

**Abstract**—Fishery-independent estimates of spawning biomass ( $B_{SP}$ ) of the Pacific sardine (*Sardinops sagax*) on the south and lower west coasts of Western Australia (WA) were obtained periodically between 1991 and 1999 by using the daily egg production method (DEPM). Ichthyoplankton data collected during these surveys, specifically the presence or absence of *S. sagax* eggs, were used to investigate trends in the spawning area of *S. sagax* within each of four regions. The expectation was that trends in  $B_{SP}$  and spawning area were positively related. With the DEPM model, estimates of  $B_{SP}$  will change proportionally with spawning area if all other variables remain constant. The proportion of positive stations (PPS), i.e., stations with nonzero egg counts—an objective estimator of spawning area—was high for all south coast regions during the early 1990s (a period when the estimated  $B_{SP}$  was also high) and then decreased after the mid-1990s. There was a decrease in PPS from the mid-1990s to 1999. The particularly low estimates in 1999 followed a severe epidemic mass mortality of *S. sagax* throughout their range across southern Australia. Deviations from the expected relationship between  $B_{SP}$  and PPS were used to identify uncertainty around estimates of  $B_{SP}$ . Because estimation of spawning area is subject to less sampling bias than estimation of  $B_{SP}$ , the deviation in the relation between the two provides an objective basis for adjusting some estimates of the latter. Such an approach is particularly useful for fisheries management purposes when sampling problems are suspected to be present. The analysis of PPS undertaken from the same set of samples from which the DEPM estimate is derived will help provide information for stock assessments and for the management of purse-seine fisheries.

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## A sudden collapse in distribution of Pacific sardine (*Sardinops sagax*) off southwestern Australia enables an objective re-assessment of biomass estimates

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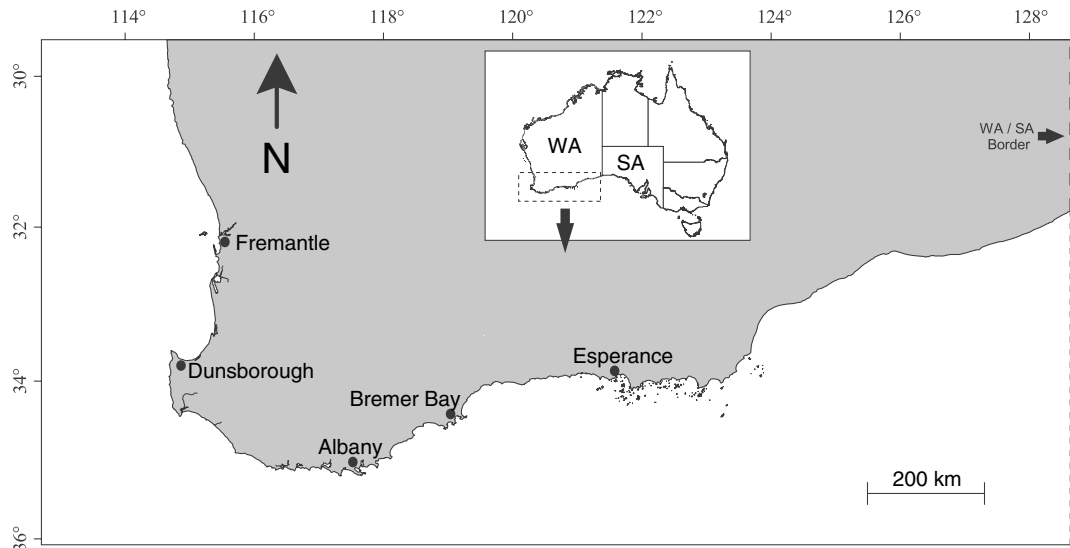
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As a stock of small pelagic fish decreases, biomass assessment becomes problematic because of such factors as patchy distribution (Fletcher and Sumner, 1999) and continuing high catchability as a result of the schooling behavior of some fish (Uphoff, 1993). In these circumstances, ichthyoplankton surveys can provide a useful means of estimating spawning biomass,  $B_{SP}$ , for some pelagic fish species. Mangel and Smith (1990) used a technique that assessed the presence or absence of sardine (*Sardinops sagax*) eggs in a known spawning area. They found that changes in adult biomass were more accurately predicted by using presence-absence of eggs in sampling surveys than mean egg abundance because of misleading results arising from the spatial patchiness of eggs. In their presence-absence analysis, the spatial distribution of eggs is the key determinant of  $B_{SP}$  estimates and is used in a model with a series of other parameters to provide an estimate of  $B_{SP}$  (Mangel and Smith, 1990). Although this technique provides an objective indication of stock size that is not subjected to the inherent problems in estimating  $B_{SP}$  with the daily egg production method (DEPM, e.g., Ward et al., 2001), the modeling requires substantial prior knowledge of adult and egg production parameters. More recently, Zenitani and Yamada (2000) developed an optimal relationship between  $B_{SP}$  and spawn-

ing area for the Japanese sardine (*Sardinops melanostictus*) using a nonlinear model that assumed patchy egg distribution. In their case, biomass was estimated by using virtual population analysis with catch-at-age data from the commercial fishery.

The purse seine fishery for *Sardinops sagax* in Western Australia (WA) operates along the south coast around the port regions of Esperance, Bremer Bay, and Albany; and on the lower West Coast in the regions of Fremantle and Dunsborough (Fig. 1). A level of spatial distinctness among adult *Sardinops* populations necessitates that three south coast regions and the west coast region be managed as separate fisheries (Gaughan et al., 2002). Unlike the case with Japanese sardine (Zenitani and Yamada, 2000), it has not been possible to estimate the  $B_{SP}$  of *Sardinops* in each fishery in WA using only an age-based approach. Although Gaughan et al. (2002) considered the catch-at-age data for the WA *Sardinops* fisheries to be reasonable, the data span a relatively short time series, commencing in 1988 at Albany and Bremer Bay and later at the other regions. Therefore, both age-structure data and estimates of spawning biomass ( $B_{SP}$ ) obtained with the DEPM have provided the biological basis for managing the *Sardinops* fisheries in WA for over a decade (Fletcher, 1991, 1995; Fletcher et al., 1996,



**Figure 1**

Map of southwestern Australia showing Pacific sardine fishing ports. North of Dunsborough to Fremantle constitutes the west coast fishery and the other regions constitute the south coast fishery. WA = Western Australia, SA = South Australia.

1996<sup>1</sup>; Cochrane<sup>2</sup>). A population model that integrates age-structure information and DEPM-derived estimates of  $B_{SP}$  ( $B_{SP-DEPM}$ ) has recently been developed by Hall (2000) for each of the three south coast regions.

Although the DEPM is able to provide relatively robust estimates of  $B_{SP}$  for a variety of species (Alheit, 1993; Hunter and Lo, 1997), it is not without problems (Cochrane, 1999; Ward et al., 2001). The  $B_{SP-DEPM}$  estimates for *Sardinops* in WA are presented at Management Advisory Committee (MAC) meetings and, in turn, are provided to the relevant government minister. The  $B_{SP-DEPM}$  estimates therefore undergo critical scrutiny by industry representatives. The shortcomings of the DEPM (e.g., sensitivity of precision for small sample sizes) are well understood by the members of the management committee; industry recognizes that onshore infrastructure and fleet capacity must be matched to long-term average  $B_{SP}$  and that industry should not capitalize at levels that require maximal stock sizes to meet financial expectations. Inasmuch, a level of conservatism has been adopted by the Management Advisory Committee when setting quotas. Nonetheless, the accu-

racy of estimates has been a contentious issue; industry members typically believe that the scientific advice presented often underestimates the  $B_{SP}$ . Likewise, wide confidence intervals around biomass estimates introduce doubt in the minds of industry members regarding the reliability of scientific advice, which can therefore stall the implementation of management measures. However, the lack of a formal and objective means of dealing with suspect and imprecise  $B_{SP-DEPM}$  estimates (e.g., because of problems with sampling spawning fish) has previously not been rigorously addressed.

Following the progression along the southern WA coast in early 1999 of a mass mortality of *Sardinops*, estimates of the quantity killed at Albany appeared to be very low (Gaughan et al., 2000). That is, very few dead *Sardinops* were found in comparison to the other regions where fisheries occur. Mortality rates for Esperance and Bremer Bay were 69.6% and 74.5% of the  $B_{SP}$ , respectively, whereas that for Albany was estimated to be only 2.4%. Estimates of the mortality rate of *Sardinops* in South Australia (SA, Fig. 1) for the same epizootic event were independently found to also be around 70% (Ward et al., 2001). The inconsistency with Albany could not be attributed to different weather conditions; the weather conditions at Albany were similar to those at Esperance and Bremer Bay and would be expected to result in equally visible evidence of mortality. Gaughan et al. (2000) contended that the true epizootic mortality rate of *Sardinops* in Albany was similar to that for the other regions, but that the very low mortality estimate was likely seen as such in view of the previous overestimation of  $B_{SP}$ .

