# DIS with protons and nuclei: Saturation based approach

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Based on:

- M. L., E. Gotsman, E. Levin, and U. Maor, *Nucl. Phys.* **A 696** (2001) 851;
- E. Gotsman, E. Levin, M. L., and U. Maor, *Eur. Phys. J.* **C** 27, (2003) 411;
- J. Bartels, E. Gotsman, E. Levin, M. L., and U. Maor, *Phys. Lett.* **B 556**, (2003) 114.

Parameterizations can be found at www.desy.de/∼lublinm/

# Research Goals:

**New Global QCD analysis based on Non-Linear QCD Evolution**
**Balitsky-Kovchegov equation (BKE)**

BKE $=$ LO BFKL $+$ Unitarization (non-linearity)

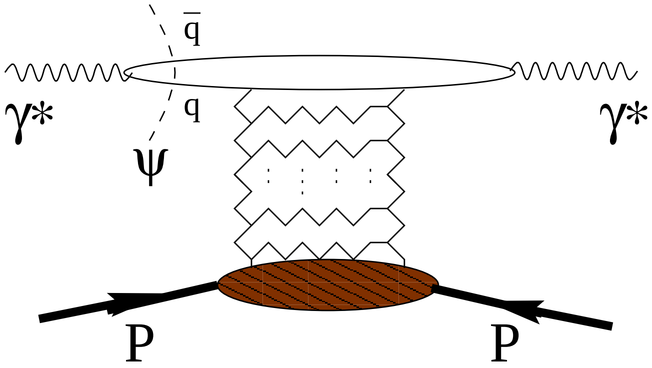
- DGLAP predicts a steep growth of parton distributions at low $x$. Violate the unitarity constraints;
- The twist OPE breaks down at low $x$
- DGLAP evolution is totally unable to describe low $Q^2$ data.

NLO corrections do not solve these problems.

**Non-linear evolution is a solution to these problems!**

- It accounts for the saturation effects due to high parton densities;
- It sums high twist contributions;
- It allows extrapolation to large distances.

# • Color Dipole Approach to DIS




$$x \;=\; Q^2 \,/\, W^2$$

The total DIS cross section:

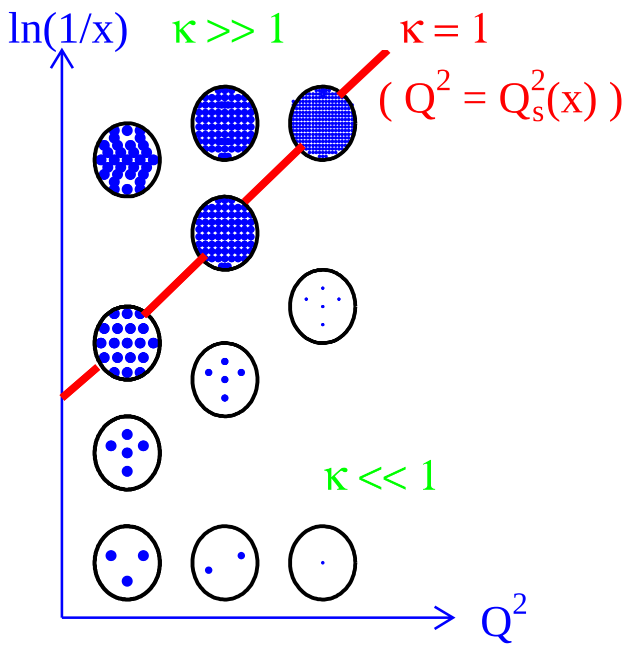
$$\sigma_{tot}^{DIS} \;=\; \int d^2 r_\perp \; d\,z \; P^{\gamma^*}(\,Q,\; r_\perp,\; z\,) \;\; \sigma_{dipole}\,(\,r_\perp,\; x\,)\,;$$

$$\sigma_{dipole} \;=\; 2 \int d^2\, b \;\; N\,(\,r_\perp,\; x;\; b\,)\,; \qquad N\,(\,r_\perp,\; x;\; b\,) \;=\; Im\, a^{el}_{dipole}\,(\,r_\perp,\; x;\; b\,)$$

pQCD: $N \;\sim\; r_\perp^2\, xG(\,r_\perp,\; x\,;b\,)$ (xG - gluon density) Unitarity: $N \;\leq\; 1$

$N \;\longleftarrow$ Balitsky-Kovchegov NLE + DGLAP corrections

## • High Density QCD, Saturation and Map of QCD




Transition to hdQCD.

The linear evolutions $\longrightarrow$ parton radiation (or dipole splitting).

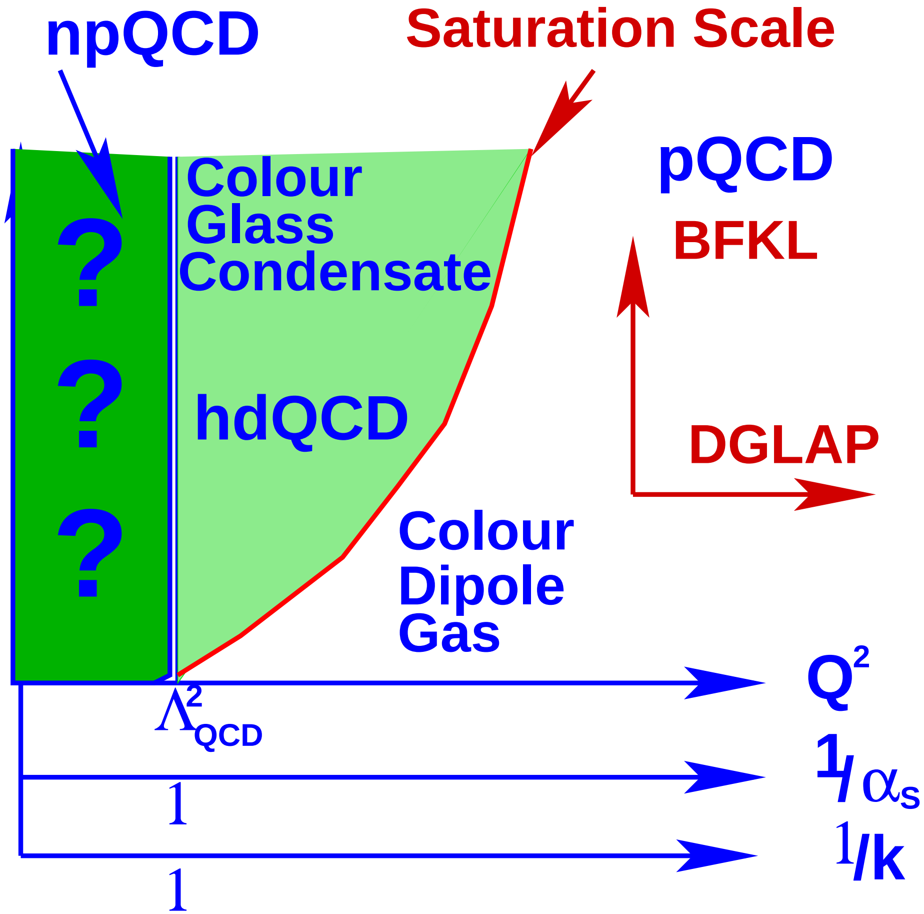
$\kappa \propto xG$ - gluon packing factor.

Transition scale $Q_s(x)$ is called saturation scale.

Parton emission $\sim$ gluon density $xG$.

Parton annihilation $\sim xG^2$

The density balance is a subject to a non-linear equation.




Phase space map of QCD. The saturation scale $Q_s(x)$ corresponds to a critical line on which the gluon packing factor $\kappa = 1$.

# New procedure for extrapolation of parton distributions

## Two steps:

- Solve the BK non-linear evolution equation.
  - it takes into account high twist contributions;
  - but only in the leading $\ln(1/x)$ approximation of pQCD;
  - and without a correct short distance description.
- Introduce a correcting function for which a DGLAP-type linear equation is proposed and solved.

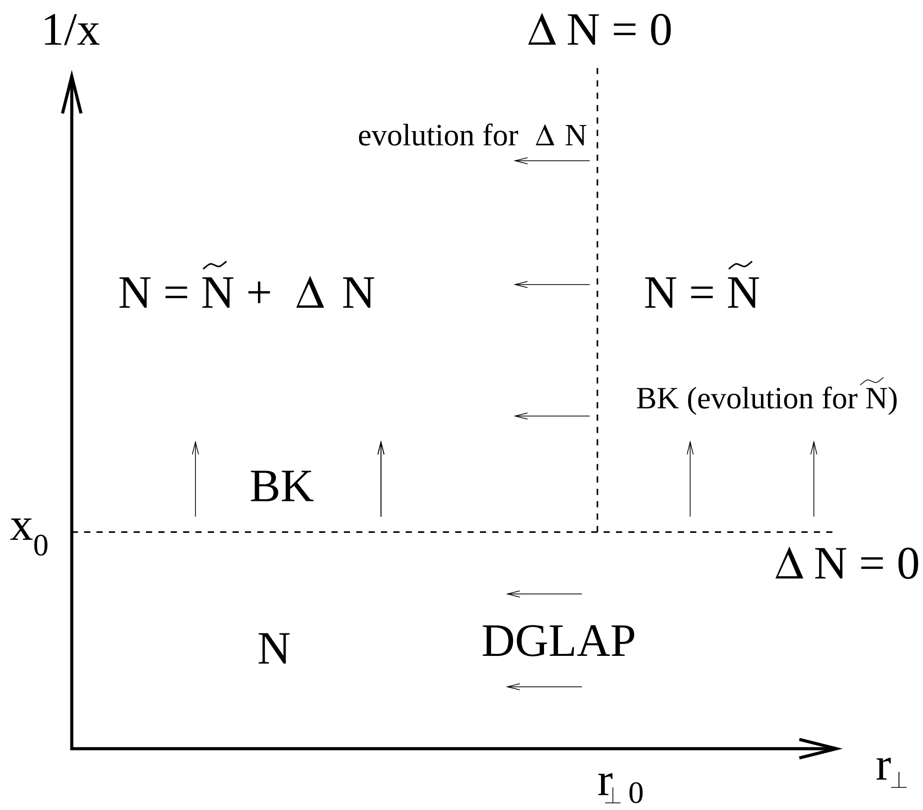
The full solution is

$$N\,(\,r_\perp\,,\,x\,;\,b\,)\;=\;\tilde{N}\,(\,r_\perp\,,\,x\,;\,b\,)\;+\;\Delta\,N\,(\,r_\perp\,,\,x\,;\,b\,)$$

