University Transportation Research Center - Region 2

Final Report

Robust Routing for Hazardous Materials Transportation with Conditional Valueat-Risk on Time-Dependent Networks

Performing Organization: University at Buffalo/SUNY

November 2012

Sponsor:
Research and Innovative Technology Administration / USDOT (RITA)

University Transportation Research Center - Region 2

The Region 2 University Transportation Research Center (UTRC) is one of ten original University Transportation Centers established in 1987 by the U.S. Congress. These Centers were established with the recognition that transportation plays a key role in the nation's economy and the quality of life of its citizens. University faculty members provide a critical link in resolving our national and regional transportation problems while training the professionals who address our transportation systems and their customers on a daily basis.

The UTRC was established in order to support research, education and the transfer of technology in the field of transportation. The theme of the Center is "Planning and Managing Regional Transportation Systems in a Changing World." Presently, under the direction of Dr. Camille Kamga, the UTRC represents USDOT Region II, including New York, New Jersey, Puerto Rico and the U.S. Virgin Islands. Functioning as a consortium of twelve major Universities throughout the region, UTRC is located at the CUNY Institute for Transportation Systems at The City College of New York, the lead institution of the consortium. The Center, through its consortium, an Agency-Industry Council and its Director and Staff, supports research, education, and technology transfer under its theme. UTRC's three main goals are:

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Project No: 49111-07-23

Project Date: November , 2012

Project Title: Robust Routing for Hazardous Materials Transportation with Conditional Valueat-Risk on Time-Dependent Networks

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Research and Innovative Technology Administration / USDOT (RITA)

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TECHNICAL REPORT STANDARD TITLE PAGE

1. Report No.	2.Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle Robust Routing for Hazardous Materials Transportation with Conditional Value-at-Risk on Time-Dependent Networks		5. Report Date November 29, 2012 6. Performing Organization Code
7. Author(s) Iakovos Toumazis and Changhyun Kwon		8. Performing Organization Report No.
9. Performing Organization Name and Address Department of Industrial and Systems Engineering University at Buffalo Buffalo, NY 14260		10. Work Unit No. 11. Contract or Grant No. 49111-07-23
12. Sponsoring Agency Name and Address Research and Innovative Technology Administration U.S. Department of Transportation 1200 New Jersey Avenue, SE Washington, DC 20590		13. Type of Report and Period Covered Final Report 14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

New methods are proposed for mitigating risk in hazardous materials (hazmat) transportation, based on Conditional Value-at-Risk (CVaR) measure, on time-dependent vehicular networks. While the CVaR risk measure has been popularly used in financial portfolio optimization problems, its application in hazmat transportation has been very recently proposed. The CVaR models are shown to be flexible and general routing models for hazmat transportation, and be solved efficiently. This research project will extend the previous research by considering CVaR for hazmat transportation on time-dependent networks.

17. Key Words	18. Distribution Statement
dynamic shortest path; time-dependent network; conditional value-at-risk; hazardous materials transportation	

19. Security Classif (of this report)	20. Security Classif. (of this page)	21. No of Pages	22. Price
Unclassified	Unclassified	25	

Form DOT F 1700.7 (8-69)

1 Introduction

During the last couple of decades industrial development resulted in the production of enormous quantities of hazardous materials. Hazardous material (hazmat), as defined by the U.S. Department of Transportation Pipeline and Hazardous Materials Agency, is a substance or material capable of posing an unreasonable risk to health, safety, or property when transported in commerce. Obviously these quantities must be transported safely to their final destination. During their transportation, the population, the environment and public structures are exposed to the risk of a potential accident.

In 2007, all commodities shipped, were estimated to be 3,344,658 million ton-miles (U.S. Census Bureau, 2007b) with hazardous materials adding up to approximately 323,457 million ton-miles (U.S. Census Bureau, 2007a). Furthermore, hazmat shipments represent about 10 percent of the total commodities shipments ton mileage, and a 5% increase in hazmat volume each year has been reported (Transportation Research Board, 2005). The average miles traveled per hazmat shipment is 96 miles (U.S. Census Bureau, 2007a) whereas the average miles per shipment independently of the nature of the load is 619 miles (U.S. Census Bureau, 2007b). These numbers show that hazmat shipments tend to travel shorter distances which along with the operational flexibility of trucks, makes them an attractive transportation mode. Despite the fact that only 42.94% of all hazmat tonnage is transported by truck, a 93.98% of individual shipments use trucks as a mode of transportation (U.S. Department of Transportation, 1998).

Hazmat accidents are rare events (low-probability incidents with the accident probabilities usually in the range of 10^{-08} to 10^{-06} per mile traveled; Abkowitz, M. and Cheng, PD, 1988) but with catastrophic consequences (high-consequence incidents) when one does occur. During the year 2011, according to the U.S. Department of Transportation Pipeline and Hazardous materials Agency, 13,908 hazmat incidents have been recorded which resulted in 145 injuries, 10 deaths and damages of total worth $104,113,342. Note that the process of hazmat transportation is divided in four phases: loading, in transit, in transit storage and unloading. This study focuses on the transit phase, since the cost of the damages caused by incidents during the transit phase had a total cost of $84,687,976 along with 70 injuries, among which 12 needed hospitalizations. Accidents during all others transportation phases resulted damages of total cost $19,425,366 and 75 injuries from which only 11 were hospitalized. From all 10 deaths that occurred in 2011, 9 of them took place in the transit phase while only one occurred during the other phases, namely the loading phase. With more than $800,000$ hazardous materials shipments performed daily in the U.S. (U.S. Department of Transportation, 1998) combined with the above statistics, the need for risk-averse route decision in the transit phase of hazmat transportation is obvious.

For the static case, when all network attributes are time-invariant, the Value-at-Risk (VaR) model and the Conditional Value-at-Risk (CVaR) model have been recently proposed as alternatives to the existing routing methods by Kang et al. (2011) and Kwon (2011), respectively. Both VaR and CVaR are popular risk measures in financial risk management. VaR is the minimum threshold value such that the probability that the hazmat risk exceeds this threshold value is less than or equals to a given probability level. CVaR is approximately the conditional expected value that the risk is greater than the VaR value, while the exact definition of CVaR will be provided. Both VaR and CVaR models provide a flexible and risk-averse framework to the decision makers for hazmat routing. This paper extends the CVaR model to the dynamic case to consider accident probabilities that vary with time.

2 Literature Review

First, a brief background on Value-at-Risk (VaR) is provided because CVaR is an extension to VaR. Even though VaR was initially introduced as a risk measure for overnight risk, it has been further developed and it is now considered as a finance industry standard for measuring financial risks. Despite its wide use and popularity, VaR has received criticism because it is not a coherent risk measure (Artzner et al., 1999; Dowd and Blake, 2006) and it might lead to an inaccurate perception of risk (Nocera, 2009; Einhorn, 2008). It is also claimed that VaR cuts off and ignores what is happening in the tail of the distribution. The VaR model provides flexibility in the risk attitude from *risk-indifferent* to *risk-averse* (Kang et al., 2011).

However the risk-indifferent attitude of the VaR model is not favorable in hazmat routing.

CVaR is a risk measure that is also broadly used in financial optimization theory. It is a computationally tractable and coherent alternative to VaR. CVaR mainly focuses on the long tail of the risk distribution to avoid extreme events, providing a risk-averse tool to the hands of the decision makers when applied in the concepts of hazmat routing. Even though in financial investment problems, this may not always lead to the optimal solution, since high risk might result in high profit, when applied in hazmat transportation, high risk cannot be traded for high-return since we are talking about public safety. In this case, a risk-averse approach appears to be more reasonable.

