A Comparison of Radio and Optical Astrometric Reduction Algorithms

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Abstract This paper examines the correspondence between two approaches to astrometric observational reductions: the approach based on angular observables used for optical observations, and the approach based on the interferometric delay observable used for very long baseline radio interferometry (VLBI) observations. Specifically, of interest here is the group of algorithms that have become standard in accounting for the physical effects traditionally called annual and diurnal aberration and gravitational light bending. These algorithms are important because they must be applied to all wide-angle astrometric measurements, whether ground-or space-based, and regardless of the distance of the objects observed.

A procedure is presented by which VLBI algorithms can be used for optical observations. This scheme can help to guarantee consistent treatment of observational results in the two regimes. It also allows for the evaluation of the precision of the algorithms. Differences between angle-based and delay-based algorithms in current use are shown to be less than 1 microarcsecond. However, the physical models used as the bases for the algorithms must be improved to reach external accuracies at such levels.

What's the Point?

The objective of wide-angle astrometry is the establishment of all-sky celestial reference frames. These reference frames are defined by the catalog coordinates (and proper motions, if measurable) of stars or extragalactic objects. All-sky astrometric reference frames are now at the 1 milliarcsecond level of accuracy in both the optical and radio regimes. There is great interest in densifying and aligning such high-quality reference frames. Among other things, this would allow multi-wavelength maps of astro-physically interesting objects to be created with some confidence that small-scale structures could be properly interpreted.

However, in constructing these reference frames, radio and optical data are collected and treated very differently.

This paper looks at the way in which two important physical effects — gravitational light bending and aberration — are treated in the two regimes. These effects are important because they must be applied to all wide-angle astrometric reductions, whether ground- or space-based, and regardless of the distance of the object observed. Are the optical and radio algorithms used consistent and precise at the increased levels of accuracy now achievable? What about proposed astrometric space systems with 5−50 microarcsecond accuracies?

The Two Regimes

Optical astrometry is usually done with conventional telescopes (pointable continuous apertures) and more or less direct measurements of angles. Observables such as transit time, CCD pixel coordinates, or plate measures are easily converted to angular quantities. Right ascension and declination are themselves angles. Various geometric and physical effects that enter into the observation reductions are treated as changing the observed object's angular position. Algorithms for gravitational light bending and aberration are functions that transform angular directions (often represented as unit vectors). For example, the algorithm for gravitational deflection has been derived using what amounts to a geometric optics construction. (The Sun is, after all, a weak gravitational lens.)

The principal high-precision astrometric technique in the radio regime is VLBI. VLBI uses very sparsely-filled (synthetic) apertures, each segment of which is separated from the others by thousands of kilometers. For VLBI, the observable is the group delay, the time interval between wavefront arrival at two antennas. The algorithm that is used to predict or interpret VLBI delay transforms time intervals, not angles. Thus, the effect of the Sun's gravity enters as a slight difference in the "Shapiro delay" at the two antennas, a slowing of the wavefront as it crosses the Sun's gravitational potential. Aberration is replaced by the "retarded baseline" effect, which accounts for the change in position of the second antenna in the interval after the wavefront hits the first antenna but before it reaches the second.

