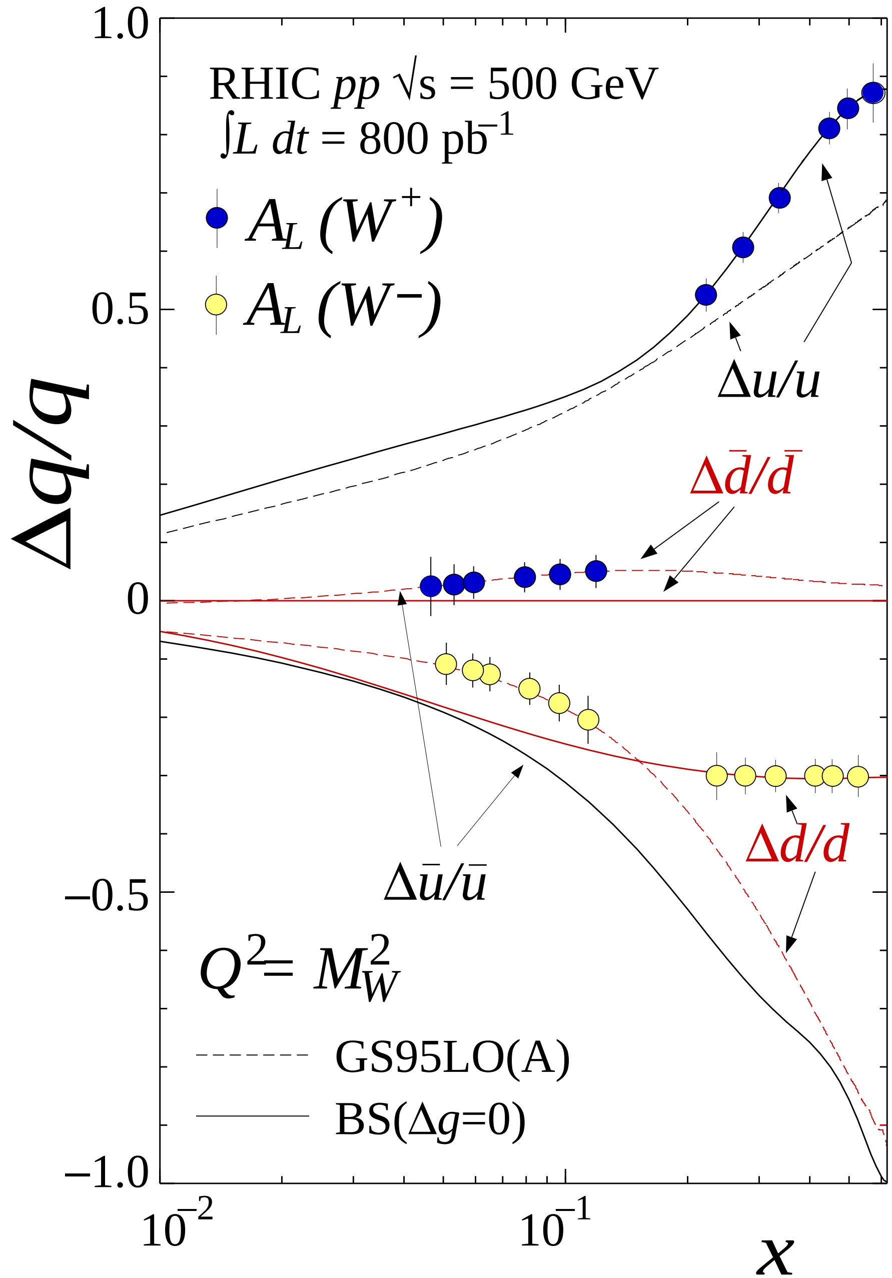
($\pm$30% normalization unc.)

