OT: Stats - Fsihers

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Fisher's Method & Number of OT Test Specimens

- In an attempt to reduce OT result variability, TTI researchers proposed reporting 3 OT specimen results from a pool of 5 tested specimens selected on the basis of minimum COV consideration.

- This approach was successful in reducing the Co-efficient of Variance (COV) of the OT cycles but the computed average OT cycles for three specimens may sometimes be considerably different from the average of all 5 specimens; see subsequent table.

Fisher's Method & Number of OT Test Specimens

District	Mix Type	Drying Method	COV (OT cycles), %	
			All 5	Best 3
Atlanta	Type D 5.2% AC	Air	68.9 **(119)***	8.5 **(92)**
		Oven	34.5 **(122)**	6.3 **(118)**
Atlanta	Type D 5.5% AC	Air	61.2 **(538)**	26.1 **(527)**
		Oven	46.7 **(396)**	19.5 **(520)**
Childress	Type D 4.9% AC	Air	57.2 **(187)**	31.6 **(176)**
		Oven	61.2 **(392)**	7.5 **(560)**
Bryan	CAM 6.7% AC	Air	17.2 **(928)**	0.0 **(1000)**
		Oven	18.4 **(856)**	6.0 **(967)**
Laredo	Type C 5.0% AC	Air	41.7 **(20)**	22.3 **(25)**
		Oven	50.4 **(36)**	15.6 **(24)**

* Values in parenthesis are average OT cycles

Fisher's Method & Number of OT Test Specimens

- The question is **"Is the reported average OT cycles of the 3 selected specimens representative of the actual cracking resistance petential of the mix?**

OR,

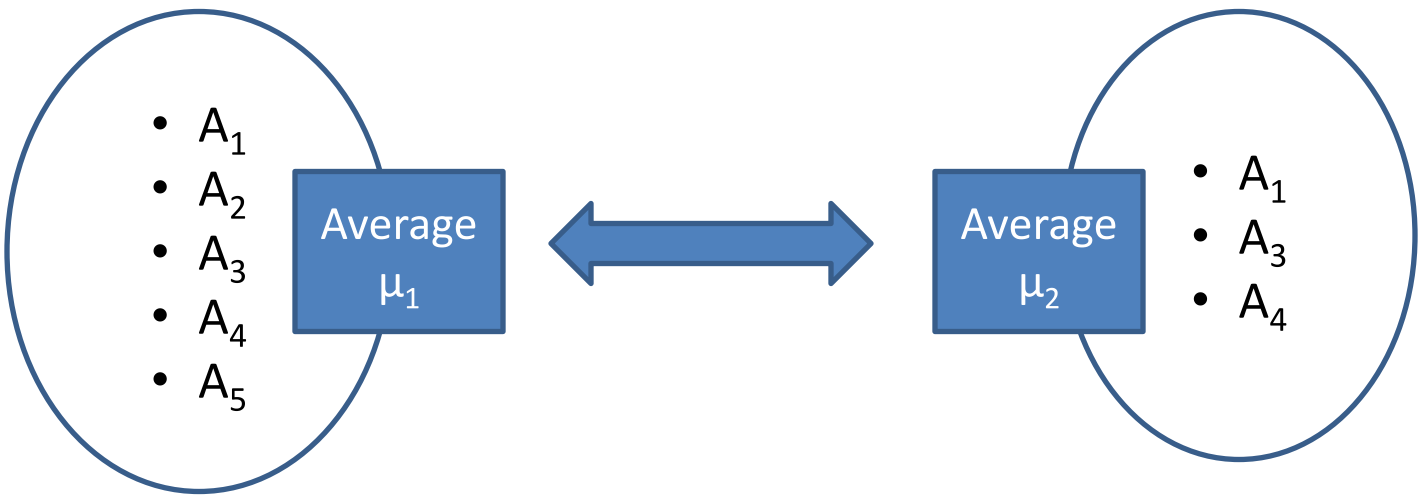
- **Is the average of the 3 selected specimens significantly different from the average of all the 5 specimens?**

