



# Mathematical Model

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# Contents

<b>1</b>	<b>Well-to-Pump</b>	<b>1</b>
1.1	Overview . . . . .	1
1.2	Resources . . . . .	2
1.3	Technologies . . . . .	3
1.4	Carbon and Sulfur Balance . . . . .	3
1.5	Input . . . . .	4
1.6	Processes . . . . .	4
1.6.1	Canonical Process . . . . .	5
1.6.2	Stationary Process . . . . .	7
1.6.3	Transportation Mode . . . . .	7
1.6.4	Transportation Process . . . . .	9
1.7	Pathways and Upstream . . . . .	10
1.8	Algorithm . . . . .	10
<b>2</b>	<b>Pump-to-Wheels Calculations</b>	<b>12</b>

# List of Figures

1.1	GREET Pyramid . . . . .	2
1.2	Process Conversion . . . . .	5
1.3	Canonical Process . . . . .	5
1.4	Crude Oil Transportation . . . . .	9

## Notations

$J$  = Joule  
 $g$  = Gram  
 $l$  = Liter  
 $B = (b_1, \dots, b_n)$  = set of  $n$  basic resources defined in the model (coal, NG, oil, uranium ore, wind, hydro, solar, etc.)  
 $AP = (ap_1, \dots, ap_m)$  = set of  $m$  air pollutants defined in the model ( $CO_2$ ,  $VOC$ , etc.)  
 $T$  = set of technologies  
 $M$  = set of transportation modes  
 $f$  = resource  
 $f_o$  = process output  
 $f_p$  = coproduct  
 $\eta$  = energy efficiency [ $J/J$ ]  
 $s$  = share [%]  
 $t$  = technology  
 $m$  = mode of transportation (rail, barge, etc.)  
 $ei$  = energy intensity [ $J/g/m$ ]  
 $d$  = distance [ $m$ ]  
 $I$  = list of all of the process inputs  
 $P$  = list of all coproducts of a process  
 $ef$  = emission factor  
 $G \subset I$  = list of all of inputs in a group (amount or efficiency group)  
 $S \subset G$  = list of all of inputs in a group which have share (amount or efficiency)  
 $a(f)$  = amount of a resource  $f$  [ $J, g, l$ ]  
 $e(f)$  = energy content of a resource  $f$  [ $J$ ]  
 $l_r(f)$  = loss rate associated with resource  $f$

$l_e(f)$  = energy loss of a resource  $f$  [ $J$ ]  
 $hv(f)$  = heating value [ $J/gal$ ] for liquids, [ $J/l$ ] for gases, [ $J/sh\text{tn}$ ] for solids  
 $\rho(f)$  = density [ $g/gal$ ] for liquids, [ $g/l$ ] for gases, does not exist for solids  
 $E = (e(b_1), \dots, e(b_n))$ ,  $b_i \in B$  = energy vector [ $J$ ]  
 $E_{up}(f)$  = energy vector associated with upstream energy to produce  $f$  [ $J/J$ ,  $J/g$ ,  $J/l$ ]  
 $Em = (a(ap_1), \dots, a(ap_m))$ ,  $ap_i \in AP$  = emissions vector [ $g$ ]  
 $Em_f = (ef(ap_1), \dots, ef(ap_m))$ ,  $ap_i \in AP$  = vector of emission factors [ $g/J$ ]  
 $Em_{up}(f)$  = energy vector associated with emissions to produce  $f$  [ $g/J$ ,  $g/g$ ,  $g/l$ ]  
 $Em_s = (s(ap_1), \dots, s(ap_m))$  = vector of shares for each air pollutant  
 $Enem = (E, Em)$  = energy-emissions vector  
 $Enem_{up}(f) = (E_{up}(f), Em_{up}(f))$  = energy-emissions upstream vector  
 $\|E\| = e(b_1) + e(b_2) + \dots + e(b_n)$  = total energy [ $J$ ]  
 $e(I) = \|E(I)\| = \left\| \sum_{f \in I} a(f) E_{up}(f) \right\|$  = process input energy [ $J$ ]  
 $e(G) = \|E(G)\| = \left\| \sum_{f \in G} a(f) E_{up}(f) \right\|$  = group energy [ $J$ ]  
 $\hat{e}(I) = \sum_{f \in I} e(f)$  = process input energy without upstream (energy used by the process) [ $J$ ]  
 $\hat{e}(G) = \sum_{f \in G} e(f)$  = group energy without upstream [ $J$ ]

# Chapter 1

## Well-to-Pump

### 1.1 Overview

In this document we describe the mathematical model used in GREET.