The concepts of VaR and CVaR applied in hazmat transportation have some significant differences from the respective models used in finance. The most notable difference is that the models, when applied to hazmat transportation, focus on measuring the risk resulted by following a specified route in the network. Hence, the investment (that is, the route) and the loss measured (that is the accident consequence) are totally different quantities, and consequently not comparable. On the contrary, in finance the measurement units of both the investment and the loss are same as monetary units. In addition, for the models addressing hazmat transportation, the risk of each road segment in a path is non-additive to each other, while losses of portfolios in financial models are additive. It is then obvious, that the models applied in hazmat routing are more complex and require application-specific analysis and computational methods.

Even though an enormous number of publications address the shortest path problem in networks, the majority of these publications apply to static networks that have fixed arc costs. However, in recent years the interest in shortest path problems has been renewed, resulting in a number of publications dealing with shortest path problems in time-dependent networks. The most notable difference between the two versions of shortest path problems is that in the dynamic case, the arc costs and the arc travel times depend on the time of entrance in the arc, whereas in the static case the arc costs and travel times are fixed constants. Dynamic shortest path problems have been a very powerful tool used for Intelligent Transportation

Systems (ITS) and for real-time dynamic management and route guidance models.

The first publication studying shortest path algorithms, proposed an extension of Bellman's principle of optimality that was computing the shortest route between every node in the network to the destination node for different time-steps (Cooke and Halsey, 1966). However, no numerical results exist to study the efficiency of the proposed method. Another approach with the same complexity as Cooke and Halsey (1966) is proposed by Dreyfus (1969), when trying to compute the shortest path from every node to the destination. Dreyfus's approach computes the shortest path for a single time step between a unique origin-destination pair. In order for this algorithm to be able to detect the shortest path, the First-In-First-Out (FIFO) condition must hold for every arc.

Another algorithm calculating the time-dependent shortest paths from all nodes in the network to the destination node is proposed by Ziliaskopoulos and Mahmassani (1993). The proposed algorithm is based on Bellman's principle of optimality and the paths are calculated while the algorithm operates backwards in a label correcting way. The most noteworthy difference of this approach is that it can deal with networks with the arc cost not necessarily being the travel times. This advantage motivated us to use the algorithm proposed by Ziliaskopoulos and Mahmassani (1993), considering the arc cost as the risk exposed by traversing each arc in the network.

3 Conditional Value-at-Risk Background

CVaR applied in the concept of hazardous materials transportation has been recently proposed Kwon (2011). Assume a directed and weighted network $G = (\mathcal{N}, \mathcal{A})$, where $\mathcal{N}$ is the set of nodes and $\mathcal{A}$ the set of directed arcs. For a path $l \in \mathcal{P}$ at the confidence level α, the CVaR for general distributions is defined as follows (Rockafellar and Uryasev, 2002; Sarykalin et al., 2008):

$$\mathsf{CVaR}^l_\alpha = \lambda^l_\alpha \mathsf{VaR}^l_\alpha + (1 - \lambda^l_\alpha)\mathbb{E}[R^l : R^l > \mathsf{VaR}^l_\alpha] \tag{1}$$

where $\lambda_\alpha^l = \big(\Pr[R^l \leq \mathsf{VaR}_\alpha^l] - \alpha\big)/(1-\alpha)$ and $\mathcal{P}$ is the set of all alternative paths, and $\mathsf{VaR}_\alpha^l = \min\{\beta : \Pr(R^l > \beta) \leq 1-\alpha\}$.

CVaR_α^l given in the form (1) cannot be used in an optimization problem because of the conditioning in the expectation. However, Rockafellar and Uryasev showed that CVaR minimization problem is equivalent to the following function (Rockafellar and Uryasev, 2000; Pflug, 2000)

$$\Phi_\alpha^l(r) = r + \frac{1}{1-\alpha}\mathbb{E}[R^l - r]^+ \approx r + \frac{1}{1-\alpha}\sum_{(i,j)\in\mathcal{A}^l} p_{ij}[c_{ij} - r]^+ \tag{2}$$

where $[x]^+ = \max(x, 0)$, when minimized by choosing a path $l \in \mathcal{P}$ at the confidence level α and p_{ij} is the accident probability on arc (i,j) and c_{ij} the accident consequences on arc (i,j). That is,

$$\min_{l\in\mathcal{P}} \mathsf{CVaR}_\alpha^l = \min_{l\in\mathcal{P}, r\in\mathbb{R}^+} \Phi_\alpha^l(r) \tag{3}$$

Note here that, the parameter r is shown to be equal to the VaR value of the proposed path for the same confidence level (Rockafellar and Uryasev, 2000). The CVaR minimization problem can be written as

$$\min_{r\in\mathbb{R}^+}\left(r + \frac{1}{1-\alpha} z_\alpha(r)\right) \tag{4}$$

where $z_\alpha(r) \equiv \min_{x\in\Omega}\sum_{(i,j)\in\mathcal{A}} p_{ij}[c_{ij} - r]^+ x_{ij}$ and

$$\Omega \equiv \left\{x : \sum_{(i,j)\in\mathcal{A}} x_{ij} - \sum_{(j,i)\in\mathcal{A}} x_{ji} = b_i \quad \forall i \in \mathcal{N}, \text{ and } x_{ij} \in \{0,1\} \quad \forall (i,j) \in \mathcal{A}\right\} \tag{5}$$

The parameter b_i has the following value:

$$b_i = \begin{cases} 1 & \text{, if node } i \text{ is the source} \\ -1 & \text{, if node } i \text{ is the sink} \\ 0 & \text{, otherwise} \end{cases} \tag{6}$$

Following the mathematical analysis of Kwon (2011) for (4), we end up with the following minimization problem:

$$\mathsf{CVaR}^*_\alpha = \min_{r=0,c_{(1)},\ldots,c_{(m)}} \left[r + \frac{1}{1-\alpha} \min_{x\in\Omega} \sum_{(i,j)\in\mathcal{A}} p_{ij}[c_{ij}-r]^+ x_{ij} \right] \tag{7}$$

which indicates that we can only search the set $\{0, c_{(1)}, ..., c_{(m)}\}$ for r, where c_i represents the i-th smallest c_{ij} value, and m is the total number of arcs (Toumazis et al., 2012). This property holds mainly because the problem becomes linear in r for any interval $[c_{(i)}, c_{(i+1)}]$ for all i, therefore an optimal solution for each interval is found at a boundary of the interval. CVaR model offers a flexible tool for decision makers whose risk attitudes range from *risk-neutral* to *risk-averse* (Toumazis, 2012). Particularly, when α is sufficiently small, the CVaR model is equivalent to the Traditional Risk model that minimizes the expected consequence, and when α is sufficiently large, the CVaR model is equivalent to the Maximum Risk model that minimizes the maximum consequence on the path.

