The impact parameter saturation model and nuclei

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Outline

- How to extend saturation model fits from protons to nuclei?
- What is the A dependence of Q_{s}?

This is (practically entirely) a literature review talk.

- K. Golec-Biernat and M. Wusthoff, *Phys. Rev.* **D59** (1999) 014017 [hep-ph/9807513].
- K. Golec-Biernat and M. Wusthoff, *Phys. Rev.* **D60** (1999) 114023 [hep-ph/9903358].
- J. Bartels, K. Golec-Biernat and H. Kowalski, *Phys. Rev.* **D66** (2002) 014001 [hep-ph/0203258].
- A. Freund, K. Rummukainen, H. Weigert and A. Schafer, *Phys. Rev. Lett.* **90** (2003) 222002 [hep-ph/0210139].
- H. Kowalski and D. Teaney, *Phys. Rev.* **D68** (2003) 114005 [hep-ph/0304189].
- H. Kowalski, L. Motyka and G. Watt, *Phys. Rev.* **D74** (2006) 074016 [hep-ph/0606272].
- N. Armesto, C. A. Salgado and U. A. Wiedemann, *Phys. Rev. Lett.* **94** (2005) 022002 [hep-ph/0407018].

Misc. other literature

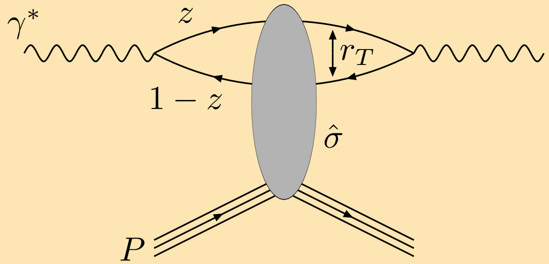
- E. Gotsman, E. Levin, M. Lublinsky, U. Maor and K. Tuchin, *Phys. Lett.* **B492** (2000) 47 [hep-ph/9911270].
- N. Armesto, A. Capella, A. B. Kaidalov, J. Lopez-Albacete and C. A. Salgado, *Eur. Phys. J.* **C29** (2003) 531 [hep-ph/0304119].
- J. Bartels, E. Gotsman, E. Levin, M. Lublinsky and U. Maor, *Phys. Rev.* **D68** (2003) 054008 [hep-ph/0304166].
- E. Iancu, K. Itakura and S. Munier, *Phys. Lett.* **B590** (2004) 199 [hep-ph/0310338].
- M. S. Kugeratski, V. P. Goncalves and F. S. Navarra, *Eur. Phys. J.* **C46** (2006) 413 [hep-ph/0511224].
- V. P. Goncalves, M. S. Kugeratski, M. V. T. Machado and F. S. Navarra, *Phys. Lett.* **B643** (2006) 273 [hep-ph/0608063].
- E. Levin and M. Lublinsky, *Nucl. Phys.* **A696** (2001) 833 [hep-ph/0104108].
- J. Bartels, E. Gotsman, E. Levin, M. Lublinsky and U. Maor, *Phys. Lett.* **B556** (2003) 114 [hep-ph/0212284].
- E. Gotsman, E. Levin, M. Lublinsky and U. Maor, *Eur. Phys. J.* **C27** (2003) 411 [hep-ph/0209074].

Basics: small-x and dipole cross section

$$\sigma_{\rm dip}(x, \mathbf{r}_T) = \int \mathrm{d}^2\mathbf{b}_T \frac{\sigma_{\rm dip}(x,\mathbf{r}_T,\mathbf{b}_T)}{\mathrm{d}^2\mathbf{b}_T}$$

$$\sigma_{\rm dip}(x, \mathbf{r}_T, \mathbf{\Delta}) = \int \mathrm{d}^2\mathbf{b}_T \frac{\sigma_{\rm dip}(x,\mathbf{r}_T,\mathbf{b}_T)}{\mathrm{d}^2\mathbf{b}_T} e^{i\mathbf{b}_T\cdot\mathbf{\Delta}},$$

$$\mathbf{\Delta}^2 = -t$$



Total $\gamma^* p$: $\sigma^{\gamma^* p}_{L,T} = \int \mathrm{d}^2\mathbf{r}_T \int \mathrm{d}z \left|\Psi^{\gamma}_{L,T}(Q^2, \mathbf{r}_T, z)\right|^2 \sigma_{\rm dip}(x, \mathbf{r}_T)$

