Entanglement, holography, and the quantum phases of matter

Fermilab, November 7, 2012

Subir Sachdev

PHYSICS
HARVARD

Lecture at the 100th anniversary Solvay conference,
Theory of the Quantum World
arXiv:1203.4565

Liza Huijse

Max Metlitski

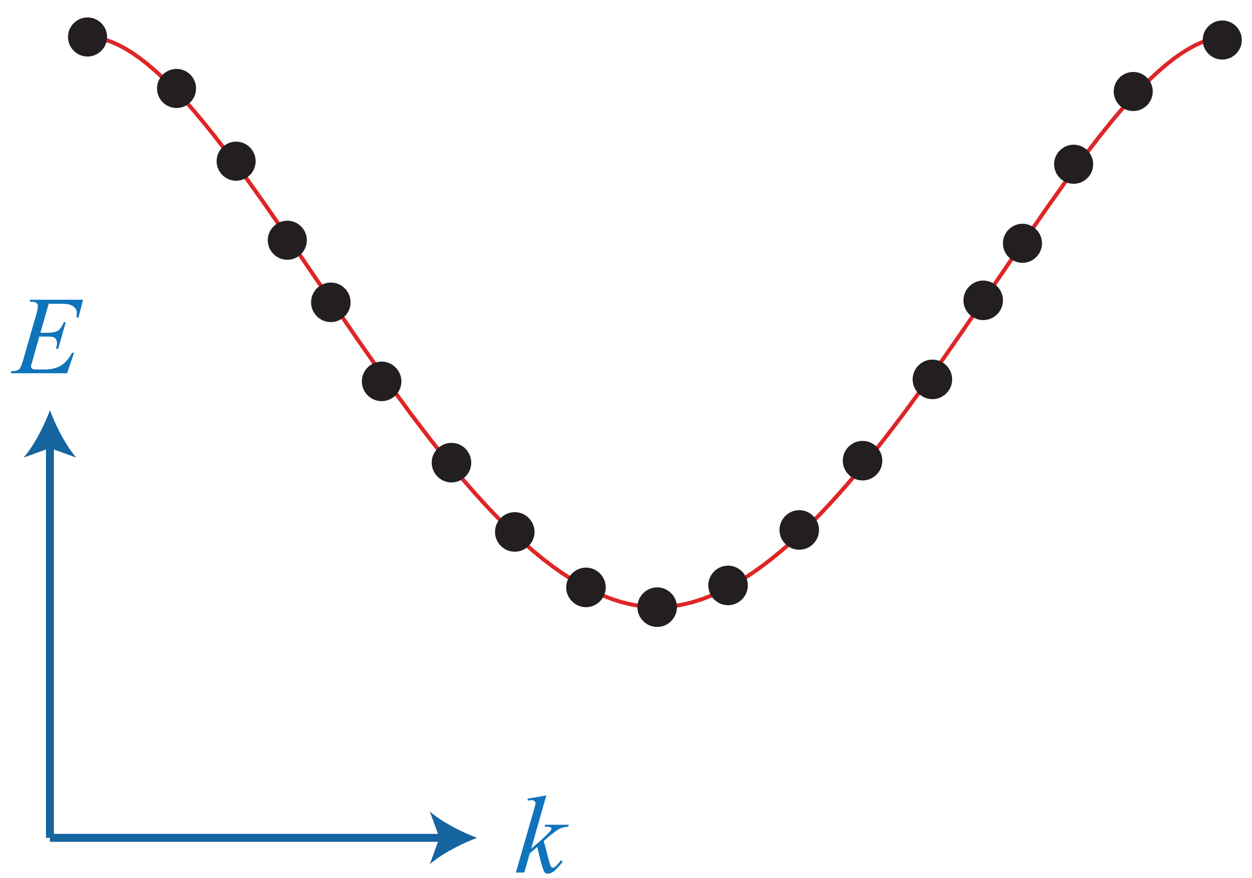
Brian Swingle

Sommerfeld-Bloch theory of
metals, insulators, and superconductors:

many-electron quantum states are adiabatically
connected to independent electron states

Sommerfeld-Bloch theory of
metals, insulators, and superconductors:
many-electron quantum states are adiabatically connected to independent electron states

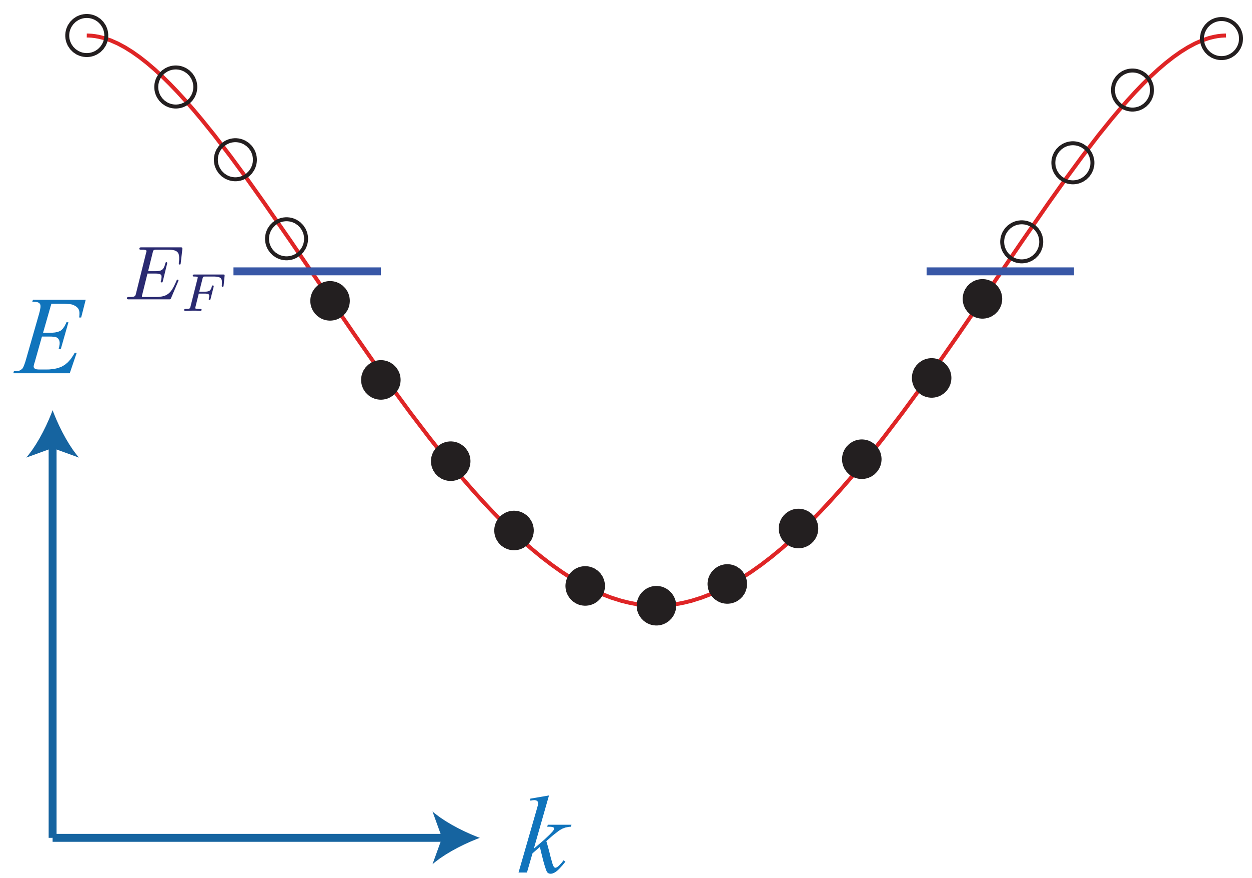
Band insulators



An even number of electrons per unit cell

Sommerfeld-Bloch theory of
metals, insulators, and superconductors:
many-electron quantum states are adiabatically
connected to independent electron states

Metals

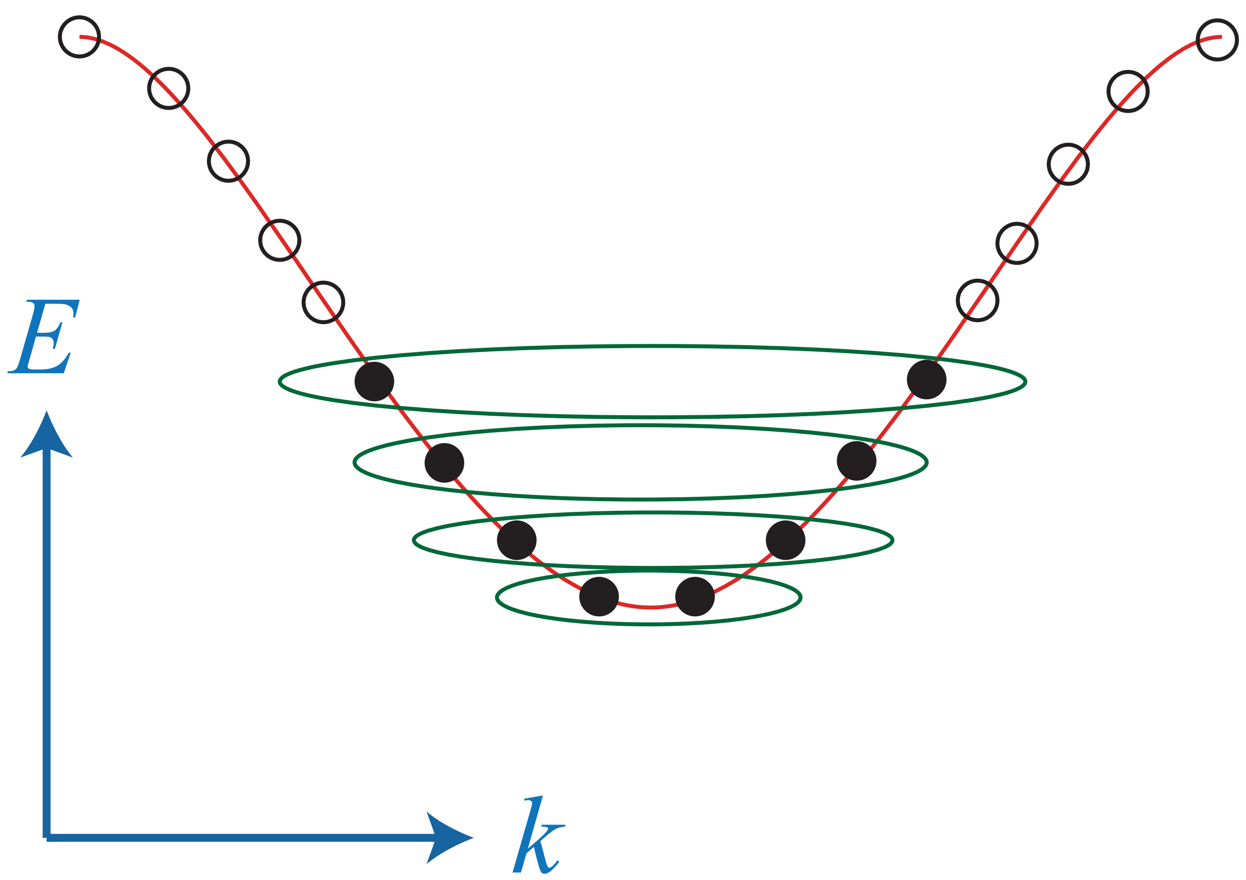


An odd number of electrons per unit cell

Sommerfeld-Bloch theory of
metals, insulators, and superconductors:

many-electron quantum states are adiabatically
connected to independent electron states

Superconductors



Modern phases of quantum matter

Not adiabatically connected

to independent electron states:

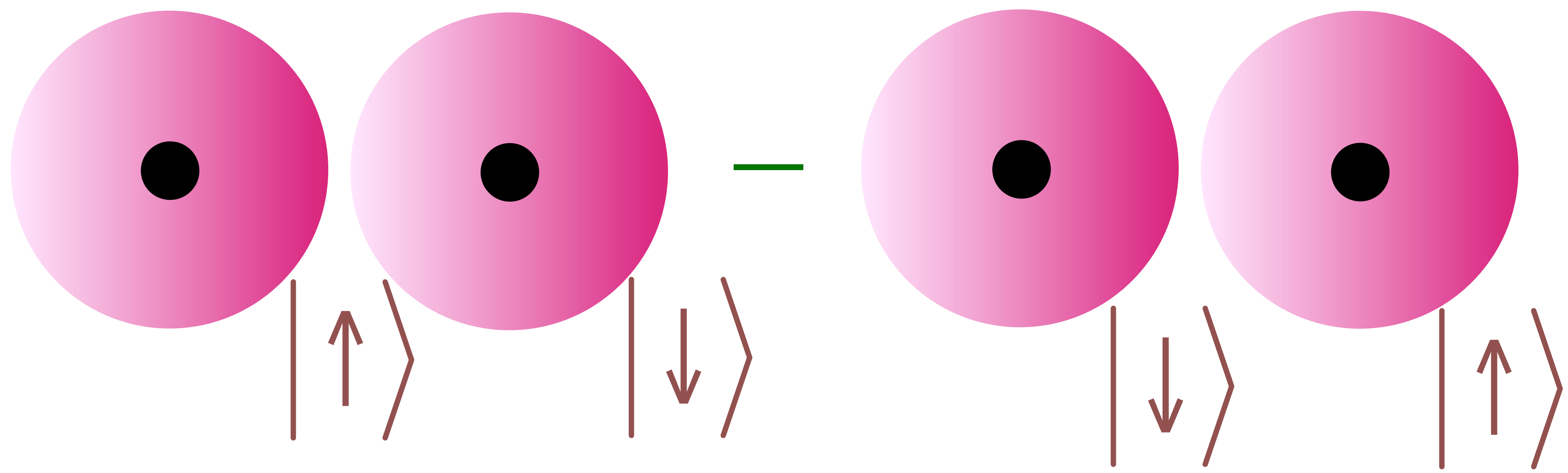
Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

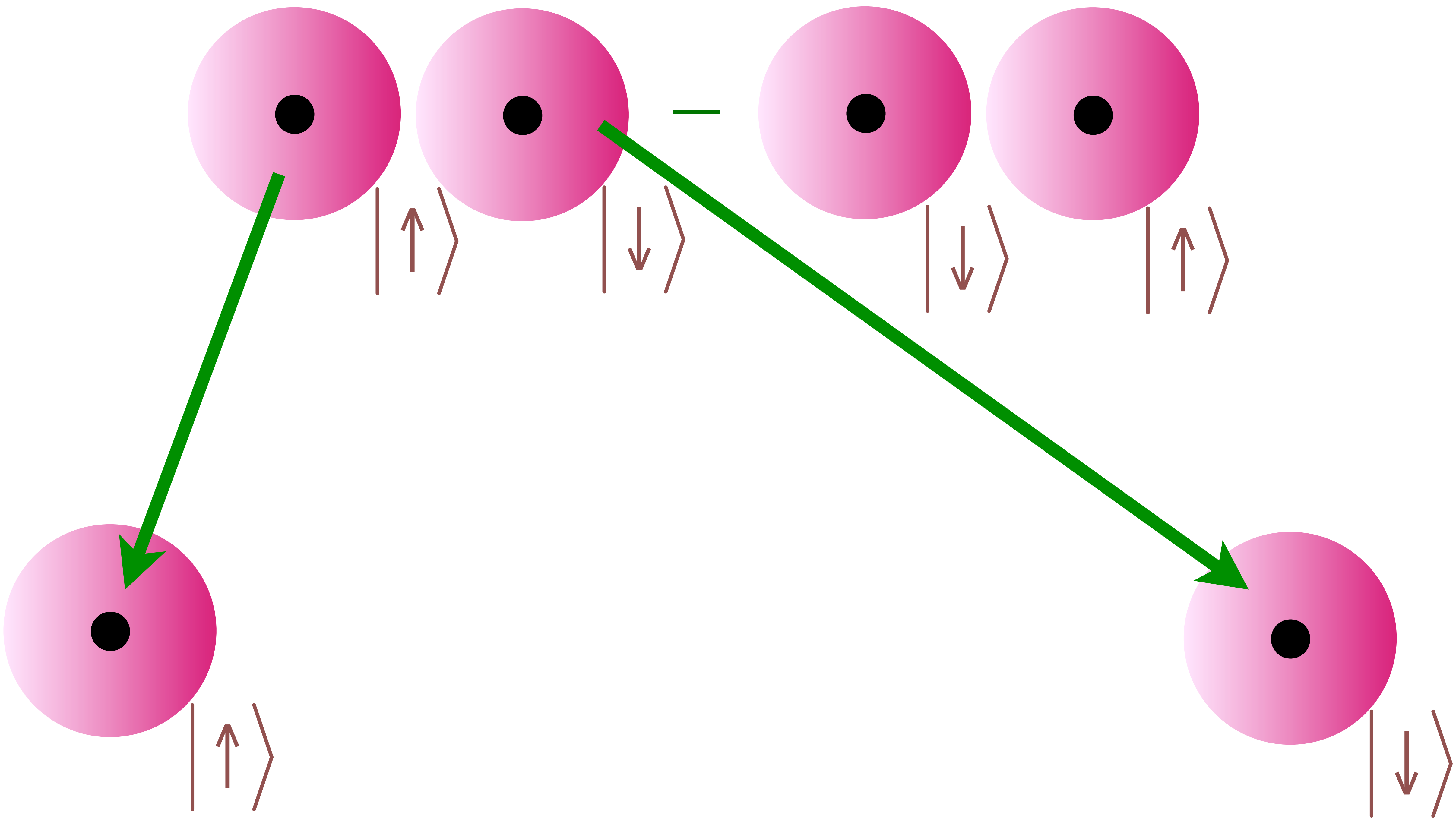
many-particle
quantum entanglement

Quantum Entanglement: quantum superposition with more than one particle

Hydrogen molecule:



Quantum Entanglement: quantum superposition with more than one particle



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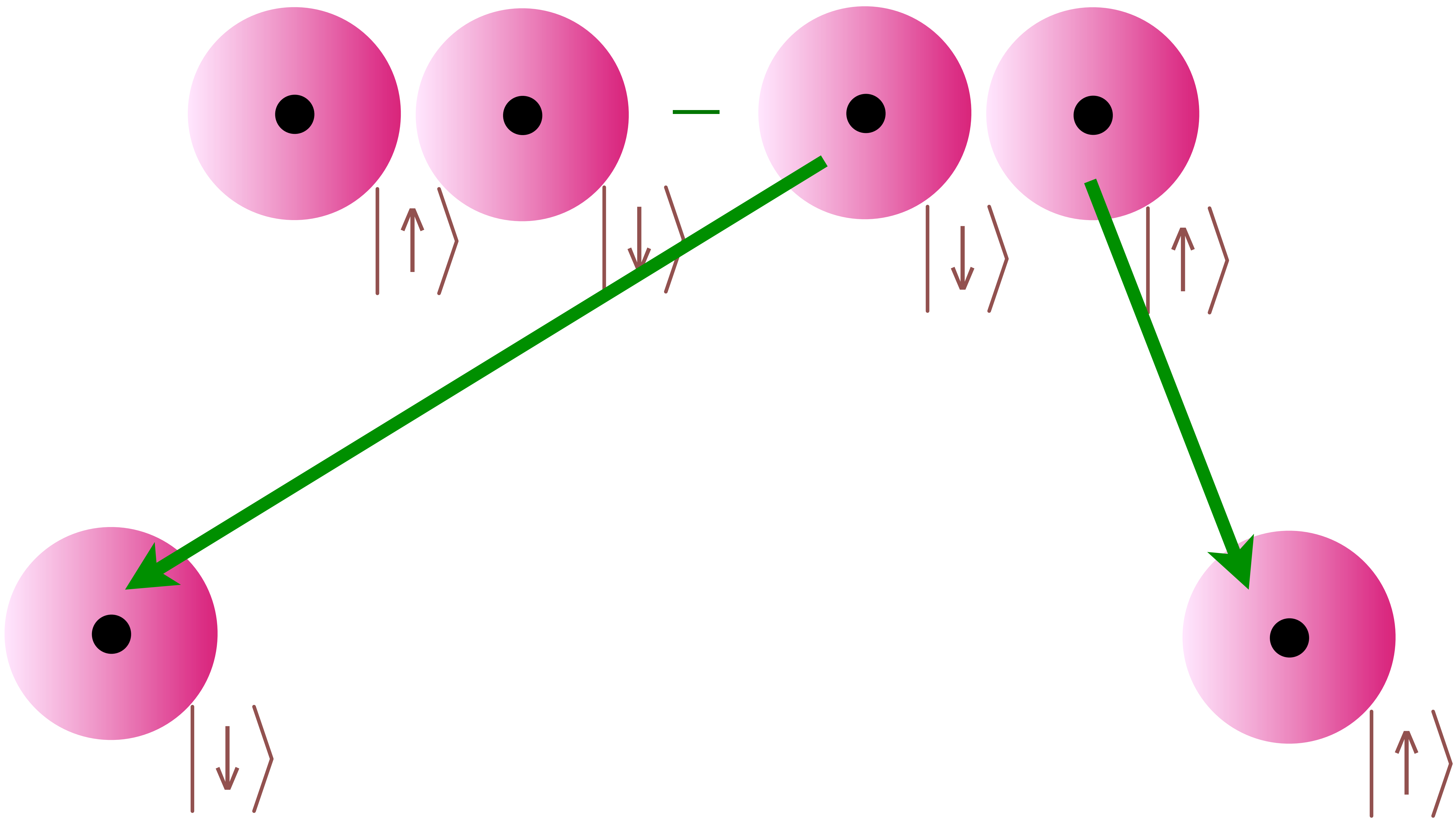

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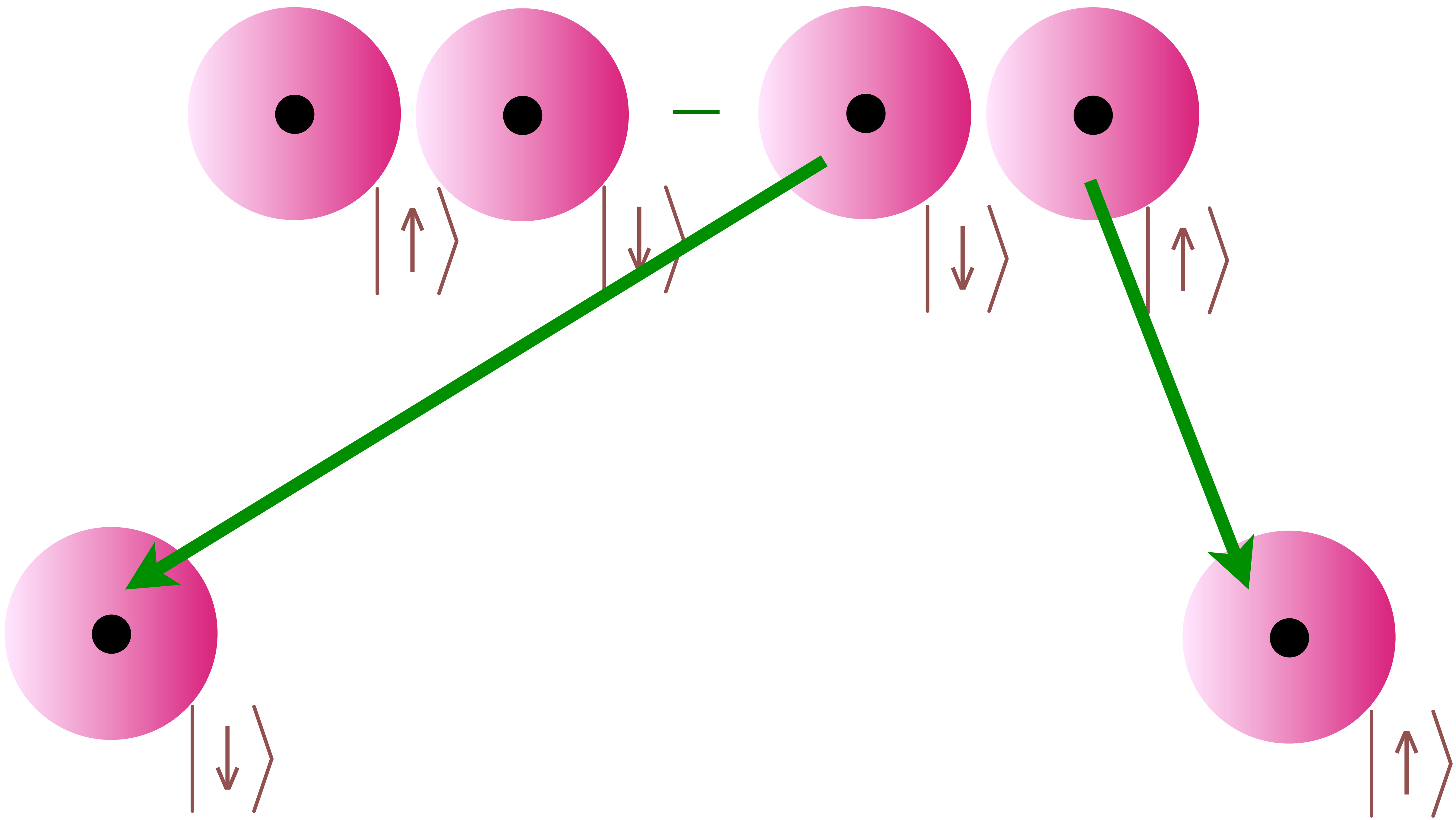
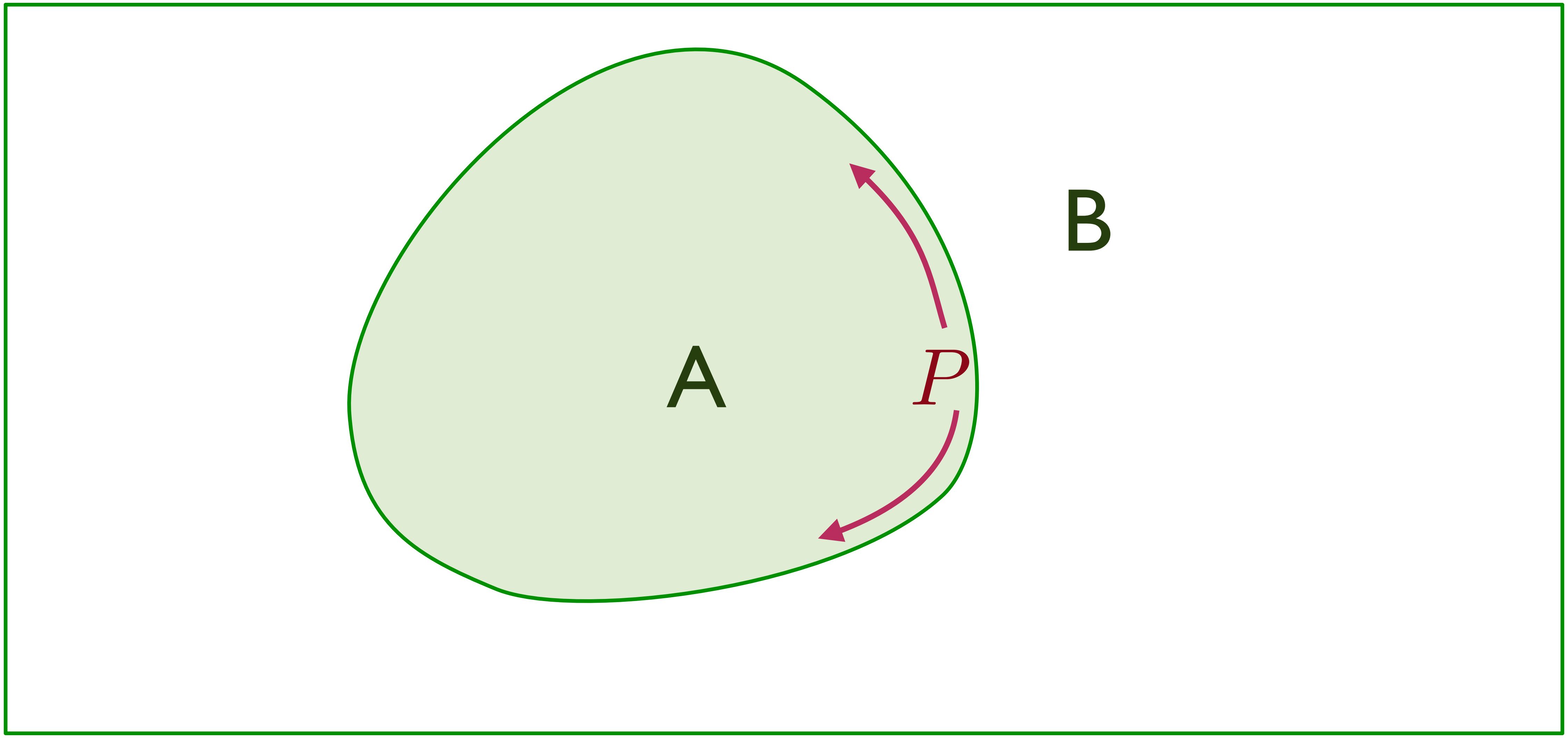


Einstein-Podolsky-Rosen "paradox": Non-local correlations between observations arbitrarily far apart

Entanglement entropy



$$|\Psi\rangle \quad \Rightarrow \quad \text{Ground state of entire system,}$$
$$\rho = |\Psi\rangle\langle\Psi|$$

$$\rho_A = \mathrm{Tr}_B \rho = \text{density matrix of region } A$$

Entanglement entropy $S_E = -\mathrm{Tr}\left(\rho_A \ln \rho_A\right)$

Entanglement entropy

$$|\Psi\rangle \quad \Rightarrow \quad \text{Ground state of entire system,}$$
$$\rho = |\Psi\rangle\langle\Psi|$$

$$\text{Take } |\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_A |\downarrow\rangle_B - |\downarrow\rangle_A |\uparrow\rangle_B\right)$$

$$\text{Then } \rho_A = \text{Tr}_B \rho = \text{density matrix of region } A$$
$$= \frac{1}{2}\left(|\uparrow\rangle_A \langle\uparrow|_A + |\downarrow\rangle_A \langle\downarrow|_A\right)$$

$$\textbf{Entanglement entropy } S_E = -\text{Tr}\left(\rho_A \ln \rho_A\right)$$
$$= \ln 2$$

"Complex entangled" states of quantum matter, *not* adiabatically connected to independent particle states

Gapped quantum matter

Spin liquids, quantum Hall states

Conformal quantum matter

Quantum critical points in antiferromagnets, superconductors, and ultracold atoms; graphene

Compressible quantum matter

Strange metals in high temperature superconductors, Bose metals

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“Complex entangled” states of quantum matter in *d* spatial dimensions

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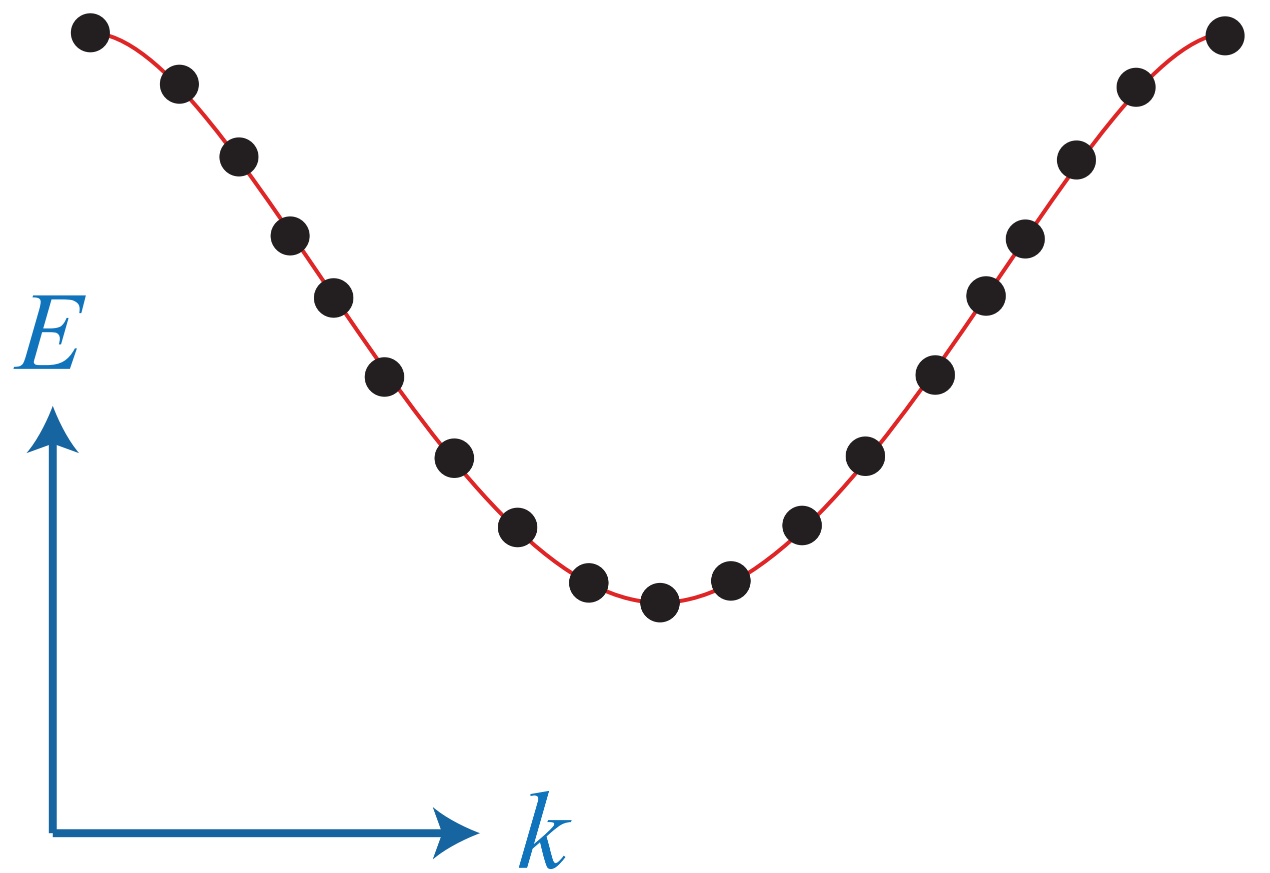
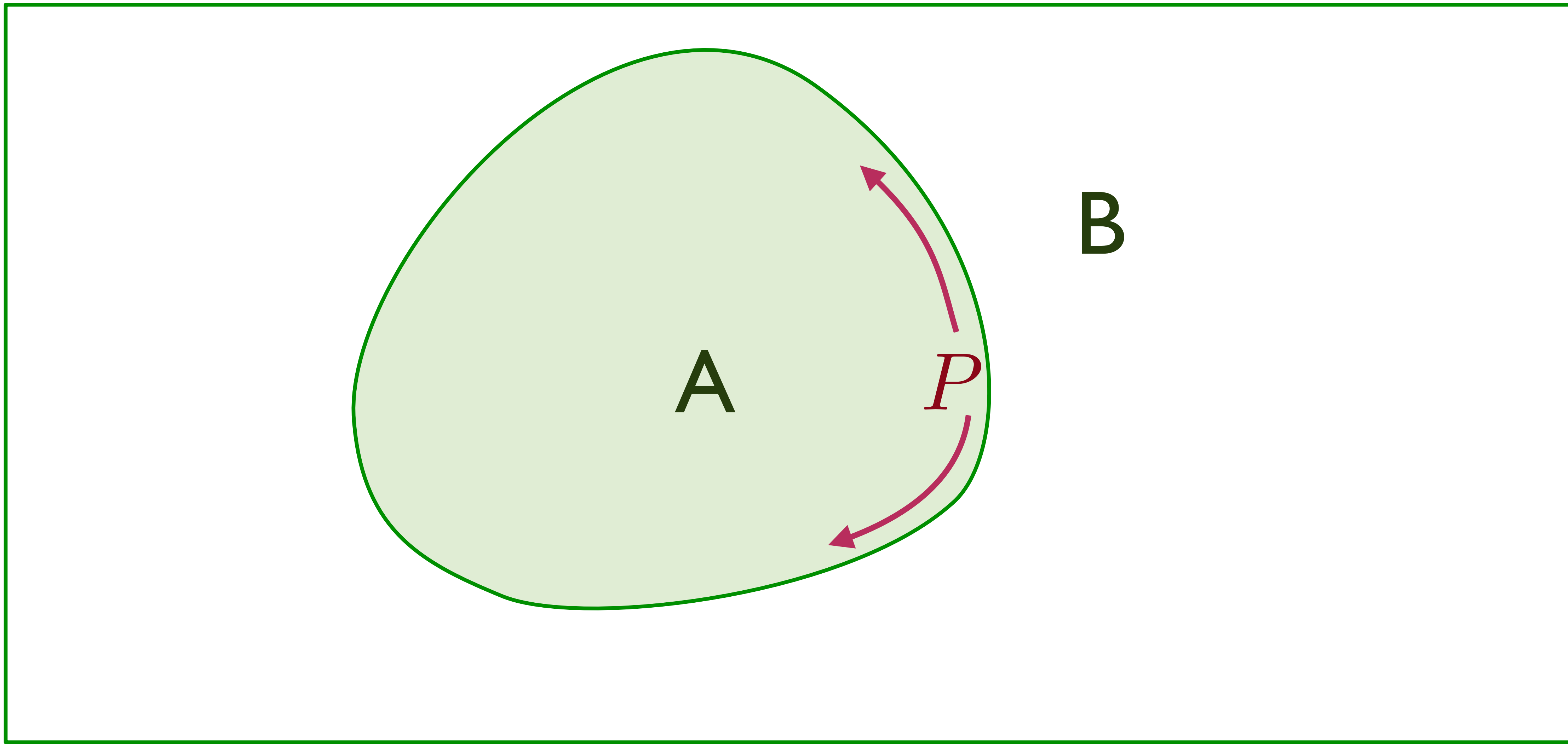
Entanglement entropy of a band insulator

Band insulators



An even number of electrons per unit cell

Entanglement entropy of a band insulator

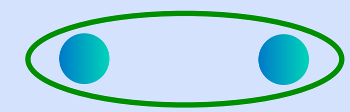


$$S_E = aP - b\exp(-cP)$$

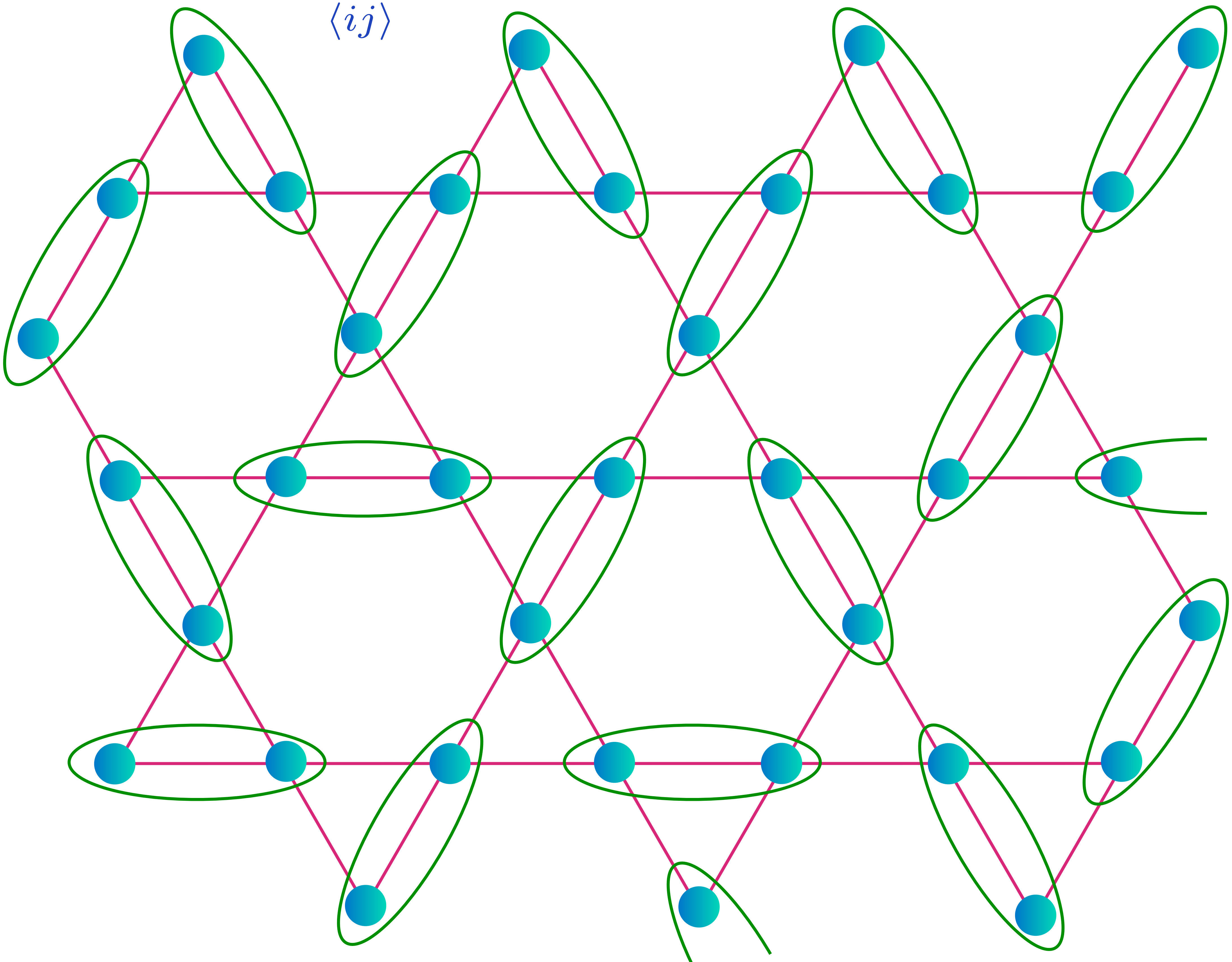
where P is the surface area (perimeter) of the boundary between A and B.