Equivalent Physical Effects Treated Differently

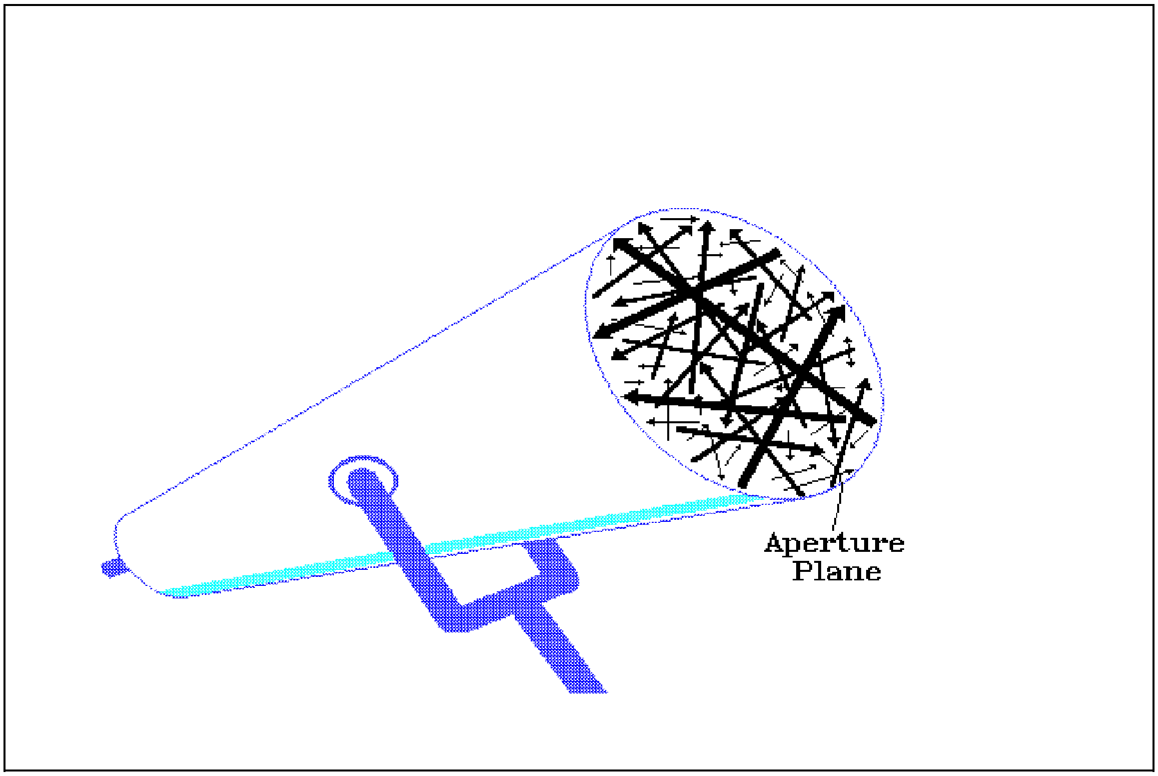
Optical		Radio
Aberration (annual and diurnal)	=	Retarded Baseline
Gravitational Light Bending	=	Differential Shapiro Delay

In both the optical and radio regimes, consensus algorithms have been developed. The preprint for this paper provides all of the formulas commonly used for the two kinds of observations. To determine whether these algorithms are consistent, we need a way to directly compare the angle-based and delay-based formulations. Because the two kinds of algorithms have been independently derived, the level of consistency also indicates the precision of the algorithms, that is, how well the mathematics represents the underlying physical model.

