

QCD saturation phenomenology: geometric scaling at HERA

Gregory Soyez

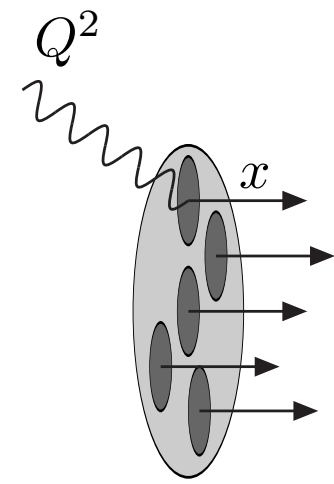
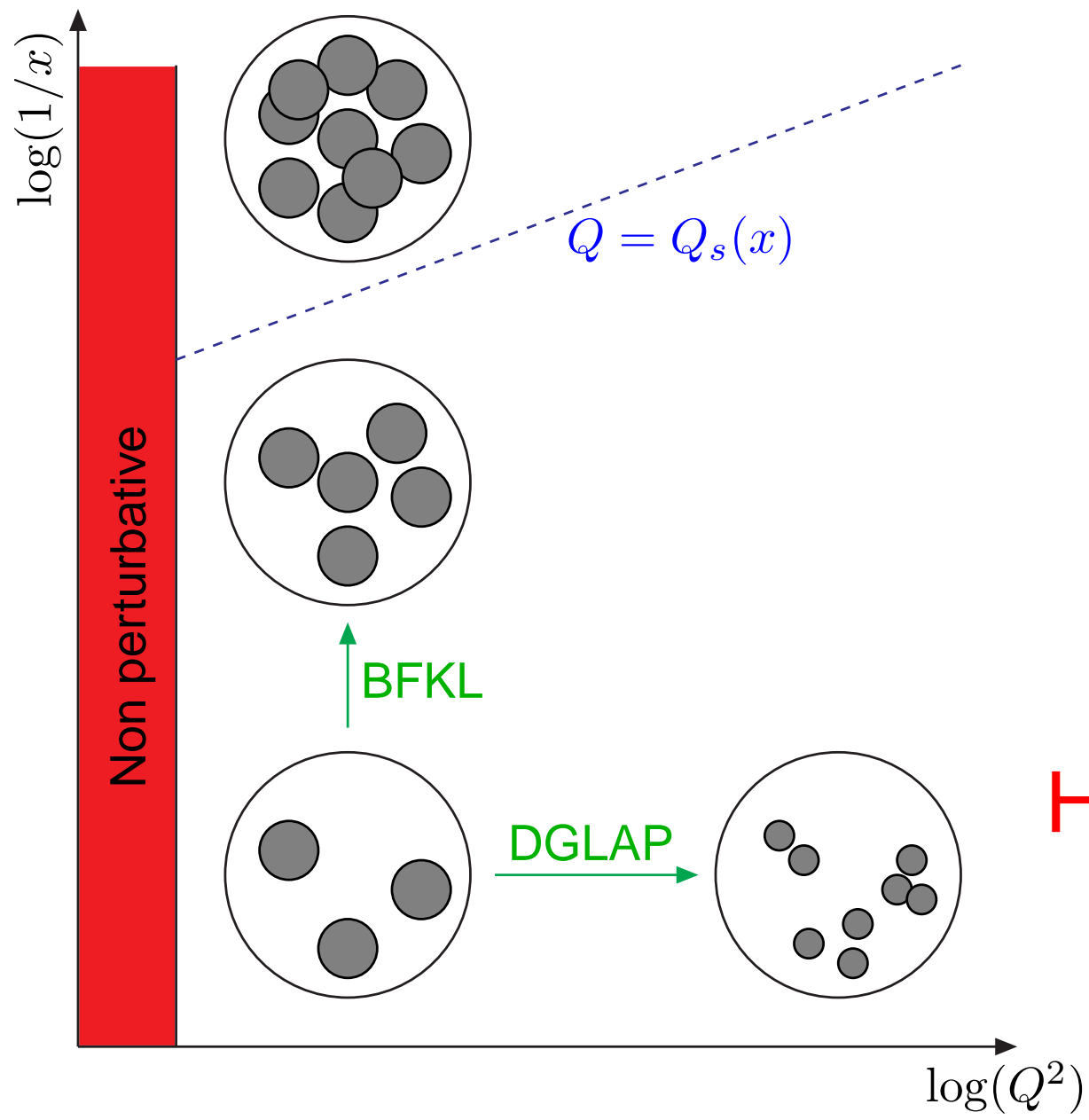
Brookhaven National Laboratory

Based on : C. Marquet, R. Peschanski, G. Soyez, Nucl.Phys.A**756** (2005) 399 [hep-ph/0502020]
C. Marquet, G. Soyez, Nucl.Phys. A**760** (2005) 208 [hep-ph/0504080]
F. Gelis, R. Peschanski, G. Soyez, L. Schoeffel, Phys.Lett.B**647** (2007) 376 [hep-ph/0610435]
C. Marquet, R. Peschanski, G.S., Phys.Rev.D**76** (2007) 034011 [hep-ph/0702171]
G.S., Phys. Lett. B**655** (2007) 32 [arXiv:0705.3672]

+ many other “external references”

- **Motivation:** QCD Bremsstrahlung & resummation → geometric scaling
- **Theoretical background:** perturbative evolution in high-energy QCD:
 - Dipole model and leading log approx.: BFKL equation
 - Unitarity/Saturation effects: Balitsky/JIMWLK and BK equation
- **Asymptotic solutions:** saturation ⇒ geometric scaling
 - impact-parameter-independent BK: mechanism for geometric scaling
 - full BK equation: geometric scaling at nonzero momentum transfer
- **Phenomenological consequences** (mainly at HERA)
 - Geometric scaling for F_2 (different approaches)
 - Geometric scaling in vector meson production and DVCS
- **Conclusion(s) & discussion(s)**

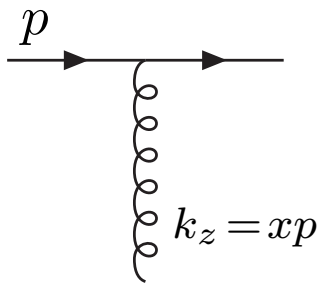
Motivation: why saturation ?



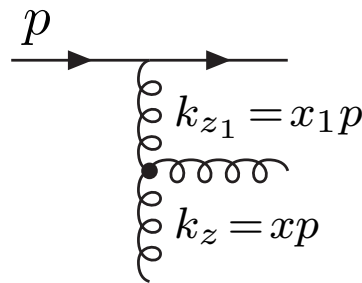
Size $\sim 1/Q$
Energy $\sim Q^2/x$

How to describe
this in QCD ?

Bremsstrahlung:

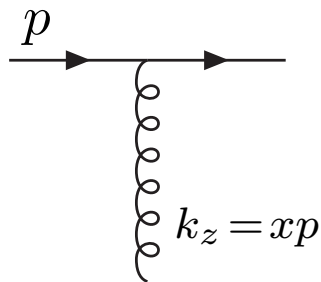


$$x \ll 1$$

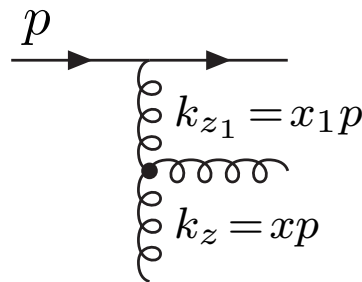


$$x \ll x_1 \ll 1$$

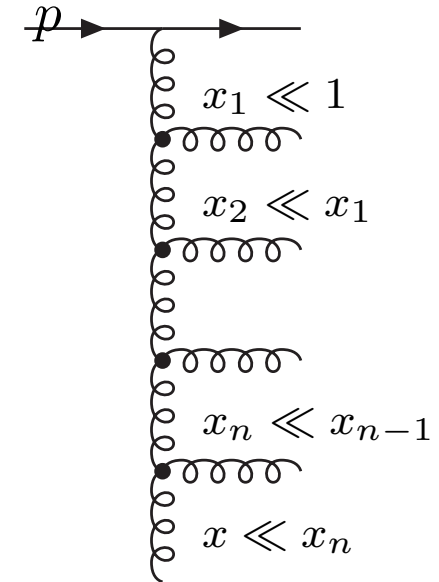
Bremsstrahlung:



$$x \ll 1$$



$$x \ll x_1 \ll 1$$



Probability of emission

$$dP \sim \alpha_s \frac{dk_{\perp}^2}{k_{\perp}^2} \frac{dx}{x}$$

In the small- x limit

$$\int_x^1 \frac{dx_n}{x_n} \dots \int_{x_2}^1 \frac{dx_1}{x_1} \sim \frac{1}{n!} \alpha_s^n \log^n(1/x)$$

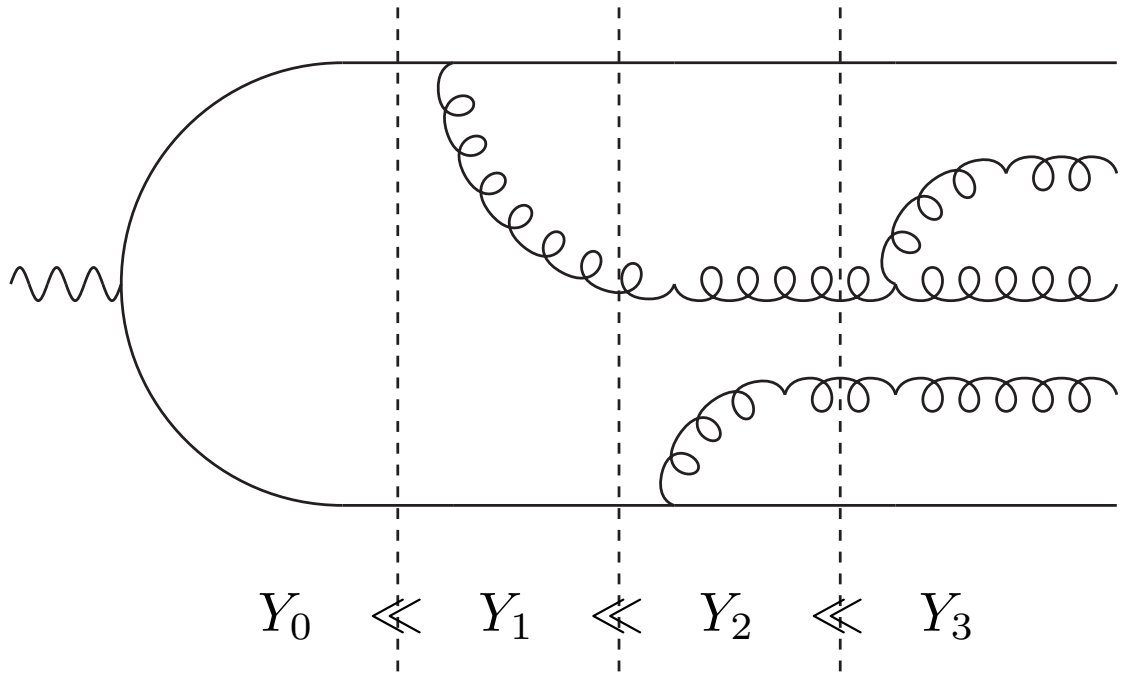
Same order when $\alpha_s \log(1/x) \sim 1$

What are we dealing with?