Mott insulator: Kagome antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$



P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

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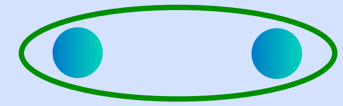

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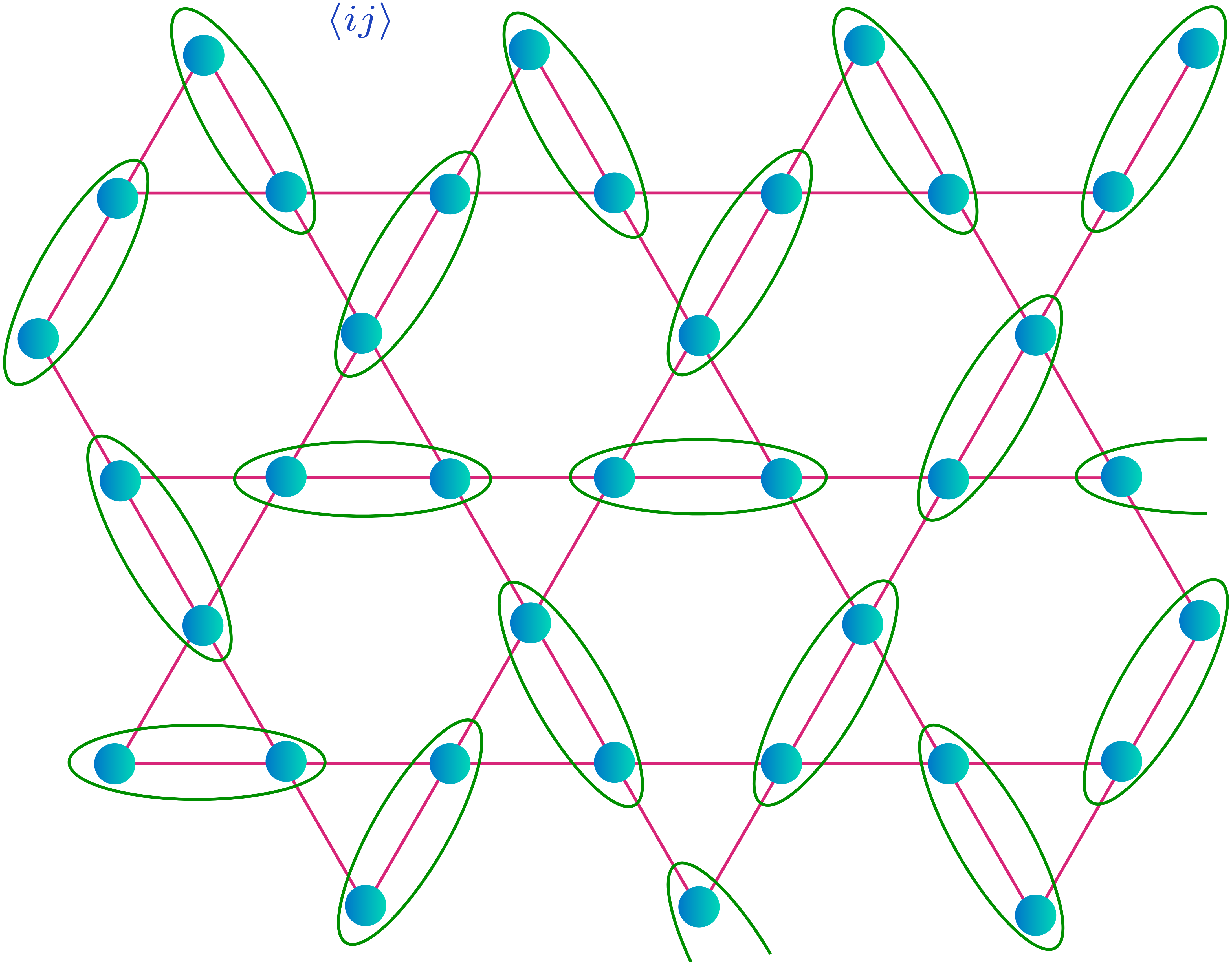

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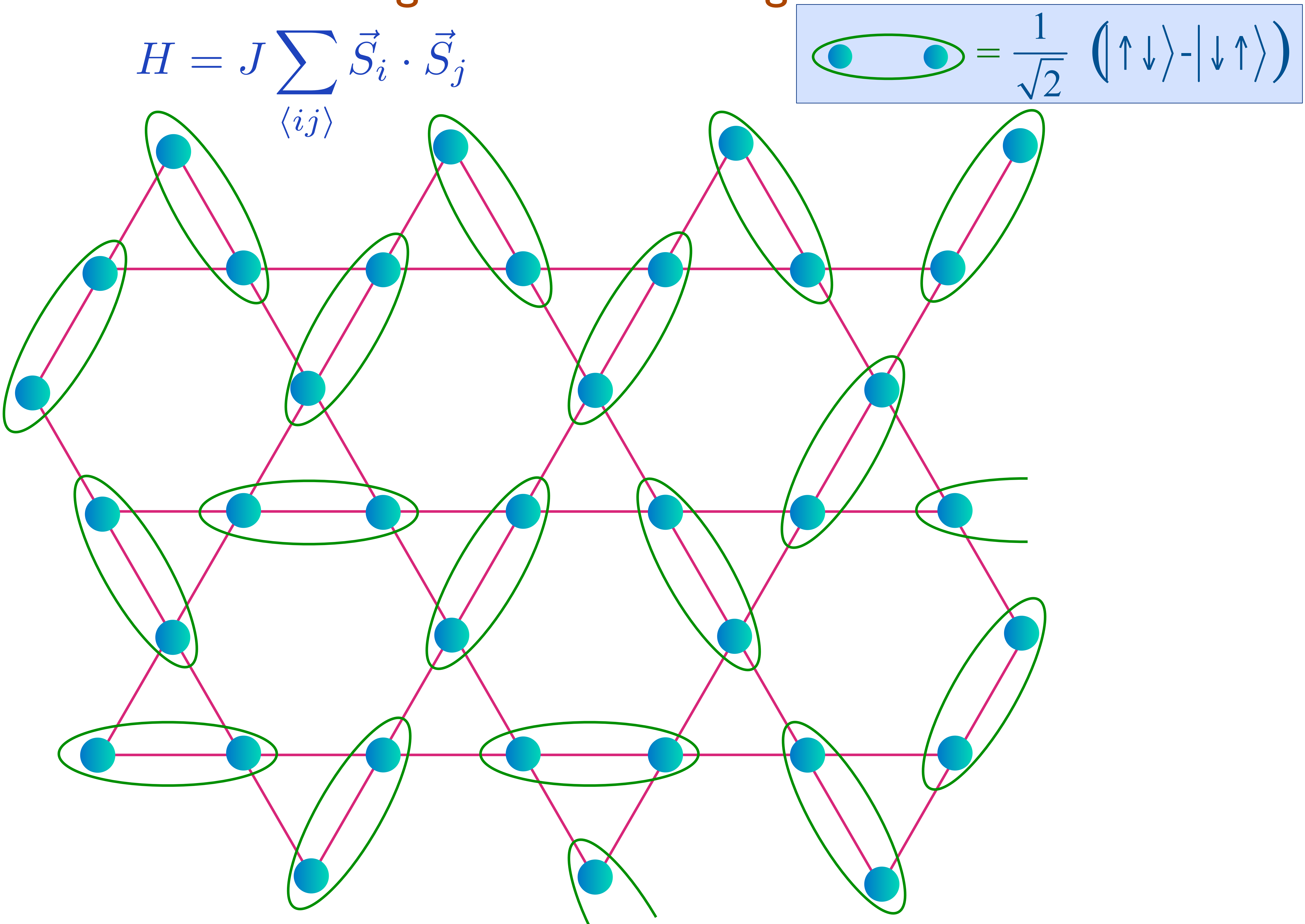

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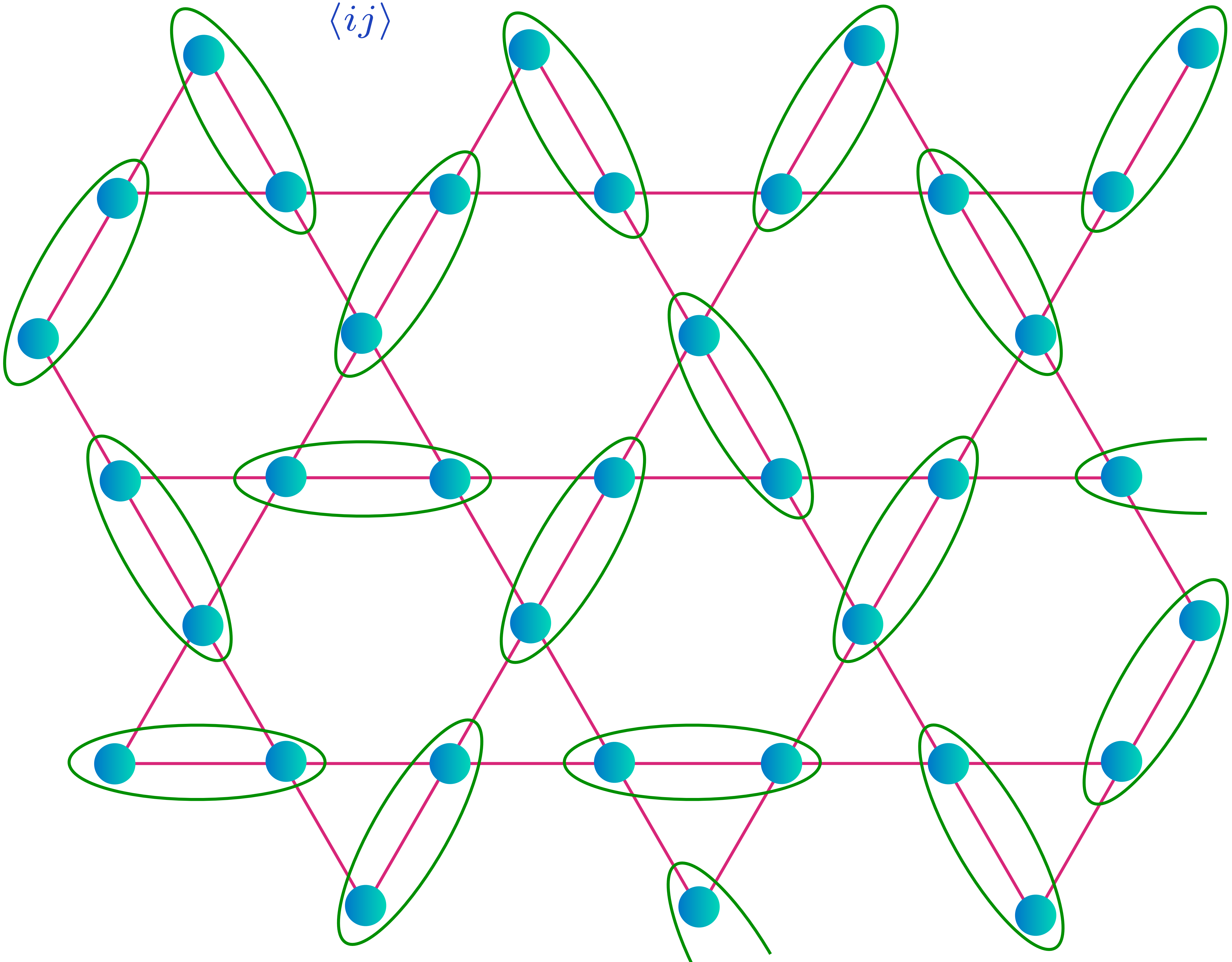

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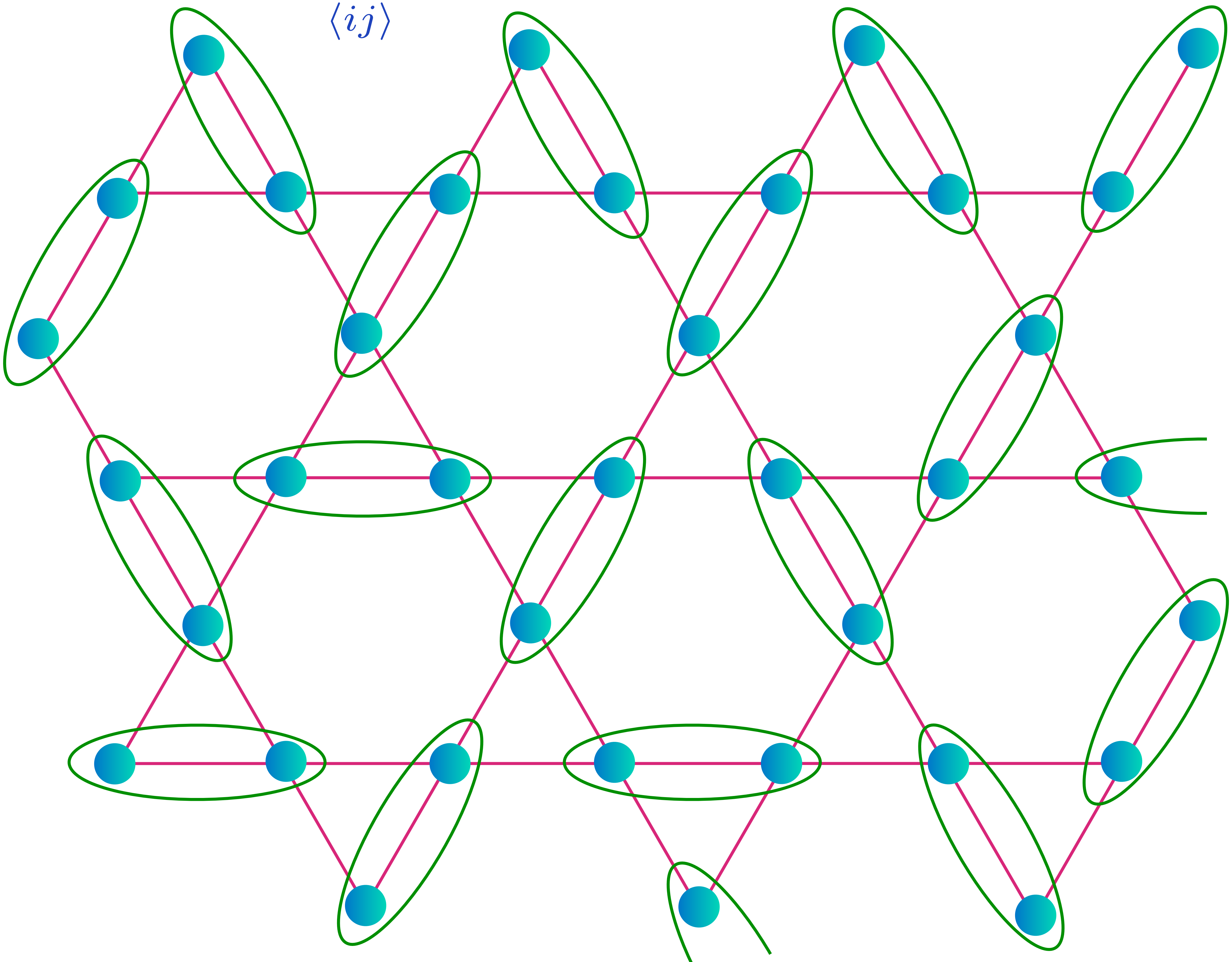

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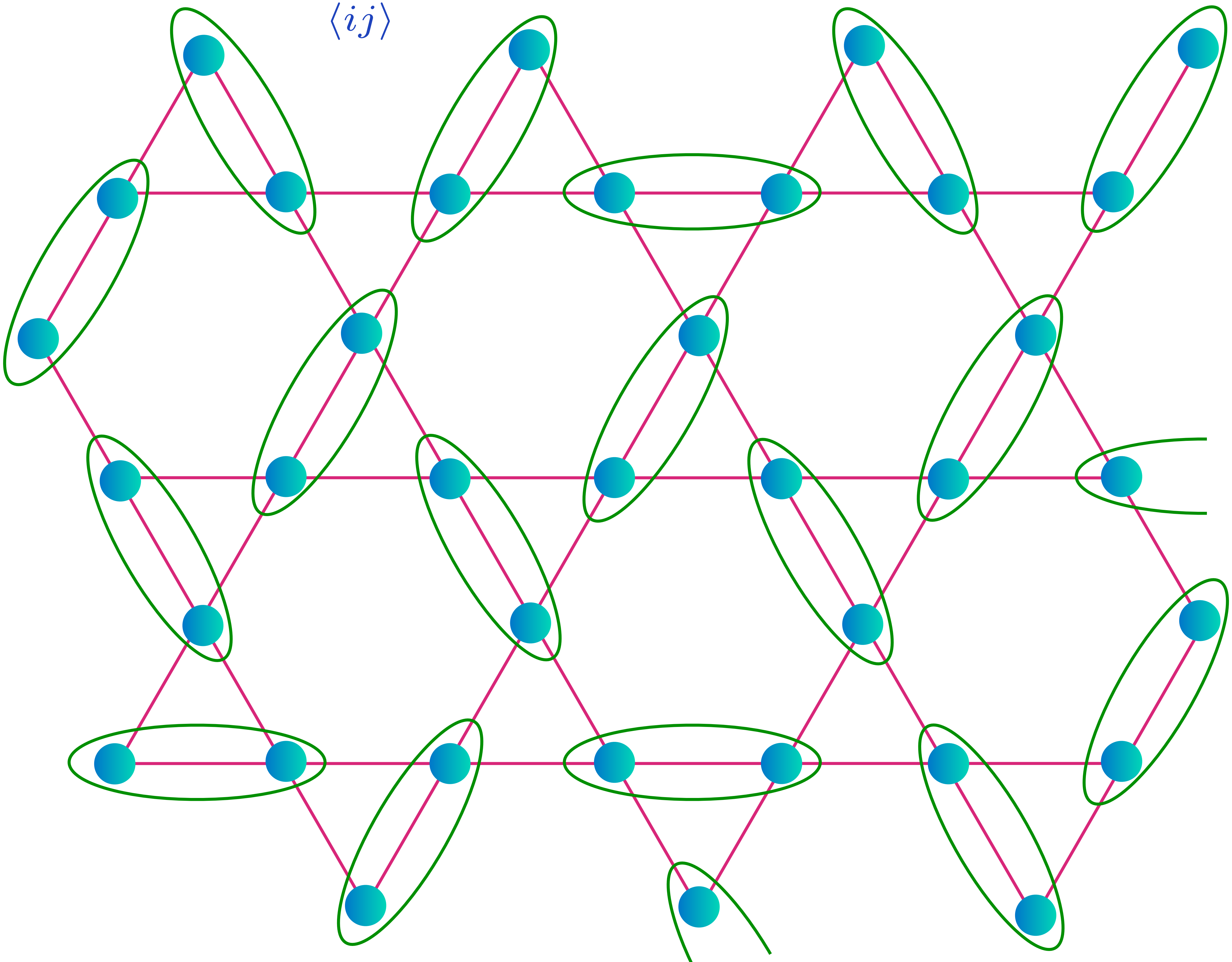

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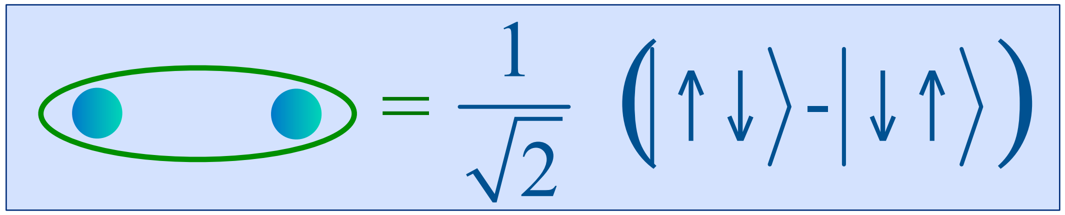
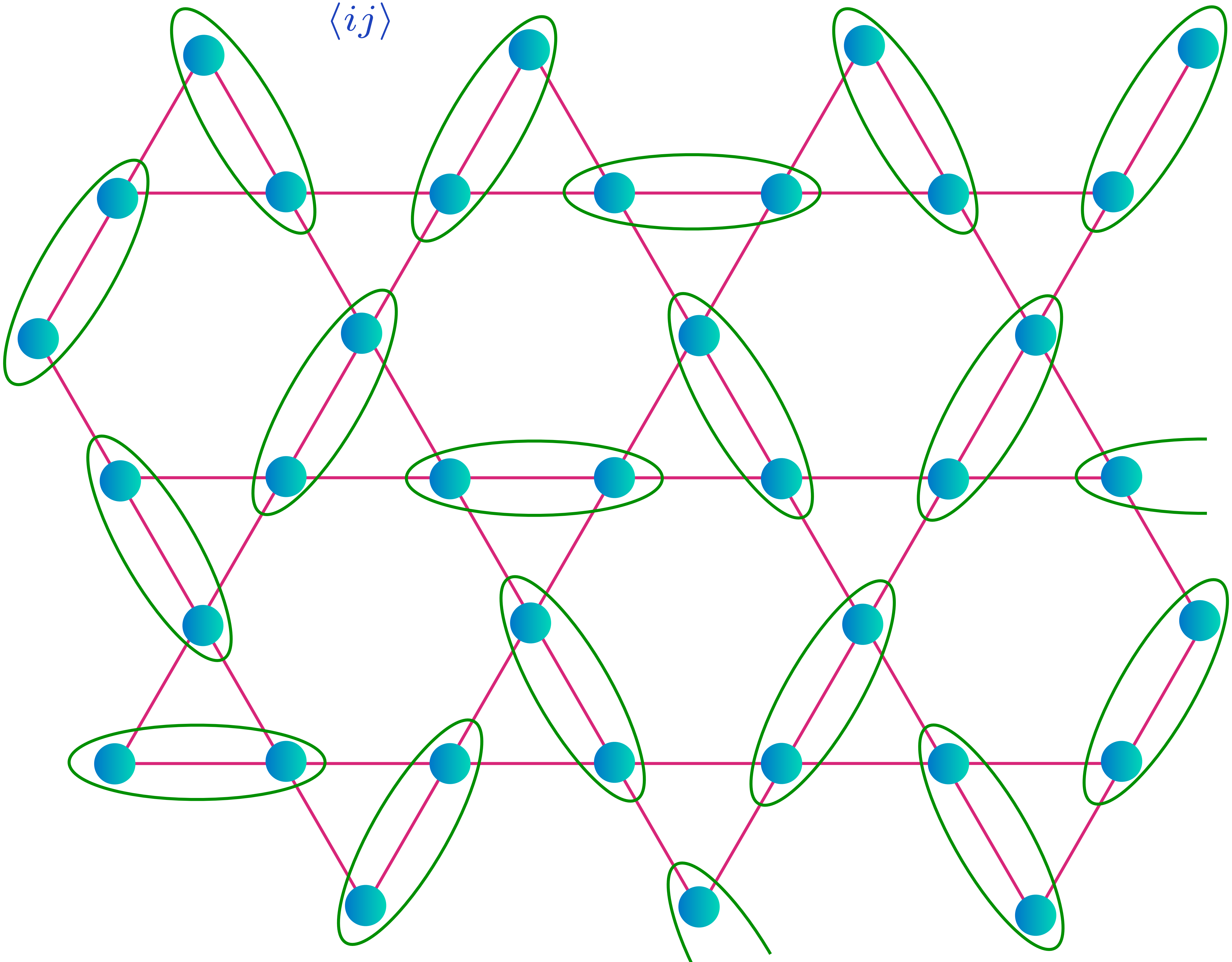
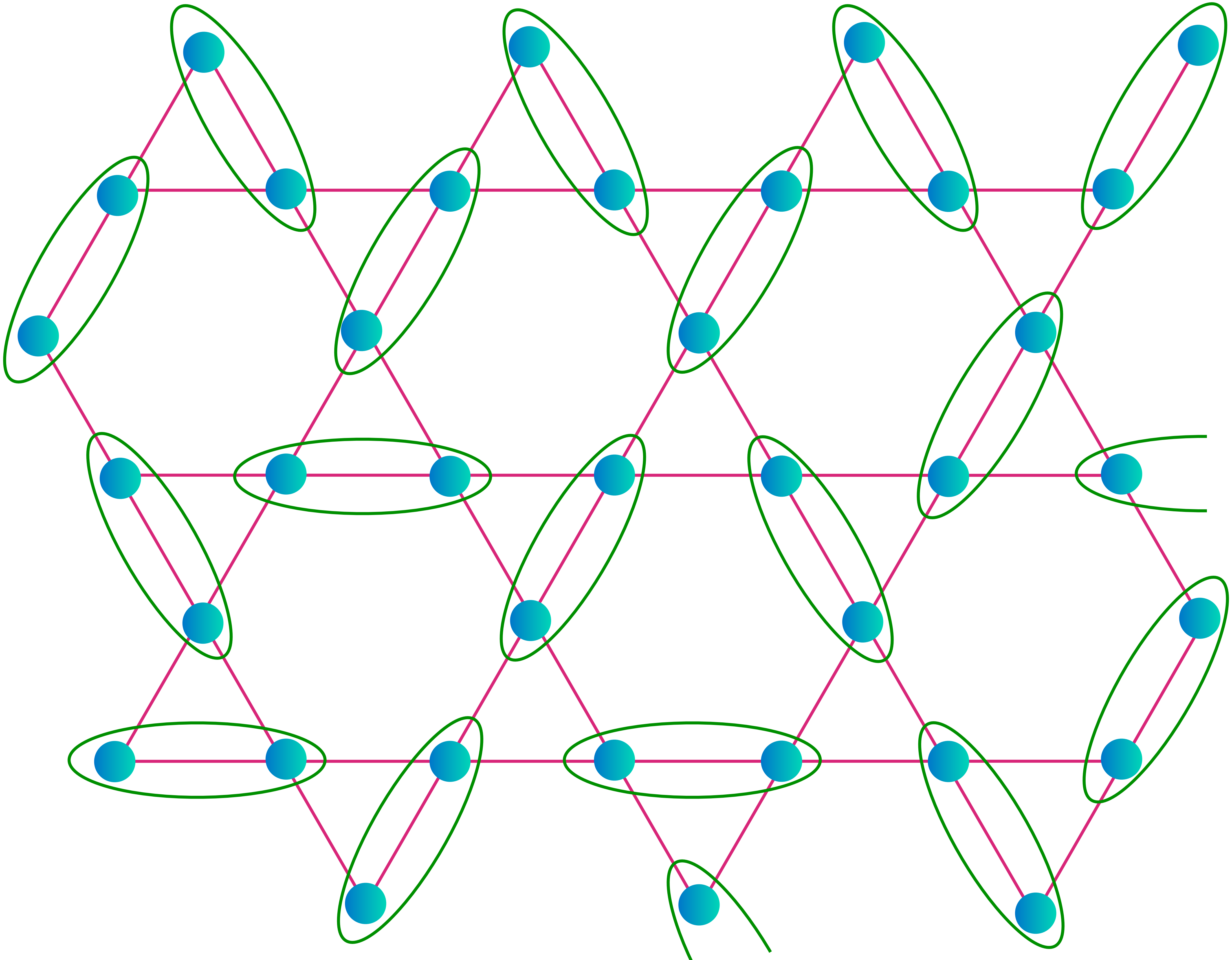




Mott insulator: Kagome antiferromagnet

Alternative view

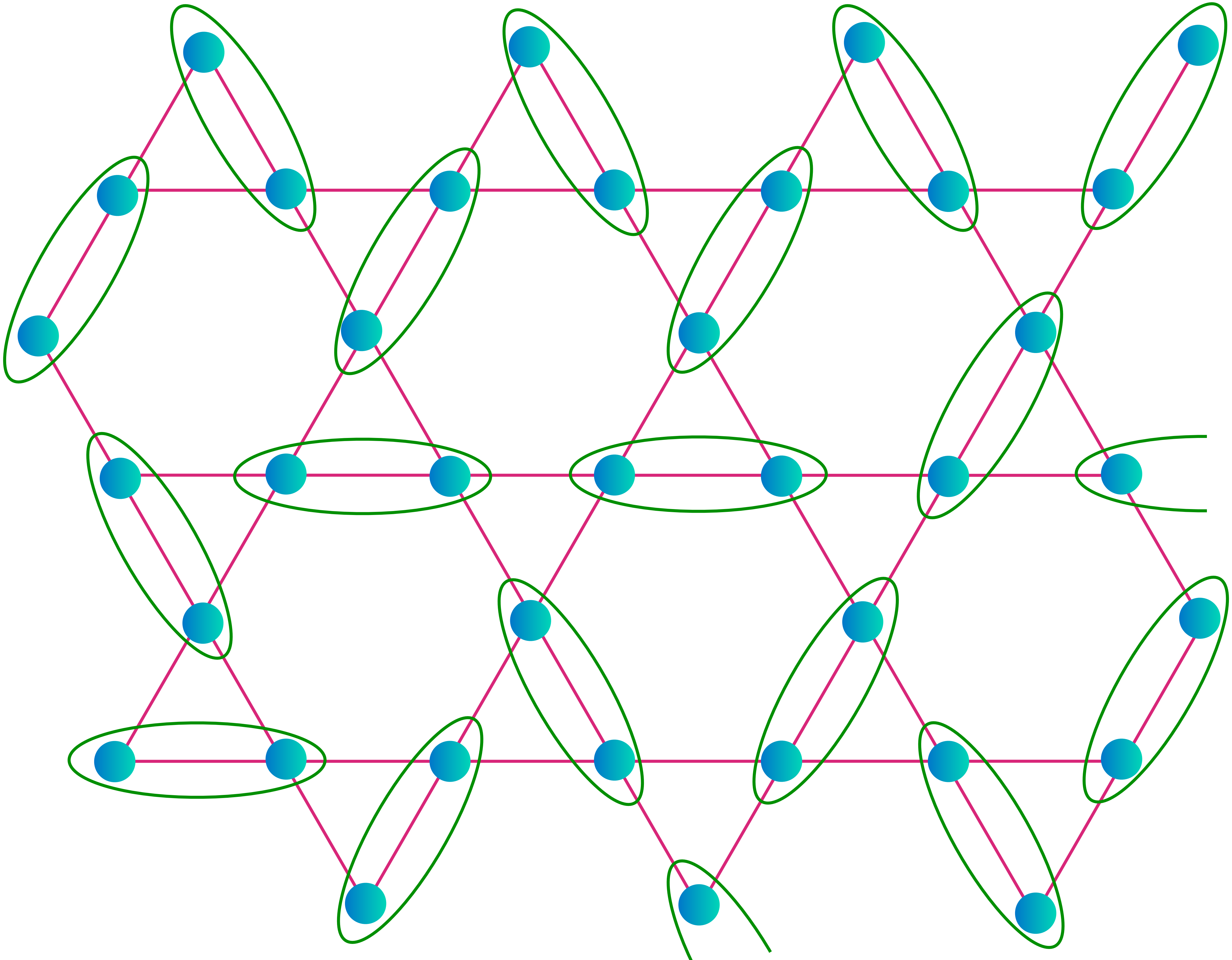
Pick a reference configuration



Mott insulator: Kagome antiferromagnet

Alternative view

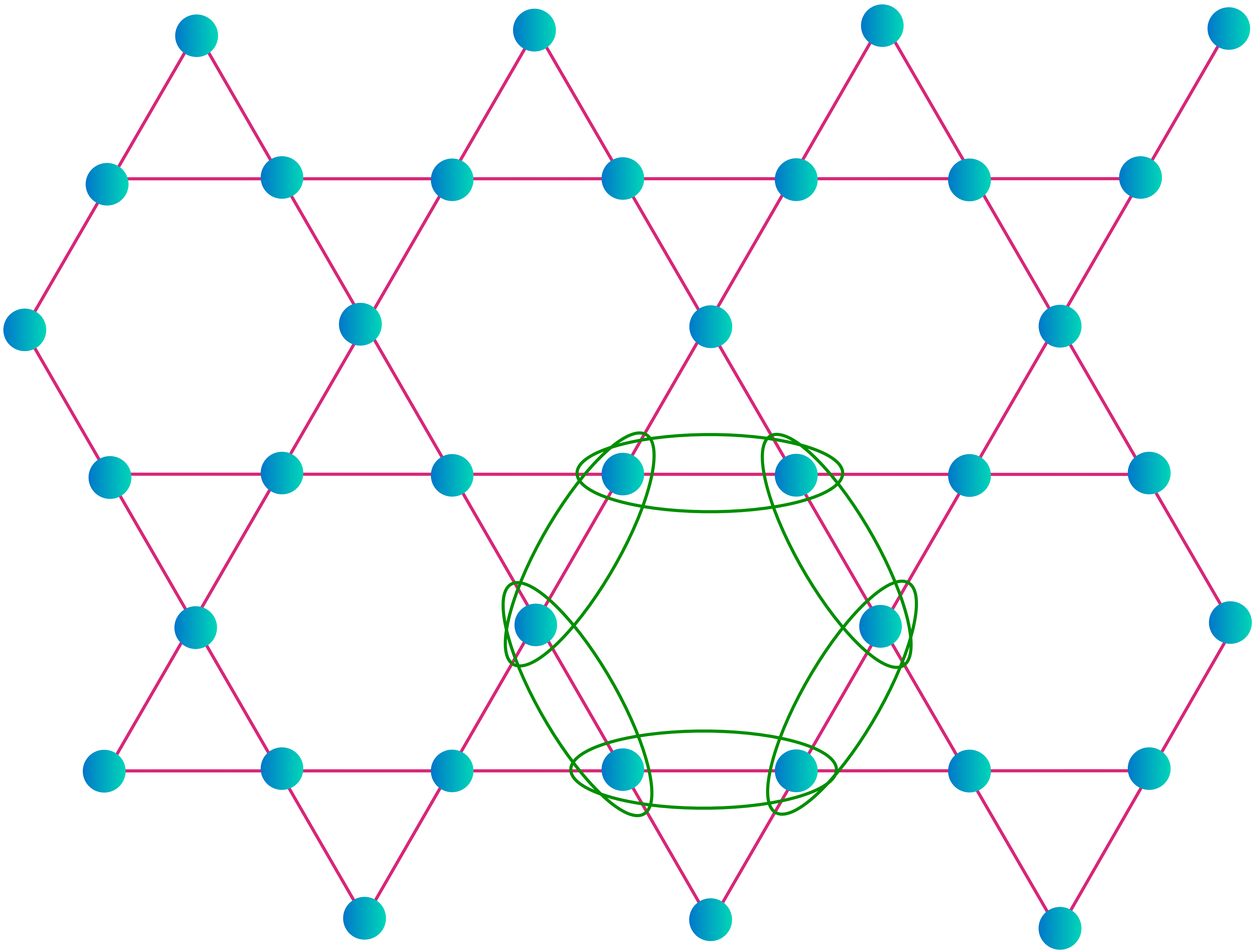
A nearby configuration



Mott insulator: Kagome antiferromagnet

Alternative view

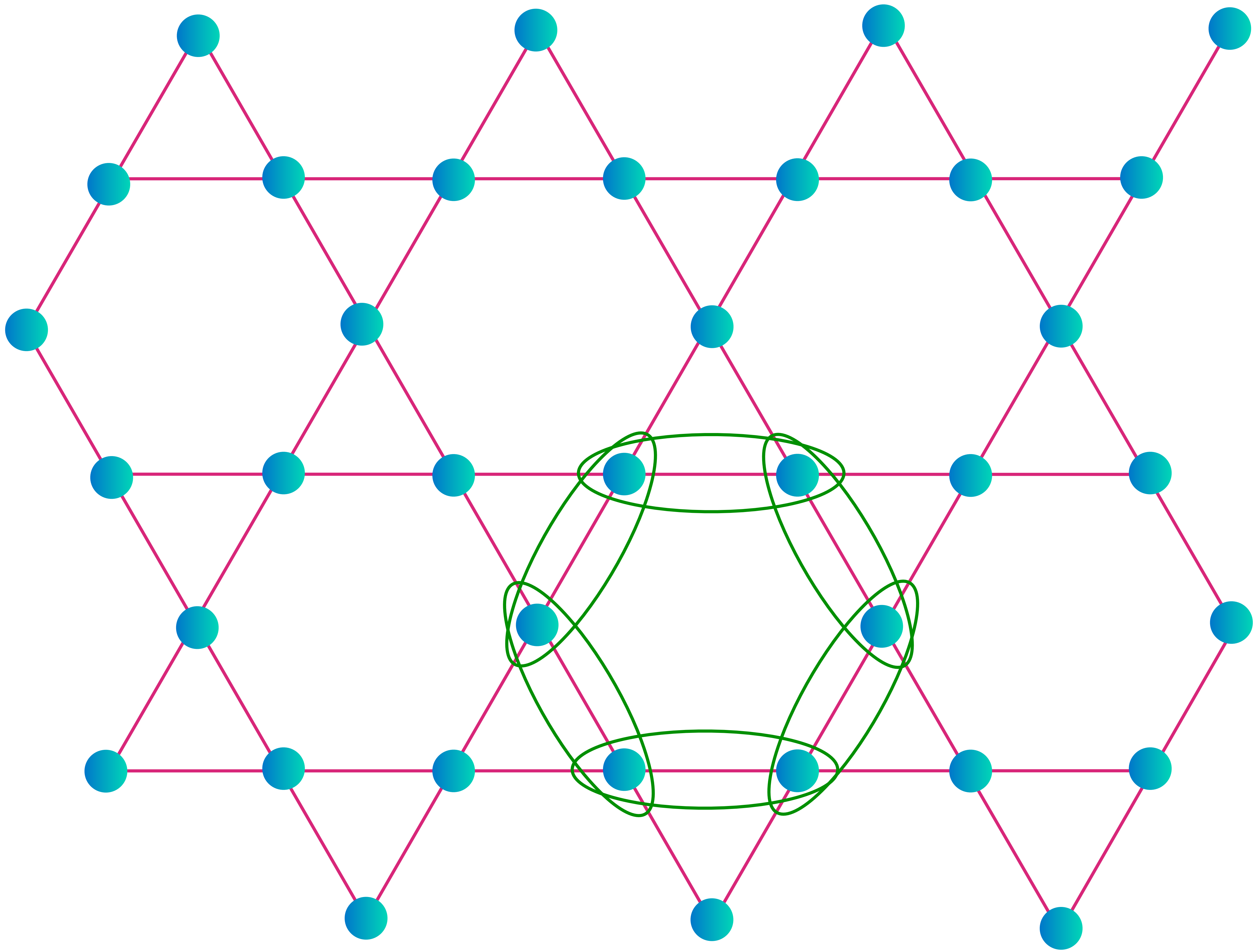
Difference: a closed loop



Mott insulator: Kagome antiferromagnet

Alternative view

Ground state: sum over closed loops



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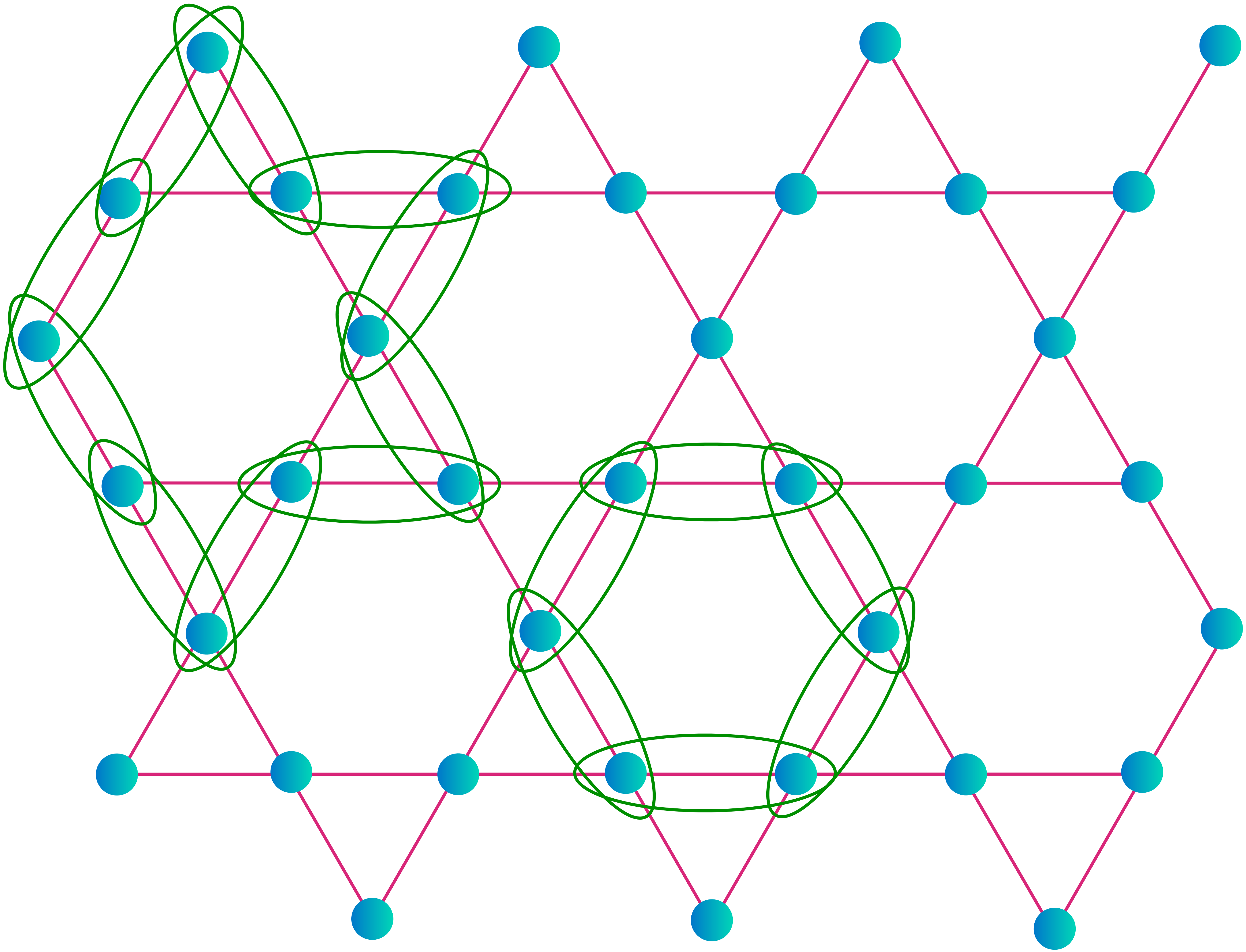