In this study we aimed to address the problem of poor precision, while also developing a technique to identify particularly poor estimates of  $B_{SP-DEPM}$ , i.e., those for

<sup>1</sup> Fletcher, W. J., K. V. White, D. J. Gaughan, and N. R. Sumner. 1996. Analysis of the distribution of pilchard eggs off Western Australia to determine stock identity and monitor stock size. Final Report to Fisheries Research and Development Corporation. Project No. 92/95, 109 p. Department of Fisheries, Government of Australia, 168-170 St. Georges Tce, Perth, WA 6000, Australia.

<sup>2</sup> Cochrane, K. L. 1999. Review of Western Australia pilchard fishery, 12–16 April 1999. Fisheries Management Paper 129, 32 p. Department of Fisheries, Government of Western Australia, 168-170 St. Georges Tce, Perth, WA 6000, Australia.

**Table 1**

Estimates of spawning biomass (metric tons) of *Sardinops sagax* obtained by using the daily egg production method ( $B_{SP-DEPM}$ ) for each of four regions in southwestern Australia. In those cases where data were sufficient to estimate a coefficient of variation (CV), the range around the  $B_{SP-DEPM}$  estimate (Min./Max.) was calculated as  $\pm 1$  standard deviation (SD); otherwise, the  $B_{SP-DEPM}$  range was calculated by using assumed (AS) values for one or more of the DEPM parameters (see text). The numbers of adult *S. sagax* and plankton samples used in these calculations are shown.

Year	Min.	$B_{SP-DEPM}$	Max.	CV	$\pm 1$ SD	Adult $n$	Plankton $n$
Albany							
1991	12,088	19,300	30,700	—	AS	10	41
1992	9006	16,994	24,981	0.44	$\pm 1$ SD	10	31
1993	16,402	23,432	30,4620	0.30	$\pm 1$ SD	9	61
1994	15,440	31,330	55,000	—	AS	10	107
1995	7720	17,544	27,368	0.56	$\pm 1$ SD	10	83
1997	13,018	18,597	24,176	0.30	$\pm 1$ SD	27	94
1999	0	89	531	4.99	$\pm 1$ SD	2	263
Bremer Bay							
1992	12,000	19,280	79,000	—	AS	—	25
1993	16,170	44,010	63,608	—	AS	—	32
1994	15,700	28,458	42,500	—	AS	—	102
1999	2161	4156	6150	0.48	$\pm 1$ SD	3	256
Esperance							
1993	14,326	32,252	61,800	—	AS	5	50
1994	10,700	20,080	40,100	—	AS	—	150
1995	10,900	31,900	45,647	—	AS	6	105
1999	3454	17,396	31,793	0.80	$\pm 1$ SD	8	257
West coast							
1993	14,500	41,250	78,000	—	AS	—	55
1994	3100	8714	29,000	—	AS	—	133
1996	43,300	60,228	77,200	0.28	$\pm 1$ SD	4	96
1998	9112	18,985	28,951	0.52	$\pm 1$ SD	28	240
1999	3948	5275	6651	0.25	$\pm 1$ SD	4	396

which accuracy was suspect. However, because of the small size of the fishery and the difficulty in securing additional research funds, no fishery-independent method of estimating biomass other than the DEPM was undertaken. We recognized that this study could not, therefore, unequivocally determine whether or not any improvement in accuracy had been achieved; therefore, we focused on improving the consistency between available data sets (in terms of the broader economic environment) in order to improve the decision-making process for what is a small-scale fishery.

We examine the relationship between relative trends in the  $B_{SP-DEPM}$  and spawning area of *Sardinops* in each of four regional fisheries. We propose a method of using the relationship between spawning area and egg presence-absence data as an indicator of  $B_{SP}$  that is simpler than the methods of Mangel and Smith (1990) and Zenitani and Yamada (2000). We specifically chose to keep the retrospective analysis simple in recognition of data limitations, i.e., short time series and low numbers of samples for some DEPM surveys. In particular, our analyses did not rely on substantial knowledge of various parameters associated with estimating  $B_{SP-DEPM}$ .

This investigation (an objective re-assessment of *Sardinops*  $B_{SP-DEPM}$ ) was undertaken because of the contrast provided by the significant collapse in distribution, coupled with the sudden and substantial decrease in  $B_{SP-DEPM}$  that followed mass mortality in 1998–99. Although problems in obtaining accurate  $B_{SP}$  estimates with the DEPM will not be resolved by the present study, greater consistency between indicators of the magnitude of  $B_{SP}$  and the development of a transparent and objective technique to identify apparent discrepancies between the two will facilitate better management of this key pelagic resource.

## Methods

### Estimates of spawning biomass with the DEPM

The estimates of  $B_{SP}$  used in this study (Table 1) were obtained by using the DEPM, which relies on ichthyoplankton surveys to estimate egg production and temporally concurrent samples of adult fish to estimate the adult parameters of fecundity, sex ratio, and weight.

The egg and adult data were subsequently combined in the DEPM model (Parker, 1985), as follows, to estimate spawning biomass:

$$B_{SP-DEPM} = (APWk)/(SFR),$$

where  $A$  = spawning area;  
 $P$  = egg production (numbers of eggs before losses due to mortality);  
 $W$  = weight of adult fish;  
 $k$  = conversion factor to bring the various units to a value in metric tons;  
 $S$  = spawning fraction; the proportion of females that spawn per day;  
 $F$  = fecundity; number of eggs produced by a female; and  
 $R$  = ratio of females to males by weight.

The DEPM provides a point estimate of spawning biomass, with upper and lower statistical bounds. In those individual surveys where all parameters could be estimated, estimates of coefficient of variation (CV) for  $B_{SP-DEPM}$  were undertaken by using the delta method to sum the CV of the component parameters (Parker, 1985). In turn, the CV was used to provide an estimate of variability around the point estimate; specifically, we used  $\pm 1$  standard deviation to indicate the upper and lower bounds around the point estimates. In several surveys, particularly when the DEPM was initially being applied in WA, few adult samples meant that values for adult parameters (spawning fraction, sex ratio, fecundity, weight) could not be estimated and therefore a CV for the final estimate of  $B_{SP-DEPM}$  could likewise not be estimated. Although sex ratio and weight could be reasonably estimated from the regular sampling of commercial catches around the survey period and fecundity could be estimated from a relatively small sample (e.g., 70–100 fish), estimating the spawning fraction was more difficult. In this latter case, the upper and lower bounds for the  $B_{SP-DEPM}$  estimate were not based on a statistical measure but rather on what were thought to be likely low and high values of spawning fraction, respectively, for *Sardinops* from other surveys in WA and elsewhere (e.g., Alheit, 1993, Fletcher et al. 1996). Prior knowledge of likely  $B_{SP-DEPM}$  values when applying the DEPM, specifically for the purpose of providing expert management advice, has recently been used successfully for *Sardinops* in South Australia (Ward et al. 2001).

### Adult samples

Twenty DEPM surveys were conducted between 1991 and 1999 to identify stocks and to estimate spawning biomass of *Sardinops* of southwestern Australia (Fletcher et al., 1996a, 1996b; Fletcher et al.<sup>3</sup>; senior author's unpubl. data). The surveys were performed during the peak spawning months for *Sardinops* off the west coast, Albany, Bremer Bay, and Esperance regions. The timing of the DEPM survey cruises in each region was based on gonadosomatic indices for samples obtained

from commercial catches, as described in Gaughan et al. (2002). The aim was to obtain samples from 35 catches of adult fish, as recommended by Alheit (1993), but this number was never achieved and in some cases no samples were obtained (Table 1). For each catch sampled, the ovaries from 15–50 females were immediately placed in 10% formalin and subsequently prepared histologically for microscopic examination. The remainder of the subsample was processed to obtain mean female weight and sex ratio by weight. Mature ovaries were retained for estimation of fecundity.

### Plankton sampling and estimation of egg production

Plankton sampling extended from nearshore waters to the edge of the continental shelf (Fig. 2). Sampling stations were generally spaced uniformly, typically 2–4 nautical miles apart, along transects perpendicular to the shore. Analysis of *Sardinops* egg distribution from surveys conducted in the early 1990s indicated that these surveys sufficiently covered the distribution of the spawning stock (Fletcher and Tregonning, 1992; Fletcher et al., 1994), and later geostatistical analyses of *Sardinops* egg distribution patterns confirmed that the spacing of transects and stations were adequate to effectively represent the spatial distribution (Fletcher and Sumner, 1999). The earlier surveys were used to refine the spatial range of subsequent surveys. The number of plankton samples taken in each survey has generally increased since the early 1990s (Table 1).

*Sardinops* eggs were collected by using vertical tows that allowed the water column to be sampled from a maximum depth of 70 m to the surface; Fletcher (1999) showed that *Sardinops* eggs off southern Australia are typically restricted to the upper 70 m. Bongo nets with diameters of either 60 or 26 cm and constructed of either 500- or 300-micron mesh were used; the change to smaller nets was made to reduce sample volume and hence sorting time, whereas the change to smaller mesh was made to increase efficiency in capturing yolk sac larvae; these changes did not affect the sampling efficiency for *Sardinops* eggs. Tow speed was standardized at 1 m/s. All samples were collected between 0630 and 1800 hours and immediately preserved in 5–10% formalin and seawater.