The input-output technique is used in GREET to account for energy consumption and emissions associated with a fuel production pathway. Each *pathway* is a sequence of *processes* (stationary and transportation). Each process has one or more *inputs* and a single *output*. Besides the main output, a process might produce *coproducts*. Every input, output, and coproduct is a *resource*. There might be a *loss* (leakage, boil off, etc.) associated with any of the inputs or outputs of a process. Within a process, inputs might be associated with *technologies*. A technology is an abstraction of a combustion or a chemical reaction process that produces emissions.

The GREET pyramid shown in Figure 1.1 shows the flow of the calculations in the model. We account for all of the resources and technologies used in the processes of a pathway and then combine them to calculate the energy and emissions associated with each pathway. Each pathway has a single main product. The calculated energy and emissions of a pathway are used as upstream values for the corresponding product when it is used as an input to any process within the model. Iterative calculations are used to resolve the circular references.

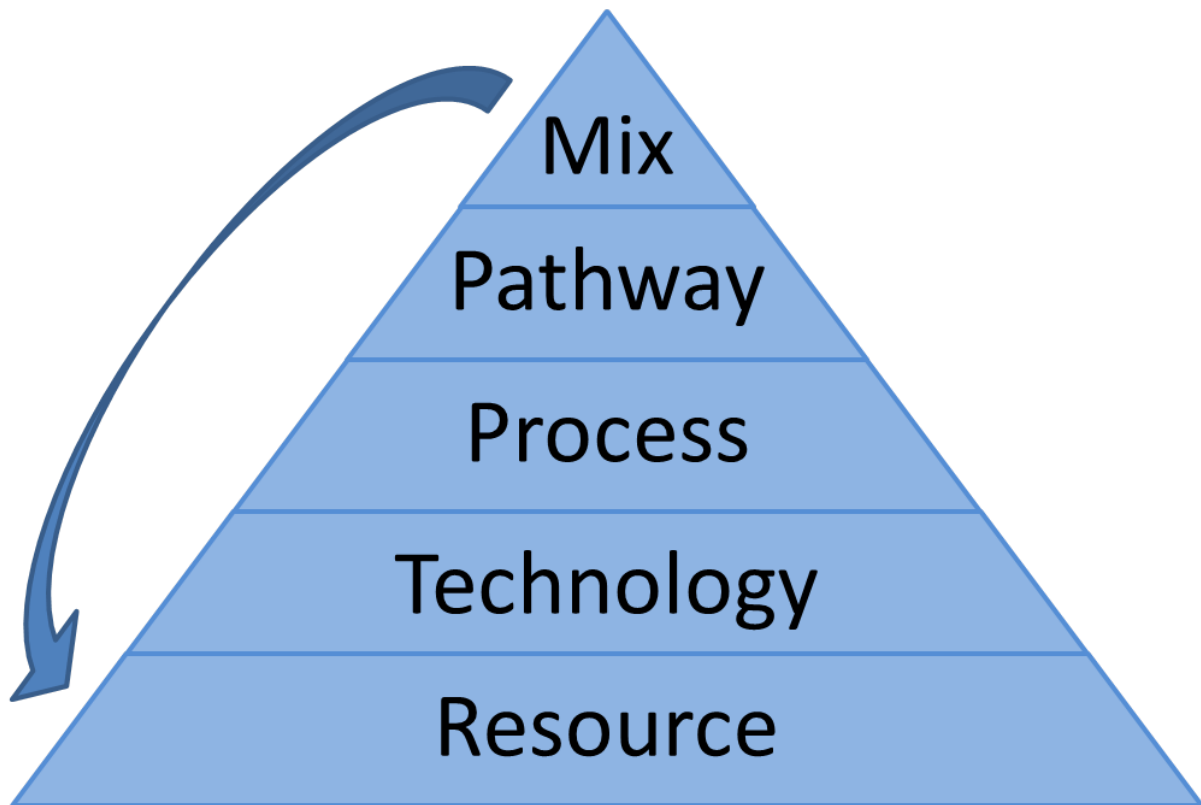


Figure 1.1: GREET Pyramid

## 1.2 Resources

Resources are inputs and outputs of the GREET processes. Resources are organized in groups. Currently there are 11 groups within GREET.

- |                     |                    |                           |
|---------------------|--------------------|---------------------------|
| 1. Petroleum Fuel   | 5. Renewable       | 9. Fertilizer             |
| 2. Natural Gas Fuel | 6. Biomass         | 10. Pesticide             |
| 3. Coal Fuel        | 7. Nuclear         | 11. Renewable Natural Gas |
| 4. Fossil Fuel      | 8. Non-Fossil Fuel |                           |

Each resource can be a member of one or more groups. You can think of groups as categories to which resources belong.