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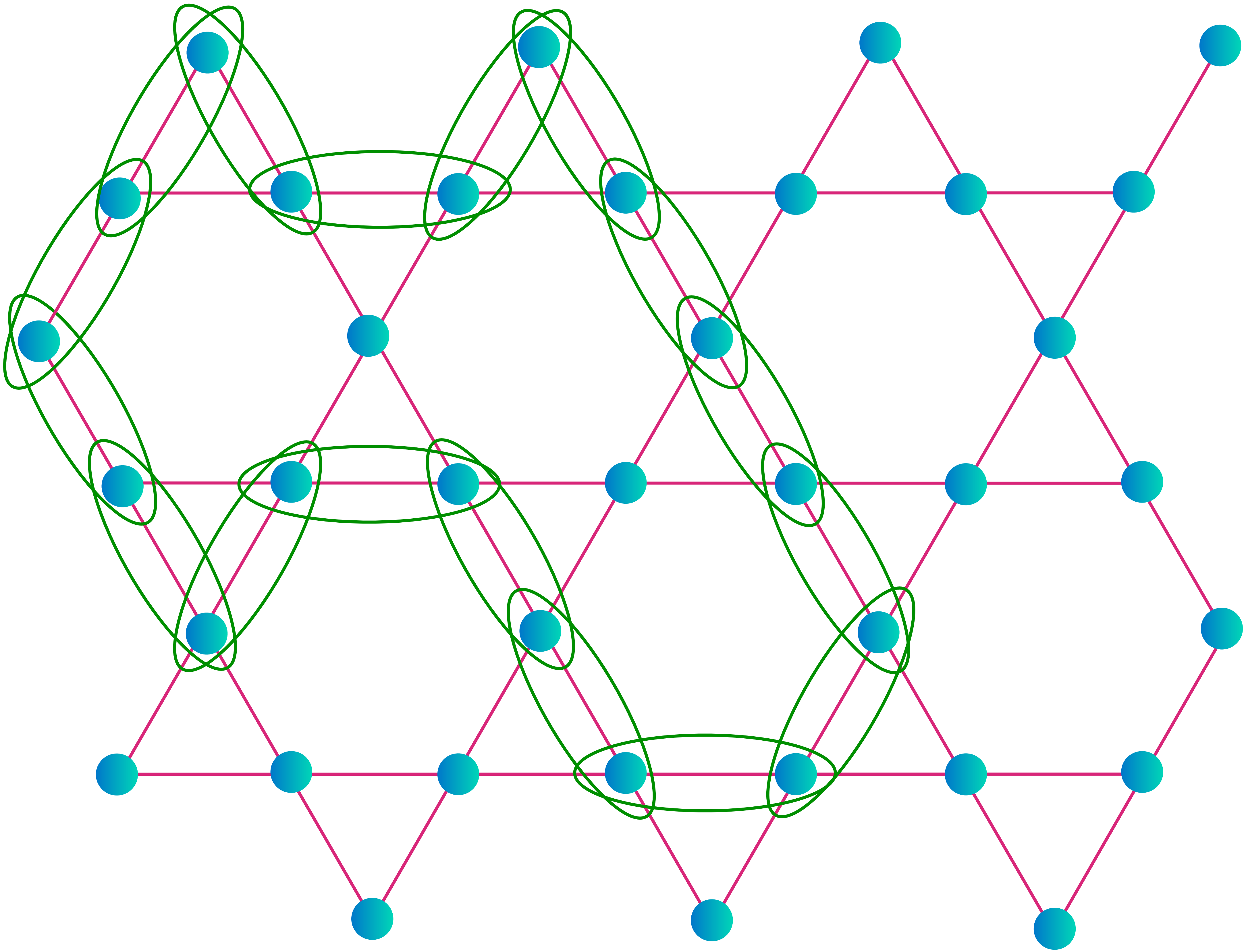

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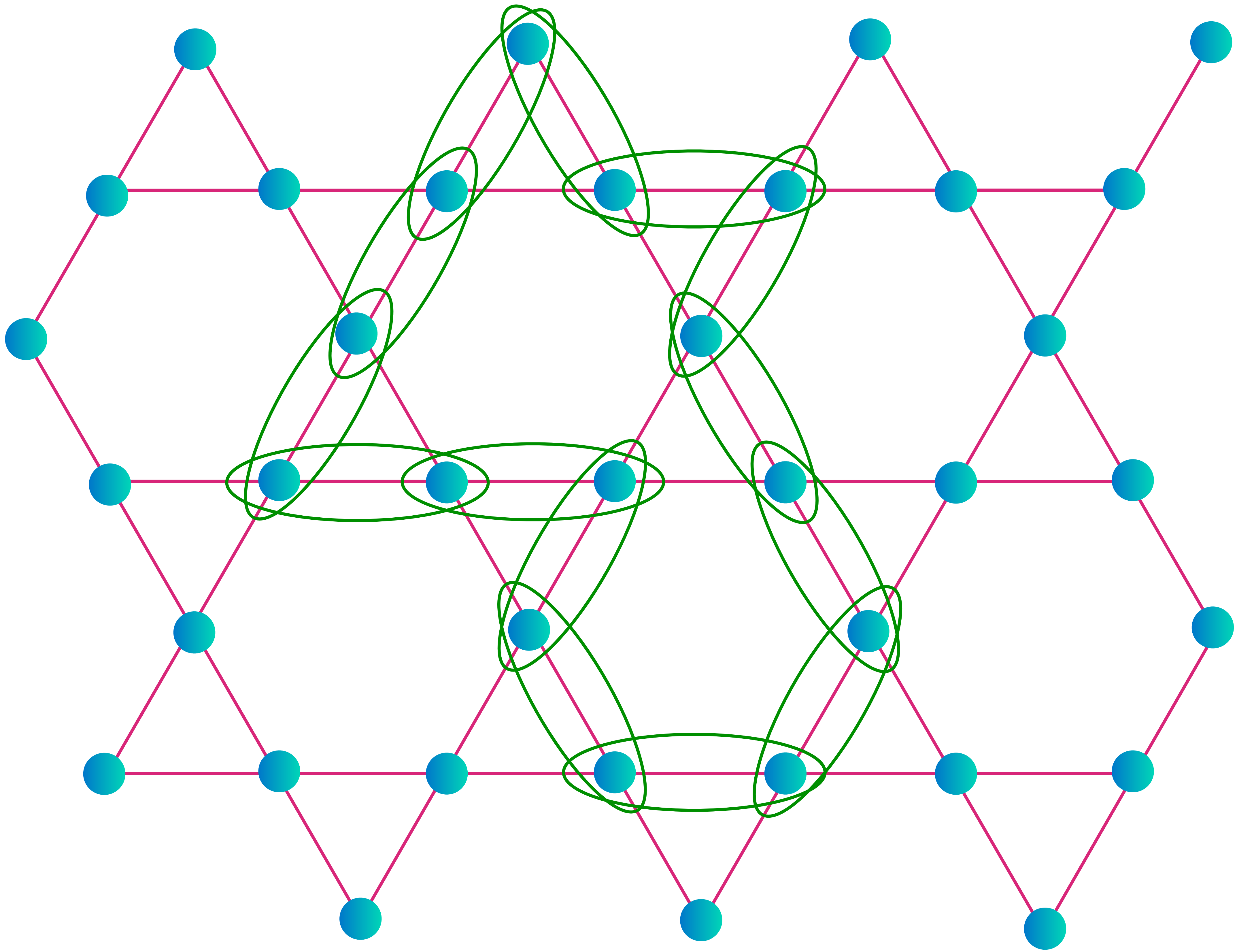
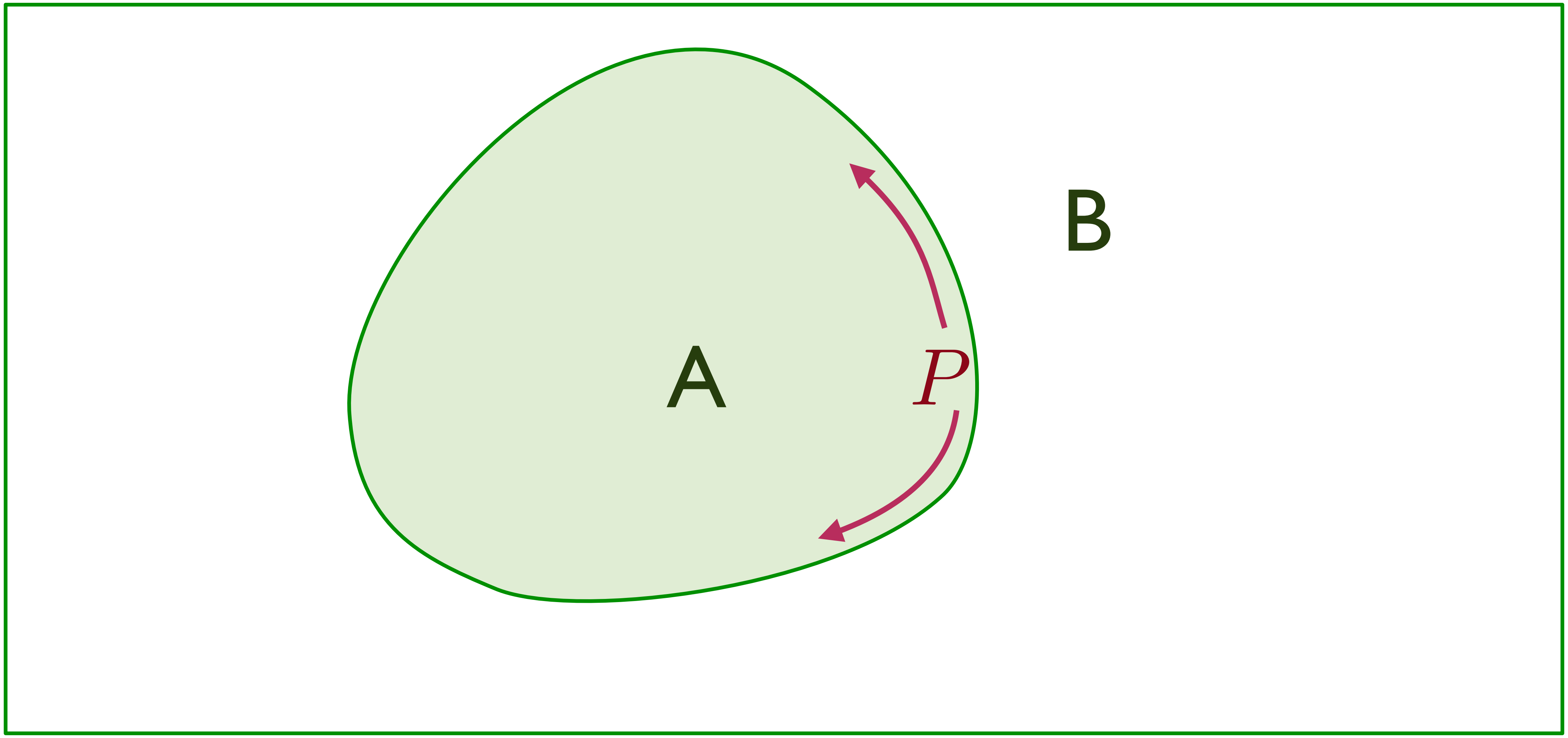


Entanglement in the Z_2 spin liquid



The sum over closed loops is characteristic of the Z_2 spin liquid, introduced in
N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991),
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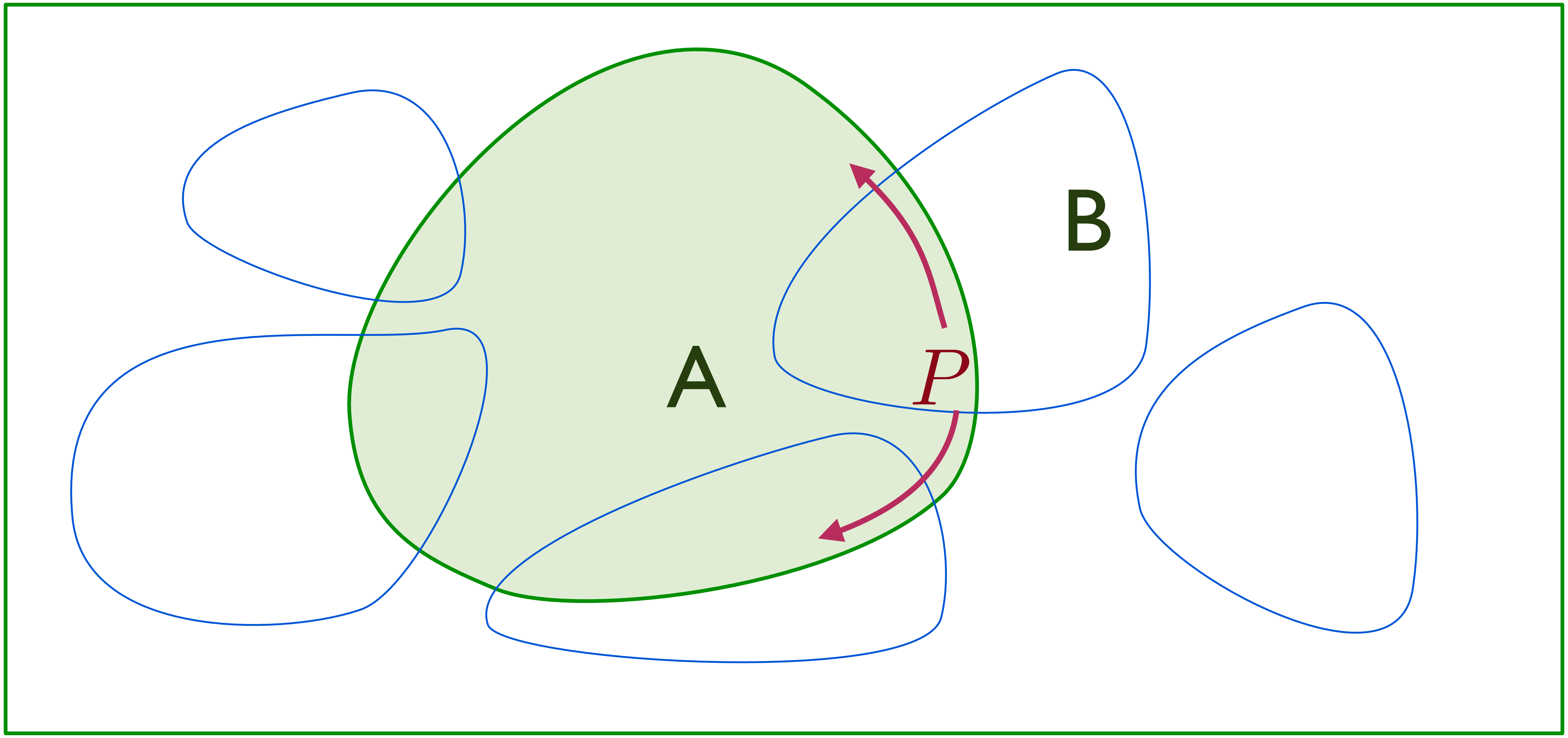
Entanglement in the Z_2 spin liquid



Sum over closed loops: only an even number of links cross the boundary between A and B

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N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991),
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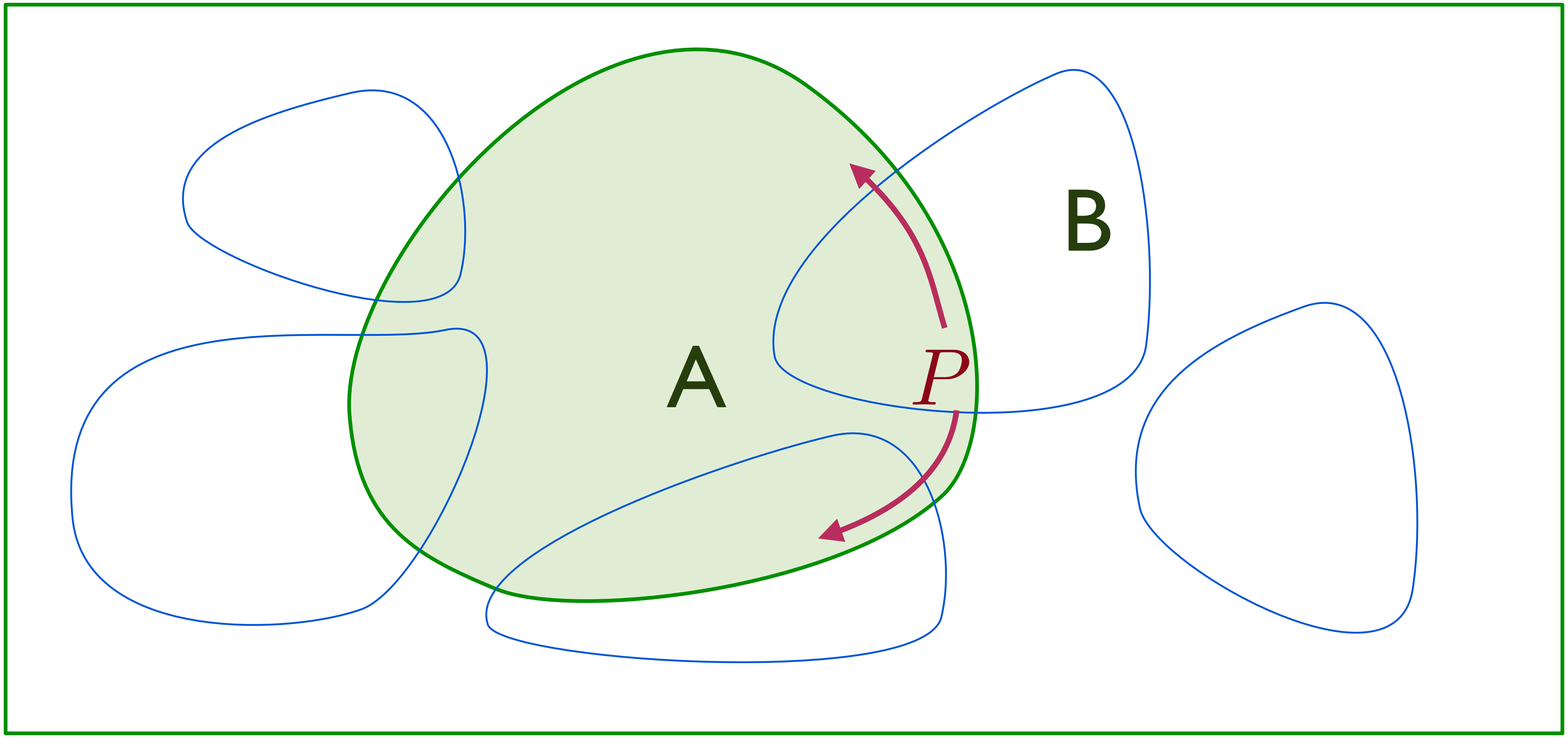
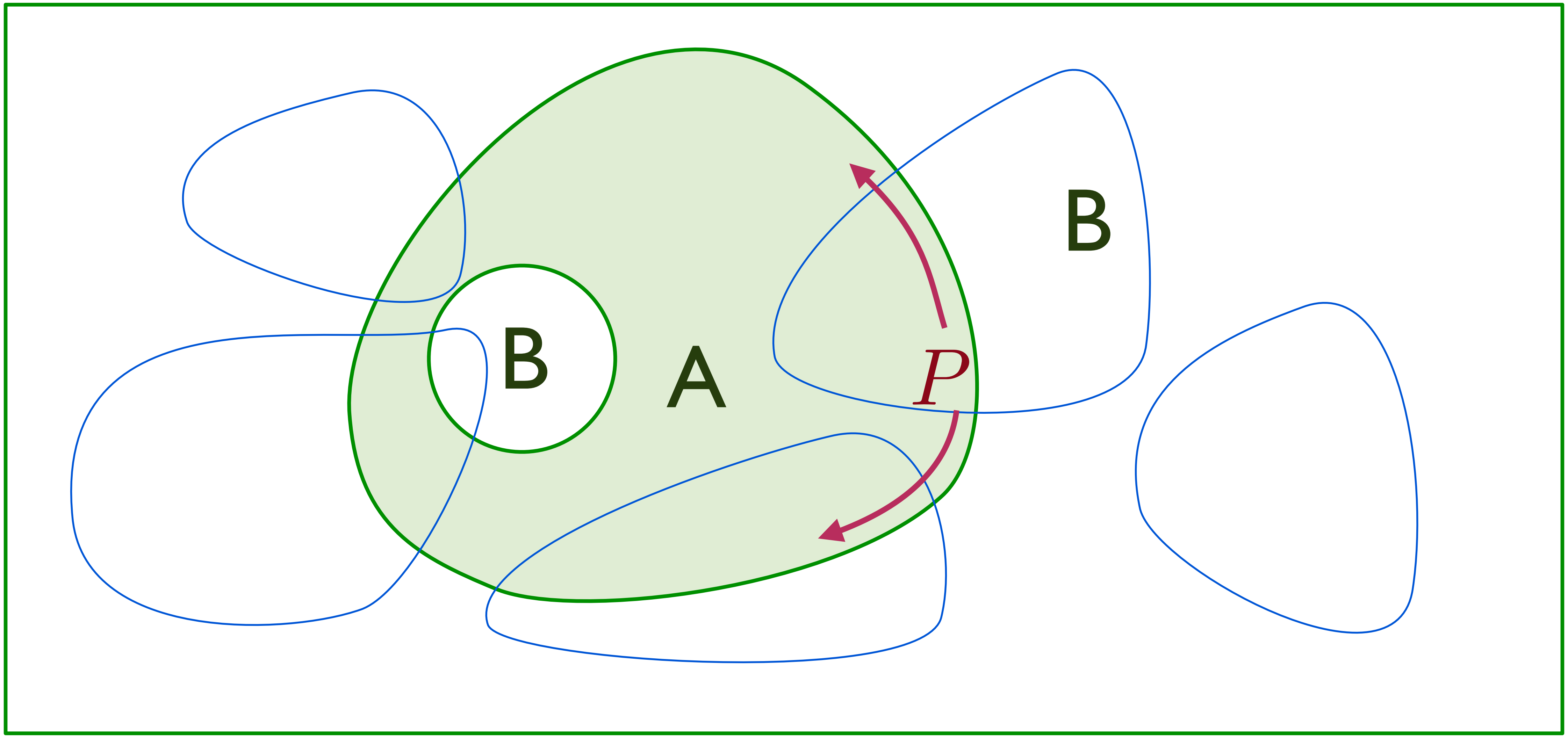


$$S_E = aP - \ln(2)$$

where P is the surface area (perimeter) of the boundary between A and B.

M. Levin and X.-G. Wen, *Phys. Rev. Lett.* **96**, 110405 (2006); A. Kitaev and J. Preskill, *Phys. Rev. Lett.* **96**, 110404 (2006);
Y. Zhang, T. Grover, and A. Vishwanath, *Phys. Rev. B* **84**, 075128 (2011).

Entanglement in the Z_2 spin liquid



$$\boxed{S_E = aP - \ln(4)}$$

where P is the surface area (perimeter) of the boundary between A and B.

M. Levin and X.-G. Wen, *Phys. Rev. Lett.* **96**, 110405 (2006); A. Kitaev and J. Preskill, *Phys. Rev. Lett.* **96**, 110404 (2006); Y. Zhang, T. Grover, and A. Vishwanath, *Phys. Rev. B* **84**, 075128 (2011).

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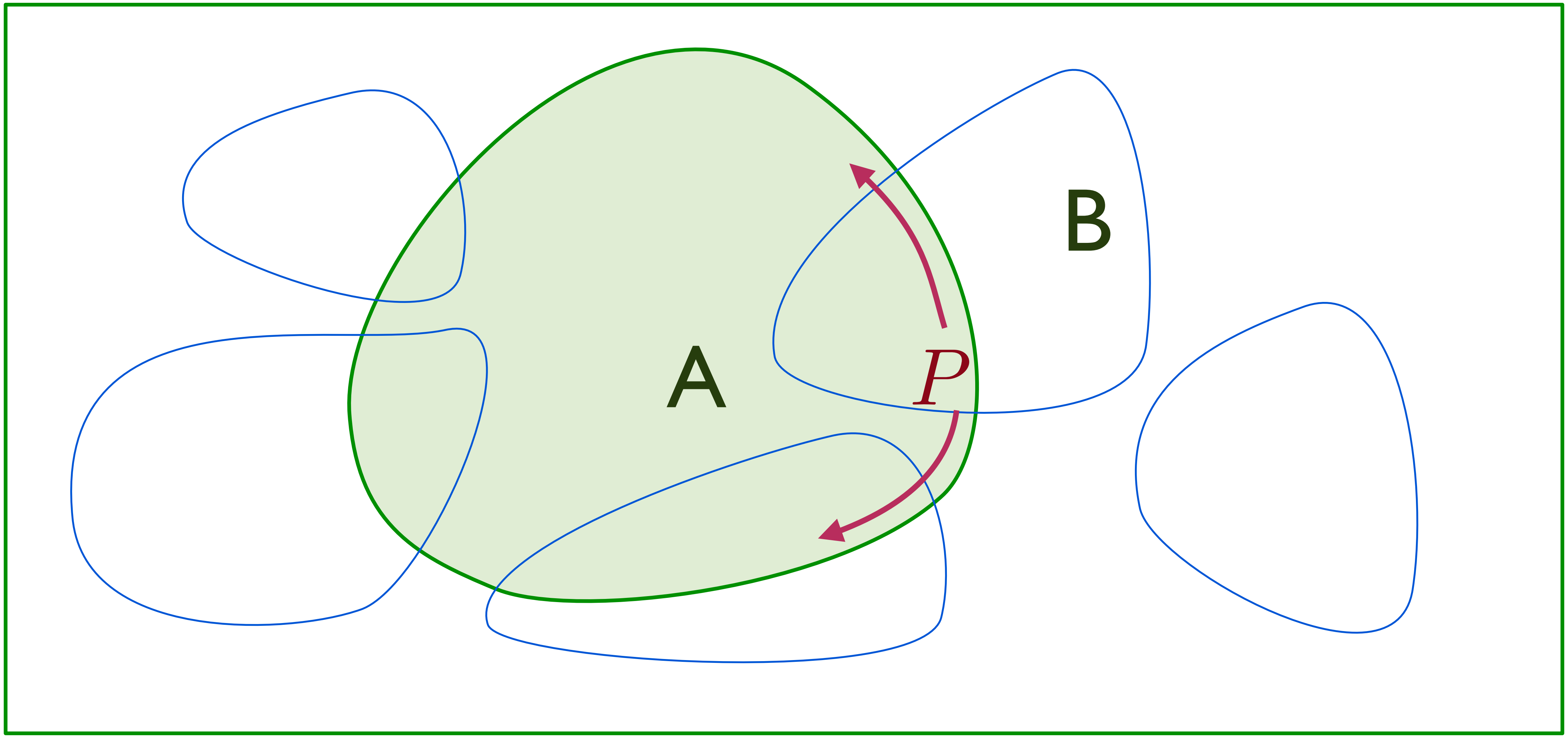

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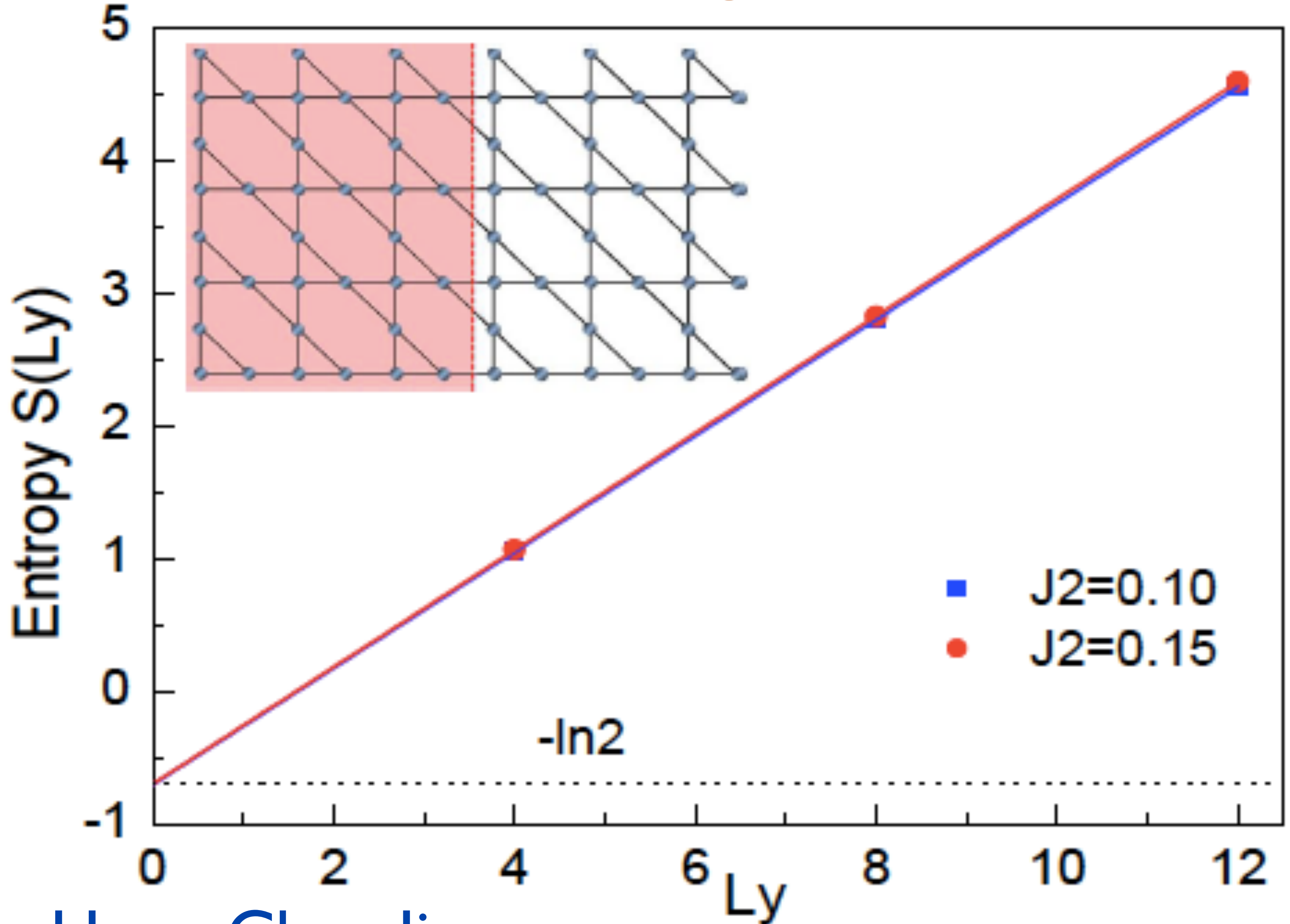
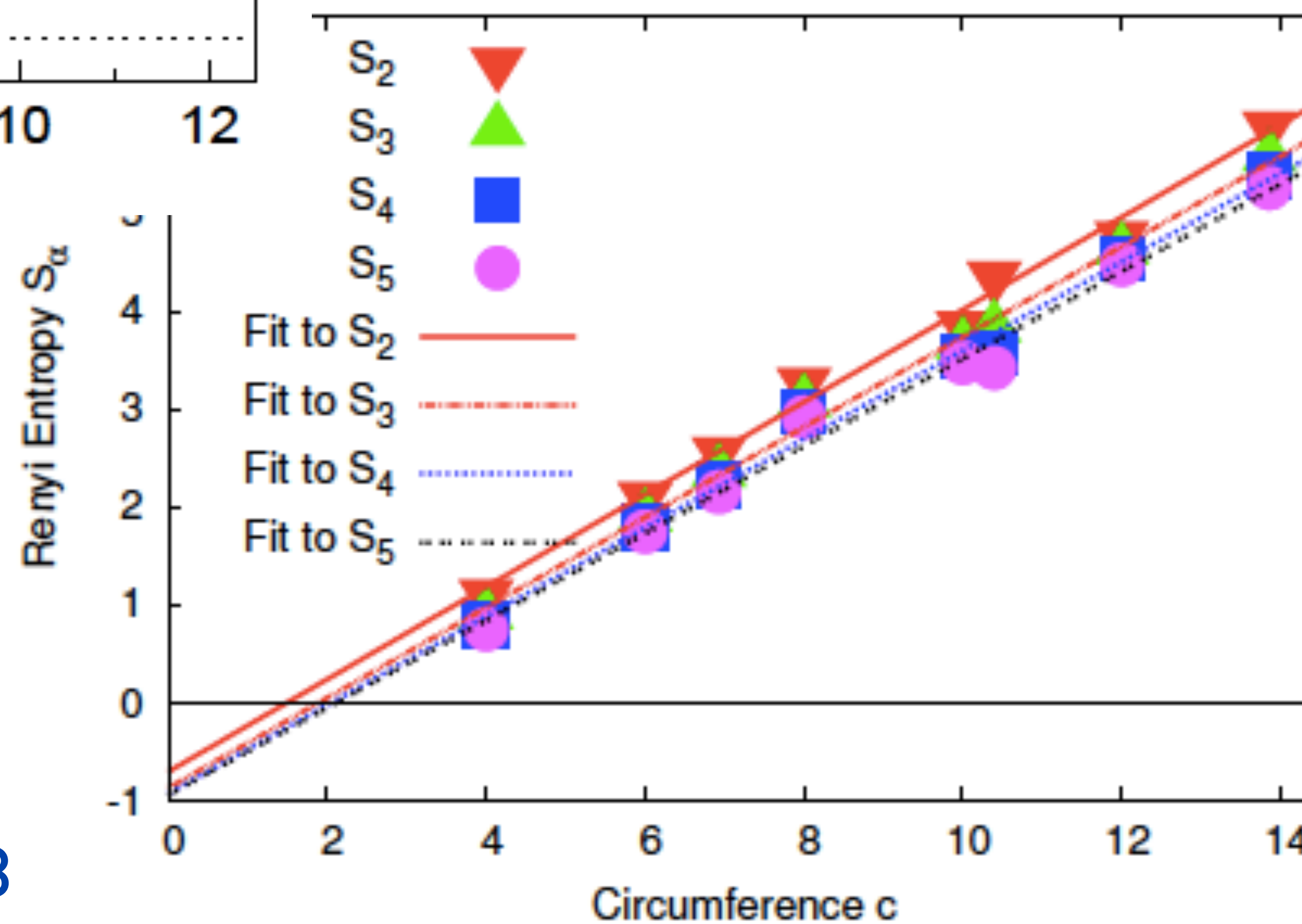
Mott insulator: Kagome antiferromagnet



Strong numerical evidence for a Z_2 spin liquid

Simeng Yan, D. A. Huse, and S. R. White, *Science* **332**, 1173 (2011).

