ROLLING WITH RUTHERFORD
TEACHER NOTES

DESCRIPTION
Students determine the radius of a ball by indirect measurement and an application of simple probability. They simply roll a "probe" ball at a row of "target" balls and count the number of times that the probe collides with the target (and the number of times that it doesn't). There are two versions of the activity here: one for middle school students and another for high school students.

STANDARDS
National Science Education Standards (U.S. National Research Council)
- Unifying Concepts and Processes:
    As a result of this activity . . . students should develop understanding and abilities aligned with the following concepts and process:
    o Evidence, models and explanation
    o Constancy, change, measurement

- Physical Science Content Standard G:
    As a result of this activity . . . students should develop an understanding of:
    o Nature of scientific knowledge.
    o Historical perspectives.

LEARNING OBJECTIVES
Students will know and be able to:
- Describe the process that Ernest Rutherford used to determine the size of the nucleus.
- Apply simple probability to experimental data.
- Use indirect measurement to determine properties difficult (or impossible) to determine otherwise.
- Create and interpret a histogram.

PRIOR KNOWLEDGE
Students must be able to keep careful records of observations and divide one integer into another.

BACKGROUND MATERIAL
Ernest Rutherford may be best known for determining that the nucleus was a tiny volume containing positive charge. He probed the nucleus by aiming alpha particles at metal foil and looking at the patterns made by the recoiling alpha particles. He found that most of them passed right through the foil.

Many objects defy direct measurement of some physical property. We can only measure these properties indirectly. One historical example is the size of the atomic nucleus. This simple experiment allows students to carry out a simple experiment and perform simpler calculation to determine the size of a marble.

IMPLEMENTATION
The student handout contains the background information that they need to carry out the experiment. It's not clear that students need a handout in this activity, but we provide two samples. These handouts do provide hints about the derivation and how to analyze the data. We provide some more detail about both on the next page of this teacher document.

| R | > 2R | R | > 2R | R | continues → |

w
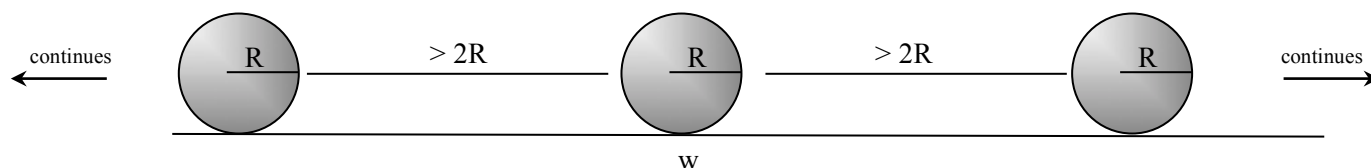
The cartoon on this page shows the simple setup. We set up the calculation in two steps.

First, we work out the probability of the probe hitting one of the target balls. You may see that they will hit when their centers are closer than the sum of their radii. The probability of this happening is the ratio of the width of the target area "covered" by the target balls to the total width of the target area. This probability increases directly as the number of target balls present.

In symbols this is $p = \dfrac{4n\sqrt{r_t r_i}}{w - 2r_i}$.

When the incident and target balls have the same radius, this simplifies to $p = \dfrac{4nr^2}{w - 2r}$.
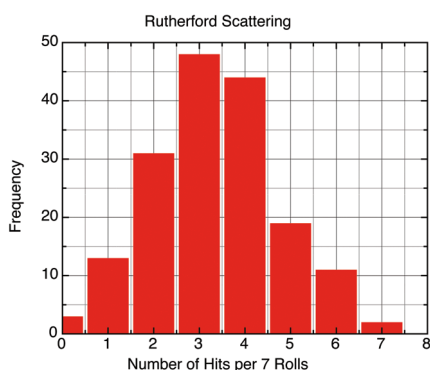
Second, we recognize that students can *measure* this probability by rolling the probe at the target area (many, many times) and counting the number of hits:

$$p = \frac{h}{t}$$

These two terms for p can be set equal and solved for the target radius r_t. There are two ways to simplify this equation. One (as noted above) is to use incident and target balls of the same radius. Another is to assume that $2r_i$ is negligibly small compared to l. All four solutions are below. Depending on their level, students should be encouraged to derive the solutions (which requires algebra and simple geometry).

	$2r_i \ll l$ **not** assumed	$2r_i \ll l$ assumed	
Different Radii	$r_t = \left[\dfrac{h(w - 2r_i)}{4nt\sqrt{r_i}} \right]^2$	$r_t = \dfrac{h^2 w^2}{16n^2 t^2 r_i}$	w is the width of the target area. n is the number of target balls. h is the number of hits. t is the number of times the probe was rolled.
Equal Radii	$r = \dfrac{w}{\frac{4nt}{h} + 2}$	$r = \dfrac{hw}{4nt}$	

Rutherford Scattering

Number of Hits per 7 Rolls

Students can also learn about (or practice making) histograms with this activity. They can divide their rolls up into a consistent number of "runs" (e.g., 12 runs of 10 rolls each yields 120 rolls). Doing so allows the class to create a group histogram of the number of hits in each run. (This will be some number between 0 and 12.) Make a table that allows students to contribute their own runs to the class set. The plot to the left shows the distribution of several hundred runs of seven turns each.
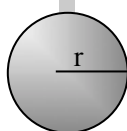
Probe – Roll towards the screened target a large number of times. Keep track of the hits and total number of trials.