Possible solution:

Statistical Variance Analysis using Fisher's Method

Fisher's Method & Number of OT Test Specimens

OPTION 1



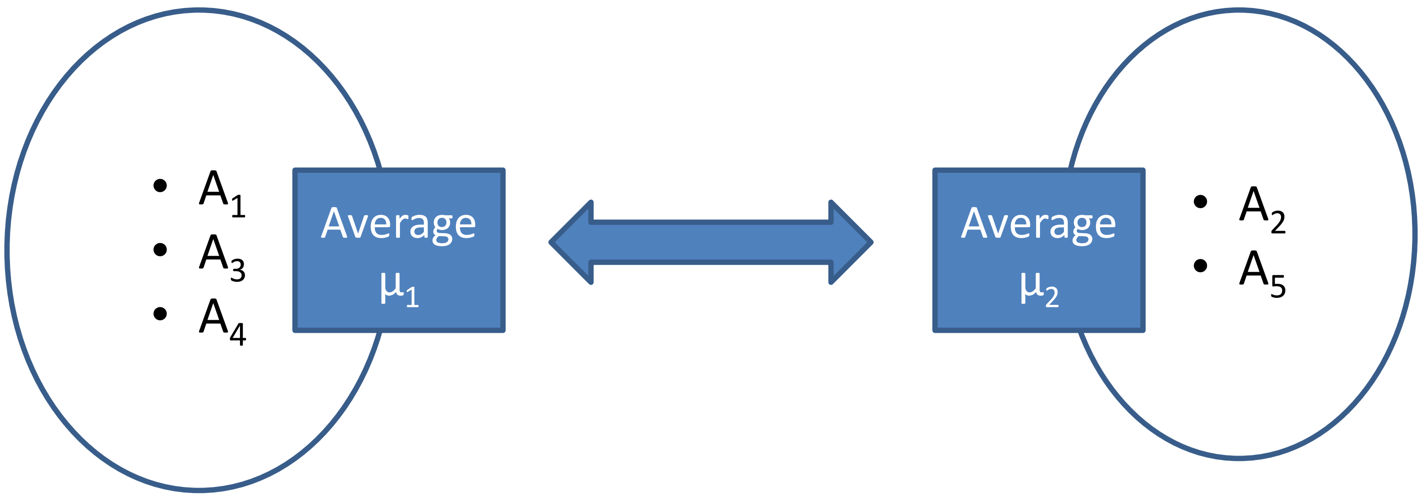
Set 1: All 5 OT specimens

Set 2: Best 3 OT specimens

- ❑ Since set 2 is a subset of set 1, any variance analysis will fail to show any statistically significant difference between μ_1 and μ_2

Fisher's Method & Number of OT Test Specimens

OPTION 2



Set 1: Best 3 OT specimens

Set 2: Discarded 2 OT specimens

- Set 1 and Set 2 are two sets of populations selected from a larger pool of 5 specimens. A variance analysis test between the two averages (μ_1 and μ_2) should be sufficient to determine whether the best 3 average statistically represents the HMA cracking behavior.

Fisher's Method & Number of OT Test Specimens

Fisher's Least Significant Difference Method*

- Fisher's LSD is a method of determining which population mean differs from the rest of 't' population means (where 't'≥3).
- If we have 't' sets of populations with means μ_1,.. μ_t (t≥3) , the first step of Fisher's LSD is to reject the null hypothesis,

$$H_o: \mu_1 = \mu_2 = \ldots\ldots\ldots = \mu_t.$$

(i.e. determining that at least one of the 't' means is statistically different than the other 't-1' mean values.)

- If the null hypothesis cannot be rejected, then Fisher's LSD analysis cannot be applied.

Fisher's LSD method cannot be applied to analyze whether the selected best 3 OT specimens are statistically different from the rejected 2 OT specimens because in this case, the number of population sets, t=2 < 3.