Hong-Chen Jiang, Z. Wang, and L. Balents, arXiv:1205.4289

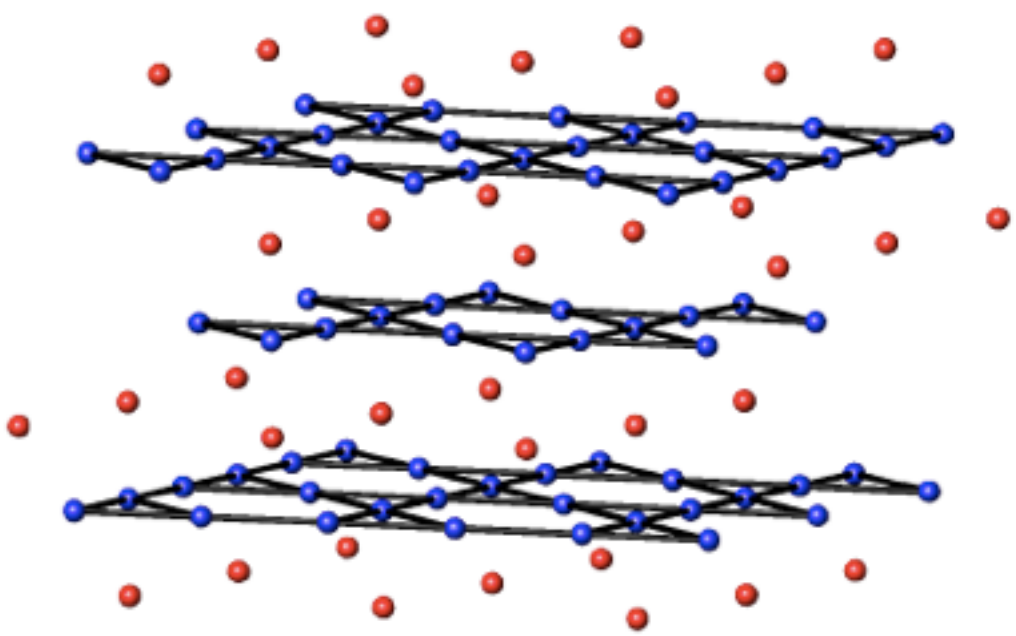


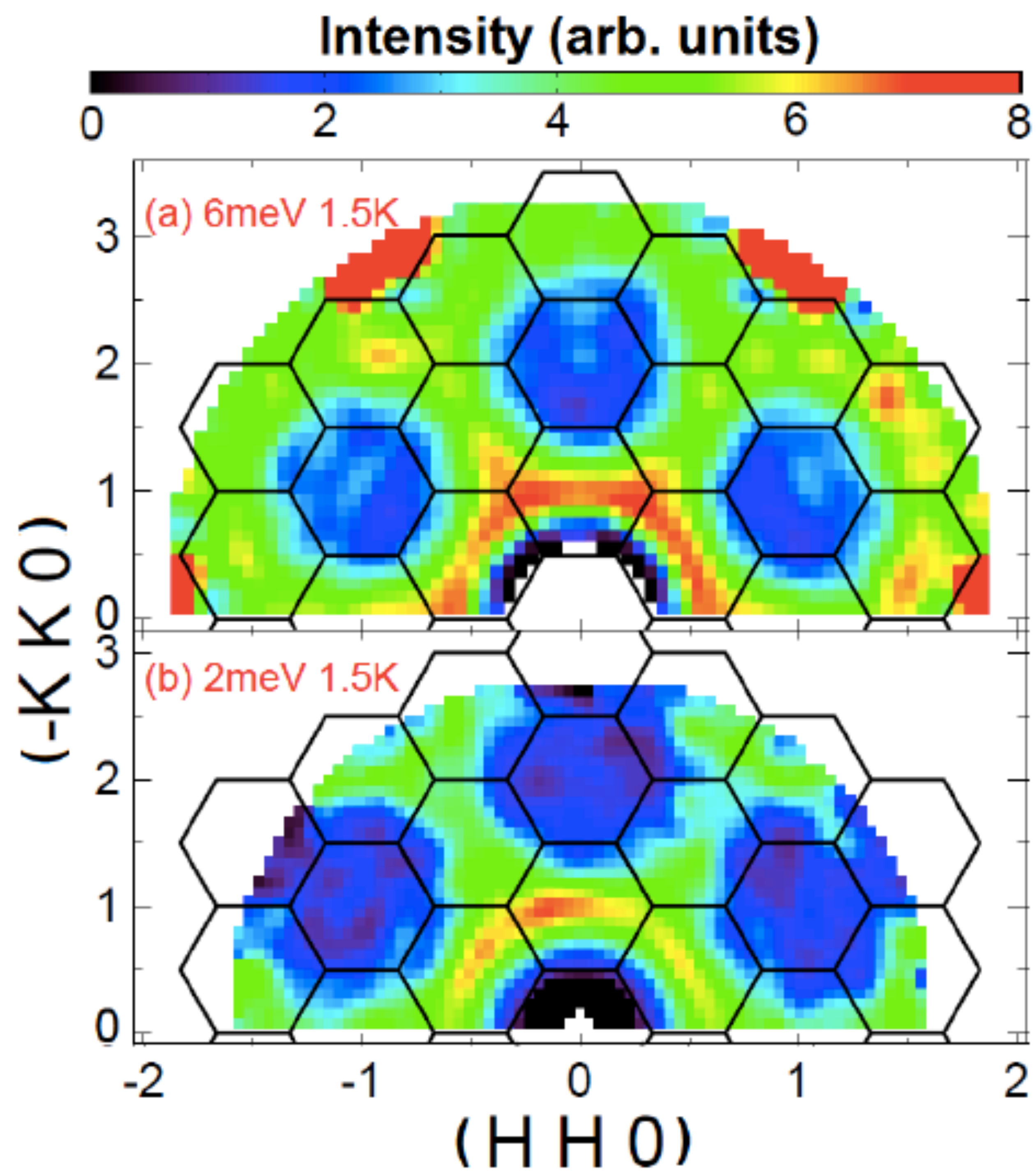
S. Depenbrock, I. P. McCulloch, and U. Schollwoeck, arXiv:1205.4858

Mott insulator: Kagome antiferromagnet

Evidence for spinons
Young Lee,
APS meeting, March 2012

$ZnCu_3(OH)_6Cl_2$ (also called Herbertsmithite)





“Complex entangled” states of quantum matter in *d* spatial dimensions

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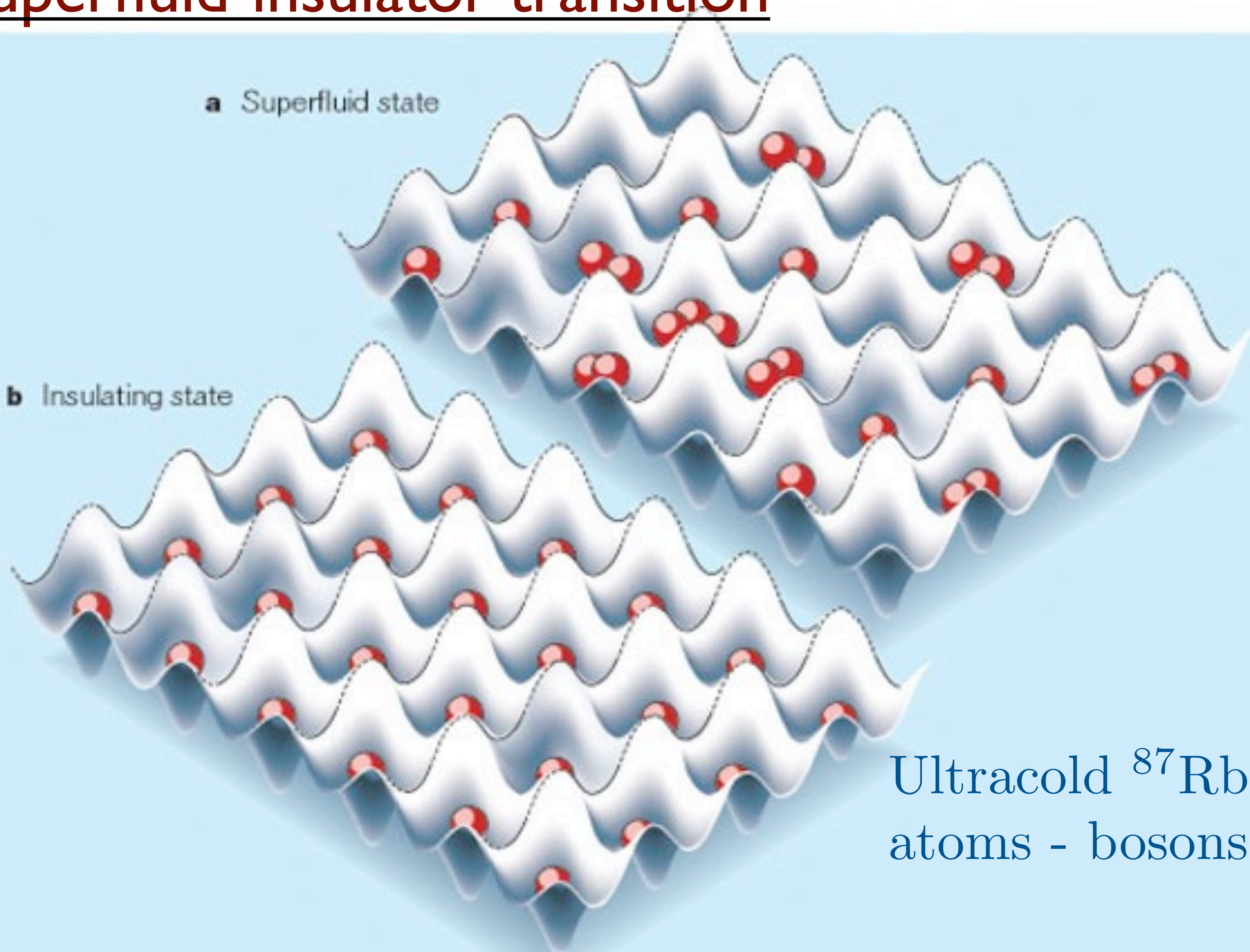
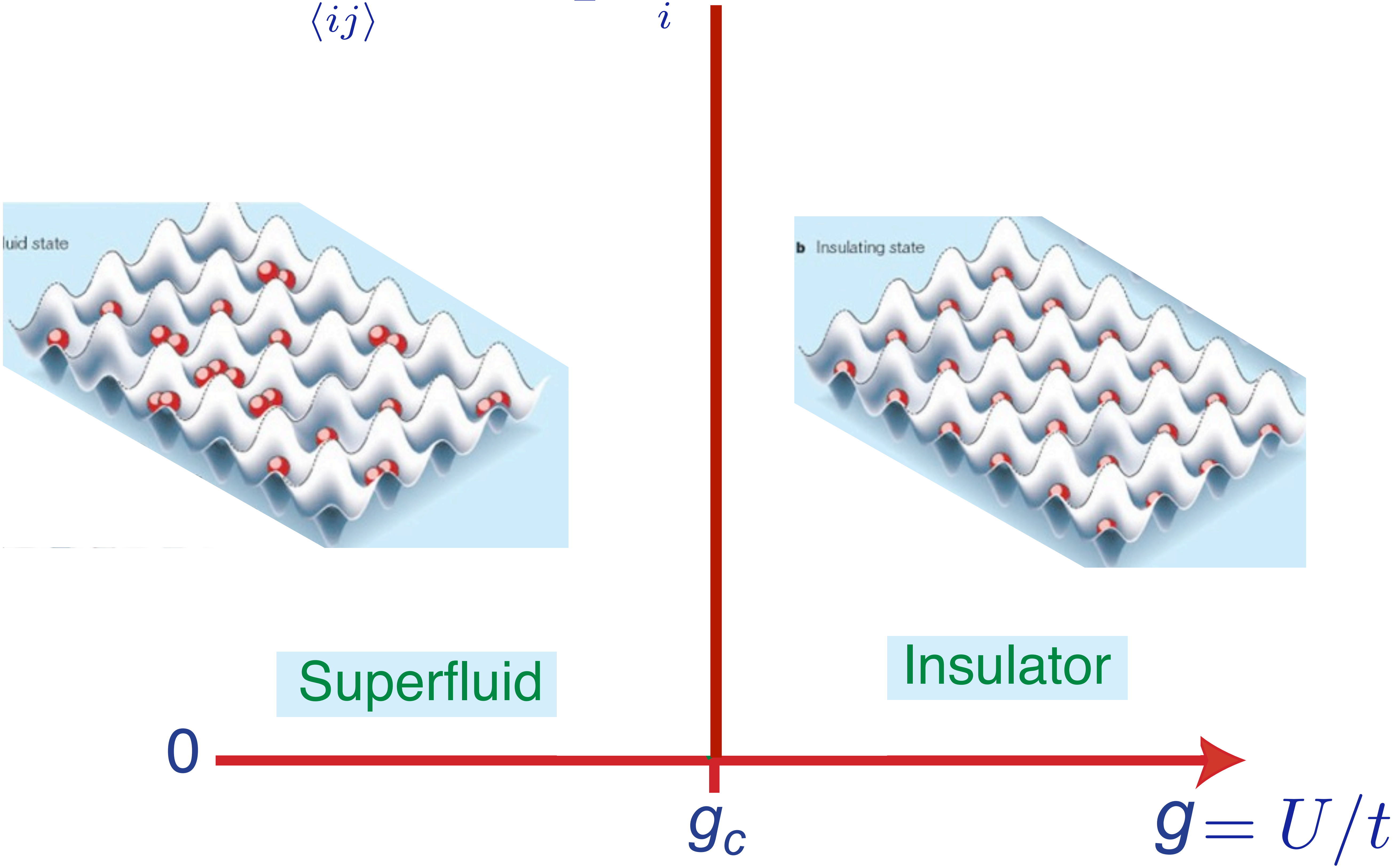


Ultracold ^{87}Rb
atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

$$H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i(n_i - 1) \quad ; \quad n_i \equiv b_i^\dagger b_i$$



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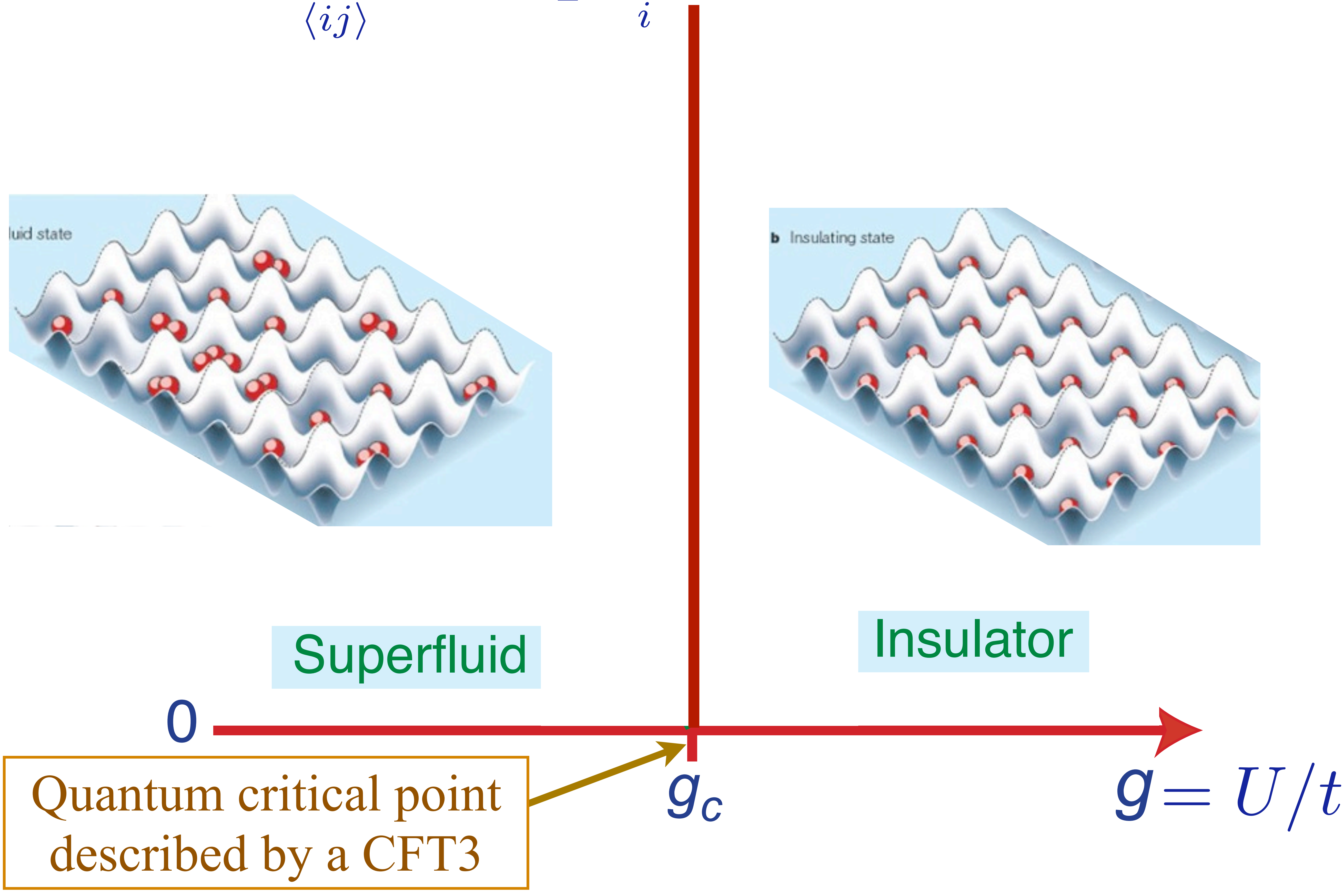
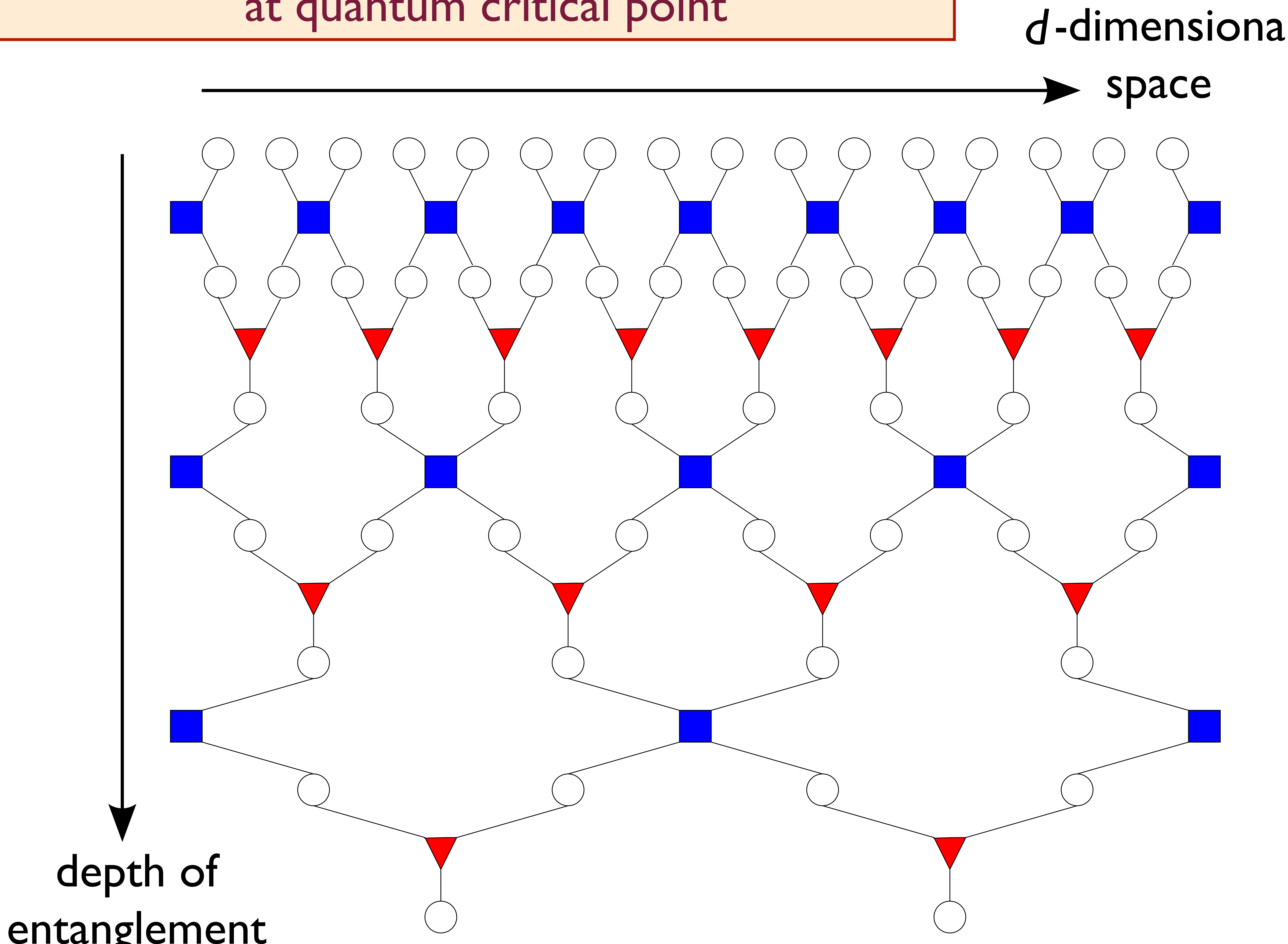


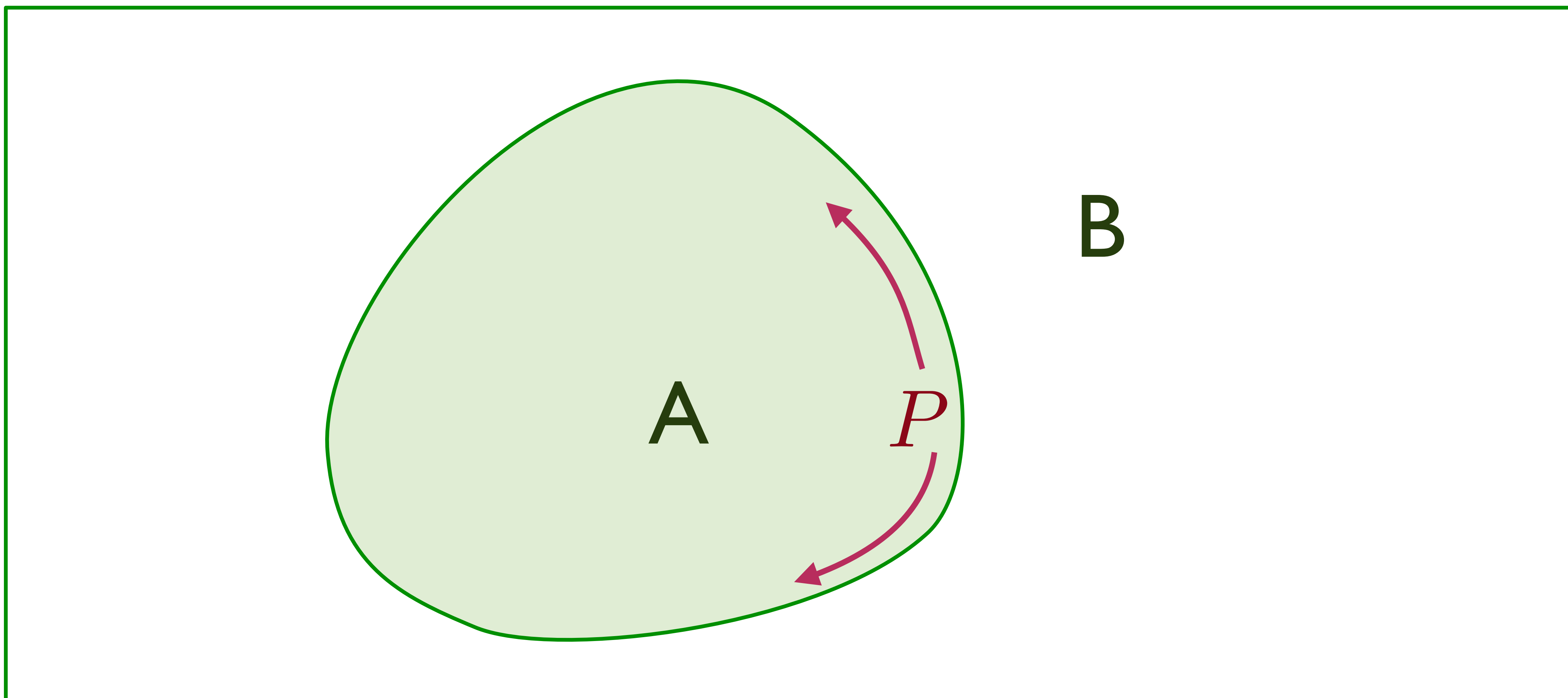
Tensor network representation of entanglement at quantum critical point



G.Vidal, Phys. Rev. Lett. 99, 220405 (2007)

Entanglement at the quantum critical point

- Entanglement entropy obeys $S_E = aP - \gamma$, where γ is a shape-dependent universal number associated with the CFT3.



M. A. Metlitski, C. A. Fuertes, and S. Sachdev, Physical Review B 80, 115122 (2009).
H. Casini, M. Huerta, and R. Myers, JHEP 1105:036, (2011)
I. Klebanov, S. Pufu, and B. Safdi, arXiv:1105.4598

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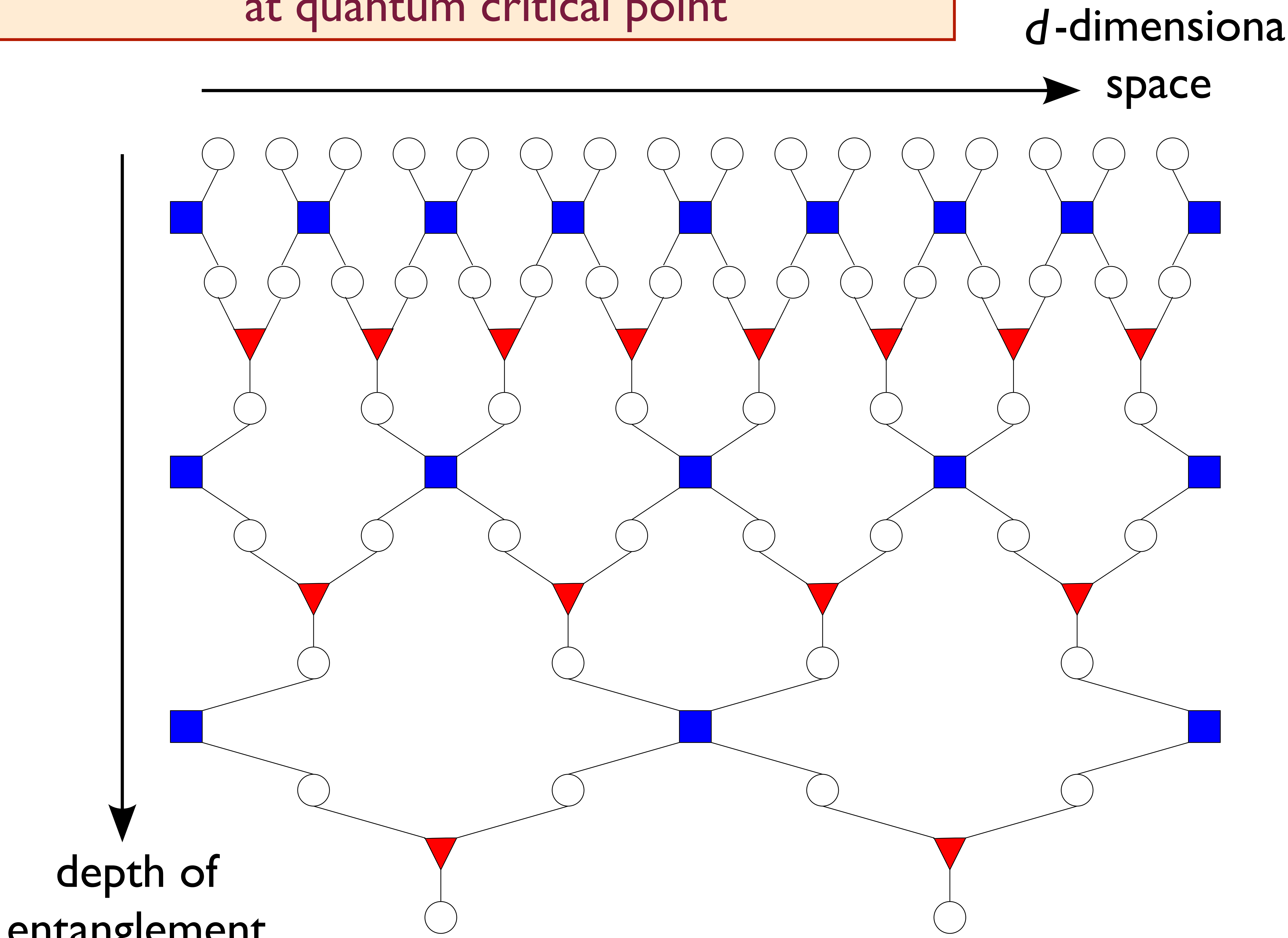

G. Vidal, Phys. Rev. Lett. 99, 220405 (2007)

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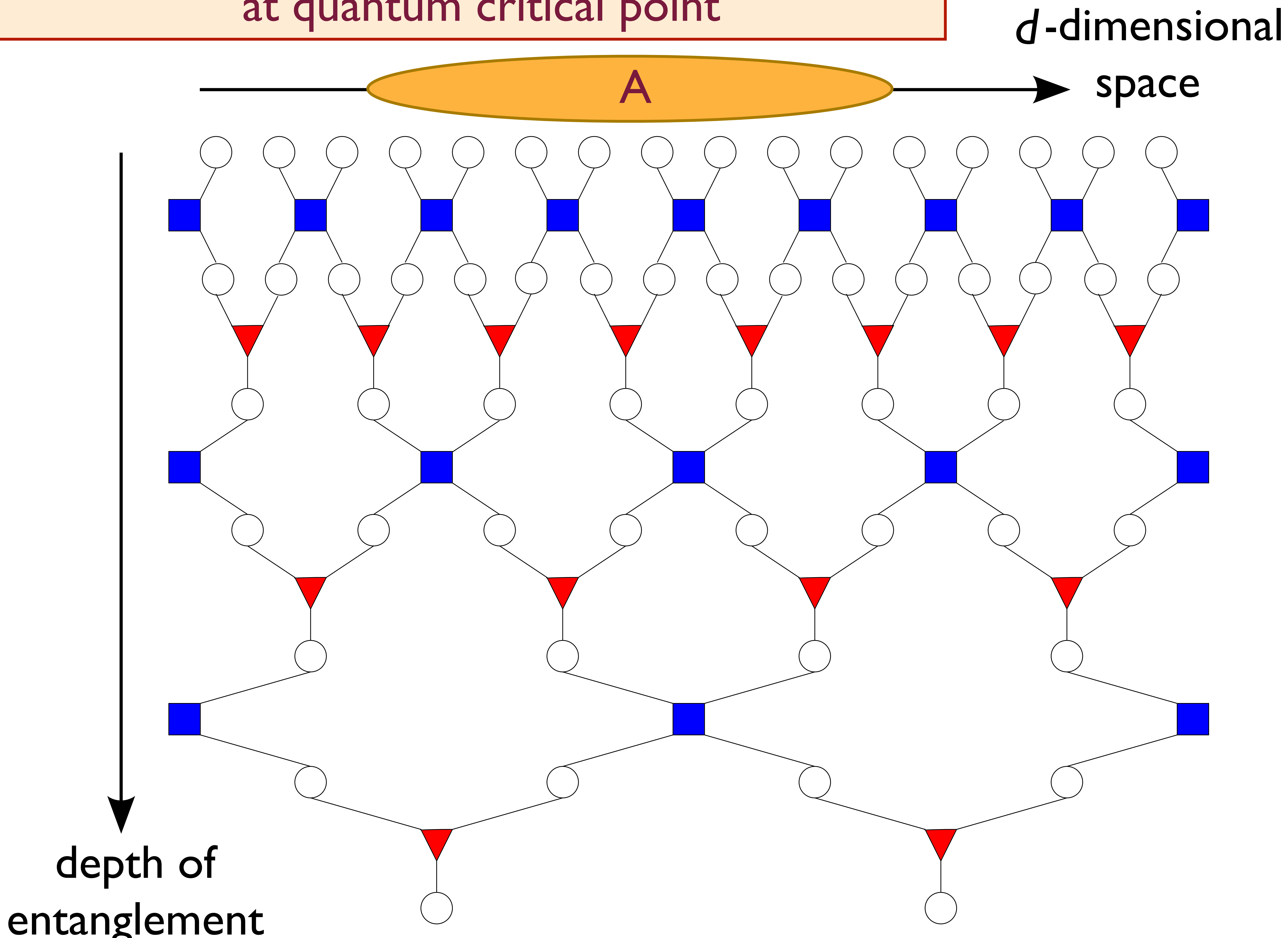

Brian Swingle, arXiv:0905.1317

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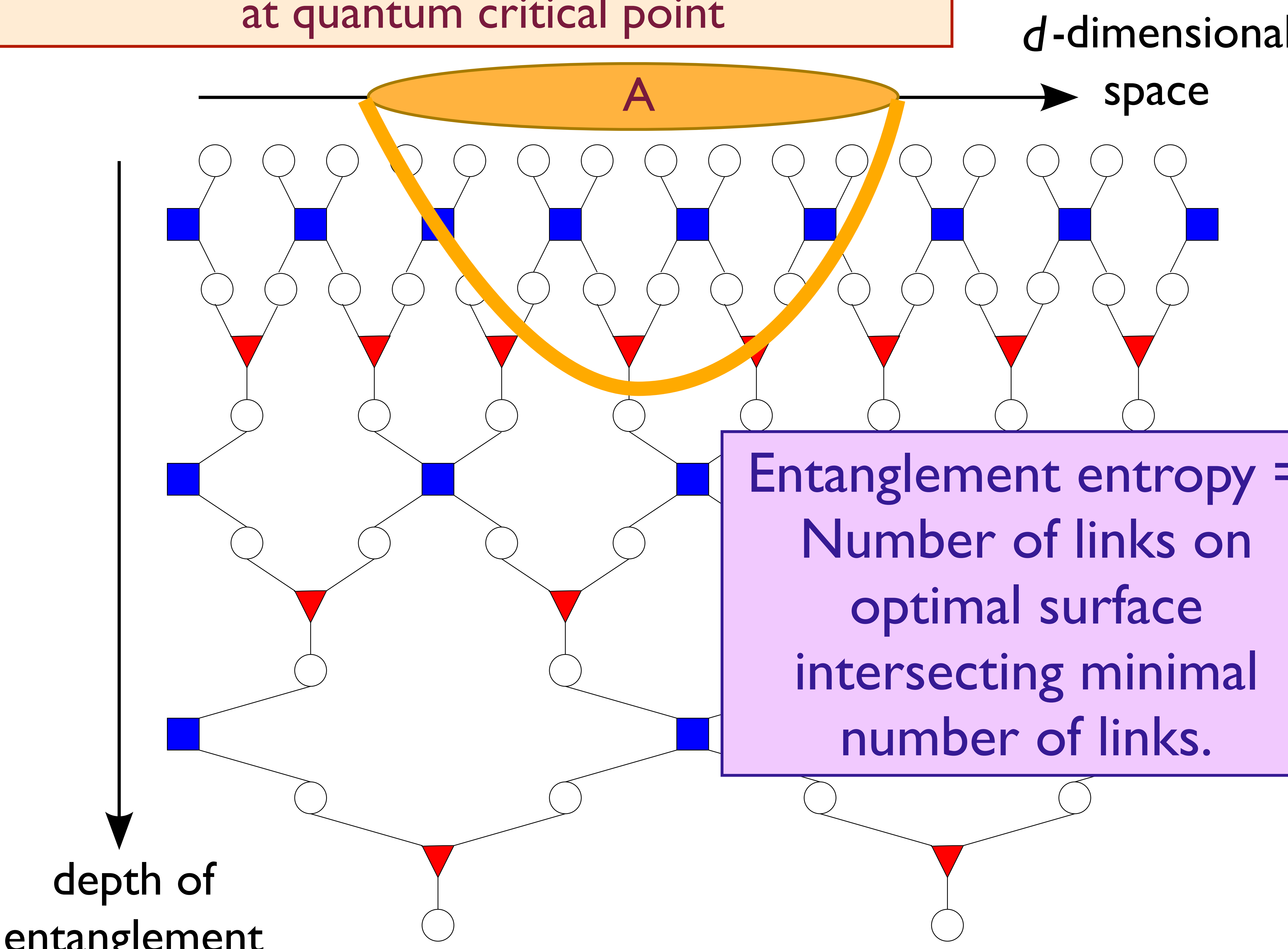
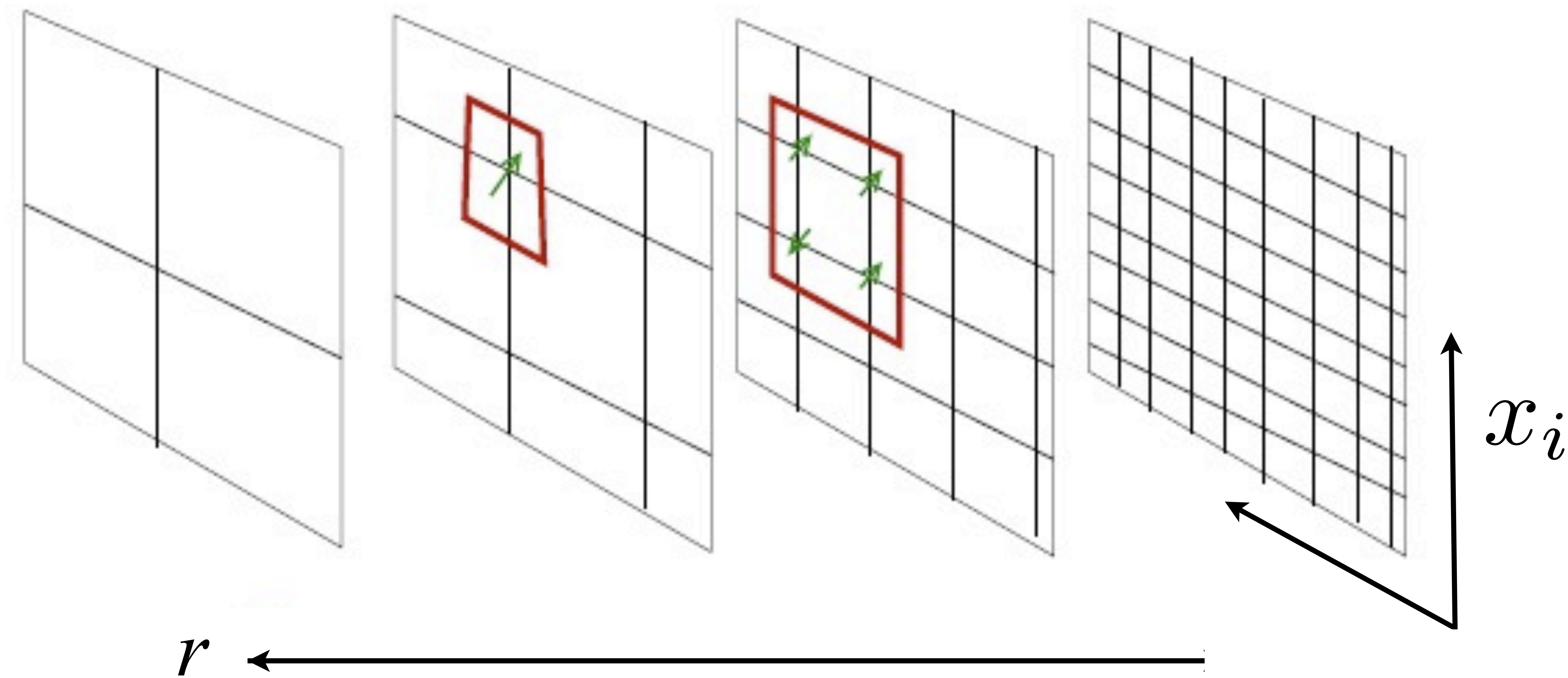


Entanglement entropy = Number of links on optimal surface intersecting minimal number of links.