Perturbative evolution in high-energy QCD

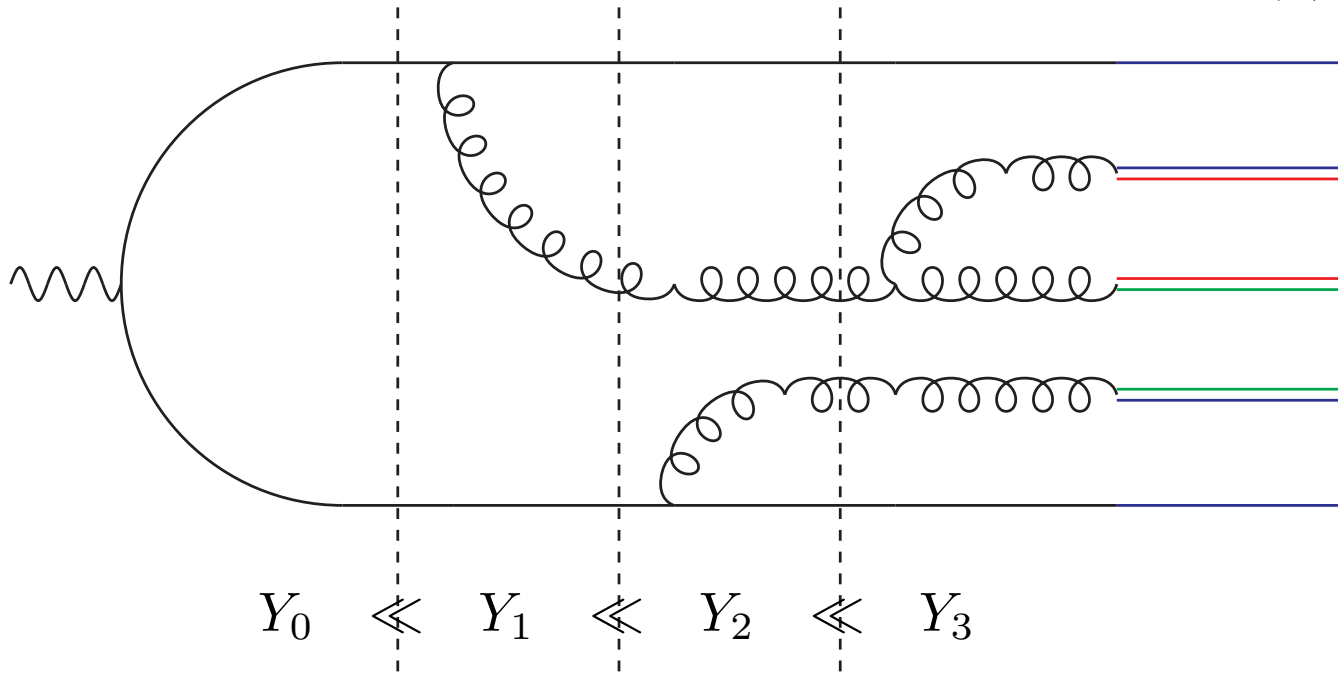
Consider a fast-moving $q\bar{q}$ dipole (Rapidity: $Y = \log(1/x)$)

[Mueller,93]



Consider a **fast-moving $q\bar{q}$ dipole** (Rapidity: $Y = \log(1/x)$)

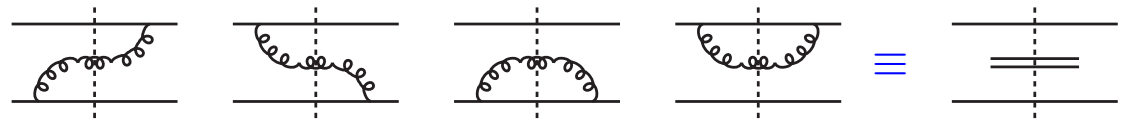
[Mueller,93]



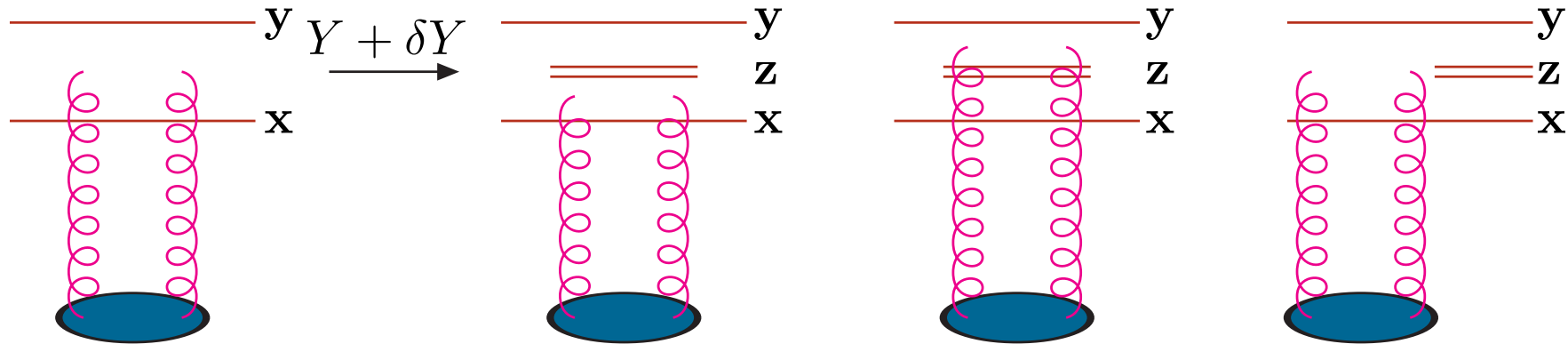
$n(r, Y)$ dipoles
of size r

- Probability $\bar{\alpha}K$ of emission
- 2 degrees of freedom: rapidity Y (longitudinal) + 2D transverse coordinate

Large- N_c approximation:



Consider a small increase in rapidity \Rightarrow **splitting**



$$\partial_Y T(\mathbf{x}, \mathbf{y}; Y)$$

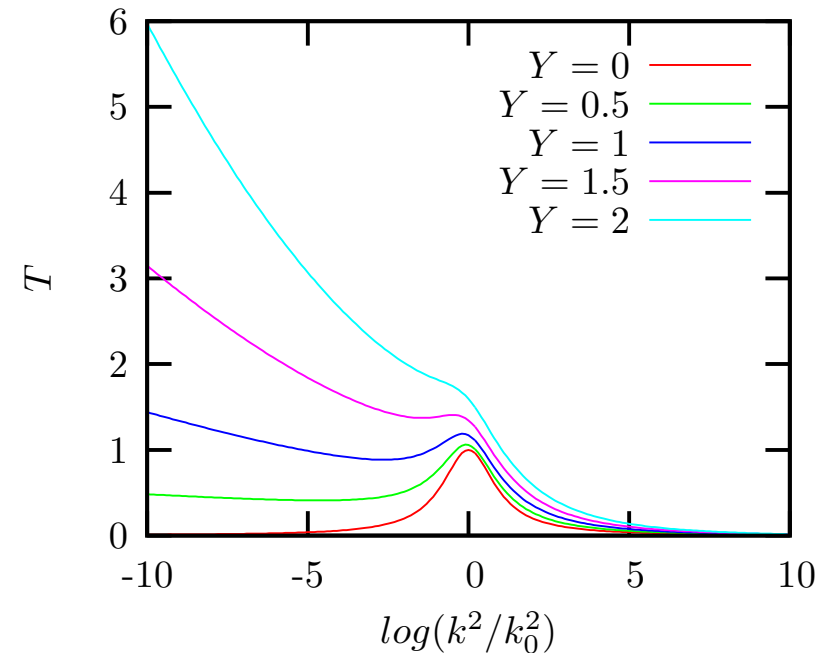
$$= \bar{\alpha} \int d^2 z \underbrace{\frac{(\mathbf{x} - \mathbf{y})^2}{(\mathbf{x} - \mathbf{z})^2 (\mathbf{z} - \mathbf{y})^2}}_{\text{Emission proba from pQCD}} \underbrace{[T(\mathbf{x}, \mathbf{z}; Y) + T(\mathbf{z}, \mathbf{y}; Y) - T(\mathbf{x}, \mathbf{y}; Y)]}_{\text{all possible interactions}}$$

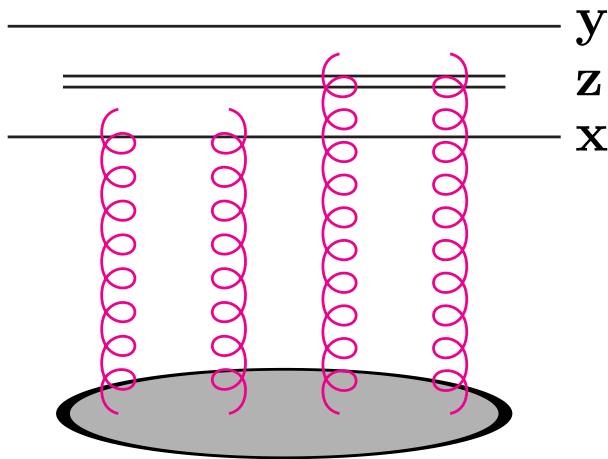
[Balitsky, Fadin, Kuraev, Lipatov, 78]

The solution goes like

$$T(Y) \sim e^{\omega Y} \sim x^{-\omega} \quad \text{with} \quad \omega = 4\bar{\alpha} \log(2) \approx 0.5$$

- Fast growth of the amplitude
- Intercept value too large (NLO, E cons.)
- Violation of the Froissart unitarity:
 $T(Y) \leq C \log^2(s) \quad T(r, b) \leq 1$
- problem of diffusion in the infrared





Multiple scattering

- ★ Proportional to T^2
- ★ important when $T \approx 1$

$\langle \cdot \rangle \equiv$ average over target field

$$\begin{aligned} & \partial_Y \langle T(\mathbf{x}, \mathbf{y}; Y) \rangle \\ &= \bar{\alpha} \int d^2 z \frac{(\mathbf{x} - \mathbf{y})^2}{(\mathbf{x} - \mathbf{z})^2 (\mathbf{z} - \mathbf{y})^2} [\langle T(\mathbf{x}, \mathbf{z}; Y) \rangle + \langle T(\mathbf{z}, \mathbf{y}; Y) \rangle - \langle T(\mathbf{x}, \mathbf{y}; Y) \rangle \\ & \quad - \langle T(\mathbf{x}, \mathbf{z}; Y) T(\mathbf{z}, \mathbf{y}; Y) \rangle] \end{aligned}$$

But

$\partial_Y \langle T(\mathbf{x}, \mathbf{y}; Y) \rangle$ contains a new object: $\langle T(\mathbf{x}, \mathbf{z}; Y) T(\mathbf{z}, \mathbf{y}; Y) \rangle$

In general: complete hierarchy

[Balitsky, 96]

$$\partial_Y \langle T^k \rangle \longrightarrow \underbrace{\langle T^k \rangle}_{\text{BFKL}}, \underbrace{\langle T^{k+1} \rangle}_{\text{saturation}}$$

- Beyond large- N_c : the hierarchy involves quadrupoles, sextupoles, ...
- Balitsky hierarchy \equiv JIMWLK eq. (Colour Glass Condensate formalism)

Mean field approx.: $\langle T_{\mathbf{xz}} T_{\mathbf{zy}} \rangle = \langle T_{\mathbf{xz}} \rangle \langle T_{\mathbf{zy}} \rangle$

$$\partial_Y \langle T_{\mathbf{xy}} \rangle = \frac{\bar{\alpha}}{2\pi} \int d^2 z \frac{(\mathbf{x} - \mathbf{y})^2}{(\mathbf{x} - \mathbf{z})^2 (\mathbf{z} - \mathbf{y})^2} [\langle T_{\mathbf{xz}} \rangle + \langle T_{\mathbf{zy}} \rangle - \langle T_{\mathbf{xy}} \rangle - \langle T_{\mathbf{xz}} \rangle \langle T_{\mathbf{zy}} \rangle]$$

[Balitsky 96, Kovchegov 99]

Simplest perturbative evolution equation satisfying unitarity constraint

What are the solutions to the BK equation?