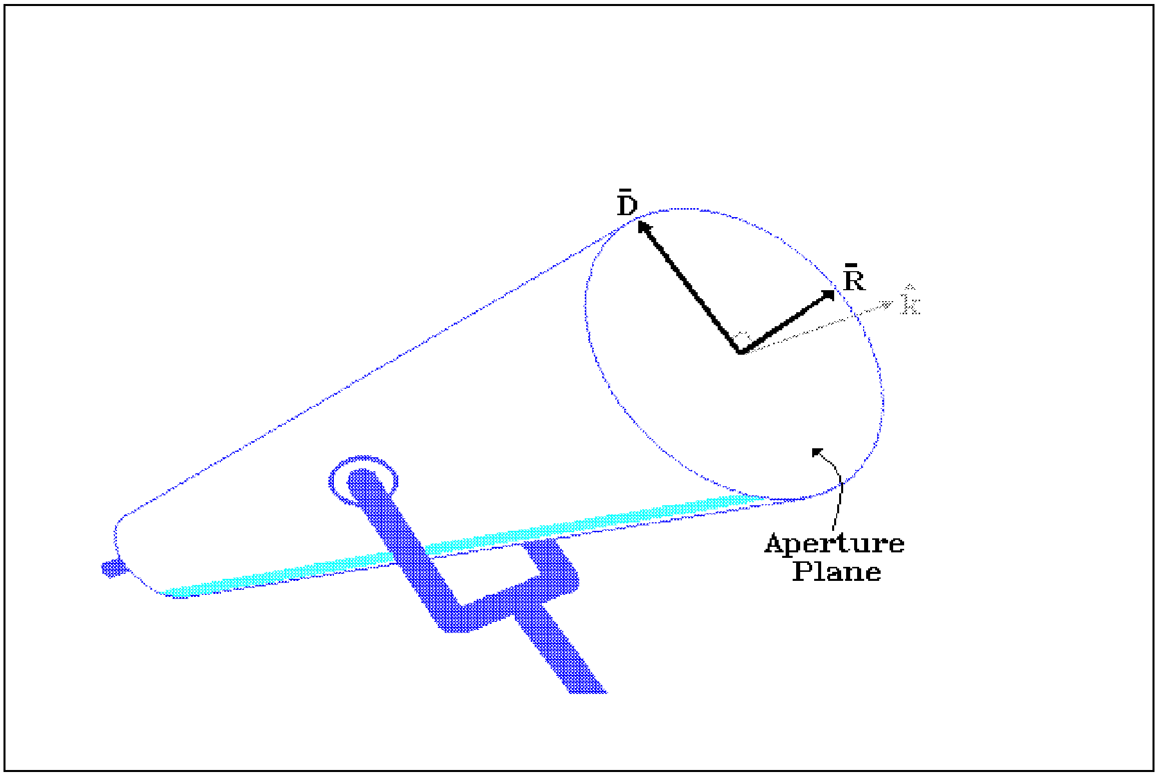
Reconciling the Two Kinds of Algorithms

We can reconcile the angle-variable and VLBI algorithms using a simple construction, illustrated by the cartoons on the next two pages. We imagine an optical telescope that is pointed in the geometric direction, **k**, of a celestial object. This is the unit vector along the line in Euclidean space that connects the center of the telescope's aperture to the object observed (in the case of solar system objects, corrected for light-time). Because of the effects we are interested in, this will not be the observed direction of the object. That remains to be computed.

We can use the standard angle-based algorithms to compute the observed direction. But, we can also imagine that the telescope's aperture is actually comprised of a large number of infinitesimal surface elements, any two of which are connected by a baseline, as illustrated on the next page. Now, these baselines are short, but that does not prevent us from using the VLBI delay algorithms on them — if we are careful to avoid some numerical degeneracies. In fact, this geometry, with the VLBI baselines orthogonal to the geometric direction of the object, simplifies the VLBI delay algorithm; a large first-order term drops out (see preprint). In applying the VLBI algorithm for delay, we also have to be careful that we compute the delay for the proper time of the telescope's reference frame.



Because the incoming wavefronts are not parallel to the aperture plane (remember, the aperture plane is orthogonal to the geometric, not the apparent, direction of the object), there will be a small but non-zero delay computed for any of these baselines. We can take any two equal-length perpendicular baselines in the aperture plane, compute the delays for each, and compute the tilt of the wavefronts — equivalently, the apparent direction, **k'**, of the observed object. The baseline geometry for this computation is shown on the next page.



For convenience, we choose the two baselines, **R** and **D**, to be in the direction of increasing right ascension and declination, respectively. So now we have a way to use the VLBI algorithms to compute the apparent right ascension and declination of the object, which can be directly compared to those same quantities computed using the standard angle-based algorithms used in optical astrometry.

How do the two kinds of algorithms compare?