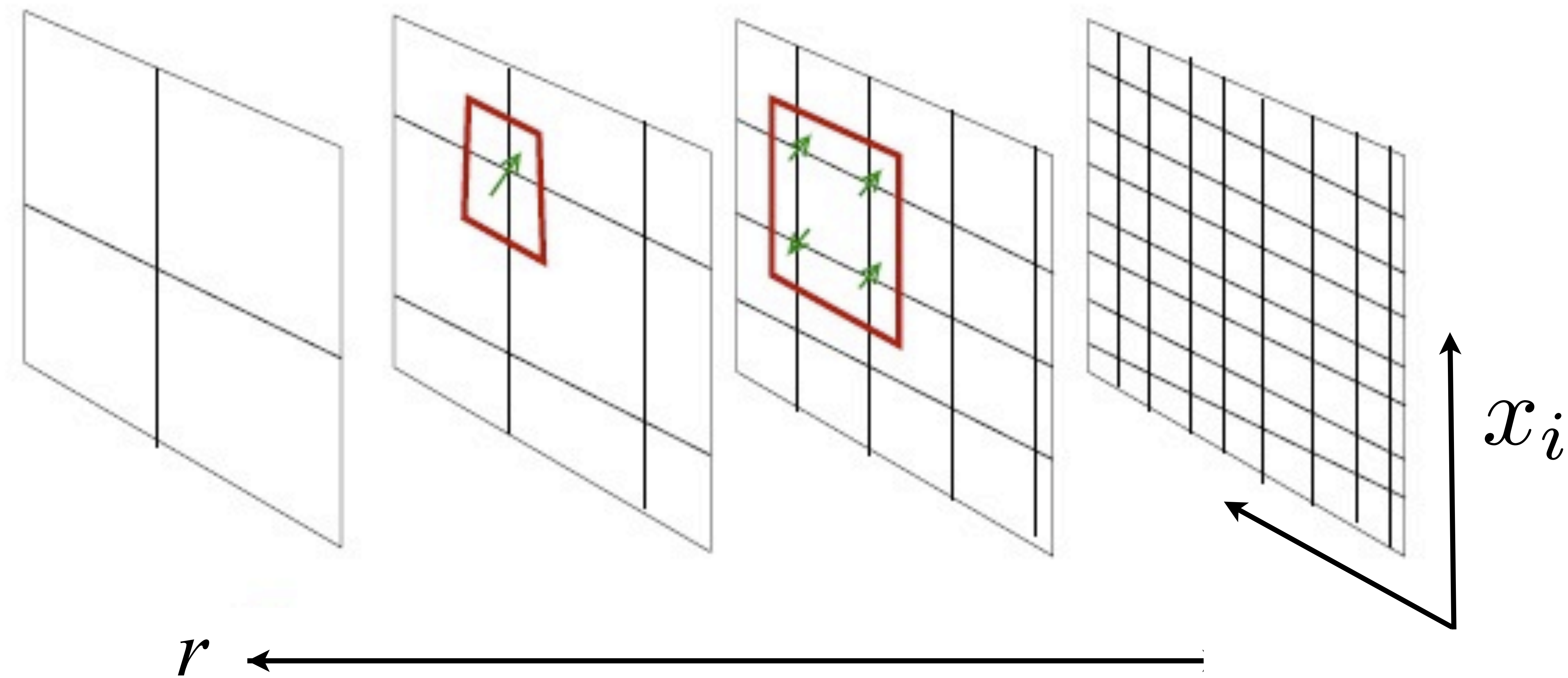
Brian Swingle, arXiv:0905.1317

Holography



Key idea: $\Rightarrow$ Implement r as an extra dimension, and map to a local theory in $d+2$ spacetime dimensions.

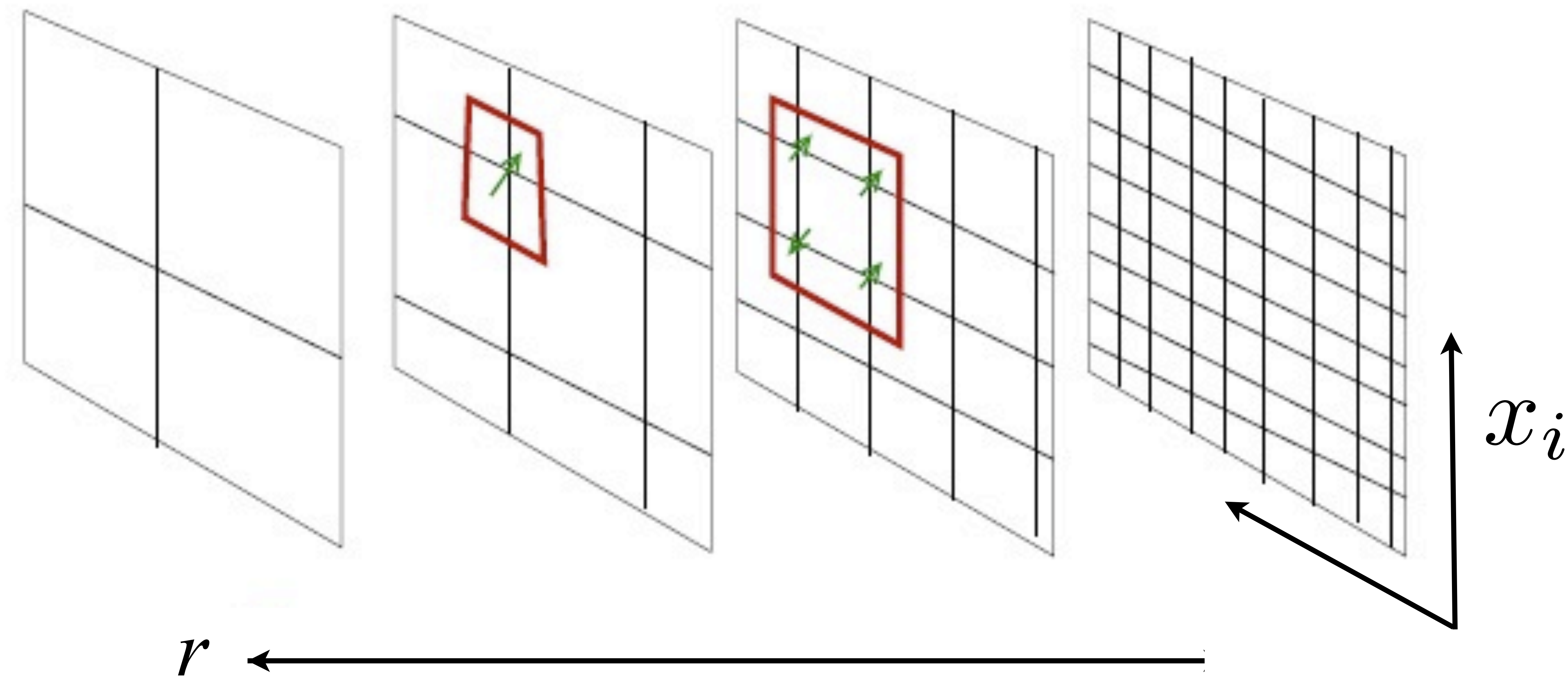
Holography



For a relativistic CFT in d spatial dimensions, the metric in the holographic space is fixed by demanding the scale transformation ($i = 1 \ldots d$)

$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad ds \to ds$$

Holography

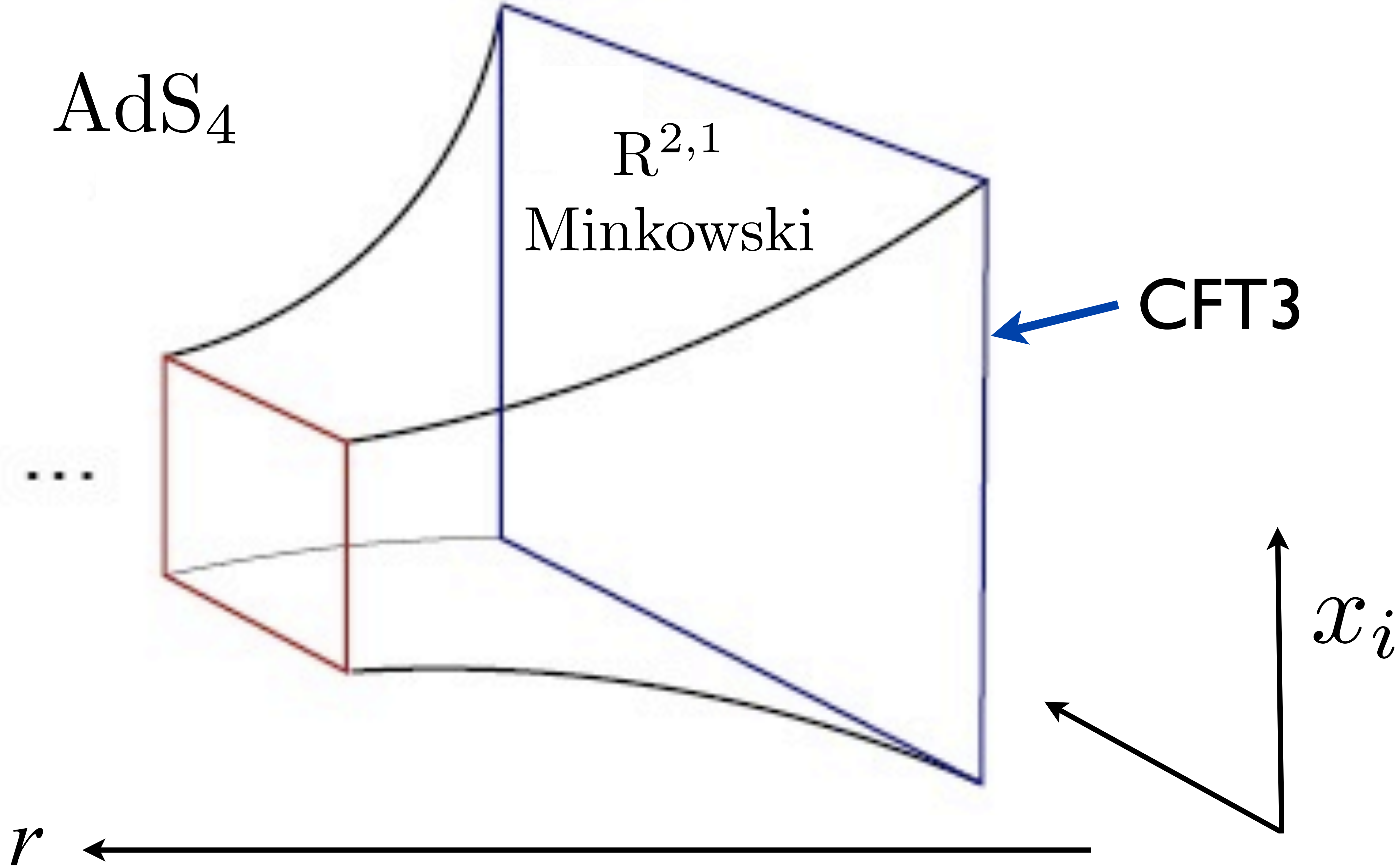


This gives the unique metric

$$ds^2 = \frac{1}{r^2}\left(-dt^2 + dr^2 + dx_i^2\right)$$

This is the metric of anti-de Sitter space AdS$_{d+2}$.

AdS/CFT correspondence



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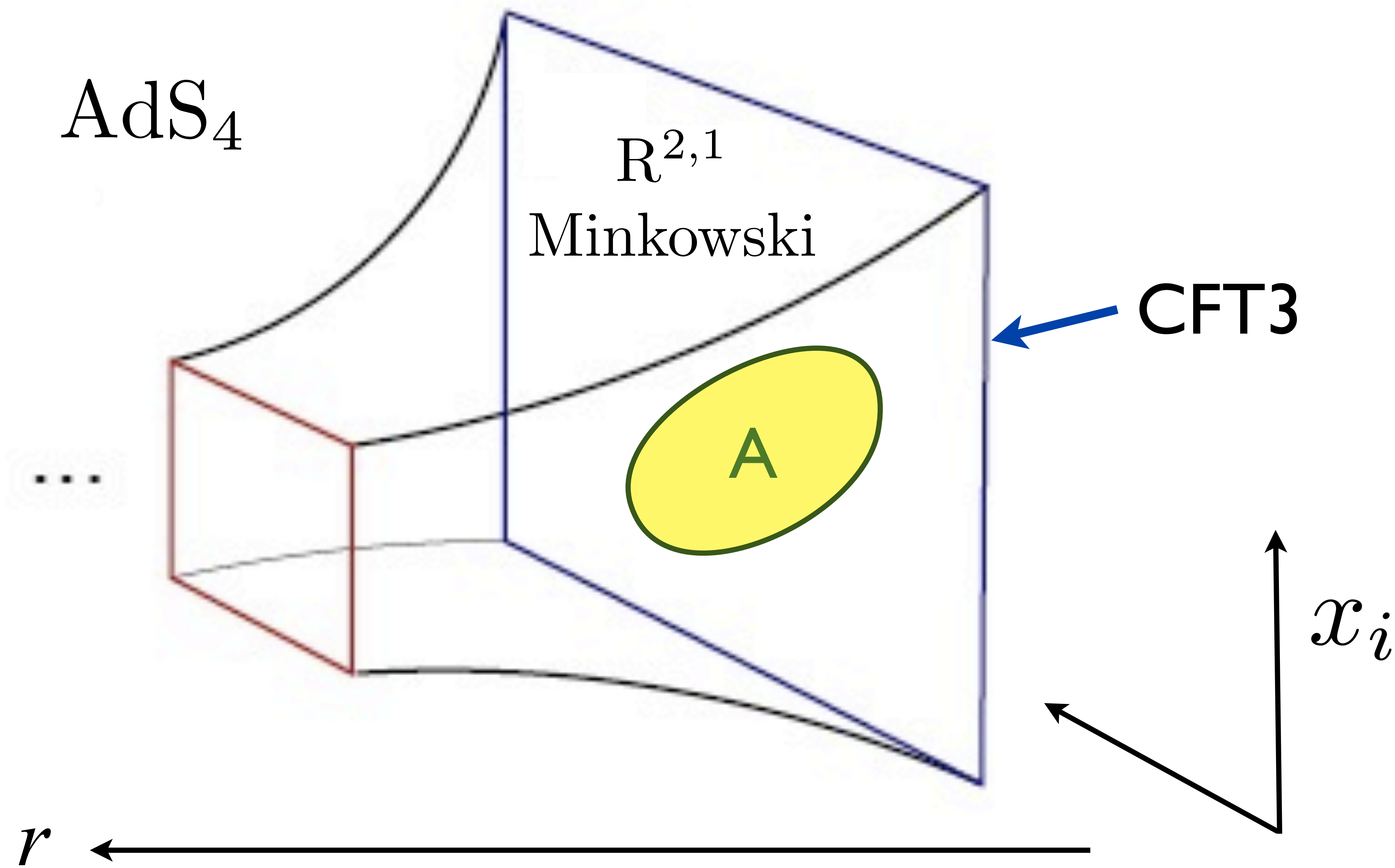

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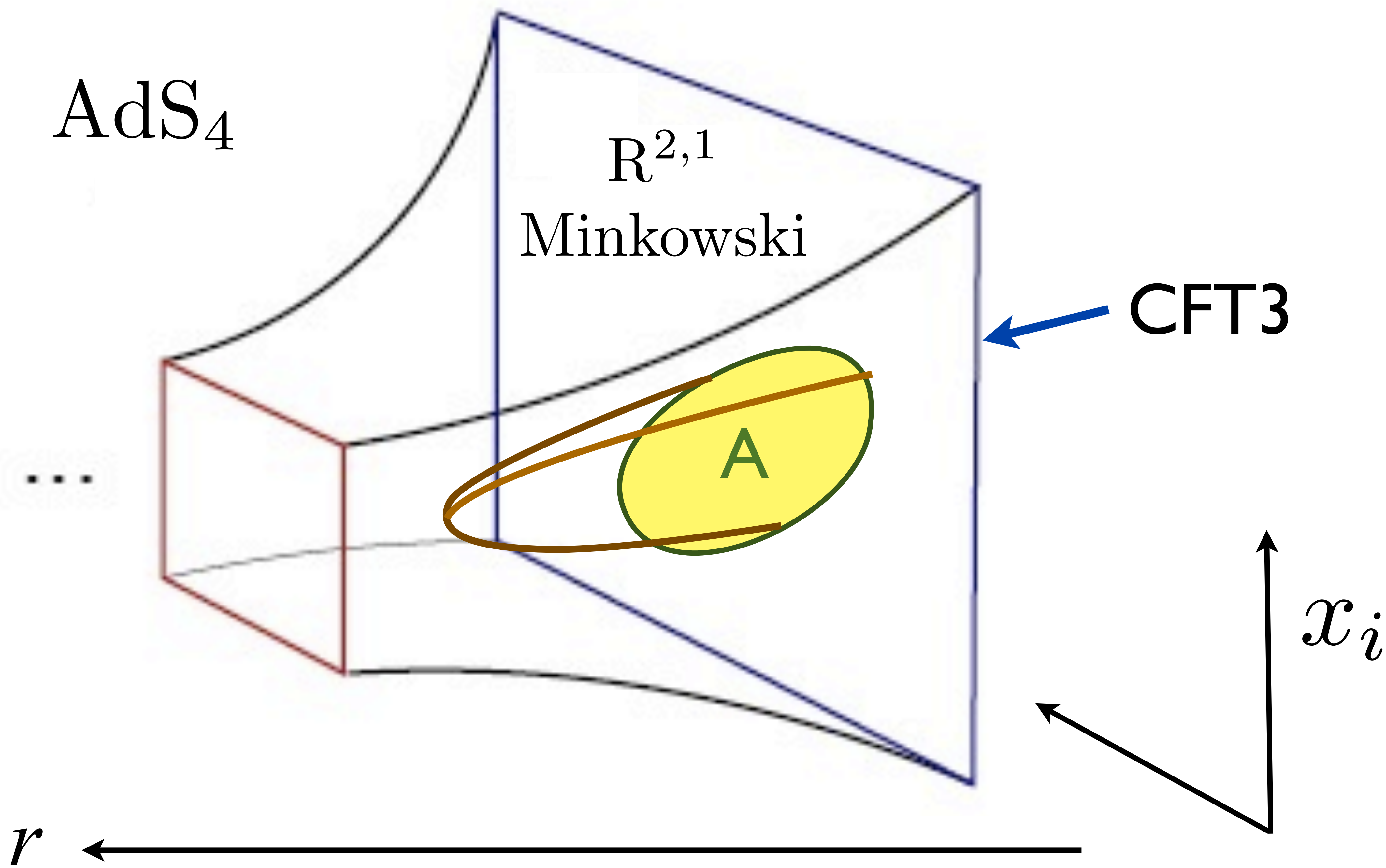
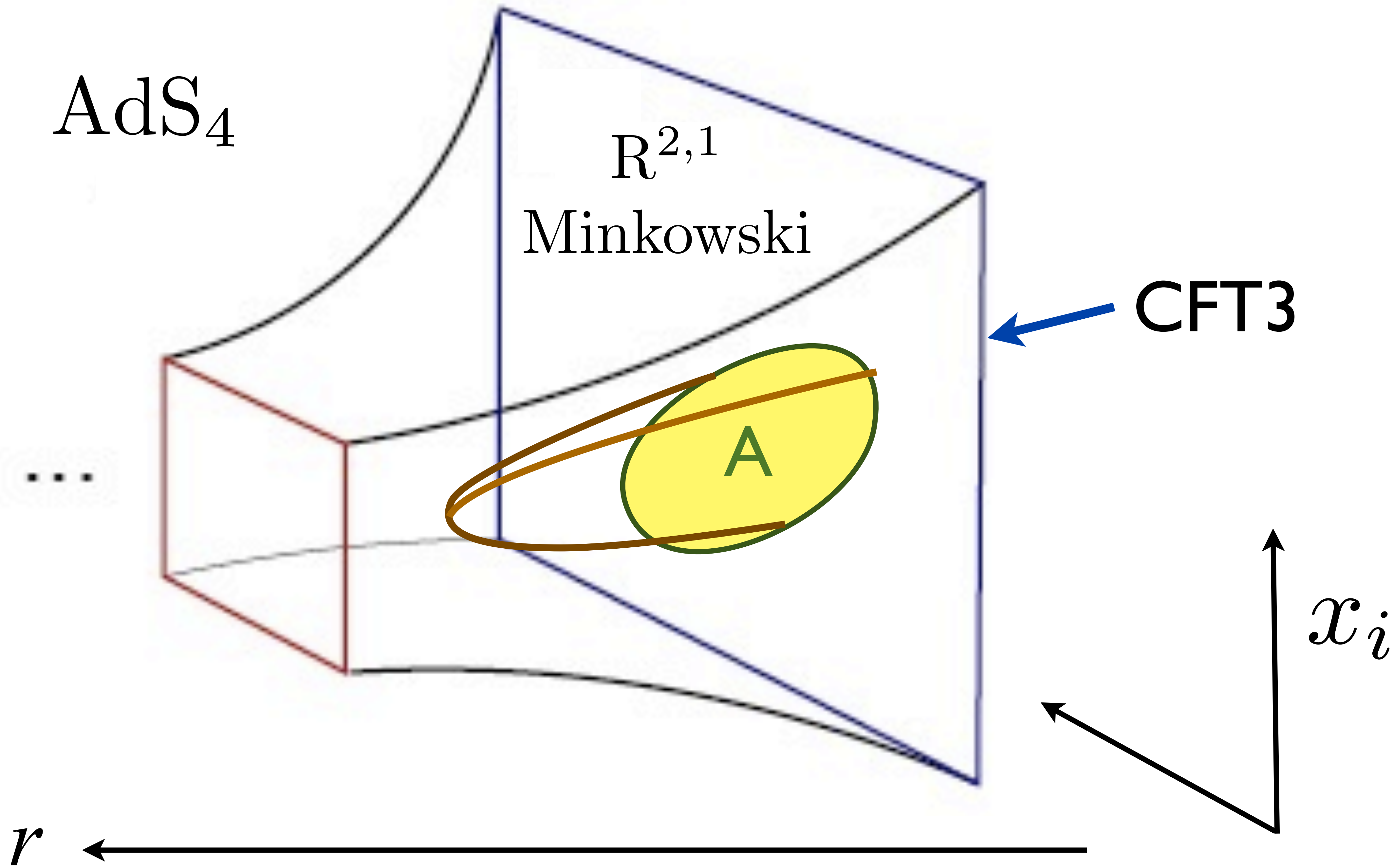


Associate entanglement entropy with an observer in the enclosed spacetime region, who cannot observe "outside" : *i.e.* the region is surrounded by an imaginary horizon.

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

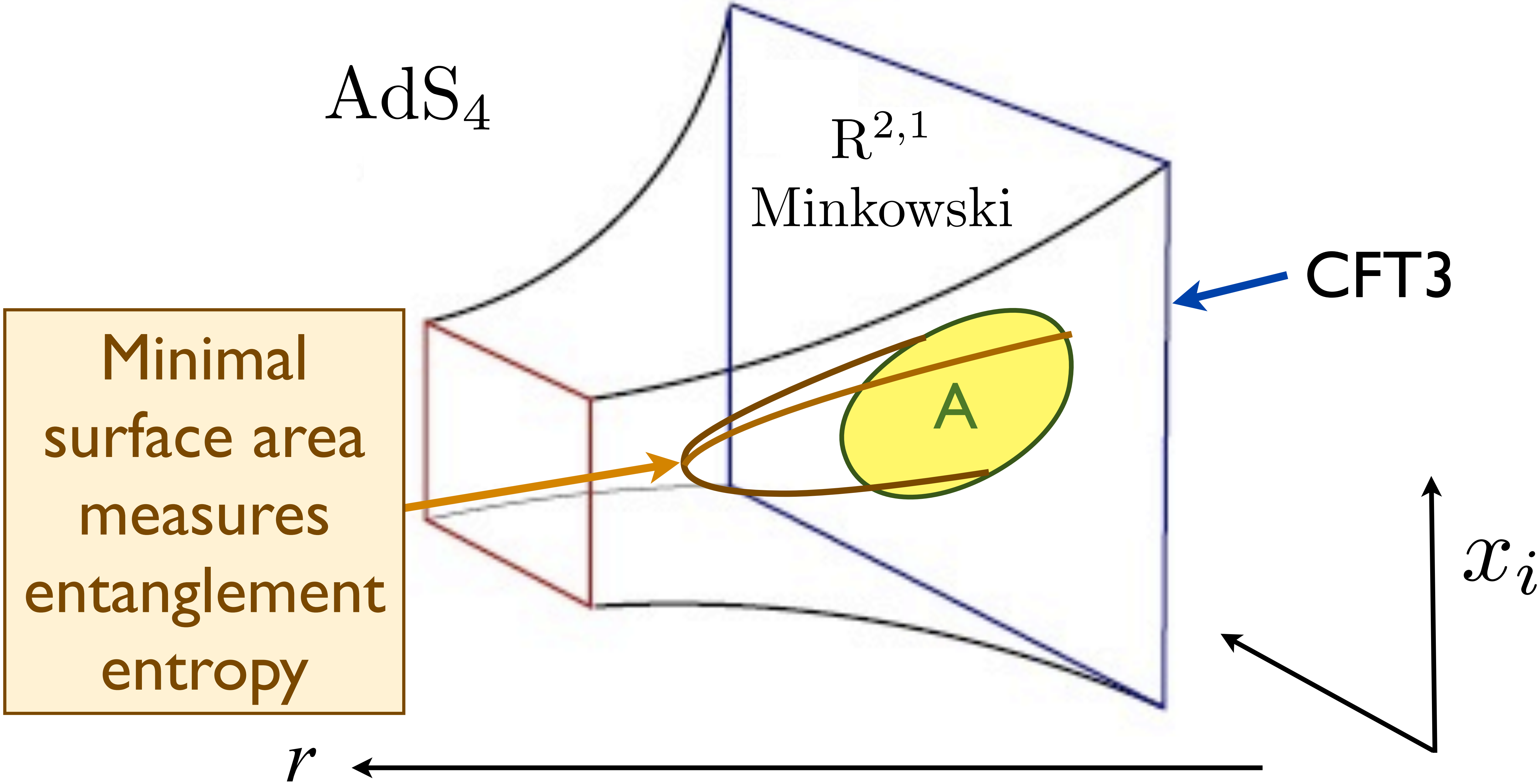
AdS/CFT correspondence



The entropy of this region is bounded by its surface area
(Bekenstein-Hawking-'t Hooft-Susskind)

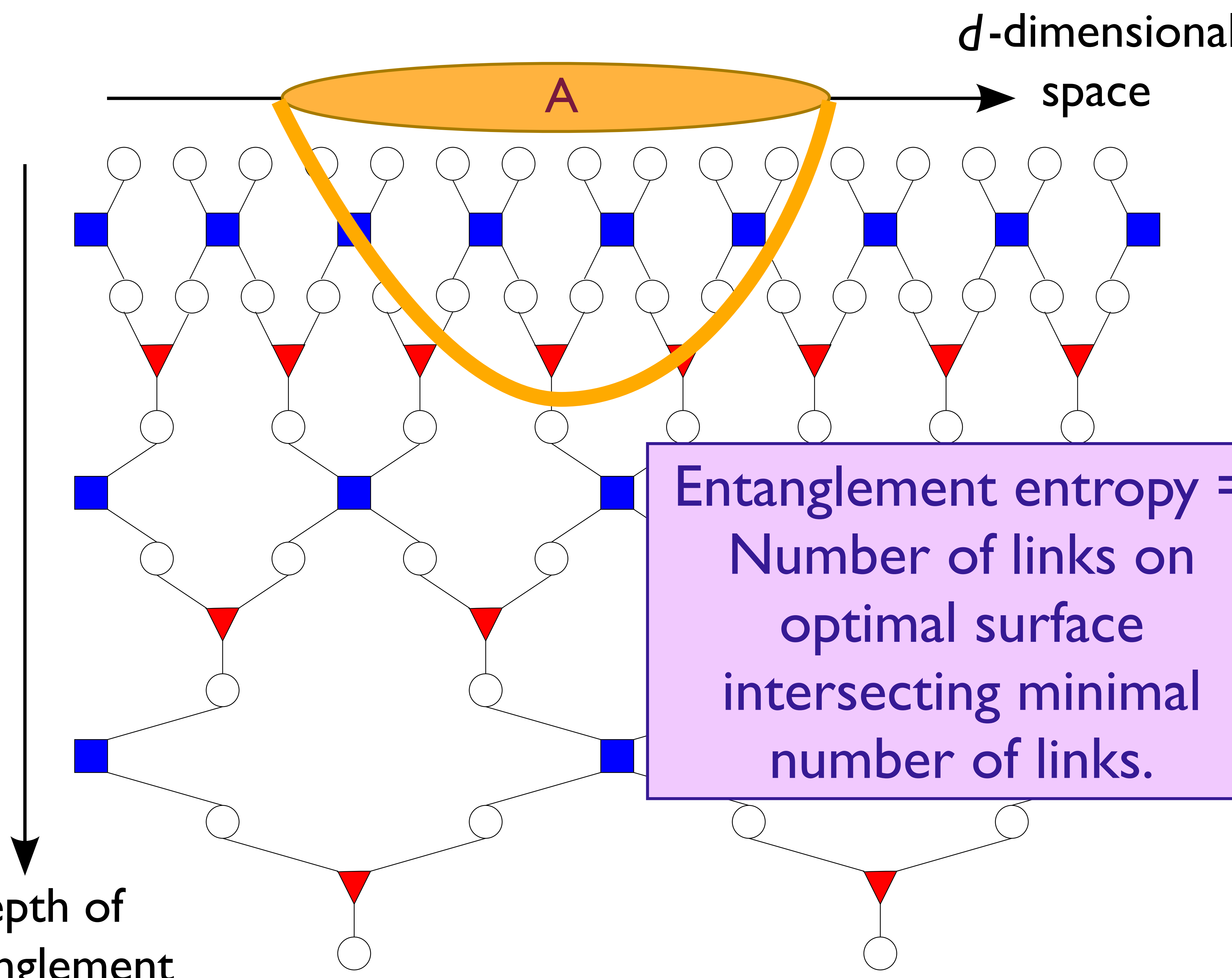
S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

AdS/CFT correspondence



S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

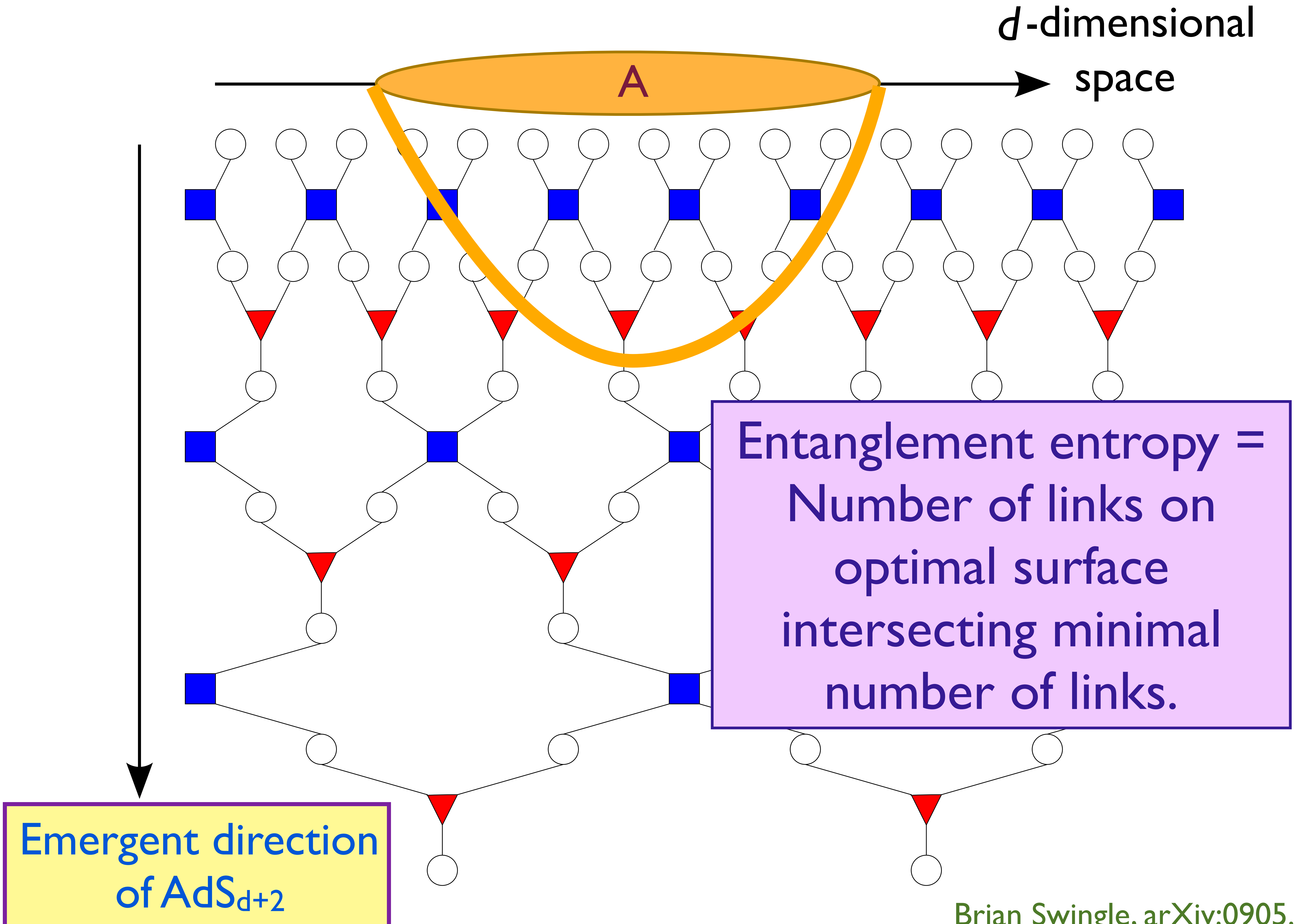
Entanglement entropy



Entanglement entropy = Number of links on optimal surface intersecting minimal number of links.

Brian Swingle, arXiv:0905.1317

Entanglement entropy



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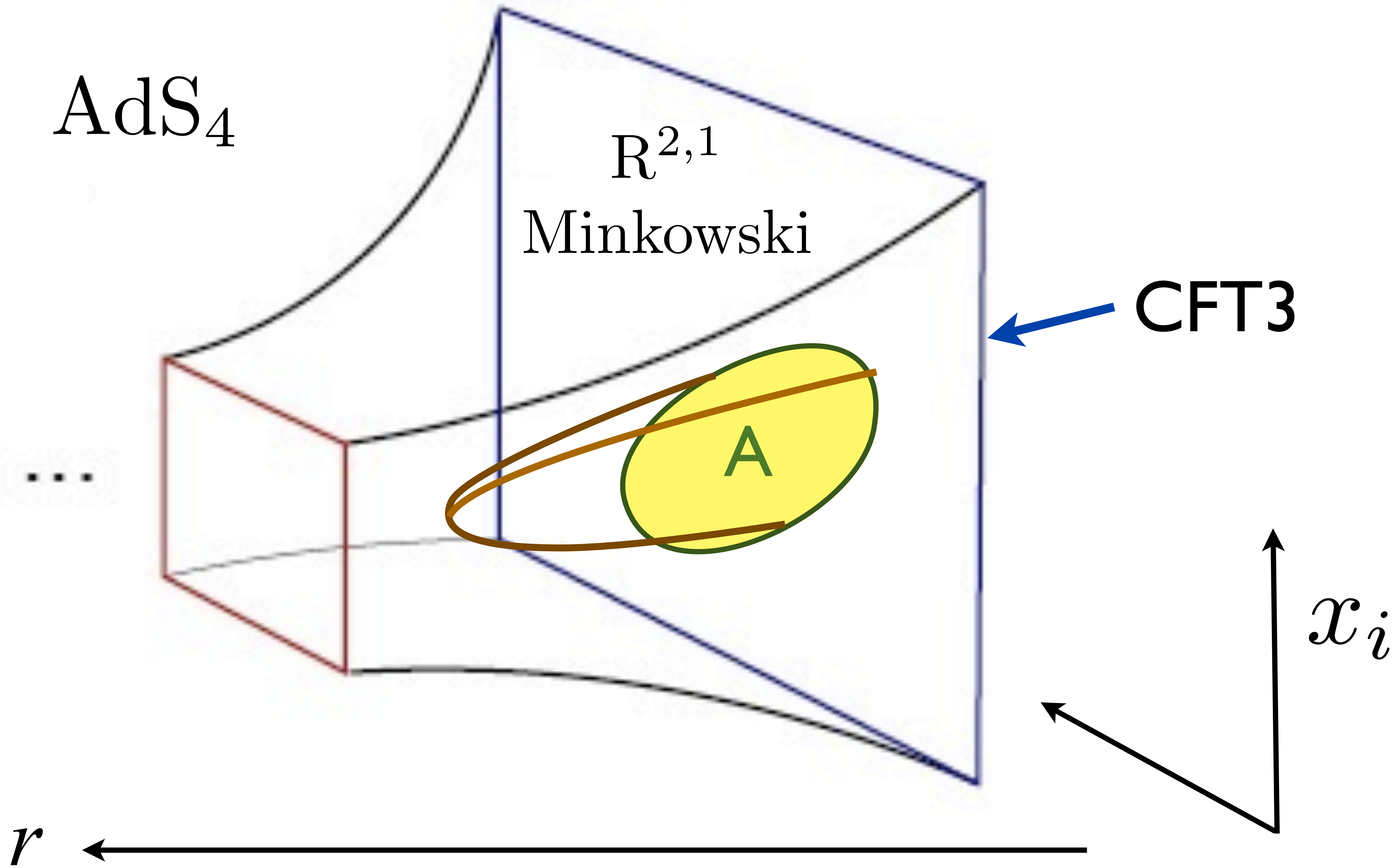


- Computation of minimal surface area yields

$$S_E = aP - \gamma,$$

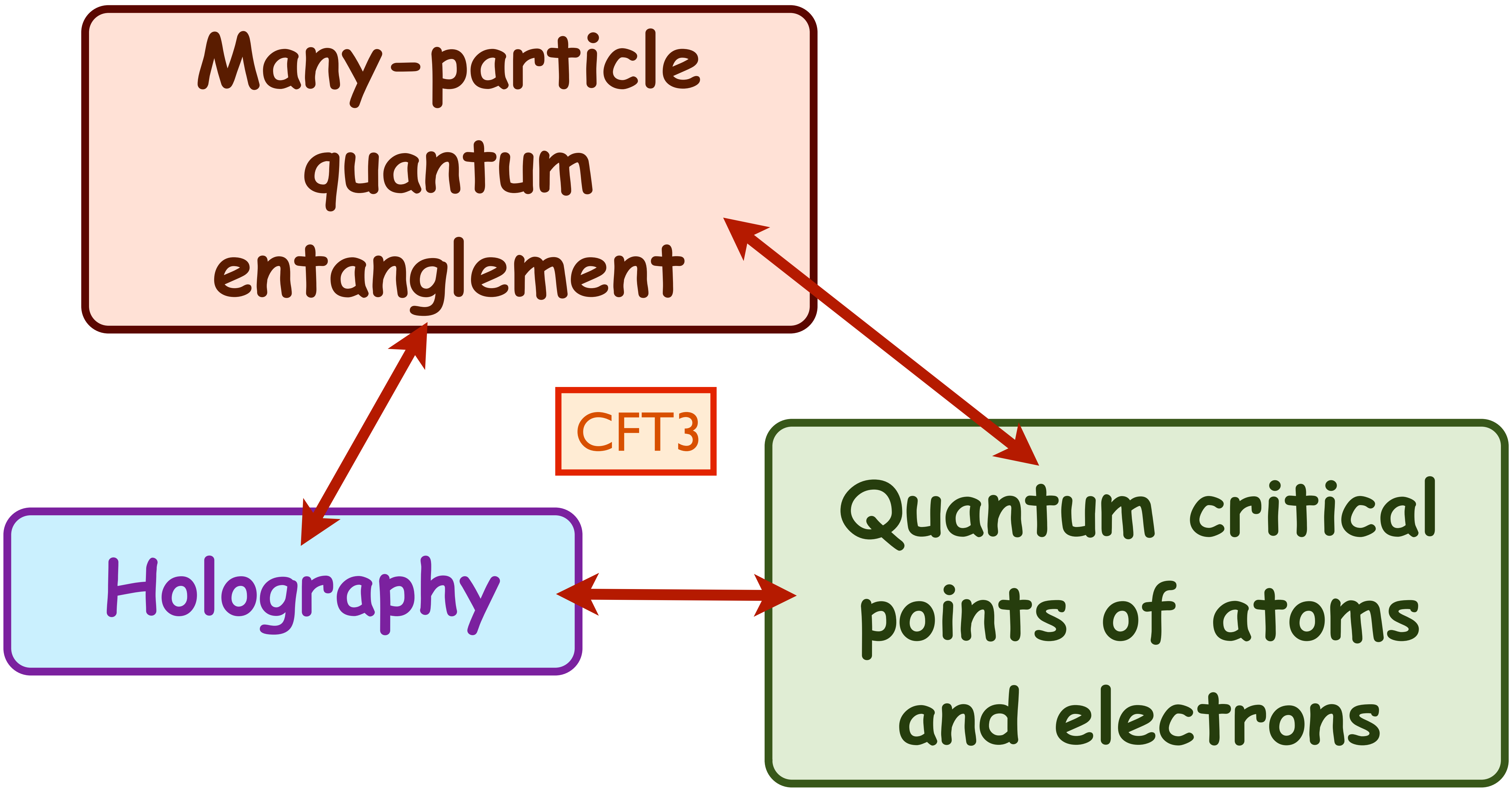
where γ is a shape-dependent universal number.

