How to measure $G(x,Q^2)$ in diffractive events using exclusive vector meson production

An attempt to create a recipe that us experimentalists can understand

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Motivation

- One of the key measurements at eRHIC is $G(x,Q^2)$
- Exclusive diffractive vector meson production is considered the most promising method
  - $\sigma \sim G(x,Q^2)^2$
- To date it is not clear to me (or others?) how the measurement is actually conducted
  - w/o this understanding we cannot realistically establish errors and quality of measurements (as a fct. of luminosity, energy, detector acceptance etc)
- We have to get away from seeing a $G(x,Q^2)$ measurement as a measurement of the ratio $R = G_{eA}/G_{ep}$
  - The assumption that things cancel out in ratios is not obvious (and as it will turn out is not justified)

This is a first attempt to learn about how $G(x,Q^2)$ could be obtained with what is measured in an eRHIC experiment

Theory(I)

[1] S. Brodsky et al., Phys.Rev.D50:3134,1994, e-Print: hep-ph/9402283

[2] L. Frankfurt et al., Phys. Rev. D 54, 3194 - 3215 (1996) (corrects above)

$$\left.\frac{d\sigma_L^{\gamma^* N \to V N}}{dt}\right|_{t=0} = \frac{12\pi^3 \Gamma_V m_V \eta_V^2 T(Q^2)}{\alpha_{em} N_c^2} \cdot \frac{\alpha_s^2(Q) \left|\left[1 + i\frac{\pi}{2}\frac{d}{d\ln x}\right] x G(x, Q^2)\right|^2}{Q^6}$$

Note: this is cross-section for a *longitudinally* polarized photon to produce a *longitudinally* polarized vector meson, i.e., it is not spin averaged over initial photon states.

Warnings (Vadim): this is a simplified version of the corresponding expression in the dipole formalism: it uses only the perturbative part of the dipole cross section and ignores complications of the final meson wave function.

TU: What's with the transversely polarized photons? Many papers say there are problems (infrared singularities).

Theory (II): Understanding the formula

$$\left.\frac{d\sigma_L^{\gamma^* N \to V N}}{dt}\right|_{t=0} = \frac{12\pi^3 \Gamma_V m_V \eta_V^2 T(Q^2)}{\alpha_{em} N_c^2} \cdot \frac{\alpha_s^2(Q)\left|\left[1 + i\frac{\pi}{2}\frac{d}{d\ln x}\right] xG(x,Q^2)\right|^2}{Q^6}$$

Γ_V: is the decay with of the vector meson into an e+e- pair

Theory (III): Understanding the formula

$$\left.\frac{d\sigma_L^{\gamma^* N\to VN}}{dt}\right|_{t=0} = \frac{12\pi^3\Gamma_V m_V \eta_V^2 T(Q^2)}{\alpha_{em} N_c^2}\cdot\frac{\alpha_s^2(Q)\left|\left[1+i\frac{\pi}{2}\frac{d}{d\ln x}\right]xG(x,Q^2)\right|^2}{Q^6}$$

η_V: effective inverse momentum of the vector meson distribution amplitude that controls the *leading twist* contribution to the lepto-production amplitude.

$$\eta_V \equiv \frac{1}{2}\frac{\int dz d^2k_T [z(1-z)]^{-1}\Phi_V(z,k_T)}{\int dz d^2k_T \Phi_V(z,k_T)}$$

$\Phi_V(z)$: wave function of longitudinal polarized vector meson

Roughly: Describes the distribution amplitudes of the longitudinal momentum fraction z of the quark in the meson.

Light mesons (ρ,φ): $\Phi_V(z) \sim 6\, z(1-z)$

Heavy mesons (J/ψ, Υ): $\Phi_V(z) \sim \delta(z-1/2)$ (non-rel. picture)

Typical values used: $\eta_\rho \approx 2$ - 5 $\eta_{J/\psi} \approx 2$ (model dep.)

Theory (IV): Understanding the formula

$$\left.\frac{d\sigma_L^{\gamma^* N \to VN}}{dt}\right|_{t=0} = \frac{12\pi^3 \Gamma_V m_V \eta_V^2 T(Q^2)}{\alpha_{em} N_c^2} \cdot \frac{\alpha_s^2(Q) \left|\left[1 + i\frac{\pi}{2}\frac{d}{d\ln x}\right] xG(x,Q^2)\right|^2}{Q^6}$$

T(Q²): Introduced in [2].