Total diff.: $\frac{\sigma^D_{L,T}}{\mathrm{d}t} = \frac{1}{16\pi} \int \mathrm{d}^2\mathbf{r}_T \int \mathrm{d}z \left|\Psi^{\gamma}_{L,T}(Q^2, \mathbf{r}_T, z)\right|^2 \sigma^2_{\rm dip}(x, \mathbf{r}_T, \mathbf{\Delta})$

X-cl. diff.: $\frac{\sigma^D_{L,T}}{\mathrm{d}t} = \frac{1}{16\pi} \left| \int \mathrm{d}^2\mathbf{r}_T \int \mathrm{d}z \left(\Psi^{\gamma}\Psi^{*V}\right)_{L,T} \sigma_{\rm dip}(x, \mathbf{r}_T, \mathbf{\Delta})\right|^2$

Assumptions:

- S-matrix real
- optical theorem

- $\Psi^{\gamma}(Q^2, \mathbf{r}_T, z) \sim K_{0,1}(\sqrt{z(1-z)}Q|\mathbf{r}_T|)$ momentum scale $Q^2 \sim 1/\mathbf{r}_T^2$
- Diffractive: t distribution is FT of $\mathbf{b}_T$ distribution.

Simple impact parameter dependence: GBW

Golec-Biernat & Wüsthoff [1,2] and many followers

For total cross section assume form where impact parameter dependence is factorized. 3 parameter ($x_0, \sigma_0 = 2\pi R_p, \lambda$) fit to:

$$\sigma_{\text{dip}}(x, \mathbf{r}_T) = 2 \int \mathrm{d}^2\mathbf{b}_T \left(1 - e^{-\mathbf{r}_T^2 Q_{\text{s}}^2(x)/4}\right) T_p(\mathbf{b}_T), \quad Q_{\text{s}}^2 = Q_0^2 (x/x_0)^{-\lambda}$$

(GBW do not choose an explicit form for $T_p(\mathbf{b}_T)$, values for R_p below are for $\theta(R_p - |\mathbf{b}_T|)$) Fit (w/ charm) gives $2\pi R_p^2 = 29\text{mb}$ ▶ $R_p \approx 0.7\text{fm}$ (no charm $\approx 0.6\text{fm}$).

For diffractive data assume constant diffractive slope

$$\sigma_{\text{dip}}(x, \mathbf{r}_T, \mathbf{\Delta}) = e^{-\frac{1}{2} B_D \mathbf{\Delta}^2} \sigma_{\text{dip}}(x, \mathbf{r}_T, \mathbf{\Delta} = 0)$$

with $B_D = 6/\text{GeV}^2 \iff$ **Gaussian** proton with rms. radius $R_p \approx 0.7\text{fm}$.

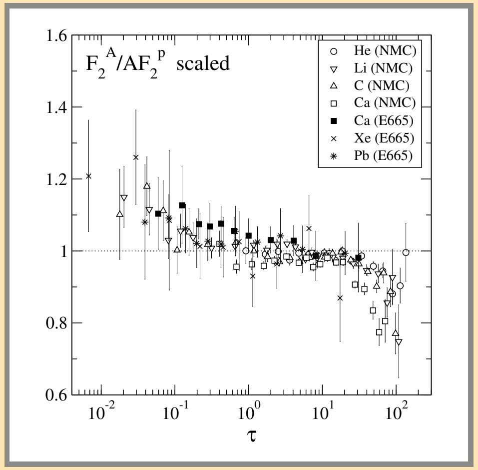
As far as I can see same is also done by Kugeratski et. al. [3].

[1] K. Golec-Biernat and M. Wusthoff, *Phys. Rev.* **D59** (1999) 014017 [hep-ph/9807513].
[2] K. Golec-Biernat and M. Wusthoff, *Phys. Rev.* **D60** (1999) 114023 [hep-ph/9903358].
[3] M. S. Kugeratski, V. P. Goncalves and F. S. Navarra, *Eur. Phys. J.* **C46** (2006) 413 [hep-ph/0511224].