Below is the list of physical properties which can be defined for each of the resources:

- |                                     |   |
|-------------------------------------|---|
| 1. Carbon Ratio                     | 5. Low-Heating Value ( $lhv(f)$ )       |
| 2. Sulfur Ratio                     | 6. State of Matter (solid, liquid, gas) |
| 3. Density ( $\rho(f)$ )            | 7. Market Value                         |
| 4. High-Heating Value ( $h hv(f)$ ) |   |

Additionally, the user can define the following properties for a resource:

- |                                 |   |
|---------------------------------|---|
| 1. Name                         | 4. Is it a Prime Resource? (i.e., a natural resource, such as oil, natural gas, coal, wind) |
| 2. List of Compatible Resources |   |
| 3. Group Memberships            | 5. Gases Created during Evaporation   |

**Carbon and sulfur ratios** need to be specified for combustible fuels. They are used to calculate the  $CO_2$  and  $SO_x$  emissions that are associated with the combustion of the corresponding fuel. The **density** and **heating value** are used to convert between mass, energy, and volume as well as to calculate the allocation ratios among the output and coproducts. The **market value** may be used for the allocation ratio calculation, as well. As discussed later in the manual, all of these properties are to be used for calculating and reporting results. A **primary resource** is one which has no upstream (energy and emissions) associated with it. In other words, it is a resource which exists in nature (crude oil, coal, uranium ore, etc.). If there are pollutant emissions associated with material losses, such as vaporization, the **gases created during the evaporation** vector specify to what pollutants they need to be attributed.

### 1.3 Technologies

Technology is an abstraction used to model emissions as a result of a combustion process or a chemical reaction. Each technology is defined by the fuel used and by the emission factors ( $Em_f(f, t)$ ). The units of measure for emission factors are  $g/J$  — grams of a pollutant per Joule of fuel used.

- |           |           |           |
|-----------|-----------|-----------|
| 1. VOC    | 4. PM10   | 7. $CH_4$ |
| 2. CO     | 5. PM2.5  | 8. $CO_2$ |
| 3. $NO_x$ | 6. $SO_x$ | 9. $N_2O$ |

The emission factors for each of the pollutants can be specified as a time series, i.e., different values can be specified for different years. Thus, emission factors for a technology is a table, in which rows are pollutants and columns are years.

**NOTE:**  $SO_x$  and  $CO_2$  emission factors do not have to be specified as model inputs. Emission factors for  $SO_x$  and  $CO_2$  can be calculated using the balancing equations. For details, see Section 1.4. If the user chooses to specify  $SO_x$  and  $CO_2$ , then balancing will not be performed.

### 1.4 Carbon and Sulfur Balance

If the  $SO_x$  emission factor is not specified for a technology, it needs to be calculated. Here is the formula for calculating the  $SO_x$  emission factor when liquid or gaseous fuel is used

$$ef(f, SO_x) = \frac{\rho(f)}{hv(f)} \frac{sr(f)}{sr(SO_2)} \quad (1.1)$$

Similarly, for a solid fuel

$$ef(f, SO_x) = \frac{1}{hv(f)} \frac{sr(f)}{sr(SO_2)} \quad (1.2)$$

Here,  $sr(f)$  is the sulfur ratio for the process fuel and  $sr(SO_2)$  is the sulfur ratio of  $SO_2$ . The ratio is in parts per million (ppm) by weight.

Here is an equivalent formula for calculating the  $CO_2$  emission factor, when liquid or gaseous fuel is used

$$ef(CO_2) = \frac{1}{cr(CO_2)} \left[ \frac{\rho(f)cr(f)}{hv(f)} - (ef(VOC)cr(VOC) + ef(CO)cr(CO) + ef(CH_4)cr(CH_4)) \right] \quad (1.3)$$

Similarly, for a solid fuel

$$ef(f, CO_2) = \frac{1}{cr(CO_2)} \left[ \frac{cr(f)}{hv(f)} - (ef(f, VOC)cr(VOC) + ef(f, CO)cr(CO) + ef(f, CH_4)cr(CH_4)) \right] \quad (1.4)$$

Here,  $cr(f)$  is the carbon ratio for the process fuel and  $cr(VOC)$ ,  $cr(CO)$  and  $cr(CH_4)$  are the carbon ratios of the corresponding pollutants.

**NOTE:** When balancing is applied, it is assumed that all of the sulfur and carbon content of the process fuel is to be attributed to emission gases, i.e., 100% of the sulfur and carbon in the fuel will be emitted into the air. Thus, it will be constant for all of the technologies associated with a particular fuel.

## 1.5 Input

As mentioned above, an input-output model is used to model each stage (process) within a fuel production pathway. In this section, we give a description of an input object. Here is the list of attributes that need to be defined for an input.

