More Analysis of Cirrus Cloud Particle Size Distributions Measured During Sparticus

Chris Schwartz
Jay Mace
University of Utah
Data provided by Paul Lawson of SPEC

Motivation

- Remote sensing of cirrus particle size distributions (PSD's)
- Maximize the posterior distribution of possible PSD params given remote observations, as given by Bayes' Rule:

$$p(\vec{x} \mid \vec{y}) \propto L(\vec{y} \mid \vec{x}) p(\vec{x})$$

- Measurement and pram vectors

$$\vec{x} = \begin{bmatrix} p_1 \\ \dots \\ p_n \end{bmatrix} \qquad \vec{y} = \begin{bmatrix} Z \\ \dots \\ Tb \end{bmatrix}$$

Focus on the Prior Distribution: $p(\vec{x})$

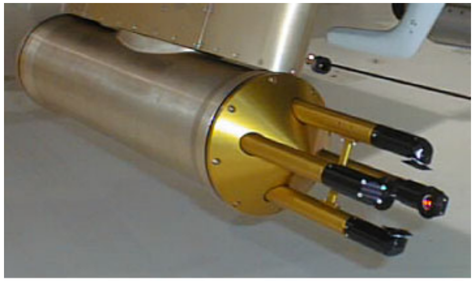
- Often assumed to be Gaussian (e.g. Rogers, 2000)

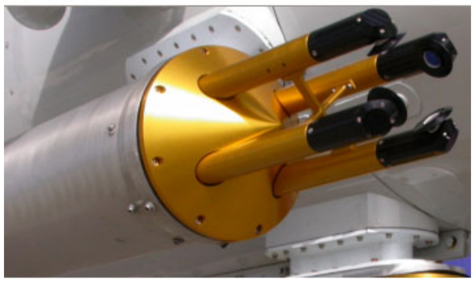
$$p(\vec{x}) = \frac{1}{(2\pi)^{n/2}|S_a|^{1/2}} \exp\left[-\frac{1}{2}(\vec{x}-\vec{x}_a)^T S_a^{-1}(\vec{x}-\vec{x}_a)\right]$$

- Covariance generally assumed to be diagonal

$$S_a = \begin{bmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \sigma_2^2 & 0 \\ 0 & 0 & \sigma_3^2 \end{bmatrix}$$

PSD Fits





- Data from 81 of Sparticus' flight legs
- 2-DS distributions fit with

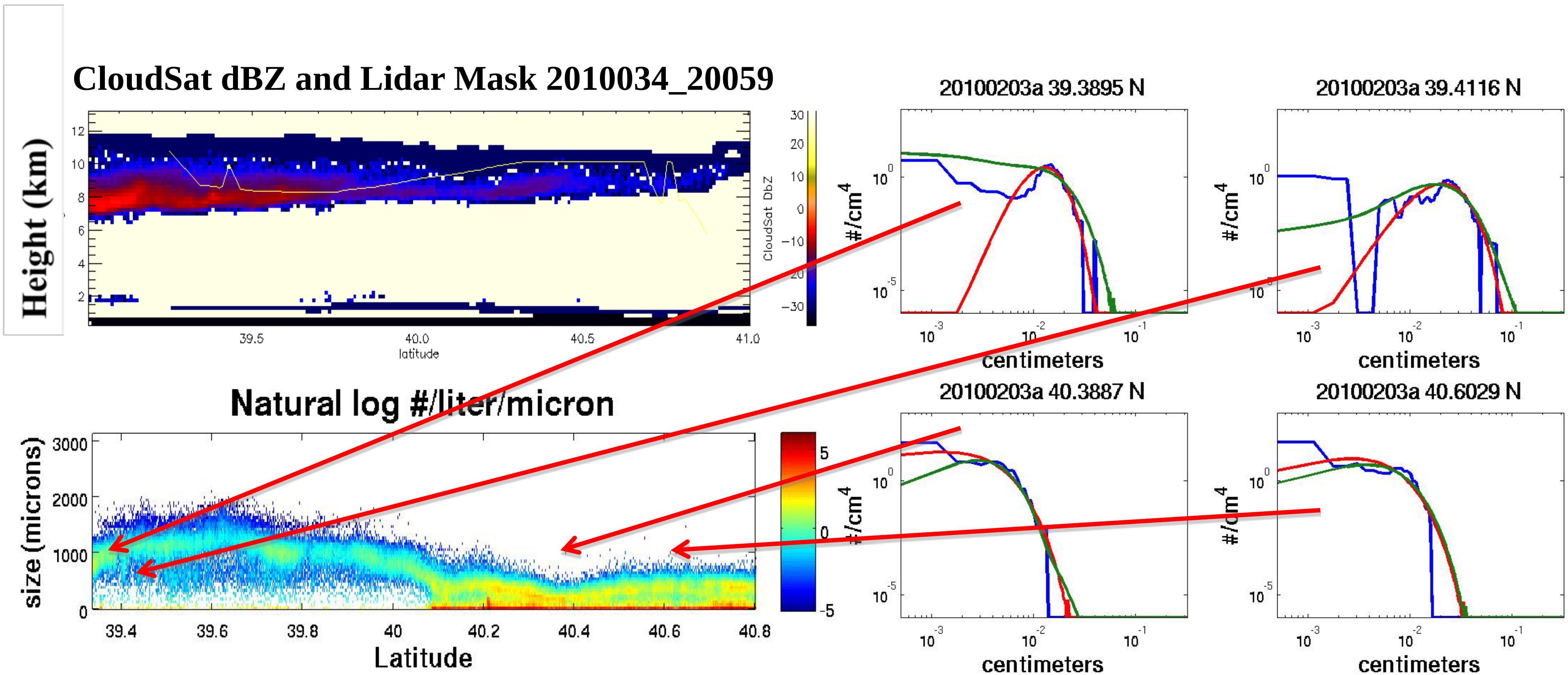
$$n(D) = N_0 (D/D_0)^{\alpha} \exp(-D/D_0); \quad \vec{x} = [N_0 \quad D_0 \quad \alpha]^T$$