Accounts for "preasymptotic" effects

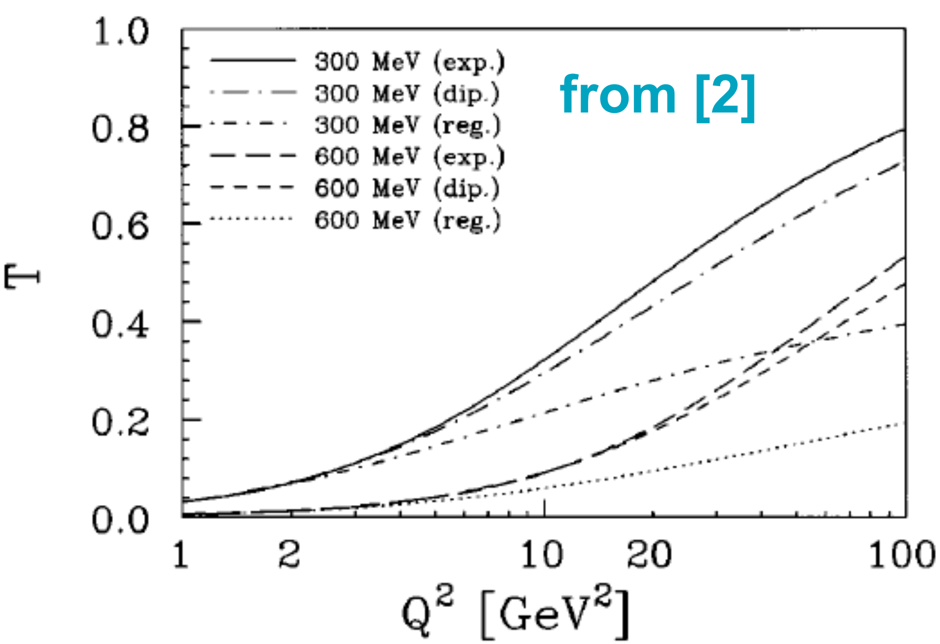
$T(Q^2 \to \infty) = 1$

Formula (w/o T) is only valid when transverse momenta in $q\bar{q}$ dipole (Fermi-motion) are neglected, i.e., at sufficiently large Q^2. Otherwise corrections are needed.