$\tilde{N}\,(\,r_\perp\,,\,x\,;\,b\,) \longleftarrow$ BK non-linear equation;

$\Delta\,N\,(\,r_\perp\,,\,x\,;\,b\,) \longleftarrow$ DGLAP-type linear equation;

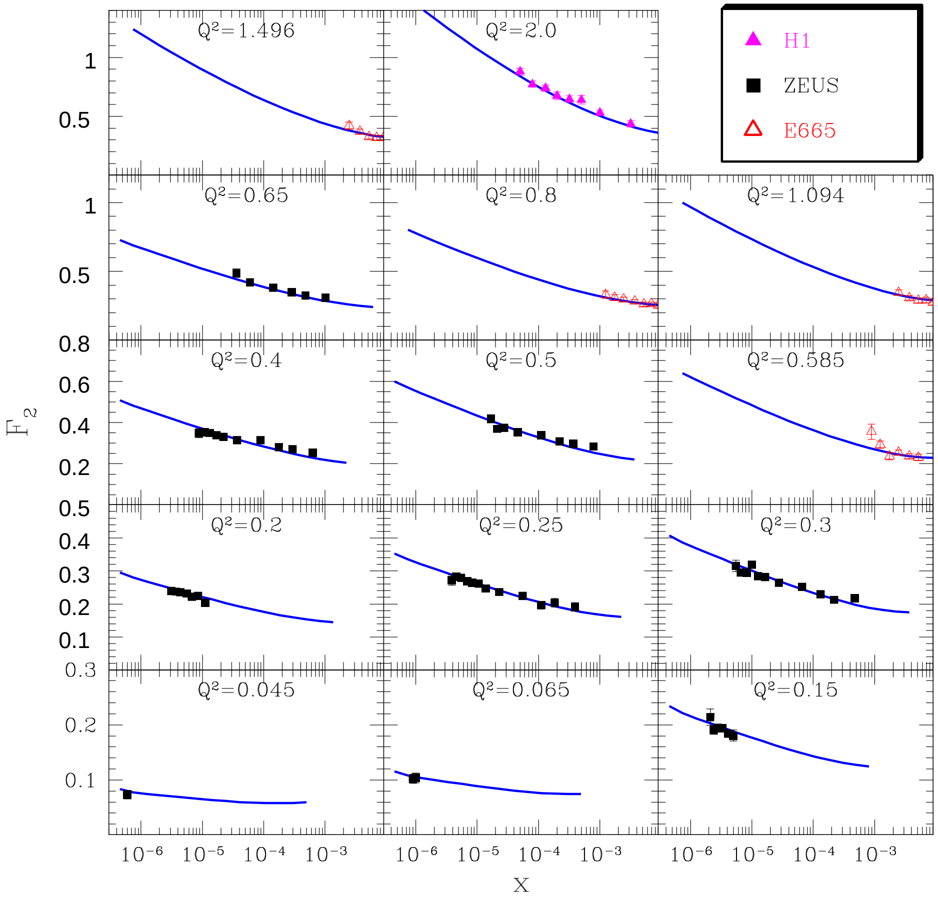
# Strategy

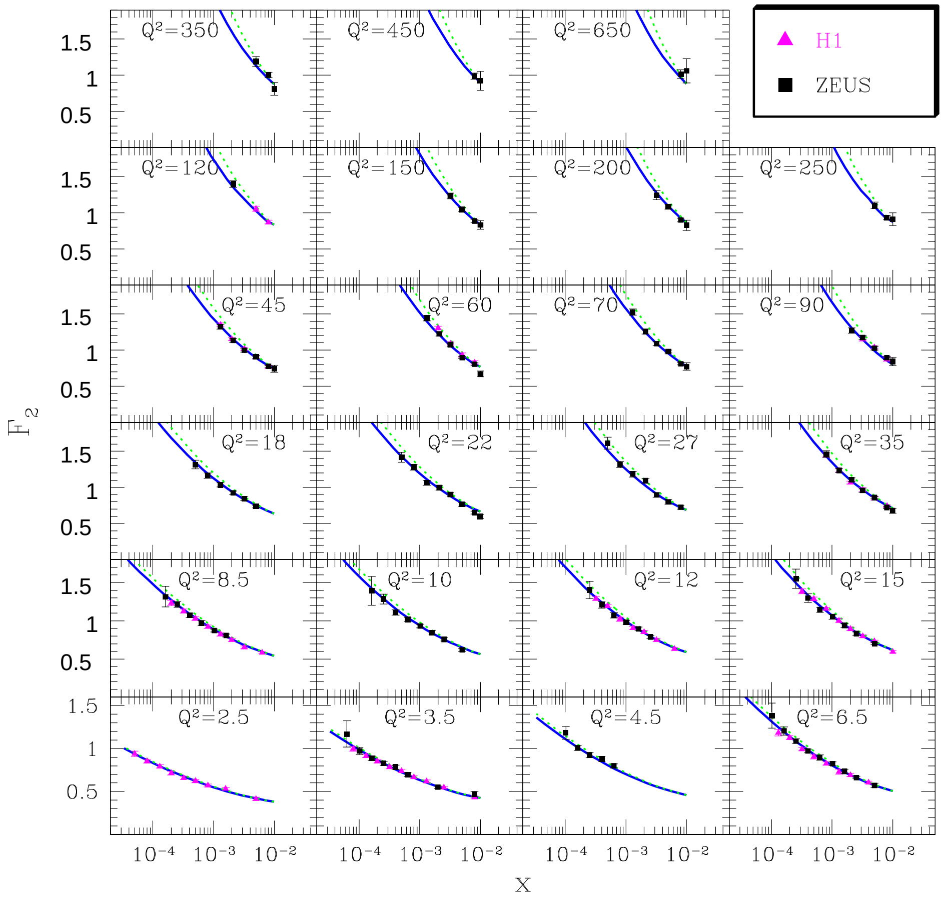



**Initial conditions:** Glauber - Mueller formula at $x_0 = 10^{-2}$:

$$\tilde{N}(r_\perp, x_0; b) \; = \; 1 \; - \; \exp\left[-\frac{\alpha_S \pi r_\perp^2}{2\, N_c\, R^2}\, x G^{DGLAP}(x_0, 4/r_\perp^2)\, S(b)\right]$$

# Results

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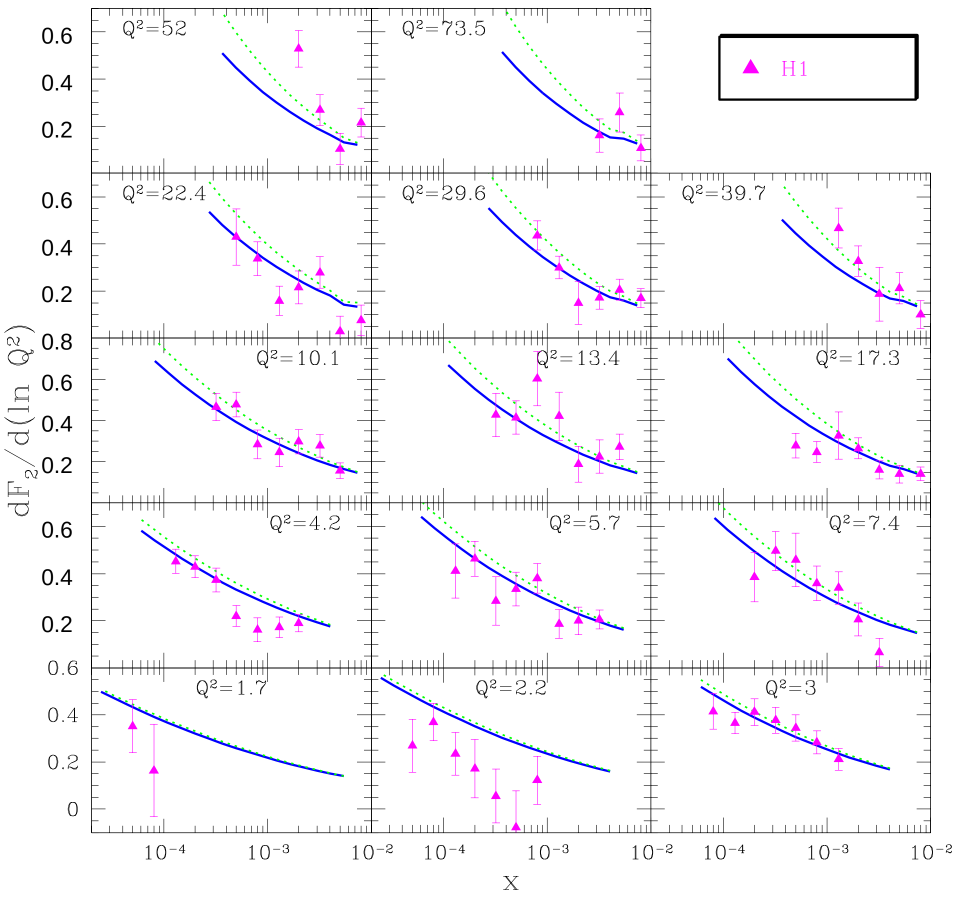
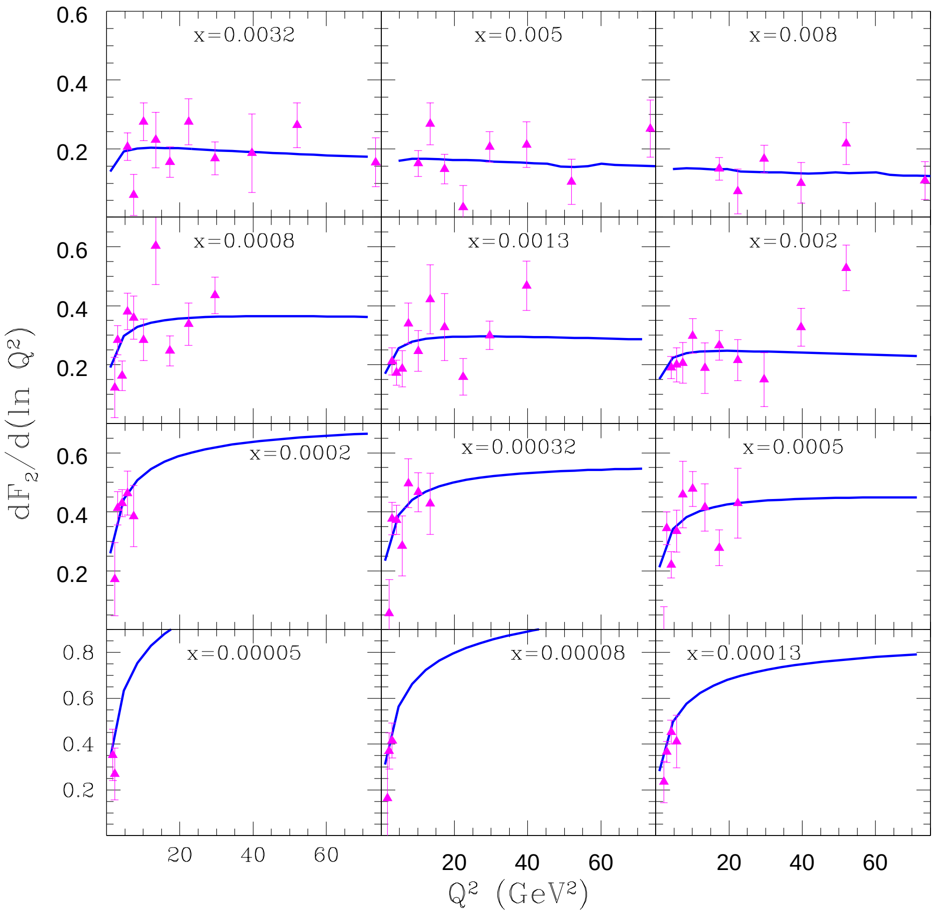
# $\partial\, \mathbf{F_2} \,/\, \partial\, \ln\, \mathbf{Q^2}$