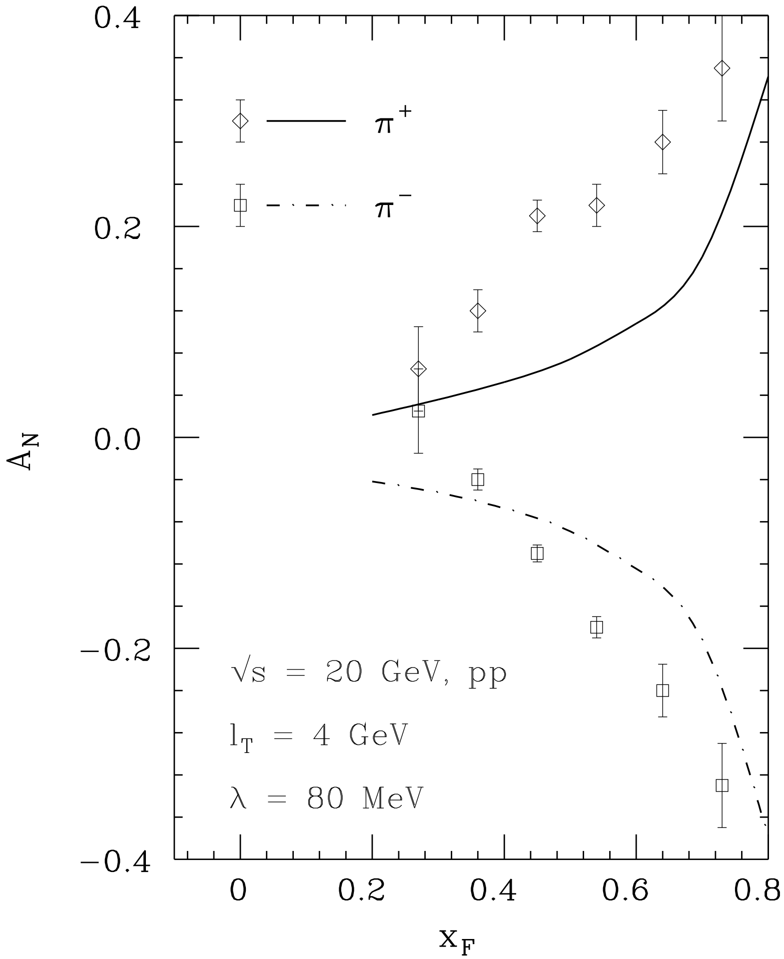


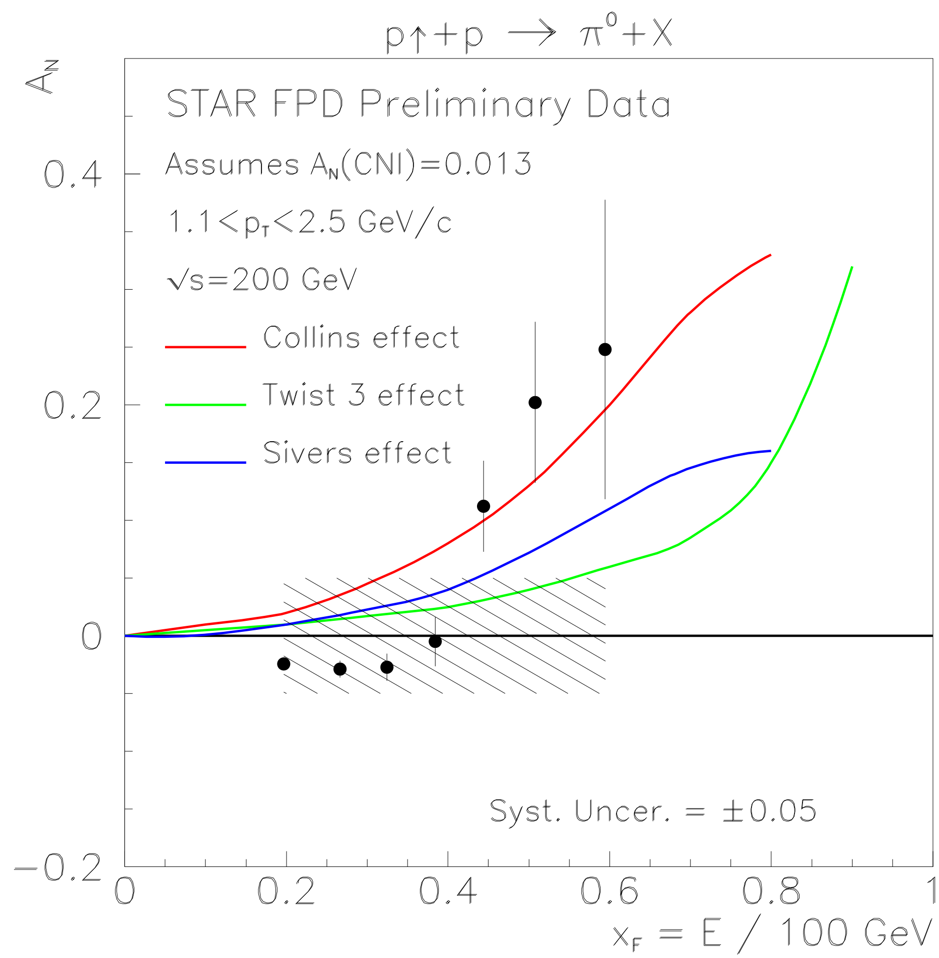
Parity violation :

$$A_{\mathrm{L}} \equiv \frac{\sigma_{\rightarrow} - \sigma_{\leftarrow}}{\sigma_{\rightarrow} + \sigma_{\leftarrow}}$$



Another example : $A_N \equiv \frac{\sigma_\uparrow - \sigma_\downarrow}{\sigma_\uparrow + \sigma_\downarrow}$





- we'll see : window to interesting QCD dynamics in nucleon structure

Today :

- Some Fundamentals About Spin
- Polarized Scattering Processes
- Lepton-Nucleon Scattering (Part I)

I. Some Fundamentals About Spin

1.1 Spin in non-relativistic quantum mechanics

- spin of particle introduced as additional angular momentum. Introduce three spin operators :

$$\vec{\mathcal{S}} = (\mathcal{S}_x , \mathcal{S}_y , \mathcal{S}_z)$$

angular momentum algebra

$$[\mathcal{S}_i , \mathcal{S}_j] = i\epsilon_{ijk}\mathcal{S}_k \qquad \epsilon_{123} = +1$$

- find

$$\left[\vec{\mathcal{S}}^2, \mathcal{S}_j\right] = 0$$

$\rightsquigarrow$ $\vec{\mathcal{S}}^2, \mathcal{S}_z$ have set of simultaneous eigenvectors;
use to label states $|S, m\rangle$

$$\vec{\mathcal{S}}^{\,2}\,|\,S,\,m\rangle \;=\; S(S+1)\,\hbar^2\,|\,S,\,m\rangle \qquad\qquad S \;=\; 0,\,\tfrac{1}{2},\,1,\,\tfrac{3}{2},\ldots$$

$$\mathcal{S}_z\,|\,S,\,m\rangle \;=\; m\,\hbar\,|\,S,\,m\rangle \qquad\qquad -S \,\le\, m \,\le\, S$$

- spin degree of freedom decoupled from kinematic d.o.f. :

$$\Psi_{\mathsf{Schr}}(\vec{r}) \;\longrightarrow\; \Psi_{\mathsf{Schr}}(\vec{r}) \;\times\; \chi_m$$

where χ_m is a $(2S+1)-$ component "spinor"

1.2 Example : Spin$-1/2$

- $2-$component spinors

$$\chi = \begin{pmatrix} a \\ b \end{pmatrix}$$

- operators may be represented by Pauli-matrices :

$$\mathcal{S}_x = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \mathcal{S}_y = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \mathcal{S}_z = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- eigenstates to $\vec{\mathcal{S}}^2$ and $\mathcal{S}_z$:

$$\chi_z^{\uparrow} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \chi_z^{\downarrow} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- eigenvalues

$$\mathcal{S}_z\, \chi_z^{\uparrow} = +\frac{1}{2}\chi_z^{\uparrow} \qquad \mathcal{S}_z\, \chi_z^{\downarrow} = -\frac{1}{2}\chi_z^{\downarrow}$$

particles in such states are "polarized in z-direction"

- general superposition

$$\chi = \begin{pmatrix} a \\ b \end{pmatrix} = a\,\chi_z^{\uparrow} + b\,\chi_z^{\downarrow} \qquad (\chi^\dagger \chi = 1)$$

$$\Rightarrow \langle \mathcal{S}_z \rangle = \chi^\dagger \mathcal{S}_z \chi = \left(+\frac{1}{2}\right)|a|^2 + \left(-\frac{1}{2}\right)|b|^2 = \frac{1}{2}\left[\,|a|^2 - |b|^2\,\right]$$