Plankton samples were examined under a dissecting microscope. *Sardinops* eggs were identified, classified into 12 developmental stages (White and Fletcher<sup>4</sup>), and

<sup>3</sup> Fletcher W. J., B. Jones, A. F. Pearce, and W. Hosja. 1997. Environmental and biological aspects of the mass mortality of pilchards (Autumn 1995) in Western Australia. Fisheries Research Report, Fisheries Department Western Australia 106, 115 p. Department of Fisheries, Government of Western Australia, 168-170 St. Georges Tce, Perth, WA 6000, Australia.

<sup>4</sup> White, K. V., and W. J. Fletcher. 1998. Identifying the developmental stages for eggs of the Australian pilchard, *Sardinops sagax*. Fisheries Research Division WA, Fisheries Research Report 103, 21 p. Department of Fisheries, Government of Western Australia, 168-170 St. Georges Tce, Perth, WA 6000, Australia.

**Table 2**

Correlations between 1) proportion of positive stations (PPS) and proportional spawning area (PSA) and 2) PPS and estimated spawning area (km<sup>2</sup>) resulting from surveys of *Sardinops sagax* eggs at four regions in southwestern Australia. PPS is the proportion of the total number of plankton sampling stations that contained at least one *S. sagax* egg that had been spawned on the previous night. PSA is the proportion of the total survey area that consisted of spawning area. The values for spawning area are also provided.

	Survey	PPS	PSA (%)	Correlation I	Area (km <sup>2</sup> )	Correlation II
Albany	Jul 91	0.46	37		1806	
	Jul 92	0.58	51		2686	
	Jul 93	0.52	36		2391	
	Jul 94	0.60	60		6672	
	Jul 95	0.33	28		1977	
	Jul 97	0.19	21		2224	
	Jul 99	0.15	1	0.94	107	0.68
Bremer Bay	Jul 92	0.61	64		2807	
	Jul 93	0.72	67		2809	
	Jul 94	0.70	71		4474	
	Jun 99	0.12	12	0.99	908	0.86
Esperance	Jul 93	0.50	44		5715	
	Jul 94	0.57	73		9796	
	Apr 95	0.61	36		5277	
	May 99	0.09	3	0.81	7840	0.83
West coast	Jul 93	0.53	62		8012	
	Jul 94	0.30	38		5199	
	Aug 96	0.23	16		2202	
	Aug 98	0.10	7		1835	
	Aug 99	0.12	10	0.98	1836	0.96

counted. Estimation of egg production was undertaken by fitting a negative exponential model (Picquelle and Stauffer, 1985) and was derived from the y-axis intercept of the regression model, representing time 0. The number of stages used to fit the model depended on the egg abundance for each stage; the best fitting model was chosen visually from an iterative sequence of fits. The best fit was not necessarily that with the smallest CV but rather that which intuitively did not violate our understanding of natural mortality rates as determined from the literature. For example, the slope of the regression model must be negative and egg mortality rates should fall within the broad range of 0.9–3.9/d (e.g., Smith et al., 1989).

#### Estimation of spawning area

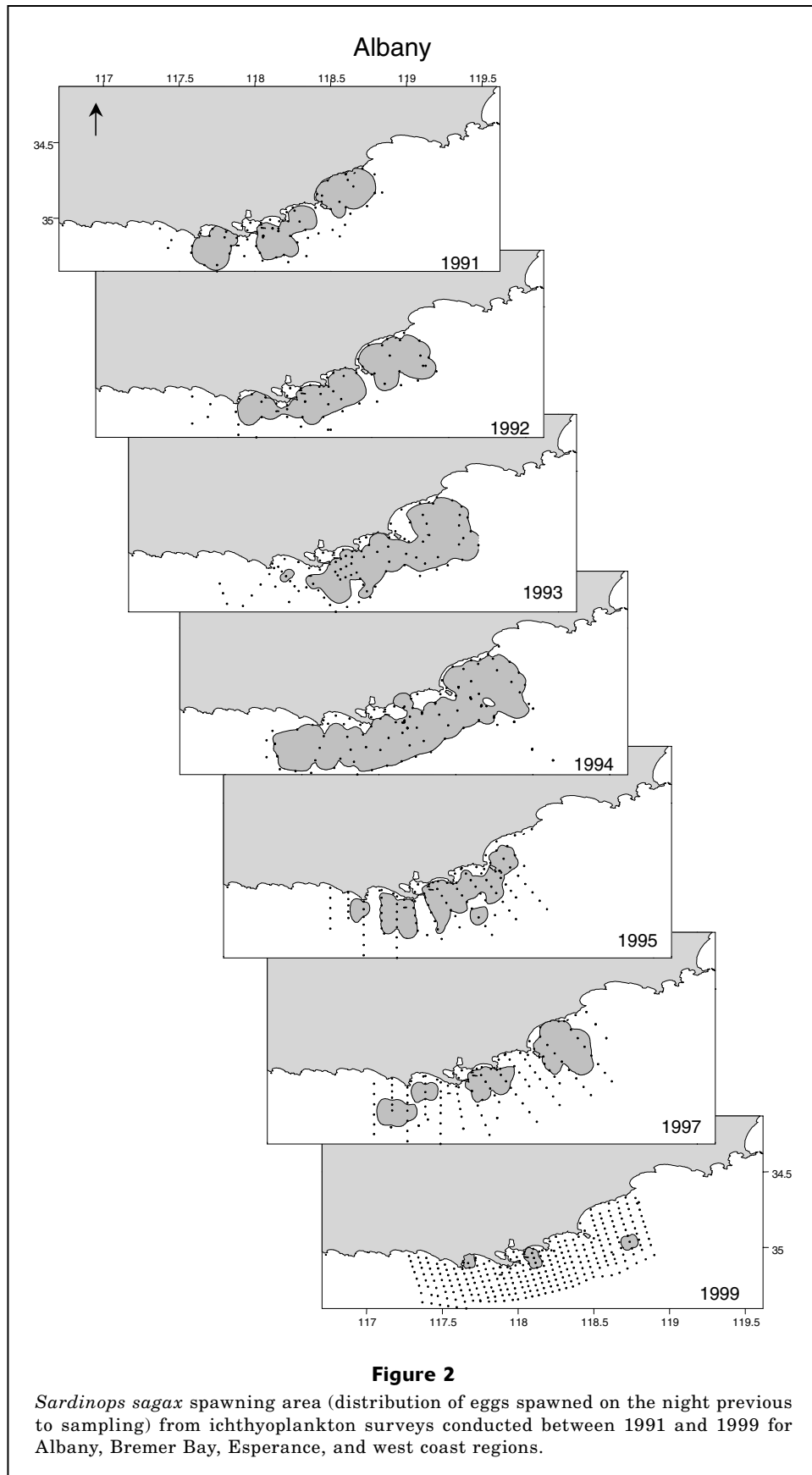
According to water temperatures during each survey and the stage of egg development, *Sardinops* eggs were determined to have been spawned either the previous night (“day-1”) or two nights previous (“day-2”) as described by Fletcher et al. (1996). The total survey area was estimated by constructing a polygon around all stations. The spawning area was defined as the area in which day-1 *Sardinops* eggs were found (Fletcher et al., 1996a). The areas of the polygons around stations that had day-1 eggs, referred to as positive stations, were summed to estimate the spawning area for each zone. When positive