- |                        |                                     |   |
|------------------------|-------------------------------------|---|
| 1. Name                | 4. Pathway Mix Reference            | 6. Sequestration Boolean Flag                 |
| 2. Amount ( $a(f)$ )   | 5. List of Technologies with Shares | 7. Sequestration Rate and Energy Requirements |
| 3. Source of the Input |                                     |   |

**Source of the input:** This attribute specifies how the upstream values need to be calculated for the input. We distinguish four different sources: Well, Pathway Mix, Previous (i.e., output of preceding process), and Pathway Feed. If Well is specified as a source, there is no upstream associated with the input. If Upstream is specified, it means that this input is a product of one or more pathways defined in the model. Previous means that the input is the output of the previous process on the pathway. The user can define a feed for a pathway that is associated with a Pathway Mix. Then, a process that is a part of a pathway for which feed is defined can have Pathway Feed to be specified as a Source. It indicates that the source for this input is the Pathway Mix, specified for the Pathway Feed.

**Pathway Mix reference:** In cases when the Source is Pathway Mix, the energy and emissions values of the corresponding Pathway Mix are used as upstream values for producing the resource. The Pathway Mix objects will be explained in more detail in Section 1.7.

The energy associated with producing an input  $f$  is calculated as

$$E(f) = a(f)E_{up}(f) \quad (1.5)$$

Emissions are calculated as follows:

$$\begin{aligned} Em(f) &= a(f)Em_{up}(f) + a(f) \sum_{t \in T} s(f, t)Ef(f, t) + Em_{other} \\ &= a(f) \left( Em_{up}(f) + \sum_{t \in T} s(f, t)Ef(f, t) \right) + Em_{other} \end{aligned} \quad (1.6)$$

Thus, the emissions are a sum of the upstream emissions, technology emissions, and non-technology related emissions, i.e., emissions associated with leakage or boil off of the input.

## 1.6 Processes

A process is the major building block in the model. Most of the calculations take place at the process level. We distinguish two types of processes in the model — *stationary* and *transportation* (see Figure 1.2). For calculation purposes, both of them are converted to a canonical input-output



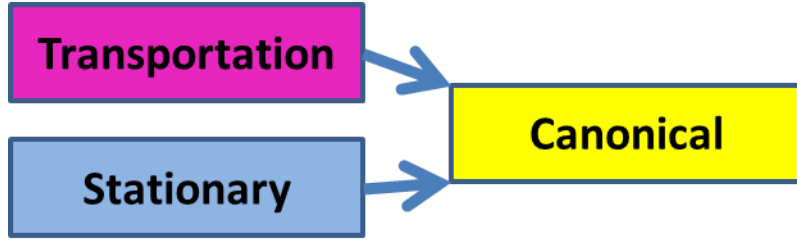


Figure 1.2: Process Conversion

representation. In the following subsections, we give a description of the canonical form and calculations performed on the canonical, as well as the conversion process form of each of two types of processes to the canonical form.

### 1.6.1 Canonical Process

The Canonical Process has a very simple input-output form as shown in Figure 1.3. There are five

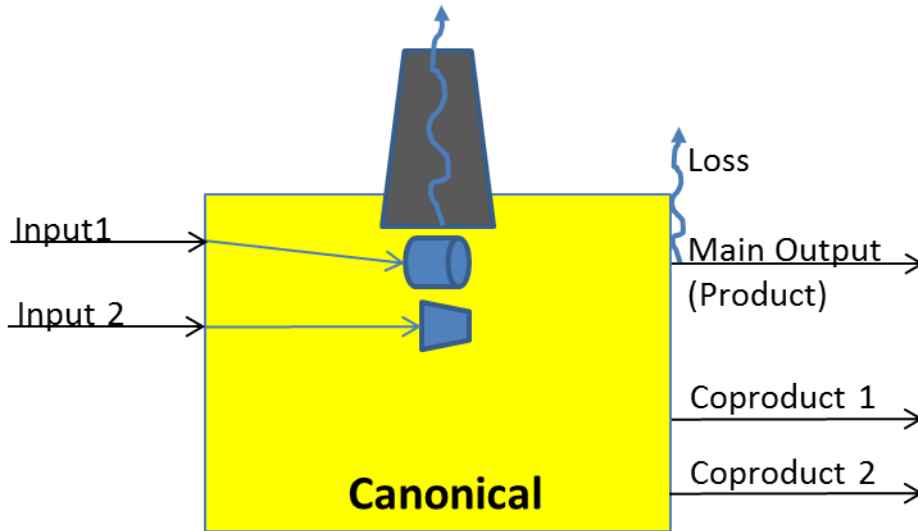


Figure 1.3: Canonical Process

components which define how energy and emissions associated with a process are accounted. Those are inputs, outputs, coproducts, and additional emissions. Below is the formula used to calculate the energy balance vector (per unit output) associated with a process.

$$E_b = \frac{E(I) - E(P)}{a(f_o)(1 - l_r(f_o))} \quad (1.7)$$

**NOTE:** The numerator of this ratio is a vector and the denominator is a scalar.

Here

$$E(I) = \sum_{f \in I} E(f) \quad (1.8)$$

and  $E(f)$  as in Equation (1.5).