4 CVaR Minimization Model for Time-Dependent Networks

In this section, CVaR model is applied in a time-dependent network. Assume a directed, weighted, discrete FIFO dynamic network $G = (\mathcal{N}, \mathcal{A})$, where $\mathcal{N}$ is the set of nodes and $\mathcal{A}$ the set of directed arcs. Let $d_{ij}(t, r) = p_{ij}(t)[c_{ij} - r]^+$ be the non-negative time-dependent arc cost, i.e. risk, from traversing the arc (i, j) when departure from node i at time t. Note that $d_{ij}(t)$ is the risk exposed to by traversing arc (i, j) when the entrance time is at t and it is a real-valued function which is defined for every $t \in S$, where $S = \{t_0, t_0+\delta, t_0+2\delta, \ldots, t_0+M\delta\}$. In addition, $\tau_{ij}(t)$ is the time needed to traverse arc (i, j) when the entering time in the arc is at time t. The earliest possible departure time, from any node of the network is defined as t_0. Constants δ and M are both user defined and represent a small time interval during which some meaningful change in the traffic conditions may occur, and a large integer value, such that the time interval from t_0 to $t_0 + M\delta$ covers the desirable time period understudy respectively.

The representation of the CVaR problem for a dynamic network is more complicated than the respective problem applied in a static network, since in this case the arc accident probabilities and consequences must be specified for every time step (M total steps). Due to the fact that shifts in population densities during the day are hard to find, as one can see from the definition of $d_{ij}(t,r) = p_{ij}(t)[c_{ij} - r]^+$, the accident consequences c_{ij} were assumed to be independent from the shipment's entrance time in arc (i,j). That means that arc consequences are fixed throughout the time horizon of interest. Also, because this project addresses the problem of transporting a single shipment through a network, with a unique origin-destination (OD) pair, it is assumed that the accident impact zone is independent from the shipment's hazmat type.

For a path l, the CVaR measure becomes as follows for any departure time t from the origin:

$$\mathsf{CVaR}^l_\alpha(t) = \min_{r \in \mathbb{R}^+} \left(r + \frac{1}{1-\alpha} \sum_{(i,j) \in \mathcal{A}^l} d_{ij}(\theta^l_{ij}(t), r) \right) \tag{8}$$

where $\theta^l_{ij}(t)$ is the time moment at which a truck enters the arc (i,j) when it departed from the origin at time t. We can express:

$$\theta^l_{i_1 j_1}(t) = t \tag{9}$$

$$\theta^l_{i_2 j_2}(t) = t + \tau_{i_1 j_1}(\theta^l_{i_1 j_1}(t)) = t + \tau_{i_1 j_1}(t) \tag{10}$$

$$\theta^l_{i_3 j_3}(t) = t + \tau_{i_2 j_2}(\theta^l_{i_2 j_2}(t)) = t + \tau_{i_2 j_2}(t + \tau_{i_1 j_1}(t)) \tag{11}$$

$$\vdots \tag{12}$$

where (i_k, j_k) represents the k-th arc in path l.

Our problem in a time-dependent network is to seek a path and a departure time that minimizes the CVaR measure of hazmat transportation risk:

$$\min_{l\in\mathcal{P},t\in S} \mathsf{CVaR}^l_\alpha(t) = \min_{l\in\mathcal{P},t\in S}\min_{r\in\mathbb{R}^+}\left(r + \frac{1}{1-\alpha}\sum_{(i,j)\in\mathcal{A}^l} d_{ij}(\theta^l_{ij}(t), r)\right) \tag{13}$$

$$= \min_{r\in\mathbb{R}^+}\Big(r + z_\alpha(r)\Big) \tag{14}$$

where

$$z_\alpha(r) = \min_{l\in\mathcal{P},t\in S}\sum_{(i,j)\in\mathcal{A}^l} d_{ij}(\theta^l_{ij}(t), r) \tag{15}$$

We note that the sub-problem (15) is a minimum cost path finding problem in a time-dependent network with the arc cost $d_{ij}(t,r)$ and the arc travel time $\tau_{ij}(t)$. Therefore, for each r, we can evaluate the function value $z_\alpha(r)$ by solving a dynamic minimum cost path problem.

We also observe that since $d_{ij}(t,r) = p_{ij}(t)[c_{ij} - r]^+$ is linear for each interval $[c_{(i)}, c_{(i+1)}]$ as in the static case. Instead of search for the entire r-space, we can only search a finite number of points to obtain a solution. That is, an optimal solution r^* is found at the following set:

$$r^* \in \{0\} \cup \{c_{ij} : (i,j) \in \mathcal{A}\} \tag{16}$$

Consequently, the optimal dynamic CVaR problem is equivalent to:

$$r^* = \arg\min_{r\in\{0\}\cup\{c_{ij}:(i,j)\in\mathcal{A}\}}\big(r + z_\alpha(r)\big) \tag{17}$$

and the optimal path and optimal departure time are the solution of the dynamic minimum cost path problem $z_\alpha(r^*)$.

In what follows, we discuss how we solve the dynamic minimum cost path problem $z_\alpha(r)$ for each given r. As stated at the beginning of this section, $G = (\mathcal{N}, \mathcal{A})$ is a discrete FIFO dynamic network. Therefore, the FIFO condition must be satisfied for each arc and every time step of the network. Therefore the following set of equations must hold for each arc and every time step:

$$\forall(i,j,t), t+\tau_{ij}(t) \leq (t+1)+\tau_{ij}(t+1) \tag{18}$$

For the purposes of this project a back-labeling algorithm (Ziliaskopoulos and Mahmassani, 1993) was used, that it is based on Bellman's principle of optimality. It calculates for every time step $t \in S$, the time-dependent shortest paths from every node i in the network, to the destination node N. Next, a slightly modified version of the algorithm is presented which effectively and efficiently solves the time-dependent CVaR minimization problem in hazmat transportation.

In order for the algorithm to work, some assumptions had to be made (Ziliaskopoulos and Mahmassani, 1993). First of all, it is assumed that $\tau_{ij}(t) = \tau_{ij}(t+M\delta)$ is constant for all $t > t_0 + M\delta$. This means that the conditions in the understudy transportation network after the peak hour stabilizes. In addition, it is assumed that, $\tau_{ij}(t) = \tau_{ij}(t_0+k\delta)$ $\forall t \in (t_0+k\delta, t_0+(k+1)\delta)$. That is, the risk from traversing arc (i,j) remains the same for any departure time within a time step. Note at this point that the first two assumptions are not restrictive, since the constants δ and M are user defined and therefore can always change. Hence, the user can increase constant M in such a way, that the time interval understudy extends to include periods with variable risk exposition on some of the arcs. Also constant δ can be set to a very small value such that the traffic conditions remain unchanged within a time step.

At each computational step of the algorithm, the total risk of the current shortest path from node i to node N at time t is denoted by $\lambda(t)$. The M-vector label $\Lambda_i = [\lambda_i(t_0), \lambda_i(t_0+\delta), \lambda_i(t_0+2\delta), \ldots, \lambda_i(t_0+M\delta)]$, contains all the labels for every time step t for node i. Using the ordered set of nodes $P_i = \{i = n_1, n_2, \ldots, n_m = n\}$, any label $\lambda_i(t)$ can be identified, from node i to the destination node N. The following formula to define $\lambda_i(t)$ is used:

$$\lambda_i(t) = \begin{cases} \min\limits_{i \neq j}\{\lambda_i(t + \tau_{ij}(t)) + d_{ij}(t,r)\} & \text{, for } i = 1, 2, \ldots, N-1 \text{ and } \forall t \in S \\ 0 & \text{, for } i = N \text{ and } \forall t \in S \end{cases} \tag{19}$$

The proposed algorithm segments the time period understudy into small discrete time intervals δ. It begins from the destination node N, and calculates the optimal routes operating in a backward label-correcting way. In order to avoid scanning all nodes of the network in every iteration, it uses a scan eligible (SE) list, which contains all the nodes of the network that might have at least one label improved. Note that, for the creation, insertion and deletion of a node in the SE list, specific rules (Dial et al., 1979) were followed. Because the algorithm operates in a label-correcting fashion, the label vectors are upper bounds to the shortest paths until the algorithm finds the optimal solution.