Comparison Results

Figures 1, 2, and 3 on the following pages are all-sky maps of the differences between the optical (angle-variable) and VLBI (delay-variable) algorithms. To create these maps, the two kinds of algorithms were used to compute the apparent positions of 16,471 imaginary stars (all with zero parallax) evenly spaced in right ascension and declination at 2° intervals. The scheme described on the preceding pages was used to produce apparent positions using the VLBI algorithms. The maps show the arc differences between the two kinds of algorithms. The three figures represent increasingly sophisticated VLBI algorithms; it is obvious that the VLBI algorithms developed over the last decade have narrowed the differences with the standard angle-variable algorithms. The precision of the VLBI algorithms has increased by orders of magnitude, and the color scales used in the three figures represent very different ranges of differences, as indicated in the figure captions.

Figures 4 and 5 show the arc differences between the VLBI and angle-based computed positions of two planets, Venus and Mars, as a function of time.

All of the five figures were computed for longitude −120°, latitude +30°, using 100 m VLBI baselines. The all-sky maps for the stellar case were computed for 1 May 1996 at 0^h TT. However, the overall pattern of algorithm differences, with respect to the Sun and the ecliptic, does not vary significantly with time or place. The length of the VLBI baseline is also not important for baselines less than a few kilometers long.

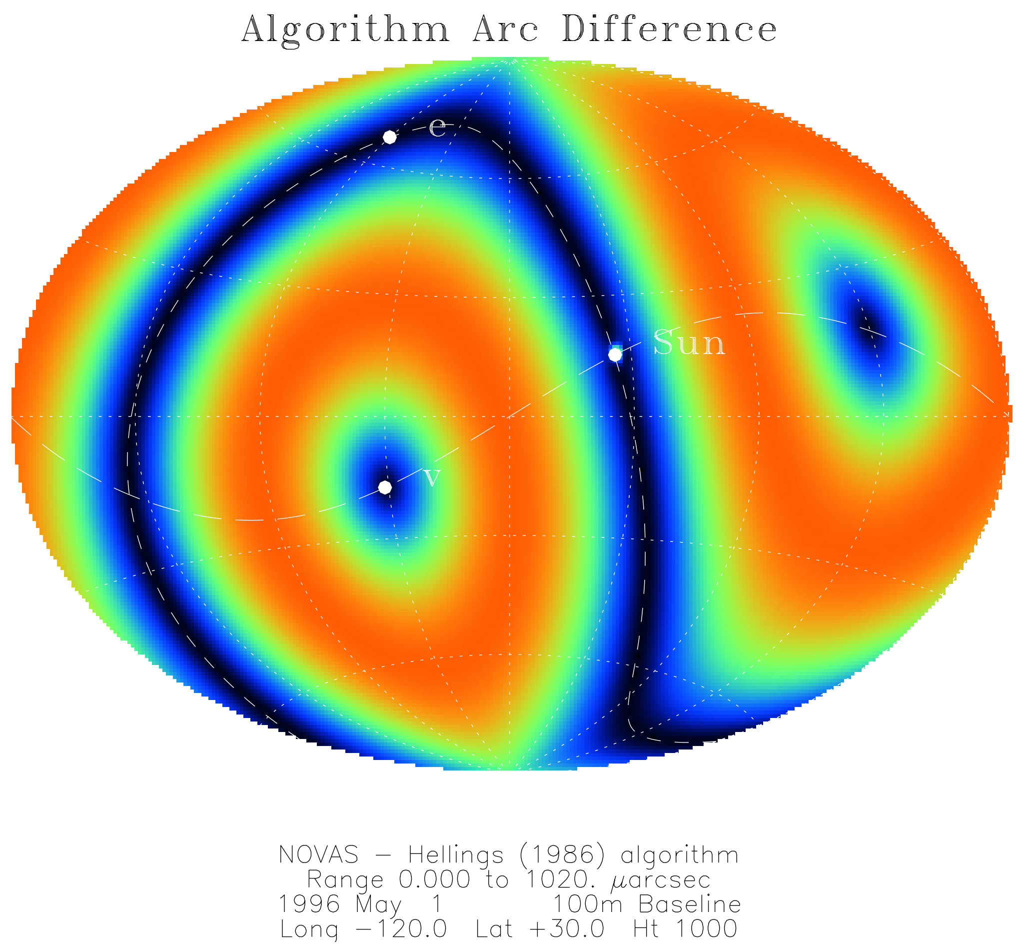


Figure 1. All-sky map showing the arc differences between star positions computed using the Hellings (1986) VLBI delay algorithm and the standard angle-variable algorithms used in optical astrometry. These computations used a grid of artificial stars at infinity and 100 m VLBI baselines. The full color scale (black through red) represents differences of 0−1 milliarcseconds. The position of the Sun, the ecliptic pole (e), and the apex of the observer's velocity (v) is indicated.