Fitting existing nuclear data à la GBW 1

Freund, Rummukainen, Weigert[4], geometric scaling fit to data from E665[5,6] and NMC[7].

$$F_2^A(x,Q^2) = \overbrace{\left(\frac{x}{x_0}\right)^{-\lambda} A^{1/3}}^{F_2 \sim Q^2\sigma} \overbrace{A^{2/3+\gamma}}^{\text{area}} F_2^p\left(x_0, \left(\frac{x}{x_0}\right)^{\lambda}\frac{Q^2}{A^\delta}\right)$$



- Expectation: $\gamma = 0$ ($\Longleftrightarrow \pi R_A^2 \sim A^{2/3}$) and $\delta = 1/3$ ($\Longleftrightarrow Q_s^2 \sim A^{1/3}$)
- Fit result: $\gamma = 0.09$ and $\delta = 1/4$

Slower growth of $Q_s^2 \sim A^{1/4}$ compensated by growth of πR_A^2.

[4] A. Freund, K. Rummukainen, H. Weigert and A. Schafer, *Phys. Rev. Lett.* **90** (2003) 222002 [hep-ph/0210139].

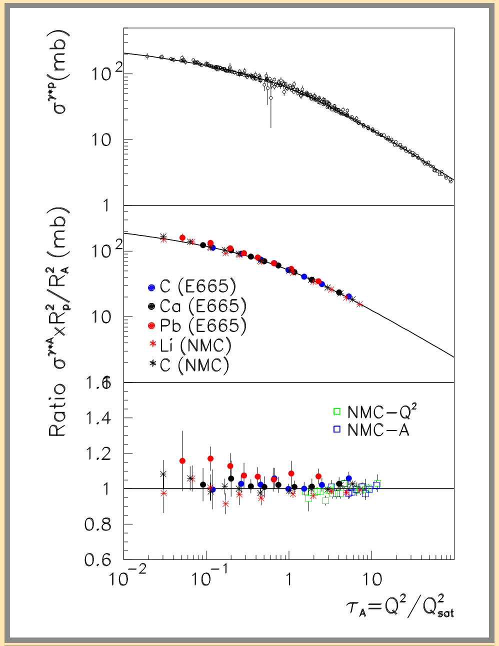
[5] **E665** Collaboration, M. R. Adams *et. al.*, *Z. Phys.* **C65** (1995) 225.

[6] **E665** Collaboration, M. R. Adams *et. al.*, *Z. Phys.* **C67** (1995) 403 [hep-ex/9505006].

[7] **New Muon** Collaboration, M. Arneodo *et. al.*, *Nucl. Phys.* **B487** (1997) 3 [hep-ex/9611022].

Fitting existing nuclear data à la GBW 2

Armesto, Salgado, Wiedemann [8]



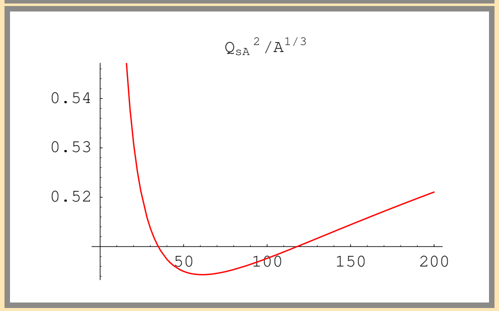
$$Q_s^{A\,2} = \left(\frac{A\pi R_p^2}{\pi R_A^2}\right)^{\alpha} (Q_s^p)^2$$

with $R_A = (1.12A^{1/3} - 0.86A^{-1/3})\text{fm}$.
Fit parameters: R_p, α, so essentially one is fitting to a form

$$Q_s^{A\,2} = C\left(\frac{A}{(A^{1/3} - 0.77A^{-1/3})^2}\right)^{\alpha} (Q_s^p)^2$$