We proposed to use the algorithm by Ziliaskopoulos and Mahmassani (1993). We use this algorithm to solve the sub-problems $z_\alpha(r)$ for each value of r.

- **Step 1:** Create the SE list and place in it the destination node N. Let: $\Lambda_N = (0, 0, \ldots, 0)$ and $\Lambda_i = (\infty, \infty, \ldots, \infty)$ for $i = 1, 2, \ldots, N-1$

- **Step 2:** Choose the first node i in the SE list, name it "Current Node", and remove it from the list. If the SE list is empty, go to Step 4. Otherwise, scan the current node i according to the following equation

  $$\lambda_i(t) = \min\left\{\lambda_i(t), d_{iN}(t,r) + \lambda_N(t + d_{iN}(t,r))\right\},\ j \in \Gamma^{-1}\{i\} \text{ for all } t \in S$$

  Namely, for every time step $t \in S$,if $\lambda_j(t)$ is greater than $d_{ji}(t,r) + \lambda_N(t + d_{ji}(t,r))$ replace $\lambda_j(t)$ in the label vector Λ_j at position i with the new value. If any of the M labels of node j has been improved, insert node j in the SE list.

- **Step 3:** Repeat Step 2.

- **Step 4:** Stop; the vector Λ_i $\forall i \in \mathcal{A}$ contain the risk of the proposed routes for each time step $t \in S$.

The following statement is proven to hold (Ziliaskopoulos and Mahmassani, 1993): At the completion of the algorithm, if every element of the vector Λ_N is an infinite number, it indicates that there is no path from this node to the destination node at the corresponding time step; if every element is a finite number, it represents the risk exposed from traversing the shortest path from this node and the corresponding time step represents the time step to the destination node.

The efficiency of the algorithm depends on the total number of scanned nodes before the completion of the algorithm. Obviously, the total number of nodes in the network, $|\mathcal{N}|$, is the lower bound of the total number of nodes scanned, with the upper bound being equal to $|\mathcal{N}|^2 M$.

5 Case Study

To illustrate the findings of this research project, a case study was developed in a portion of Buffalo's, NY transportation network. The network used had 90 nodes, 149 arcs, a unique origin-destination pair and a single hazmat shipment. The time period understudy was from 8am to 11am and the reason for this choice obviously was to capture the morning road congestion. The time period of interest was divided into 5 minute time steps; therefore $\delta = 5$ minutes and in order to cover the 3 hour time horizon, $M = 36$. The colored background in Figure 1 represents the population densities in the Buffalo area in 2010. As shown in the legend, the darker the color the higher the population density in the area. The proposed model was implemented and run in Matlab R2010a on a 3.10GHz intel Core i5-2400 CPU computer system, with computation time less that 2 seconds.

For each arc, we had to calculate the accident consequences and the probability that an accident would occur. The accident probabilities depend on the traffic volume of arc (i, j), and therefore the probabilities of a hazmat accident had to be computed for each time step.

For the computation of the accident probabilities, the Poisson probability generating function was used, with the value of the parameter γ being equal to the product of accident

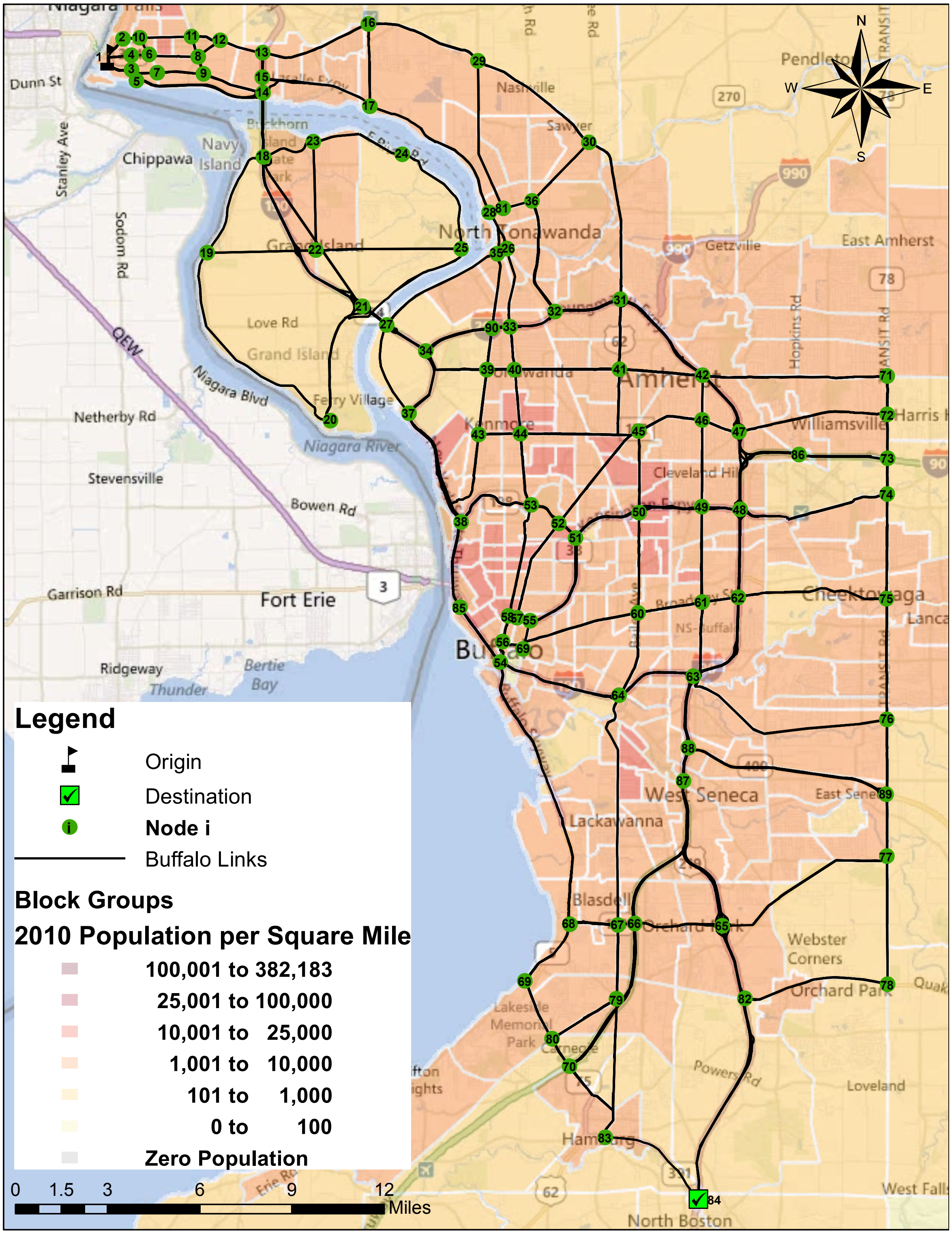

Figure 1: Case Study Network

rate, arc length and traffic volume. The use of the Poisson probability generating function was permissible, because of the assumption, that the number of hazmat accidents follows Poisson distribution with parameter γ.