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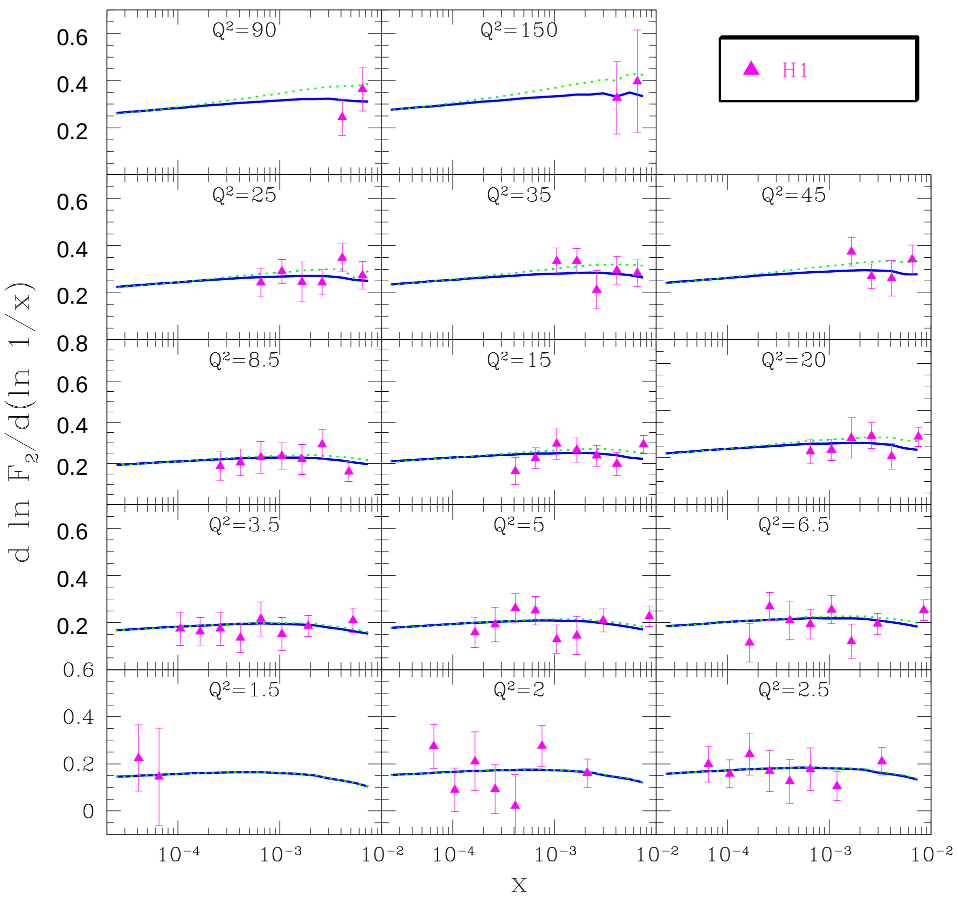
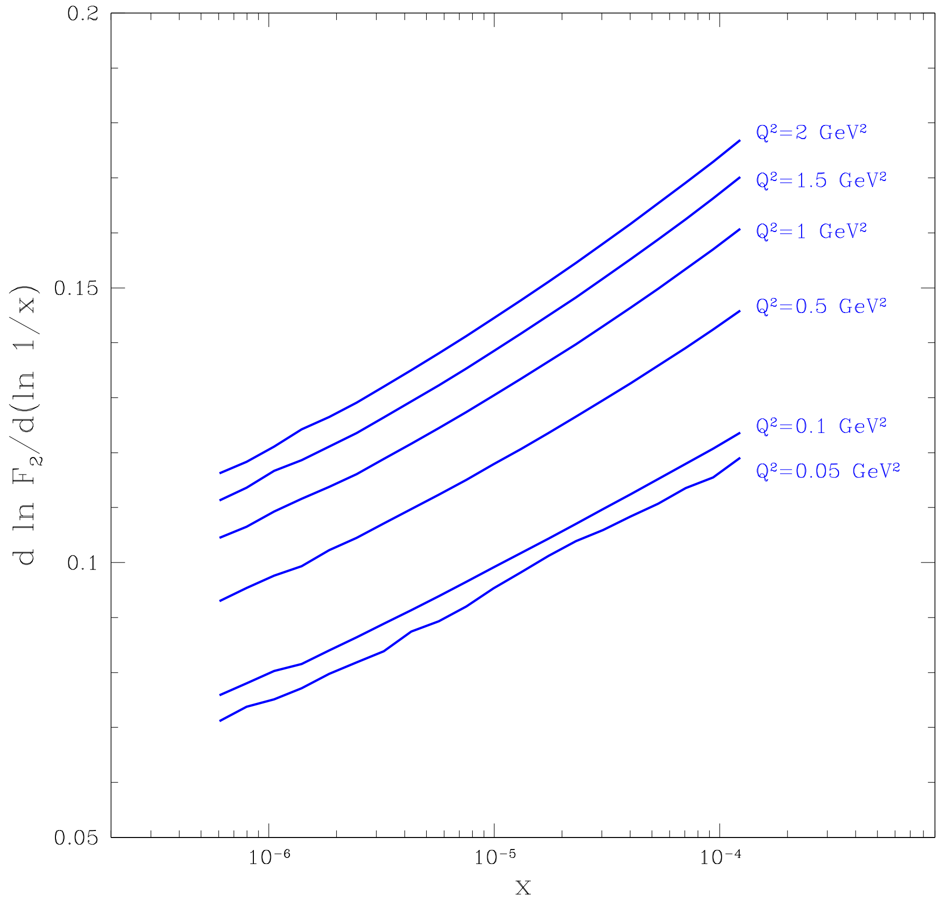
$$\lambda = \partial \ln \mathbf{F_2} / \partial \ln (\mathbf{1}/\mathbf{x})$$