- Example : $a = b = 1/\sqrt{2}$

$$\chi = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \Rightarrow \langle \mathcal{S}_z \rangle = 0$$

- why ? Because this χ is eigenstate to $\mathcal{S}_x$ (with eigenvalue $+1/2$)
- in other words,

$$\chi_x^{\uparrow} = \frac{1}{\sqrt{2}}\left[\chi_z^{\uparrow} + \chi_z^{\downarrow}\right] \qquad \text{“polarized in } x\text{-dir.”}$$

- arbitrary direction $\vec{n}$ with $|\vec{n}| = 1$:

$$\vec{n} \cdot \vec{S} \; = \; n_x \mathcal{S}_x \; + \; n_y \mathcal{S}_y \; + \; n_z \mathcal{S}_z \; = \; \frac{1}{2} \begin{pmatrix} n_z & n_x - i n_y \\ n_x + i n_y & -n_z \end{pmatrix}$$

- state that is an eigenstate to this : "polarized in $\vec{n}$-direction"
  $\vec{n}$ – "polarization vector"

- eigenvalues $\pm 1/2$

- how to find these eigenstates : use projection operators

  note that

$$\left(\vec{n}\cdot\vec{S}\right)^2 = \frac{1}{4}\mathbb{1} = \frac{1}{4}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

  Therefore,

$$(\vec{n}\cdot\vec{S})\left[\left(\frac{\mathbb{1}}{2} + \vec{n}\cdot\vec{S}\right)\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right] = +\frac{1}{2}\left[\left(\frac{\mathbb{1}}{2} + \vec{n}\cdot\vec{S}\right)\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right]$$

  Hence,

$$\left(\frac{\mathbb{1}}{2} + \vec{n}\cdot\vec{S}\right)\begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \text{is an eigenstate}$$

  Projection operators are $\frac{\mathbb{1}}{2} \pm \vec{n}\cdot\vec{S}$

- physical significance : for the eigenstate with eigenvalue $+1/2$

$$\begin{aligned} 2\langle\mathcal{S}_x\rangle &= n_x = \mathcal{P}(\uparrow_x) - \mathcal{P}(\downarrow_x) \\ 2\langle\mathcal{S}_y\rangle &= n_y = \mathcal{P}(\uparrow_y) - \mathcal{P}(\downarrow_y) \\ 2\langle\mathcal{S}_z\rangle &= n_z = \mathcal{P}(\uparrow_z) - \mathcal{P}(\downarrow_z) \end{aligned}$$

- all this information contained in

$$\rho = \frac{1}{2}\begin{pmatrix} 1+n_z & n_x - in_y \\ n_x + in_y & 1-n_z \end{pmatrix}$$

- this is the "spin density matrix" for the state

- its importance : describes also the spin of a more complex system
  for example, an ensemble of particles that is not in a pure state

1.3 A little more on the density matrix . . .

- pure state $\leftrightarrow$ state vector – a superposition of eigenstates, say, to $\mathcal{S}_z$

take
$$|\Psi\rangle = \sum_{m=-S}^{S} c_m \, |S, m\rangle$$

- expectation value of any operator in this state :

$$\langle \mathcal{O} \rangle_\Psi = \sum_{m,m'} c^*_{m'} \, \langle S, m' \, |\mathcal{O}| \, S, m \rangle \, c_m = \sum_{m,m'} \underbrace{c_m \, c^*_{m'}}_{\rho_{m,m'}} \, \mathcal{O}_{m',m}$$

$$\equiv \mathrm{Tr} \left[\mathcal{O} \, \rho \right]$$

- $\rho \leftrightarrow$ state $\leftrightarrow$ expectation value for any operator

- for spin$-1/2$ and polarization in $\vec{n}$:
$$\rho = \frac{1}{2} \begin{pmatrix} 1 + n_z & n_x - i n_y \\ n_x + i n_y & 1 - n_z \end{pmatrix}$$

- non-pure state : state is one of statistical ensemble of possibilities
  $\rightarrow$ incoherent mixture of i pure states with probabilities $p^{(i)}$

- define

$$\rho_{mm'} = \sum_i p^{(i)} \, c_m^{(i)} \left(c_{m'}^{(i)} \right)^*$$

- mean value of operator over *ensemble*

$$\langle \, \mathcal{O} \, \rangle_{\text{ensemble}} = \sum_i p^{(i)} \sum_{m,m'} c_m^{(i)} \left(c_{m'}^{(i)} \right)^* \mathcal{O}_{m',m} = \mathrm{Tr} \left[\mathcal{O} \, \rho \right]$$

- for spin$-1/2$ find

$$\rho = \frac{1}{2} \begin{pmatrix} 1 + \eta_z & \eta_x - i\eta_y \\ \eta_x + i\eta_y & 1 - \eta_z \end{pmatrix}$$

where $\eta_i = \langle n_i \rangle = 2 \, \mathrm{Tr} \left[\mathcal{S}_i \, \rho \right]$

$$\vec{\eta}^{\,2} \leq 1$$

- we'll see that for nucleon $\eta_z \sim \Delta q(x)/q(x), \;\; \eta_{x,y} \sim \delta q(x)/q(x)$!

1.4 Spin in the relativistic theory

- physics invariant under Lorentz boosts, rotations, and translations in space and time
- $\rightarrow$ Poincaré group, has 10 generators :

$$\mathcal{P}^{\mu} \, , \quad \mathcal{M}^{\mu\nu}$$

- pure rotations $J_i = -\frac{1}{2}\epsilon_{ijk}\mathcal{M}^{jk}$ pure boosts $\mathcal{K}_i = \mathcal{M}^{i0}$
- find $[\, J_i \, , J_j \,] = i\,\epsilon_{ijk}\,J_k$ total angular momentum

Spin ?