A challenging problem was the computation of the accident consequences. Three different approaches were tested. The first one uses the λ-neighborhood concept (Batta and Chiu, 1988) with λ being the equal to the hazmat spread radius. The second one uses the λ-neighborhood concept divided by the arc length. The results from these methods are available in Toumazis (2012).

In this study, the accident consequences were computed assuming that the endangered area would be a circle of radius λ as presented in Figure 2. That is,

$$c_{ij} = \pi\lambda^2\rho_{ij} \tag{20}$$

The parameter ρ_{ij}, that is the average population density, has different values along the arc. Instead of calculating the population density for each point of the arc, it is assumed that the ρ_{ij} has fixed value along the length of each arc and equal to the average population density around the arc. That is, the population density inside a circle of radius λ and center point any point on arc (i, j) is equal to the average population density ρ_{ij}.

At this point it should be emphasized that, because the accident consequences are independent from the entrance time in the arc, the c_{ij}'s for each arc (i, j), are the same for every time step. Hence, unlike the computation of the accident probabilities where the probabilities of an accident were computed for each time step, the accident consequences are only computed once.

Note at this point, that for the purposes of this study only the population densities of the areas around the arcs were considered, to compute the accident consequences. However, a case study is briefly described, where the main infrastructures of the Buffalo area like the Buffalo/Niagara International airport and the two bridges connecting Grand Island with Buffalo and Niagara City are also considered. The results from this case study are presented in Section 5.1.

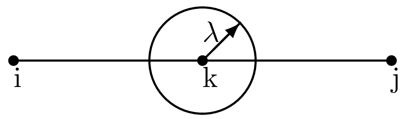

Figure 2: Hazmat Accident Endangered Area Described by a circle of radius λ

Furthermore, assume that the number $\mathcal{E}$ of hazmat accidents, follows Poisson distribution with parameter:

$$\gamma = (\text{hazmat accident rate per mile/vehicle}) \times (\text{arc length}) \times (\text{hourly traffic volume}) \quad (21)$$

Hence,

$$\mathcal{E} \sim \text{Poisson}(\gamma(t)) \quad (22)$$

where hazmat accident rate per mile/vehicle $= 3.19922 \times 10^{-07}$ (Federal Motor Carrier Safety Administration, 2001). This assumption was made after considering the nature of hazmat accidents. It is known that the Poisson distribution is the most appropriate distribution for rare events, and therefore for hazmat accidents as well. In addition, if we examine closely the selection of the parameter $\gamma_{ij}(t)$ we will see that it is nothing else but the expected number of accidents for each link at every time step t. In other words the parameter $\gamma_{ij}(t) = E(\mathcal{E}_{ij})$. Using the latter assumption, allows us to use the Poisson probability generating function to compute the time-dependent accident probabilities for every arc and every time step using the following formula:

$$p_{ij}(t) = 1 - \Pr\{\text{No accident occurs.}\} = 1 - \frac{\gamma(t)^{\epsilon}}{\epsilon!} e^{-\gamma(t)} \quad (23)$$

where $\epsilon = 0$. The expression (23) represents that the probability of one accident or more is

Table 1: Optimal paths given by the CVaR model for various confidence levels α using a circle of radius λ for the computation of the accident consequences

Confidence Level α	Departure Time Time	Optimal CVaR Route
$[0, 0.9938]$	9:55 am	1,3,7,9,14,18,23,24,25,21,27,34,39, 43,44,53,58,56,54,67,69,80,70,83,84
$[0.9939, 0.9941]$	9:55 am	1,3,5,14,18,21,27,34,39,43,44,53,58, 56,54,67,69,80,70,83,84
$[0.9942, 0.9954]$	9:55 am	1,3,5,14,18,21,27,34,39,43,44,53,52, 57,58,56,54,67,69,80,70,83,84
$[0.9955, 0.9992]$	8:00 am	1,2,10,11,12,13,16,29,30,31,42,71,72, 73,74,48,62,75,76,89,77,65,82,84
$[0.9993, 0.9994)$	8:00 am	1,2,10,11,12,13,16,29,30,31,42,47,72, 73,74,48,62,75,76,89,77,65,82,84
$[0.9994, 0.9995]$	8:00 am	1,2,10,11,12,13,16,29,30,31,42,47,48, 62,63,88,87,65,82,84
$[0.9996, 1)$	8:00 am	1,3,5,14,18,21,27,34,39,43,38,85,54, 67,69,80,70,83,84

equal to 1 minus the probability of no accident on arc (i, j).

The model applied in the Buffalo network, results in 7 different optimal routes for the transportation of the hazmat shipment for different α values. The proposed routes and their respective departure times are shown in Table 1. As you can see from this table, the proposed route maintains its optimality for specific confidence level intervals. Also note that the model alters its proposed route, frequently as the confidence level value approaches to one. While the proposed route remains unchanged for confidence level up to 0.9938, beyond that point the proposed route changes repeatedly. This is happening because the accident probabilities are very small, in the range of 10^{-08} to 10^{-06} Abkowitz, M. and Cheng, PD (1988) and the accident consequences of such events are extremely high. Therefore these events in order to be captured by the model, the confidence level value should be very close to 1.

The graphical representation of the proposed optimal path for each one of the confidence intervals shown in Table 1 are provided in Figure 3.

As shown in Toumazis (2012), when the confidence level α value is close to zero, the CVaR model is equivalent to the Traditional Risk (TR) model that minimizes the expected risk. In