Theory (V): Understanding the formula

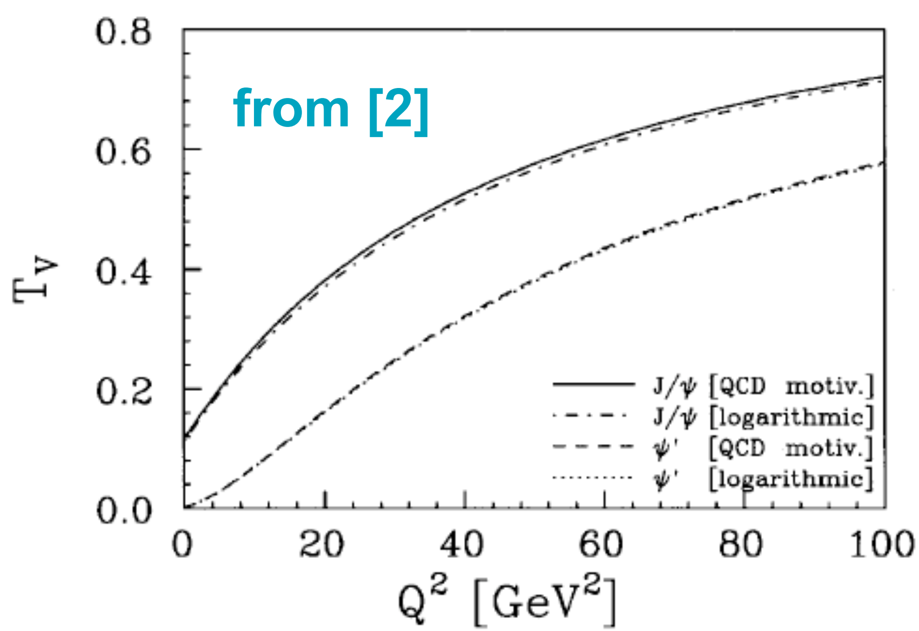
Light quark vector mesons

$$T(Q^2) = \left(\frac{\int_0^1 dz \int_0^{Q^2} d^2k_t \psi_V(z,k_t)\left(-\frac{1}{4}\Delta_t\right)\left[\frac{Q^4}{Q^2+\frac{k_t^2+m^2}{z(1-z)}}\right]}{\int_0^1 \frac{dz}{z(1-z)} \int_0^{Q^2} d^2k_t \psi_V(z,k_t)} \right)^2 .$$



Heavy quark vector mesons

$$T_V(Q^2) = \left(\frac{\int d^3k\, \psi_V(k)\left(-\frac{1}{16}\Delta_t\right)\left[\frac{(Q^2+4m_c^2)^2}{Q^2+\frac{k_t^2+m_c^2}{z(1-z)}}\right]}{\int d^3k\, \psi_V(k)\, \frac{k^2+m_c^2}{k_t^2+m_c^2}} \right)^2$$



Theory (VI): more questions

$$\left.\frac{d\sigma_L^{\gamma^* N\to VN}}{dt}\right|_{t=0} = \frac{12\pi^3\Gamma_V m_V \eta_V^2 T(Q^2)}{\alpha_{em} N_c^2} \cdot \frac{\alpha_s^2(Q)\left|\left[1+i\frac{\pi}{2}\frac{d}{d\ln x}\right]xG(x,Q^2)\right|^2}{Q^6}$$

- In eA: which $\alpha_s(Q^2)$ to use when $Q < Q_s$ ($\alpha_s(Q_s^2)$?)
- What's with the term [*1+i* $\pi/2$ *d/dln x*] ?
  - In [1] the alternative form is offered:

$$\left.\frac{d\sigma_L^{\gamma^* N\to VN}}{dt}\right|_{t=0} = \frac{12\pi^3\Gamma_V m_V \eta_V^2 T(Q^2)}{\alpha_{em} N_c^2} \cdot \frac{\alpha_s^2(Q)|xG(x,Q^2)|^2}{Q^6}$$

Claim is to use the former at small x and large Q^2.
What's large/small in the context of eRHIC?
Cyrille: "10% uncertainty in omitting the real part"

(confused TU: why is the the term containing the i the real part?
P.S.: I know about the optical theorem ☺)

Theory (VII): even more questions

$$\left.\frac{d\sigma_L^{\gamma^* N \to VN}}{dt}\right|_{t=0} = \frac{12\pi^3 \Gamma_V m_V \eta_V^2 T(Q^2)}{\alpha_{em} N_c^2} \cdot \frac{\alpha_s^2(Q) |[1 + i\frac{\pi}{2}\frac{d}{d\ln x}] xG(x,Q^2)|^2}{Q^6}$$

- Diffractive slope b?
- What's about the transversely polarized part?
- Balance between σ_L and σ_T ?
    - Separately study σ_L and σ_T?
    - Cannot study $d\sigma_L/dt$ and $d\sigma_T/dt$ in e+A
- What's with the hard scale $\bar{Q}^2$ at which the gluon density is probed? (see I.Ivanov APP B Vol. 39, 2373 (2008))
    - Confusing statements in literature
    - heavy quarkonia: $\bar{Q}^2 \approx (Q^2+m_V^2)/4$
    - light quarks: $\bar{Q}^2 \approx 0.1(Q^2+m_V^2)$
    - $\bar{Q}^L$ and $\bar{Q}^T$ are expected to be different

Theory (VIII): transversely polarized case

[3] L. Frankfurt et al., Phys.Rev.D57:512,1998, hep-ph/9702216

$$\left.\frac{d\sigma^{\gamma^* N\to VN}}{dt}\right|_{t=0} = \frac{12\pi^3\Gamma_V m_V^3}{\alpha_{em}}\cdot\frac{\alpha_s^2(Q)|[1+i\frac{\pi}{2}\frac{d}{d\ln x}]xG(x,Q^2)|^2}{(Q^2+4m^2)^4}\cdot\left(1+\epsilon\frac{Q^2}{m_V^2}\right)\mathcal{C}(Q^2)$$

where

$$\mathcal{C}(Q^2) = \left(\frac{\eta_V}{3}\right)^2\left(\frac{Q^2+4m^2}{Q^2+4m_{run}^2}\right)^4 T(Q^2)\frac{R(Q^2)+\epsilon(Q^2/m_V^2)}{1+\epsilon(Q^2/m_V^2)}$$