- **two** group invariants ($\rightsquigarrow$ fundamental observables) :

$$\mathcal{P}_\mu \, \mathcal{P}^\mu = \mathcal{P}^2 = m^2$$

$$\mathcal{W}_\mu \, \mathcal{W}^\mu \qquad \text{where} \quad \mathcal{W}_\mu = -\frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \mathcal{M}^{\nu\rho} \mathcal{P}^\sigma \qquad \text{Pauli-Lubanski}$$

- $\mathcal{W}_\mu$ satisfies $\left[\mathcal{W}_\mu \, , \mathcal{W}_\nu \right] = i \, \epsilon_{\mu\nu\rho\sigma} \mathcal{W}^\rho \, \mathcal{P}^\sigma$
- if acting on states at rest : $\left[\mathcal{W}^i \, , \mathcal{W}^j \right] = i \, m \, \epsilon_{ijk} \mathcal{W}^k$ ✓
- then : identify $\mathcal{S}_i \equiv \frac{1}{m} \mathcal{W}^i = J_i$

  $\rightsquigarrow$ particle at rest with non-zero angular momentum !
- $\mathcal{W}_\mu \, \mathcal{W}^\mu$ has eigenvalues $m^2 \, S \, (S+1)$
- procedure : start with rest states and apply Lorentz transformations

1.5 **Example : back to spin$-1/2$!**

- relativistic case : free spin$-1/2$ particle obeys Dirac equation

$$(i\,\partial_\mu\,\gamma^\mu \;-\; m)\;\Psi(x)\;=\;0$$

- the γ^μ are the Dirac matrices. They are 4×4 matrices.

- $\rightsquigarrow$ solutions are $4-$component spinors

$$\Psi(x) = \begin{cases} \mathrm{e}^{-i\,p\cdot x}\;u(p) & \text{positive energy} \;\longrightarrow\; \text{particle} \\ \mathrm{e}^{+i\,p\cdot x}\;v(p) & \text{negative energy} \;\longrightarrow\; \textit{anti}\text{particle} \end{cases}$$

each with two solutions – “spin up/down”

- recall : previously constructed eigenstates to $\vec{\mathcal{S}}^2$ and $\vec{n} \cdot \vec{\mathcal{S}}$
- Dirac solutions eigenstates to

$$\mathcal{W}_\mu \mathcal{W}^\mu \, | p, S \rangle = m^2 S(S+1) \, | p, S \rangle \qquad S = \tfrac{1}{2}$$

$$-\frac{W \cdot n}{m} \, | p, S \rangle = \pm \frac{1}{2} \, | p, S \rangle$$

- $\Pi \equiv -\frac{W \cdot n}{m}$ "polarization operator"
- n "covariant polarization vector" $n^2 = -1\,, \quad n \cdot p = 0$
- find for particle solutions

$$\Pi = \frac{1}{2} \gamma_5 \not{n} \qquad (\not{n} = \gamma_\mu n^\mu)$$

- $\leadsto$ projection operators $\frac{1}{2} (\mathbb{1} \pm \gamma_5 \not{n})$

Two important cases :

(a) longitudinal polarization : $\vec{n} = \vec{p}/|\vec{p}|$

- find

$$\Pi = \frac{1}{2}\gamma_5 \not{n} = \frac{\vec{J}\cdot\vec{p}}{|\vec{p}|}$$

- eigenvalues $\pm 1/2$

$$\frac{\vec{J}\cdot\vec{p}}{|\vec{p}|}u_\pm(p) = \pm\frac{1}{2}u_\pm(p) \equiv \frac{\lambda}{2}u_\pm(p) \qquad \rightsquigarrow \lambda \text{ “helicity”}$$

- states labelled as $|\,\vec{p},\ \lambda\rangle$
- massless particles : $\dfrac{\vec{J}\cdot\vec{p}}{|\vec{p}|} \to \gamma_5$ helicity = chirality
- $m \to 0$: helicity becomes Lorentz-invariant

(b) transverse polarization : $n = (0, \vec{n}_\perp, 0)$ (for $\vec{p}$ in z direction)

- find after some work

$$\Pi = \gamma_0 \vec{J} \cdot \vec{n} = \gamma_0 J_\perp$$

- this is not the same as $J_\perp$!
- hence, "transversity" , not "transverse spin"
- eigenvalues $\pm 1/2$

$$\gamma_0 J_\perp u_{\uparrow\downarrow}(p) = \pm \frac{1}{2} u_{\uparrow\downarrow}(p)$$

- as in non-relativistic theory :

$$u_\uparrow^{(x)} = \frac{1}{\sqrt{2}} \left[u_+ + u_- \right]$$

- transverse polarization invariant under boosts along $\vec{p}$

Note :

- by "tuning n" can create general polarization with longitudinal and transverse components $s_{\parallel} \sim \lambda\,, \quad s_{\perp} \sim n_{\perp}$
- at high energies find :

$$n^{\mu} \;=\; s_{\parallel}\,\frac{p^{\mu}}{m} \;+\; s_{\perp}^{\mu}$$

- appears as if transverse spin unimportant at high energies
- however, not true : in high-energy processes only

$$\frac{1}{2}\not{p}\left[\,\mathbb{1} \;-\; s_{\parallel}\,\gamma_5 \;+\; \gamma_5\,\not{s}_{\perp}\,\right]$$

- density matrix :

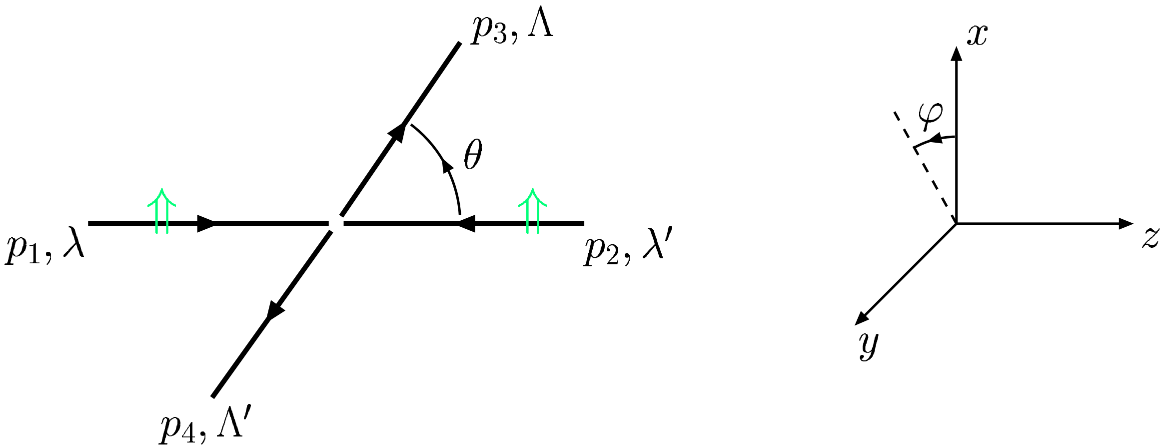
$$\rho \;=\; \frac{1}{2}\begin{pmatrix} 1+s_{\parallel} & s_x - i s_y \\ s_x + i s_y & 1 - s_{\parallel} \end{pmatrix}$$