- and

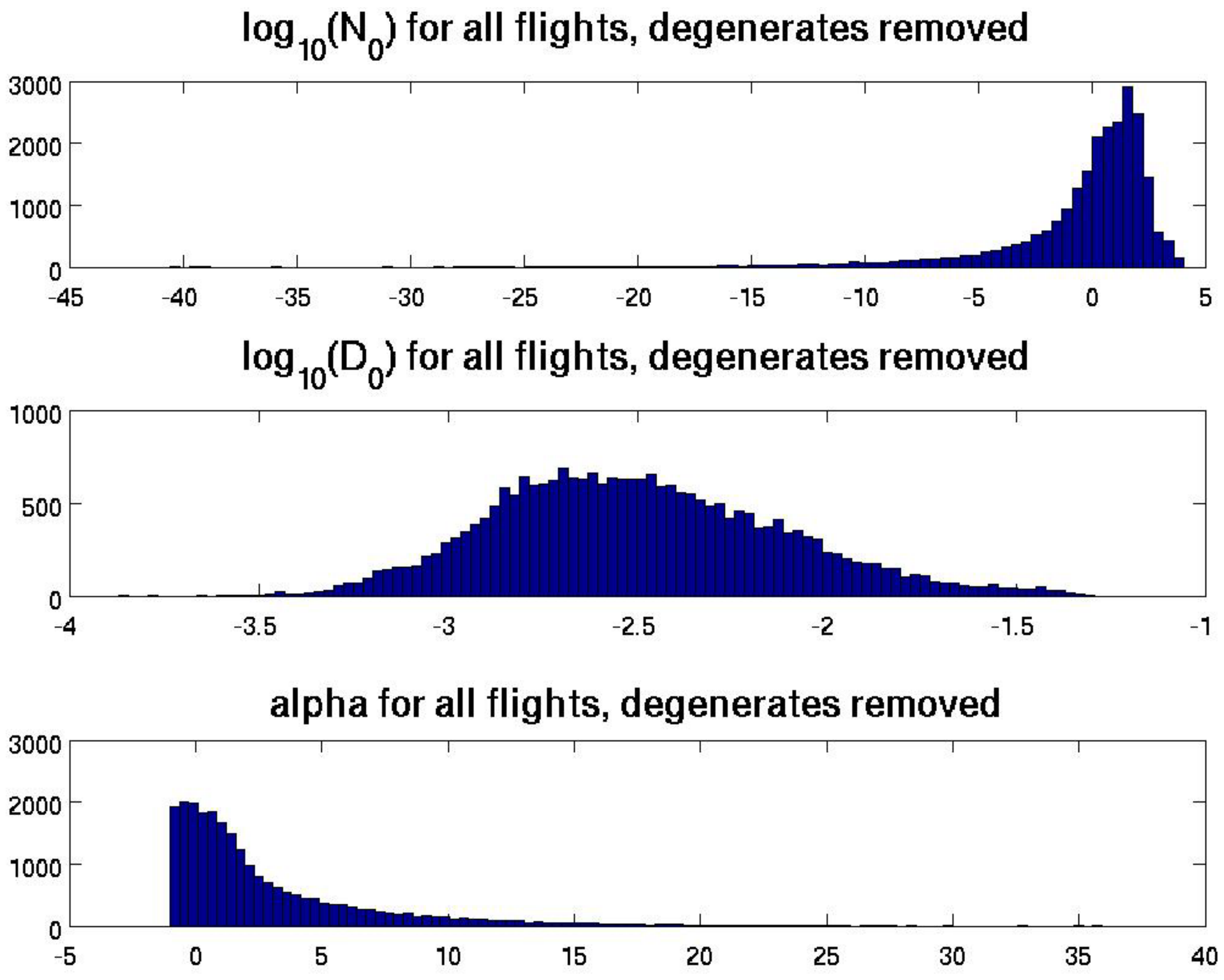
$$n(D) = n_l(D) + n_s(D); \quad \vec{x} = \begin{bmatrix} N_{0l} & D_{0l} & \alpha_l & N_{0s} & D_{0s} & \alpha_s \end{bmatrix}^T$$