$l_r(f_o)$  is the loss rate defined for the main output,  $0 \leq l_r < 1$ . The user is allowed to specify  $l_r \geq 1$ . However, the calculated results would be **meaningless**.

Similarly,

$$Em_b = \frac{Em(I) - Em(P) + Em_{other}}{a(f_o)(1 - l_r(f_o))} \quad (1.9)$$

$Em_{other}$  are user specified emissions, usually associated with losses.

### Calculating $E(P)$ and $Em(P)$

A coproduct's energy and emission burden can be accounted in two different ways, using either allocation or displacement method.

$$E(P) = \sum_{f_p \in P} E(f_p) \quad (1.10)$$

$$Em(P) = \sum_{f_p \in P} Em(f_p) \quad (1.11)$$

$E(f_p)$ , depends on what treatment method is used.

### Allocation

$$E(f_p) = (1 - r_e)E(I)$$

$$Em(f_p) = (1 - r_{em})Em(I)$$

Here,  $r_e$  and  $r_{em}$  are the ratios of energy and emissions correspondingly to be allocated to the production of the main output. Depending on the physical properties defined for the process output and the coproduct, the following allocation methods can be used.

- Energy
- Mass
- Market Value
- Volume

Here is how the  $r$  is calculated for energy allocation

$$r_e = r_{em} = \frac{e(f_o)}{e(f_p) + e(f_o)}. \quad (1.12)$$

When a non-energy allocation is used, we define an intermediate share  $s$ , based on the amount of main output and coproduct as

$$s = \frac{a(f_o)}{a(f_o) + a(f_p)}. \quad (1.13)$$

**NOTE:** The amount of main product and coproduct dimensions must be the same!

Then, the  $r_{em} = s$  and the energy ratio  $r_e$  are the solution of the following equation

$$(e(I) - e(f_o) - e(f_p))s + e(f_o) = r_e e(I) \quad (1.14)$$

If we simplify the above equation, we get

$$r_e = s - \frac{e(f_o)}{e(I)} \left( \frac{s}{s_e} - 1 \right), \quad (1.15)$$

Here

$$s_e = \frac{e(f_o)}{e(f_p) + e(f_o)} \quad (1.16)$$

### Displacement

Displacement assumes that a conventional product  $f_c$  is displaced by a process coproduct. It means that at least one pathway for  $f_c$  needs to be defined in the model. The upstream of the  $f_c$  is used to calculate  $E(f_p)$  and  $Em(f_p)$  as follows.

$$E(f_p) = a(f_c)E_{up}(f_c) \quad (1.17)$$

$$Em(f_p) = a(f_c)Em_{up}(f_c) \quad (1.18)$$

Here  $a(f_c) = a(f_p)r(f_p, f_c)$ . The  $r(f_p, f_c)$  is a displacement ratio, which is specified in the model for each coproduct.

### 1.6.2 Stationary Process

Some of the stationary processes are modeled using energy efficiency. We denote it by  $\eta$ .

$$\eta = \frac{a(f_o)}{\sum_{f_i \in G} a(f_i)} \left[ \frac{J}{J} \right]$$

$$\frac{1}{\eta} = \frac{\sum_{f_i \in G} a(f_i)}{a(f_o)}$$

**NOTE:** It is assumed that input losses are accounted in the efficiency and output losses are not.

A stationary process has a very similar structure to a canonical process, with one exception. A stationary process might have a group object. A group object is used to specify inputs in a slightly different way. Each group has either an amount or an efficiency attribute. The efficiency attribute can be converted into an amount.

$$a(G) = \frac{a(f_o)}{\eta} = \sum_{f_i \in G} a(f_i) \quad (1.19)$$

The group amount is distributed among the inputs within the group in the following way. The set of inputs within the group for which the amount is defined is denoted by  $G_1$

$$\hat{a}(G) = a(G) - \sum_{f \in G_1} a(f) \quad (1.20)$$

and

$$a(I) = a(G) + \sum_{f \in I-G} a(f_i) \quad (1.21)$$

The rest of the input amounts are defined by shares, i.e., an input has a share instead of having an amount attribute. The amount for those inputs is calculated as

$$a(f) = s(f)\hat{a}(G), \text{ for } f \in G - G_1 \quad (1.22)$$

Once an amount attribute for each input in a group is calculated, a stationary process conforms to the canonical process form.