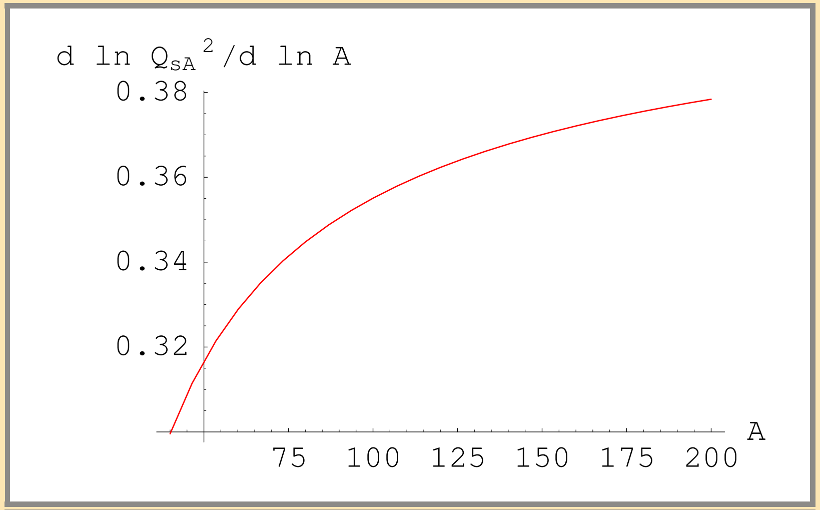
with C and α determined by the fit.
Expectation: $\alpha = 1$
Fit result: $R_p = 0.7\text{fm}$, $\alpha = 1.25$.

- **Statement: "Favors $Q_s^{A\,2} \sim A^{4/9}$"**



[8] N. Armesto, C. A. Salgado and U. A. Wiedemann, *Phys. Rev. Lett.* **94** (2005) 022002 [hep-ph/0407018].

ASW fit: discussion



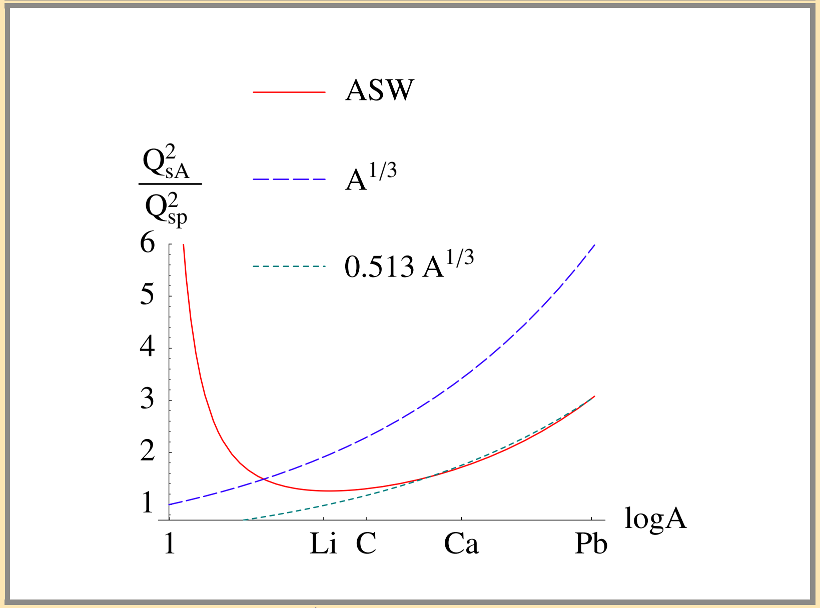
Statement: "Favors ${Q_s^A}^2 \sim A^{4/9}$"

$$
{Q_s^A}^2 = C \left(\frac{A}{(A^{1/3} - 0.77 A^{-1/3})^2} \right)^{\alpha} (Q_s^p)^2
$$

But assumed nuclear radius dependence makes interpretation more complicated for realistic size nuclei ($A \lesssim 200$).

Note normalization: ${Q_s^A}^2 \sim 0.5 A^{1/3} {Q_s^p}^2$

- back to this in a second.