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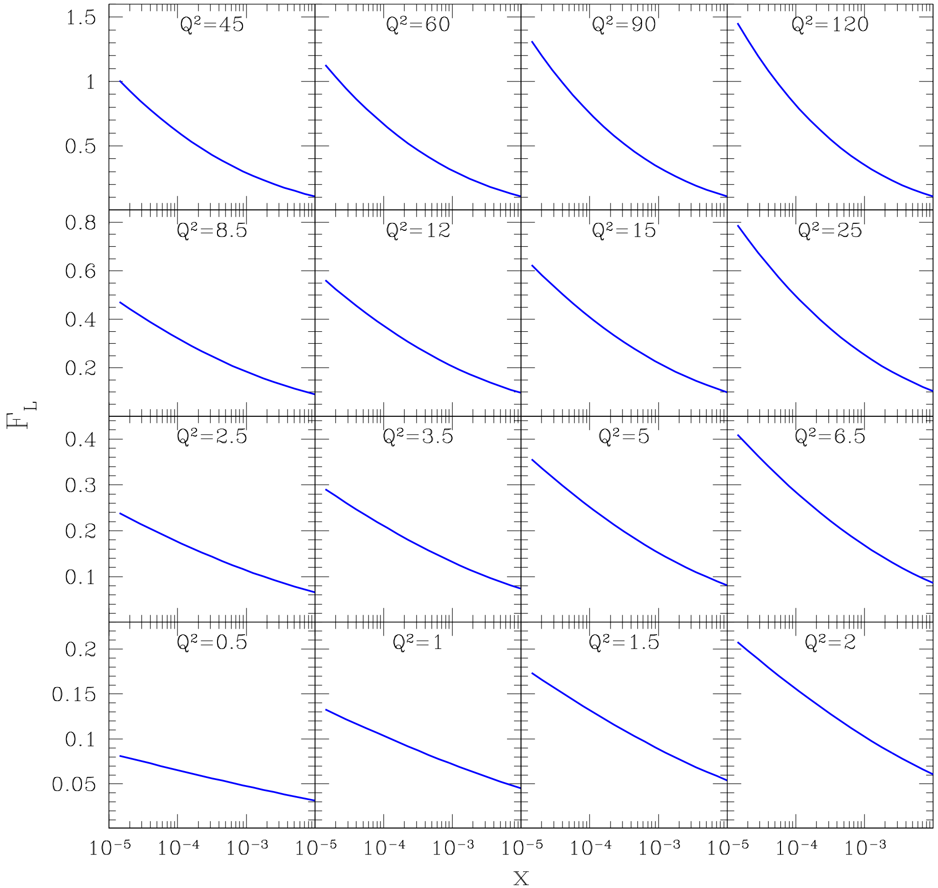
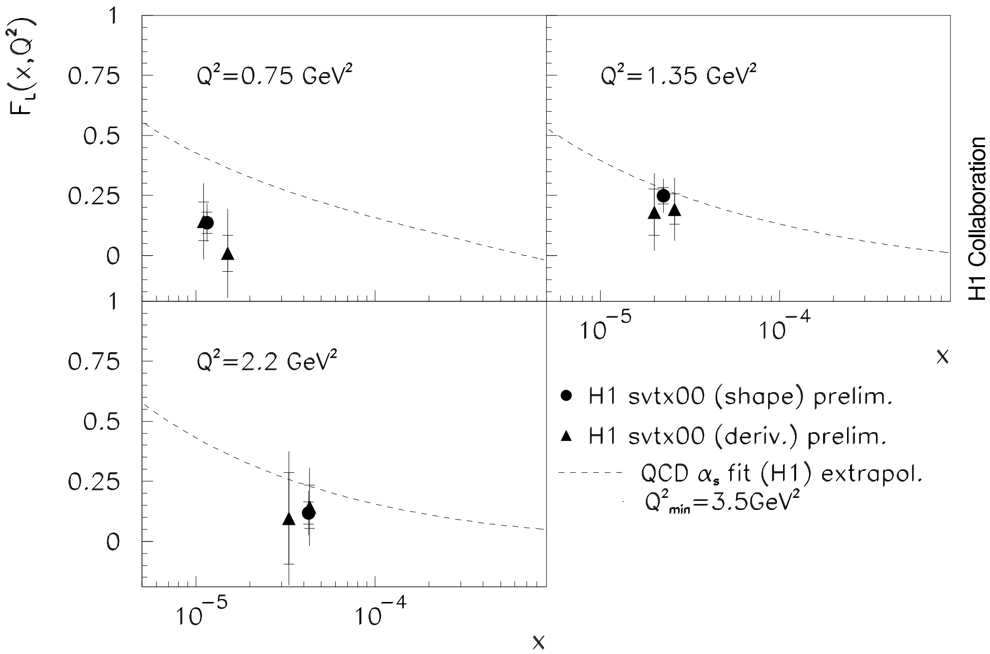
# $F_L$

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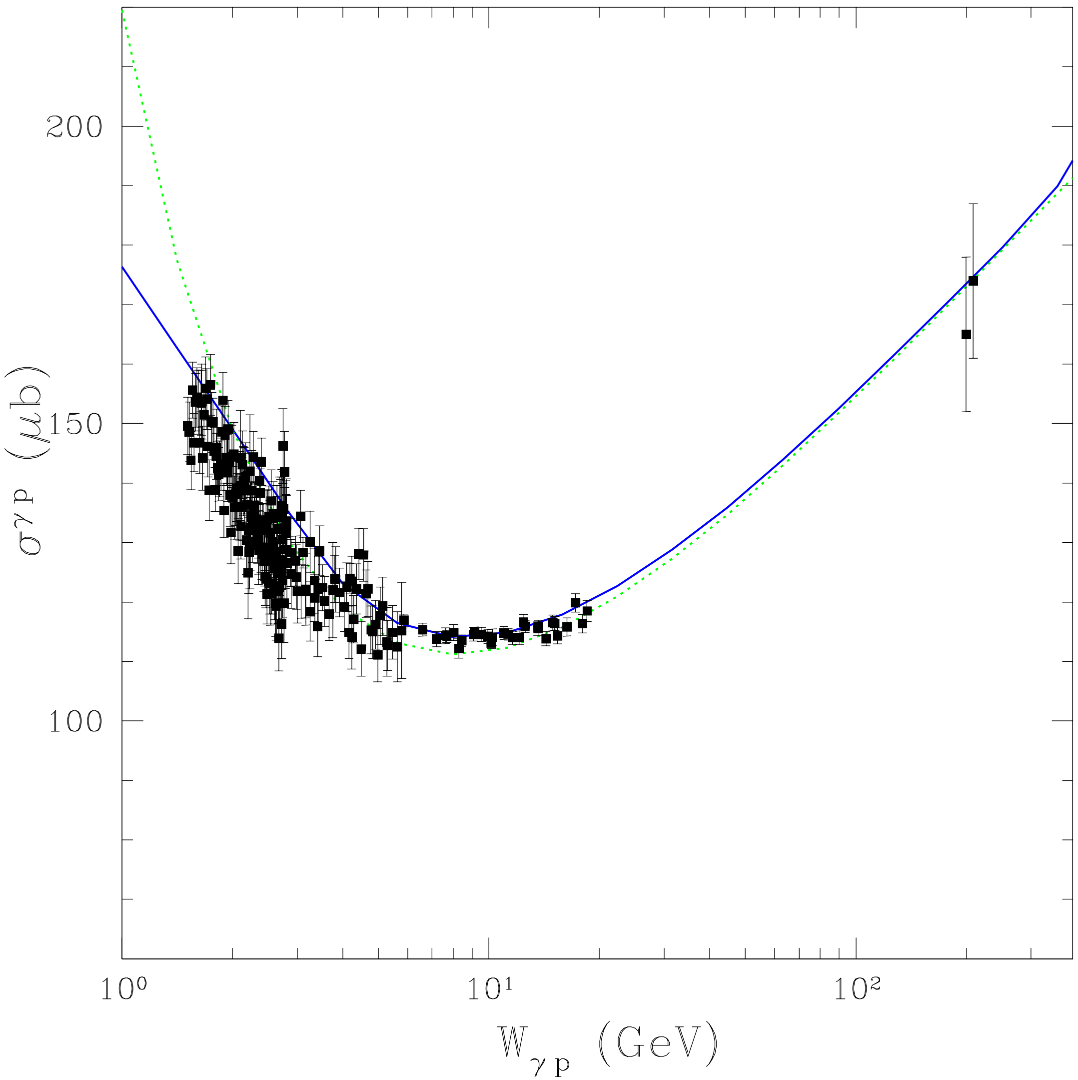
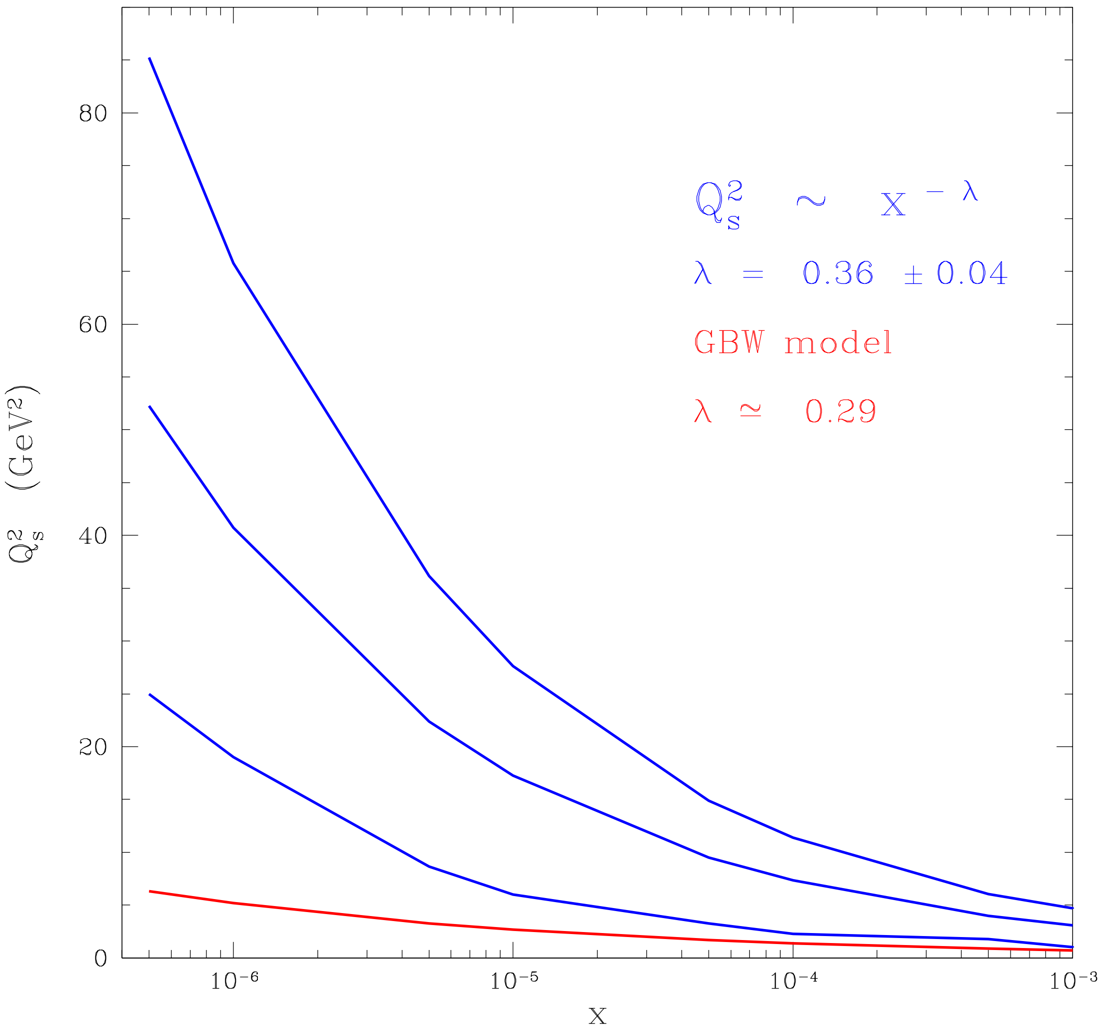
# Photoproduction