### 1.6.3 Transportation Mode

Before we discuss the transportation process model, we describe a model of a transportation mode, which is a building block for every transportation process. Each transportation mode is defined by three parameters:

1. Energy intensity  $\left[ \frac{J}{kg \times m} \right]$
2. Process fuel (fuel used to propel an engine)
3. Emissions factors for each process fuel

A user can define more than one process fuel for a transportation step. Each process fuel then needs to be assigned a share. For example, there are four process fuels defined for the Pipeline mode in the model:

- |                       |                      |
|-----------------------|----------------------|
| 1. Diesel - 20%       | 3. Natural Gas - 24% |
| 2. Residual Oil - 50% | 4. Electricity - 6%  |

In other words, it means that on average, in 20% of the cases, a diesel engine is used to create a pressure in the pipeline; in 50% of the cases it creates residual oil, etc. It also means that different types of engines (technologies) are used for different types of process fuels and emission factors must be defined in the models for each of the engine types.

Energy intensity is a measure of efficiency of a transportation mode, given the engine type and the material transported. There are five modes of transportation that are supported in the default distribution of GREET:

- |                 |             |
|-----------------|-------------|
| 1. Ocean Tanker | 4. Pipeline |
| 2. Barge        | 5. Rail     |
| 3. Truck        |             |

The energy intensity (*ei*) is an input for **Rail** and **Pipeline** and it is a calculated value for **Ocean Tanker**, **Barge**, and **Truck**. Below, the formulas used for energy intensity calculations are given.

$$ei(f_t, f) = \frac{ec(f)hp(f_t) \times LoadFactor}{Payload(f_t) \times Speed}$$

$ec(f)$  = Energy Consumption  $\left[ \frac{J}{hph} \right]$   
 $hp$  = horse power required  $[hp]$   
 $f_t$  = resource transported  
 $f$  = process fuel  
 $LoadFactor$  = scaling factor ( $\leq 1$ ), constant  
 $Speed$  = average travel speed, constant  $\left[ \frac{km}{h} \right]$   
 $[hph]$  is a horsepower hour (unit of energy) and equals to  $2.685 \times 10^6 [J]$  or  $0.7457 [kWh]$   
 $hp$  is calculated as follows:

For the **Ocean Tanker**:

$$hp(f_t) = 9070[hp] + 0.101 \left[ \frac{hp}{g} \right] Payload(f_t)[g]$$

For the **Barge**:

$$hp(f_t) = \frac{5600}{22500} \left[ \frac{hp}{g} \right] Payload(f_t)[g]$$

To calculate  $ec$  - energy consumption the following formulas are used:

**Ocean Tanker:**

$$ec(f) = 0.7457 \left[ \frac{Wh}{hph} \right] \left( \frac{14.42 \left[ \frac{g}{Wh} \right]}{LoadFactor} + 150 \left[ \frac{g}{Wh} \right] \right) \frac{hv(f)}{\rho(f)} \left[ \frac{J}{g} \right]$$

**Barge:**

$$ec(f) = 0.7457 \left( \frac{14.42}{LoadFactor} + 350 \right) \frac{hv(f)}{\rho(f)}$$

**Truck:**

$$ei(f_t, f) = \frac{ec(f) \left[ \frac{J}{km} \right]}{Payload(f_t)[g]}$$

$$ec(f) = \frac{hv(f) \left[ \frac{J}{l, g} \right]}{FuelEconomy \left[ \frac{km}{l, g} \right]}$$

For **Pipeline** and **Rail** the  $ei$  is an input and it is not a calculated value.

**NOTE:** In the default distribution of the GREET model, the **Pipeline**  $ei$  depends on what type of material is transported (liquid, solid, etc.). For **Rail**, the  $ei$  is the same for any transported material.

Energy intensity is a measure of energy required to transport one gram of material for one kilometer. To model emissions produced by using a specific transportation mode, there is an emission factor ( $Em_f$ ) defined for each mode and each mode technology (engine type).

### 1.6.4 Transportation Process

In this section, we describe a model of a transportation process and describe how it can be converted to a canonical form. This transportation process is defined by transportation steps. Each step is defined by a transportation mode (Pipe, Truck, Rail, Barge, or Ocean Tanker), distance, and possibly by a shared mode; if several modes can be used to transport to the same destination.