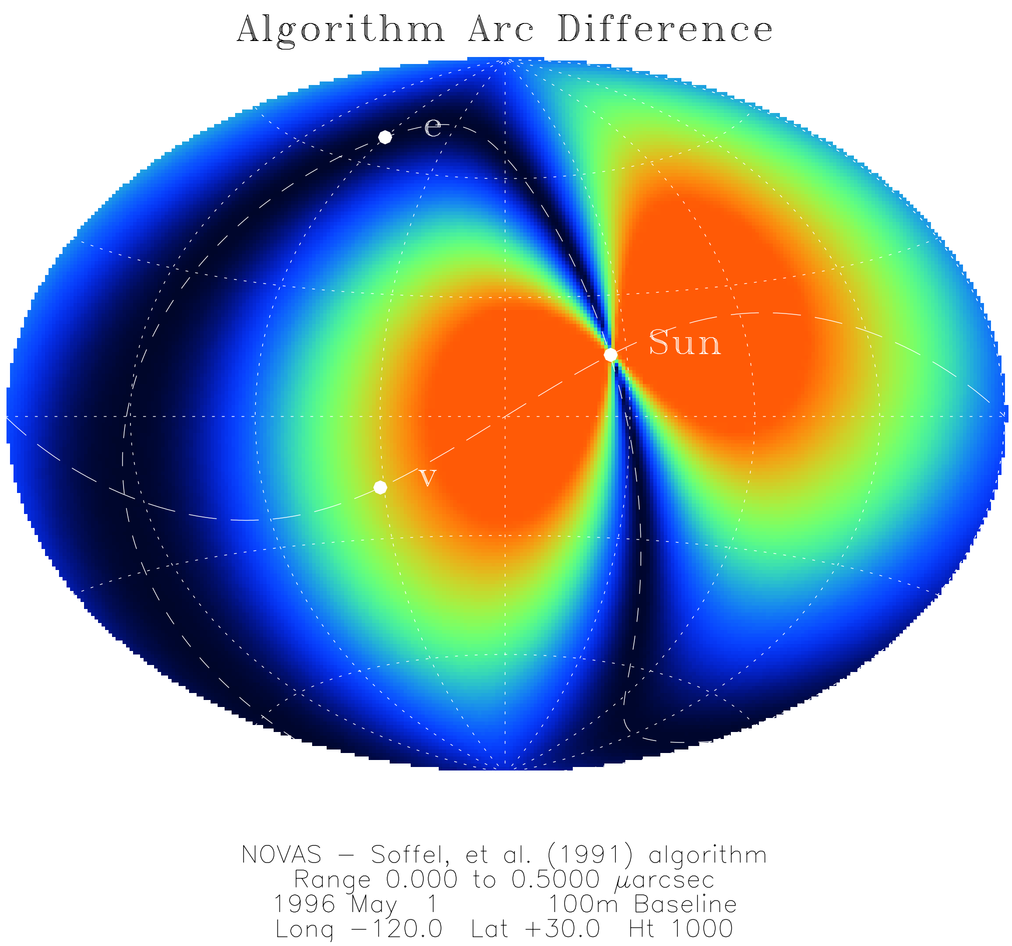


Figure 2. All-sky map similar to Figure 1, except that the Soffel *et al.* (1991) VLBI delay algorithm was used, and the full color scale (black through red) represents differences of 0−0.8 microarcseconds.