What are the consequences of saturation?

Case 1: BK equation with no impact parameter dependence

$$T_{\mathbf{xy}} \rightarrow T \left(\mathbf{r} = \mathbf{x} - \mathbf{y}, \mathbf{b} = \frac{\mathbf{x} + \mathbf{y}}{2} \right) \rightarrow T(r)$$

Introduce $\rho = \log(r_0^2/r^2)$ (or $\rho = \log(k_\perp^2/k_0^2)$) in momentum space)

Note:

- all arguments work for $T(r)$ or its Fourier transform $\tilde{T}(k)$
- for \tilde{T} , the non-linear term in the BK equation is simply $-\tilde{T}^2(k)$

$$\text{BK equation: } \partial_Y T = \underbrace{\chi(-\partial_\rho) T}_{\text{BFKL}} - T^2$$

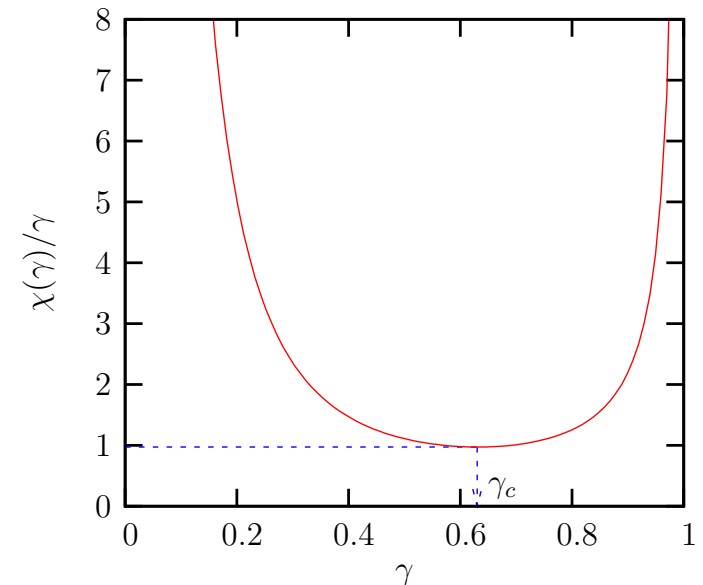
When $T \ll 1$ BFKL works: $\partial_Y T = \chi(-\partial_\rho) T$

Solution known:

$$\begin{aligned} T(k) &= \int \frac{d\gamma}{2i\pi} T_0(\gamma) \exp [\chi(\gamma) \bar{\alpha} Y - \gamma \rho] \\ &= \int \frac{d\gamma}{2i\pi} T_0(\gamma) \exp \left[-\gamma \left(\rho - \frac{\chi(\gamma)}{\gamma} \bar{\alpha} Y \right) \right] \end{aligned}$$

\Rightarrow Wave of (ρ -)slope γ travels at speed $v = \chi(\gamma)/\gamma$

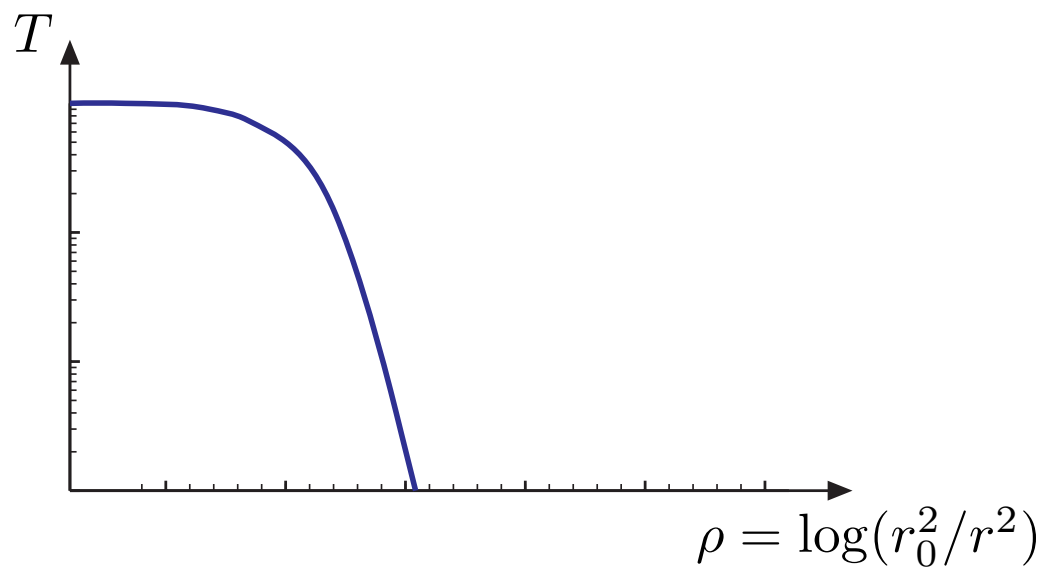
[S.Munier,R.Peschanski,03]



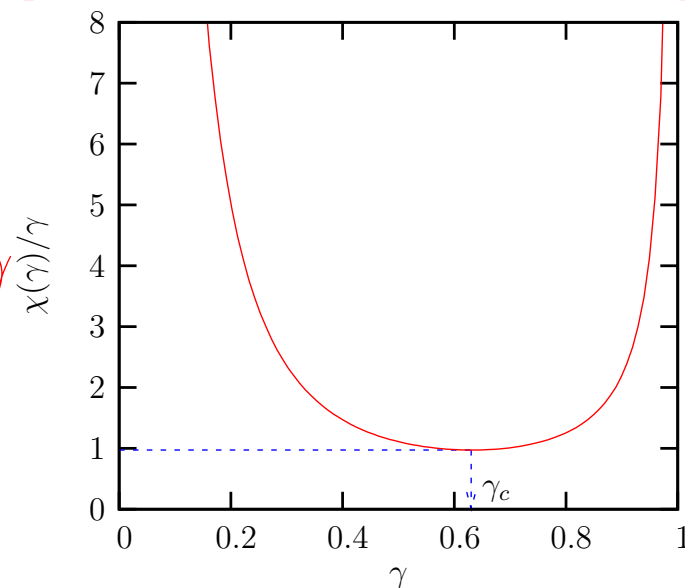
$\frac{\chi(\gamma)}{\gamma}$ min. when $\gamma = \gamma_c$

BK equation: $\partial_Y T = \underbrace{\chi(-\partial_\rho) T}_{\text{BFKL}} - T^2$

⇒ Wave of (ρ -)slope γ travels at speed $v = \chi(\gamma)/\gamma$



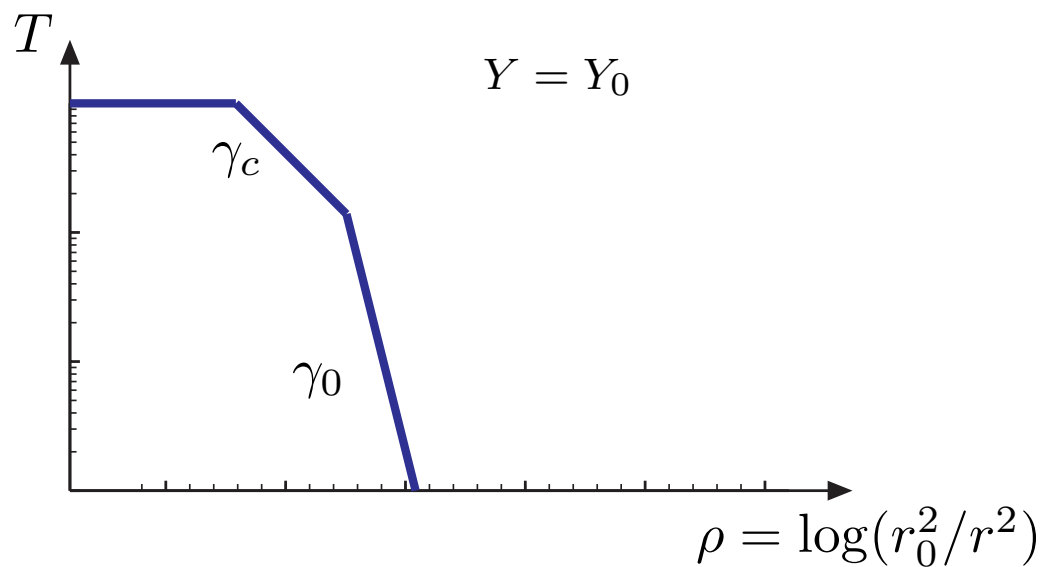
[S.Munier,R.Peschanski,03]



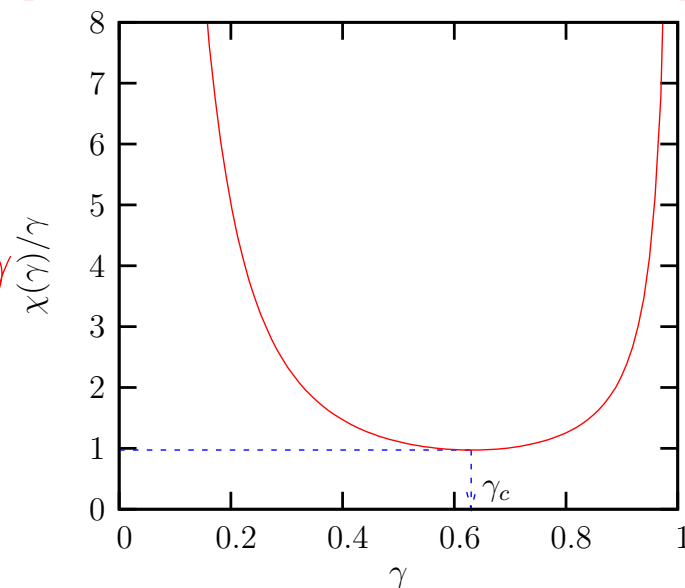
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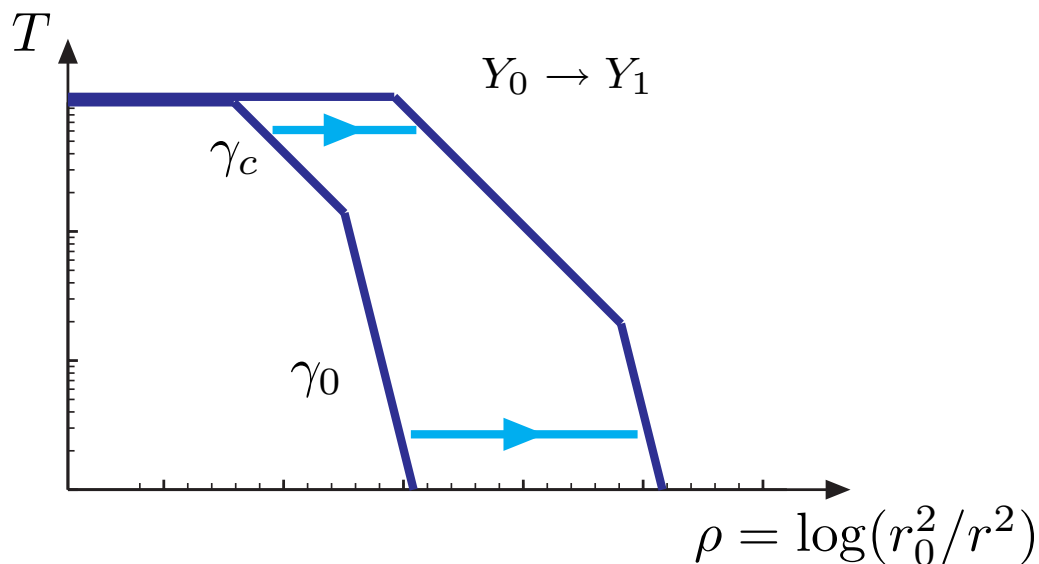
[S.Munier,R.Peschanski,03]