* Reference: An Introduction to Statistical Methods and Data Analysis by R.L. Ott and M. Longnecker

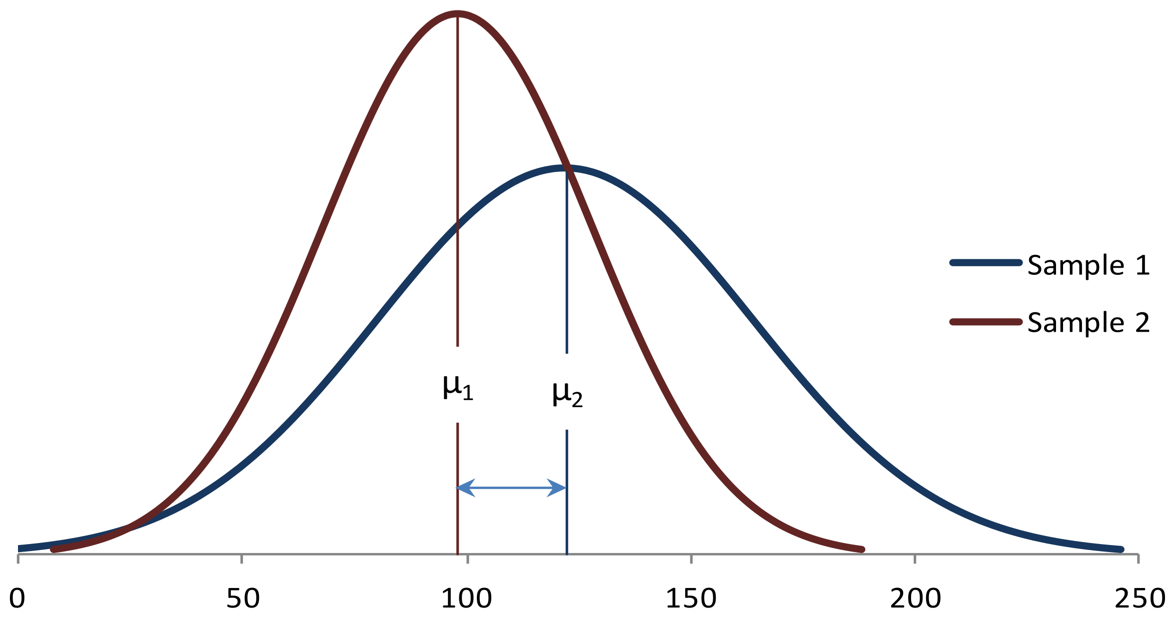
Fisher's Method & Number of OT Test Specimens

Recommendation

- Alternate approach to Fisher's Least Significant Difference Method to compare the two sets of populations (Best 3 and Rejected 2): Interference Analysis when number of population is 2.

- Analysis ongoing.

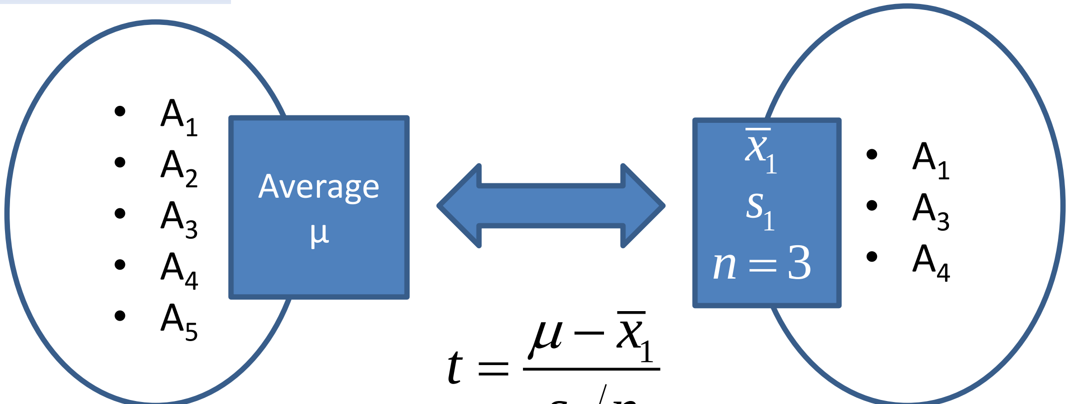
‘t’ test & Number of OT Test Specimens



A ‘t-test’ compares the means of two groups and determine the likelihood that the difference of the two means occurred by chance.