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

Many-particle quantum entanglement

Holography

Quantum critical points of atoms and electrons

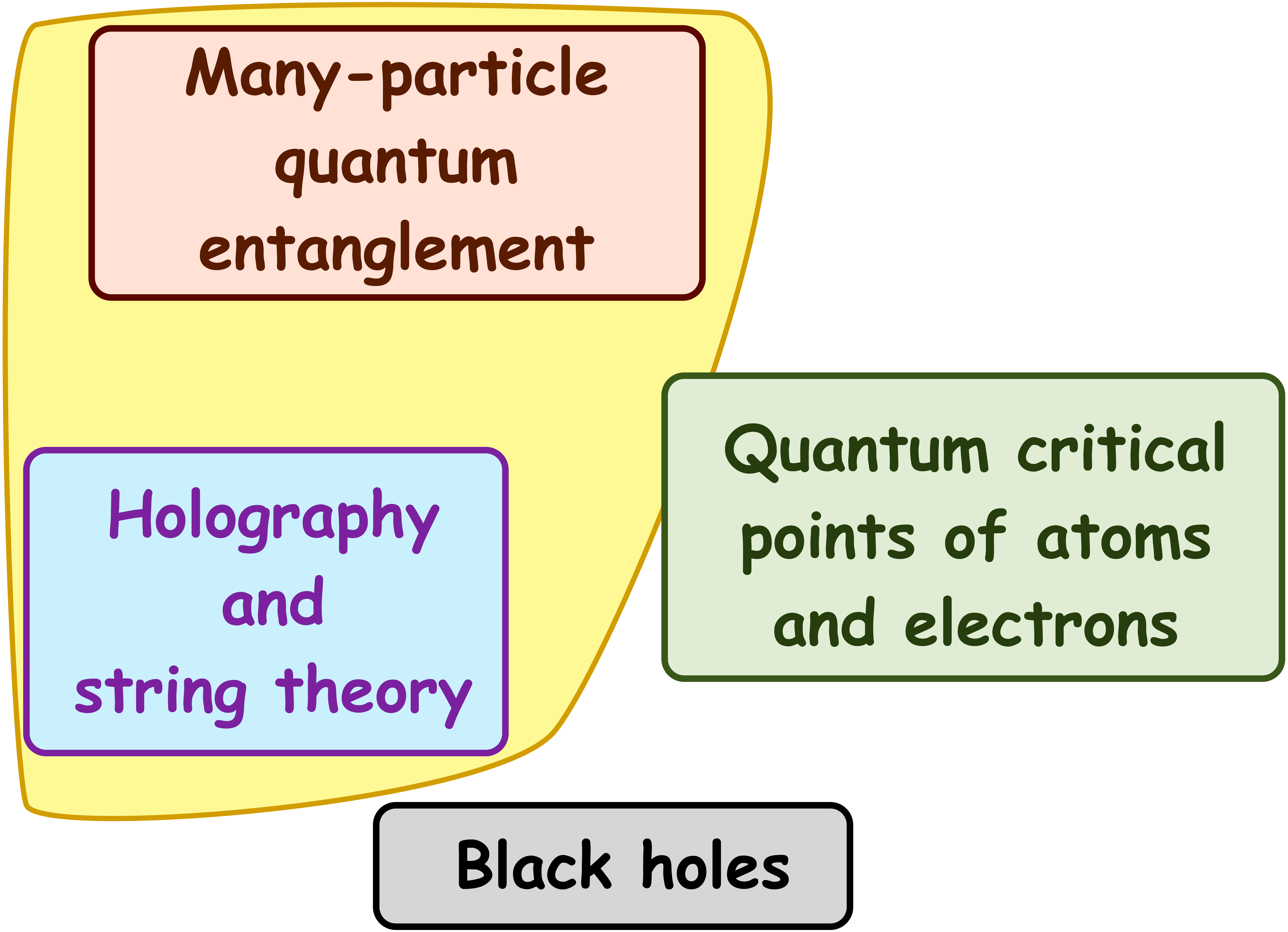

Many-particle quantum entanglement
CFT3
Holography
Quantum critical points of atoms and electrons

Many-particle quantum entanglement

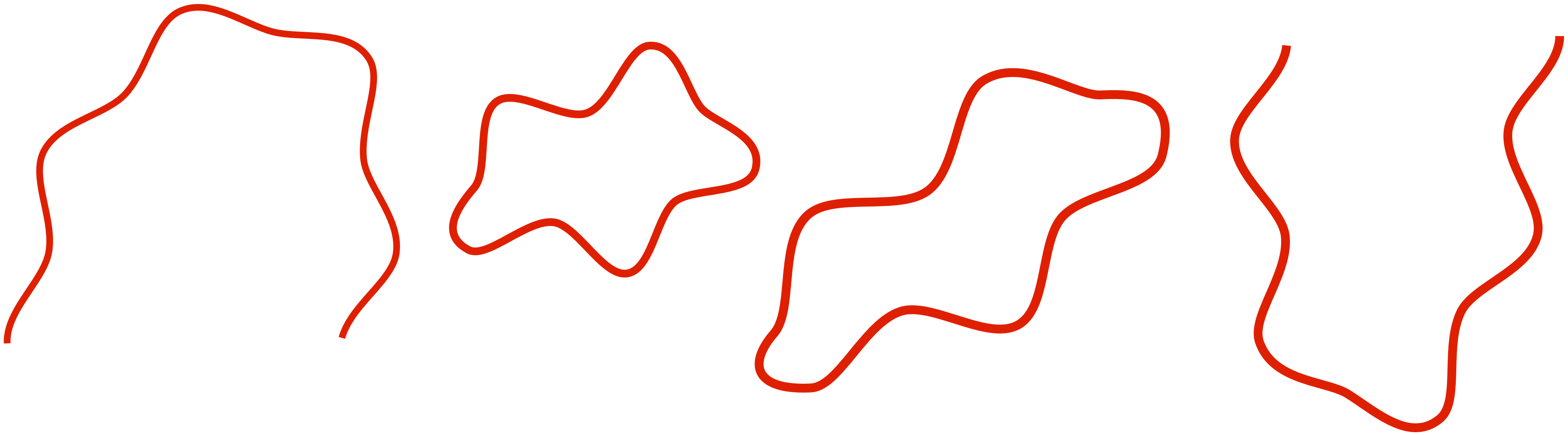
Quantum critical points of atoms and electrons

Holography and string theory

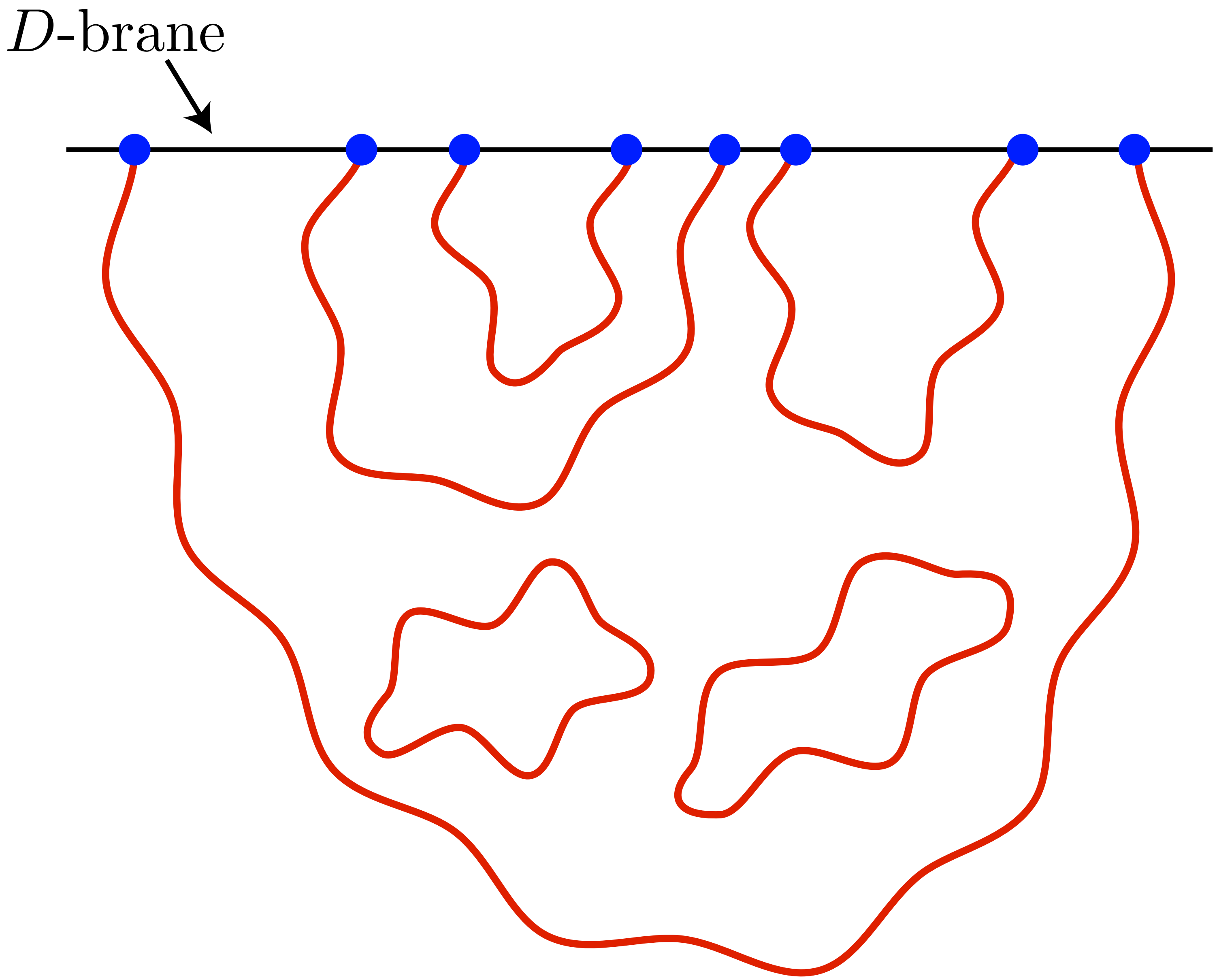
Black holes


Many-particle quantum entanglement
Holography and string theory
Quantum critical points of atoms and electrons
Black holes

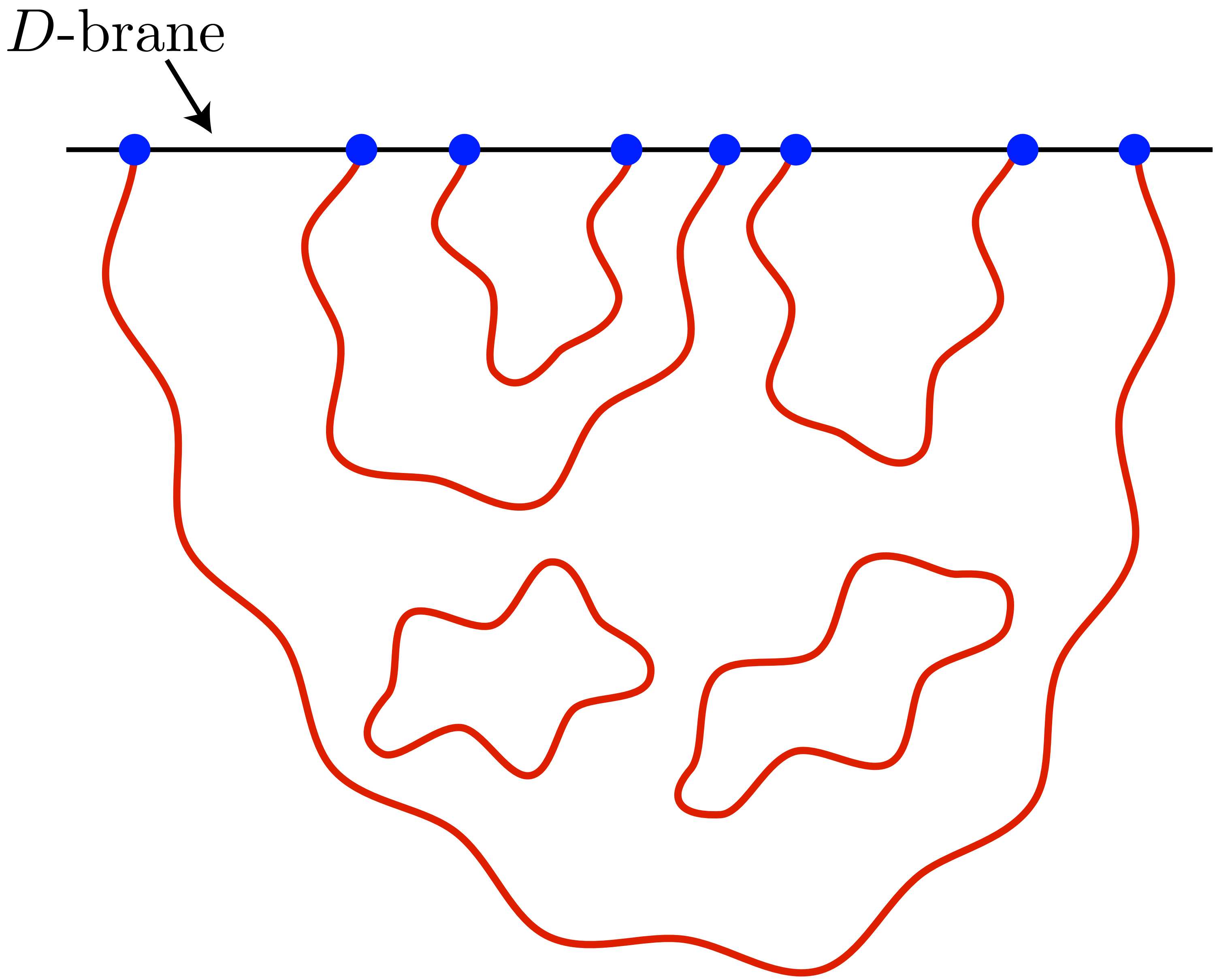
String theory



- Allows unification of the standard model of particle physics with gravity.
- Low-lying string modes correspond to gauge fields, gravitons, quarks ...



- A D-brane is a d-dimensional surface on which strings can end.



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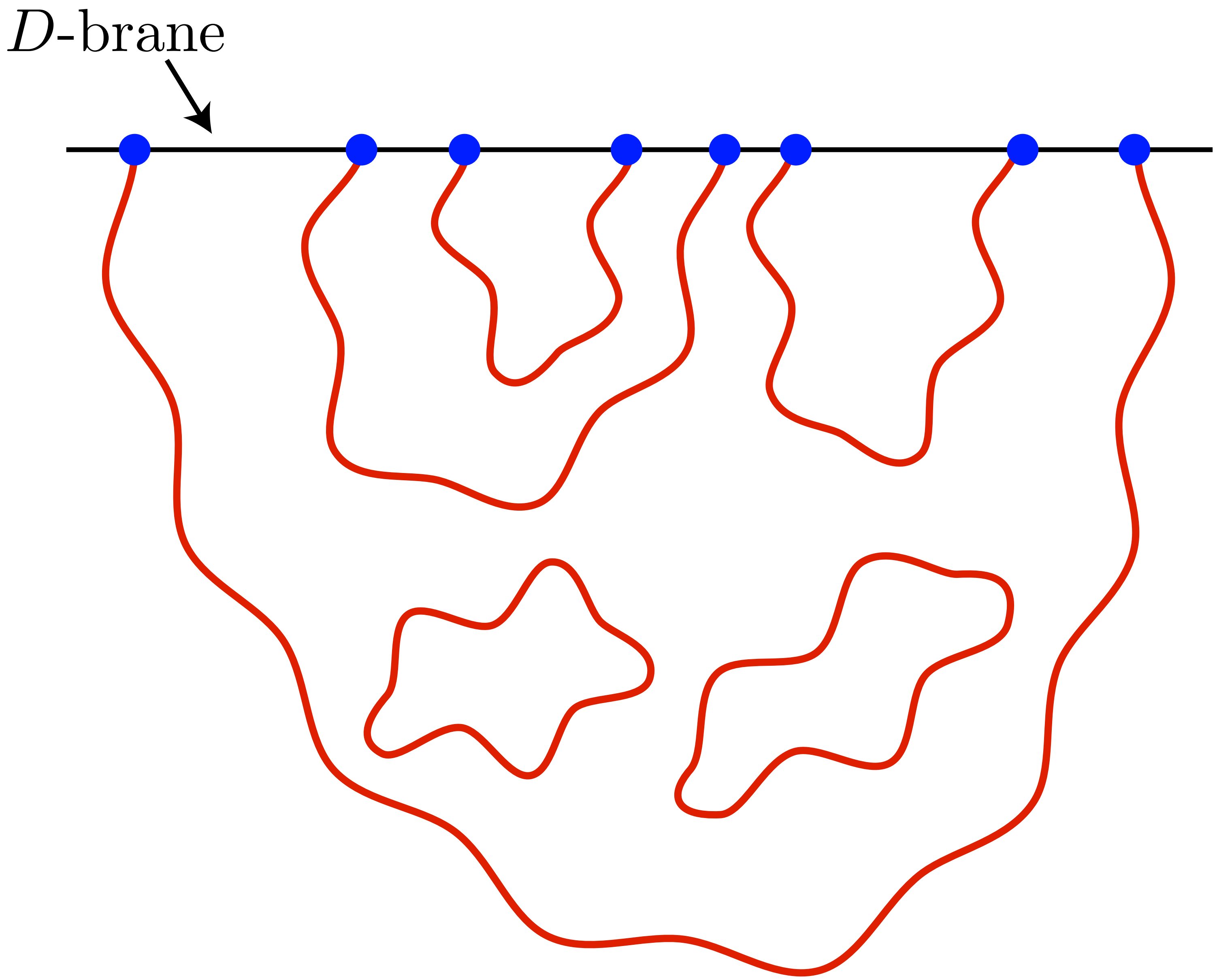
- The low-energy theory on a D-brane has no gravity, similar to theories of entangled electrons of interest to us.

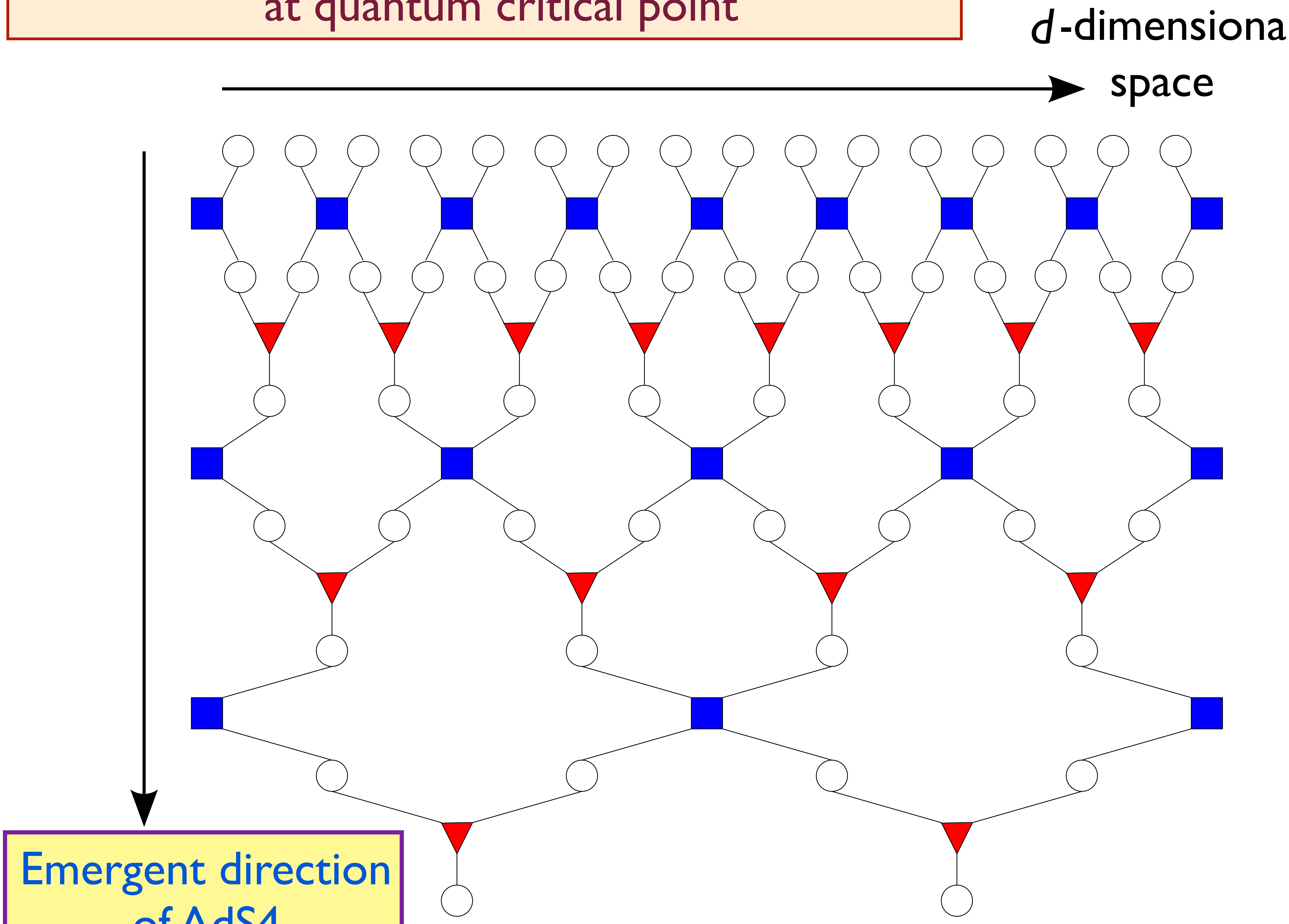


- A D-brane is a d-dimensional surface on which strings can end.

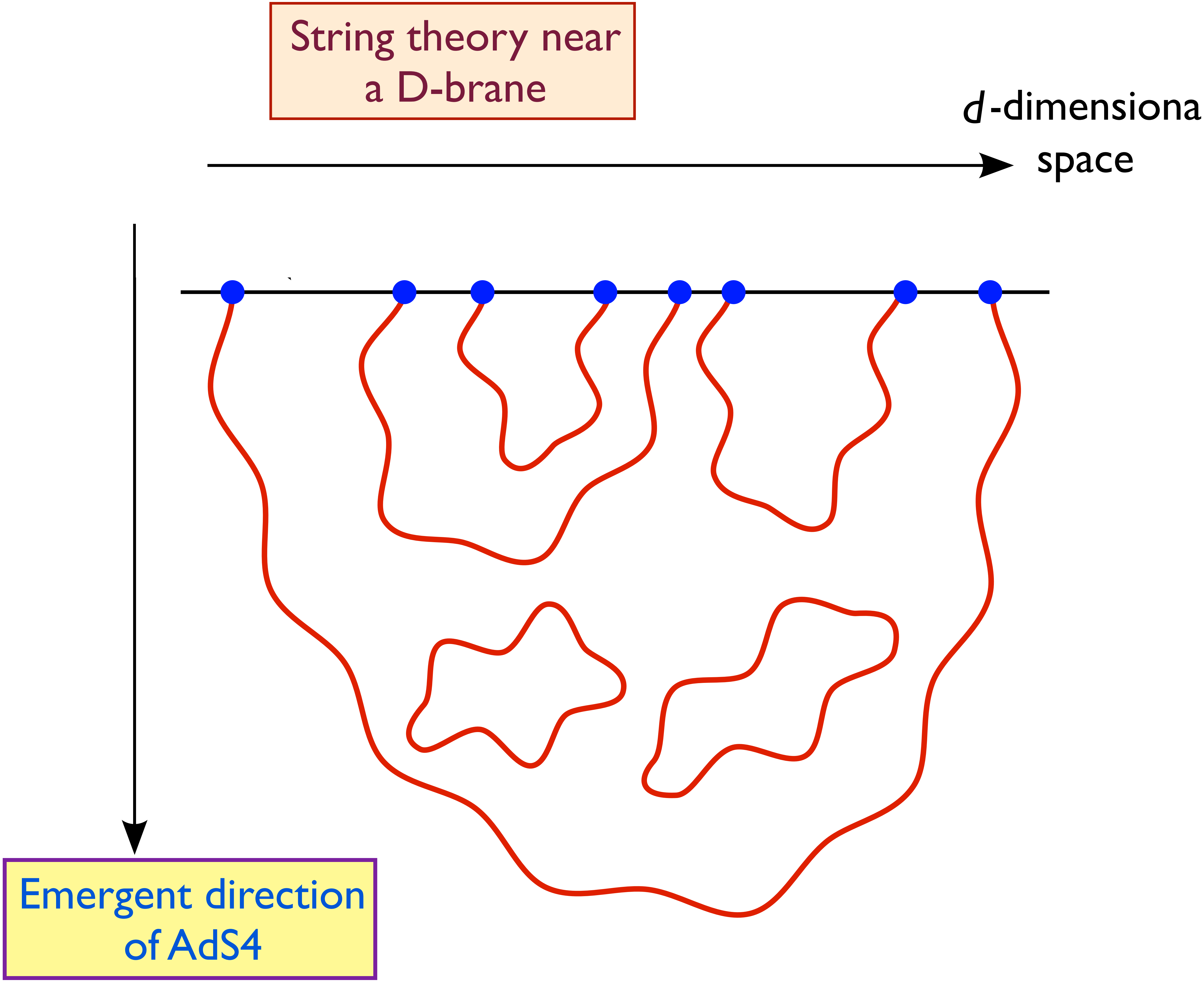
- The low-energy theory on a D-brane has no gravity, similar to theories of entangled electrons of interest to us.

- In $d = 2$, we obtain strongly-interacting **CFT3**s. These are "dual" to string theory on anti-de Sitter space: **AdS4**.

Tensor network representation of entanglement at quantum critical point



Brian Swingle, arXiv:0905.1317

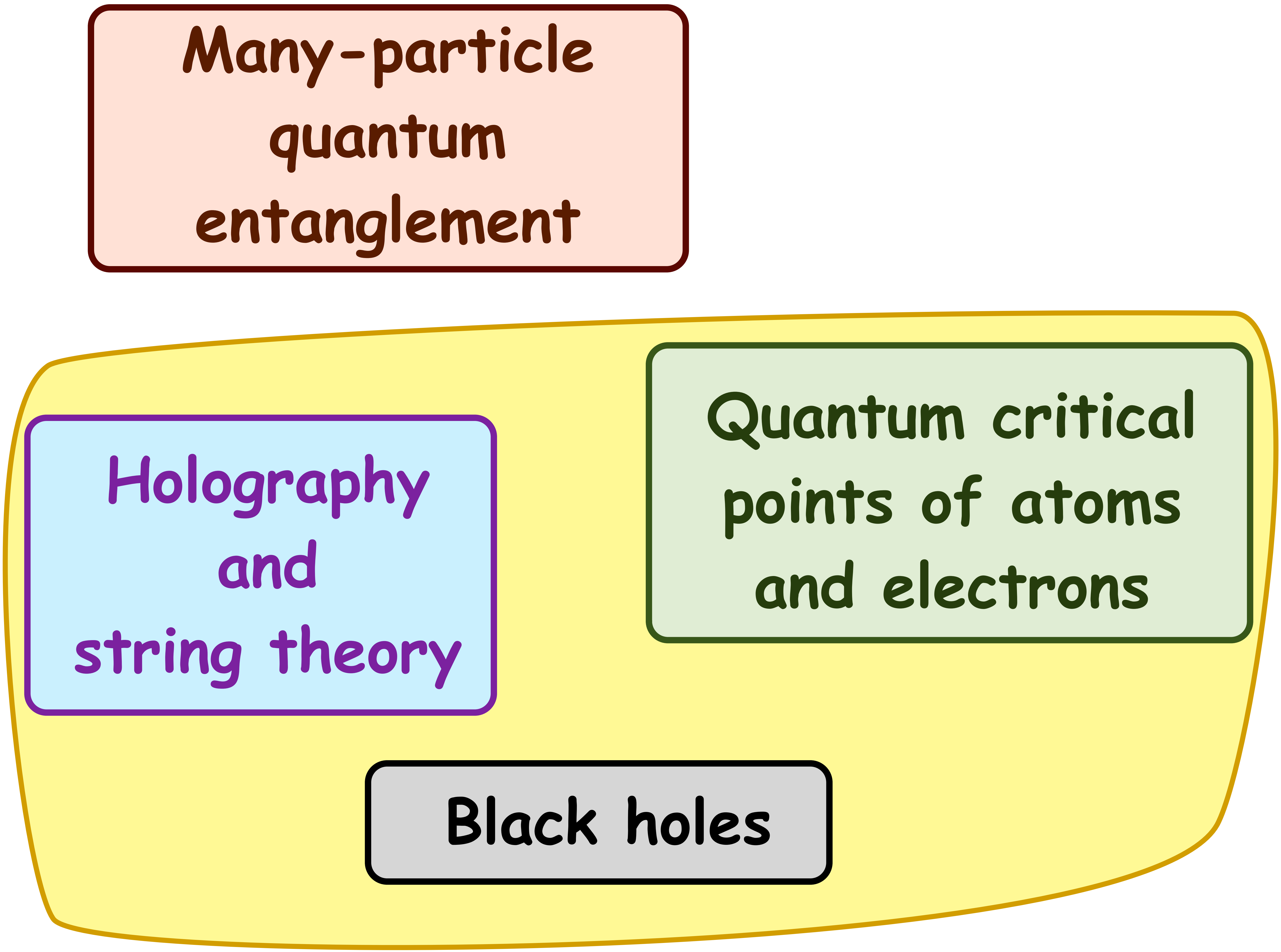

String theory near a D-brane
d-dimensional space
Emergent direction of AdS4

Many-particle quantum entanglement

Holography and string theory

Quantum critical points of atoms and electrons

Black holes


Many-particle quantum entanglement
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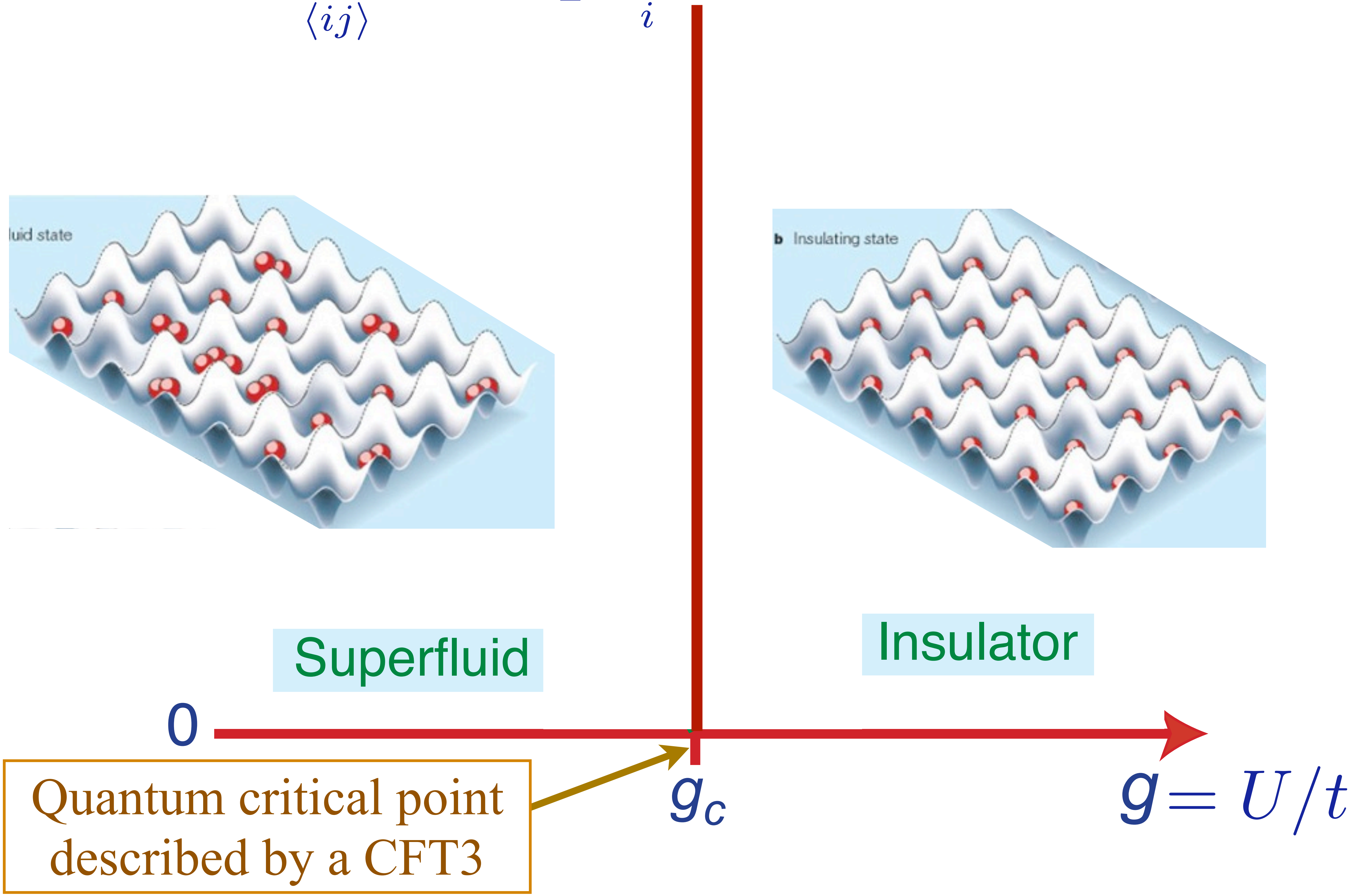
Black holes

$$H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i (n_i - 1) \quad ; \quad n_i \equiv b_i^\dagger b_i$$



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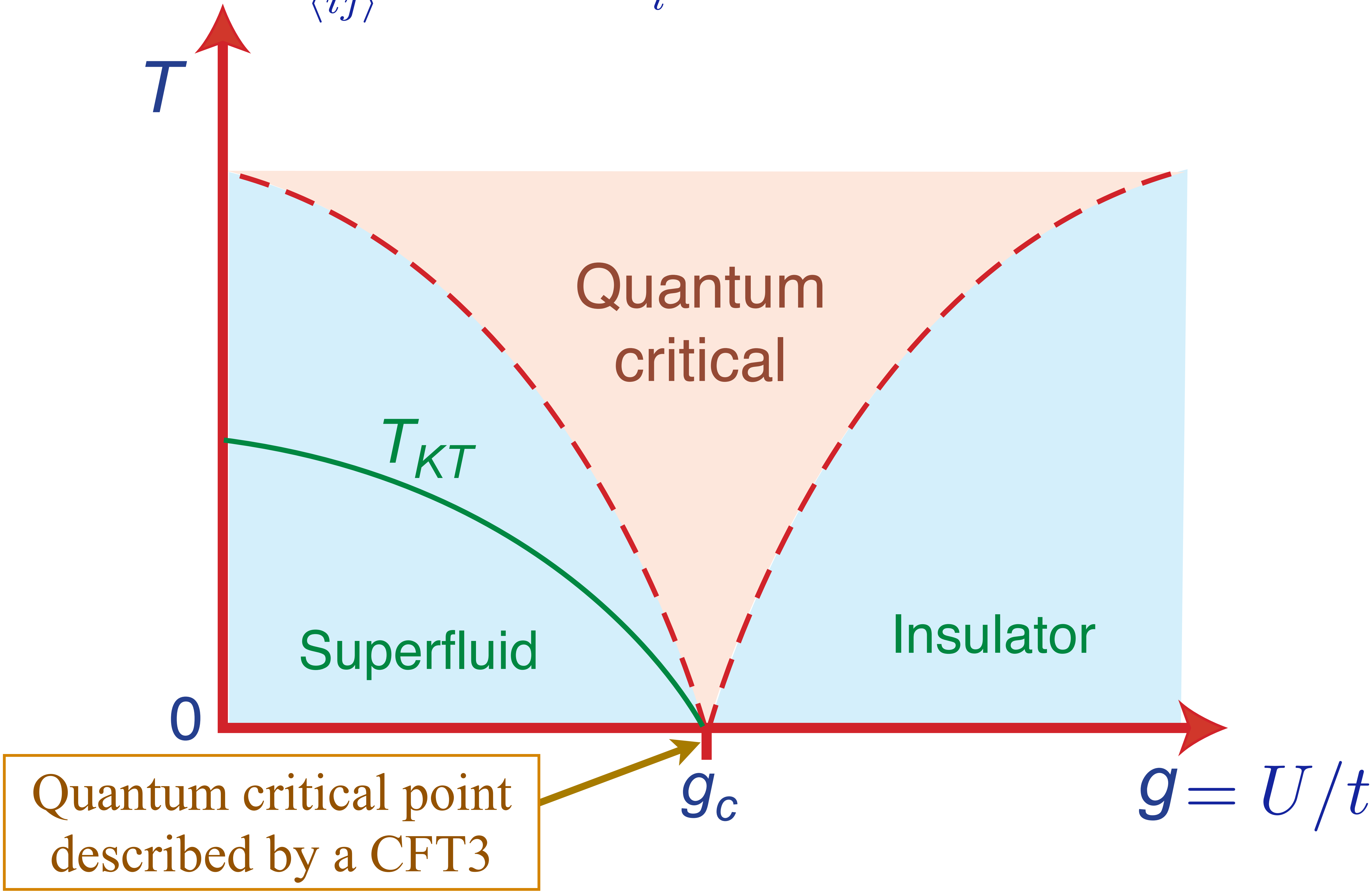