# Saturation Scale

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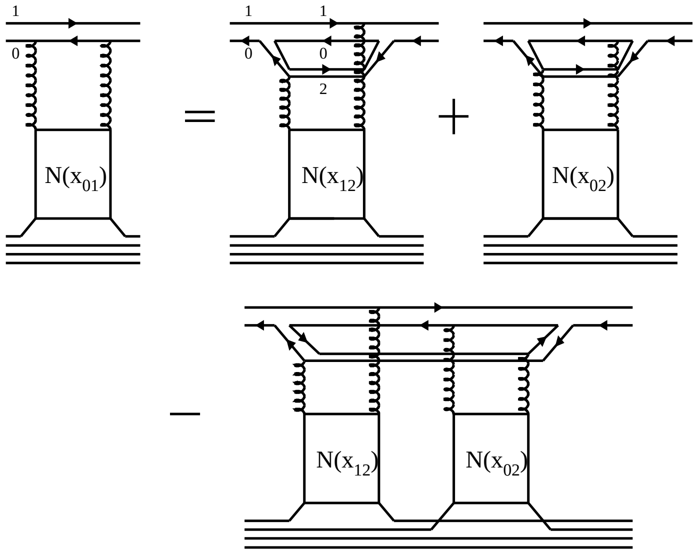
# Non-linear evolution




**GLR (81)**
**Mueller & Qiu (86)**
**..........**
**Balitsky (95)**
**Kovchegov (99)**
**Braun (2000)**
**Iancu, Leonidov & McLerran (2000)**

$$\frac{d\tilde{N}\,(\,x_{01}\,,\,y\,;\,b\,)}{dy} \;=\; -\,\frac{2\,C_F\,\alpha_S}{\pi}\;\tilde{N}\,(\,x_{01}\,,\,y\,;\,b\,)\;\ln\frac{x_{01}^2}{\rho^2}\;+$$

$$\frac{C_F\,\alpha_S}{\pi^2}\times\int_\rho d^2\,x_2\,\frac{x_{01}^2}{x_{02}^2\,x_{12}^2}\;\Big(2\,\tilde{N}\,(\,x_{02}\,,\,y\,;\,b\,)\;-\;\tilde{N}\,(\,x_{02}\,,\,y\,;\,b\,)\,\tilde{N}\,(\,x_{12}\,,\,y\,;\,b\,)\Big)$$

Approximations: LLog $1/x$; $N_c \to \infty$; $\alpha_s$ - const,; $b$ - large.

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## Correcting Function $\Delta N$

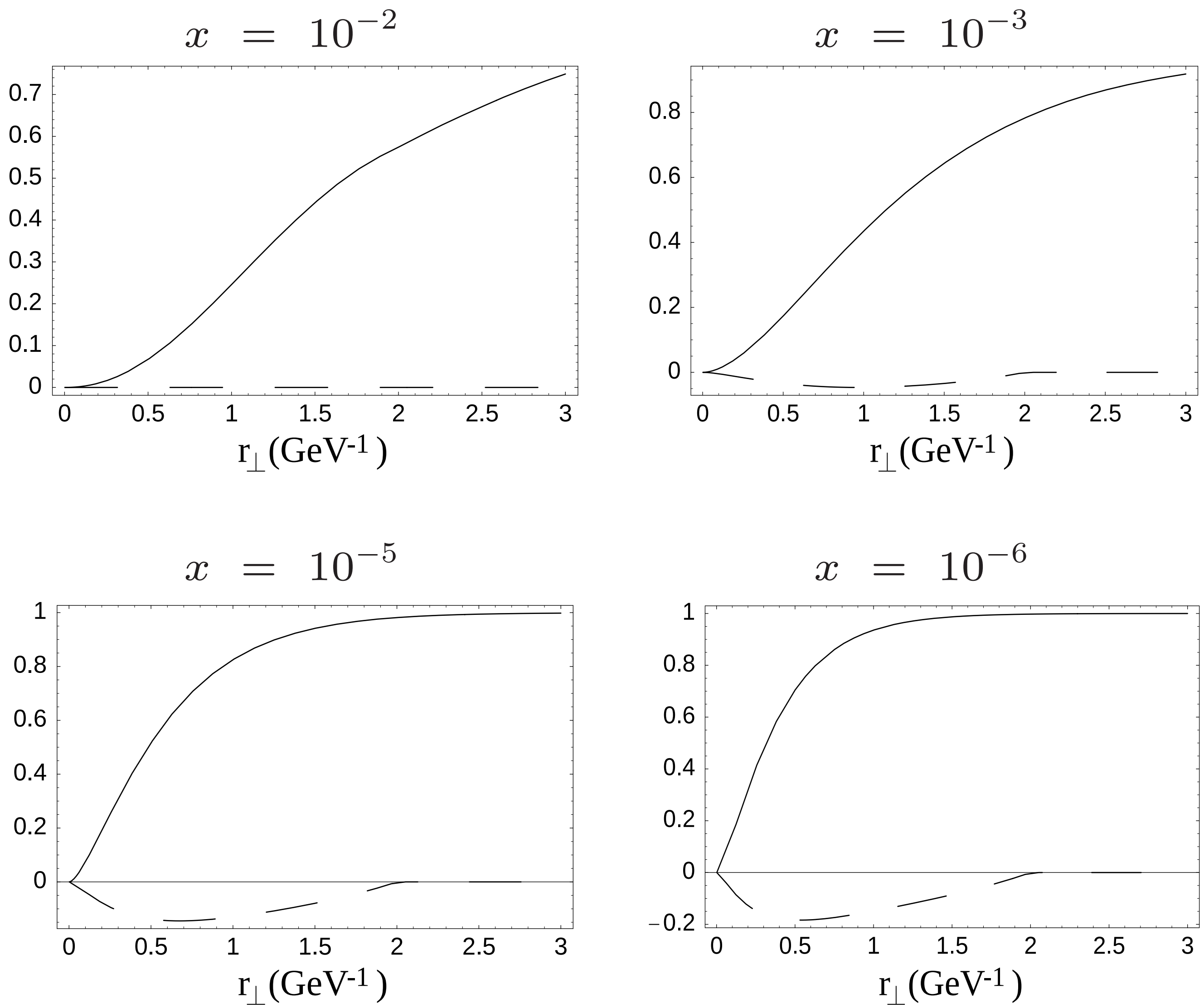
- $$\tilde{n} \equiv \tilde{N}/(\alpha_s\, r_\perp^2)\,; \quad n \equiv N/(\alpha_s\, r_\perp^2)\,; \quad \Delta n \equiv \Delta N/(\alpha_s\, r_\perp^2)\,.$$

- $$\frac{\partial \Delta n(r_\perp, x)}{\partial\, \ln(1/r_\perp^2)} = \frac{C_F \alpha_S}{\pi} \int_{x/x_0}^{1} P_{g\to g}(z)\ \Delta n(r_\perp, \frac{x}{z})\, dz -$$
$$\frac{2\, C_F\, \alpha_s}{\pi} \int_{x/x_0}^{1} \frac{dz}{z}\, \tilde{N}(r_\perp, \frac{x}{z}) \Delta n(r_\perp, \frac{x}{z})\ +$$
$$\frac{C_F\, \alpha_s}{\pi} \int_{x/x_0}^{1} \left( P_{g\to g}(z) - \frac{2}{z} \right) \tilde{n}(r_\perp, \frac{x}{z})\, dz\ -$$
$$\frac{\partial \tilde{n}(r_\perp, x_0)}{\partial\, \ln(1/r_\perp^2)} + \frac{C_F\, \alpha_s}{\pi} \int_{x}^{x/x_0-} P_{g\to g}(z)\ n(r_\perp, \frac{x}{z})\, dz\,.$$

$et$ where

$$P_{g\to g}(z) = 2 \left[ \frac{1-z}{z} + \frac{z}{(1-z)_+} + z(1-z) + \left( \frac{11}{12} - \frac{n_f}{18} \right) \delta(1-z) \right]$$

# Solutions of the equations ($b = 0$)



- $\alpha_s$ - running $\alpha_s^{max} \simeq 0.5$
- $xG^{DGLAP} \longleftarrow$ LO CTEQ6
- $R^2 = 3.1\ (\mathrm{GeV}^{-2})$
- $r_{\perp\,0} = 2\ (\mathrm{GeV}^{-1})$

# Predictions for Nuclei

**Results on nuclei can be predicted only from information on proton structure and without any additional fitting parameters!**

Two approaches:

- BK equation for a nucleus target.

$$\frac{d\tilde{N}_A}{dy} = Ker \otimes (\tilde{N}_A - \tilde{N}_A \tilde{N}_A) .$$

Initial conditions: Glauber - Mueller formula at $x_0 = 10^{-2}$ :

$$\tilde{N}_A(r_\perp, x_0; b) = 1 - \exp\left[-\sigma^N(r_\perp, x_0)\, S_A(b)\right]$$

$\sigma^N$ - dipole - nucleon cross section.

$\sigma^N(r_\perp, x_0) \longleftarrow$ $F_2$ data on proton.