- ~25,000 fits

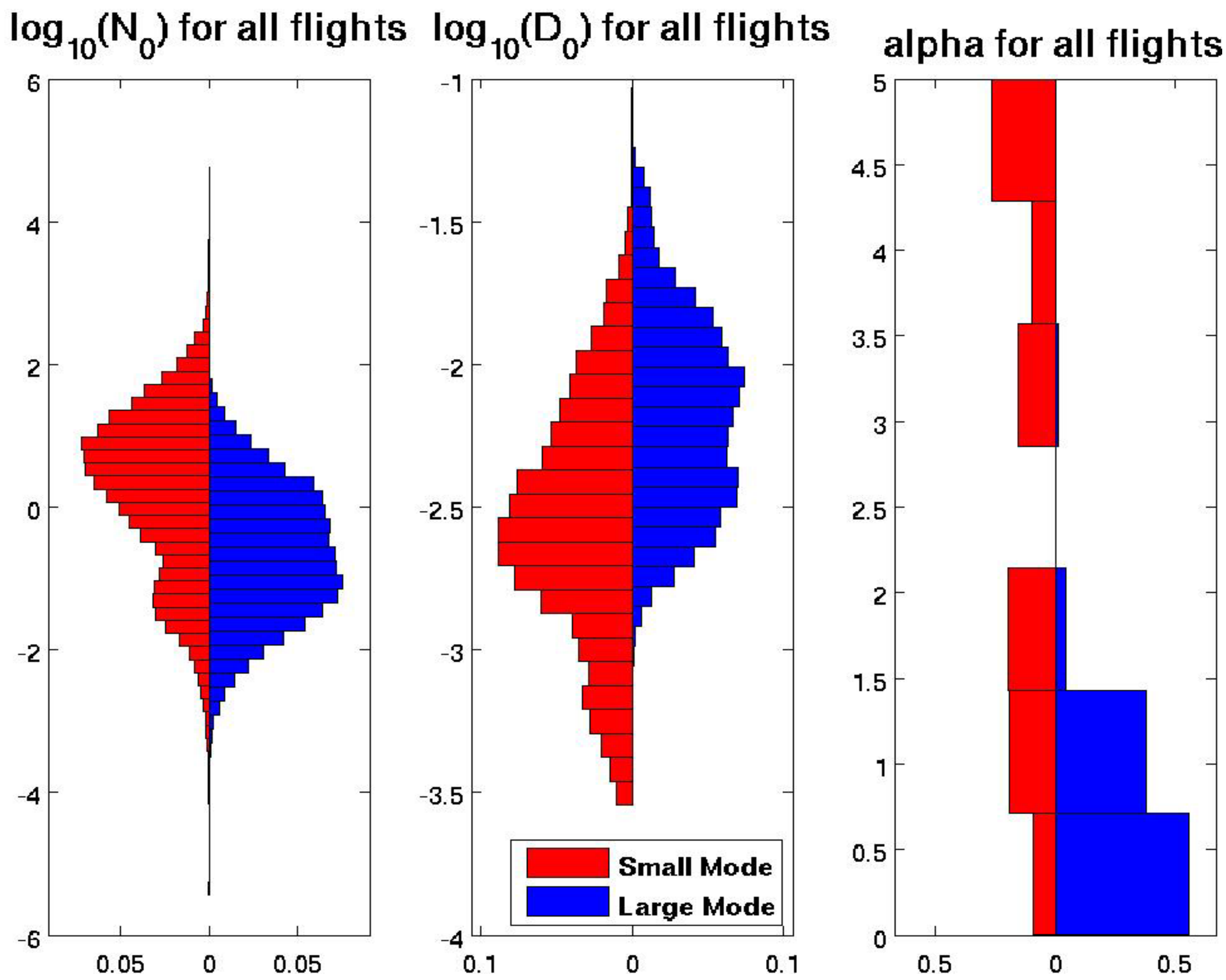
CloudSat Overpass of Sparticus, 2/3/10



Unimodal Distribution Fit with Maximum Likelihood Algorithm



Bimodal Distribution Fit with Method of Moments/Excess Mass Algorithm

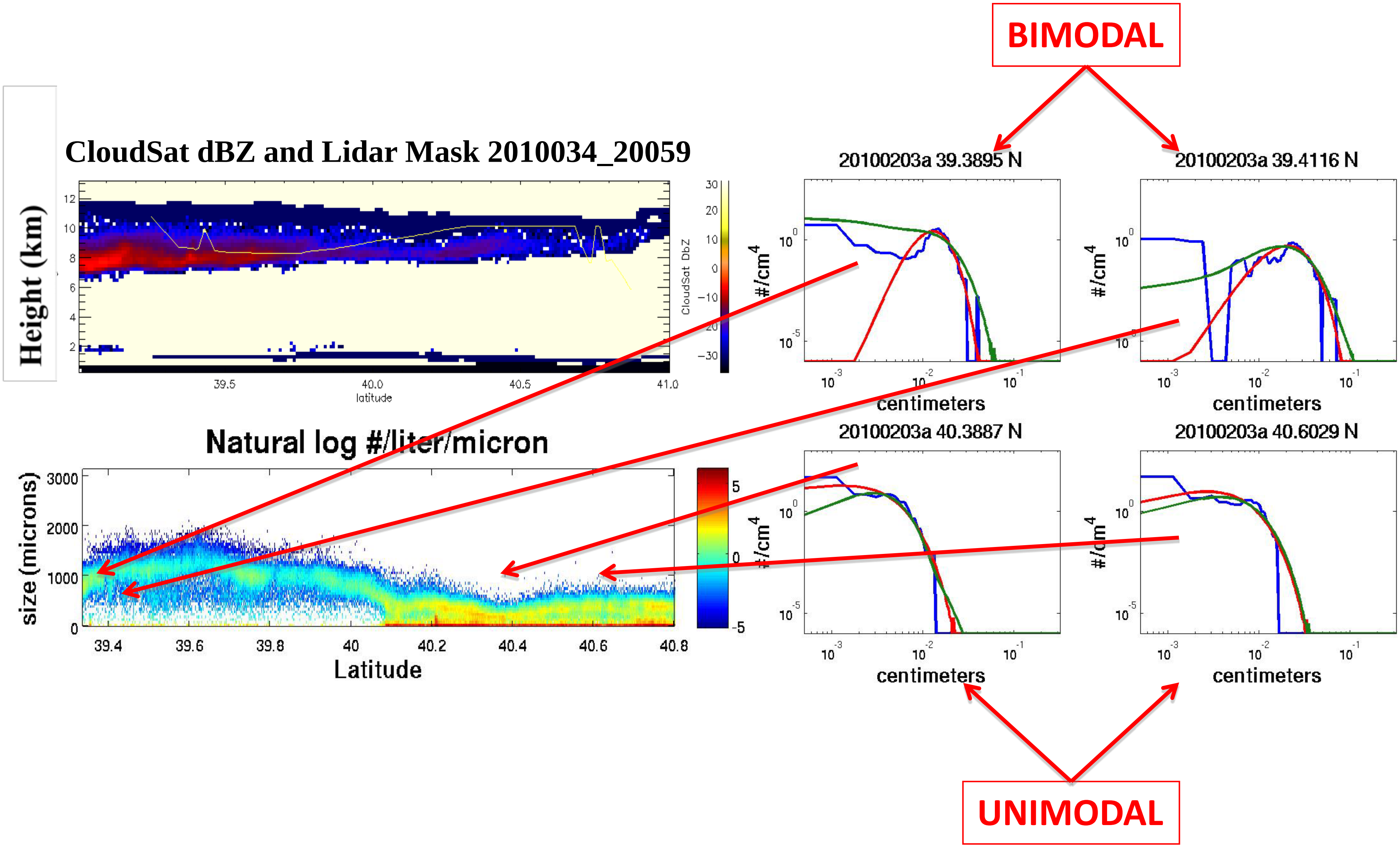


Covariance Analysis for Both Fits

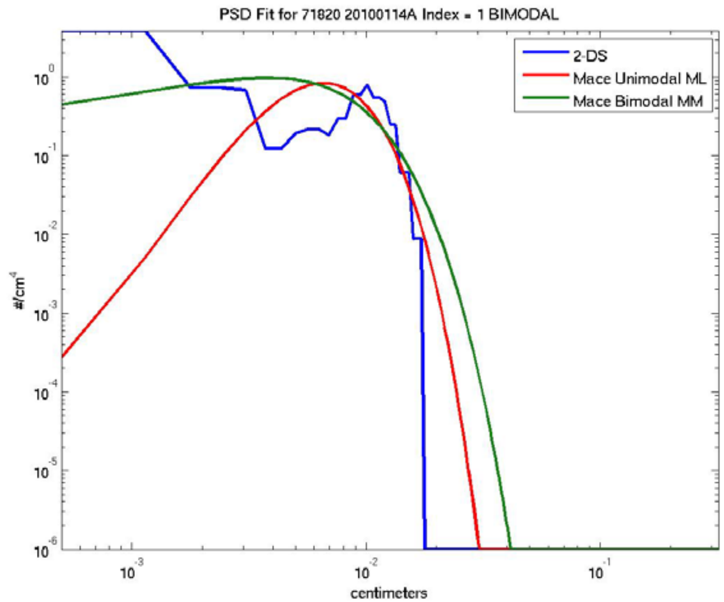
$$S_{uni} = \operatorname{cov}\begin{bmatrix} \log_{10}(N_0) \\ \log_{10}(D_0) \\ \alpha \end{bmatrix} = \begin{bmatrix} 12.18 & 0.22 & -13.35 \\ .022 & 0.14 & -0.61 \\ -13.35 & -0.61 & 16.43 \end{bmatrix}$$

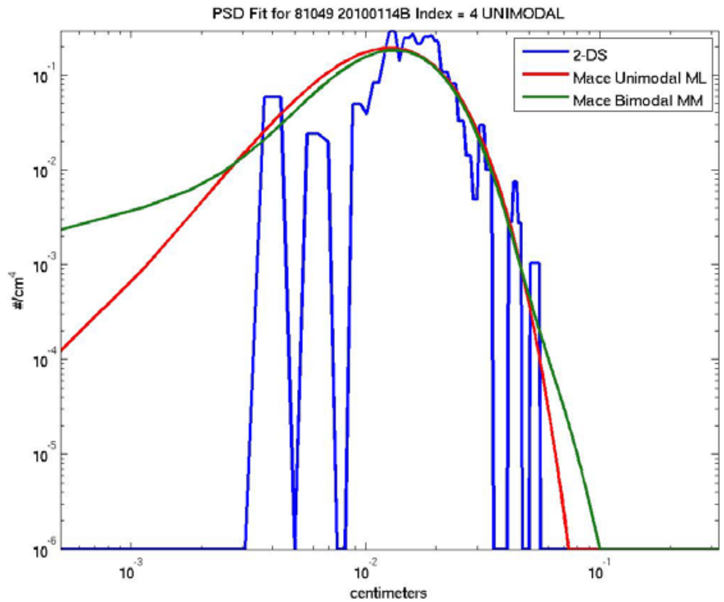
$$S_{bi} = \operatorname{cov}\begin{bmatrix} \log_{10}(N_l) \\ \log_{10}(D_l) \\ \alpha_l \\ \log_{10}(N_s) \\ \log_{10}(D_s) \\ \alpha_s \end{bmatrix} = \begin{bmatrix} 0.89 & -0.22 & -0.14 & 0.33 & -0.21 & 0.38 \\ -0.22 & 0.12 & -0.03 & -0.04 & 0.12 & -0.30 \\ -0.14 & -0.03 & 0.42 & -0.01 & -0.002 & 0.02 \\ 0.33 & -0.04 & -0.01 & 1.34 & -0.10 & -1.06 \\ -0.21 & 0.12 & -0.002 & -0.10 & 0.17 & -0.43 \\ 0.38 & -0.30 & 0.02 & -1.06 & -0.43 & 2.86 \end{bmatrix}$$

Whether the Unimodal or Bimodal Fit is More Likely Correct Tested Using a Likelihood Ratio Test