ASSESSMENT
You might consider asking the students questions like:
- Would you expect more or fewer collisions if you increased the number of balls in the target area? Why?
- Why is the relationship between radius and the number of hits direct?

ROLLING WITH RUTHERFORD
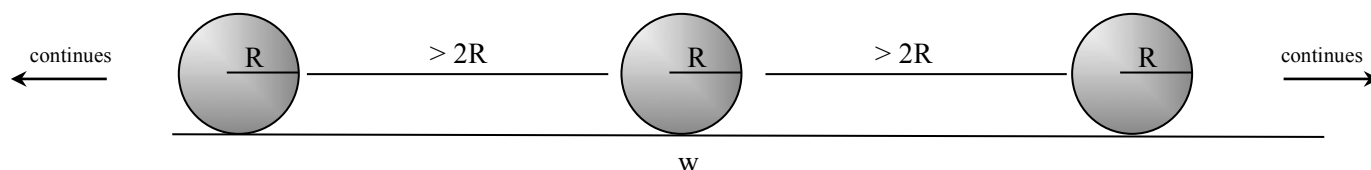
continues ←

R — > 2R — R — > 2R — R

continues →

w

Ernest Rutherford may be best known for determining that the nucleus was a tiny volume containing positive charge. He probed the nucleus by aiming alpha particles at metal foil and looking at the patterns made by the recoiling alpha particles. He found that most of them passed right through the foil.

Rutherford Scattering

Frequency vs. Number of Hits per 7 Rolls
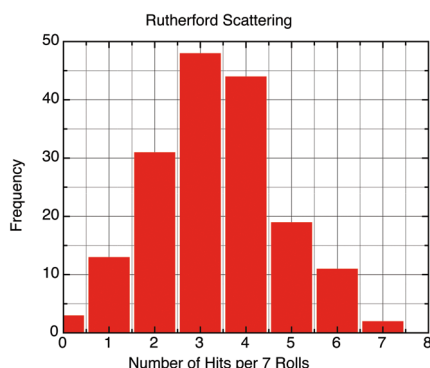
You can simulate Rutherford's work by rolling a marble at a line of marbles and counting the number of times that there is a collision. (Count only one collision per roll.) The spacing of the marbles in the line will affect the shape of your histogram. It will also affect the location of the peak. Closely spaced marbles will yield more collisions than marbles farther apart. We provide a sample of one setup in the chart on the left.

One can also calculate the radius of the balls by looking at the probability of a collision. If your students repeat a large number of rolls at similarly spaced targets, they have also measured this probability and can calculate the radius (R) from the equation below.
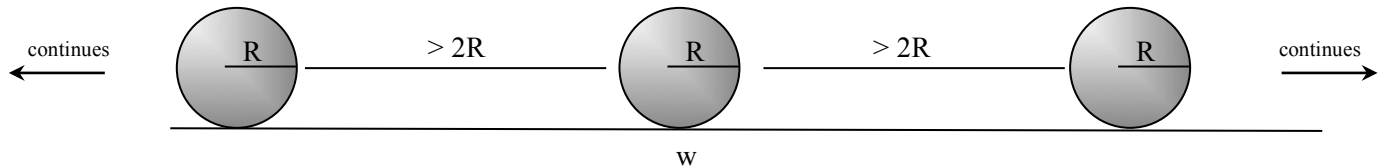
$$R = \frac{hw}{2tn} - r$$

h is the number of hits.
w is the width of the target region.
t is the number of trials.
n is the number of target balls.
r is the radius of the probe.

One can write this much more simply if the target and the probe are the same size.

r
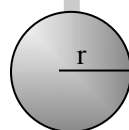
Probe – Roll towards the screened target a large number of times. Keep track of the hits and total number of trials.

ROLLING WITH RUTHERFORD

R   > 2R   R   > 2R   R

w

Ernest Rutherford may be best known for determining that the nucleus was a tiny volume containing positive charge. He probed the nucleus by aiming alpha particles at metal foil and looking at the patterns made by the recoiling alpha particles. He found that most of them passed right through the foil. Students can simulate Rutherford's work with ball bearings and a meter stick. The end result will be an indirect calculation of the radius of one of the balls in the array above.

We first need to work out the probability of the rolling ball striking one of the target balls. The two will hit when their centers are closer than the sum of their radii. The probability of this happening is the ratio of the summed radii to the total width of the target area. If more than one target ball is present, then the probability increases by a factor of the number of balls present. In symbols this is:

$$P = N\frac{2(R + r)}{L}$$

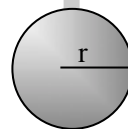We can also measure the probability by rolling the ball at the target area and counting the number of hits:

$$P = \frac{H}{T}$$

These two terms for P can be set equal and solved for the target radius R:

$$R = \frac{HL}{2TN} - r$$

H is the number of hits.
L is the length of the target region.
T is the number of trials.
N is the number of target balls.
r is the radius of the probe.

One can write this much more simply if the target and the probe are the same size.

r

Probe – Roll towards the screened target a large number of times. Keep track of the hits and total number of trials.