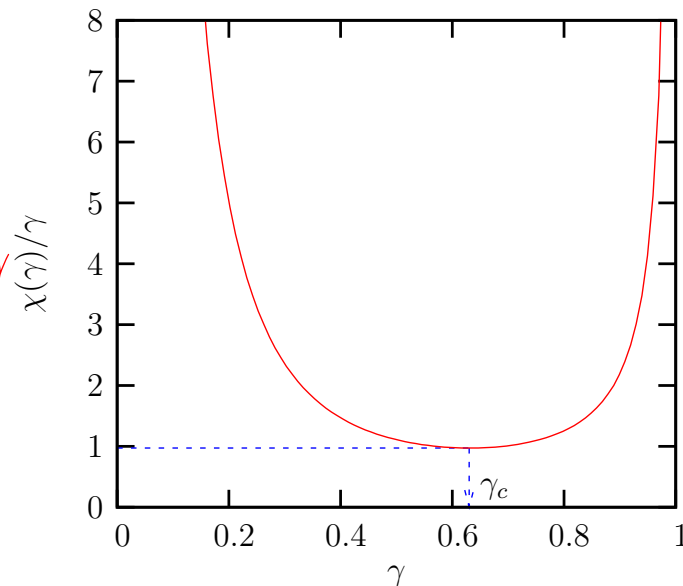
$\frac{\chi(\gamma)}{\gamma}$ min. when $\gamma = \gamma_c$

BK equation: $\partial_Y T = \underbrace{\chi(-\partial_\rho) T}_{\text{BFKL}} - T^2$

⇒ Wave of (ρ -)slope γ travels at speed $v = \chi(\gamma)/\gamma$



[S.Munier,R.Peschanski,03]



$\frac{\chi(\gamma)}{\gamma}$ min. when $\gamma = \gamma_c$

The minimal speed is selected during evolution

This is:

$$T \propto \exp[-\gamma_c (\rho - v_c \bar{\alpha} Y)]$$

Consequence: **geometric scaling** ($Q_s \equiv$ saturation scale \equiv front position)

$$T(r, Y) = T[rQ_s(Y)] \quad \text{with} \quad Q_s^2(Y) = v_c \bar{\alpha} Y$$

$$\stackrel{rQ_s \ll 1}{=} \underbrace{[r^2 Q_s^2(Y)]^{\gamma_c}}_{\text{slope } \gamma_c} \underbrace{\exp \left[\frac{\log^2(r^2 Q_s^2)}{2\chi''(\gamma_c) \bar{\alpha} Y} \right]}_{\text{scaling window}}$$

$$|\log(r^2 Q_s^2)| \lesssim \sqrt{\chi''(\gamma_c) \bar{\alpha} Y}$$

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- Generic arguments: exponential rise + saturation \Rightarrow select γ_c
- Parameters fixed by linear kernel only
- Saturation effects even though $T \ll 1$
- Came initially from a analogy between BK and F-KPP in stat. phys.

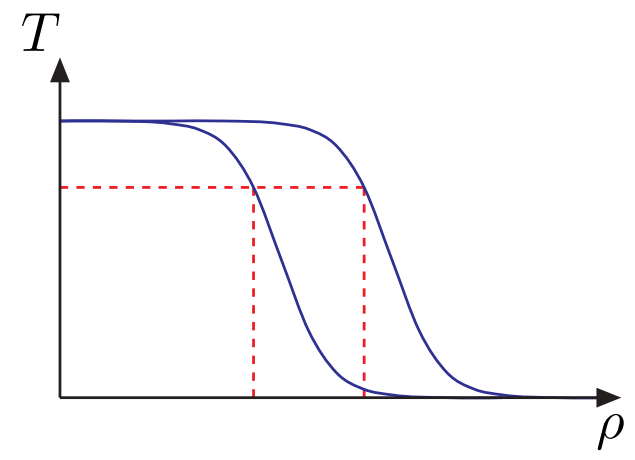
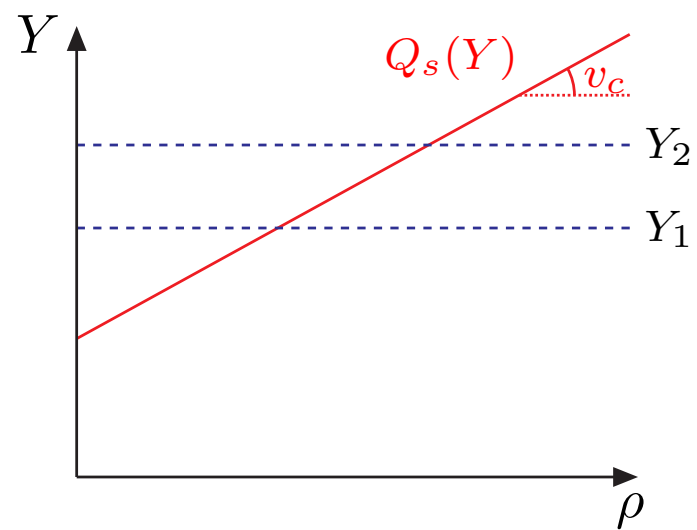
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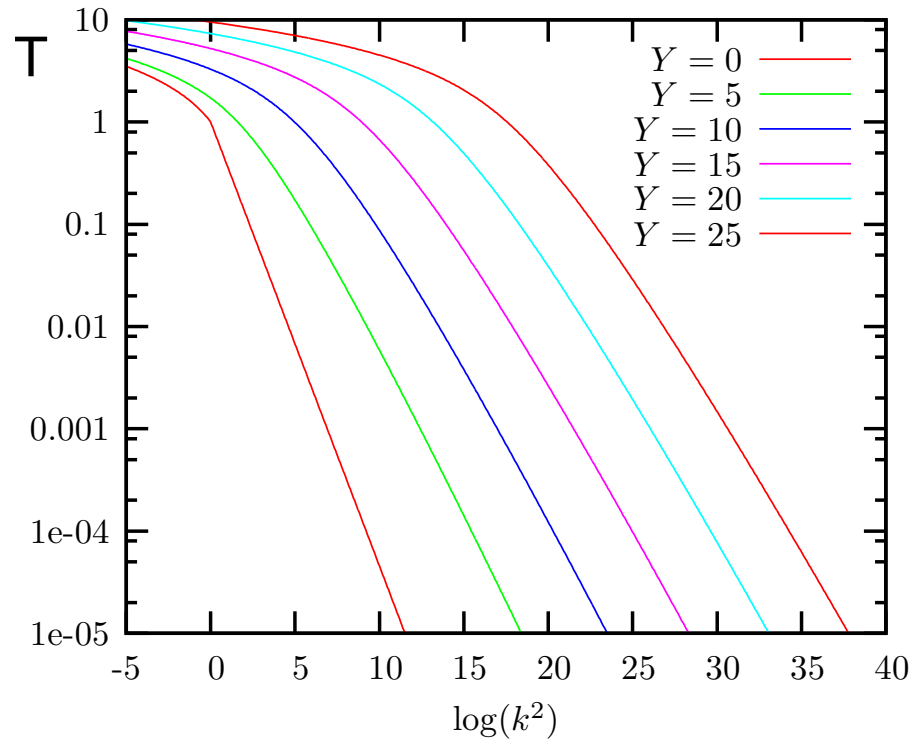
$$|\log(r^2 Q_s^2)| \lesssim \sqrt{\chi''(\gamma_c) \bar{\alpha} Y}$$

Interpretation: **invariance along the saturation line**

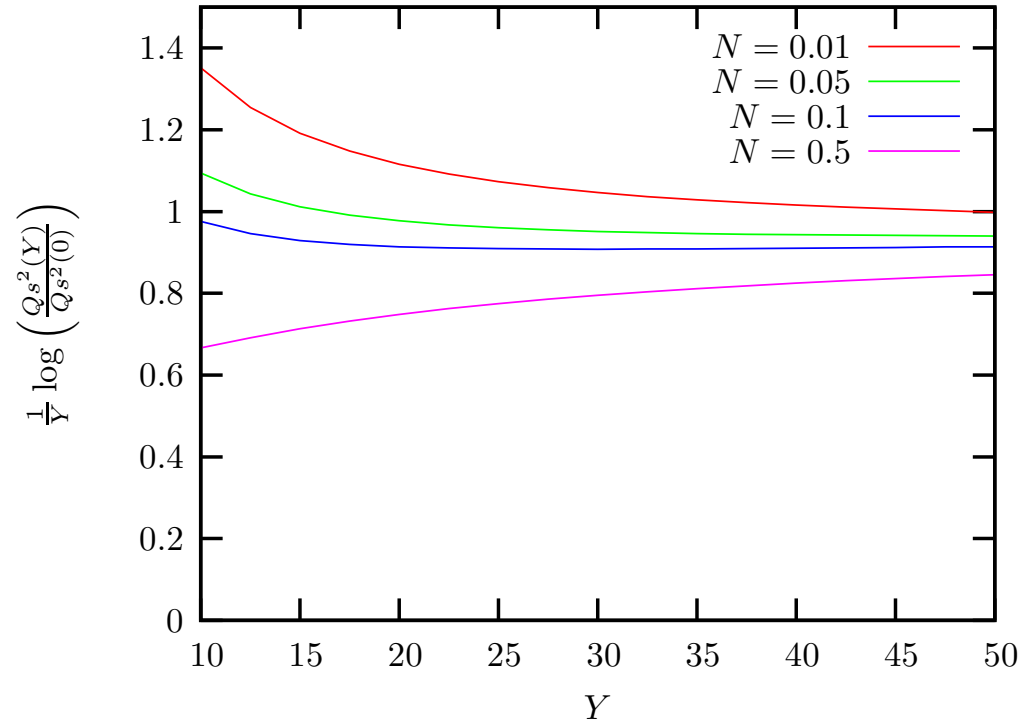
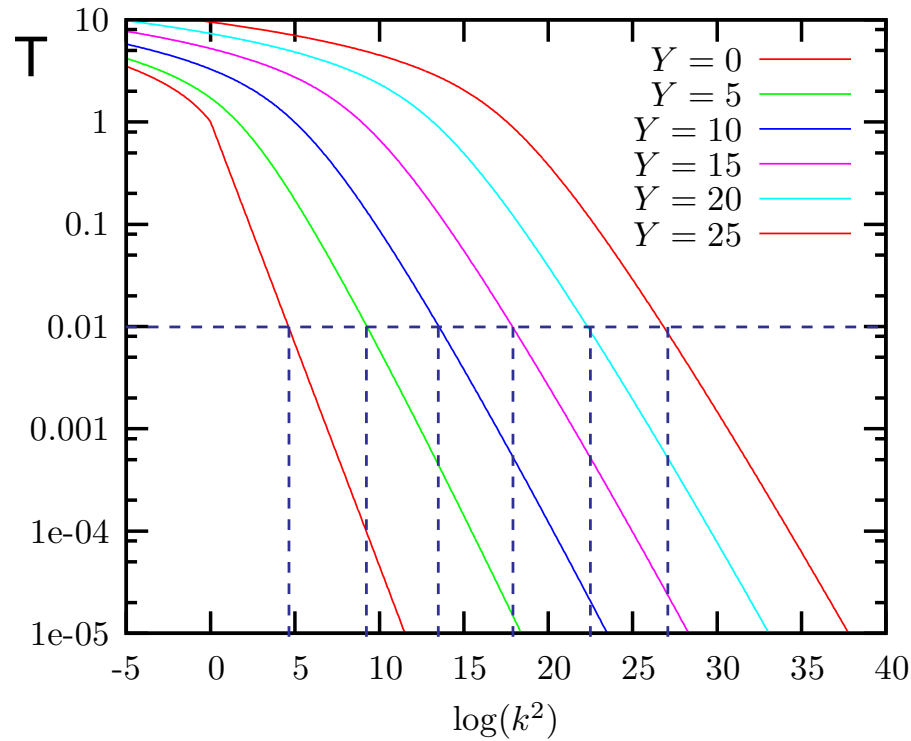