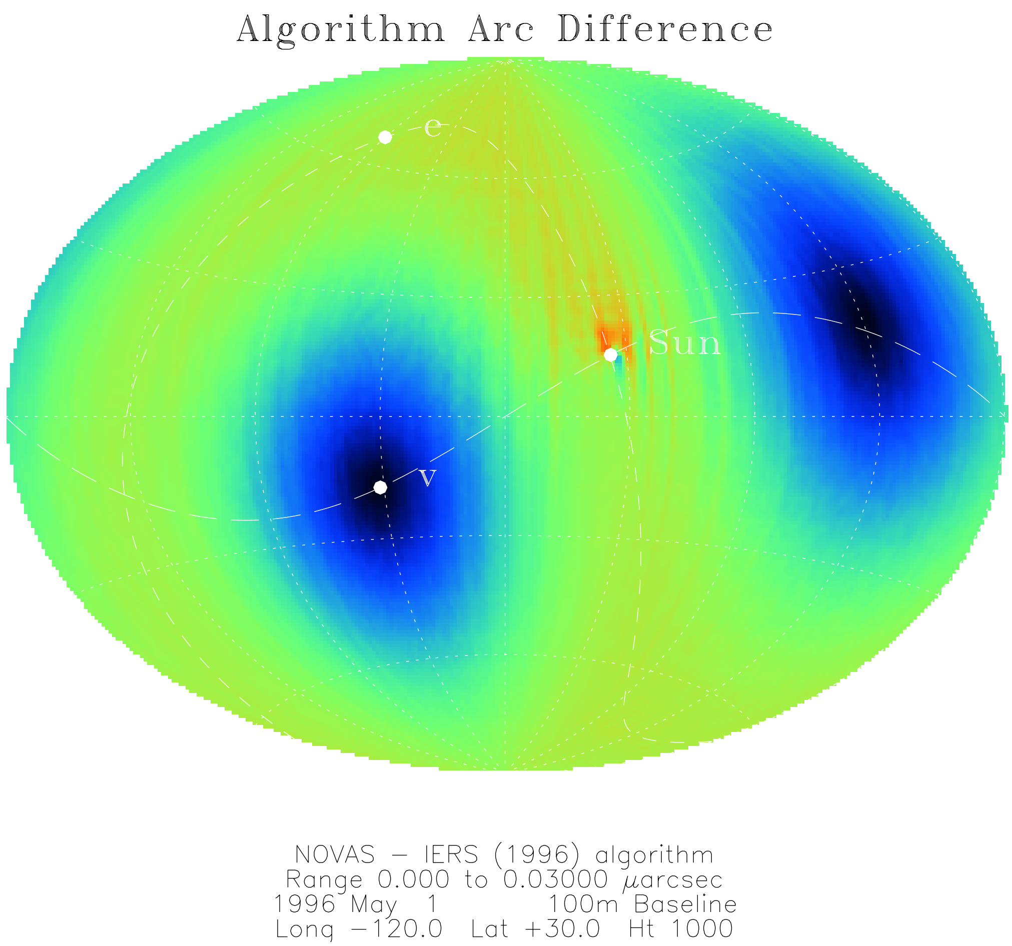


Figure 3. All-sky map similar to Figure 1, except that the International Earth Rotation Service (IERS) "consensus" VLBI delay algorithm (McCarthy 1996) was used, and the full color scale (black through red) represents differences of 0−0.03 microarcseconds.