$S_A(b) \longleftarrow$ real distribution from Nuclear Data Tables or Wood-Saxon distribution.

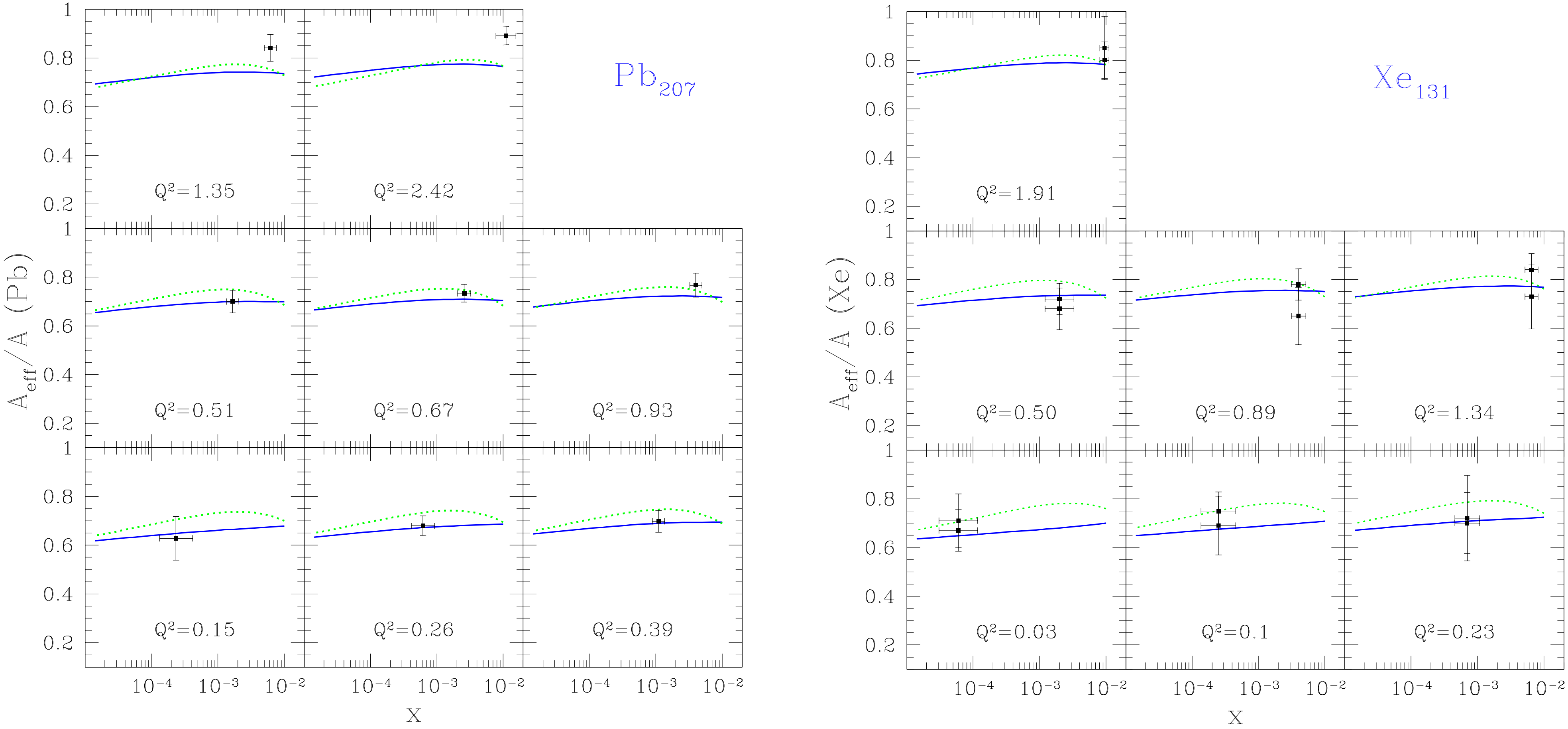
BKE works fine for dense nuclei but has problems in application when nucleus is dilute.

- Glauber formula for a nucleus target.

$$\tilde{N}_A(r_\perp, x; b) = 1 - \exp\left[-\sigma^N(r_\perp, x)\, S_A(b)\right]$$

Reasonable approximation for dilute nuclei.

# Results on Nuclear Shadowing

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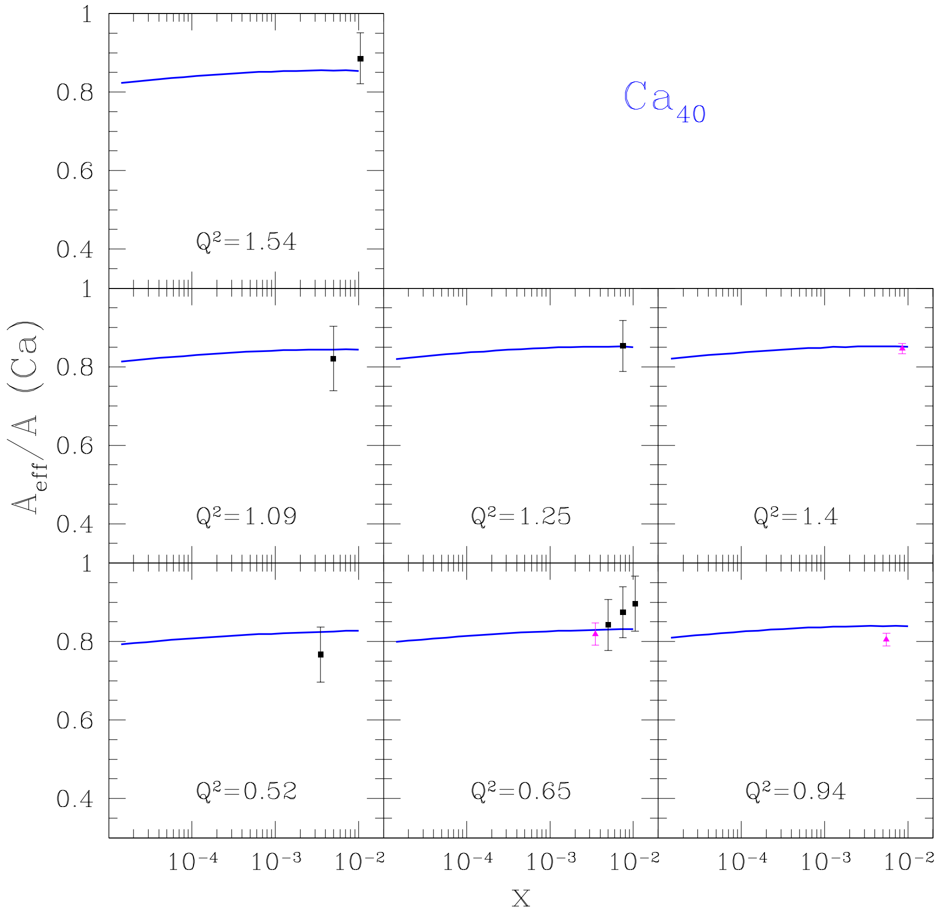
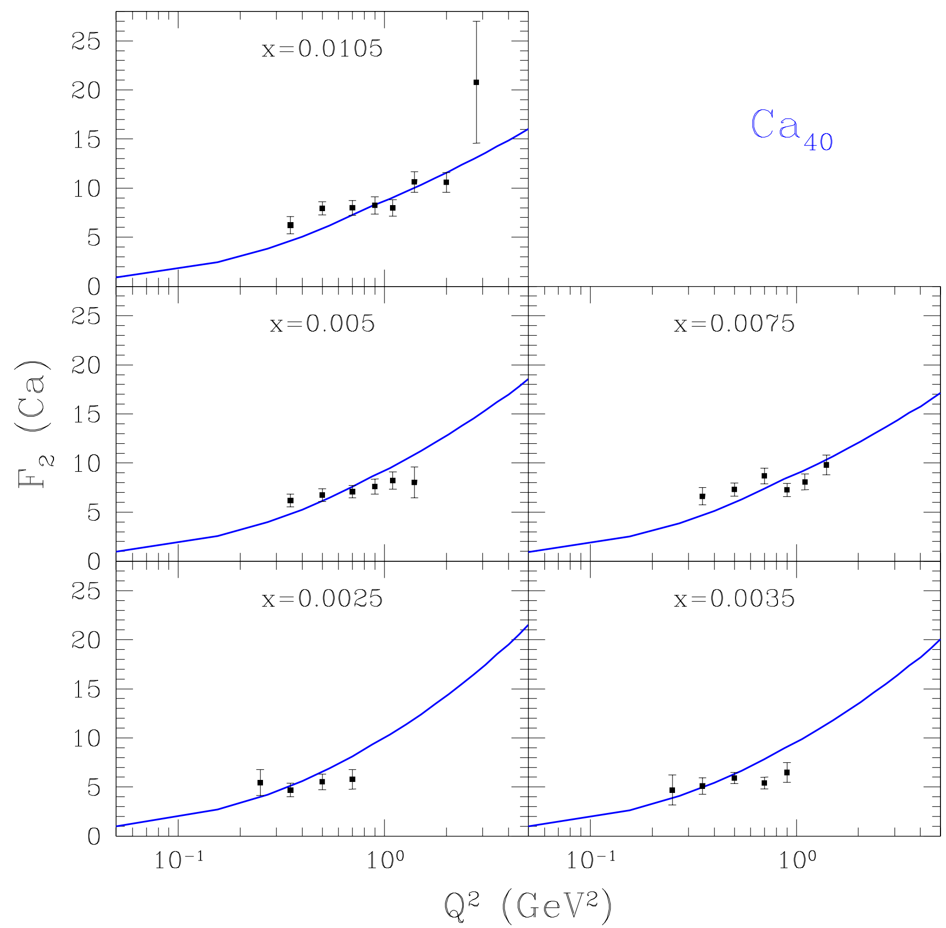