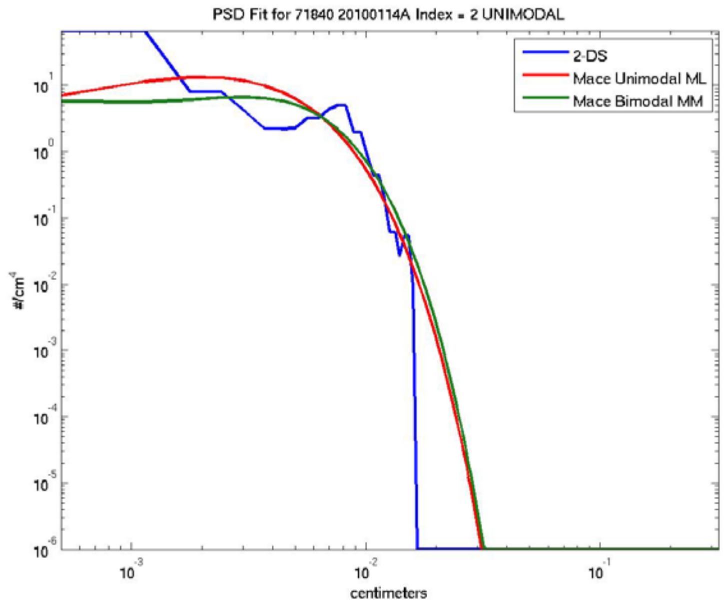


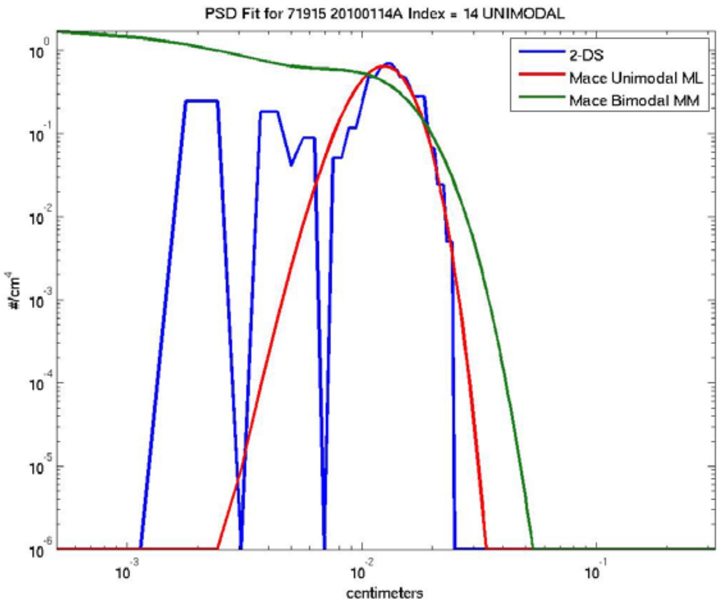
Common Examples of Bimodal and Unimodal Fits, Correctly Flagged