[Iancu, Itakura, McLerran, 02]

Numerical simulations:



Numerical simulations:



$$T(k, Y) = T\left(\frac{k^2}{Q_s^2(Y)}\right) \approx \left(\frac{k^2}{Q_s^2(Y)}\right)^{-\gamma_c}$$

$$Q_s^2(Y) \propto \exp(v_c Y)$$

Case 2: including impact parameter

Go to momentum space: use momentum transfer \mathbf{q}

$$\tilde{T}(\mathbf{k}, \mathbf{q}) = \int d^2x d^2y e^{i\mathbf{k}\cdot\mathbf{x}} e^{i(\mathbf{q}-\mathbf{k})\cdot\mathbf{y}} \frac{T(\mathbf{x}, \mathbf{y})}{(\mathbf{x} - \mathbf{y})^2}$$

new form of the BK equation

$$\begin{aligned} \partial_Y \tilde{T}(\mathbf{k}, \mathbf{q}) &= \frac{\bar{\alpha}}{\pi} \int \frac{d^2k'}{(k - k')^2} \left\{ \tilde{T}(\mathbf{k}', \mathbf{q}) - \frac{1}{4} \left[\frac{k^2}{k'^2} + \frac{(q - k)^2}{(q - k')^2} \right] \tilde{T}(\mathbf{k}, \mathbf{q}) \right\} \\ &- \frac{\bar{\alpha}}{2\pi} \int d^2k' \tilde{T}(\mathbf{k}, \mathbf{k}') \tilde{T}(\mathbf{k} - \mathbf{k}', \mathbf{q} - \mathbf{k}') \end{aligned}$$

[C.Marquet, R.Peschanski, G.S., 05]

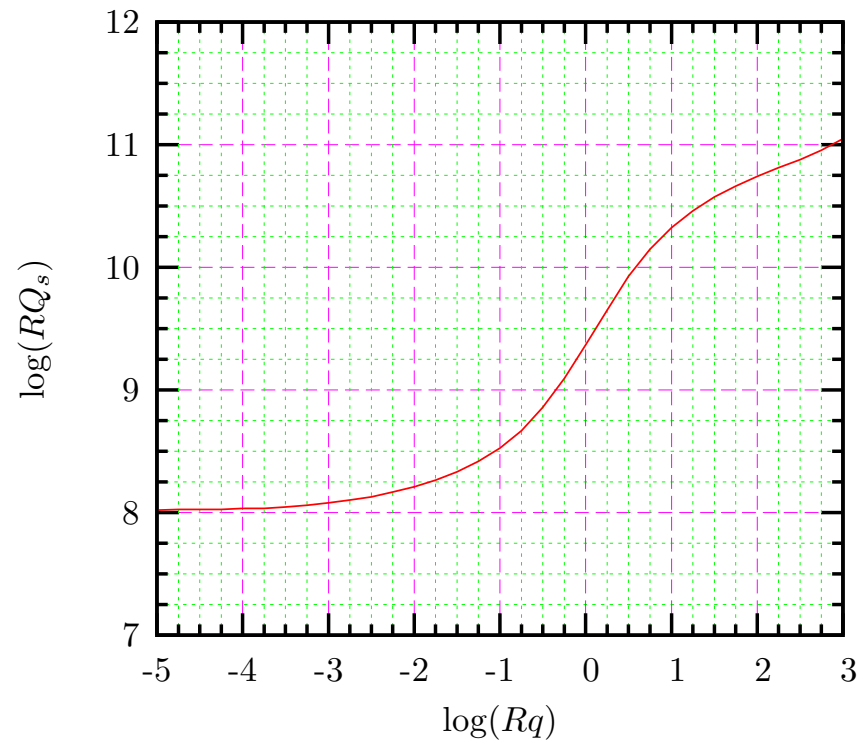
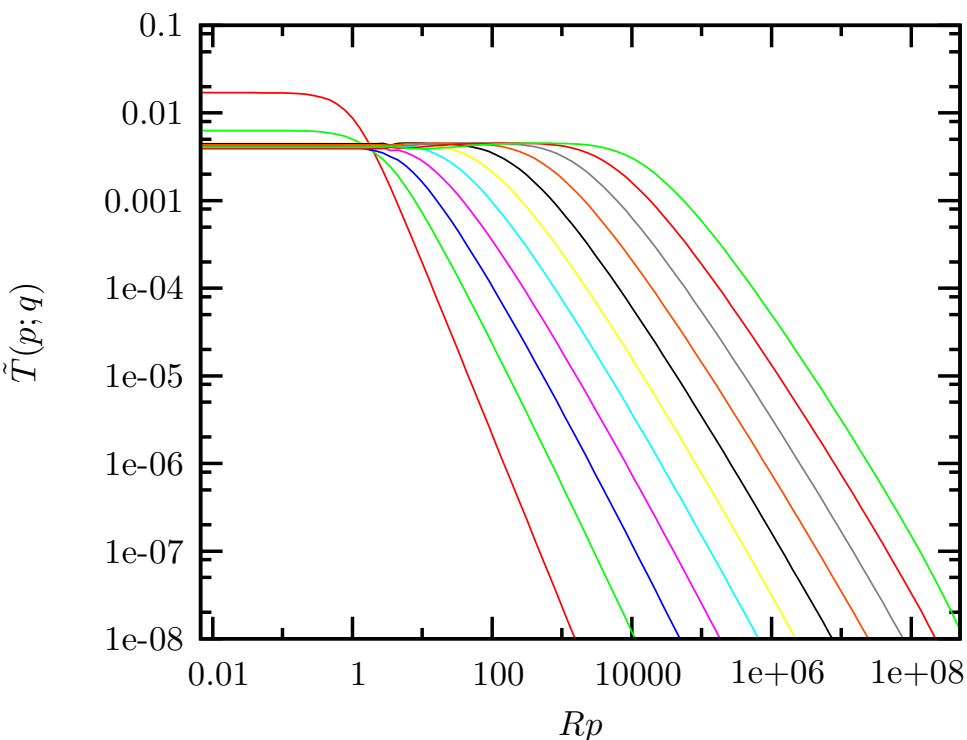
1. Study BFKL with both k and q dependences
2. Look for power decreases in the tail ($k \gg q, k_0, k_0 \equiv$ target scale)

2 possible situations:

- $q \gg k_0 \Rightarrow$ tail given by $e^{\bar{\alpha}\chi(\gamma)Y} (k^2/q^2)^{-\gamma}$
- $q \ll k_0 \Rightarrow$ tail given by $e^{\bar{\alpha}\chi(\gamma)Y} (k^2/k_0^2)^{-\gamma}$

\Rightarrow same selection mechanism with different reference scale:

$$T(k, q; Y) \propto \left[\frac{k^2}{Q_s^2(q, Y)} \right]^{-\gamma_c} \quad \text{with } Q_s^2(q, Y) = \begin{cases} k_0^2 e^{v_c Y} & \text{if } q \ll k_0 \\ q^2 e^{v_c Y} & \text{if } q \gg k_0 \end{cases}$$



One can prove **analytically** that:

- traveling wave at large k : BFKL \Rightarrow **same** γ_c, v_c
- q dependence: $Q_s^2(q, Y) = \min(k_0^2, q^2) \exp(v_c Y)$

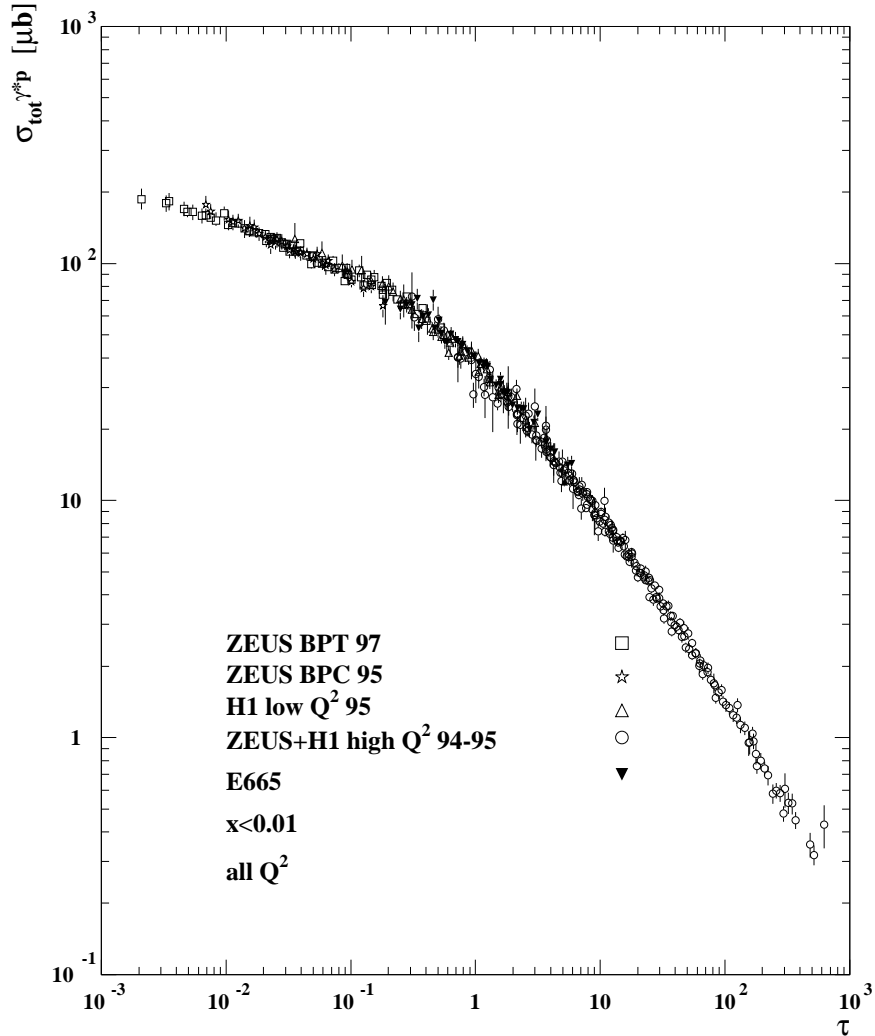
Predicts geometric scaling for t -dependent processes

Phenomenology

1. Geometric scaling in F_2

1.1. direct observation

[A.Stasto, K.Golec-Biernat, Kwiecinski, 01]

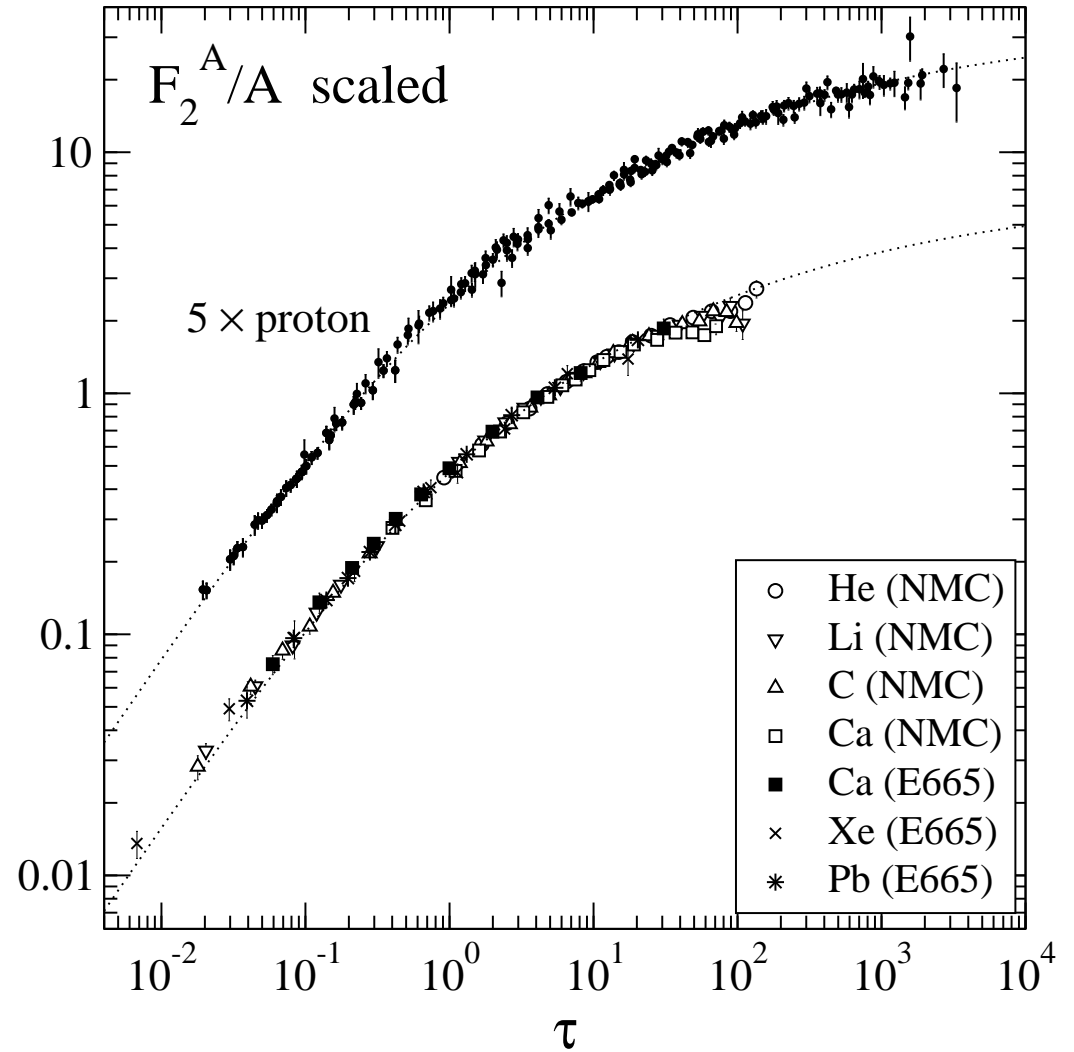


$$\sigma^{\gamma^* p}(Q^2, x) = \sigma(\tau)$$

$$\tau = \tau(Q^2, x) = Q^2 / Q_s^2(x)$$

[Freund, Rummukainen, Weigert, Schafer, 03]

Also observed in
 eA collisions



[F.Gelis, R.Peschanski, L.Schoeffel, G.S., 07

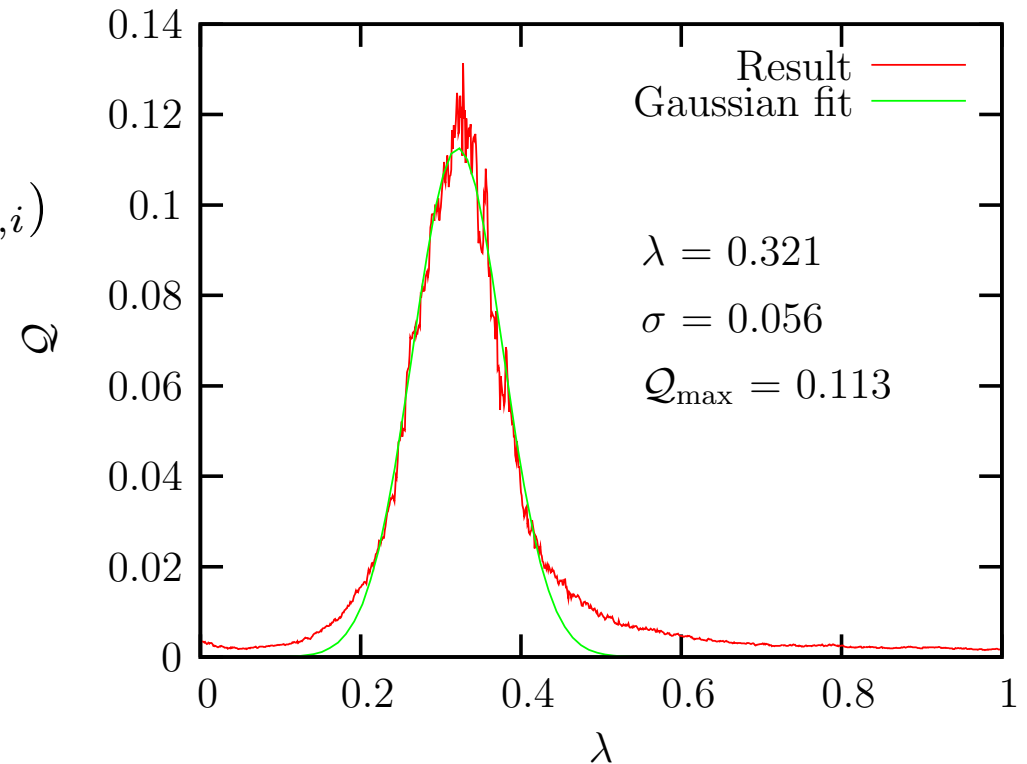
Systemtic approach to find scaling relations ?

Starting point:

Set of points: $(\tau_i = \tau(Q_i^2, Y_i), F_{2,i})$

→ Quality factor: Q

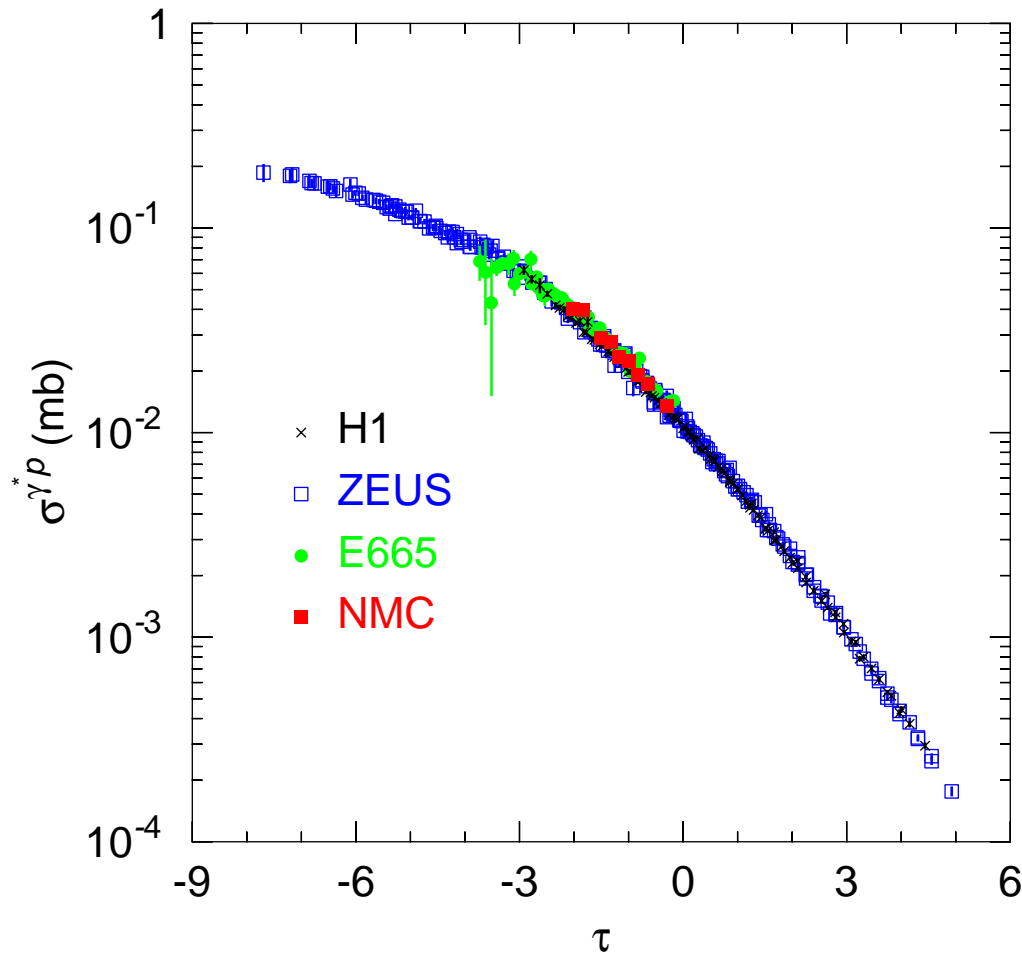
Large when points on a curve.



Example: $\tau = \log(Q^2) - \log(Q_s^2(Y))$ with $\log(Q_s^2(Y)) = \lambda Y$

Scan in $\lambda \Rightarrow \lambda \approx 0.321$

[F.Gelis, R.Peschanski, L.Shoeffel, G.S., 07]



$$\sigma^{\gamma^* p}(Q^2, x) = \sigma(\tau)$$

$$\tau = \log(Q^2) - \log(Q_s^2(Y))$$

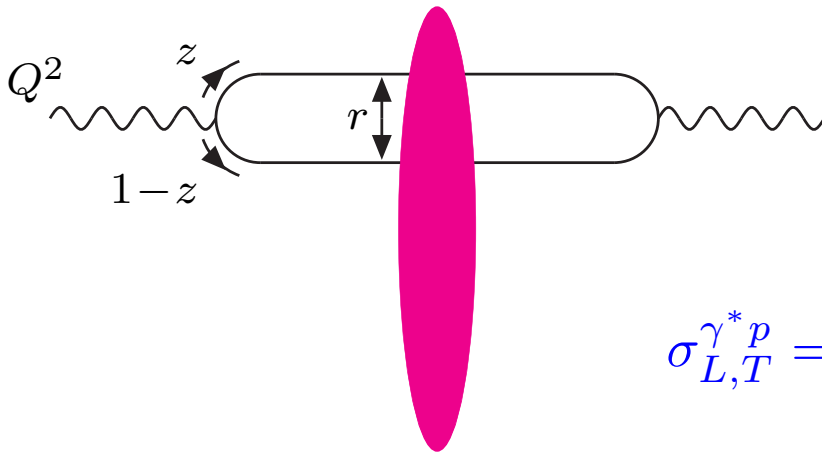
- BK with fixed coupling:
 - $\log(Q_s^2(Y)) = \lambda Y$
 - $\lambda \approx 0.32$
- BK with running coupling:
 - $\log(Q_s^2(Y)) = \lambda \sqrt{Y}$
 - $\lambda \approx 1.62$

Phenomenology

- 1. Geometric scaling in F_2**
 - 1.2. QCD/saturation description**

Factorisation formula:

[E.Iancu, K.Itakura, S.Munier, 03]



$$\sigma_{L,T}^{\gamma^*p} = \int d^2r \int_0^1 dz \overbrace{|\Psi_{L,T}(z, r; Q^2)|^2}^{\text{from pQED}} 2\pi R_p^2 T(\mathbf{r}; Y)$$

param for dipole amplitude: scaling variable $\tau = \log(r^2 Q_s^2/4)$

$$T(r; Y) = \begin{cases} T_0 \exp\left(\gamma_c \tau - \frac{\tau^2}{2\bar{\alpha}\chi_c'' Y}\right) & \text{if } rQ_s < 2 \quad (\text{travelling wave}) \\ 1 - \exp[-a(\tau + b)^2] & \text{if } rQ_s > 2 \quad (\text{dense BK}) \end{cases}$$

$Q_s^2(Y) = k_0^2 \exp(\lambda Y) \longrightarrow \gamma_c, \chi_c''$ from LO BFKL, 3 parameters: λ, k_0, R_p
 $\Rightarrow \lambda \approx 0.25$, in agreement with NLO BFKL predictions.

Phenomenology

2. Massive quarks effects

- No heavy quark in the IIM model
- Some other model does but
 - are not fully QCD-based
 - have clearly lower saturation momentum

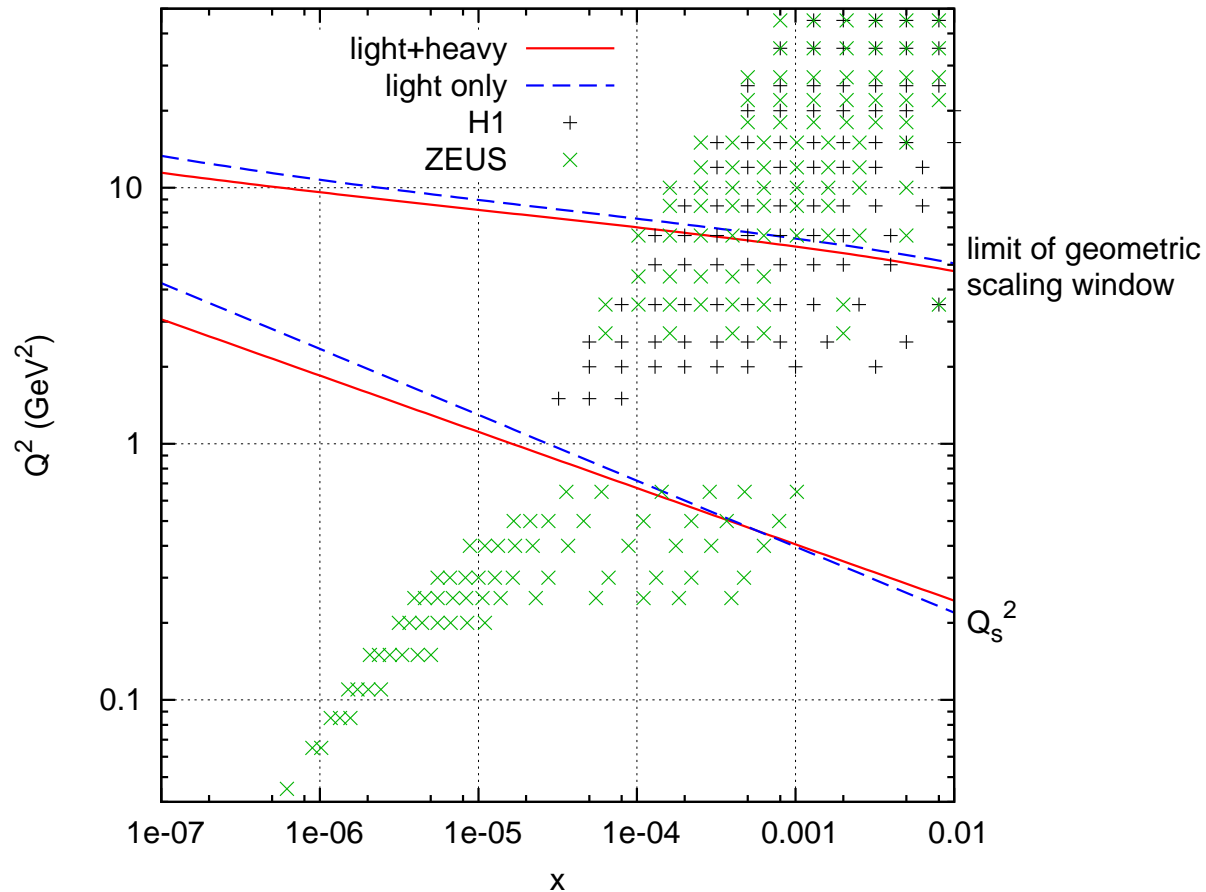
Aim: Include the charm in the IIM model ?

Key issue: alloc γ_c to vary !

Data: ZEUS and H1 (5% renorm.) Domain: $x \leq 0.01$, $Q^2 \leq 150 \text{ GeV}^2$

model	γ_c	v_c	x_0	R_p	χ^2/n
IIM	0.6275	0.253	$2.67 \cdot 10^{-5}$	3.250	≈ 0.9
IIM+c	0.6275	0.195	$6.42 \cdot 10^{-7}$	3.654	1.109
	0.7065	0.222	$1.19 \cdot 10^{-5}$	3.299	0.963

Note: $\gamma_c \approx 0.7$ is in better agreement with NLO BFKL predictions



- Saturation scale ~ 1 GeV
- Saturation important up to the limit of the geometric scaling window
i.e. well outside the “saturated” region

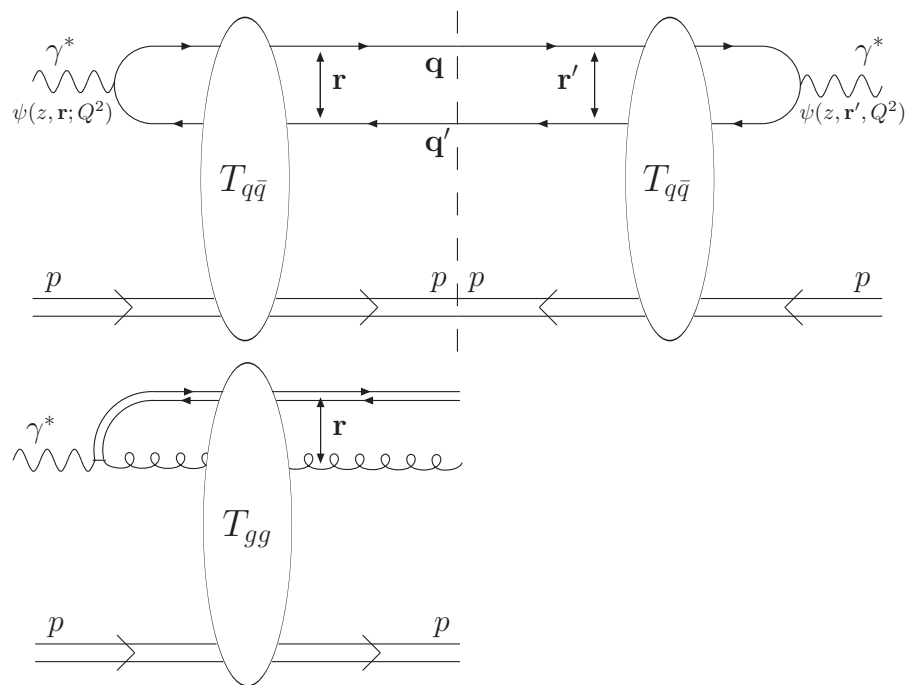
Phenomenology

3. Diffraction and exclusive processes

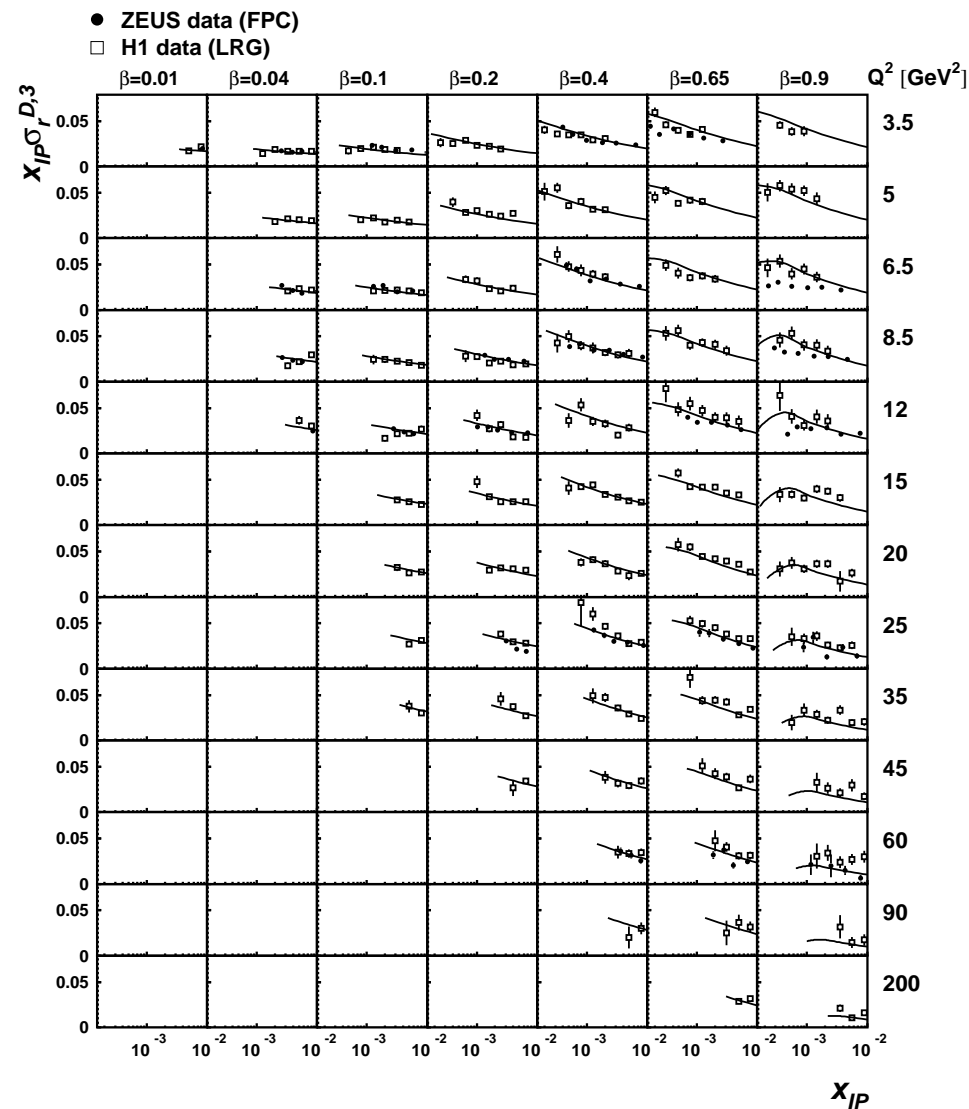
3.1. Diffractive structure function

Same kind of factorisation
but more contribs:

$$F_2^D = F_2^{D(q\bar{q})} + F_2^{D(q\bar{q}g)} + \dots$$



Basically: $F_2^D \propto T^2$
with the same T as for F_2



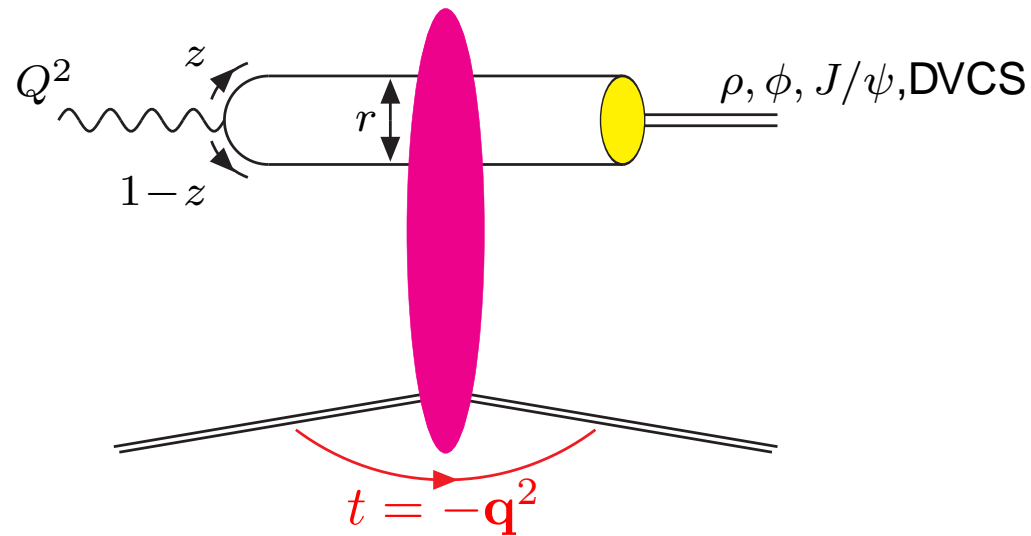
[Marquet, 07]

Phenomenology

3. Diffraction and exclusive processes

3.2. Geometric scaling in vector-meson production

[C.Marquet, R.Peschanski, G.S., 07]



Factorisation formula:

$$\mathcal{A}_{L,T}^{\gamma^* p \rightarrow V p} = i \int d^2 r \int_0^1 dz \Psi_{L,T}(z, r; Q^2) \Psi_{L,T}^*(z, r, q; M_V^2) e^{i z \mathbf{q} \cdot \mathbf{r}} \sigma_{\text{dip}}(\mathbf{r}, \mathbf{q}; Y)$$

\$\rightarrow \frac{d\sigma}{dt}, \sigma_{\text{el}}\$ for \$\rho, \phi, J/\psi, \text{DVCS}\$

- photon wavefunction: from QED
Vector-mesons wavefunction: Boosted-Gaussian or Light-cone Gaussian
- dipole amplitude:

$$\sigma_{\text{dip}}(r, q; Y) = 2\pi R_p^2 e^{-b|t|} T_{\text{GS}}(r, Q_s^2(q, Y))$$

- Normalisation: only one slope b (no Q^2 dependence)
- T -matrix: t -dependent saturation scale from theoretical predictions:

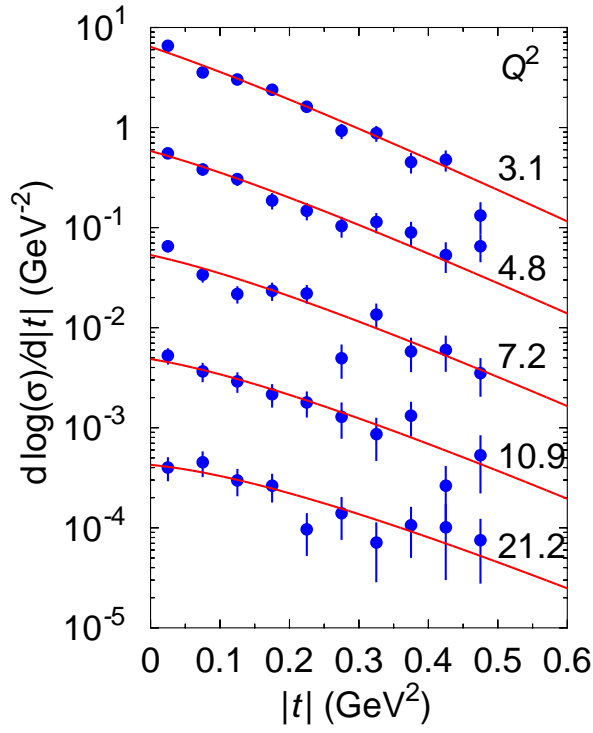
$$Q_s^2 = k_0^2 e^{\lambda Y} \quad \rightarrow \quad Q_s^2 = k_0^2 (1 + c|t|) e^{\lambda Y}$$

Hence:

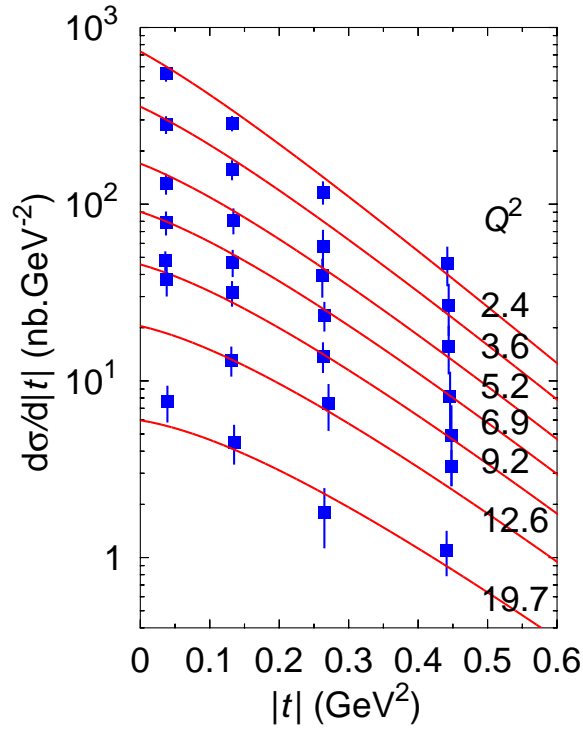
$$b, c \quad \rightarrow \quad \left. \frac{d\sigma}{dt}, \sigma_{\text{el}} \right|_{\rho, \phi, J/\Psi} \quad (269 \text{ data})$$

Example: differential cross-section:

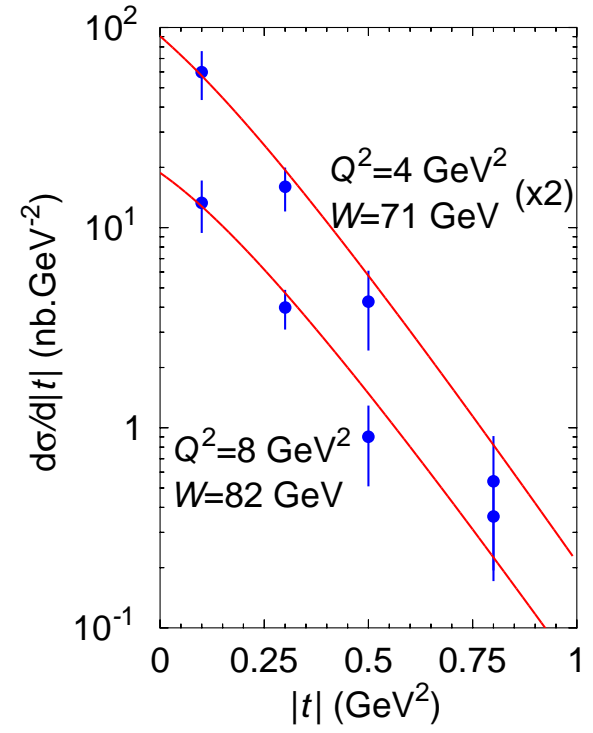
$\gamma^* p \rightarrow \rho p$



$\gamma^* p \rightarrow \phi p$



pred. for DVCS



Part 1: Evolution equations towards high-energy

- The BK equation contains both BFKL exchanges and unitarity/saturation corrections
- Recently: pomeron-loops equations

Part 2: Solutions for scattering amplitudes

- BK equation \Rightarrow geometric scaling
 - At fixed impact parameter
 - At nonzero momentum transfer
- Recently: pomeron-loops equations \Rightarrow geometric + diffusive scaling

Part 3: Phenomenological consequences

We focused on the HERA phenomenology:

- geometric scaling for F_2 :
 - Direct analysis of the data (*Quality factor*)
 - Iancu-Itakura-Munier model + new extension with charm
- geometric scaling in vector-meson production and DVCS
 t -dependence from pQCD instead of b -dependence postulated

▷▷ indications for saturation

▷▷ can help understanding for other experiments

Question 1: Is that really a saturation effect?

i.e. can we have geometric scaling from “something else” e.g. DGLAP evolution or energy-conservation in BFKL or non-perturbative effects?

- DGLAP & E -cons in BFKL have 2 problems:
 - They apply only to a limited phase-space at large Q^2
 - They come from UV physics (while Q_s is a IR regulator)
- We go further than non-perturbative effects since Q_s can be a perturbative scale

Question 2: What about RHIC physics?

- CGC & Q_s are present in many RHIC analysis
- Studies of particle production in dA collisions (e.g. KKT model)
- No combined HERA+RHIC analysis so far (**under study**)

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Questions ≥ 3 : ...?