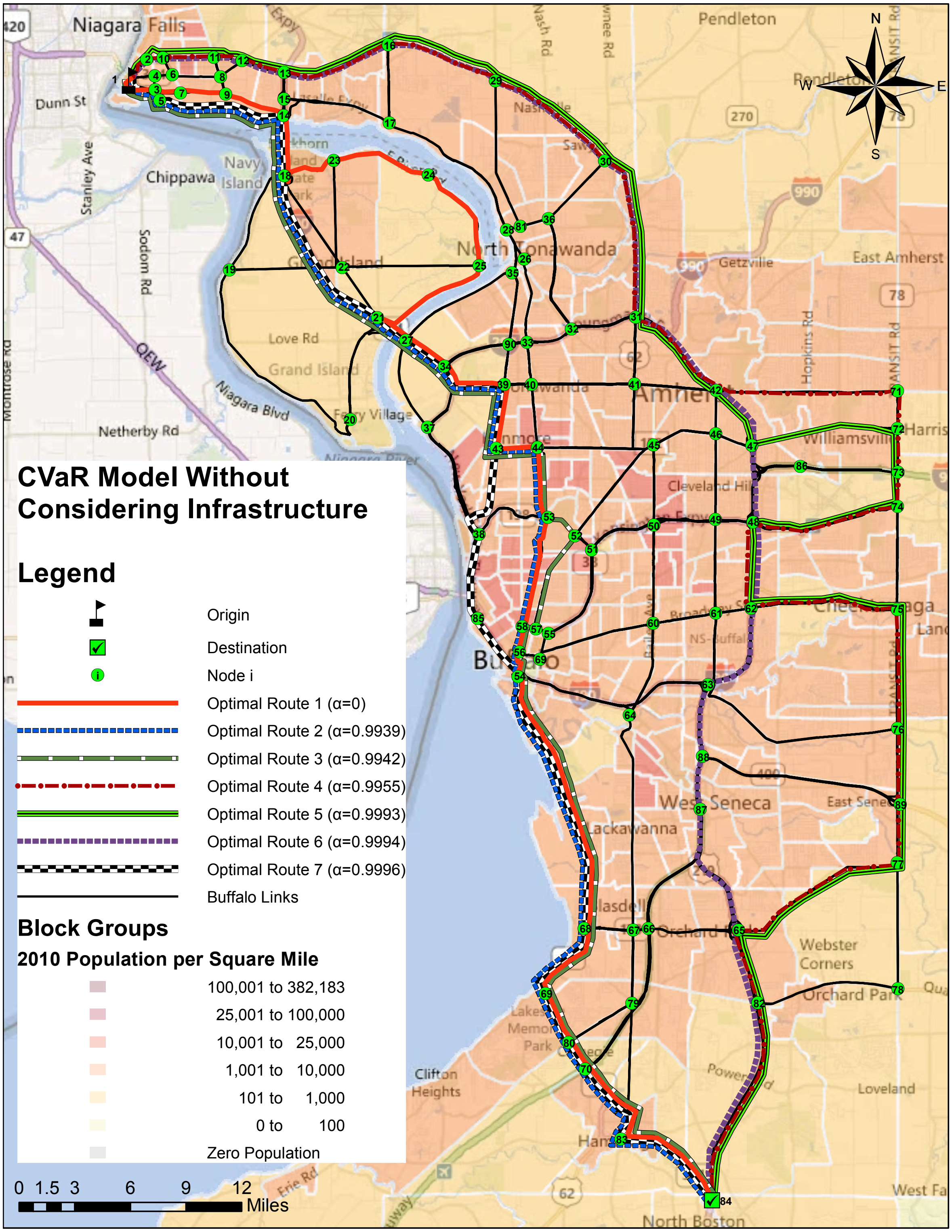

Figure 3: Optimal Routes resulted using a circle of radius λ for the computation of the accident consequences

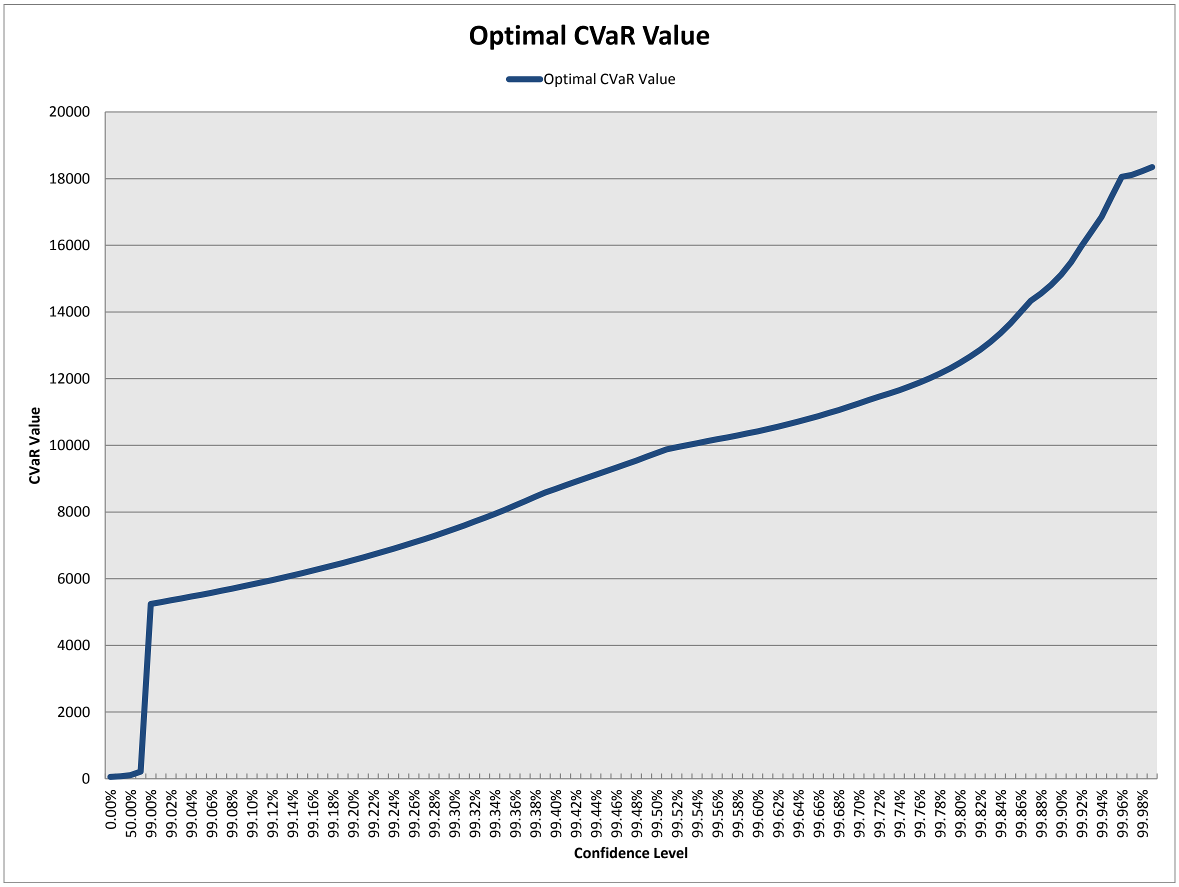

Figure 4: Optimal CVaR value using a circle of radius λ for the computation of the accident consequences

this case study, we found that the optimal route proposed by the CVaR model for confidence level in the interval $[0, 0.9938]$ is same as the one proposed by the TR model.

Since TR model computes the expected value of the risk along a path and manipulates it as a risk measure, the CVaR model in this case has a risk-neutral behavior. The proposed route passes through the highly populated area of downtown Buffalo. The reason for this is that the model fails to capture the extreme events from the risk distribution, since the CVaR measure is located at the mean of the distribution. Note that, because the accident probabilities are very small, the upper bound of the interval, 0.9938 is considered very small in hazmat transportation.

The routes proposed for greater values of the confidence level keeps improving as α approaches one. For confidence level in the interval $[0.9939, 0.9941]$ the model proposes Route

2 and for confidence level in the interval $[0.9942, 0.9954]$ Route 3. Both routes are identical throughout their entire trail, excluding a small segment in the middle. In particular both routes depart from the origin following the waterside road before passing the bridge to Grand Island. Both of them follows the same path through Grand Island and then through Kenmore before the shipment reaches Delaware Park. At that point, Route 2 continues going through Delaware Avenue where Route 3 detours on Scajaquada Expressway up to Main Street. Then it heads South on Main Street until Edward St at which point the shipment is heading west before meeting Route 2 on Delaware Avenue. Then the shipment is going south on Delaware for each one of the above routes, before going on Buffalo Skyway. Then both routes continue south up to Camp Rd. in Athol Springs area. At that point it continues heading east on Camp Rd and then South on Buffalo St. When reaching East Main St the shipment continues east and it finally reaches its destination.

The CVaR model for confidence levels in the intervals $[0.9955, 0.9992]$, $[0.9993, 0.9994)$ and $[0.9994, 0.9995]$ proposes Routes 4, 5 and 6 respectively. All three of these routes begin on Main St in Niagara Falls, heading Northeast up to U.S. 62. Then it follows US-62 all the way until I-290. At that point all three routes continue on I-290 heading southeast. Route 4 exits I-290 following Sheridan Dr, up to Transit Rd and then it continues south. Route 5 continues on I-290 until Main St, on which it then continues going West until Transit St. There it merges with Route 4 continuing south. Both routes then go through Genesse St before continuing on I-90 for a while and then exit on Broadway. When they reach Transit Rd, continue their course heading south. After diverging on US-20, the two routes head south on US-219 before reaching their destination. Route 6 on the other hand, continues southeast on I-290 and then I-90 all the way to the point where it reaches US-219. Beyond that point, it continues south on US-219 until the destination.

