Dark matter in 3D

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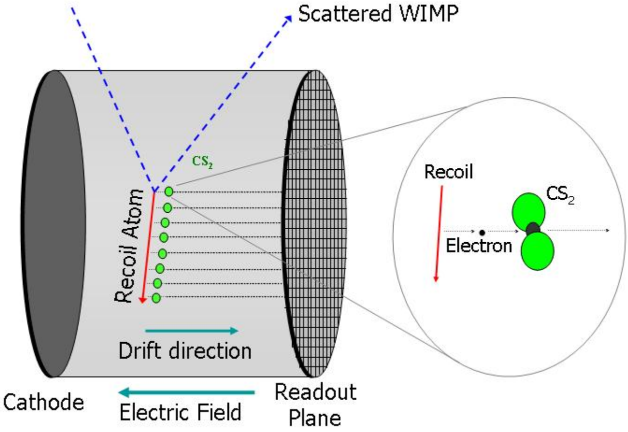
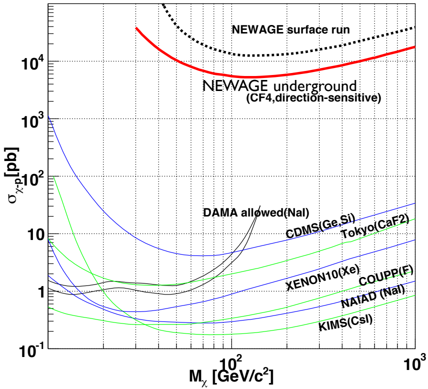
arXiv:1204.5487v1

Outline

Directional detection



Current results and prospects



Miuchi et al.
arXiv:1002.1794v1

- Currently not competitive with XENON or CDMS for dark matter discovery
- Provide much more information about dark matter kinematics through recoil directions
- Unique information about the dark matter halo in the post discovery era

1. Directional detection : overview

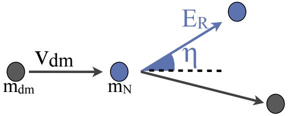
2. Reconstructing the dark matter distribution function
   - Kinematics: overview
   - Continuous velocity distributions
   - Parameterizing the distribution function

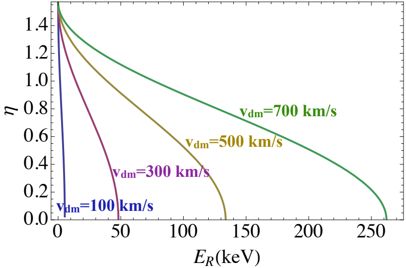
3. Distribution function and statistics

4. Test using N-body simulations

Recoil energies and angles

$$\cos\eta = \frac{v_{min}(E_R)}{v_{DM}}$$

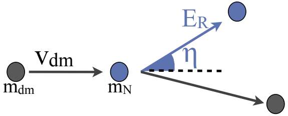


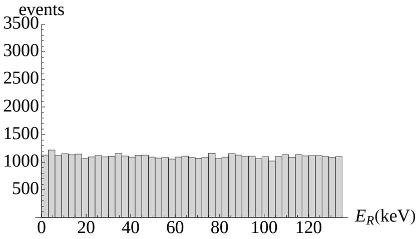


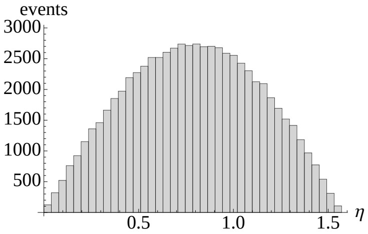
$\eta \sim 0$ for high E_R, wide angle for low E_R