Figure 1.4 is an example of a transportation process for Crude Oil. This graph can be interpreted as

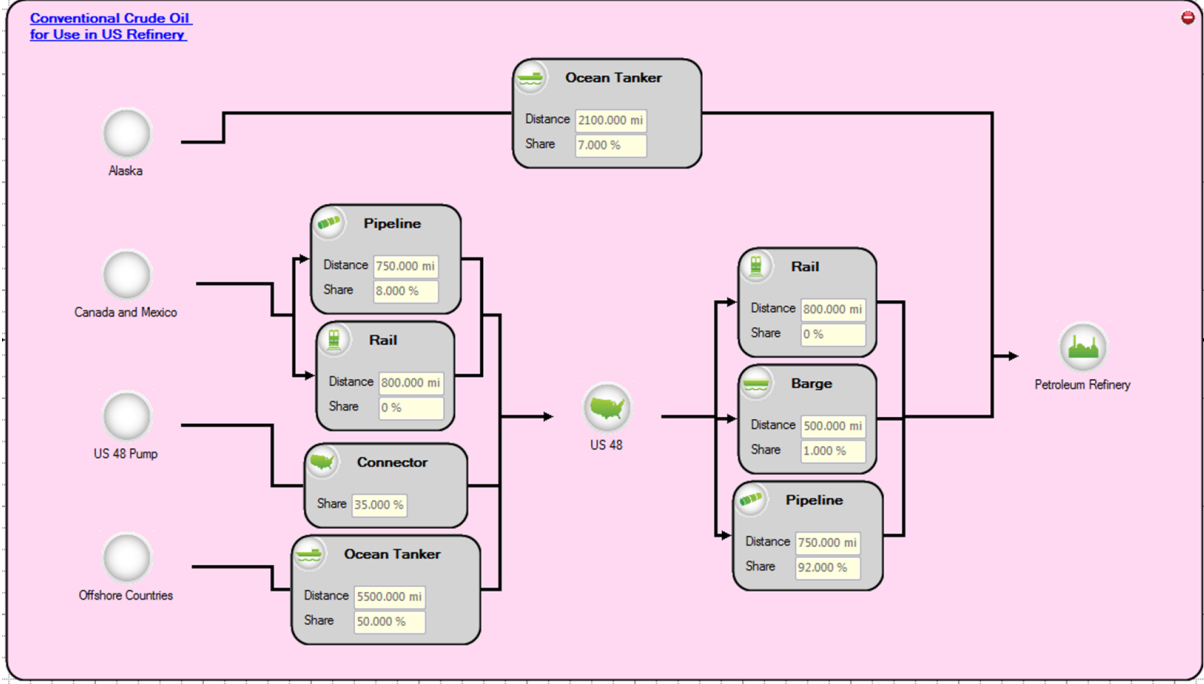


Figure 1.4: Crude Oil Transportation

follows. There are a few ways Crude Oil can be transported to a refinery. On average, 7% of Crude Oil is transported from Alaska by Ocean Tankers, the rest (93%) is transported from a US Terminal (US 48). The transportation between the terminal and refinery is done by mostly Pipeline (92%) and Barge (1%). To get to a US Terminal, oil is transported from Canada and Mexico (8%), from within the US (35%), and the rest comes from Offshore Countries (50%).

Energy intensity ( $ei$ ) for each mode is used to calculate the amount of energy consumed by the corresponding mode in a step.  $ei$  is either an input to a model or a calculated parameter. In the Subsection 1.6.3 we describe how  $eis$  are calculated for different transportation modes.

**NOTE:** Energy intensity depends on the fuels used to propel the engine and the resource transported.

To convert a transportation process to a canonical form, we need to convert each step into an input and technology associated with it. Below we show a conversion process for each individual step. Say, the step has distance  $d$ , share  $s$ , and a mode that has two technologies and two corresponding process fuels with shares  $t_1$ ,  $f_1$ ,  $s(t_1, f_1)$  and  $t_2$ ,  $f_2$ ,  $s(t_2, f_2)$ . This step will be converted into two inputs of a canonical process  $f_1$  and  $f_2$  and each input will have one technology associated with it,  $t_1$  and  $t_2$ , correspondingly. The energy requirement is calculated using the  $ei$  of the mode

$$e(f_1) = d \times ei(f_t, f_1) \times s \times s(f_1, t_1) \times l_r(f_t) \times b$$

$$e(f_2) = d \times ei(f_t, f_2) \times s \times s(f_2, t_2) \times l_r(f_t) \times b$$

We convert each step into canonical process inputs. Then, we add fuel transported  $f_t$  as input and output to the canonical process. The source of  $f_t$  is always "Previous."

In the equation above  $l_r(f_t)$  is the loss rate specified by the user. The user can specify the rate using the following dimensions

- dimensionless ( $b = 1$ )
- $1/m$  ( $b = d$ )
- $1/(sec \times m)$  ( $b = d \times time$ )

Here  $time = d/speed$  is the transportation step duration. If there is no  $speed$  defined for the mode, then  $time = 1\ sec$  by default.

## 1.7 Pathways and Upstream

Pathway is nothing but a sequence of processes. The output of the last process of a pathway is the pathway output. A pathway output is called a product. Thus, the *upstream* of a product is nothing but energy and emission balance values which are calculated as shown in equations (1.7) and (1.9), correspondingly.