II. Polarized Scattering Processes

2.1 Some general remarks . . .

- measurements of spin phenomena may
  - help explore & understand a theory
  - test a theory
  - deduce nature of new dynamics & constrain a new theory
- in all cases : need to separate properties that are
  - "generic"
  - specific to the theory

2.2 An example : the process $\vec{e}\,\vec{\mu} \rightarrow e\,\mu$



- let's look at it in terms of helicity states $|\,\vec{p}_1\,,\ \lambda\rangle$ etc.
- amplitude for given helicity configuration : $H_{\lambda\lambda';\,\Lambda\Lambda'}(\theta,\,\varphi)$

  16 "helicity amplitudes"
- helicity states have definite properties under rotations by α around φ :

$$|\,\vec{p}_1\,,\ \lambda\rangle \ \rightarrow\ \mathrm{e}^{\,i\,\alpha\,\lambda/2}\,|\,\vec{p}_1\,,\ \lambda\rangle \qquad \text{etc.}$$

- tied to angular-momentum conservation

- this leads to :

$$H_{\lambda\lambda';\,\Lambda\Lambda'}(\theta,\,\varphi) \;=\; \mathrm{e}^{\frac{i\,\varphi}{2}\,(\lambda-\lambda')}\; h_{\lambda\lambda';\,\Lambda\Lambda'}(\theta)$$

- from this, we can construct cross section for arbitrary polarization.

  Recall,

$$\chi_x^{\uparrow} \;=\; \frac{1}{\sqrt{2}}\,\left[\,\chi_z^{\uparrow} + \chi_z^{\downarrow}\,\right] \qquad\qquad \text{etc.}$$

- note, cross section

$$\frac{d\sigma}{d\Omega} \;\propto\; \left|\, H_{\ldots}(\theta,\varphi)\,\right|^2$$

- for simplicity, let's sum over final-state polarizations $\Lambda,\, \Lambda'$

- find most general form :

$$\frac{d\sigma}{d\Omega} \propto$$

$$\left(1 + s_{\parallel}\, s'_{\parallel}\right) R_1 + \left(1 - s_{\parallel}\, s'_{\parallel}\right) R'_1 + \left(s_{\parallel} + s'_{\parallel}\right) R_2 + \left(s_{\parallel} - s'_{\parallel}\right) R'_2$$

$$+ s_{\perp} \left\{ \cos(\varphi)\, R_3 - \sin(\varphi)\, R_4 \right\} + s'_{\perp} \left\{ \cos(\varphi)\, R'_3 + \sin(\varphi)\, R'_4 \right\}$$

$$+ s'_{\parallel}\, s_{\perp} \left\{ \cos(\varphi)\, R_5 - \sin(\varphi)\, R_6 \right\} + s_{\parallel}\, s'_{\perp} \left\{ \cos(\varphi)\, R'_5 + \sin(\varphi)\, R'_6 \right\}$$

$$+ s_{\perp}\, s'_{\perp} \left\{ R_7 + \cos(2\varphi)\, R_8 - \sin(2\varphi)\, R_9 \right\}$$

- has all the azimuthal-angular dependence !
- all $R_i \equiv R_i(\theta)$ are given in terms of the $h_{\lambda\lambda';\,\Lambda\Lambda'}$

Here they are . . .

$$R_1 = \sum_{\Lambda,\Lambda'} \left[|h_{++;\Lambda\Lambda'}|^2 + |h_{--;\Lambda\Lambda'}|^2 \right] \qquad R'_1 = \sum_{\Lambda,\Lambda'} \left[|h_{+-;\Lambda\Lambda'}|^2 + |h_{-+;\Lambda\Lambda'}|^2 \right]$$

$$R_2 = \sum_{\Lambda,\Lambda'} \left[|h_{++;\Lambda\Lambda'}|^2 - |h_{--;\Lambda\Lambda'}|^2 \right] \qquad R'_2 = \sum_{\Lambda,\Lambda'} \left[|h_{+-;\Lambda\Lambda'}|^2 - |h_{-+;\Lambda\Lambda'}|^2 \right]$$

$$R_{\substack{3\\4}} = \sum_{\Lambda,\Lambda'} 2 \left\{ \begin{array}{c} \mathrm{Re} \\ \mathrm{Im} \end{array} \right\} \left[h_{++;\Lambda\Lambda'}\, h^*_{-+;\Lambda\Lambda'} \pm h_{--;\Lambda\Lambda'}\, h^*_{+-;\Lambda\Lambda'} \right]$$

$$R_{\substack{5\\6}} = \sum_{\Lambda,\Lambda'} 2 \left\{ \begin{array}{c} \mathrm{Re} \\ \mathrm{Im} \end{array} \right\} \left[h_{++;\Lambda\Lambda'}\, h^*_{-+;\Lambda\Lambda'} \mp h_{--;\Lambda\Lambda'}\, h^*_{+-;\Lambda\Lambda'} \right]$$

$$R_7 = 2 \sum_{\Lambda,\Lambda'} \mathrm{Re} \left[h_{++;\Lambda\Lambda'}\, h^*_{--;\Lambda\Lambda'} \right] \qquad R_8 = 2 \sum_{\Lambda,\Lambda'} \mathrm{Re} \left[h_{+-;\Lambda\Lambda'}\, h^*_{-+;\Lambda\Lambda'} \right]$$

$$R_9 = 2 \sum_{\Lambda,\Lambda'} \mathrm{Im} \left[h_{+-;\Lambda\Lambda'}\, h^*_{-+;\Lambda\Lambda'} \right]$$

What more can we say ?