Ca$_{40}$
$A_{eff}/A$ (Ca)
$Q^2$=1.54
$Q^2$=1.09
$Q^2$=1.25
$Q^2$=1.4
$Q^2$=0.52
$Q^2$=0.65
$Q^2$=0.94
X


Ca$_{40}$
$F_2$ (Ca)
x=0.0105
x=0.005
x=0.0075
x=0.0025
x=0.0035
$Q^2$ (GeV$^2$)

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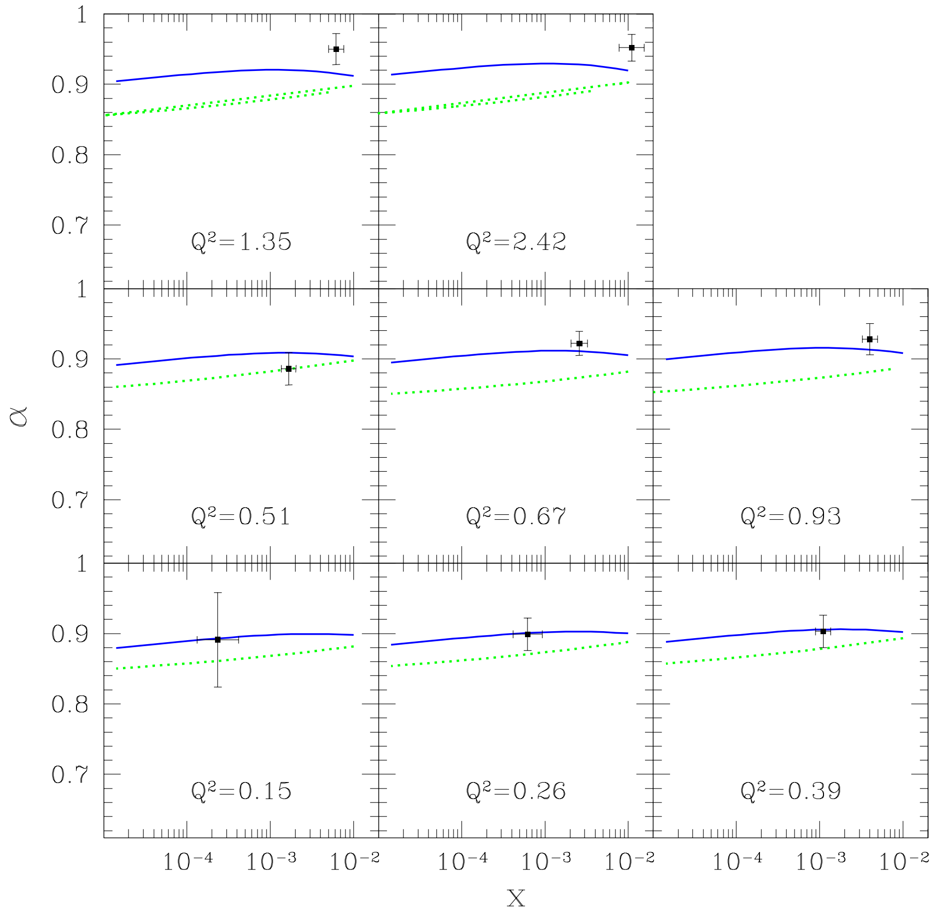
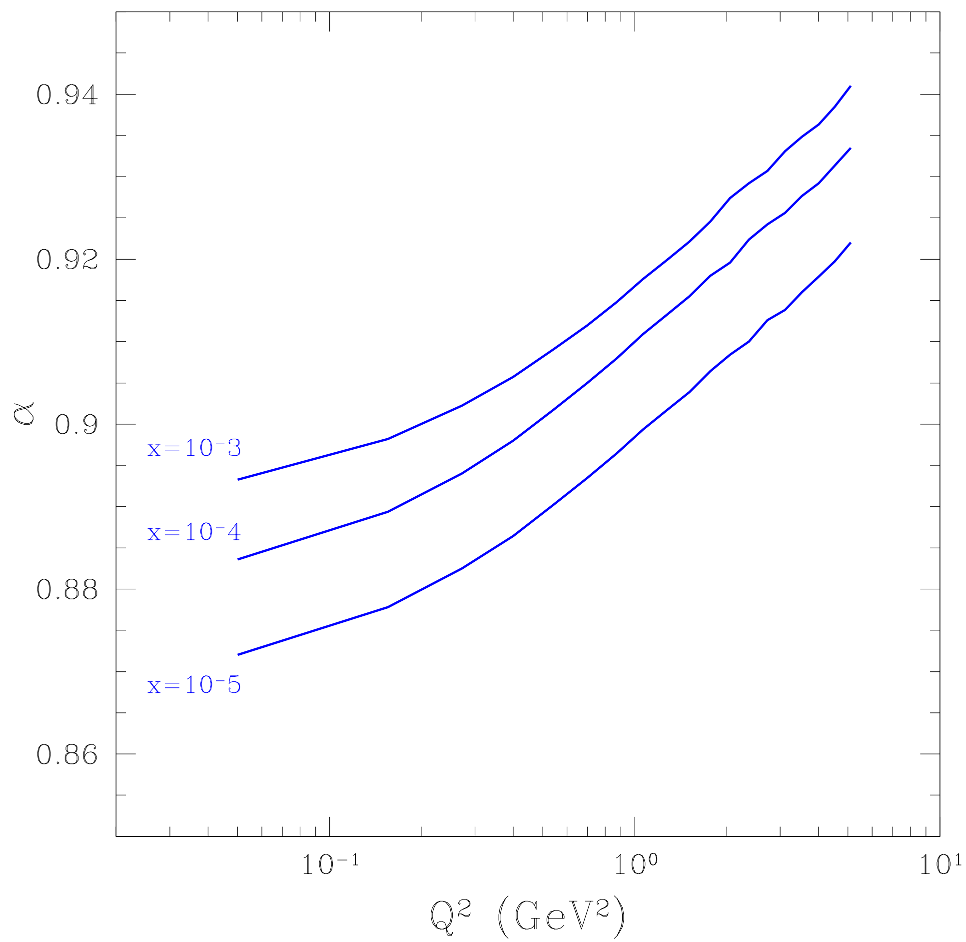
$$A_{eff} \sim A^{\alpha}$$