Scattering and form factors

$$\frac{dR}{dE_R d\Omega} \propto |F(E_R)|^2 |F_{DM}(E_R)|^2 \delta(\ldots)$$







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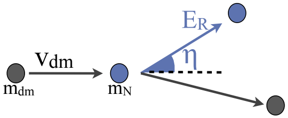
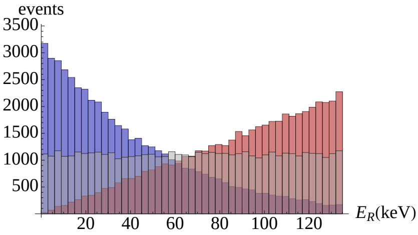
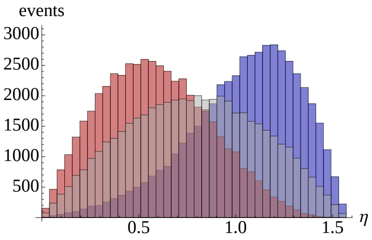


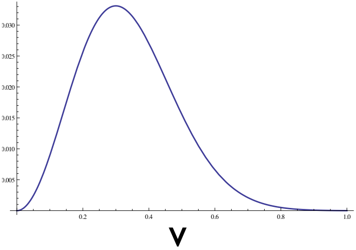




Examples of velocity distributions

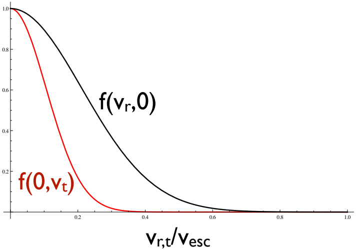
Maxwellian distribution

$$f(v) = \left(e^{\frac{v_{esc}^2 - v^2}{v_0^2}} - 1 \right) \Theta(v_{esc}^2 - v^2)$$



Michie distribution

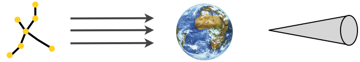
$$f(v) = \left(e^{\frac{v_{esc}^2 - v^2}{v_0^2}} - 1 \right) e^{-\alpha \frac{v_t^2}{v_0^2}} \Theta(v_{esc}^2 - v^2)$$

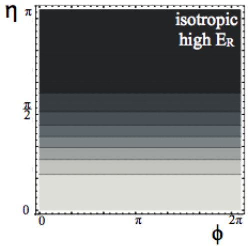


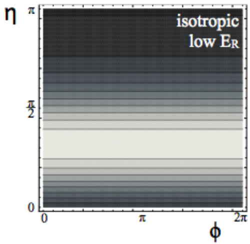
$\eta - \phi$ profiles of the rate – Isotropic case

Flux from Cygnus

Preferred recoil directions



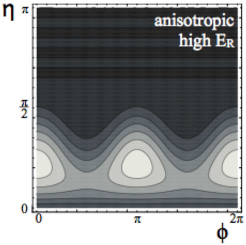


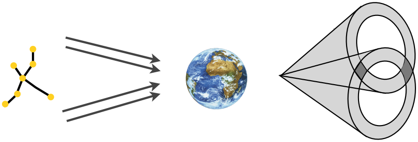


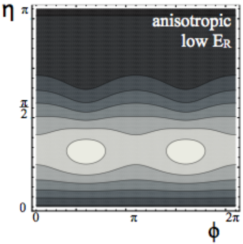
$\eta - \phi$ profiles of the rate – Anisotropic case

Flux from Cygnus

Preferred recoil directions







Dark matter galactic halo

Dark matter halo characterized by a phase space distribution function

$$P(\vec{r},\vec{v}) = f(\vec{r},\vec{v}) d^3v d^3r$$

Dark matter density profile

$$\rho(\vec{r}) = \int f(\vec{r},\vec{v}) d\vec{v}$$

Local velocity distribution at $\vec{r}_0$

$$f_{\vec{r}_0}(\vec{v}) = f(\vec{r}_0,\vec{v})$$

From local to global

A detector on Earth measures

$$f_{\oplus}(\vec{v}) = f(\vec{r}_{\oplus}, \vec{v}).$$

How to get $f(\vec{r}, \vec{v})$ for any position in the galaxy?