- now use properties of the underlying theory (here, QED)
- Parity invariance :
  helicity $\vec{J} \cdot \vec{p}/|\vec{p}|$ is pseudoscalar under space reflection

$$\hat{\mathrm{P}}\,|\,\vec{p},\,\lambda\,\rangle \;=\; \eta\,|\,-\vec{p},\,-\lambda\,\rangle \qquad (\,\eta \;=\; \text{intrinsic parity} \times \text{phase}\,)$$

- one can show that for our amplitudes :

$$h_{-\lambda,-\lambda';-\Lambda,-\Lambda'}(\theta) \;=\; (-1)^{(\lambda-\lambda'-\Lambda+\Lambda')/2}\; h_{\lambda\lambda';\Lambda,\Lambda'}(\theta)$$

- therefore,

$$h_{--;--} \;=\; h_{++;++}(\theta)$$
$$h_{-+;-+} \;=\; h_{+-;+-}(\theta)$$
$$h_{-+;--} \;=\; -\,h_{+-;++}(\theta) \qquad \text{etc.}$$

- $16 \;\longrightarrow\;$ 8 helicity amplitudes

Careful inspection reveals :

$$R_2 = R_2' = R_3 = R_3' = R_6 = R_6' = R_9 = 0$$

Therefore,

$$\frac{d\sigma}{d\Omega} \propto$$

$$\left(1 + s_{\|}\, s_{\|}'\right) R_1 + \left(1 - s_{\|}\, s_{\|}'\right) R_1' + \xcancel{\left(s_{\|} + s_{\|}'\right) R_2} + \xcancel{\left(s_{\|} - s_{\|}'\right) R_2'}$$

$$+ s_{\perp} \left\{ \xcancel{\cos(\varphi)\, R_3} - \sin(\varphi)\, R_4 \right\} + s_{\perp}' \left\{ \xcancel{\cos(\varphi)\, R_3'} + \sin(\varphi)\, R_4' \right\}$$

$$+ s_{\|}'\, s_{\perp} \left\{ \cos(\varphi)\, R_5 - \xcancel{\sin(\varphi)\, R_6} \right\} + s_{\|}\, s_{\perp}' \left\{ \cos(\varphi)\, R_5' + \xcancel{\sin(\varphi)\, R_6'} \right\}$$

$$+ s_{\perp}\, s_{\perp}' \left\{ R_7 + \cos(2\varphi)\, R_8 - \xcancel{\sin(2\varphi)\, R_9} \right\}$$

- Time reversal invariance :
  relates $A + B \to C + D$ to $C + D \to A + B$
  Here we have elastic reaction $A + B \to A + B$

- one can show that for our amplitudes :

$$h_{\Lambda,\Lambda';\lambda,\lambda'}(\theta) \;=\; (-1)^{(\lambda-\lambda'-\Lambda+\Lambda')/2} \; h_{\lambda\lambda';\Lambda,\Lambda'}(\theta)$$

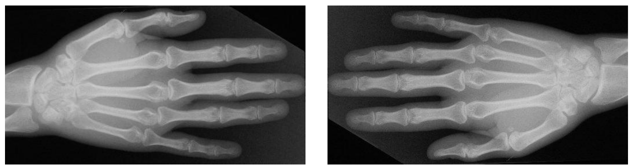
- therefore,

$$\begin{aligned} h_{+-;--} &= -h_{--;+-}(\theta) \\ h_{+-;++} &= -h_{++;+-}(\theta) \end{aligned}$$

- $8 \to 6$ helicity amplitudes

Chirality (or helicity-conservation) :

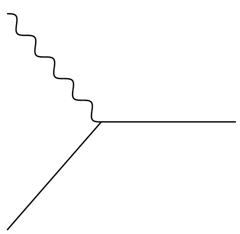
- saw that at high energies helicity = chirality



$$\Psi_{R,L} = \frac{1}{2}\left[\,\mathbb{1} \pm \gamma_5\,\right]\Psi$$

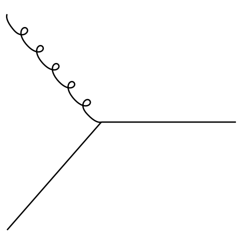
- preserved in all vector and axial-vector current interactions

- preserved by all perturbative QCD and electroweak interactions



$\gamma,\, W^{\pm},\, Z^0$:

$$\begin{cases} \bar{\Psi}\gamma^\mu\Psi = \bar{L}\gamma^\mu L + \bar{R}\gamma^\mu R \\ \bar{\Psi}\gamma^\mu\gamma^5\Psi = \bar{L}\gamma^\mu L - \bar{R}\gamma^\mu R \end{cases}$$

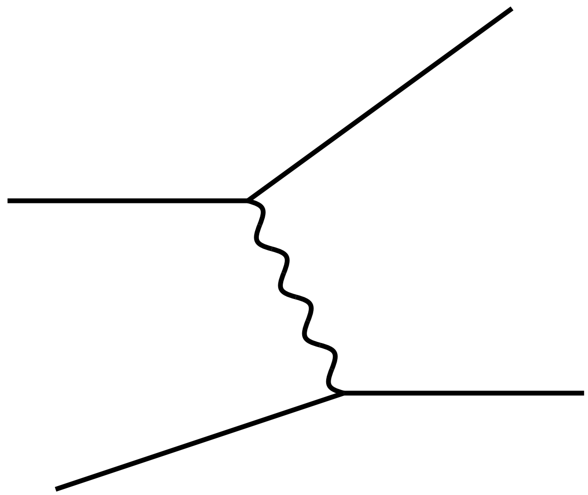


g :

$$\bar{\Psi}\gamma^\mu\frac{\lambda}{2}\Psi = \bar{L}\gamma^\mu\frac{\lambda}{2}L + \bar{R}\gamma^\mu\frac{\lambda}{2}R$$

- violated for non-standard ((pseudo)scalar,tensor) interactions

- Feynman diagram for $e\,\mu$ scattering :