$\varepsilon=0$ purely transverse polarized (real photons $Q^2 = 0$)
$\varepsilon=1$ equal mix

This is getting a bit out of hand now

How do I measure all this at eRHIC
Let's see what the experimentalists say...
This will also clarify the meaning of ε and R

Experimental Side (I)

[4] ZEUS, Eur.Phys.J.C6:603,1999, hep-ex/9808020
[5] ZEUS, Nucl.Phys.B695:3,2004, hep-ex/0404008
[6] H1 Eur.Phys.J.C13:371,2000, hep-ex/9902019

Experiments measure ep (eA) cross-section not virtual photo-production cross-sections

In Born approximation:

$$\frac{d^2\sigma^{ep}}{dydQ^2} = \underbrace{\Gamma_T(y,Q^2)}\underbrace{(\sigma_T^{\gamma*p} + \epsilon\sigma_L^{\gamma*p})}$$

Measured → $\frac{d^2\sigma^{ep}}{dydQ^2}$

theory → $\sigma_L^{\gamma*p}$

$\Gamma_T(y,Q^2)$: Flux of *transverse* virtual photons

$(\sigma_T^{\gamma*p} + \epsilon\sigma_L^{\gamma*p})$: transverse and longitudinal virtual photoproduction cross-section

Recall: $Q^2 = sxy$

Experimental Side (II)

$$\frac{d^2\sigma^{ep}}{dydQ^2} = \Gamma_T(y,Q^2)(\sigma_T^{\gamma*p} + \epsilon\sigma_L^{\gamma*p})$$

ε is the ratio of long. and transv. virtual photon flux

$$\epsilon = \frac{2(1-y)}{1+(1-y)^2}$$

typically 0.8 - 1

and the transverse photon flux is:

$$\Gamma_T = \frac{\alpha_{em}}{2\pi}\frac{1+(1-y)^2}{yQ^2}$$

together:

$$\frac{d^2\sigma^{ep}}{dydQ^2} = \frac{\alpha_{em}}{\pi Q^2 y}[(1-y+\frac{y^2}{2})\sigma_T^{\gamma*p} + (1-y)\sigma_L^{\gamma*p}]$$

Experimental Side (III)

The virtual photon cross-section

$$\sigma^{\gamma^* p} \equiv \sigma_T^{\gamma^* p} + \epsilon \sigma_L^{\gamma^* p}$$

can be used to evaluate the total exclusive cross-section

$$\sigma_{tot}^{\gamma^* p} \equiv \sigma_T^{\gamma^* p} + \sigma_L^{\gamma^* p}$$

What?

through:

$$\sigma_{tot}^{\gamma^* p} = \frac{1+R}{1+\epsilon R} \sigma^{\gamma^* p}$$

where

$$R = \frac{\sigma_L^{\gamma^* p}}{\sigma_T^{\gamma^* p}}$$

Experimental Side (IV)

In our case:

$$\sigma^{\gamma^* p \to pJ/\psi} \equiv \sigma_T^{\gamma^* p} + \epsilon \sigma_L^{\gamma^* p}$$