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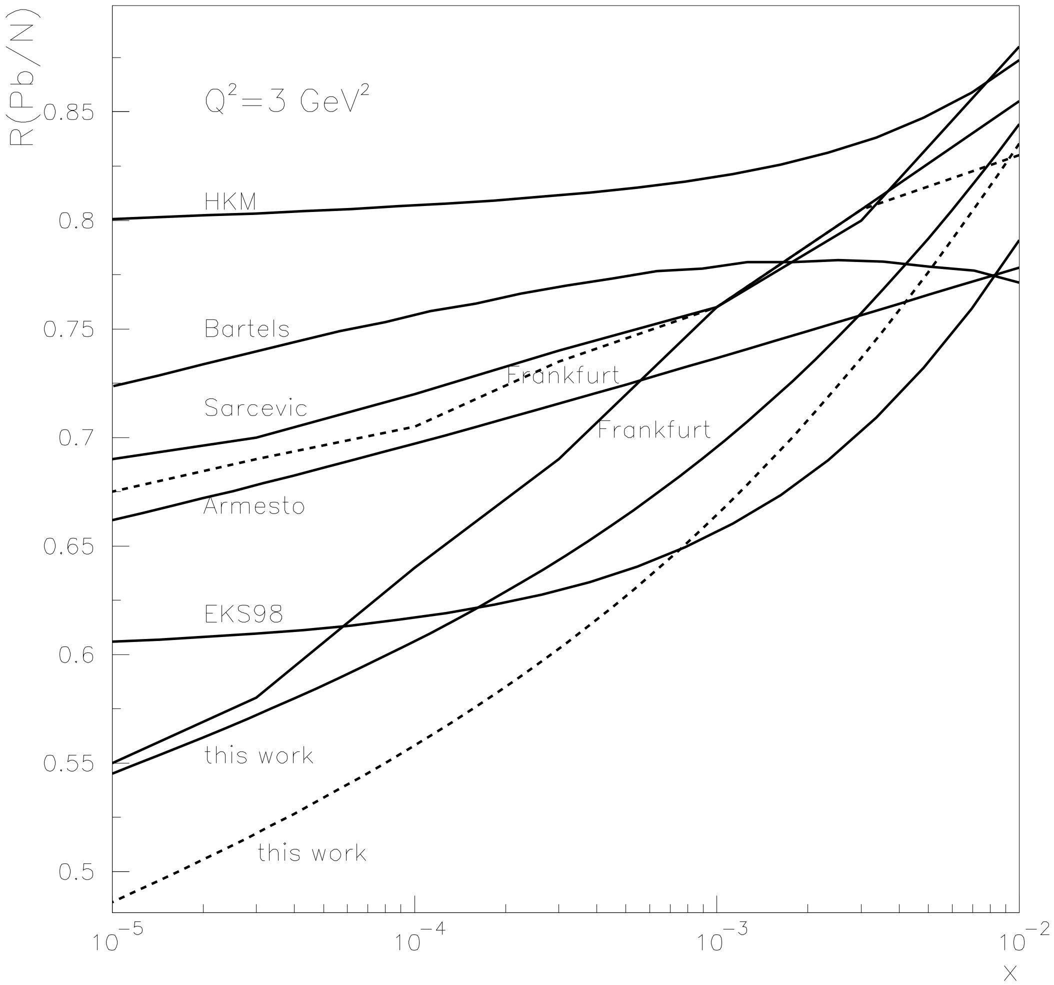
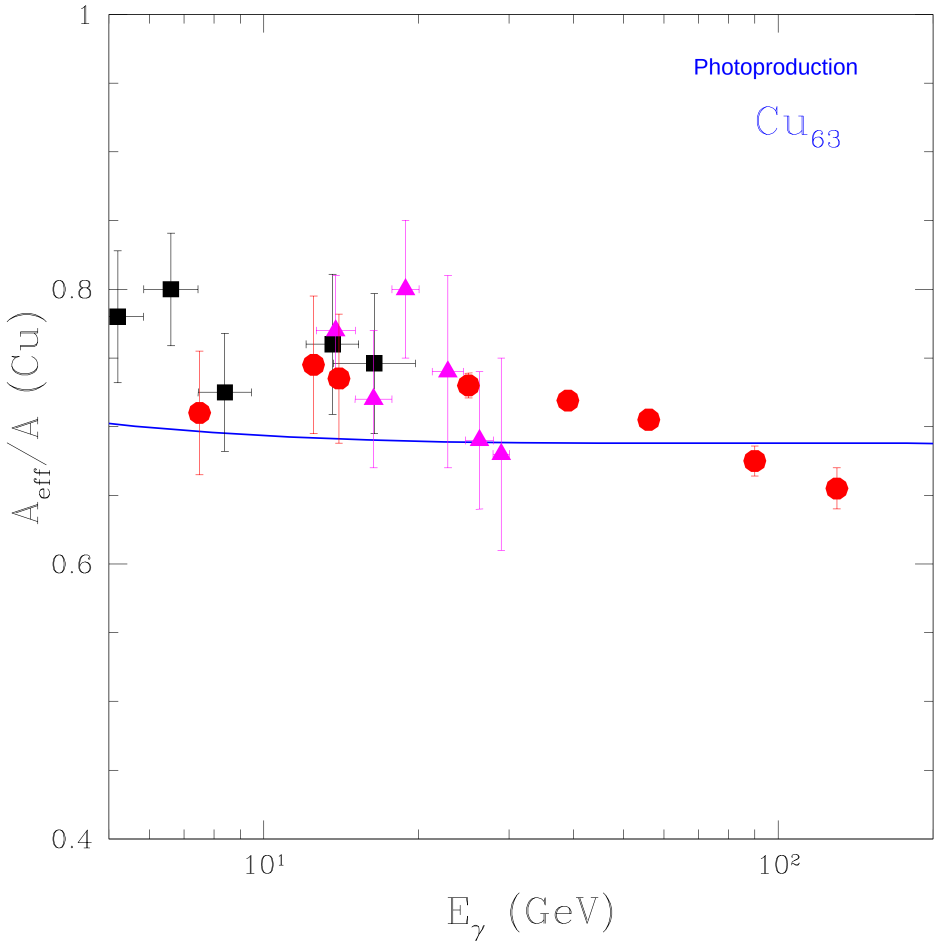
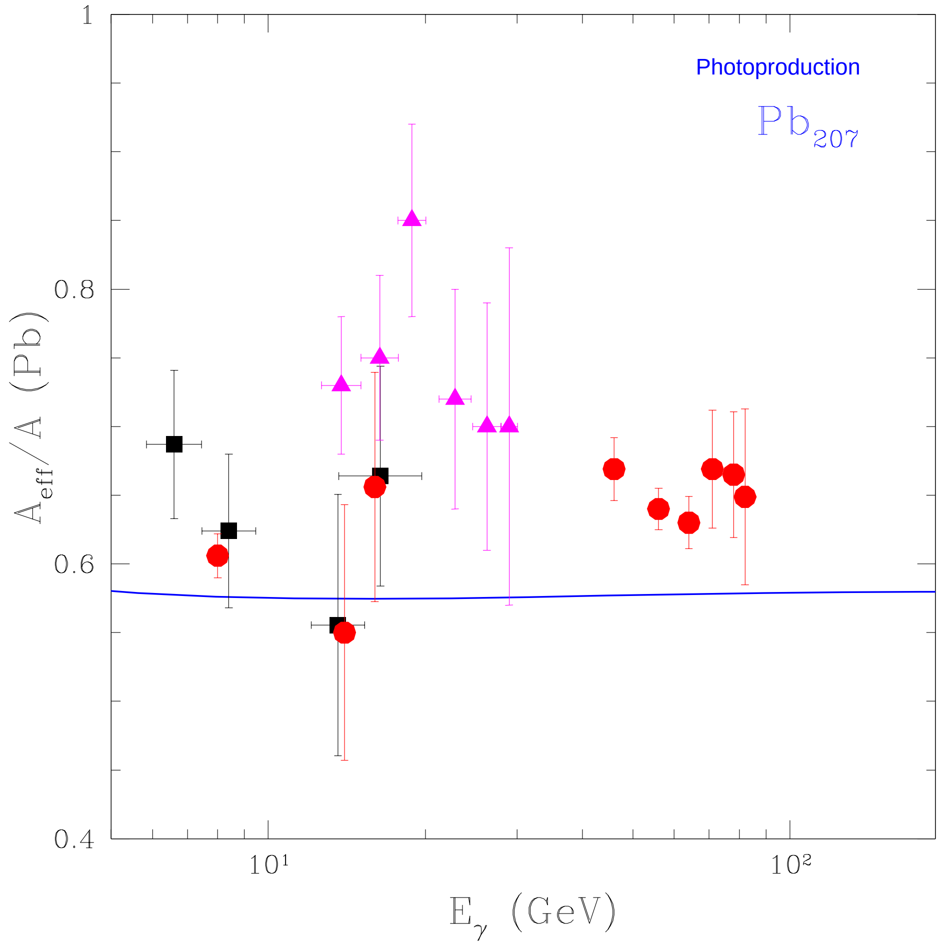



N. Armesto J.Phys.G32:R367-R394,2006

# Photo-Production on Nuclei

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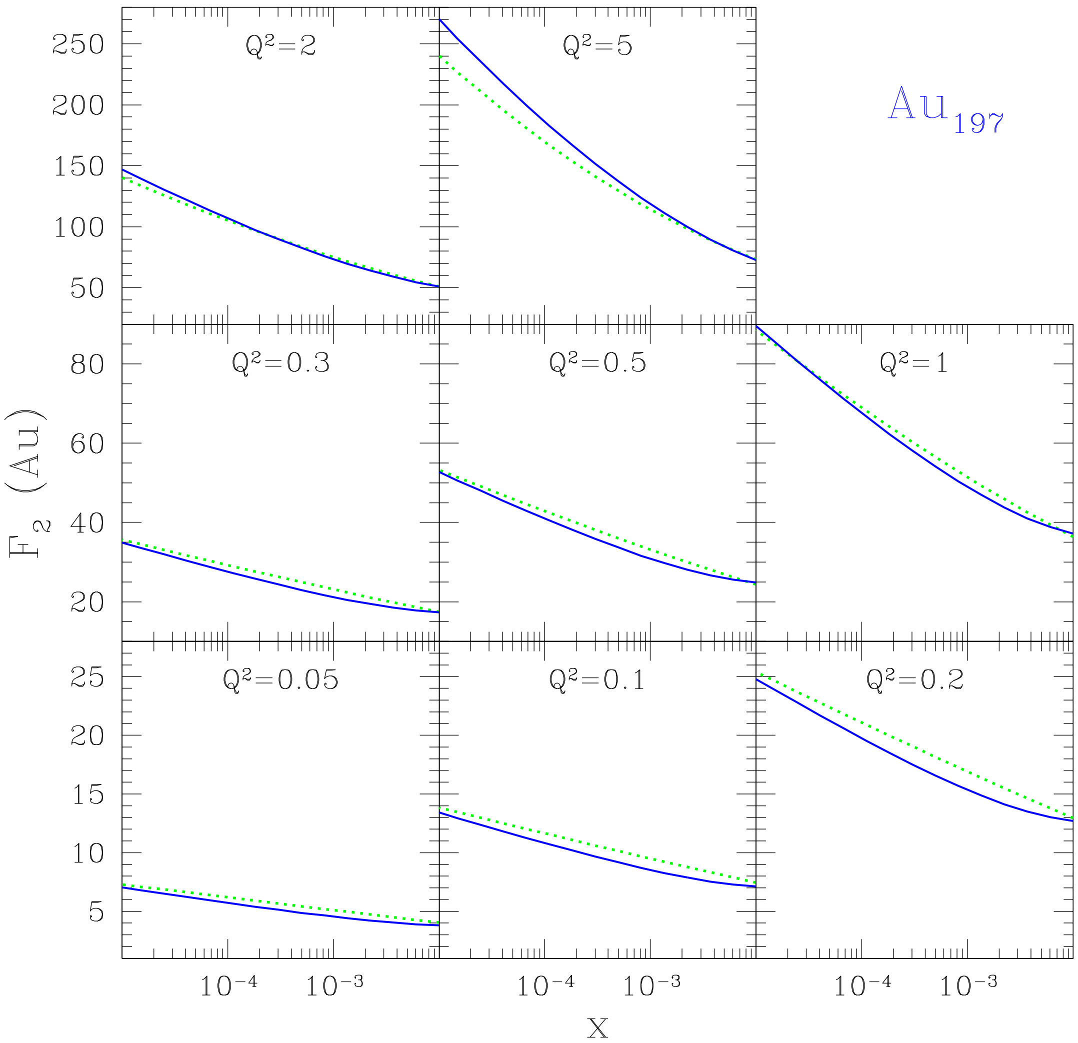
# The Structure Function $F_2$



# Summary

- A new approach to global QCD analysis based on BKE is developed;
- Low $x$ data on the $F_2$ structure function is reproduced using two fitting parameters only; Resulting $\chi^2/ndf\ = 1$;
- Our method allows extrapolation of the parton distributions to the LHC energies as well as very low photon virtualities $Q^2 \ll 1\,GeV^2$;
- We find $\lambda\ \simeq\ 0.25 - 0.4$ at large $Q^2$ (hard BFKL pomeron) while $\lambda\ \simeq\ 0.08 - 0.1$ at very low $x$ and $Q^2$ well below $1\,GeV^2$. A result which agrees with the "soft pomeron" intercept without soft physics involved;
- Results on nuclei can be predicted from information on proton structure without any additional fitting parameters;
- The $A$-dependence of the structure function $F_{2\,A}$ is computed and successfully compared with the data.