- requires $\Lambda = \lambda$, $\Lambda' = \lambda'$, or

$$h_{\lambda\lambda';\Lambda\Lambda'} \longrightarrow h_{\lambda\lambda';\lambda\lambda'}$$

This gives :

$$R_4 = R_4' = R_5 = R_5' = R_7 = R_8 = 0$$

Therefore (with parity),

$$\frac{d\sigma}{d\Omega} \propto$$

$$\left(1 + s_{\parallel}\, s'_{\parallel}\right) R_1 + \left(1 - s_{\parallel}\, s'_{\parallel}\right) R_1' + \cancel{\left(s_{\parallel} + s'_{\parallel}\right) R_2} + \cancel{\left(s_{\parallel} - s'_{\parallel}\right) R_2'}$$

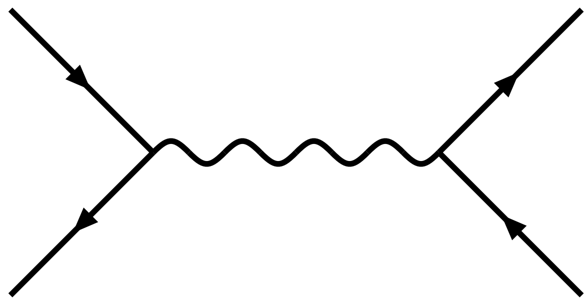
$$+ s_{\perp} \left\{ \cancel{\cos(\varphi)\, R_3} - \cancel{\sin(\varphi)\, R_4} \right\} + s'_{\perp} \left\{ \cancel{\cos(\varphi)\, R_3'} + \cancel{\sin(\varphi)\, R_4'} \right\}$$

$$+ s'_{\parallel}\, s_{\perp} \left\{ \cancel{\cos(\varphi)\, R_5} - \cancel{\sin(\varphi)\, R_6} \right\} + s_{\parallel}\, s'_{\perp} \left\{ \cancel{\cos(\varphi)\, R_5'} + \cancel{\sin(\varphi)\, R_6'} \right\}$$

$$+ s_{\perp}\, s'_{\perp} \left\{ \cancel{R_7} + \cancel{\cos(2\varphi)\, R_8} - \cancel{\sin(2\varphi)\, R_9} \right\}$$

- However, note : before using chirality, we had assumed nothing about specifics of the reaction !

- For $e^- e^+ \rightarrow \mu^- \mu^+$ things would be different :



- requires $\lambda' = -\lambda$, $\Lambda' = -\Lambda$, or

$$h_{\lambda\lambda';\Lambda\Lambda'} \longrightarrow h_{\lambda,-\lambda;\,\Lambda,-\Lambda}$$

In this case :

$$R_1 = R_4 = R_4' = R_5 = R_5' = R_7 = 0$$

Therefore (with parity),

$$\frac{d\sigma}{d\Omega} \propto$$

$$\left(1 + s_{\parallel}\, s_{\parallel}'\right) R_1 + \left(1 - s_{\parallel}\, s_{\parallel}'\right) R_1' + \left(s_{\parallel} + s_{\parallel}'\right) R_2 + \left(s_{\parallel} - s_{\parallel}'\right) R_2'$$

$$+ s_{\perp} \left\{ \cos(\varphi)\, R_3 - \sin(\varphi)\, R_4 \right\} + s_{\perp}' \left\{ \cos(\varphi)\, R_3' + \sin(\varphi)\, R_4' \right\}$$

$$+ s_{\parallel}'\, s_{\perp} \left\{ \cos(\varphi)\, R_5 - \sin(\varphi)\, R_6 \right\} + s_{\parallel}\, s_{\perp}' \left\{ \cos(\varphi)\, R_5' + \sin(\varphi)\, R_6' \right\}$$

$$+ s_{\perp}\, s_{\perp}' \left\{ R_7 + \cos(2\varphi)\, R_8 - \sin(2\varphi)\, R_9 \right\}$$

The upshot is :

- Parity invariance (if it holds) forbids
  - single-longitudinal spin asymmetries of the form

$$A_L = \frac{\sigma_{\rightarrow} - \sigma_{\leftarrow}}{\sigma_{\rightarrow} + \sigma_{\leftarrow}} \equiv \frac{\sigma_{+} - \sigma_{-}}{\sigma_{+} + \sigma_{-}}$$

  - certain azimuthal-angular dependences for transverse polarization

- single-transverse spin asymmetries of the form

$$A_N = \frac{\sigma_{\uparrow} - \sigma_{\downarrow}}{\sigma_{\uparrow} + \sigma_{\downarrow}}$$

are allowed, but require helicity flips and complex amplitudes

(actually, require "Im{ non-flip $\times$ single-flip* }")

- double-longitudinal spin asymmetries,

$$A_{LL} = \frac{\sigma_{++} + \sigma_{--} - \sigma_{+-} - \sigma_{-+}}{\sigma_{++} + \sigma_{--} + \sigma_{+-} + \sigma_{-+}}$$

are allowed. Reduce under parity to

$$A_{LL} = \frac{\sigma_{++} - \sigma_{+-}}{\sigma_{++} + \sigma_{+-}}$$

- double-transverse spin asymmetries,

$$A_{TT} = \frac{\sigma_{\uparrow\uparrow} - \sigma_{\uparrow\downarrow}}{\sigma_{\uparrow\uparrow} + \sigma_{\uparrow\downarrow}}$$