$4/9 \approx 0.44$

Kowalski-Teaney impact parameter saturation model: protons[9]

2 improvements over GBW

- Include DGLAP evolution: improves fit for large Q^2, smoother large x limit. (Starting in Bartels et. al.[10], Gotsman, Levin, Lublinsky [11,12,13], see Mischa's talk.)
- Consistently use same impact parameter dependence for both total and diffractive cross sections (also here I don't claim that KT were necessarily the first ones to do this, see e.g. Mischa's talk.)

$$\sigma_{\mathrm{dip}}(x, \mathbf{r}_T) = 2 \int \mathrm{d}^2\mathbf{b}_T \left(1 - \exp \overbrace{\left\{ -\frac{\pi^2}{2N_{\mathrm{c}}} \alpha_{\mathrm{s}}(\mu^2) x g(x, \mu^2) T_p(\mathbf{b}_T) \mathbf{r}_T^2 \right\}}^{-\mathbf{r}_T^2 Q_{\mathrm{s}}^2/4} \right),$$

with $\mu^2 = \frac{C}{\mathbf{r}_T^2} + \mu_0^2$.

$xg(x, \mu^2)$ is evolved with DGLAP with initial condition $A_g x^{-\lambda_g}(1-x)^{5.6}$
Best fit result $\lambda_g < 0$: increase of Q_{s} for small x comes entirely from DGLAP!

[9] H. Kowalski and D. Teaney, *Phys. Rev.* **D68** (2003) 114005 [hep-ph/0304189].
[10] J. Bartels, K. Golec-Biernat and H. Kowalski, *Phys. Rev.* **D66** (2002) 014001 [hep-ph/0203258].
[11] E. Levin and M. Lublinsky, *Nucl. Phys.* **A696** (2001) 833 [hep-ph/0104108].
[12] J. Bartels, E. Gotsman, E. Levin, M. Lublinsky and U. Maor, *Phys. Lett.* **B556** (2003) 114 [hep-ph/0212284].
[13] E. Gotsman, E. Levin, M. Lublinsky and U. Maor, *Eur. Phys. J.* **C27** (2003) 411 [hep-ph/0209074].

KT: nuclei

Straightforward generalization to nuclei:

$$\sigma_{\mathrm{dip}}^{A}(x,\mathbf{r}_T) = 2\int \mathrm{d}^2\mathbf{b}_T \left(1 - \exp\left\{-\frac{\pi^2}{2N_\mathrm{c}}\alpha_\mathrm{s}(\mu^2)xg(x,\mu^2)\sum_{i=1}^{A} T_p(\mathbf{b}_T - \mathbf{b}_{Ti})\mathbf{r}_T^2\right\}\right),$$

where $\mathbf{b}_{Ti}$ are the positions of the nucleons, drawn from a Woods-Saxon density as in the Monte Carlo Glauber model we all know and love.

T_p gives a proton radius $\sim 0.6\mathrm{fm}$, much less than the nucleon-nucleon distance. So this is a **lumpy** nucleus, very different from $\sum_{i=1}^{A} T_p(\mathbf{b}_T - \mathbf{b}_{Ti}) \longrightarrow T_A(\mathbf{b}_T)$.

Assuming $\mathbf{b}_{Ti}$ are uncorrelated this can be resummed to

$$\sigma_{\mathrm{dip}}^{A}(x,\mathbf{r}_T) = 2\int \mathrm{d}^2\mathbf{b}_T \left[1 - \left(1 - \frac{T_A(\mathbf{b}_T)}{2}\sigma_{\mathrm{dip}}^{p}\right)^A\right]$$

$$\approx_{A\to\infty} 2\int \mathrm{d}^2\mathbf{b}_T \left[1 - e^{-\frac{AT_A(\mathbf{b}_T)}{2}\sigma_{\mathrm{dip}}^{p}}\right]$$

($\int \mathrm{d}^2\mathbf{b}_T T_A(\mathbf{b}_T) = 1$ ▶ $T_A(\mathbf{0}_T) \sim A^{-2/3}$)

$A^{1/3}$ from the KT model?

$$\frac{\mathrm{d}\sigma^A_{\mathrm{dip}}(x,\mathbf{r}_T,\mathbf{b}_T)}{\mathrm{d}^2\mathbf{b}_T} \approx 2\left[1 - e^{-\frac{A T_A(\mathbf{b}_T)}{2}\sigma^p_{\mathrm{dip}}}\right] \quad (1)$$