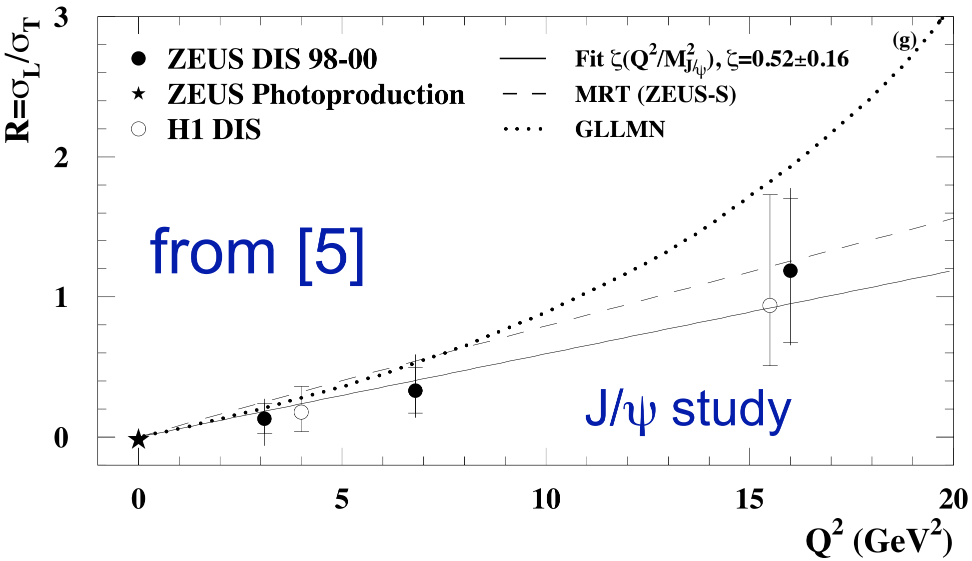
can be used to obtain:

$$\begin{aligned} \sigma_{tot}^{\gamma^* p \to pJ/\psi} &\equiv \sigma_T^{\gamma^* p \to pJ/\psi} + \sigma_L^{\gamma^* p \to pJ/\psi} \\ &= \frac{1+R}{1+\epsilon R} \sigma^{\gamma^* p \to pJ/\psi} \end{aligned}$$

What is the value for R and on what does it depend?

Experimental Side (V)

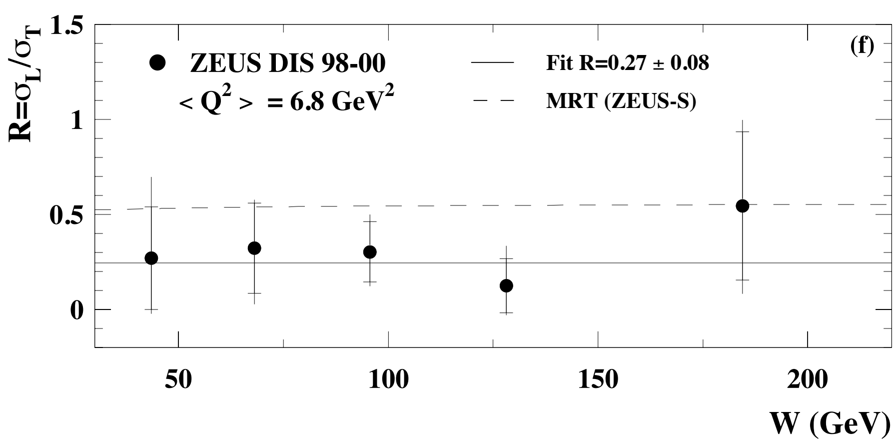
- Model predictions: e.g. $R = 0.5 \cdot (Q^2/M_{J/\psi})$
- Helicity structure of VM production can be used to get R

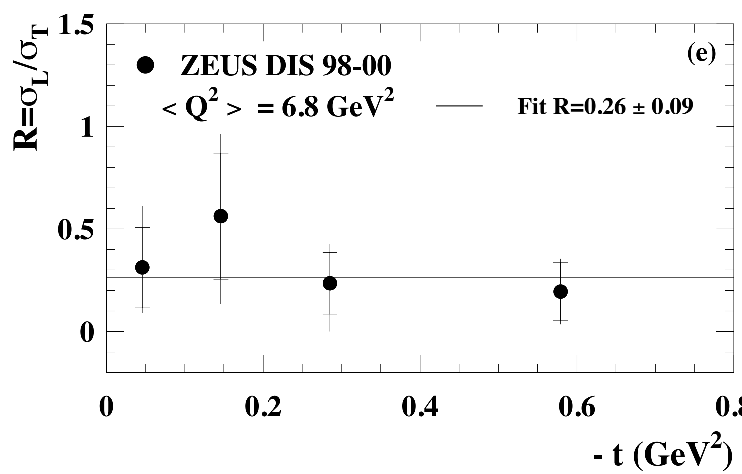


R from polar angle distributions:

$R = 0.52 \pm 0.16\ (Q^2/M_{J/\psi})$

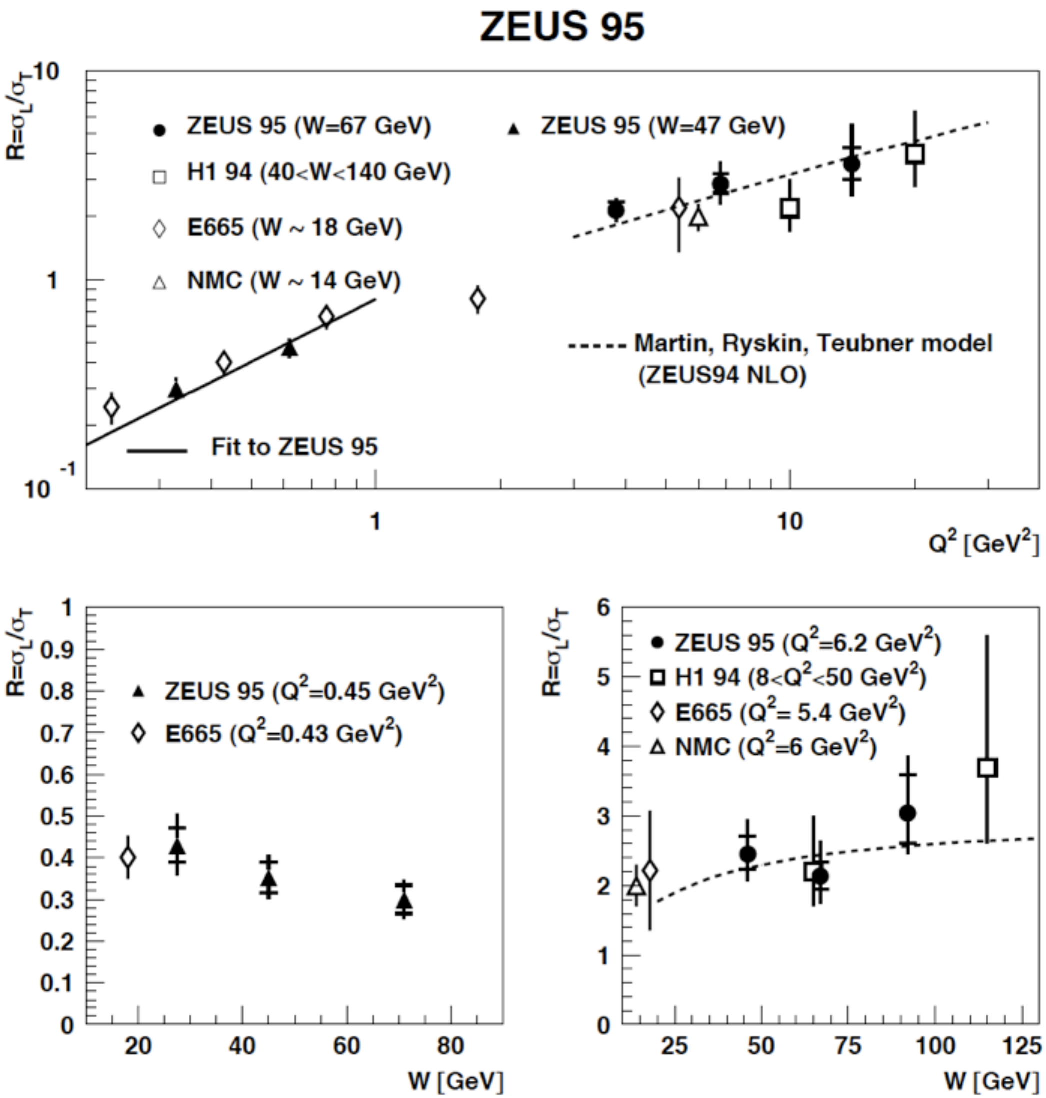
No W, t dependence?!

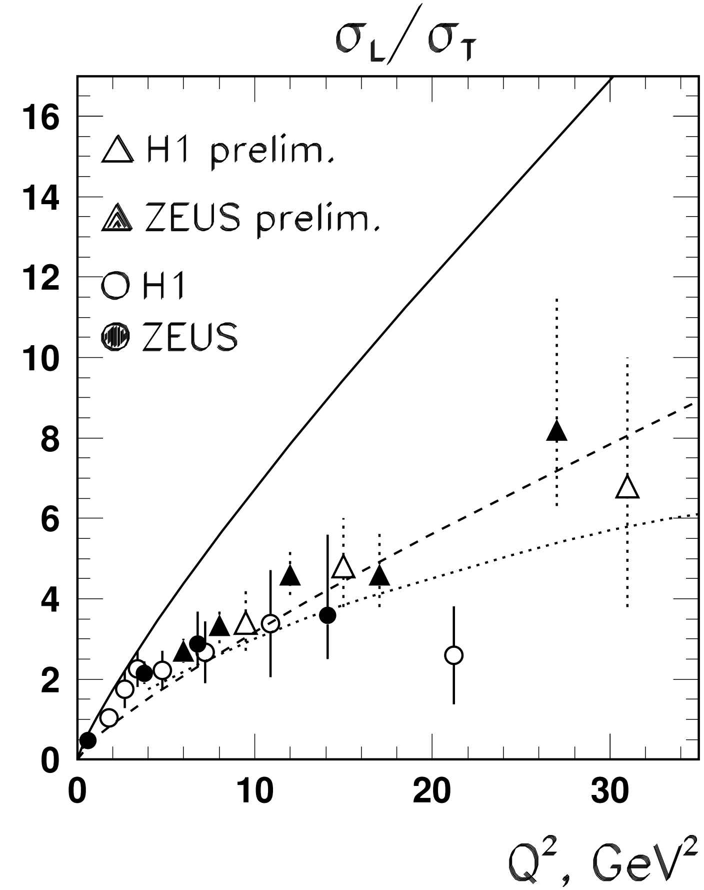




Experimental Side (VI)

Much bigger (and more uncertain) for ρ





Model prediction deviate big time

Comparing Theory with Experiment

In order to compare results with calculations and thus relate measured value with $G(x,Q^2)$ we need:

$$\left.\frac{d\sigma_L^{\gamma^* p}}{d|t|}\right|_{t=0} = \frac{R}{1+R} \cdot \frac{b}{1-e^{-b|t|_{max}}} \cdot \sigma_{tot}^{\gamma^* p}$$

since $$\frac{d\sigma}{d|t|} \propto e^{-b|t|}$$

In e+A at eRHIC we are not going to measure any *t*-dependence

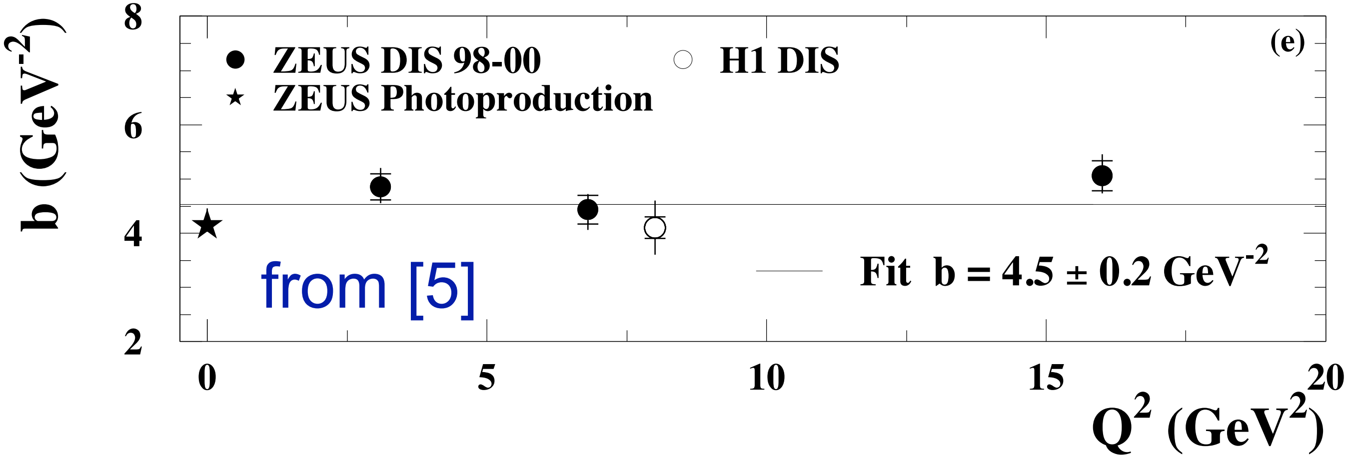
So what is b? What is t_{max}?

Guess t_{max} will be related to the point where incoherent sets in?

We can get an estimate from e+p - is that good enough?

More on b

J/ψ: no significant Q dependence   b = 4.5±0.2 GeV^{-2}



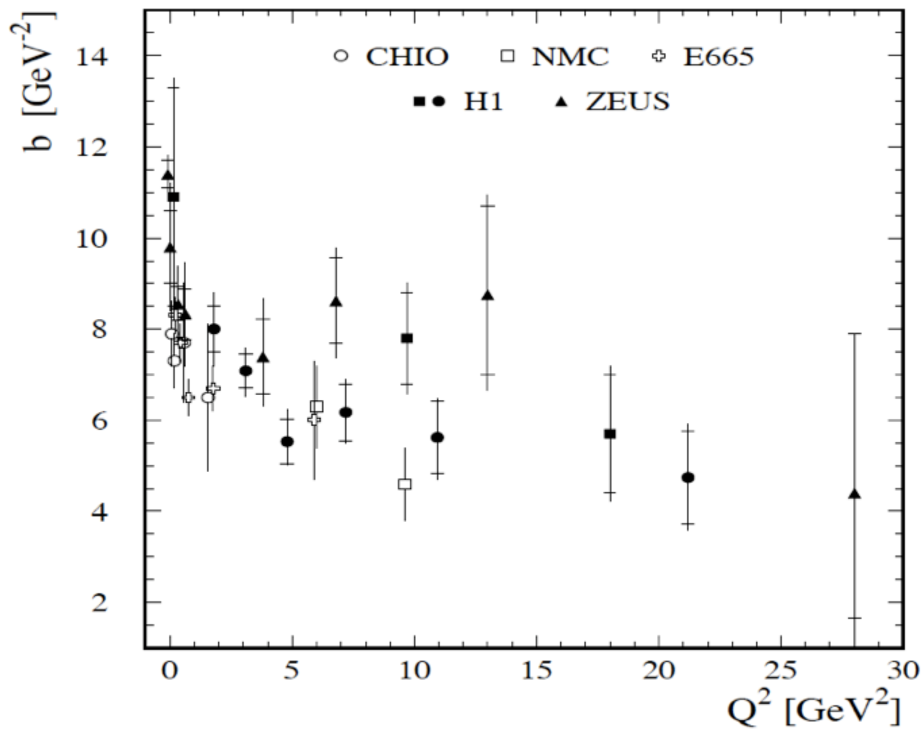
ρ: carful about what is said here:

while for the J/ψ photo and electroproduction give the same b this is not true for the ρ

At times authors are not careful in their statements

Even more on b for the ρ

from [6] elastic ρ production



b = 2.5±1.0 GeV^{-2}

diffractive component with proton dissociation and the non-resonant two-pion background. The elastic component is fitted with a free slope parameter b, whereas the contribution of diffractive ρ events with proton dissociation, which amounts to $11 \pm 5\%$ of the elastic signal, has a fixed slope parameter $b_{pd} = 2.5 \pm 1.0$ GeV^{-2} (see section 3.2.2).[11] The non-resonant background,

Questions instead of Conclusion

$$\left.\frac{d\sigma_L^{\gamma^* N \rightarrow VN}}{dt}\right|_{t=0} = \frac{12\pi^3 \Gamma_V m_V \eta_V^2 T(Q^2)}{\alpha_{em} N_c^2} \cdot \frac{\alpha_s^2(Q) |[1 + i\frac{\pi}{2}\frac{d}{d\ln x}]xG(x,Q^2)|^2}{Q^6}$$

- Is this a reasonable calculation to work with ?
  - Vadim expressed doubts
  - Is there anything better ?
- Is the long. + trans. calculation OK,
  - or is it better to deal with long. calculation only and fix it experimentally (appears not to be equivalent)
- What do we do with b in e+A?