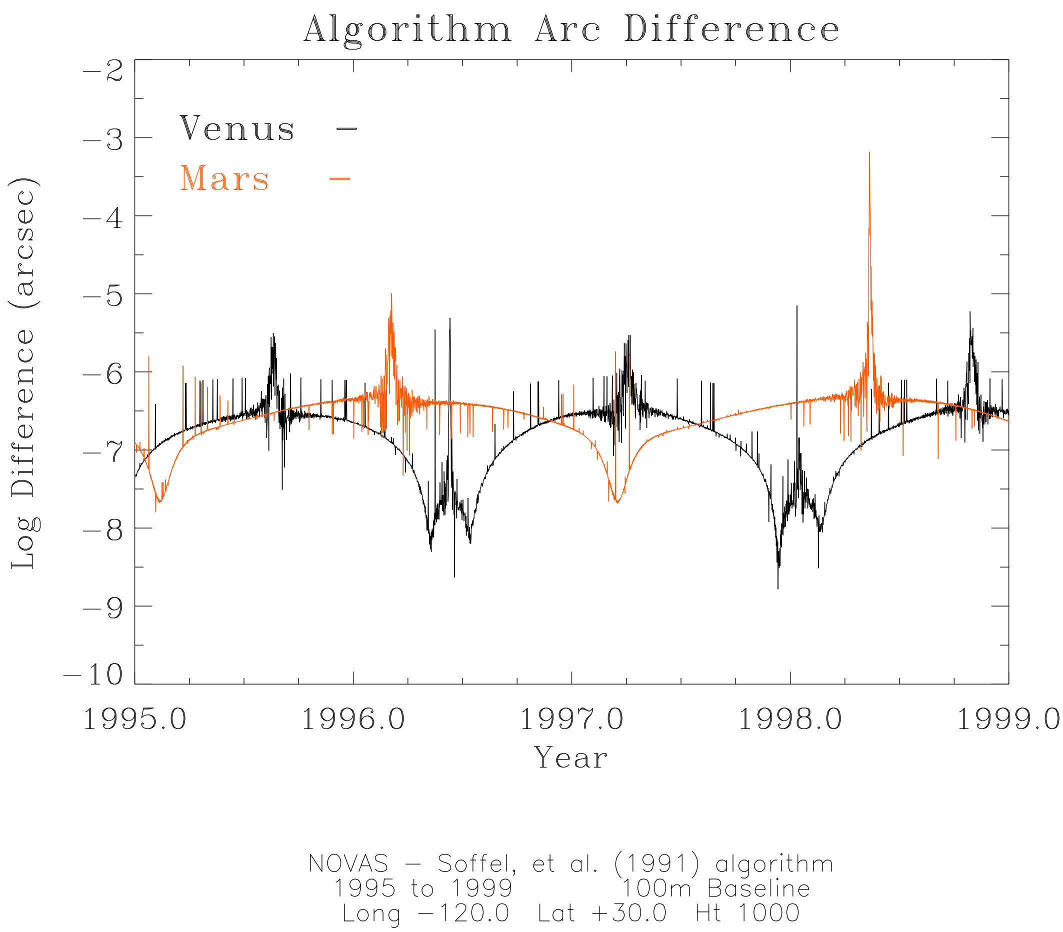


Figure 4. Plot of differences between planet positions computed using the Soffel *et al.* (1991) VLBI delay algorithm and the standard angle-variable algorithms used in optical astrometry. Differences for Venus are shown in black and those for Mars are shown in red. This comparison required that the usual VLBI formula for differential gravitational (Shapiro) delay be generalized for use with solar system objects.