Jeans theorem

Any steady-state solution of the collisionless Boltzmann equation depends on the phase-space coordinates only through integrals of motion. Conversely, any function of the integrals of motion is a steady-state solution of the collisionless Boltzmann equation.

Jeans theorem

A detector on Earth measures

$$f_{\oplus}(\vec{v}) = f(\vec{r}_{\oplus}, \vec{v})$$

Jeans theorem states

$$f(\vec{r}, \vec{v}) = f\left(I_1[\vec{r}, \vec{v}], I_2[\vec{r}, \vec{v}], I_3[\vec{r}, \vec{v}]\right)$$

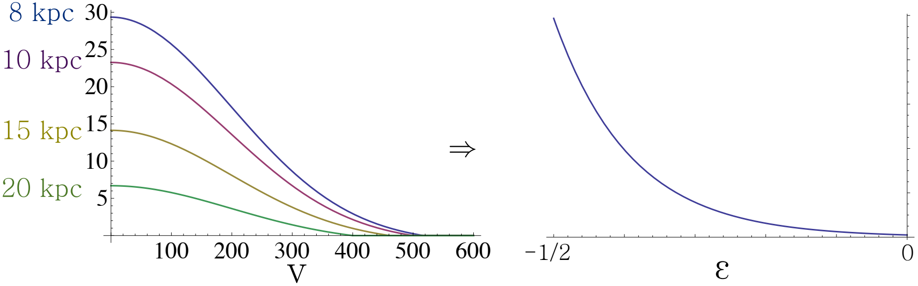
$$\Rightarrow f_{\oplus}(\vec{v}) = f\left(I_1[\vec{r}_{\oplus}, \vec{v}], I_2[\vec{r}_{\oplus}, \vec{v}], I_3[\vec{r}_{\oplus}, \vec{v}]\right)$$

$f_{\oplus}(\vec{v})$ allows to reconstruct the global distribution function!

Example : Maxwellian distribution

$$f(\vec{r}, v) = \left(e^{\frac{v_{esc}^2(\vec{r}) - v^2}{v_0^2}} - 1 \right) \Theta(v_{esc}^2 - v^2) \Rightarrow f(\mathcal{E}) = \left(e^{-\frac{\mathcal{E}}{\mathcal{E}_0}} - 1 \right) \Theta(-\mathcal{E})$$

$$\mathcal{E}(\vec{r}, v) = \frac{v^2 - v_{esc}^2(\vec{r})}{2}$$

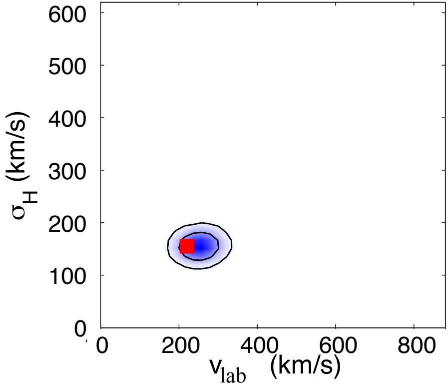


Parameterizing the distribution function

Standard procedure:

- Assuming an ansatz for $f(\vec{v})$ (e.g. Maxwellian)
- Fit the detection signal to get v_0, etc...

- Only local results
- Strong model dependance



Peter et al., arXiv:1202.5035v1

Parameterizing the distribution function

Find a *global* and *model-independant* parameterization of $f(\vec{r}, \vec{v})$:

- global : $f(\vec{r}, \vec{v}) \to f(\mathcal{E}, L_z, |L|, \ldots)$
- model-independant : use series expansions

$$f(\vec{r}, \vec{v}) = \sum_{ijk} c_{ijk} g_i(\mathcal{E}) h_j(L_z) \ldots$$

Model independent parameterization of the distribution function

$$f(\vec{r},\vec{v}) = f(\mathcal{E}, L_t, L_z) = f_1(\mathcal{E}) f_2(L_t) f_3(L_z)$$

$$L_t = \sqrt{L^2 - L_z^2}$$

$$f_1(\mathcal{E}) = \sum_i c_i P_L^{(i)}\left(\frac{\mathcal{E}}{\mathcal{E}_{lim}}\right) \quad \text{Legendre series}$$

$$f_2(L_t) = \sum_i d_i \cos\left(i\pi \frac{L_t}{L_{max}}\right) \quad \text{Fourier series}$$

$$f_3(L_z) = \sum_i f_i \cos\left(i\pi \frac{L_z}{L_{max}}\right) \quad \text{Fourier series}$$

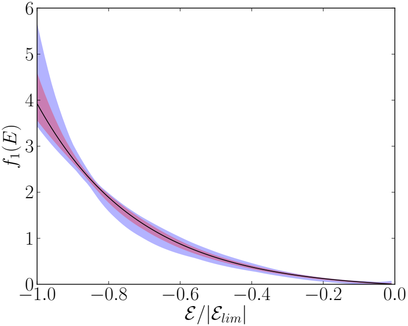
Halo model and detector

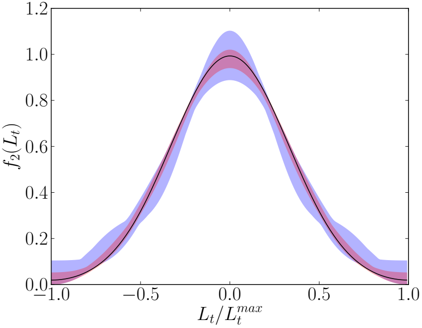
- Michie distribution

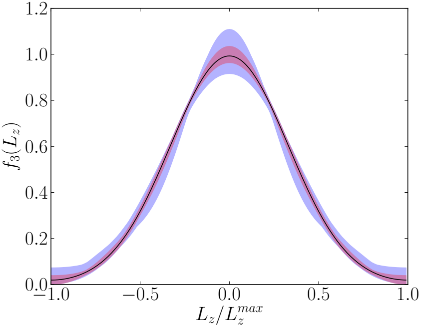
$$f(\mathcal{E}, L_t, L_z) = \left(e^{-\mathcal{E}/\mathcal{E}_0} - 1\right) e^{-\alpha(L_t^2+L_z^2)/L_0^2}\Theta(-\mathcal{E})$$

- $L_0 = r_\oplus v_0$, $\mathcal{E}_0 = v_0^2/2$, $v_0 = 280$ km/s, $\alpha = 1$
- $v_{esc} = 600$ km/s, kept fixed
- CS_2 target
- $m_{DM} = 6$ GeV
- Fit the detection rate for 1000 and 10000 signal events

Michie distribution
Light dark matter
Sulphur nuclear target
10^3 and 10^4 signal events

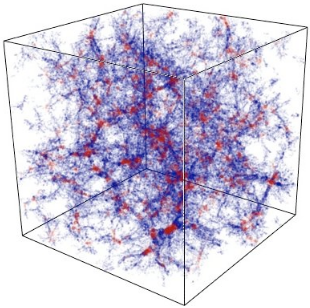


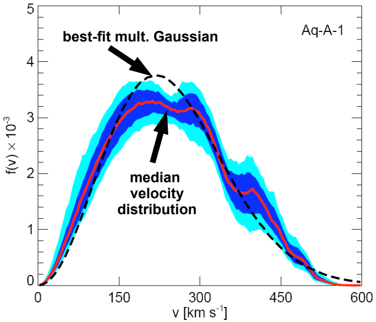




N-body simulations

Best current possible estimates of $f(\vec{r}, \vec{v})$
Via Lactea II: 10^9 particles, each of mass $\sim 10^3 M_{Sun}$

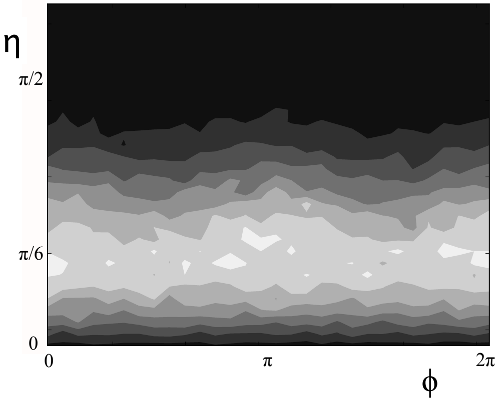




Vogelsberger et al., Aquarius

$\eta - \phi$ profile of the detection rate

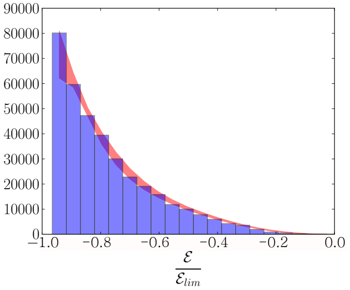
$m_{DM} = 6$ GeV – Sulphur nuclear target – 10000 signal events

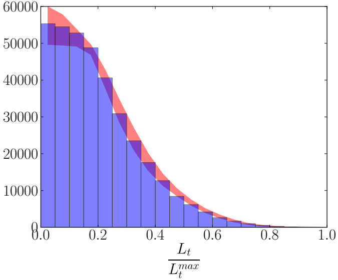


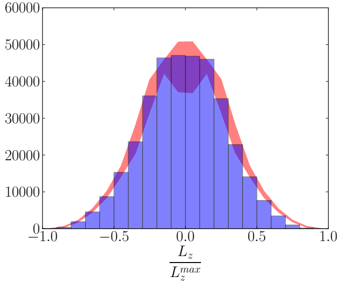
Hotspots in 0 $[\pi]$ showing that $f_2(L_t)$ is suppressed

$\mathcal{E}$, L_t, L_z distributions near the Earth

10000 signal events
Particles taken in a 200 pc thick spherical shell
8 kpc away from the Sun







VLII simulation and Jeans theorem

Check that $f(\mathcal{E}, L_t, L_z)$ is the same everywhere in the galaxy.

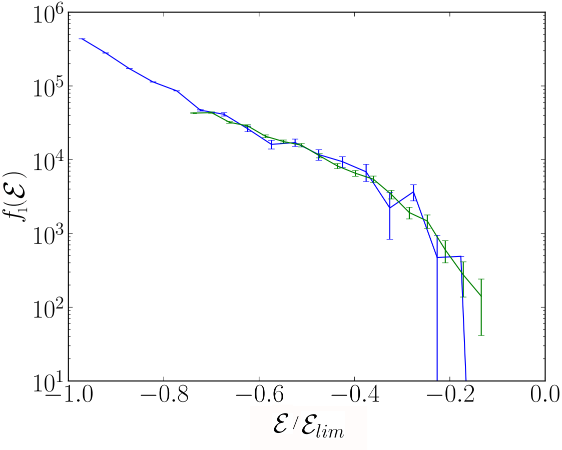
- Get $f_1(\mathcal{E})$ directly from binned data using

$$f_1(\mathcal{E}_i) = \frac{N(\mathcal{E}_i, 0, 0)}{\sqrt{2\mathcal{E}_i + v_{esc}^2}}$$

  but lack of statistics for $f_2(L_t)$ and $f_3(L_z)$

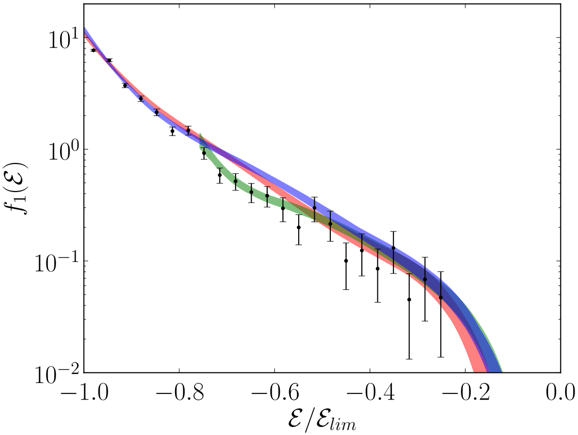
- Fit the velocity distributions at different positions : $r = 4.5$, 8 and 30 kpc and compare the results

$f_1(\mathcal{E})$ from binned data at 8 and 30 kpc



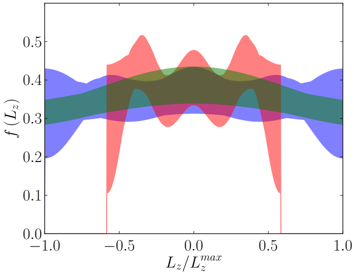
Fits of $f(\mathcal{E}, L_t, L_z)$

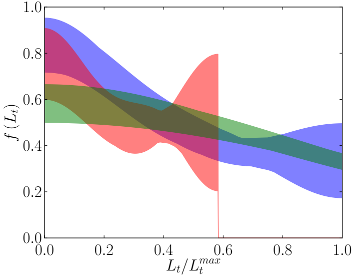
$f_1(\mathcal{E})$ at 4.5, 8 and 30 kpc away from the center of the galaxy



Fits of $f(\mathcal{E}, L_t, L_z)$

$f_2(L_t)$ and $f_3(L_z)$ at 4.5, 8 and 30 kpc
away from the center of the galaxy





Good agreement at 1 σ, small discrepancies at large distances probably due to our choice of integrals of motion

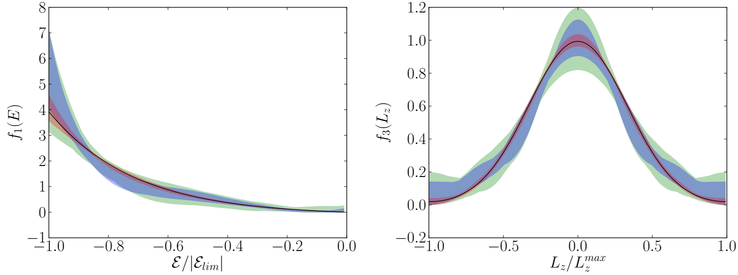
Conclusion

- Directional sensitivity is necessary to understand the kinematic properties of the dark matter halo
- The local velocity distribution near the Earth gives direct access to the galactic dark matter distribution function using Jeans theorem
- Series expansions allow to parameterize the distribution function in a model independent way
- Multidimensional fitting techniques allow to get a reasonable estimate of the shape of the DF with about 1000 events
- Very good fits of the local velocity distribution function from the VLII simulation using our ansatz
- Jeans theorem seems verified near the center of the galaxy for VLII data

Thank you

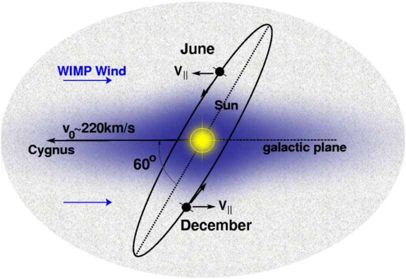
Future work

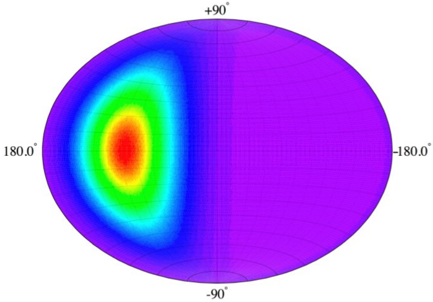
- Find the optimal number of terms in the series expansion without overfitting
- Get a better estimate of the error on the distribution function
- Study performance of the algorithm with background
- Study performance of the algorithm in presence of streams
- Get a better understanding of N-body simulation results and of the symmetries of the gravitational potential



$f_1(E)$
$\mathcal{E}/|\mathcal{E}_{lim}|$
8
7
6
5
4
3
2
1
0
−1
−1.0
−0.8
−0.6
−0.4
−0.2
0.0
$f_3(L_z)$
L_z/L_z^{max}
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−1.0
−0.5
0.0
0.5
1.0

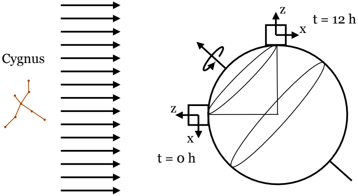
Annual modulation

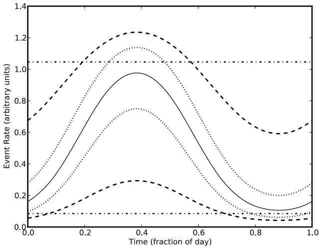




- Annual modulation of the dark matter flux
- Anisotropy of the recoil directions for *any* velocity distribution

Daily modulation





Daily modulation of the recoil direction (about 45%)

Current experiments

Experiment	Target	Energy threshold (keV)	V(m^3)
DRIFT	CS_2	~ 20	1
NEWAGE	CF_4	~ 100	0.03
DMTPC	CF_4	~ 50	0.01
MIMAC	He_3/CF_4	< 1	0.00013
Emulsions	$AgBr$	N/A	N/A

- Very low fiducial volumes
- Low pressure (drift length limited by diffusion)
- Large spin targets (for spin dependent scattering)
- High lower energy threshold for most experiments

Integrals of motion and symmetries

Gravitational potential $\psi(\vec{r}, t)$, related to the phase space density as

$$\Delta\psi = -4\pi\rho$$

Possibles integrals of motion according to the symmetries of ψ:

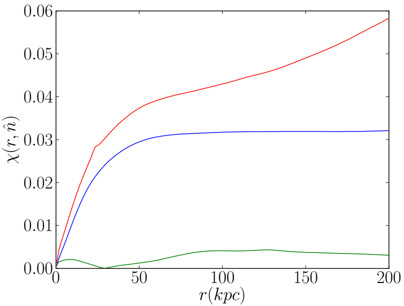
- $E = \frac{v^2}{2} - \psi(\vec{r})$: $\psi(\vec{r}, t) = \psi(\vec{r})$ (good for haloes at equilibrium)
- $L_{x,y,z} = (\vec{r} \times \vec{v}_{x,y,z})$: spherically symmetric potentials, $\psi(\vec{r}) = \psi(r)$
- L_z : axisymmetric potentials
- I_3 : flattened axisymmetric potentials, no analytical expression
- I_2 : planar non-axisymmetric potentials, no analytical expression

Assumptions about $f(\vec{r}, \vec{v})$

$$\chi(r, \hat{n}) = \frac{\psi(r\hat{n}) - \psi_s(r)}{\psi(r\hat{n})}$$

$\mathcal{E}$, L_t and L_z are integrals of motion

- Halo at equilibrium : $\mathcal{E}$ is an integral of motion
- Approximate spherical symmetry of $\psi(\vec{r})$ until $r \sim 30kpc$



Assumptions about $f(\vec{r}, \vec{v})$ – Separation of variables

If $f(\mathcal{E}, L_t, L_z) = f_1(\mathcal{E}) f_2(L_t) f_3(L_z)$

$$G(\mathcal{E}, L_t, L_z) = \frac{g^2 f(\mathcal{E}, L_t, L_z)}{g_{\mathcal{E}}(\mathcal{E}) g_{L_t}(L_t) g_{L_z}(L_z)} = 1$$

with

$$g_X(X) = \int \int f(X, Y, Z) dY dZ$$

$$g = \int f(\mathcal{E}, L_t, L_z) d\mathcal{E} dL_t dL_z$$

For more statistics, use $\bar{G}_X = \frac{1}{\Delta Y \Delta Z} \int \int G(X, Y, Z) dY dZ$

Assumptions about $f(\vec{r}, \vec{v})$ – Separation of variables

$\bar{G}_{\mathcal{E}}, \bar{G}_{L_t}, \bar{G}_{L_z}$