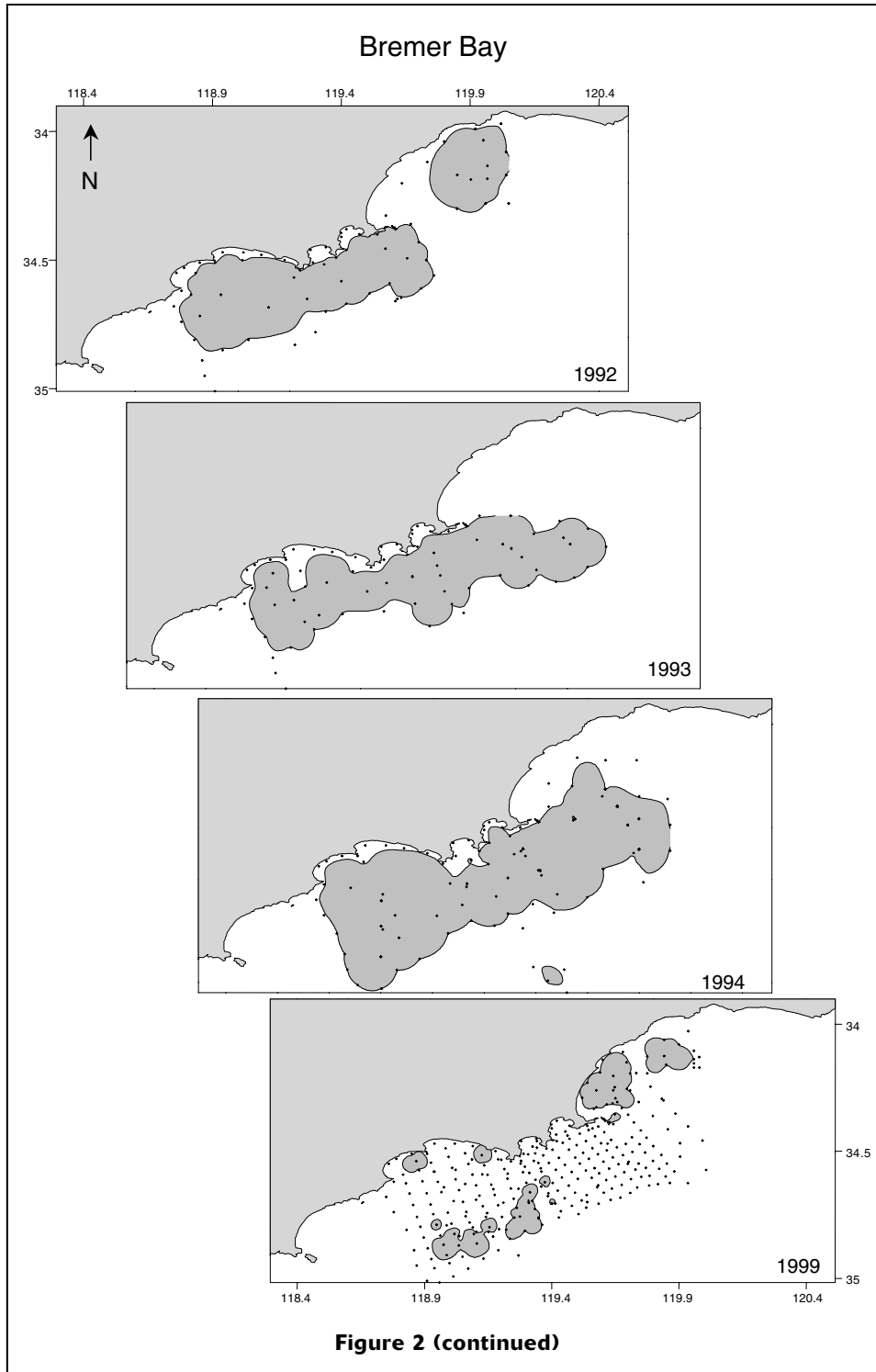
stations occurred on the margin of the sampling area, polygons for these positive stations were drawn as for the embedded positive stations, but the areas of these polygons were extended by a standardized amount beyond the sampling areas (Wolf and Smith, 1986).

The proportion of positive stations (PPS) was calculated for each survey. The proportion of the survey area (PSA) that consisted of spawning area was also evaluated in each case. PPS and PSA were positively correlated at each region (Table 2); this result was expected and indicated that PPS provides a realistic representation of changes in spawning area. The relationships between PPS and the areal estimates of spawning area were not as strong, but these latter estimates suffered as potential predictors of biomass in our study because of the large differences in numbers of plankton samples collected between surveys (Table 1). PPS is thus not only an objective measure but can also be considered as an index of spawning area.

#### Modeling of spawning biomass

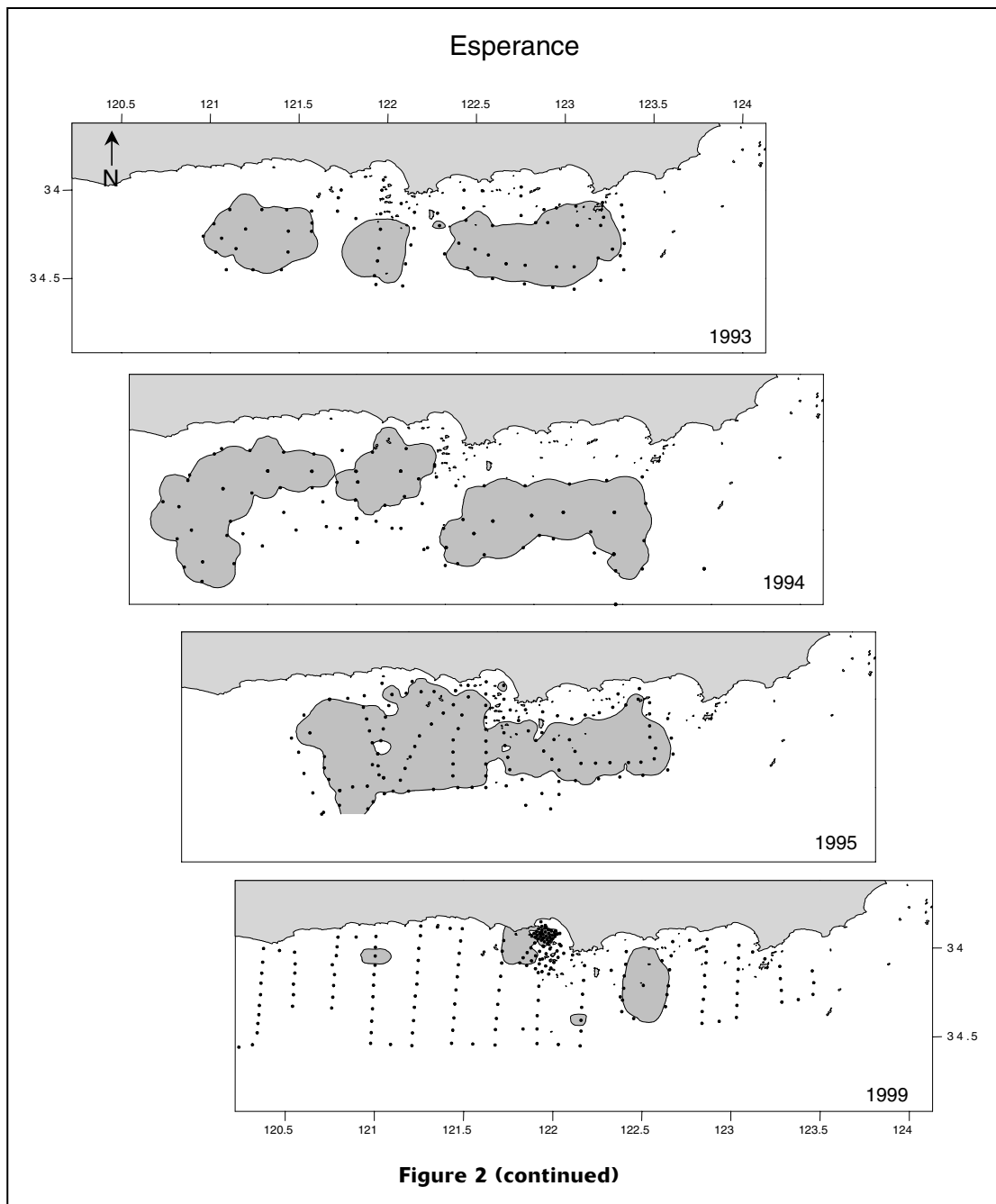
The collapse in distribution of *Sardinops* at each of four locations in southern Western Australia in 1999 is shown by the decline in spawning area (Fig. 2). The importance of this collapse in providing contrast for model fitting in otherwise poor data sets (few points with either flat or clumped distributions) is evident from linear fits of





$B_{SP-DEPM}$  against PPS for the south coast locations (Fig. 3). Because we wished to examine the relationship between trends in DEPM-based estimates of  $B_{SP}$  and PPS with the aim of improving estimates of  $B_{SP}$ , the development of an appropriate model is described here from first principles, followed by a selectivity analysis

of error variance to choose the optimal estimator of  $B_{SP}$ . Given that we did not have a means of assessing the level of accuracy of the “adjusted” estimates, and the aim was therefore to improve consistency between data sets for the purpose of enhancing the decision-making process, our criteria in choosing an optimal estimator was to



minimize variance. In terms of improving management of the *Sardinops* fisheries in southwestern Australia, we considered this approach appropriate because of the relatively conservative exploitation rates that have been adopted by the Management Advisory Committee.