‘t’ test & Number of OT Test Specimens

OPTION 1



$$t = \frac{\mu - \overline{x}_1}{s_1 / n}$$

Population: All 5 OT specimens

Sample: Best 3 OT specimens

d.f. (n-1)	t-distribution for Probability level				
	10%	5%	2%	1%	0.1%
1	6.314	12.706	31.821	63.657	636.619
2	2.920	**4.303**	6.965	9.925	31.598
3	2.353	3.182	4.541	5.841	12.941
4	2.132	2.776	3.747	4.604	8.610
5	2.015	2.571	3.365	4.032	6.859

A comparison between the total **population** (all 5 specimens) set and the selected **sample** (best 3 specimens) set to determine whether the two means are statistically different or not to determine whether the best 3 average statistically represents the HMA cracking behavior..

‘t’ test & Number of OT Test Specimens

OPTION 2

- A_1
- A_3
- A_4

$\bar{x}_1$

s_1

$n = 3$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{SE}$$

$\bar{x}_2$

s_2

$n = 2$

- A_2
- A_5

$$SE = S_P \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

Set 1: Best 3 OT specimens

$$S_P^2 = \frac{(n_1 - 1)\, S_1^2 + (n_2 - 1)\, S_2^2}{n_1 + n_2 - 2}$$

Set 2: Discarded 2 OT specimens

d.f. $(n_1 + n_2 - 2)$	t-distribution for Probability level 10%	5%	2%	1%	0.1%
1	6.314	12.706	31.821	63.657	636.619
2	2.920	4.303	6.965	9.925	31.598
3	2.353	**3.182**	4.541	5.841	12.941
4	2.132	2.776	3.747	4.604	8.610
5	2.015	2.571	3.365	4.032	6.859

Set 1 and Set 2 are two **sample** sets selected from a larger pool (**population**) of 5 specimens. A variance analysis test between the two averages (μ_1 and μ_2) to determine whether the best 3 average is statistically different from the average of the discarded 2.

‘t’ test & Number of OT Test Specimens

Mix	Drying Method	Average Cycle			t- value		Significantly different?	
		All 5	Best 3	Discarded 2	Option 1	Option 2	Option 1	Option 2
Atlanta Type D	Air	119	92	161	5.980	0.896	yes	no
	Oven	122	118	126	0.932	0.182	no	no
Childress Type D	Air	187	176	202	0.343	0.234	no	no
	Oven	392	560	140	6.928	5.497	yes	yes
Laredo Type C	Air	20	25	12	1.553	2.945	no	no
	Oven	36	24	53	5.551	3.095	yes	no
Chico TypeD	Air	304	167	510	5.962	5.392	yes	yes
	Oven	158	210	80	1.723	3.322	no	yes
Bryan CAM	Oven	856	967	691	3.314	5.968	no	yes
Chico (2 from 4.5”)	Oven	171	230	84	1.639	3.059	no	no

‘t’ test & Number of OT Test Specimens

- For around 60% of the analyzed cases, picking the best three OT specimens produce a result that is not significantly different from ‘All five’ or ‘discarded two’. Therefore, the proposed approach of picking the best 3 based on the lowest COV is not unreasonable!!

- However, it is interesting to note that the t-analysis is valid when the population follows a ‘normal distribution’. For the OT test, the population size is 5 which is not likely to follow the normal distribution. Therefore, applicability of any statistical test in this case will be somewhat questionable.