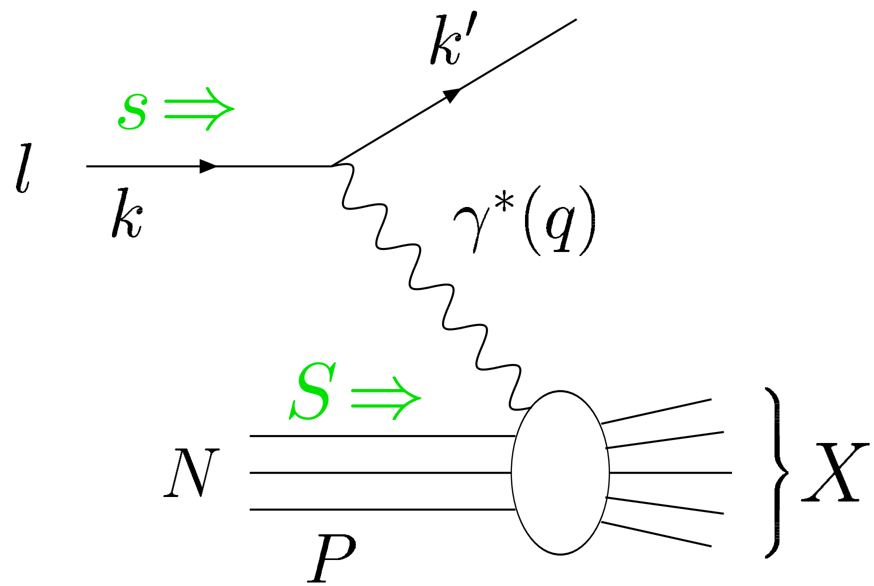
are allowed, but require helicity flip or annihilating fermion lines

- [occasionally may use symmetry constraints for identical particles (e.g., $pp \to pp$)]

We'll encounter all this . . .

III. Lepton-Nucleon Scattering

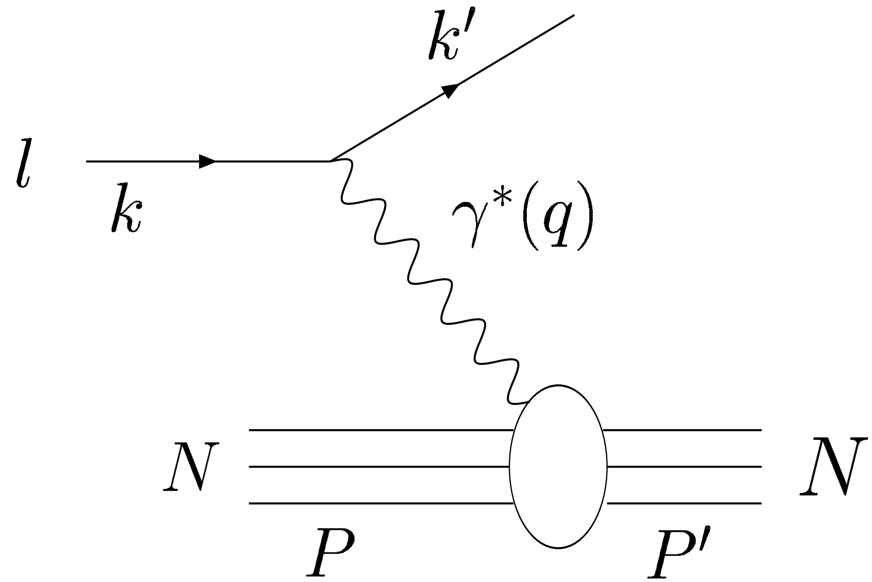
$l\,N \;\to\; l'\,X$: standard 'microscope' to explore nucleon structure !



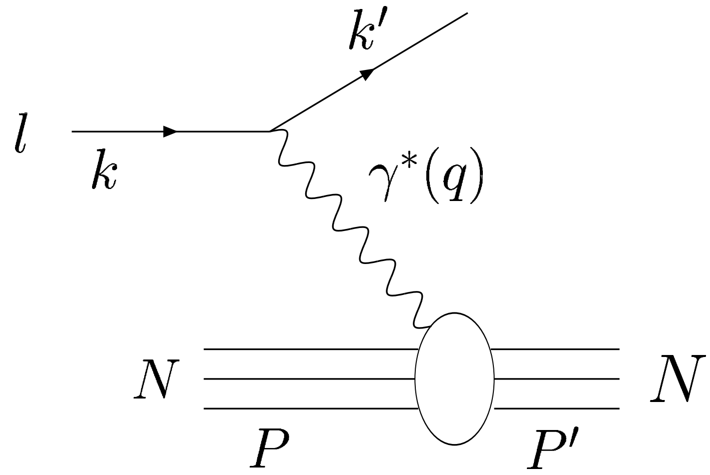
$Q \;\equiv\; \sqrt{-q^2} \;\;\to$ "Resolution" $\quad \frac{\hbar}{Q} \;=\; \frac{2\times 10^{-16}\,\mathrm{m}}{Q/\mathrm{GeV}} \qquad r_N \;\sim\; 10^{-15}\,\mathrm{m}$

"inelasticity" $\quad x \;\equiv\; \frac{Q^2}{2P\cdot q} \;\equiv\; \frac{Q^2}{2m\nu} \;=\; \frac{Q^2}{Q^2+M_X^2-m^2}$ Bjorken var.

3.1 Elastic lepton-nucleon scattering (brief . . .)



- $x = 1$ for elastic case
- if proton had no structure : would be same as $e\,\mu \to e\,\mu$!



- amplitude

$$\mathcal{M} = -e^2 \left(\frac{i g_{\mu\nu}}{Q^2} \right) \bar{u}(k') \, \gamma^\nu \, u(k) \quad \langle \, P' \mid J^\mu(0) \mid P \, \rangle$$

- find structure

$$\langle \, P' \mid J^\mu(0) \mid P \, \rangle = \bar{u}(P') \left[\gamma^\mu \, \mathcal{F}_1(Q^2) + \frac{i \, \sigma_{\mu\nu} q^\nu}{2m} \, \mathcal{F}_2(Q^2) \right] u(P)$$

- $\mathcal{F}_1(Q^2)$, $\mathcal{F}_2(Q^2)$ "nucleon elastic form factors"

- Sachs form factors :

$$G_E(Q^2) = \mathcal{F}_1(Q^2) - \frac{Q^2}{4m^2}\mathcal{F}_2(Q^2)$$
$$G_M(Q^2) = \mathcal{F}_1(Q^2) + \mathcal{F}_2(Q^2)$$

- cross section measures G_E^2, G_M^2
- $\longrightarrow$ access to nucleon properties, e.g.

$$G_E(0) = e_N \qquad G_M(0) = \kappa_N$$
$$\langle r^2 \rangle_E = -6 \frac{dG_E(Q^2)}{dQ^2}\bigg|_{Q^2=0} \qquad \text{“charge radius”}$$

- with spin : different combinations of G_E, G_M in cross section

Toward increasing masses $W = M_X$: