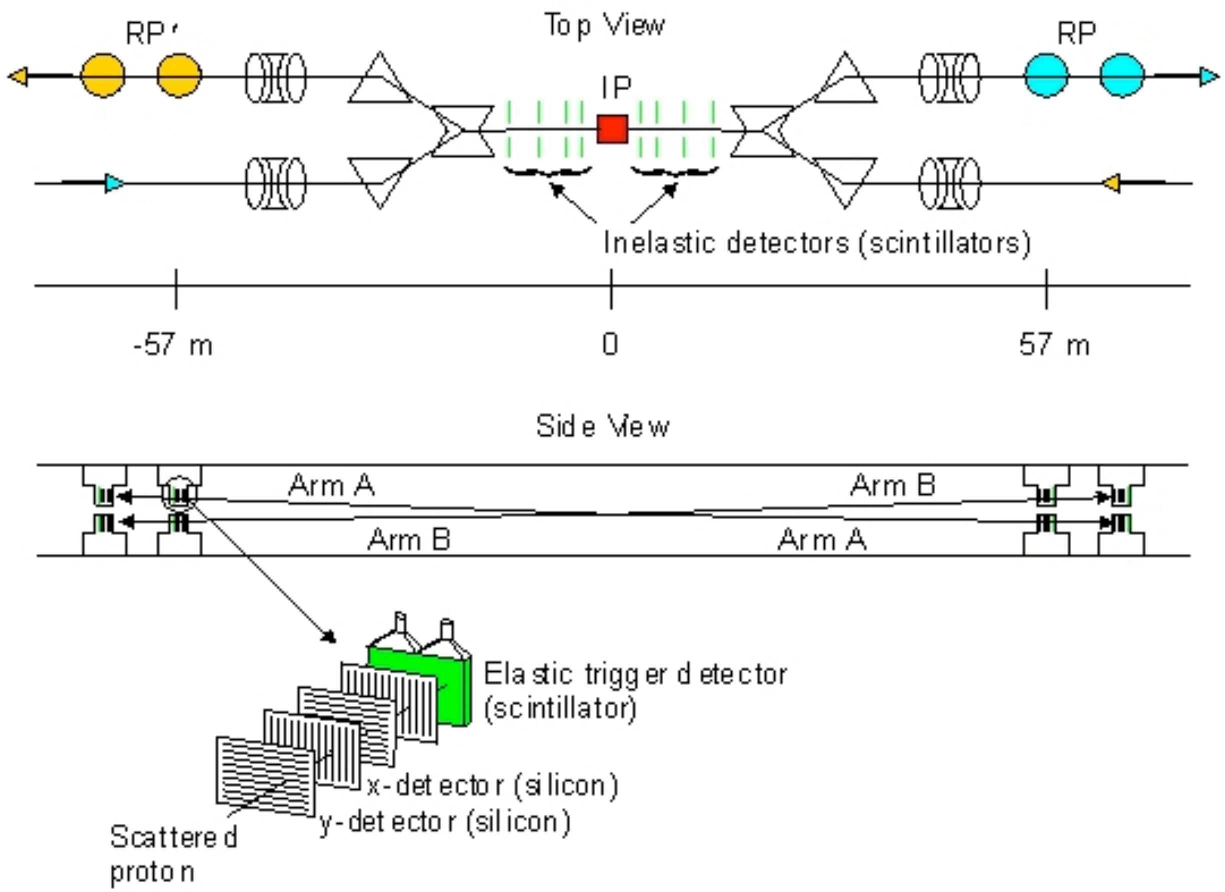
Measuring small scattering angles at the colliders



- The protons collide at the interaction region (IR) in a local coordinate system at a vertical distance $y*$ from the reference orbit and scatter with an angle θ_y*.

- Since the scattering angles are small, protons follow trajectories determined by the lattice of the accelerator until they reach the detector, which measures the positions of the scattered particles with respect to the reference orbit.

- Hence, the known parameters of the accelerator lattice can be used to calculate the deflection $y*$ and the scattering angle θ_y* at the interaction point, knowing the deflection y at the detector.

- Proton momenta are not parallel, the angular spread is given by the beam angular divergence:

$$\theta = \sqrt{\frac{\varepsilon}{6\pi\beta^*}}$$

- The beam size at the collision point is given by:

$$\sigma_{x,y} = \sqrt{\frac{\varepsilon\beta^*}{6\pi}}$$

4. At a point where the phase advance from the interaction point is Ψ and the betatron function is β, y is given by:

$$y = \sqrt{\frac{\beta}{\beta^*}}\left(\cos\Psi + \alpha^*\sin\Psi\right)y^* + \sqrt{\beta\beta^*}\sin\Psi\theta_y^*,$$

5. Where α^* is the derivative of the betatron function β^* at the interaction point. We have considered a lattice configuration such that α^* is very close to zero. Eq. (4) can be rewritten as:

$$y = a_{11}y^* + L_{eff}\theta_y^*, \quad \text{with} \quad a_{11} = \sqrt{\frac{\beta}{\beta^*}}\left(\cos\Psi + \alpha^*\sin\Psi\right) \quad \text{and} \quad L_{eff} = \sqrt{\beta\beta^*}\sin\Psi$$

6. The optimum experimental condition is $a_{11} = 0$ and L_{eff} as large as possible, as the displacement y becomes independent of the coordinate y^* at the IR in the transverse plane of the accelerator, and largest displacements at the detection point are obtained for a given scattering angle. This occurs when Ψ is an odd multiple of $\pi/2$. Then, the expression for the y coordinate at the detection point simplifies to:

$$\theta^*_{\min} = \frac{d_{\min}}{L_{eff}}, \text{ with } \quad d_{\min} = k\sigma_y + d_0$$

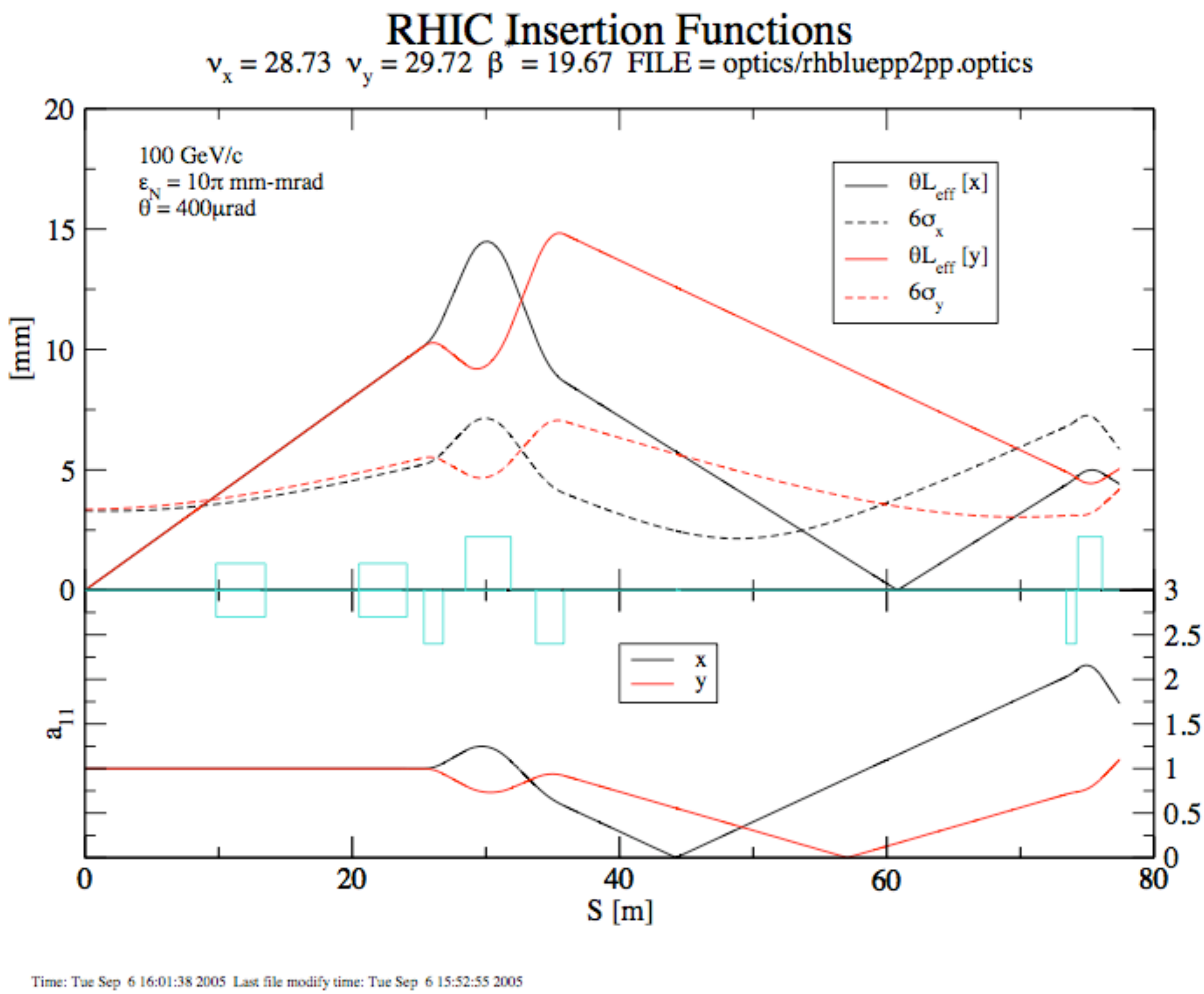
$$y = L_{eff}\theta_y^*,$$

and the scattering angle is determined just from the measurement of the displacement alone. With the above condition satisfied, rays that are parallel to each other at the interaction point are focused onto a single point at the detector, commonly called "parallel-to-point focusing."