However, there might be more than one pathway associated with the same product. The *upstream* object is used to combine several pathways for a given product. Each pathway is assigned a share. All of the shares within the *upstream* sum up to 100%.

## 1.8 Algorithm

Below is the pseudo code that outlines how energy and emission results are calculated for each of the pathways.

**Input:**  $max\_it \geq 0$

**Input:**  $tol \geq 0$

```

while  $diff \geq tol$  or  $it \leq max\_it$  do
     $diff = 0$ 
    for each resource  $res$  do
        for each pathway  $path$  that has  $res$  as output do
            if  $path$  has  $feed$  then
                 $Enem_{prev} = Enem_{up}(feed)$ 
            else
                 $EM_{prev} = 0$ 
            end if
            for each  $proc$  in the  $path$  do
                 $E_b = 0, Em_b = 0, Enem = (E_b, Em_b)$ 
                for each input  $f$  in  $proc$  do
                    if source of  $f$  is previous then
                         $Enem_{up}(f) = Enem_{prev}$ 
                    else if source of  $f$  is well then
                         $Enem(f) = e(f)v(f)$ 
                    else if source of  $f$  is Pathway Mix then
                         $Enem_{up}(f) = Enem_{up}(mix)$ 
                    else if source of  $f$  is Pathway then
                         $Enem_{up}(f) = Enem_{up}(pathway)$ 
                    end if
                    calculate  $E(f)$  and  $Em(f)$  as shown in 1.5 and 1.6, correspondingly
                end for
                calculate  $E_b$  and  $Em_b$  as shown in 1.7 and 1.9, correspondingly
            end for
             $Enem_{prev}(path) = Enem(path)$ 
             $Enem(path) = (E_b, Em_b)$ , energy and emission balance of the last process of the pathway
             $diff = diff + (Enem_{prev}(path) - Enem(path))^2$ 
        end for
    end for
     $it+ = 1$ 

```

**end while**

The vector  $v(f)$  can be defined for a basic resource as follows it has zeros in all positions except for the position that corresponds to index of  $f$  in the vector fo basic fuels.  $v(f)$  has one in this position.

$$v(f) = (0, 0 \dots 0, 1, 0 \dots, 0)$$

## Chapter 2

# Pump-to-Wheels Calculations

There are two types of base fuels for vehicles in GREET — gasoline (spark-ignition engine) and diesel (compression-ignition engine). For the gasoline and diesel vehicles, there are given  $fe$  (fuel economy) and emission factors as model inputs. For vehicles using non-base fuels, the  $fe$  is given as a scaling factor to the gasoline vehicle  $fe$  and emission factors are given as a scaling factor for the gasoline or diesel vehicle, depending on the engine ignition type. Thus, if a calculation is performed for a non-baseline vehicle, first the  $fe$  and emission factors need to be scaled. Once the  $fe$  and emission factors are calculated, here is how the energy and emission factors associated with the vehicle usage can be calculated. The general formula for calculating *vehicle operation energy* per unit of distance traveled is

$$e_{op} = \frac{\sum_{f \in F} s(f) hv(f)}{fe} \left[ \frac{J}{km} \right]$$

$F$  = fuels in the blend

$s(f)$  = volumetric fuel share.

The formula assumes that the vehicle fuel is a blend and no electricity is used to propel the engine.

When electricity is used to propel the engine, there are two operational modes — charge depletion (CD) and charge sustaining (CS). Below are the formulas used in the GREET model to calculate energy use associated with each of the modes.

### Energy use in CD (charge depletion) mode:

Electric energy use per unit of distance traveled is given by:

$$e_{el} = B \frac{3.412}{\eta}$$

$B$  = Electricity Consumption in CD Mode  $\left[ \frac{J}{km} \right]$

$\eta$  = charger efficiency (currently default value for all of the vehicles is 85%)

total energy is given by:

$$e_{CD} = e_{el} + e_f$$

$e_f$  = fuel energy use

### Energy use in CS (charge sustaining) mode:

$$e_{CS} = \frac{\sum_{f \in F} s(f) hv(f)}{fe}$$

To combine CD and SC modes we need to calculate the **Utility Factor** (UF). The UF is the ratio of vehicle miles traveled (VMT) in CD mode to total VMT (in CD and CS modes combined).

$$UF(ER) = -7.73 \times 10^{-9} ER^4 + 2.63 \times 10^{-6} ER^3 - 3.65 \times 10^{-4} ER^2 + 2.66 \times 10^{-2} ER$$

Here  $ER$  is the electric range of the vehicle.



Then combine CD and CS energy use for PHEVs is given by

$$e_{op} = UF \times e_{CD} + (1 - UF)e_{CS}$$