Q_s is not directly a parameter of the model any more. Define

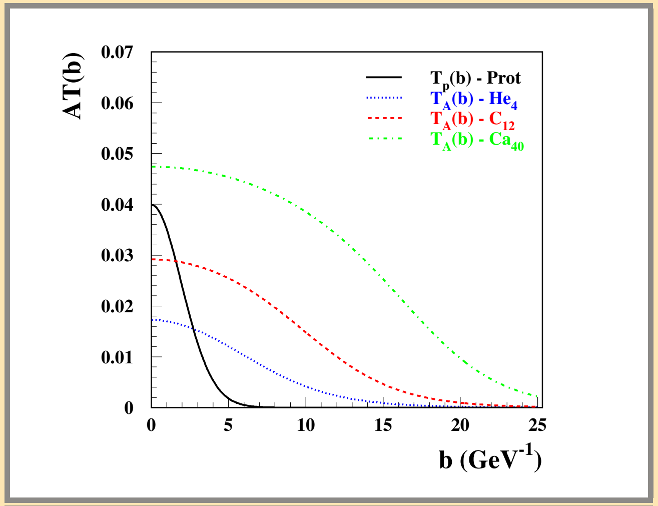
$$\frac{\mathrm{d}\sigma_{\mathrm{dip}}(x,\mathbf{r}_T,\mathbf{b}_T)}{\mathrm{d}^2\mathbf{b}_T} = 2(1-e^{-1/4}), \text{ when } \mathbf{r}_T^2 = 1/Q_\mathrm{s}^2$$

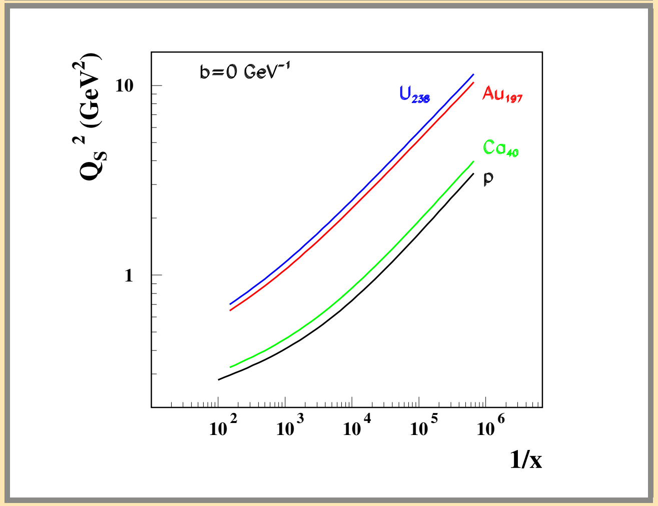
If we simplify (1) by approximating $\sigma^p_{\mathrm{dip}}(\mathbf{r}_T) = 2\pi R_p^2 e^{-\mathbf{r}_T^2 (Q^p_\mathrm{s})^2/4}$ and $T_A(\mathbf{b}_T) = \theta(R_A - |\mathbf{b}_T|)/(\pi R_A^2)$

- for $A \gg 1$

$$Q_\mathrm{s}^{A\,2} \approx \frac{A R_p^2}{R_A^2} Q_\mathrm{s}^{p\,2} \approx 0.3 A^{1/3} Q_\mathrm{s}^{p\,2}$$

with $R_p = 0.6\mathrm{fm}$ and $R_A = 1.1A^{1/3}\mathrm{fm}$





Additional increase from DGLAP

"Small constant times $A^{1/3}$" gives most of KT A-dependence, but there is an additional effect: the Au to Ca – ratio is $\lesssim 3$, more than $(197/40)^{1/3} \approx 1.7$