For the measurement of the smallest measurable value of $|t|$, $t_{\mathbf{min}}$, must be optimized. $t_{\min} = (p\theta^*_{\mathbf{min}})^2$ is determined by the smallest scattering angle measured $\theta^*_{\mathbf{min}}$, whichis given by:

$$t_{\min} = \frac{k^2 \varepsilon p^2}{\beta^*}.$$

It follows also that special beam conditions are needed to reach smallest t: $t_{\mathbf{min}}$ is reached by having β∗ as large as possible and by reducing the *k*-factor and the emittance ε, i.e. by optimizing the beam scraping.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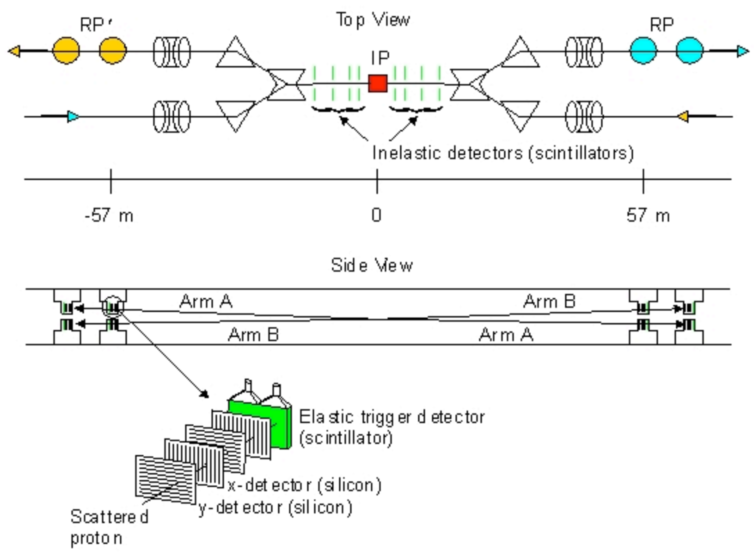
Principle of the Measurement



- Protons scattered at very small scattering angle θ^*, hence beam transport magnets determine trajectory scattered protons
- The optimal position for the detectors is where scattered protons are well separated from beam protons
- Need Roman Pot to measure scattered protons close to the beam without breaking accelerator vacuum

Beam transport equations **relate measured position at the detector to scattering angle**.

$$\begin{pmatrix} x_D \\ \Theta_D^x \\ y_D \\ \Theta_D^y \end{pmatrix} = \begin{pmatrix} a_{11} & L_{eff}^x & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & L_{eff}^y \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \begin{pmatrix} x_0 \\ \Theta_x^* \\ y_0 \\ \Theta_y^* \end{pmatrix}$$

x_0, y_0: Position at Interaction Point

Θ^*_x Θ^*_y : Scattering Angle at IP

x_D, y_D : Position at Detector

Θ^x_D, Θ^y_D : Angle at Detector

Reconstruction of the Proton Momentum Loss ξ

1. Need to measure vector at the detection point, hence two RPs are needed on each side of STAR.

2. For a proton, which scatters with Θ and ξ we have:

$$x_1 = a_1 x_0 + L_1 \Theta_x + \eta_1 \xi\,; \quad \textbf{detection point 1}$$
$$x_2 = a_2 x_0 + L_2 \Theta_x + \eta_2 \xi\,; \quad \textbf{detection point 2}$$

⇐ Accelerator transport

$$\begin{pmatrix} \Theta_x \\ \xi \end{pmatrix} = \frac{1}{Det} \begin{pmatrix} \eta_2; & -\eta_1 \\ -L_2; & -L_1 \end{pmatrix} \begin{pmatrix} x_1 - a_1 x_0 \\ x_2 - a_2 x_0 \end{pmatrix}$$

$$M_X = \sqrt{\xi_1 \xi_2 s} \approx 2\xi \cdot p \Rightarrow \text{For } M_X = 2 \text{ GeV } \xi = 0.01$$

Because Θ and ξ are small special focusing is needed

The pp2pp Experimental Setup