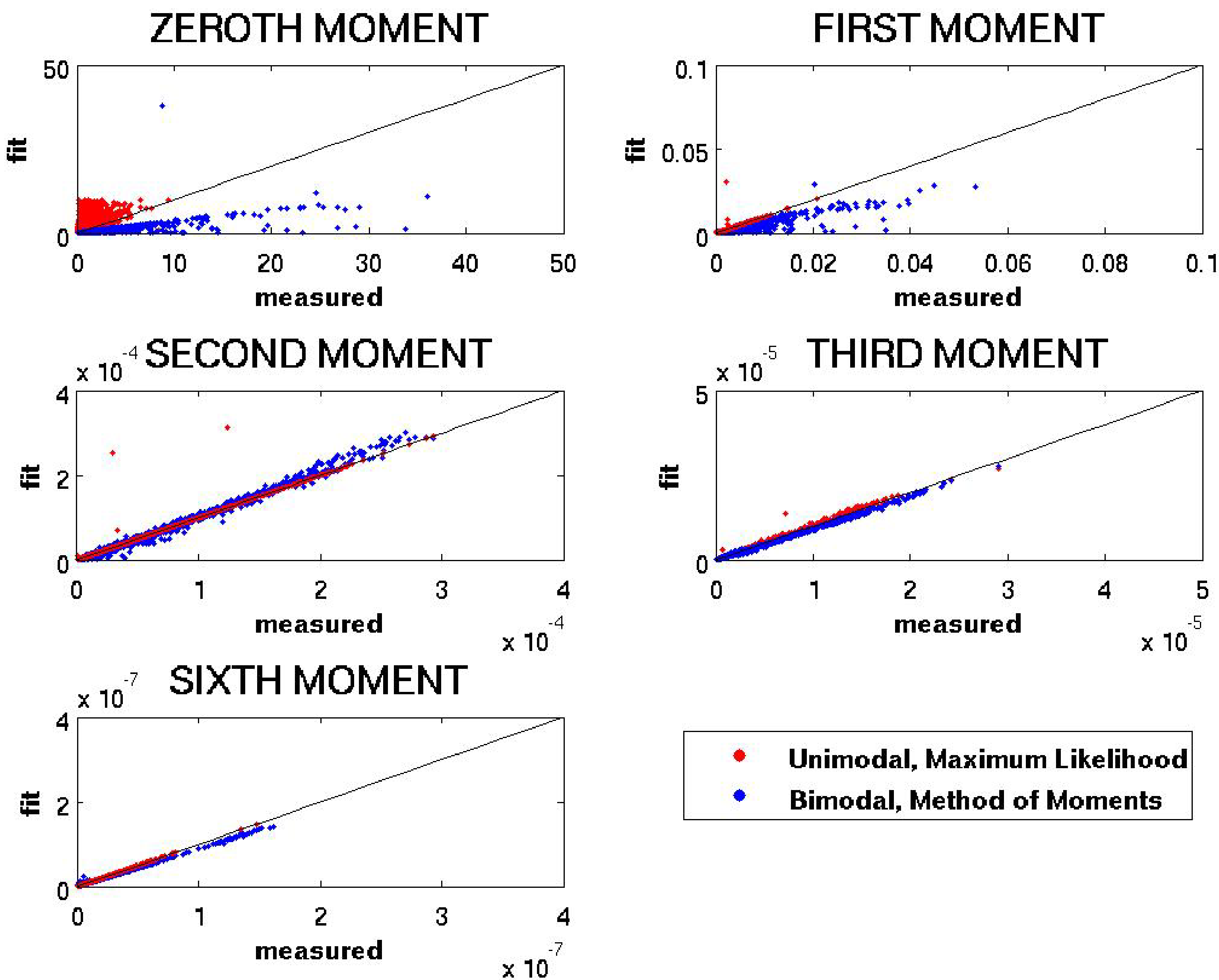


Some Counter-Examples

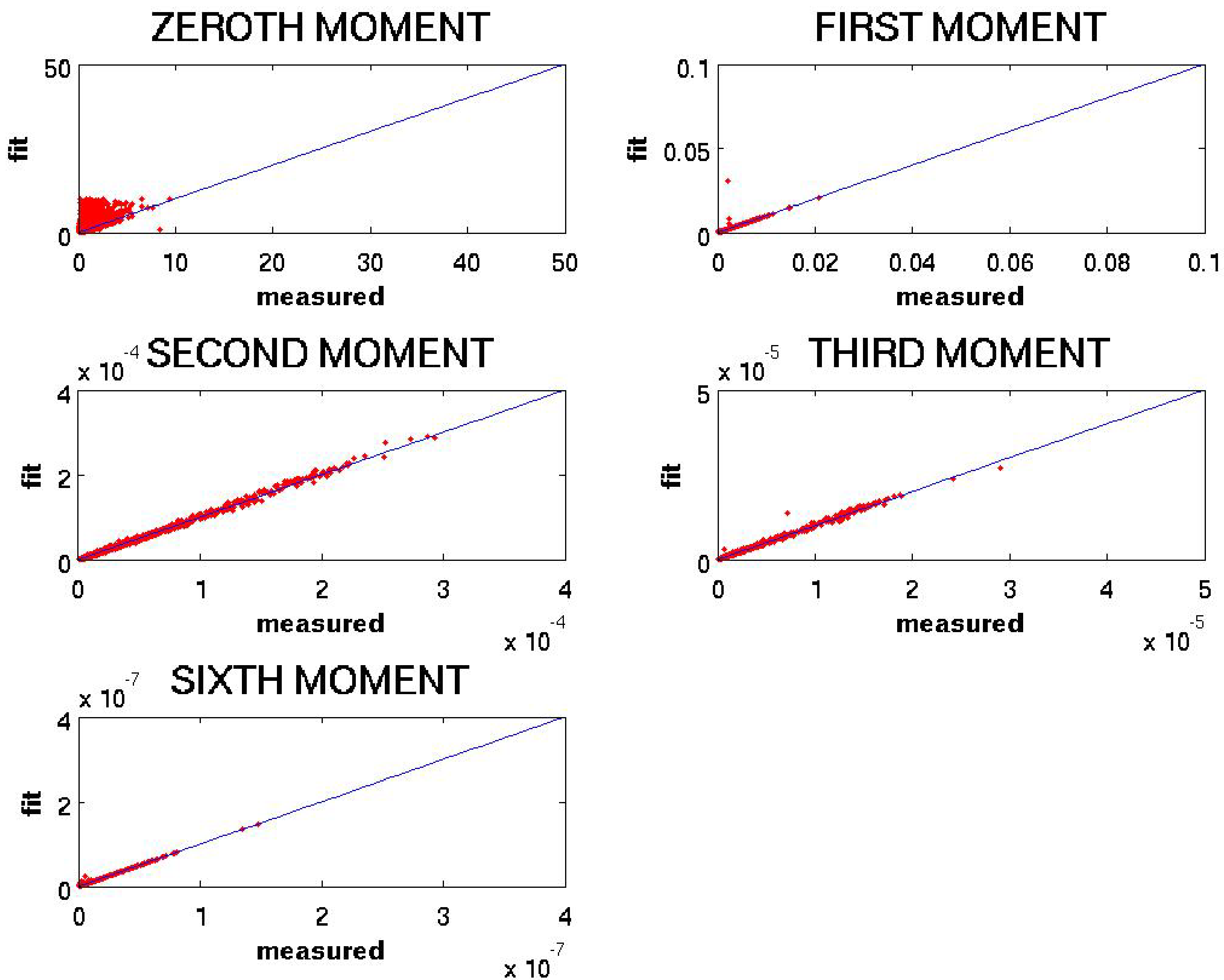




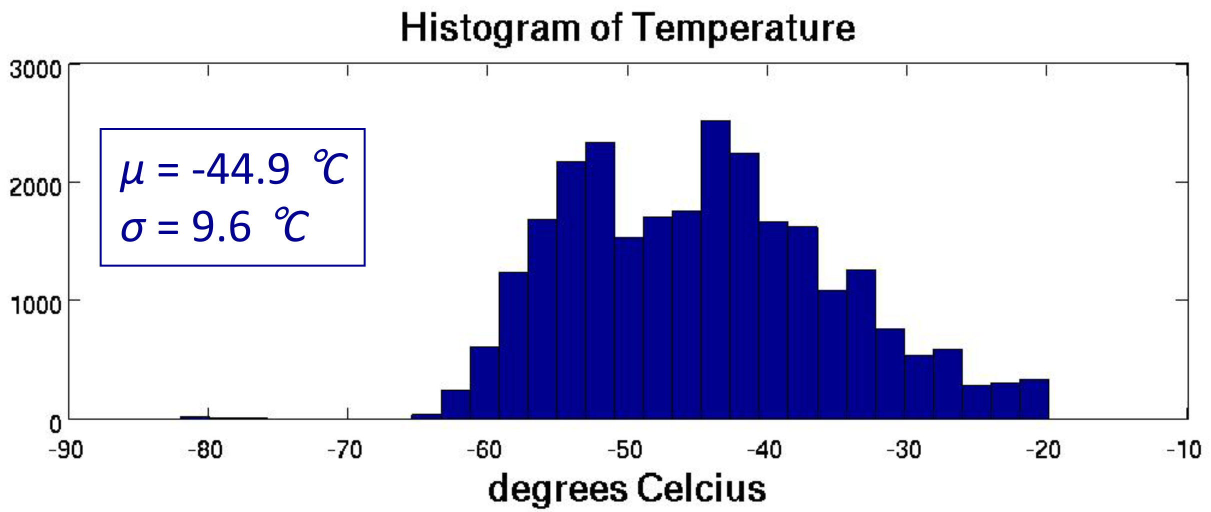
Scattered Moments from the Two Fit Distributions

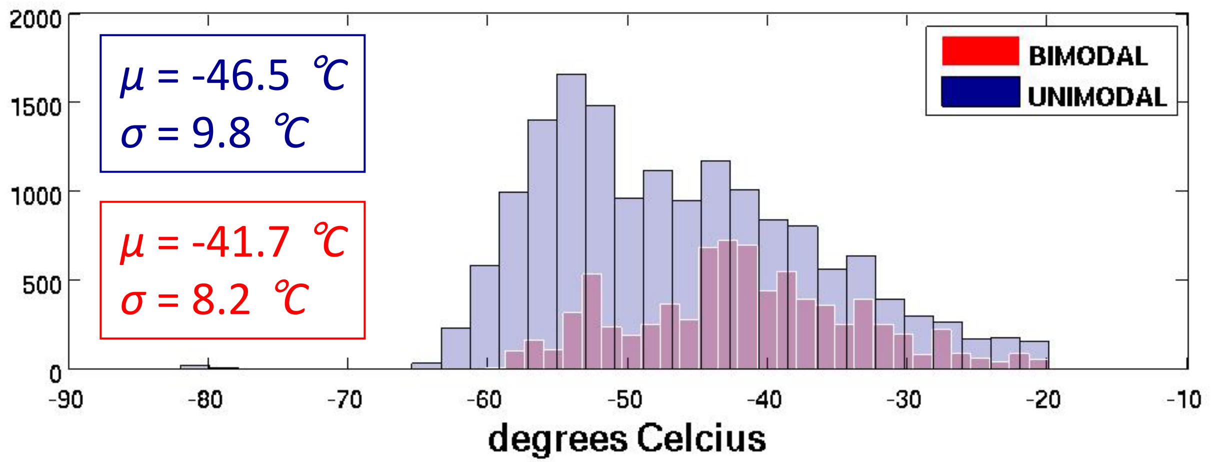


Scattered Moments from Either Distribution



Bimodality and Temperature





- Two-sample Kolmolgorov-Smirnov test confirms that the two temperature histograms were drawn from different distributions

Summary and Conclusion

- Fit with a uni- or bimodal distribution function, the prior distribution of PSD parameters is not Gaussian and does not have a diagonal covariance matrix.
- A likelihood ratio was used to determine whether fit was more appropriate.
- The bimodal model is not always better than the unimodal model, even in cases that exhibit bimodality
- If a bimodal distribution is to be used, a more sophisticated model needs to be developed.
- Cirrus PSD retrievals can be informed by meteorological situation.