#### Model selection

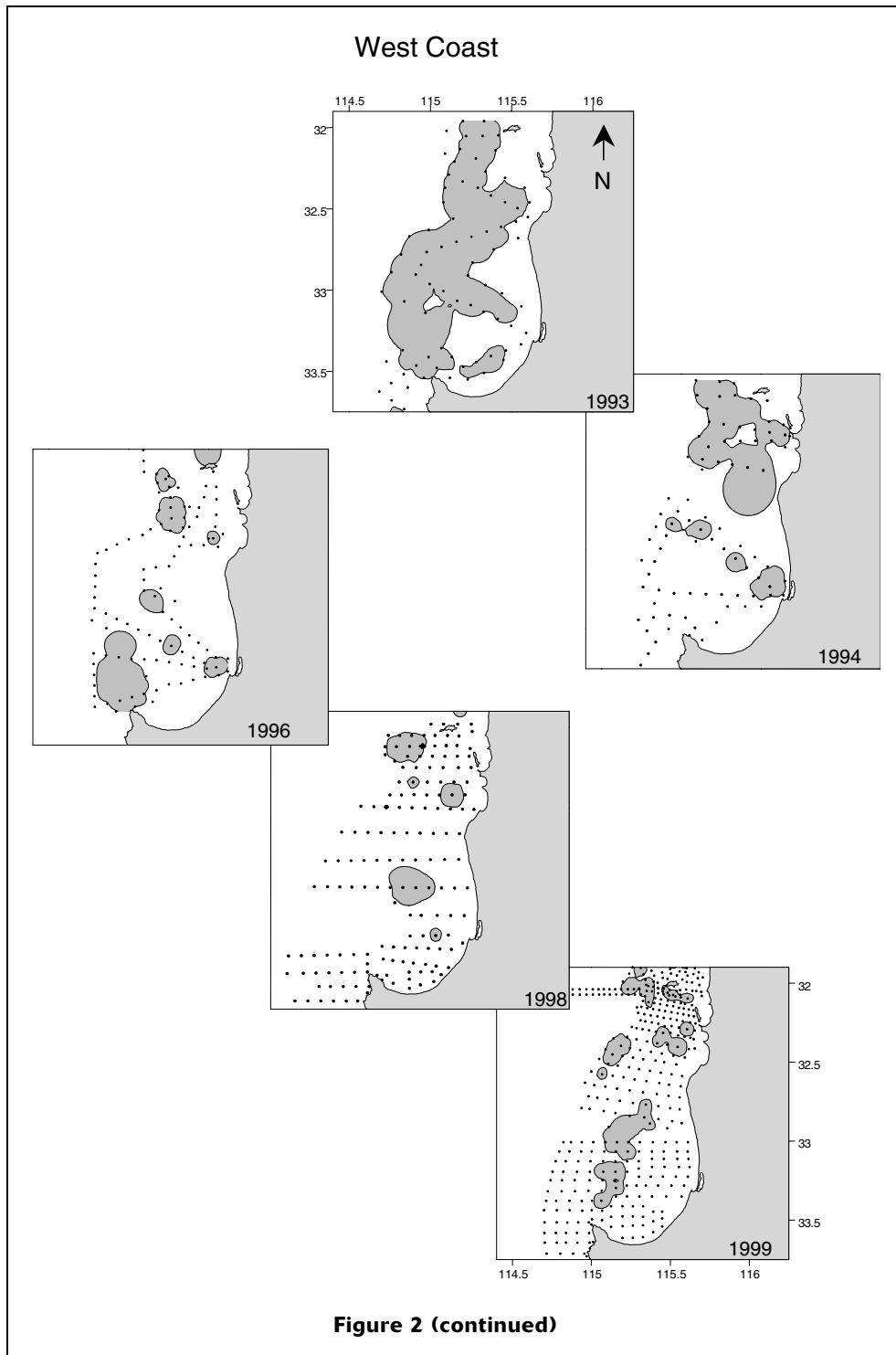
The procedure used in the present study was to first invoke a general model and then use the data to drive a simplification process in order to avoid a specification error. Thus, for each of the fishery regions, we first

considered the following general relationship between  $B_{SP-DEPM}$  and PPS holding over time:

$$(B_{SP-DEPM})_t = \alpha_0 (PPS_t)^\lambda + \alpha_2 + e_t \quad (1)$$

where  $\lambda$  is common for all areas,  $\alpha_0$  and  $\alpha_2$  are area specific constants, and the error term  $e_t$  is independent, homoscedastic and normal with a mean of zero. This model was chosen specifically in order to be amenable to Taylor series expansion during the simplification process. In satisfying dimensional and conservational





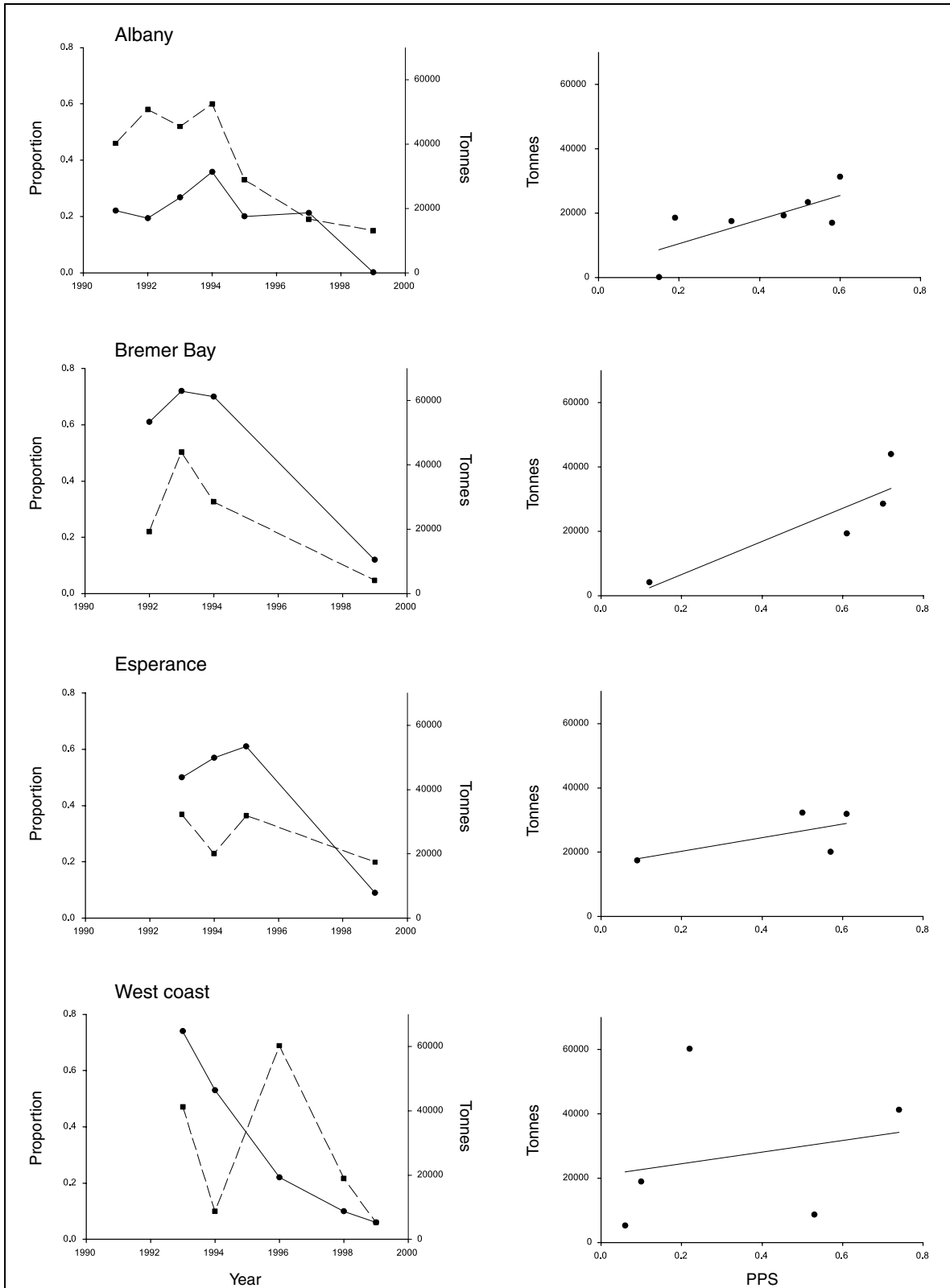
arguments,  $\alpha_2$  must be zero; therefore the above family of models reduces to

$$(B_{SP-DEPM})_t = \alpha_0(PPS)_t^\lambda + e_t \quad (2)$$

An attempt to fit the linear regression model, composed of the natural logarithms of either side of the above rela-

tion, to the observed data was unsuccessful because of heterogeneity of error variance.

Direct estimation of model II with a nonlinear regression procedure gave estimates of  $\lambda$  that were near 1 and with large standard errors on account of the small size of the data sets. However, residual diagnostics were satisfactory. Because the observed PPS values fell between



**Figure 3**

Plots of the proportions of positive stations (—●—) and  $B_{SP-DEPM}$  estimates (---■---) for the four *Sardinops sagax* fisheries in southwestern Australia. The right-hand panel shows linear fits of the relationship between proportion of positive stations and  $B_{SP-DEPM}$  estimates.

**Table 3**

Parameter estimates for two models (III and IV, see text for details) of *Sardinops sagax* spawning biomass, including tests for zero coefficients, at each of four regions in southwestern Australia.

Model III					Model IV				
<b>Albany</b>					<b>Albany</b>				
SE of estimate: 6011					SE of estimate: 6343				
	$B_{SP}$	$SE$ of $B_{SP}$	$t(22)$	$P$ -level		$B_{SP}$	$SE$ of $B_{SP}$	$t(23)$	$P$ -level
Intercept	-4597.15	2419.94	-1.90	0.070661	PPS	45,910.11	3552.36	12.92	4.96E-12
PPS	56,780.19	6638.90	8.55	1.91E-08					
<b>Bremer Bay</b>					<b>Bremer Bay</b>				
SE of estimate: 4016					SE of estimate: 3994				
	$B_{SP}$	$SE$ of $B_{SP}$	$t(14)$	$P$ -level		$B_{SP}$	$SE$ of $B_{SP}$	$t(18)$	$P$ -level
Intercept	-1436.88	1573.94	-0.91	0.37674	PPS	42,784	2308.30	18.53	9.47E-12
PPS	45,341.85	3638.2	12.46	5.75E-09					
<b>Esperance</b>					<b>Esperance</b>				
SE of estimate: 4639					SE of estimate: 10,214				
	$B_{SP}$	$SE$ of $B_{SP}$	$t(20)$	$P$ -level		$B_{SP}$	$SE$ of $B_{SP}$	$t(21)$	$P$ -level
Intercept	15,840.52	1751.38	9.045	0.000317	PPS	48,377.13	5095.09	9.49	4.76E-09
PPS	17,790.27	4097.73	4.34	1.66E-08					
<b>West coast</b>					<b>West coast</b>				
SE of estimate: 15,280					SE of estimate: 15,330				
	$B_{SP}$	$SE$ of $B_{SP}$	$t(32)$	$P$ -level		$B_{SP}$	$SE$ of $B_{SP}$	$t(33)$	$P$ -level
Intercept	5316.10	4826.58	1.10	0.278929	PPS	84,615.52	12,615.76	6.71	1.22E-07
PPS	63,192.69	23,161.29	2.73	0.010251					

zero and one, and  $\lambda$  was also close to 1, model II was able to be recast in a more tractable form by using the Taylor series expansion of the RHS of model II about  $PPS = 1$ , leading to the relationship

$$(B_{SP-PPS})_t = \alpha_0 PPS_t + \delta + e_t, \tag{3}$$

where the expected value of  $\delta$  is approximately  $-0.25\alpha_0(\lambda-1)$ . Details of the derivation are provided in Appendix 1.

Fitting the regression model III to the DEPM-based estimates of  $B_{SP}$  gave the estimated coefficients shown in the left hand column of Table 3. Residual diagnostics showed that model III was satisfactory. Because none of the intercept terms were significantly different from zero, the parsimonious model

$$(B_{SP-PPS})_t = \alpha_0 (PPS)_t + e_t \tag{4}$$

was fitted, giving the results in the right hand column of Table 3. Residual diagnostics were also satisfactory for these models.

**Optimal estimation of spawning biomass**

We now have available two unbiased estimates of  $B_{SP}$ : estimator 1 (i.e.,  $B_{SP-DEPM}$ ) with associated error  $e'$ ,

which has an expected value of 0 and variance  $\text{Var}(e') = \sigma_1^2$ ; and estimator 2 (i.e.,  $B_{SP-PPS}$ ) which model IV of the previous section fitted to the values of  $B_{SP-DEPM}$  with error  $e$ , which had an expected value of 0 and variance  $\text{Var}(e) = \sigma^2$ . Thus estimator 2 can be seen to be unbiased and with full error term ( $e+e'$ ). In order to obtain an optimal predictor, i.e., with minimum variance, of spawning biomass ( $B_{SP-Optimal}$ ), we considered the weighted average of the two estimators above:

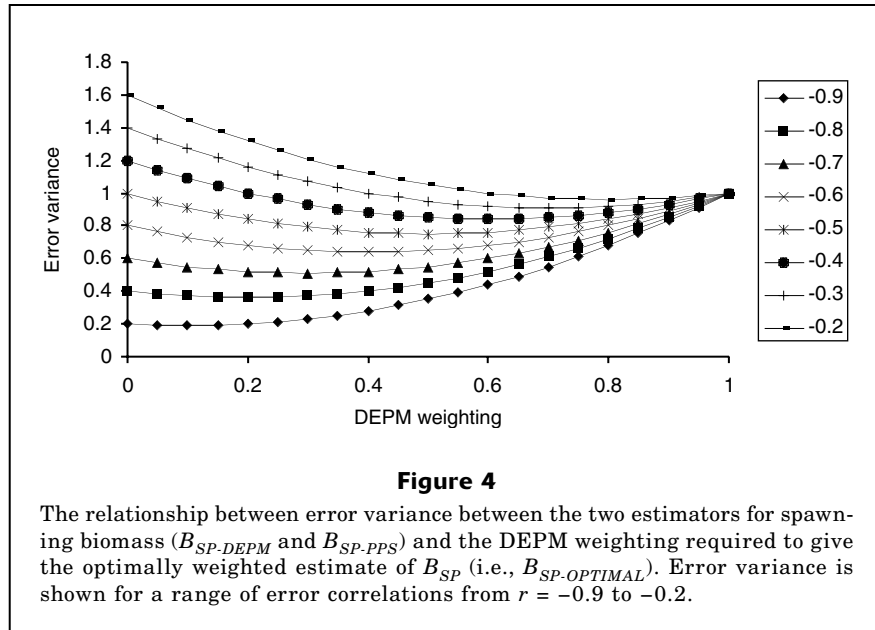
$$B_{SP-Optimal} = ((w) \text{ estimator 1} + (1-w) \text{ estimator 2}),$$

with weight  $w: 0 < w < 1$ .

We must choose the weight  $w$  of estimator 1 in order to minimize the variance ( $\text{Var}(B_{SP-Optimal})$ ) of the estimator  $B_{SP-Optimal}$ .

$$\begin{aligned} \text{Var}(B_{SP-Optimal}) &= \text{Var}(we+(1-w)(e+e')) \\ &= \text{Var}(e+(1-w)e') \\ &= \text{Var}(e) + (1-w)^2 \text{Var}(e') + 2(1-w) \\ &\quad \text{covariance}(e,e') \\ &= \sigma^2 + (1-w)^2 \sigma_1^2 + 2(1-w) \sigma \sigma_1 \rho \end{aligned}$$

where  $\rho$  = correlation between  $e$  and  $e'$ . For  $\text{Var}(B_{SP-Optimal})$  to be a minimum, the  $w$  derivative must be zero, yielding



$$0 = (1-w)\sigma_1 + \sigma\rho,$$

which requires  $w = 1 + \sigma\rho/\sigma_1$ .

In the event of  $\rho = -\sigma_1/\sigma$ ,  $B_{SP-Optimal}$  will have  $w = 0$ , i.e., the optimal estimator will just be estimator 2 alone.

The limited sample information available indicates that  $\sigma_1$  is approximately equal to  $\sigma$ , which we therefore assume in order to simplify the next analysis. Because  $B_{SP-PPS}$  is based on estimates of PPS, which can be estimated with more confidence than  $B_{SP-DEPM}$ , it must be expected that often, if not always,  $\text{Variance}(B_{SP-PPS}) < \text{Variance}(B_{SP-DEPM})$ .

i.e.,  $\text{Var}(e+e') < \text{Var}(e) + \varepsilon$ , for small  $\varepsilon > 0$ ;

i.e.,  $\sigma^2 + \sigma_1^2 + 2\rho\sigma_1\sigma < \sigma^2 + \varepsilon$ ;

i.e.,  $2\rho < (-\sigma_1^2 + \varepsilon)/\sigma_1\sigma$ .

This requires  $\rho < -0.5$  when  $\sigma_1 = \sigma$ , and  $\varepsilon$  is small. This relation has an important role in our decision of what is the best estimator.

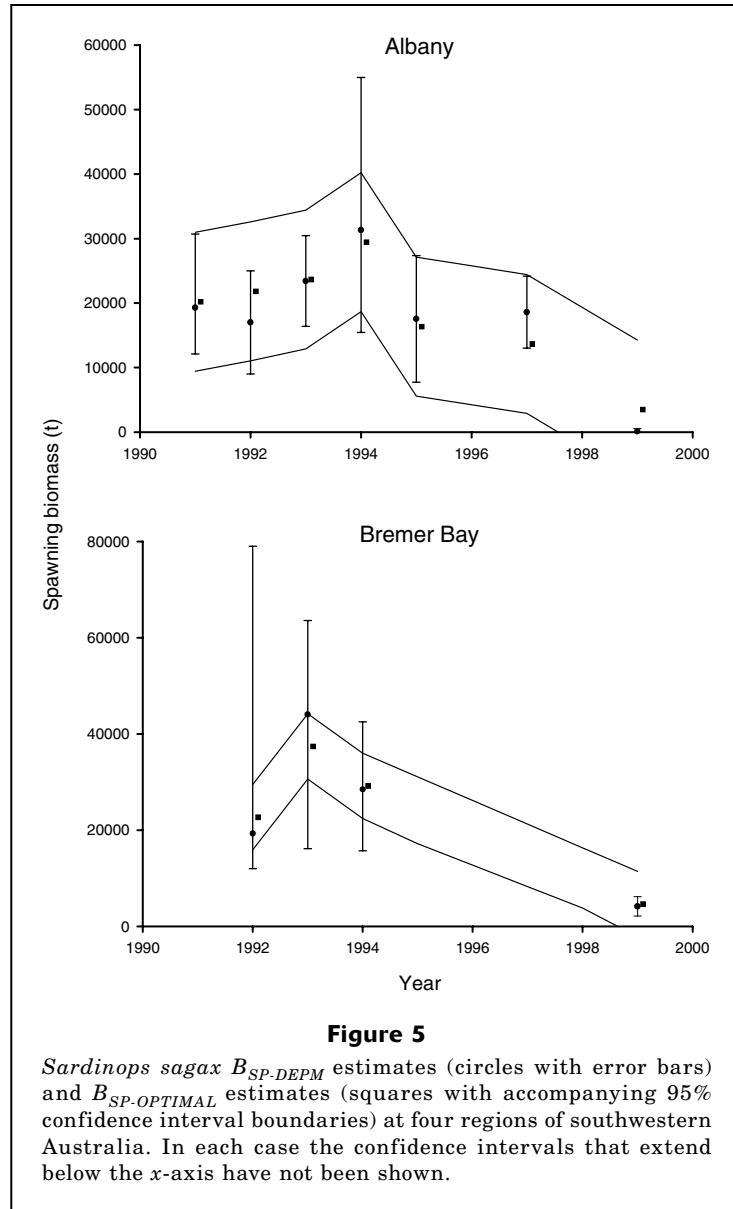
In Figure 4 the error variance for the estimator  $B_{SP-OPTIMAL}$  is shown for various DEPM weightings and a theoretical range of error correlations (i.e., between  $e$  and  $e'$ ) from  $r = -0.9$  to  $-0.2$ . Our aim was to choose a DEPM weighting that provides minimal error variance along the most stable regions of the suite of error correlation curves, i.e., where the error correlation curves are flattest. The error correlation curves from  $-0.4$  to  $-0.7$  were the most stable and across these the DEPM weightings from 0.3 to 0.7 had the smallest error variance. Therefore we choose 0.5 as our preferred DEPM weighting, which lies centrally within a stable part of the range of theoretical error correlations.

## Results

The decline in spawning area in each region (Fig. 2) corresponded to declines in  $B_{SP-DEPM}$  (Table 1), which in turn were reflected by the  $B_{SP-OPTIMAL}$  estimates (Fig. 5). We recognize that imbalance in the intensity of samples between years poses a problem for the interpolation of data between sampling stations but we contend that the collapse in distribution observed is of sufficient contrast to be a reliable reflection of the estimated 70% decrease in *Sardinops* biomass that resulted from the 1998–99 epidemic (Gaughan et al., 2000; Ward et al., 2001). Note that we have used Albany (Fig. 2A) as the primary support for this contention because of the larger data set. The same pattern was observed at all regions, although it was not so marked for the west coast *Sardinops* (Fig. 2D) because estimated  $B_{SP}$  (this term hereafter is used generically) had already declined substantially between 1996 and 1998.

Despite sometimes large intervals between consecutive surveys, there were two broad patterns in the trends for *Sardinops*  $B_{SP}$  during the 1990s (Fig. 5). Within each region on the south coast (Albany, Bremer Bay, and Esperance),  $B_{SP-DEPM}$  remained relatively high in the early to mid 1990s before decreasing substantially by 1999. In contrast to the results from south coast DEPM surveys, the west coast estimated  $B_{SP-DEPM}$  fluctuated widely (Table 1). This fluctuation resulted in a relatively poor fit of the optimal model and correspondingly wide CIs. Since 1996, when substantially more samples were routinely collected during each survey on the west coast, there has also been a decrease in  $B_{SP}$  consistent with that observed on the south coast.

Inconsistency in the determination of variability estimates around some  $B_{SP-DEPM}$  estimates precludes any definitive statements about the relative precision of the



$B_{SP-OPTIMAL}$  estimates. Notwithstanding this, the CIs for the optimal estimates always encompassed the DEPM point estimates. Because the CIs were so broad in relation to the point estimates, only the point estimates for  $B_{SP-DEPM}$  and  $B_{SP-OPTIMAL}$  are further compared.

The estimated  $B_{SP-OPTIMAL}$  indicates that for Albany the spawning biomasses were underestimated in 1992 and 1999 and overestimated in 1997 and that the difference between estimates in each case was greater than 25% (Fig. 5, Table 4). Although the DEPM estimated that the Albany  $B_{SP}$  remained steady between 1995 (17,544 t) and 1997 (18,597 t), the PPS almost halved from 0.33 to 0.19 for these same surveys (Tables 1 and 2). For Bremer Bay the estimates for  $B_{SP-DEPM}$  and the  $B_{SP-OPTIMAL}$  were within 20% (Fig. 5, Table 4). In Esperance the  $B_{SP-OPTIMAL}$  estimate indicates that the DEPM

underestimated  $B_{SP}$  by 19% in 1994, but overestimated  $B_{SP}$  by 37% in 1999.

The DEPM estimates of *Sardinops*  $B_{SP}$  on the west coast had the poorest fit against PPS. Thus, the optimal estimates of  $B_{SP}$  differed by >30% in four of the five DEPM-based  $B_{SP}$  estimates. In particular, the 1994 and 1999 DEPM estimates were too low, and those for 1996 and 1998 were too high.

## Discussion

Egg presence-absence analysis, i.e., proportion of positive stations (PPS), was used to objectively assess changes in the spawning area of *Sardinops* along the south and lower west coasts of WA between 1991 and 1999. The

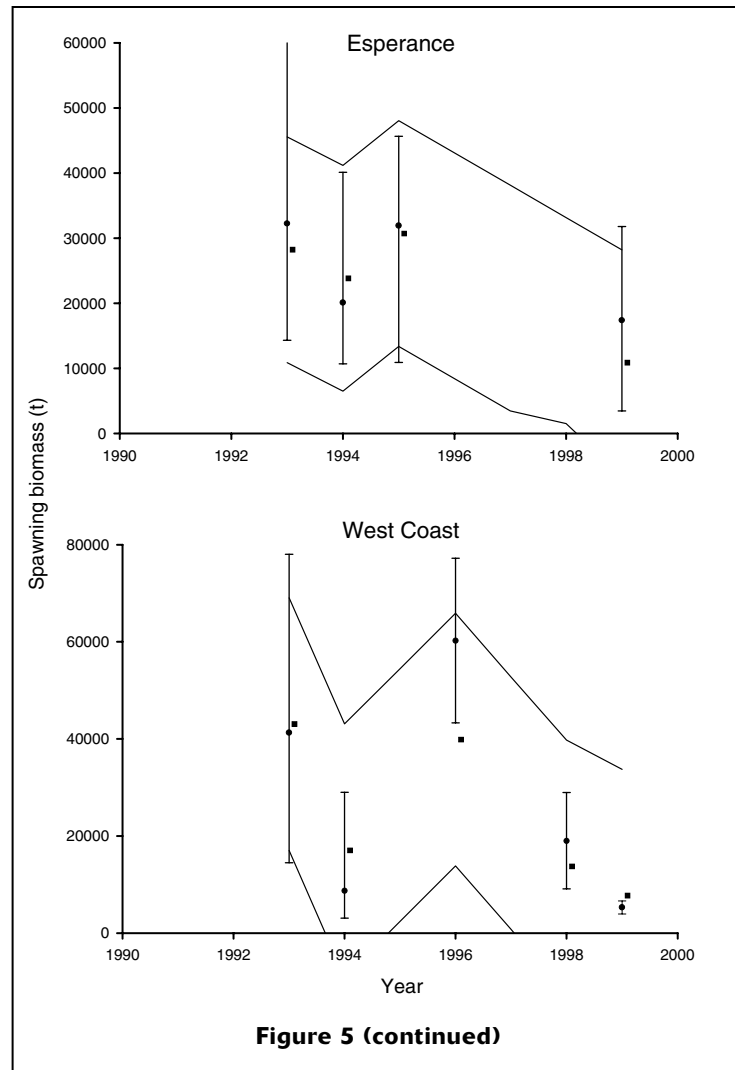


Figure 5 (continued)

collapse in distribution was evident in 1999 for three of the four regions examined and has been attributed to a combination of fishing mortality, several years of poor recruitment, and two mass mortality events (Murray and Gaughan, 2003). The spawning stock in Albany and Bremer Bay decreased to a point where the annual total allowable catch (TAC) in these fisheries was reduced to zero. The concurrent decreases in  $B_{SP-DEPM}$  and PPS at the south coast regions in 1999, estimated shortly after the progression of an epidemic mass mortality (Gaughan et al., 2000), indicates a positive relationship between  $B_{SP-DEPM}$  and PPS. This widespread response provides support for the concept of using the  $PPS-B_{SP-DEPM}$  relationship to objectively detect, albeit retrospectively, particularly suspect estimates of  $B_{SP-DEPM}$ .

The marked decline in  $B_{SP}$  in 1999 to a very low level at Albany provided sufficient contrast in the time series of data to allow detection of an overestimation of spawning biomass in 1997. Although the difference may not appear to be overly large, the critical factor in this particular case is that the  $B_{SP}$  of 18,597 metric tons (t)

was seen to be healthy, whereas an estimate of 13,660 t would have clearly indicated to the Management Advisory Committee a downward trend in *Sardinops*  $B_{SP}$ . In turn, such a result would have supported the contention that the stock was in decline, which was expected because of several years of poor recruitment, as evidenced by catch-at-age data (Gaughan et al., 2002). Furthermore, in 1998, during the 6 months prior to the mass mortality, the purse-seine fleet in Albany experienced significant difficulties in meeting catch expectations, which also indicated that the stock was at a low level. Although we cannot assess precision of the revised estimates of  $B_{SP}$ , it is likely that the  $B_{SP-OPTIMAL}$  for 1997 still overestimates the actual stock size.

The evidence for a decline in  $B_{SP}$  at Bremer Bay from 1994 to 1999, as suggested by the decline in PPS, was supported by trends in catch curves for that period, which showed very low levels of recruitment (Gaughan et al., 2002). The recruitment trends ensured that the annual TACs for Bremer Bay after the mid 1990s did not increase but instead were gradually reduced. The

**Table 4**

Comparison of estimates of spawning biomass for *Sardinops sagax* from four regions in southwestern Australia. Estimates obtained by using the daily egg production method (DEPM) were re-estimated by using a model that considered the proportion of positive stations (PPS, see text and Table 2) during each of the DEPM surveys. In turn, a weighted, or optimal, estimate was derived from the previous two estimates. The difference and ratio between the optimal and the DEPM estimates are provided for comparison. Optimal estimates that fall outside the 95% confidence intervals for the DEPM estimates and optimal:DEPM ratios that differ from 1 by greater than 0.25 are shown in bold type.

	DEPM estimate	PPS estimate	Optimal estimate	DEPM-PPS estimate	Optimal:DEPM
<b>Albany</b>					
1991	19,300	20,190	20,209	-890	1.05
1992	16,994	25,456	21,811	-8462	<b>1.28</b>
1993	23,432	22,823	23,653	609	1.01
1994	31,330	26,334	29,438	4996	0.94
1995	17,544	14,484	16,347	-1197	0.93
1997	18,597	8339	13,660	10,258	<b>0.73</b>
1999	89	6584	<b>3488</b>	-6495	<b>39.19</b>
<b>Bremer Bay</b>					
1992	19,280	21,346	22,689	-2066	1.18
1993	44,010	25,195	37,407	-6603	0.85
1994	28,458	24,495	29,204	3963	1.03
1999	4156	4199	4645	-43	1.12
<b>Esperance</b>					
1993	32,252	23,542	28,220	-4032	0.88
1994	20,080	26,838	23,827	-6758	1.19
1995	31,900	28,721	30,705	3179	0.96
1999	17,396	4238	10,875	13,158	<b>0.63</b>
<b>West coast</b>					
1993	41,250	48,318	43,048	-7068	1.04
1994	8714	27,350	17,049	-18,636	<b>1.96</b>
1996	60,228	20,968	<b>39,845</b>	39,260	<b>0.66</b>
1998	18,985	9117	13,723	9868	<b>0.72</b>
1999	5725	10,940	<b>7714</b>	-5665	<b>1.35</b>

very poor fit for Esperance may reflect the low sample size or may be indicative of a certain level of decoupling of  $B_{SP}$  and PPS not evident in the other south coast regions.

The 1996 estimate for the west coast was hampered by poor estimation of adult parameters resulting from a low number of adult samples obtained; the  $B_{SP-DEPM}$  estimate for that year appeared to be much too high and, intuitively, was not used as the basis for making management decisions at that time. The precautionary decision to use the lower bound rather than the "best" estimate from the 1996 west coast DEPM survey was therefore justified. In contrast, the estimate of  $B_{SP-DEPM}$  of 8714 t in 1994 for the west coast *Sardinops* stock appears to have been too low. The lack of an obvious collapse in distribution off the west coast was partly due to the marked changes in the intensity and distribution of sampling after 1996. Another contributing factor may have been a change in the distribution of the spawning adults because of the anomalously warm water in the Indian Ocean in the late 1990s (Webster et al., 1999) during the last major La Niña. The PPS

of only 0.10 in 1998, before the epidemic mortality, may therefore have been the result of behaviorally mediated changes in the distribution of *Sardinops* in response to the warmer than average water temperatures (Gaughan et al., 2000). We recognize that other factors may also have influenced the distribution of *Sardinops* off the west coast but our relatively short time series of data precluded development of more definitive, alternative hypotheses at this time. The potential for unusual environmental conditions to influence spawning behavior applies equally to the south coast *Sardinops*; interpretation of PPS data therefore also requires consideration of environmental conditions in each case. As our time series of biomass estimates is extended through further DEPM surveys, hypotheses regarding the influence of the environment will be further developed. Preliminary hypotheses have already been presented to the Management Advisory Committee and thus form part of current management deliberations.

The results from this retrospective analysis will immediately be used to reassess the  $B_{SP}$  estimates obtained for *Sardinops* in WA during the 1990s before

refitting them to Hall's (2000) integrated models for the three adult assemblages on the south coast of WA. The integrated model is tuned with  $B_{SP-DEPM}$  estimates. Therefore, replacing  $B_{SP-DEPM}$  estimates with  $B_{SP-OPTI-MAL}$  estimates will result in a model that better simulates the size of the *Sardinops* stocks off southern WA. Although the changes may appear trivial, it is important that re-estimating the most deviant estimates of  $B_{SP-DEPM}$  can be undertaken in a manner that satisfies demands by stakeholders, including industry, for openness and clarity in the provision of scientific advice.

As further DEPM surveys are conducted to assess the status of the *Sardinops* stocks in the five to six years following the 1998–99 mortality event, more reliable relationships between PPS and  $B_{SP-DEPM}$  will be developed. To assist this process, the relative merits of the data for individual DEPM surveys can also be re-examined, particularly those data that this study has indicated to have resulted in poor estimates of  $B_{SP-DEPM}$ . An ongoing iterative approach that employs retrospective analyses will be undertaken in an attempt to continuously reduce the variance of the PPS- $B_{SP-DEPM}$  relationship. This approach will permit further refinement of Hall's (2000) integrated model, a process already in progress (Stephenson et al.<sup>5</sup>) and will therefore contribute to increased confidence in the scientific advice that is provided for management of the *Sardinops* fisheries in WA. Eventually, PPS alone may be sufficient to provide an indication of spawning biomass with an acceptable level of precision.

Besides contributing to the integrated model, the  $B_{SP-OPTIMAL}$  point estimates obtained over nearly a decade in each of four management regions now provide a clearer indication of potential maximum biomass levels against which industry members can plan their businesses. Because of the highly variable recruitment of many small pelagic fish, purse-seine businesses that target fish such as *Sardinops* should not invest at levels that require an economic return based on maximum biomass sizes. For the purse-seine fishing zones in southern WA the maximum spawning biomass from which purse-seine industry members can expect their TACS to be determined are as follows: west coast 40,000 t, Albany 29,000 t, Bremer Bay 37,000 t, and Esperance 30,000 t. Although these values provide an upper limit to business planning, maximum biomasses should not represent investment targets. These values provide an indication of the maximum size for the industry but, because of the "natural and social disarray" that can result "from harvesting marine fish species at the crest of their production" (Smith 2000), the industry should be structured at a level that focuses on longer-term average biomass and that includes industry's ability to survive during periods of low stock size. Maximum and average  $B_{SP}$  for *Sardinops* at each of the four management regions

in southwestern Australia will be further investigated during ongoing development of the integrated model and as more information becomes available.

## Conclusion

Even large numbers of plankton samples can result in imprecise estimates of egg production for use in DEPM calculations (e.g., Mangel and Smith, 1990). Relative trends in spawning area that can be obtained from the same survey by using egg presence-absence analysis provide a secondary means of assessing trends in the status of stocks. This egg presence-absence analysis will be particularly useful for stocks already assessed by using DEPM surveys and more so for those that do not have large amounts of ancillary information, such as long time-series of catch-at-age data, or meaningful effort data.

Detection of either upwards or downwards bias in estimates of  $B_{SP}$  will be considered in the integrated model and also communicated to industry members to increase their understanding of the stock in each region. Although this review of biomass trends of *Sardinops* during the 1990s cannot change how *Sardinops* were managed during that period, an increased understanding of both the stock sizes and the science behind the biomass assessments will facilitate ongoing management processes.

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## Appendix 1—Derivation of model III from model II

We start with model II

$$B_{SP\_DEPM} = \alpha_0 PPS^\lambda + e_t$$

For brevity,  $y$  is used to denote PPS. We first note that  $y^\lambda = y y^{(\lambda-1)}$ . Now we obtain the Taylor series expansion of  $y^{(\lambda-1)}$  about  $y=1$  giving

$$y^{(\lambda-1)} = 1 + (\lambda-1)(y-1) + (\lambda-1)(\lambda-2)/2!(y-1)^2 + (\lambda-1)(\lambda-2)(\lambda-3)/3!(y-1)^3 + \dots$$

Multiplying this expression through by  $y$  gives

$$y^\lambda = y + y(y-1)(\lambda-1) + y[\text{terms involving second and higher powers of } (y-1)].$$

Because  $y$  is a proportion, it satisfies  $0 < y < 1$  so that the higher powers of  $(y-1)$  will be individually and collectively small. If  $y$  is close to 0.5, a further algebraic simplification of the second term is possible, giving the identity

$$y(y-1) = -0.25 + (y-0.5)^2.$$

When  $y$  is in the range  $0.25 < y < 0.75$ , the right-hand side of this identity remains close to  $-0.25$ . Thus model II may be simplified to

$$B_{SP\_PPS} = \alpha_0(PPS) - 0.25\alpha_0(\lambda-1) + e_t$$

which is the form of model III.