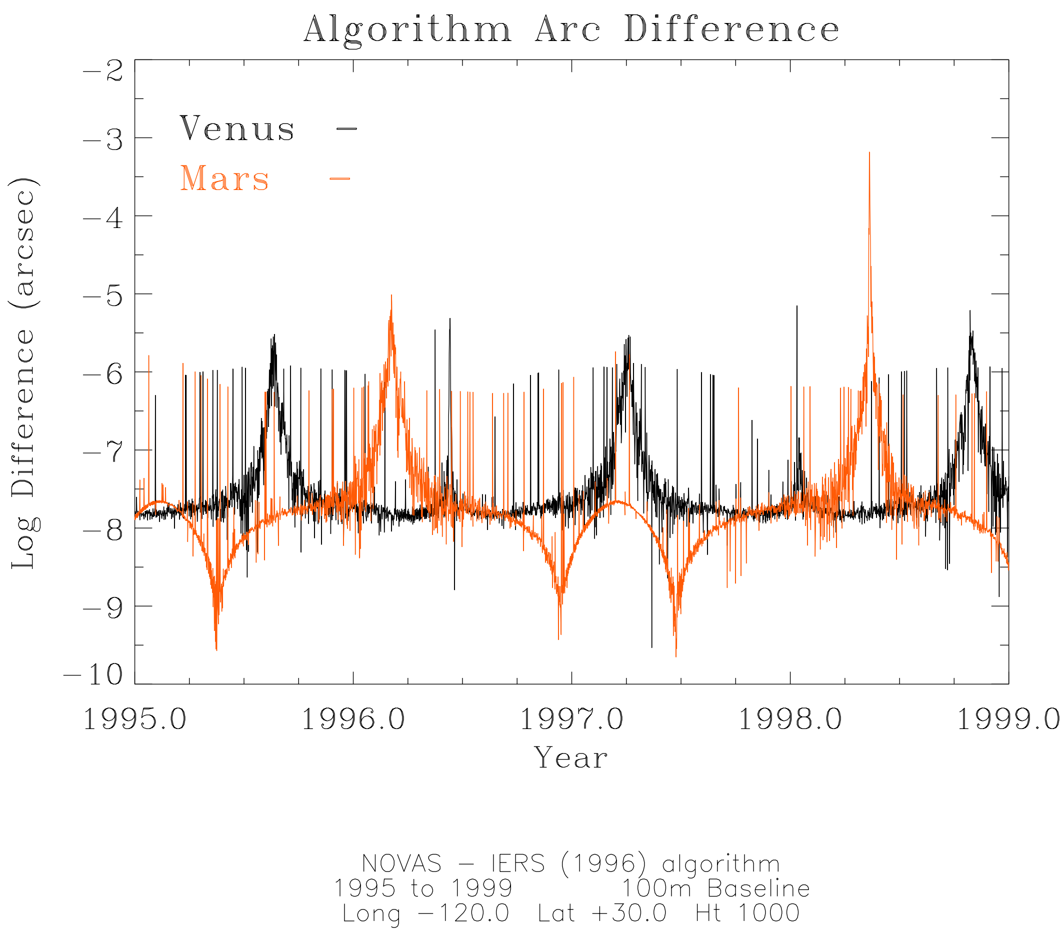


Figure 5. Plot similar to Figure 4, except that the IERS "consensus" VLBI algorithm (McCarthy 1996) was used. Note that the difference "floor" is lower by about an order of magnitude, making this plot appear noisier than Figure 4.

Conclusions

This analysis (described in more detail in the preprint) has shown that VLBI delay algorithms have become substantially more sophisticated over the last decade and that the VLBI and optical algorithms now correspond at the microarcsecond level or better over most of the sky. The correspondence between these algorithms provides information on their precision, which, as used here, refers to how well a mathematical representation of some effect corresponds to the physical model constructed to account for it. What has not been addressed is the accuracy of these algorithms, that is, how well the physical model corresponds to reality. These algorithms are known to be incomplete at the microarcsecond level, although the incompleteness involves only the gravitational deflection of rays passing very near the limbs of solar system bodies.

To carry out the comparison of algorithms, a procedure was developed whereby VLBI delay algorithms can be used to generate angular positions. The software developed for this, called WAAAV (Wide Angle Astrometry Algorithms from VLBI), is available from the author.

Perhaps the principal value of the algorithm comparison scheme developed here is its utility in validating extensions to the underlying physical models. The preprint provides a specific example. Further independent development of the algorithms in the two regimes can now be directly compared. Such a comparison is a useful diagnostic tool in implementing new algorithms required by increasingly sophisticated and accurate observing systems.

References

Hellings, R. W. 1986, AJ, 91, 650

Soffel, M. H., Műller, J., Wu, X., & Xu, C. 1991, AJ, 101, 2306

McCarthy, D. D. 1996, editor, IERS Conventions (1996), IERS Tech. Note 21 (Observatoire de Paris)