Note that the final route proposed in this case by the model, namely Route 7 shown in Figure 3, starts from the origin following the same path as Routes 2 and 3 until it reaches Military Rd on Kenmore. Then instead of leaving Military Rd and continues heading east, Route 7 keeps heading south on Military Rd, and then on I-190. Then, before taking Buffalo

Skyway it continues heading south on I-190. After getting on Buffalo Skyline, Route 7 follows the same path as Routes 1, 2 and 3. Note that Route 7 has its biggest part right next to water. Hence, the risk exposure by following this route is basically half for the section of the route that is next to water, since the population is located only at the one side of the path. Note that because accident consequences were computed considering a circle of radius λ, which was assumed equal to 1 mile, the proposed route appears to be close to highly populated areas. However, the risk from the whole path has the smallest value than any alternative path from the origin to the destination.

Figure 4 demonstrates how the optimal value of the model's risk measure CVaR increases, as the confidence level value increases. The reason for this behavior is because the model captures more information from the risk distribution as the confidence level value increases approaching one. Therefore, since there are more extreme events that are taken under consideration, the risk is increasing. In addition, the value of the optimal CVaR constantly changes, even in cases where the proposed route maintains its optimality. This is because the CVaR model's objective function also depends on the confidence level value as shown in Formula (13).

5.1 Accident Consequences Considering Infrastructure

In the previous section, the only factor considered to affect accident consequences was the areas' population density. However, there are many other factors that must be taken under consideration in order to achieve the safest route. One of these parameters is the infrastructure in the endangered area. For the remaining of this section, the numerical results of a case study considering both population densities and infrastructures for the computation of the accident consequences are presented. The only infrastructures considered, are the two bridges that connect Grand Island with Buffalo and Niagara City, and the Buffalo/Niagara International airport. We also assumed that each main infrastructure is equivalent to a $20,000$ population density. We expressed the infrastructure cost in terms of population density, to use the same unit for the computation of the accident consequences. Note that the number $20,000$ was arbitrary chosen.

The network used for this part of the project was exactly the same as before, with the only difference being the different c_{ij}'s values for arcs $(14, 18)$, $(21, 27)$ which represents the two bridges connecting Grand Island with Niagara City and Buffalo respectively, and arcs $(72, 73)$, $(73, 74)$, $(73, 86)$, $(74, 48)$, $(74, 75)$ and $(86, 48)$ which are the links surrounding the Buffalo/Niagara International airport.

The proposed algorithm, when all the above were taken under consideration, resulted in 7 different paths with their respective departure times. The results are given in Table 2 and the graphical representation of the proposed paths are provided in Figure 5.

Figure 5 demonstrates how the proposed routes avoid the two bridges, i.e. arcs $(14, 18)$ and $(21, 27)$, as well as the arcs near the airport. In the paths proposed by the model described earlier, every route was passing through either the bridges or/and near the airport.

Similarly to the CVaR model without infrastructure, the increase in the optimal CVaR value as the confidence level value increases follows the same pattern as Figure 4. As noted before, the increase in the CVaR value is relatively small when the confidence level values are small, and drastically increases for greater α values.

6 Conclusions and Future Work

This paper extends the newly proposed CVaR model to apply hazmat transportation in time-dependent networks. The objective is to minimize the risk experienced by the hazmat shipment transport in any given transportation network.

The flexibility of the model was authenticated. CVaR model provides the opportunity to the decision makers to retrieve alternative paths for different confidence levels. They can alter the model's approach from risk-neutral, by setting the α-value close to zero, to risk-averse, with α close to one. This flexibility of the CVaR model addressing hazmat transportation is what the existing methods for hazmat transportation lack.

This study suggests that CVaR is a proper risk measure for hazmat route decision making in time-dependent networks. Since the model provides feasible solution relatively fast, it can

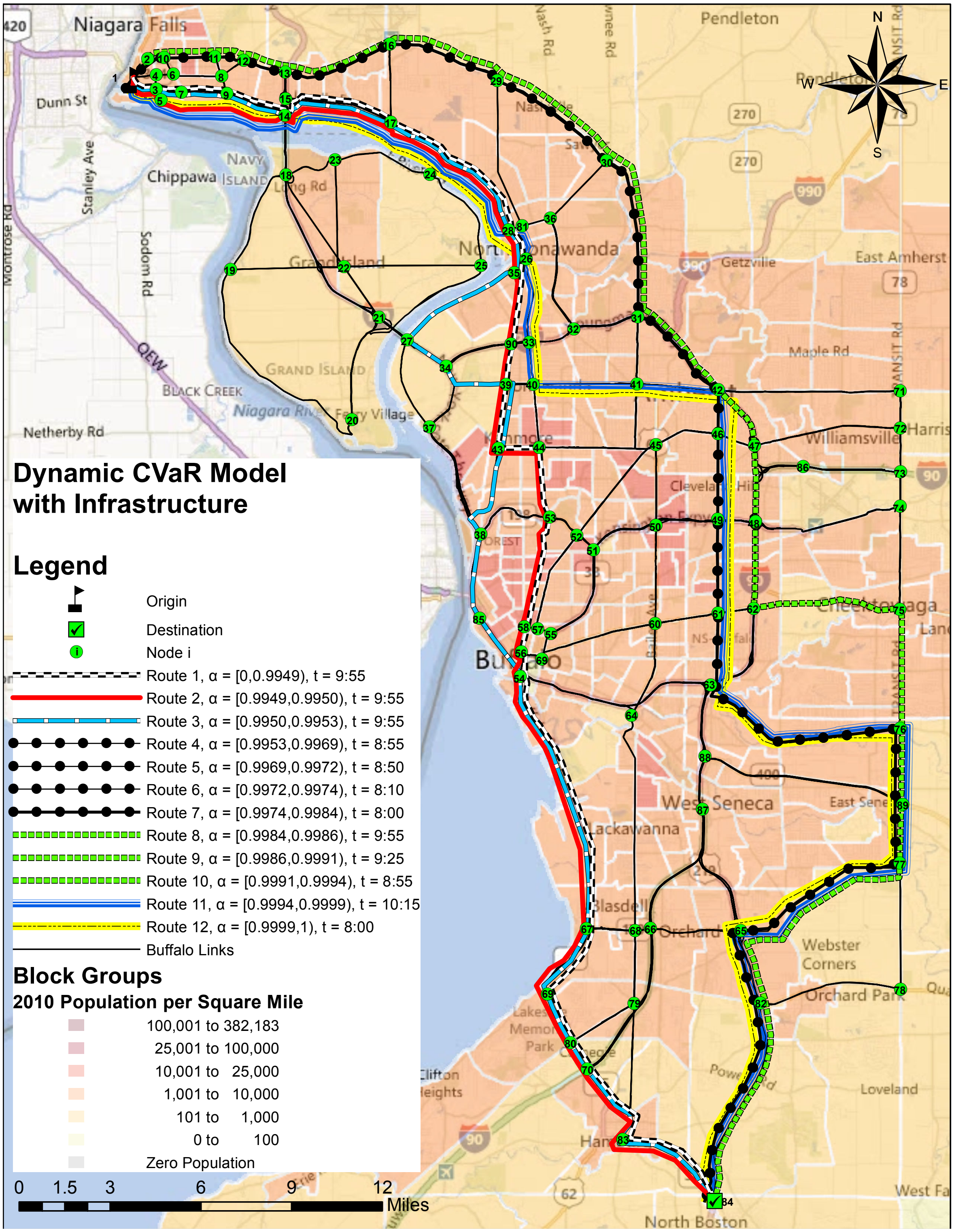

Figure 5: Optimal Routes resulted using a circle of radius λ for the computation of the accident consequences considering Infrastructure