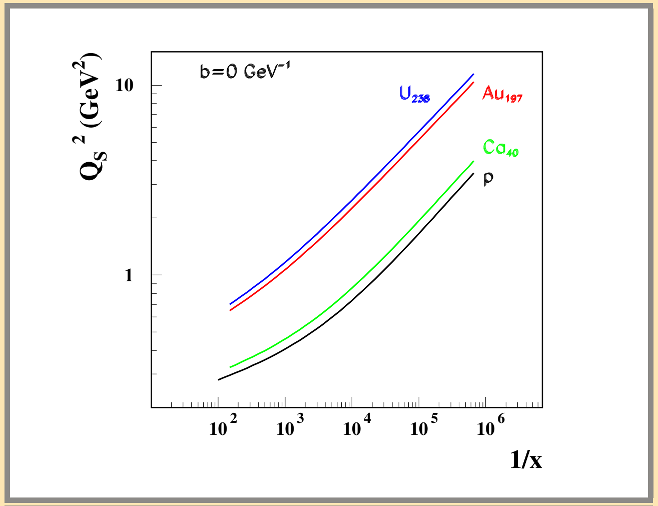
Previous estimate: neglected increase of $xg(x, C/\mathbf{r}_T^2 + \mu_0^2)$ with increasing $1/\mathbf{r}_T^2$.

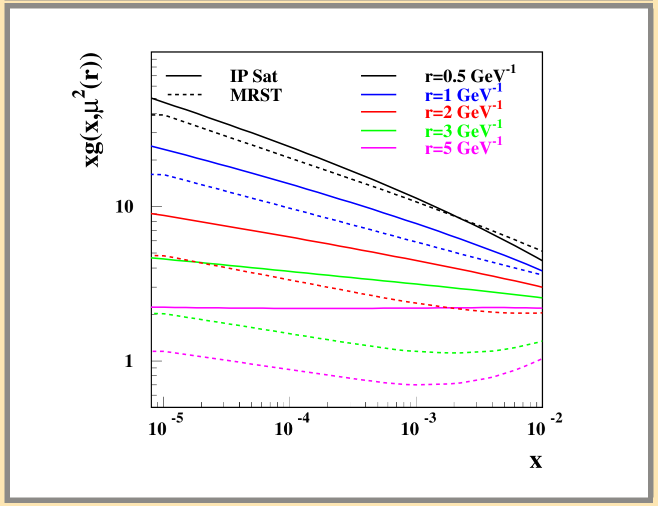
Take nuclei A and B. Saturation condition

$$\frac{\overbrace{A T_A(\mathbf{b}_T)}^{\sim A^{1/3}} xg(x, Q_\mathrm{s}^{A\,2})}{(Q_\mathrm{s}^A)^2} = \frac{\overbrace{B T_B(\mathbf{b}_T)}^{\sim B^{1/3}} xg(x, Q_\mathrm{s}^{B\,2})}{(Q_\mathrm{s}^B)^2}$$

$$\frac{Q_\mathrm{s}^{A\,2}}{Q_\mathrm{s}^{B\,2}} \sim \left(\frac{A}{B}\right)^{1/3} \times \frac{xg(x, Q_\mathrm{s}^{A\,2})}{xg(x, Q_\mathrm{s}^{B\,2})}$$

Gold/Calcium: $2.6 \approx 1.7 \times 1.5$ (around $x = 10^{-4}$?)
Bigger effect for smaller x.





Conclusions: what is the nuclear "oomph" factor?

Impact parameter dependence and "lumpiness" of nucleus: comparing nuclear and proton Q_{s} is not necessarily apples to apples.

What $\mathbf{b}_T$ in proton does one compare to? Gold/Calcium or other such ratio better.

- Pay a price for diluteness of nucleus: $R_p < A^{-1/3} R_A$
- For larger nuclei (as in $Q_{\mathrm{s}}^{\mathrm{Au}}/Q_{\mathrm{s}}^{\mathrm{Ca}}$) expect something like $A^{1/3}$
- DGLAP increase of $xg(x, Q^2)$ with Q^2 will **add** to this.
- Also nuclear surface effect less important at large A: small increase in Q_{s} from deviation from $T_A(\mathbf{b}_T = \mathbf{0}_T) \sim A^{-2/3}$

Further things to do:

- Lumpiness and other nuclear effects on diffraction (too small t to be measured?, $1/R_A \sim 0.03\mathrm{GeV}$)
- More blackness locally and in rare events: fluctuations?