Table 2: Optimal paths given by the CVaR model for various confidence levels α considering Infrastructure

Confidence Level α	Departure Time	Optimal CVaR Route Considering Infrastructure
$[0, 0.9944]$	9:55 am	1,3,7,9,14,17,28,35,90,39,43,44,53, 58,56,54,67,69,80,70,83,84
$[0.9945, 0.9946)$	9:55 am	1,3,5,14,17,28,35,90,39,43,44,53, 52,57,58,56,54,67,69,80,70,83,84
$[0.9946, 0.9967]$	8:00 am	1,2,10,11,12,13,16,29,30,31,42,46, 49,61,62,75,76,89,77,65,82,84
$[0.9968, 0.9978]$	8:00 am	1,2,10,11,12,13,16,29,30,31,42,46, 49,61,76,89,77,65,82,84
$[0.9979, 0.9993]$	8:00 am	1,2,10,11,12,13,16,29,30,31,42,47, 48,62,75,76,89,77,65,82,84
$[0.9994, 0.9995]$	8:00 am	1,2,10,11,12,13,16,29,30,31,42,47, 48,62,63,88,87,65,82,84
$[0.9996, 1)$	8:00 am	1,3,5,14,17,28,35,27,34,39,43,38, 85,54,67,69,80,70,83,84

be utilized for real time hazmat routing decisions. In addition, CVaR model has the potentials to be used for other known low-probability high-consequence events for risk mitigation.

Hazmat routing is very important for public safety. Regulators are trying to find the safest routes for trucks transporting hazmat and working towards that direction they are not allowing hazmat trucks to use some important roads. One of the most common constraints is the restriction of use of some major bridges according to the class of the hazardous material that is transported. In that way, regulators control the routes that can be used for hazmat transportation. Therefore, CVaR model can be used for determining which links of a transportation network will not be allowed for the transport of hazardous materials.

This paper focuses on a network with a unique origin-destination (OD) pair and a single hazmat shipment. It is in our near future plans to extend this proposed model to a network with multiple origin-destination pairs and a variety of hazmat shipments with different types of hazmat. That will obviously affect the accident consequences that would not be the same for each shipment.

In addition, in this research we assumed that the probability of an accident is the same at any point in each arc including intersections (nodes). Clearly, the probability of an accident occurring in an intersection like traffic lights or stop signs is much greater than the probability of an accident occurring in a straight road like the interstates and highways. This improvement will make the model a better representation of a realistic case.

For the implementation of the model, the accident probabilities and accident consequences were assumed to be known. But in reality, this is not the case. Due to the fact that hazmat accidents rarely happen, there are not enough data out from which accurate estimates for these parameters can be calculated. Therefore the computation of the optimal route for hazmat transportation including data uncertainty remains a complicated and challenging issue that needs to be addressed. In a future paper, a model addressing hazmat transportation under data uncertainty would be proposed.

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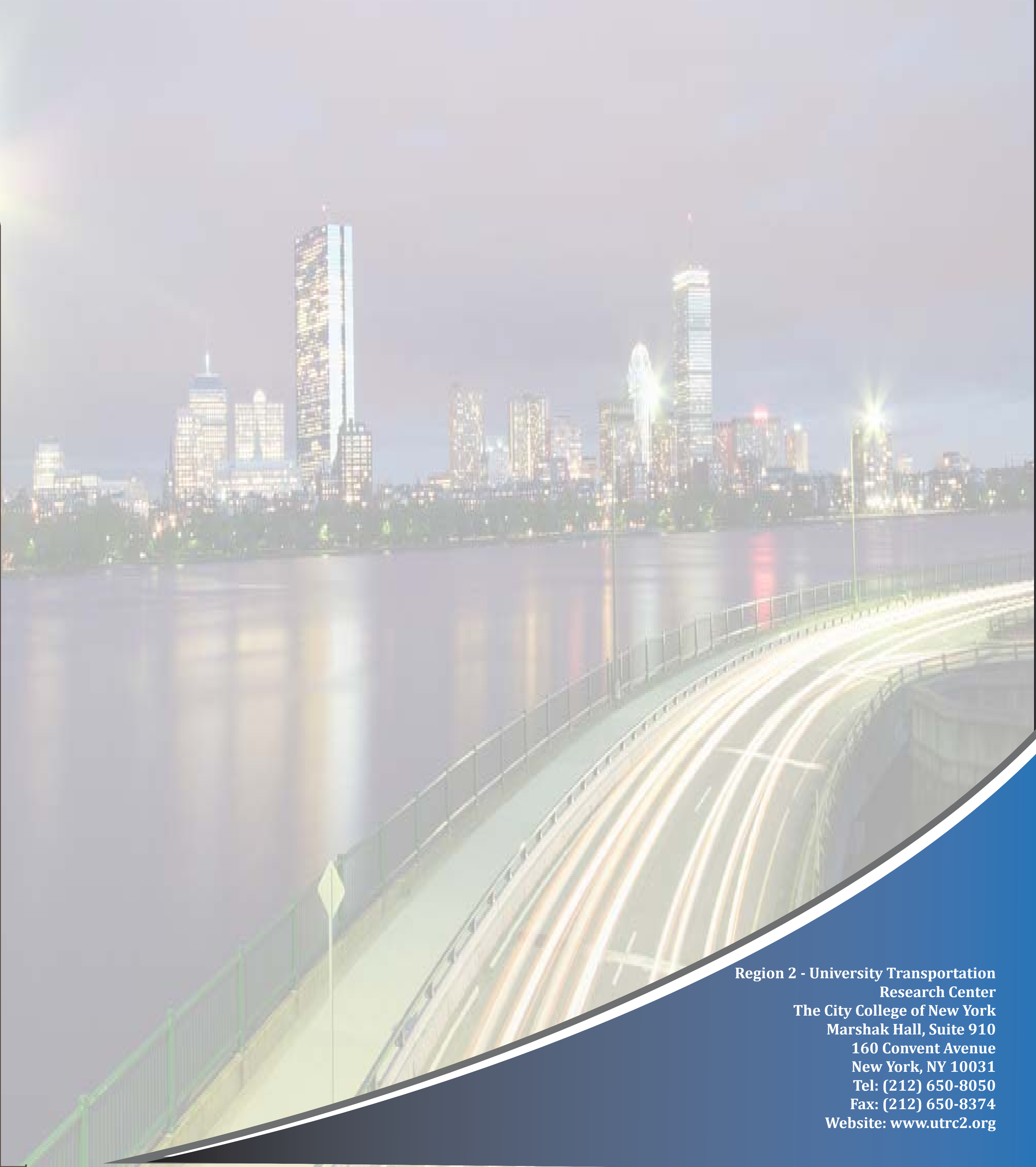

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