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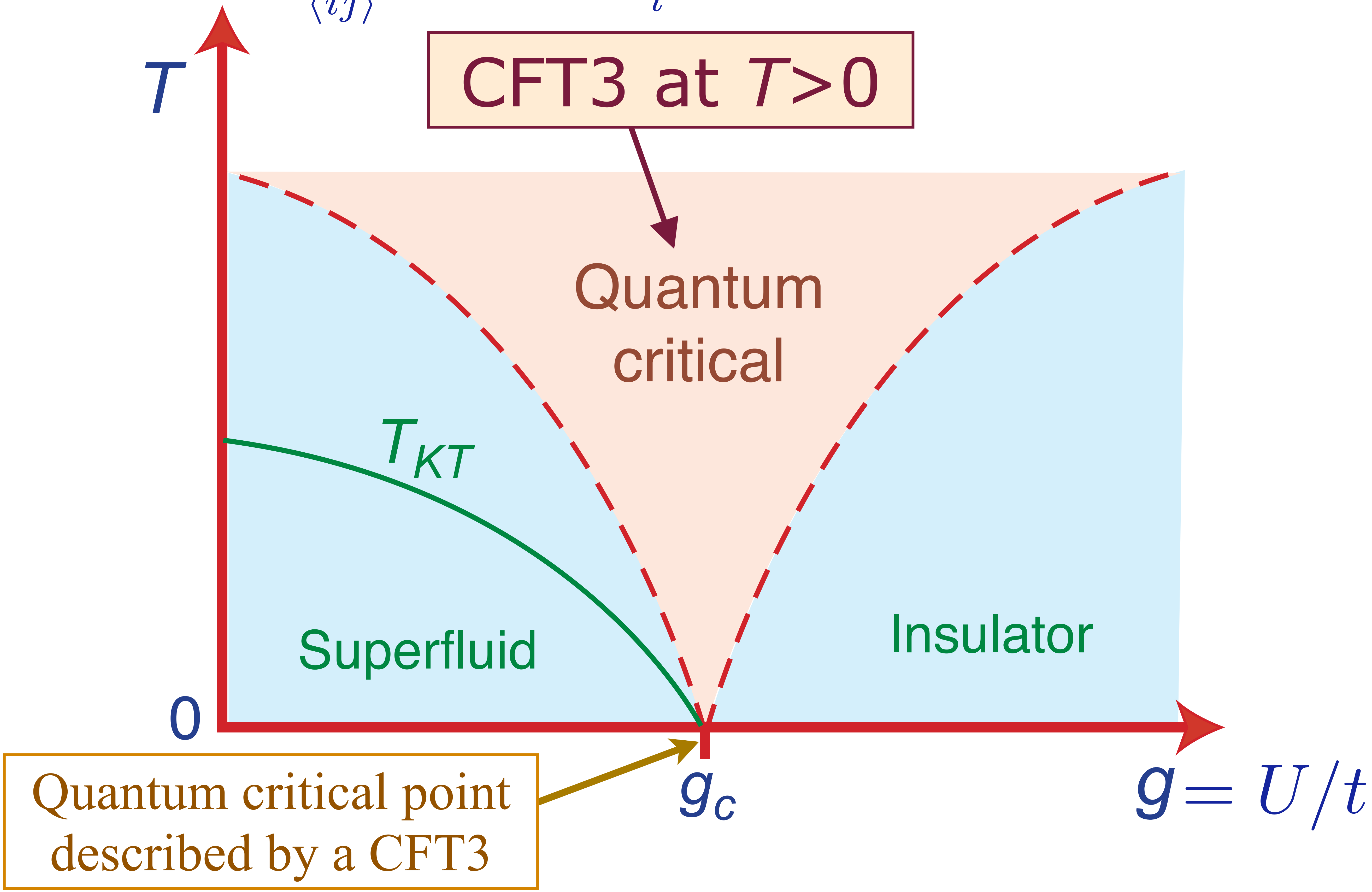

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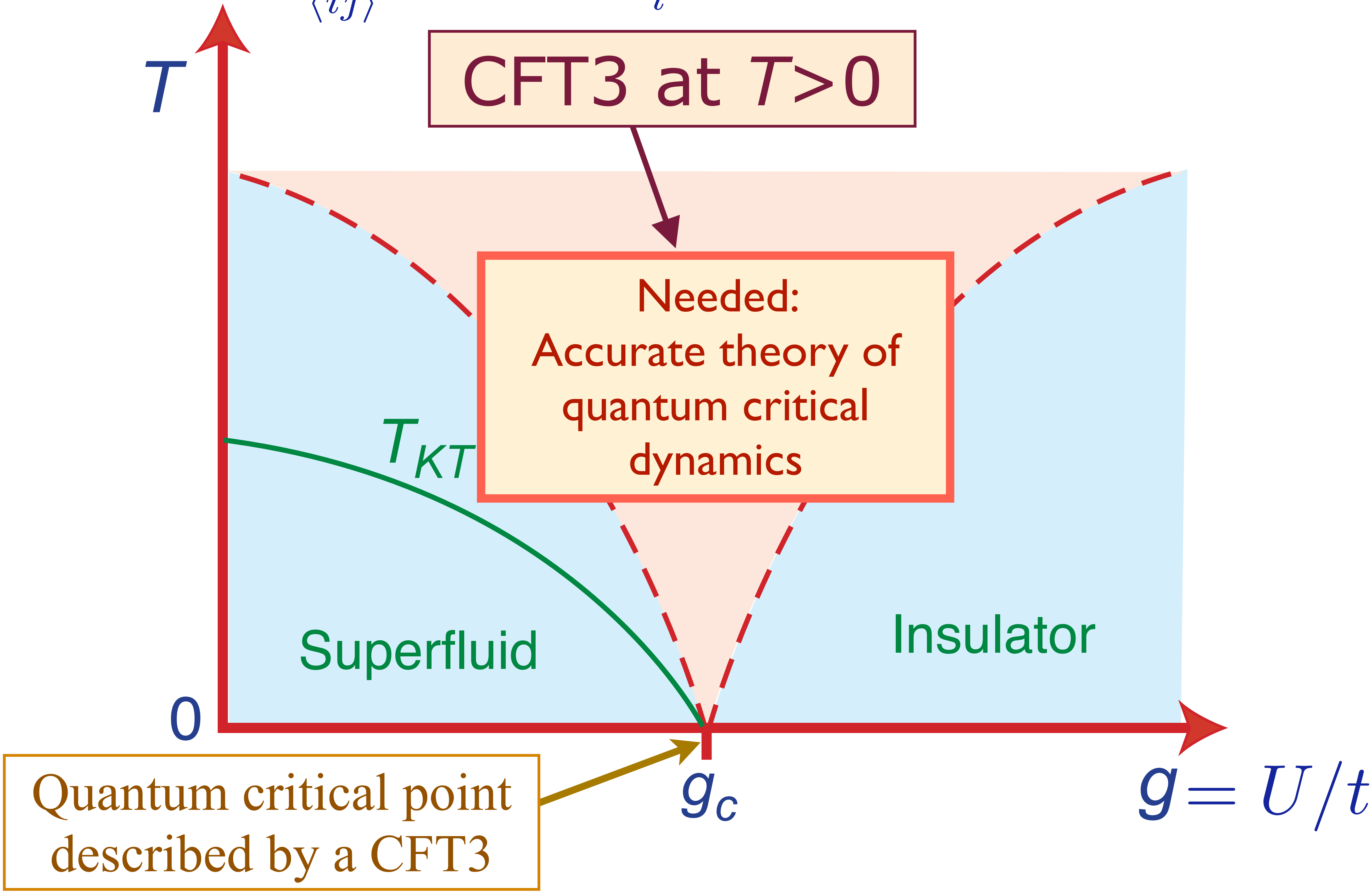
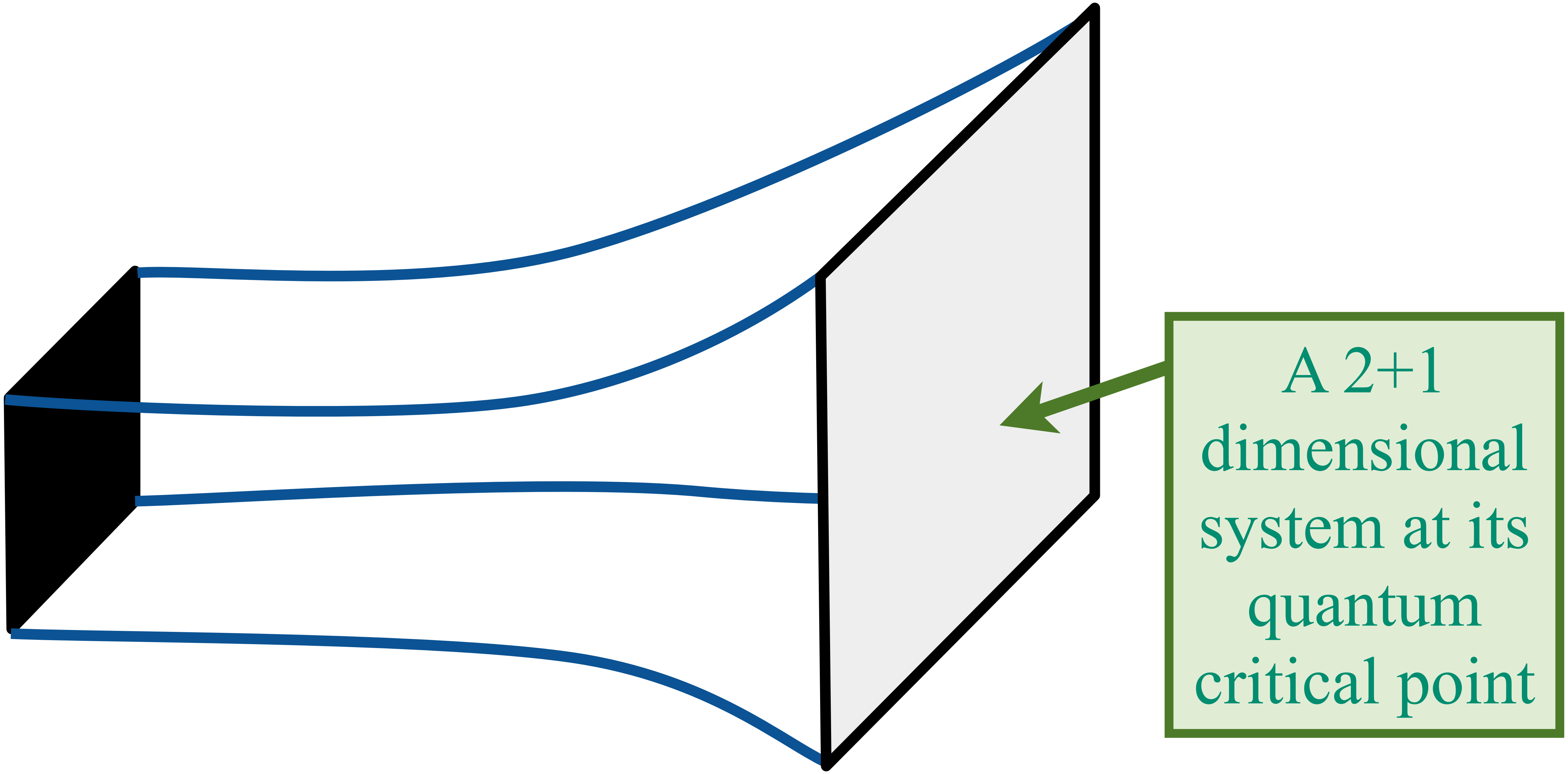
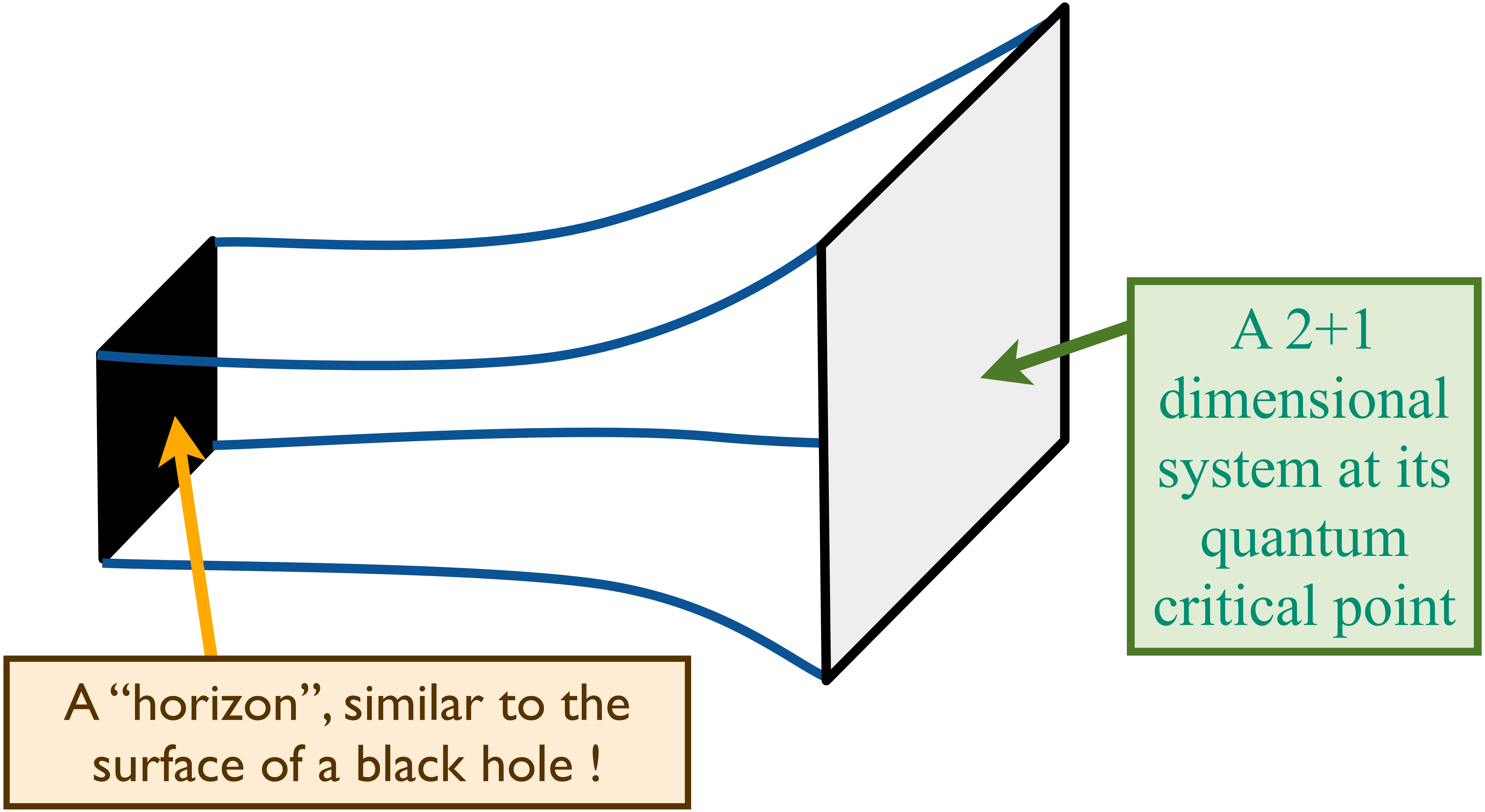


String theory at non-zero temperatures

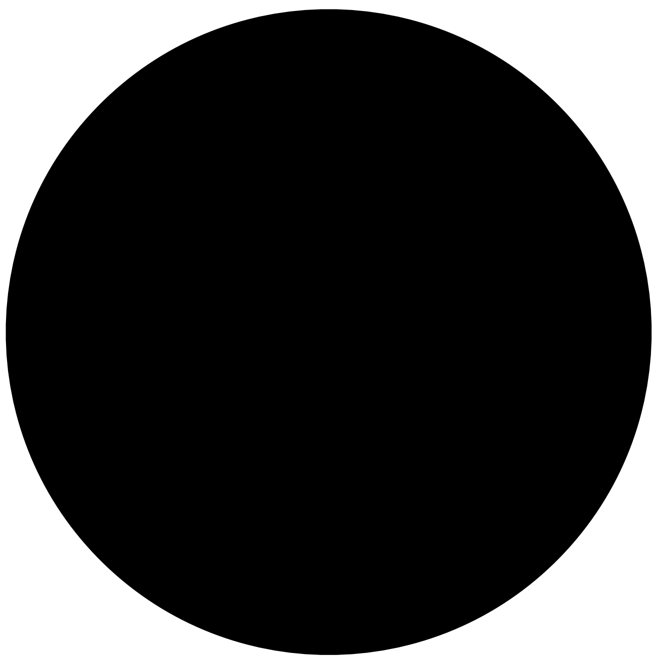


String theory at non-zero temperatures



Black Holes

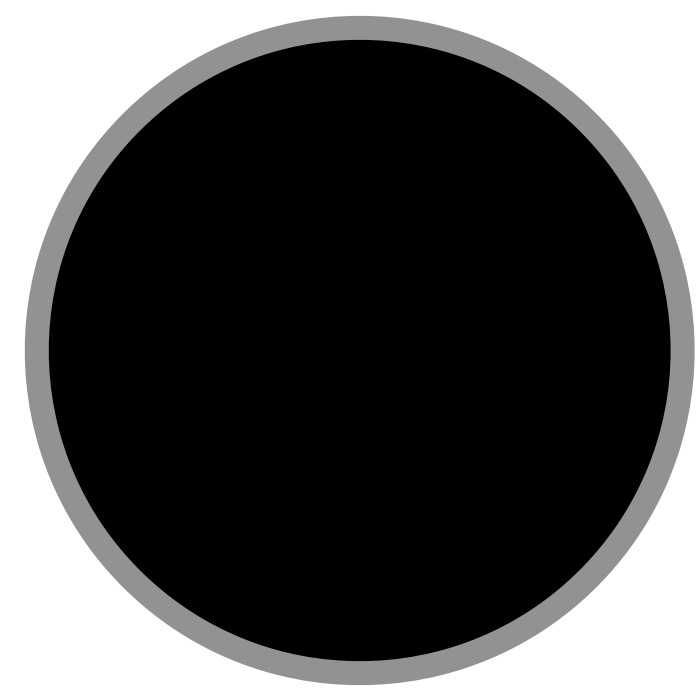
Objects so massive that light is gravitationally bound to them.



Black Holes

Objects so massive that light is gravitationally bound to them.

In Einstein's theory, the region inside the black hole horizon is disconnected from the rest of the universe.

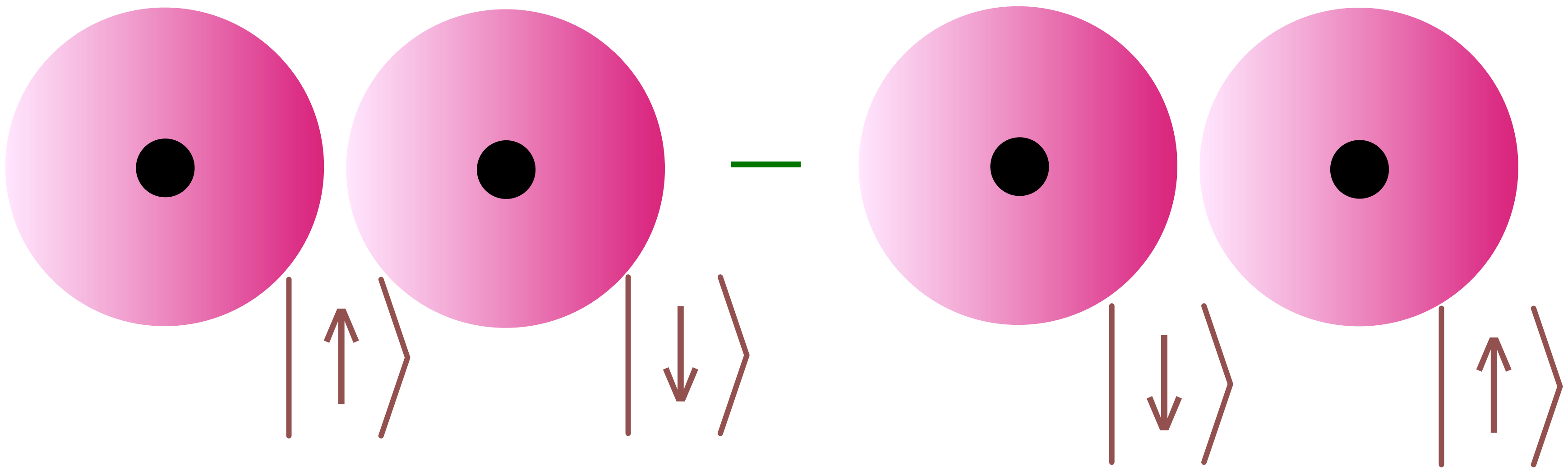


$$\text{Horizon radius } R = \frac{2GM}{c^2}$$

Black Holes + Quantum theory

Around 1974, Bekenstein and Hawking showed that the application of the quantum theory across a black hole horizon led to many astonishing conclusions

Quantum Entanglement across a black hole horizon



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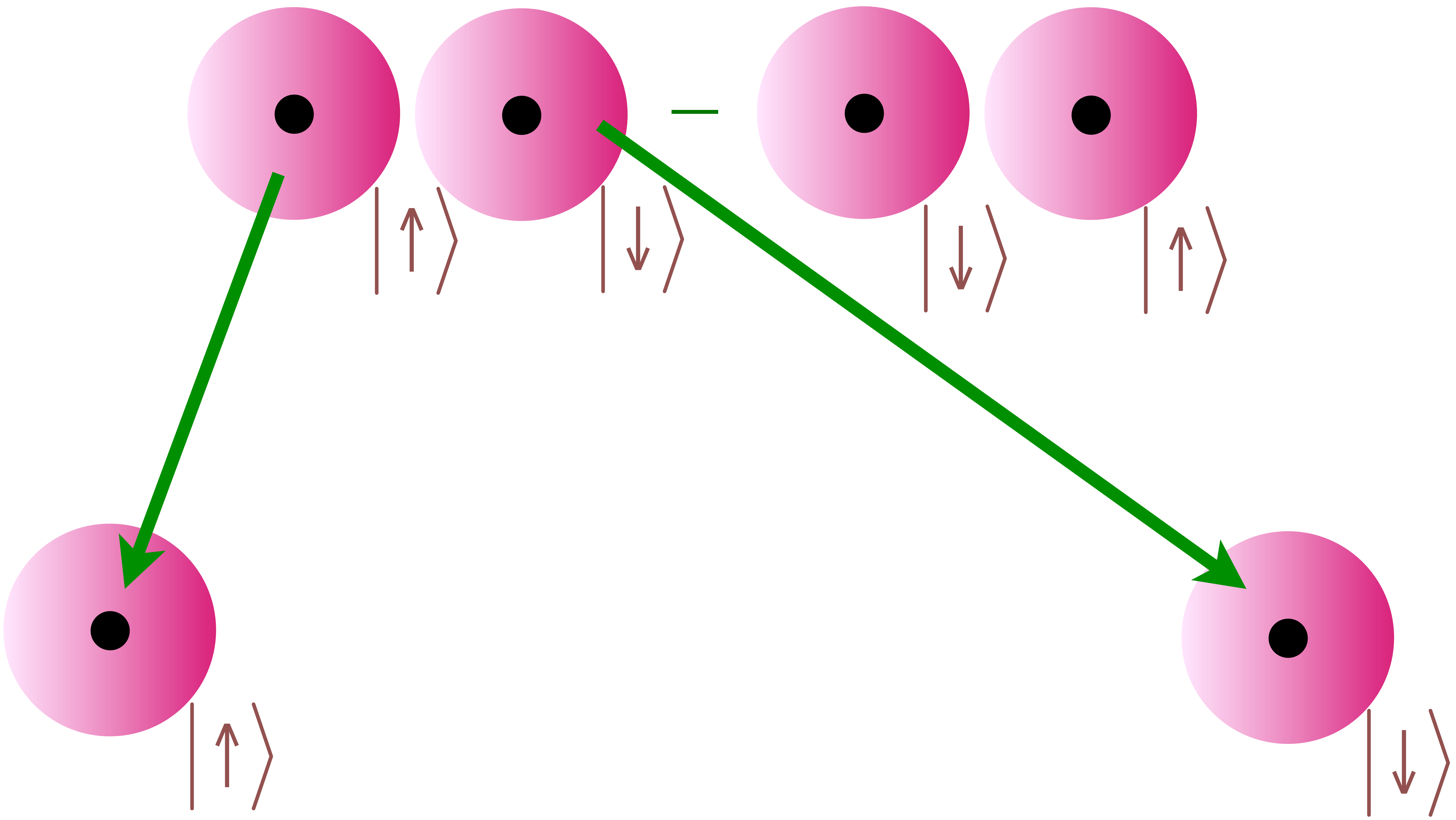
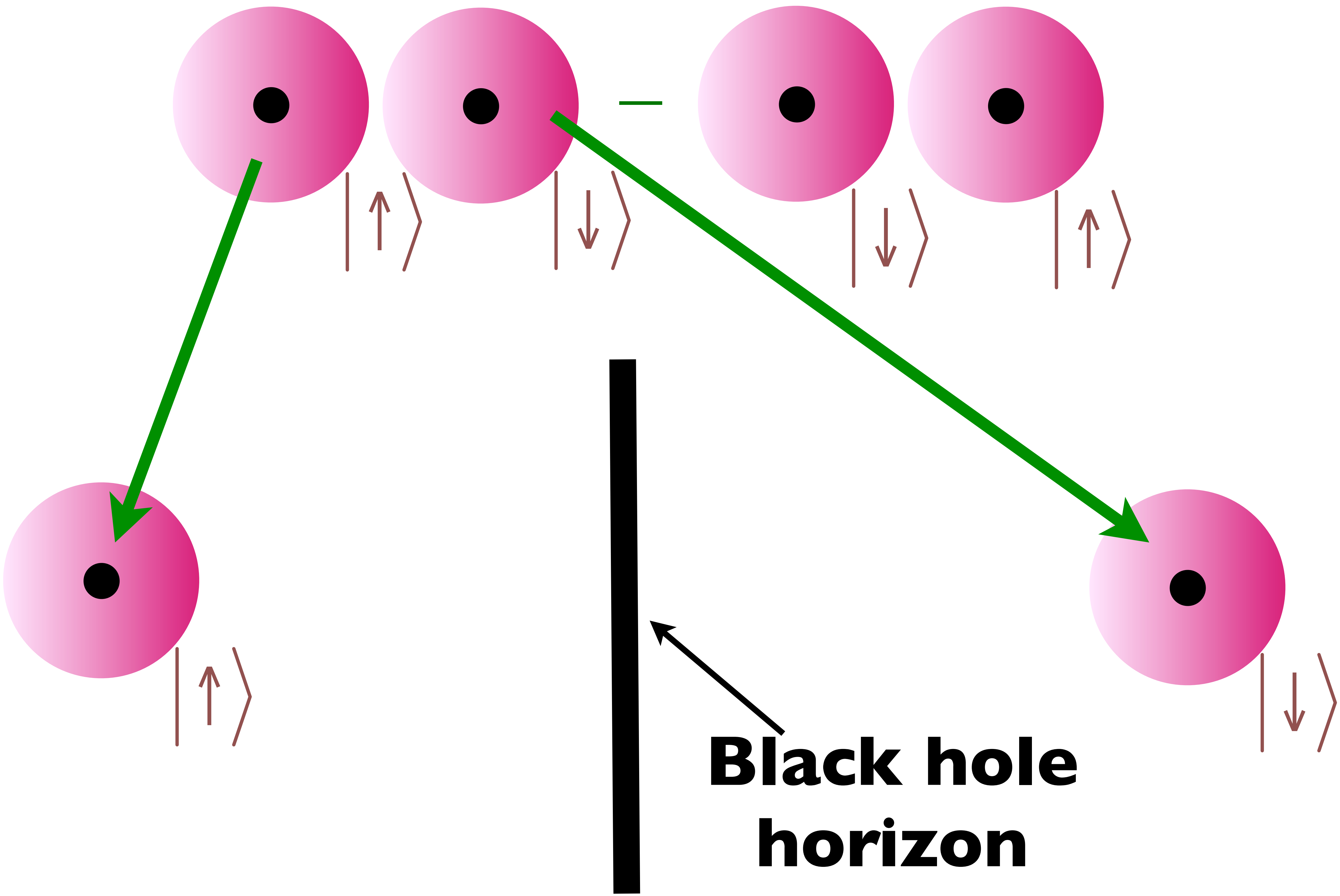


Quantum Entanglement across a black hole horizon



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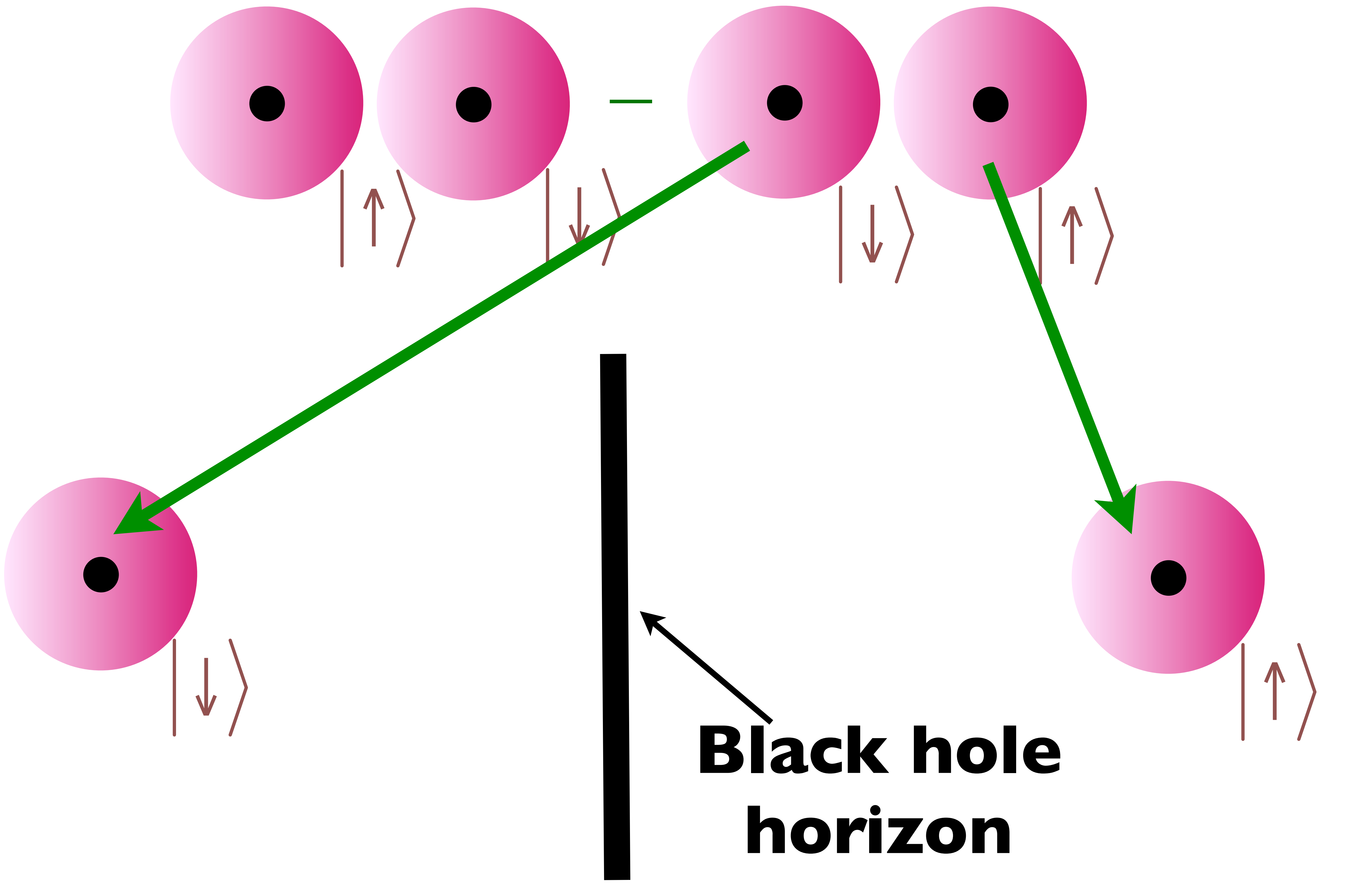
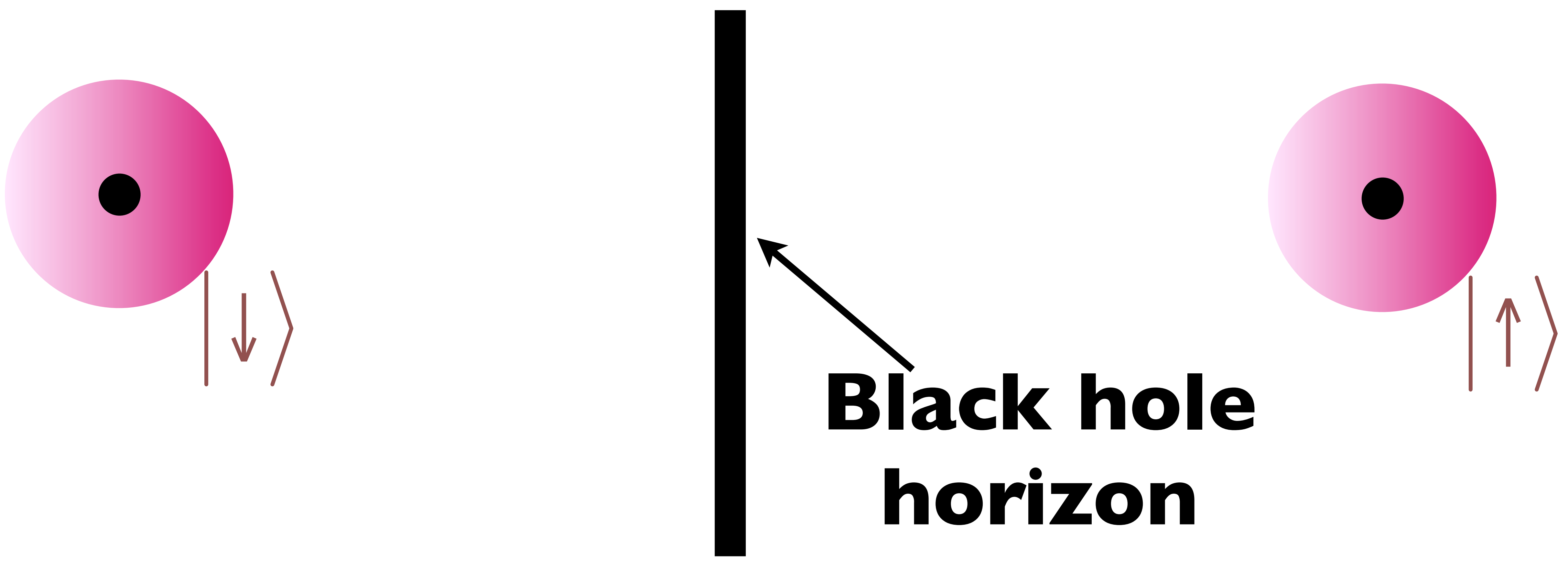


Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole



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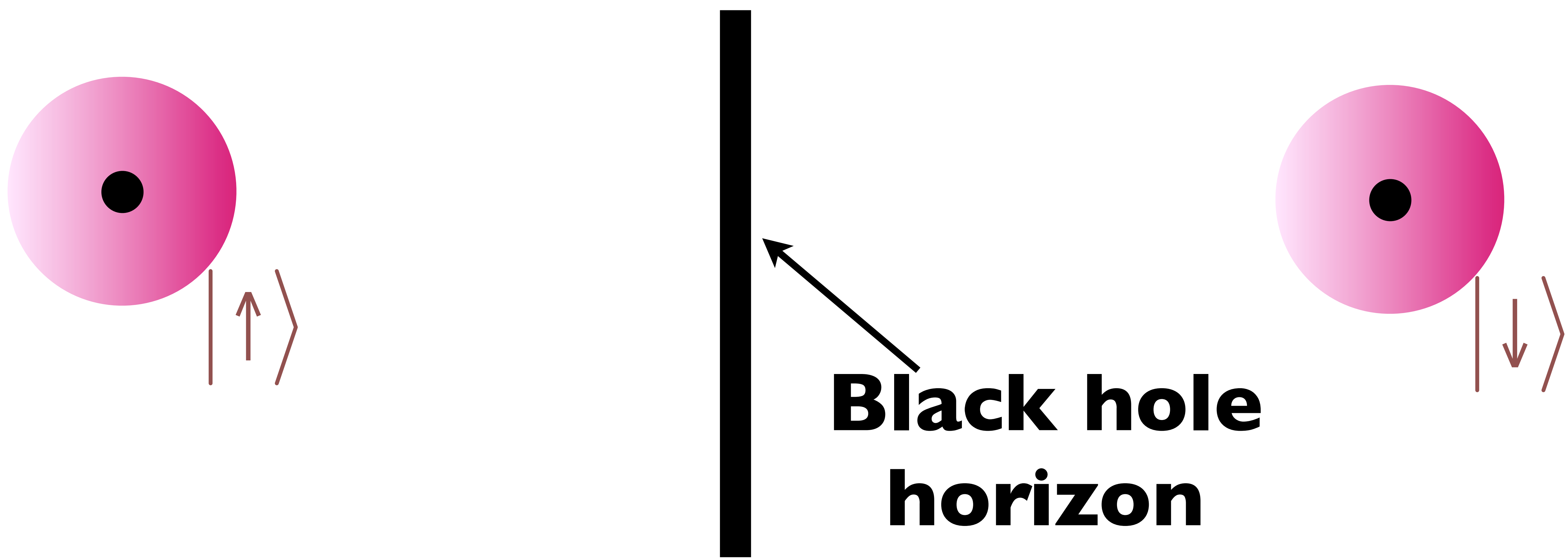
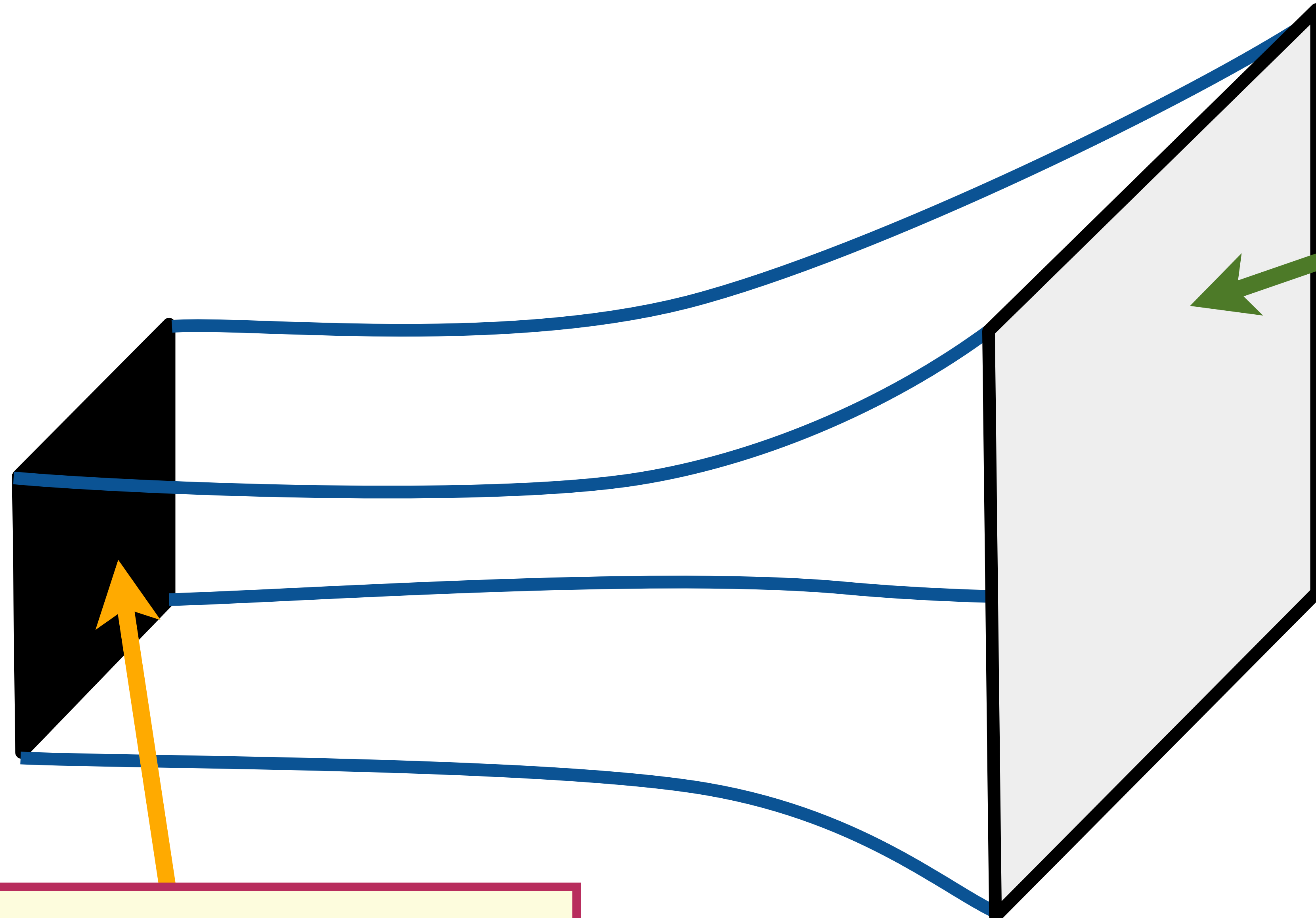


Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole

This entanglement leads to a black hole temperature (the Hawking temperature) and a black hole entropy (the Bekenstein entropy)

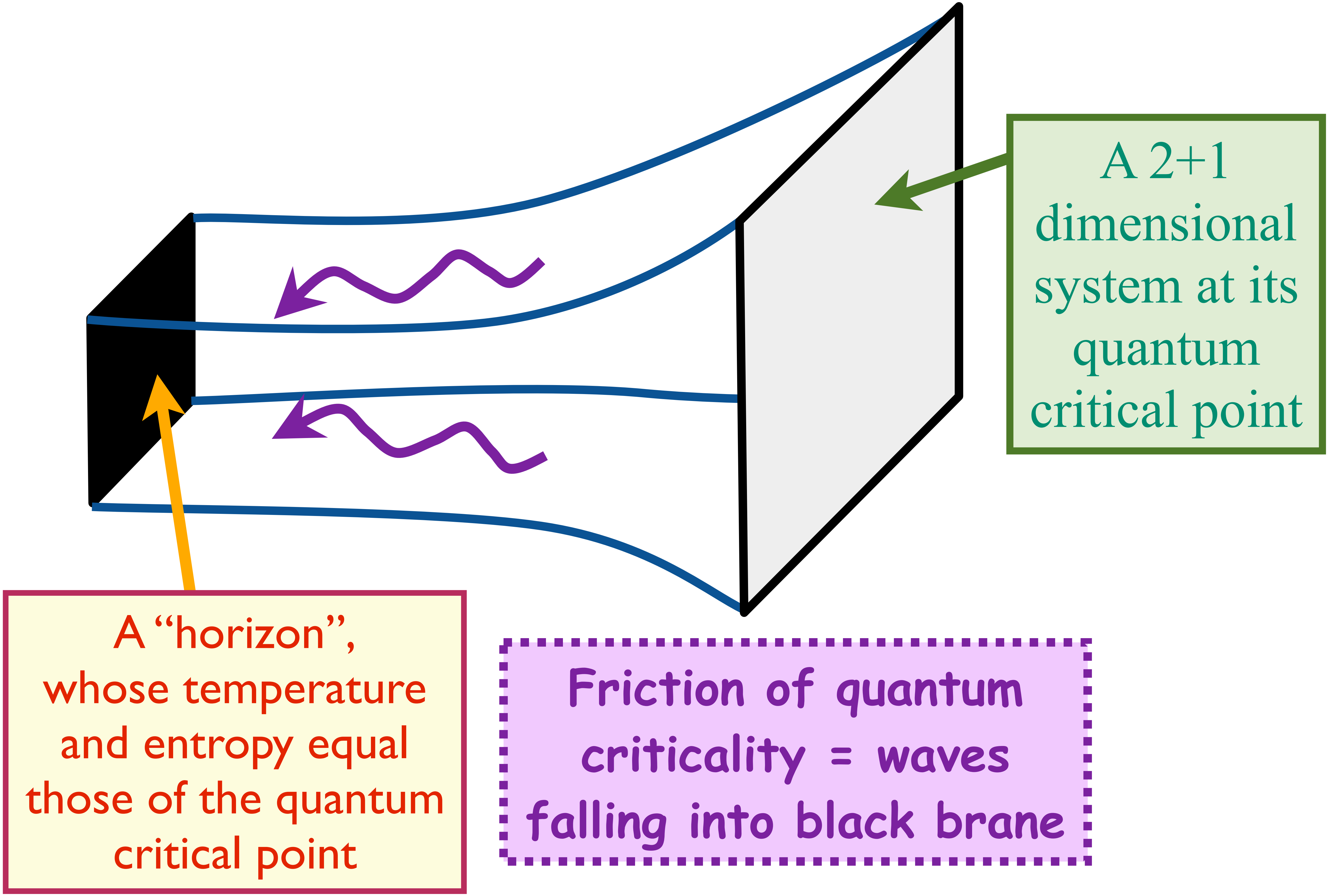
String theory at non-zero temperatures



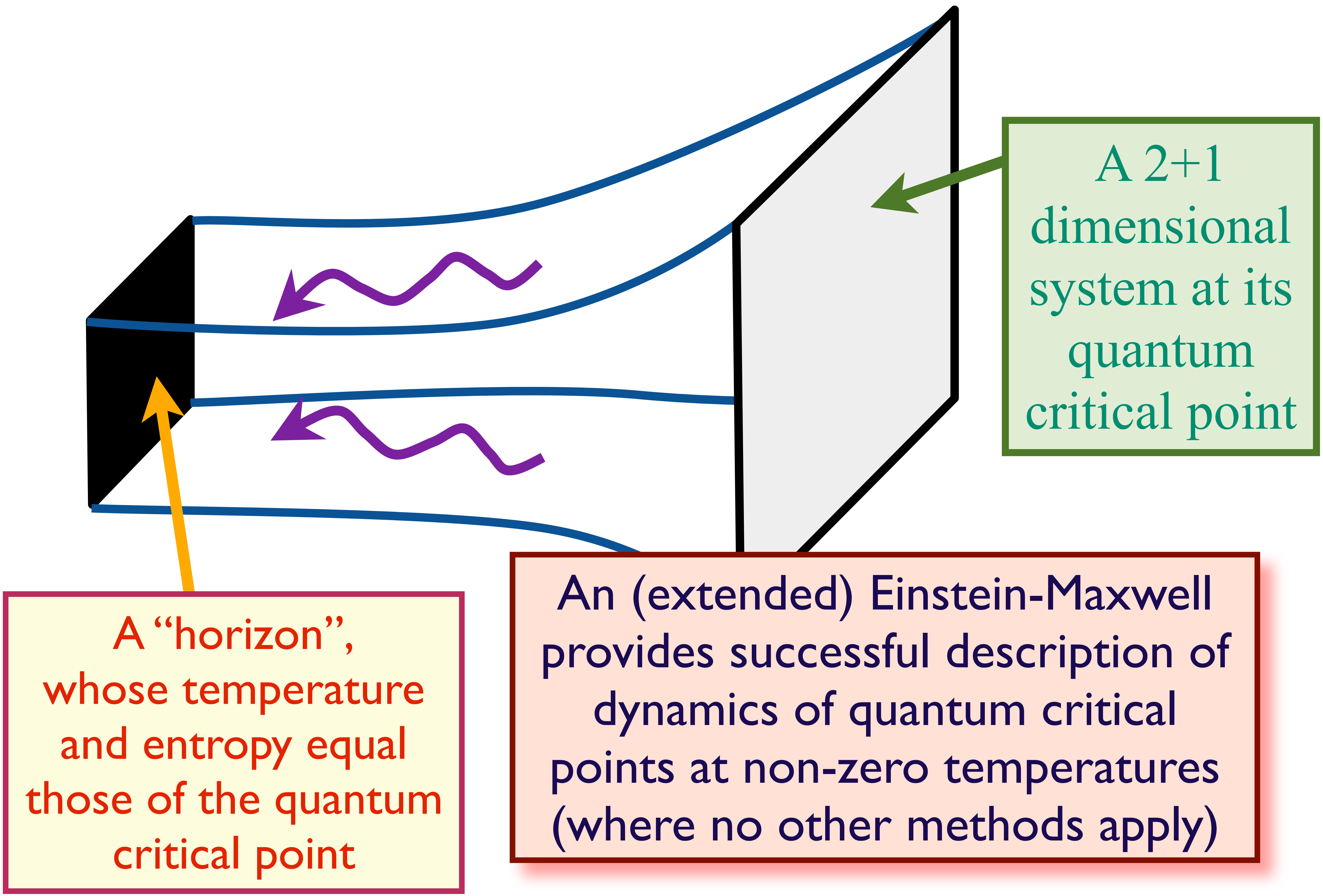
A 2+1 dimensional system at its quantum critical point

A “horizon”, whose temperature and entropy equal those of the quantum critical point

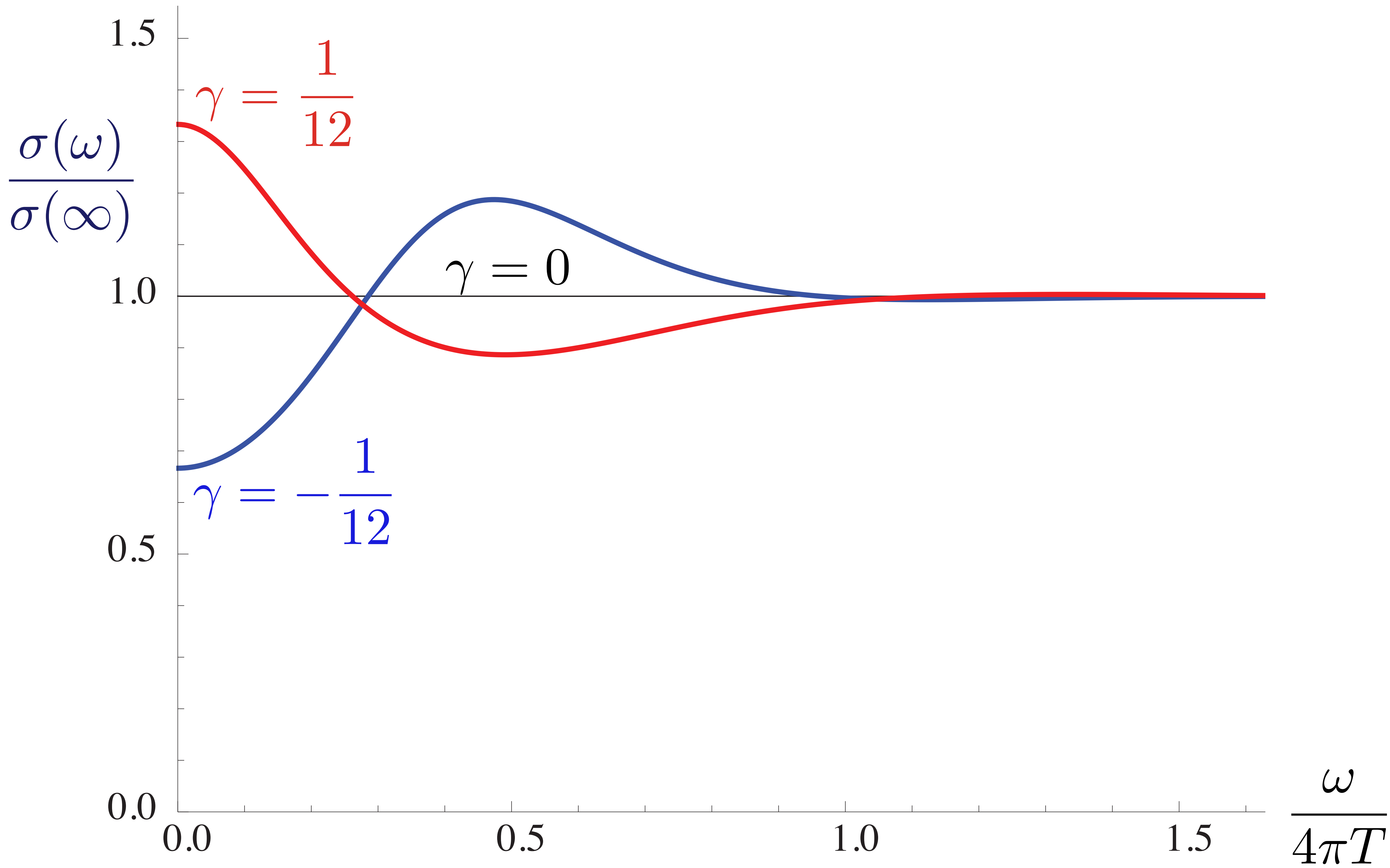
String theory at non-zero temperatures



String theory at non-zero temperatures

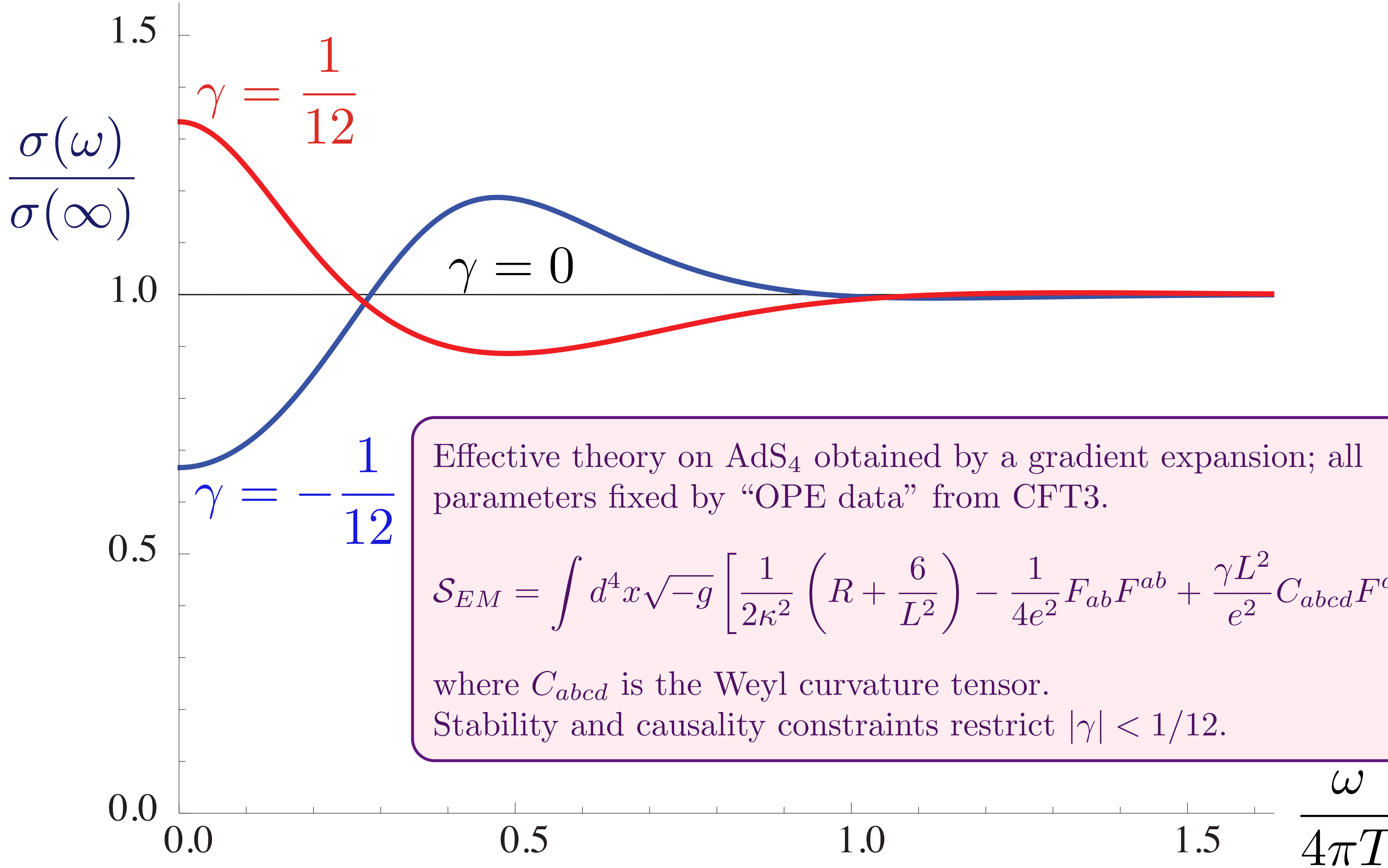


AdS$_4$ theory of charge transport in a CFT3



R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)
W. Witczak-Krempa and S. Sachdev, arXiv:1210.4166
D. Chowdhury, S. Raju, S. Sachdev, A Singh, and P. Strack, arXiv:1210.5247

AdS$_4$ theory of charge transport in a CFT3



Effective theory on AdS$_4$ obtained by a gradient expansion; all parameters fixed by "OPE data" from CFT3.

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right) - \frac{1}{4e^2}F_{ab}F^{ab} + \frac{\gamma L^2}{e^2} C_{abcd}F^{ab}F^{cd}\right],$$

where C_{abcd} is the Weyl curvature tensor.
Stability and causality constraints restrict $|\gamma| < 1/12$.

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)
W. Witczak-Krempa and S. Sachdev, arXiv:1210.4166
D. Chowdhury, S. Raju, S. Sachdev, A Singh, and P. Strack, arXiv:1210.5247

“Complex entangled” states of quantum matter in *d* spatial dimensions

Gapped quantum matter

Spin liquids, quantum Hall states

Conformal quantum matter

Quantum critical points in antiferromagnets, superconductors, and ultracold atoms; graphene

Compressible quantum matter

Strange metals in high temperature superconductors, Bose metals

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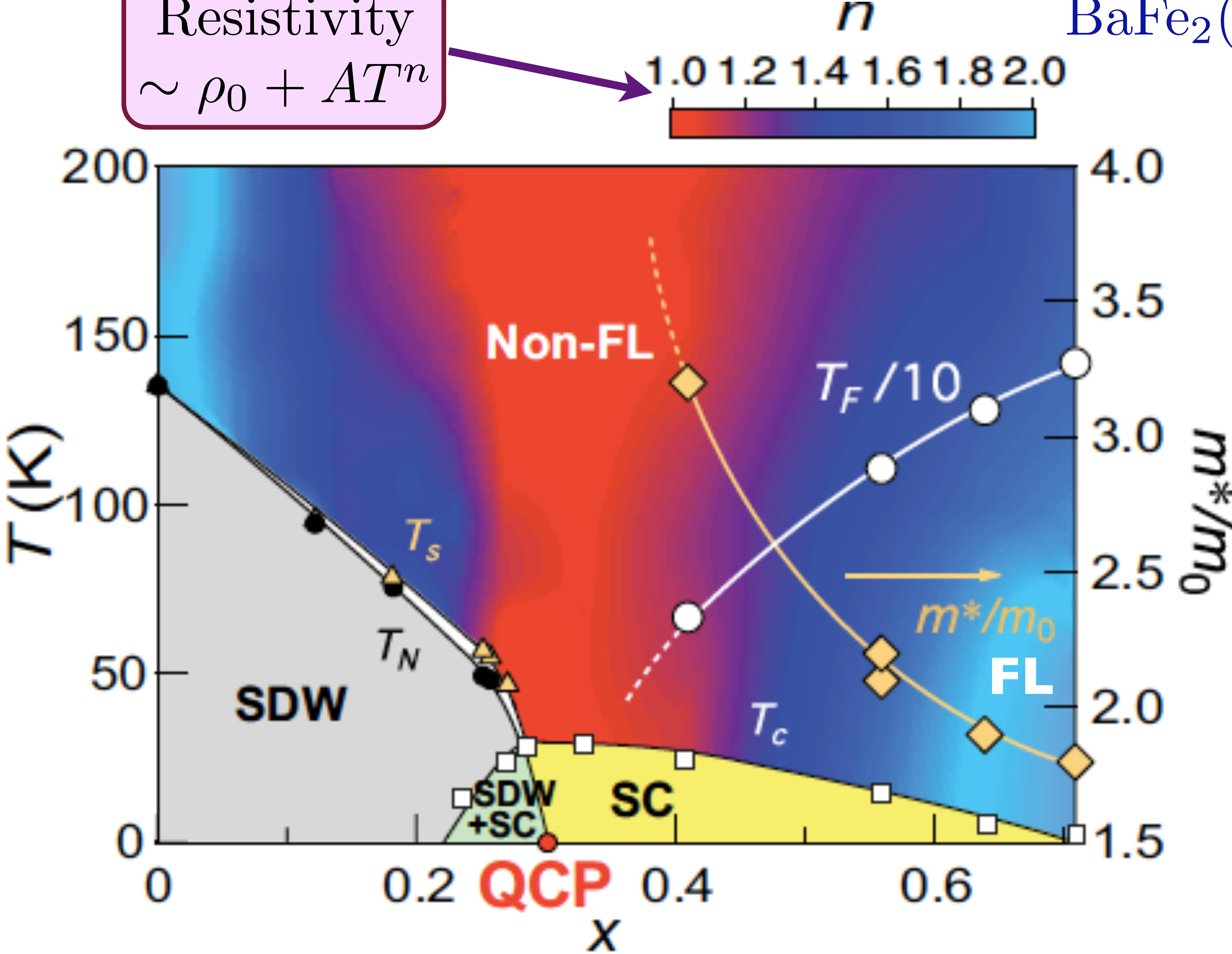
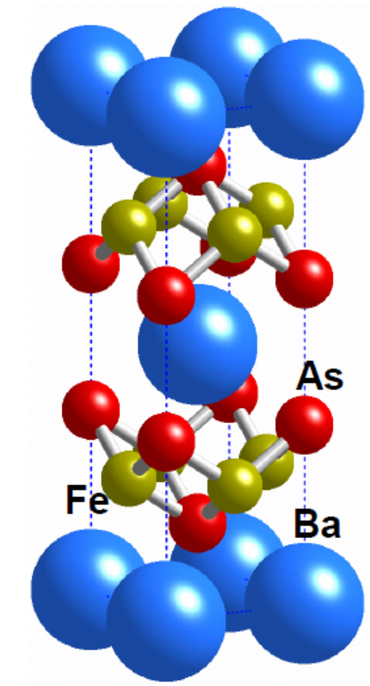
Strange metals in high temperature superconductors, Bose metals

Resistivity $\sim \rho_0 + AT^n$

$BaFe_2(As_{1-x}\,P_x)_2$

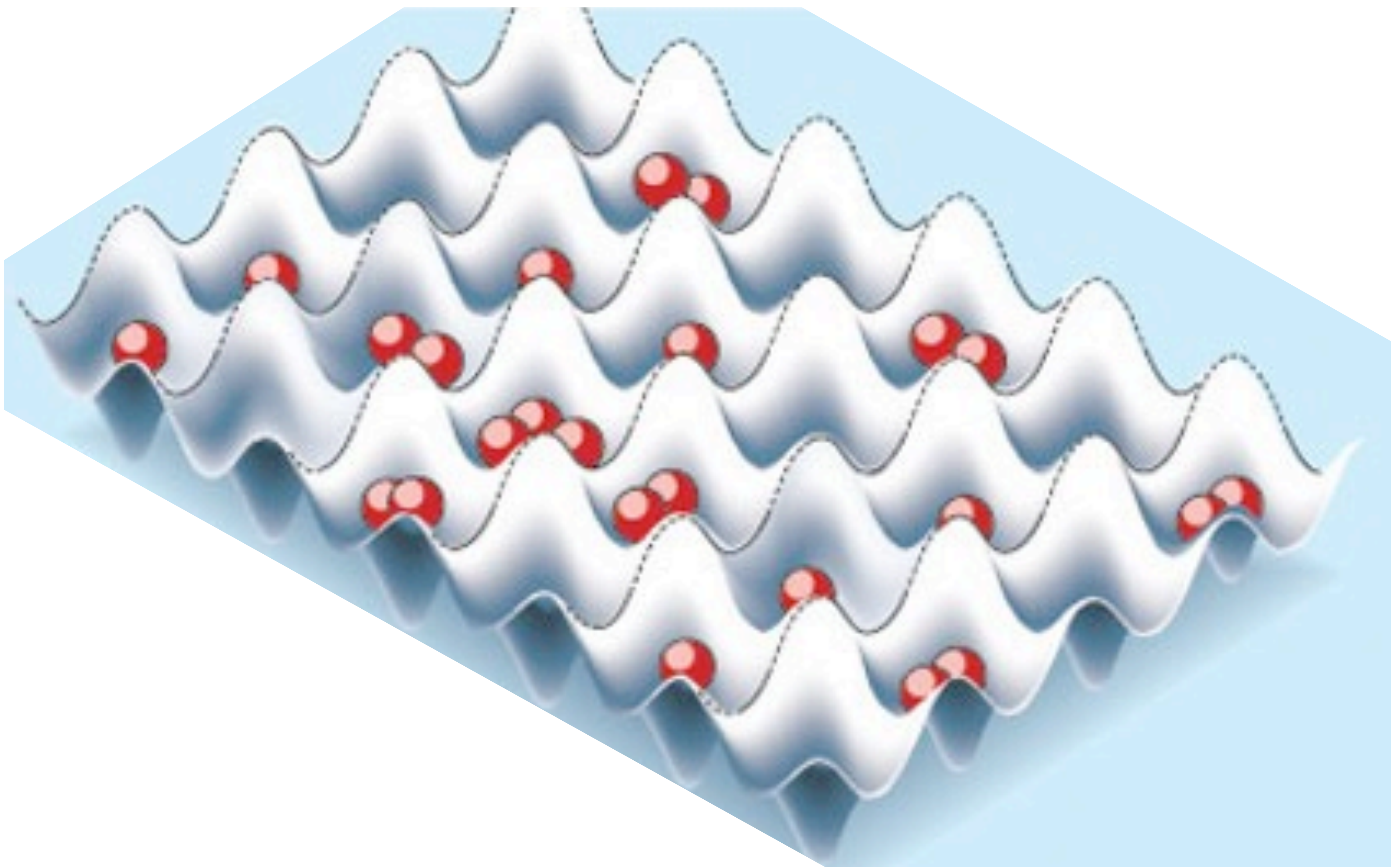




K. Hashimoto, K. Cho, T. Shibauchi, S. Kasahara, Y. Mizukami, R. Katsumata, Y. Tsuruhara, T. Terashima, H. Ikeda, M. A. Tanatar, H. Kitano, N. Salovich, R. W. Giannetta, P. Walmsley, A. Carrington, R. Prozorov, and Y. Matsuda, *Science* **336**, 1554 (2012).

Bosons with correlated hopping

$$H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i(n_i - 1) + w \sum_{ijk\ell \in \square} b_i^\dagger b_k^\dagger b_j b_\ell$$



A *Bose metal*: a compressible phase of bosons which breaks no symmetries.

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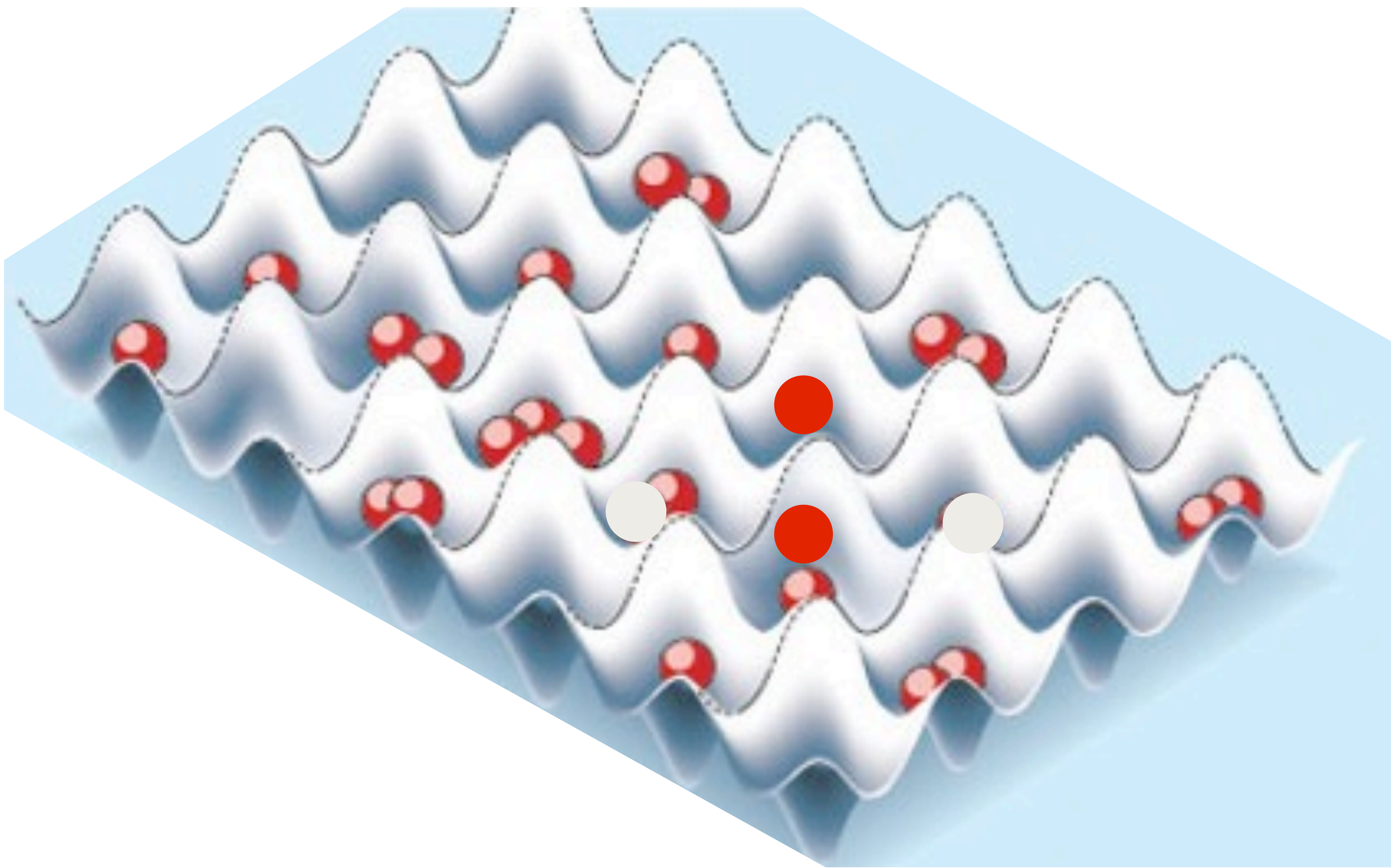

A *Bose metal*: a compressible phase of bosons which breaks no symmetries.

Bosons with correlated hopping

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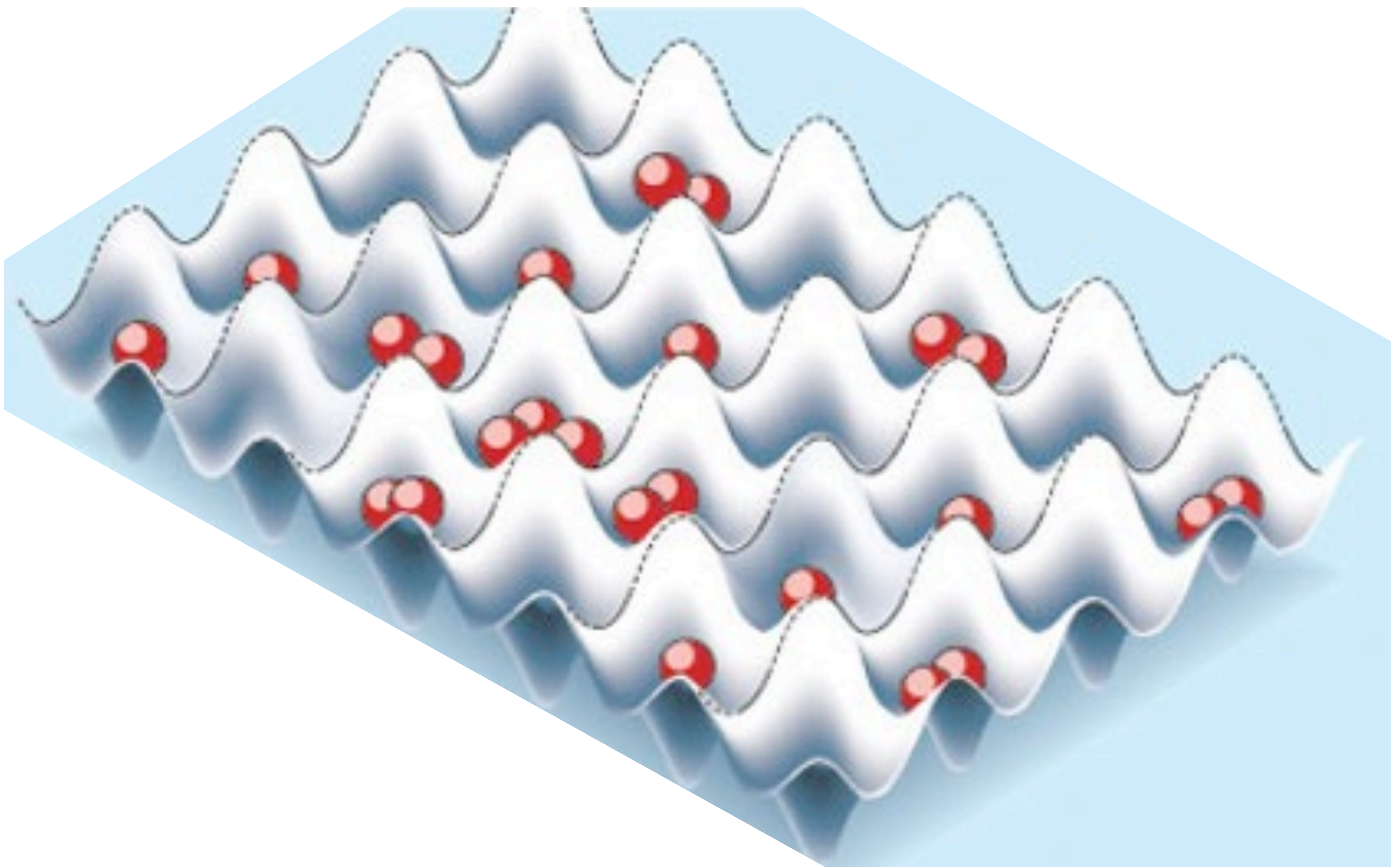
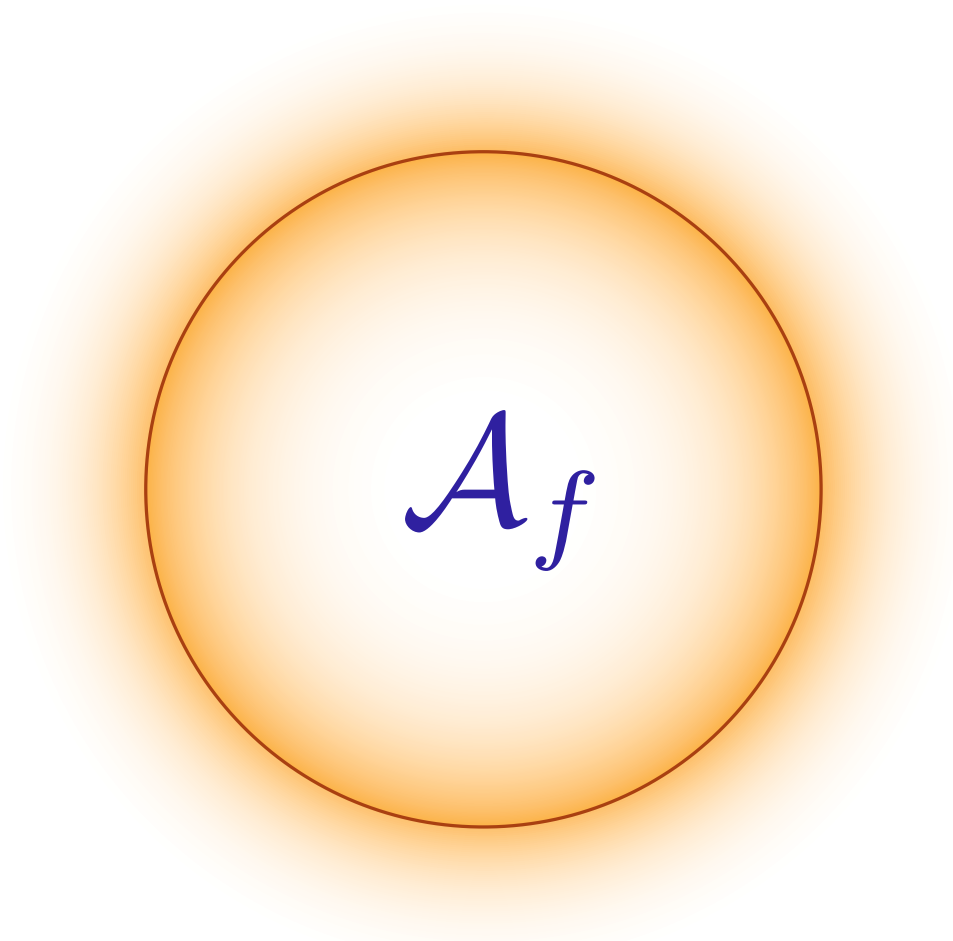


A *Bose metal*: a compressible phase of bosons which breaks no symmetries.

- *Bose metal*: the boson, b, fractionalizes into (say) 2 fermions, f_1 and f_2 ("*quarks*"), each of which forms a Fermi surface. Both fermions necessarily couple to an emergent gauge field, and so the Fermi surfaces are "*hidden*".



$$\mathcal{Q} = b^\dagger b$$

$$\mathcal{A}_f = \langle \mathcal{Q} \rangle$$

O. I. Motrunich and M. P. A. Fisher,
Physical Review B **75**, 235116 (2007)
L. Huijse and S. Sachdev,
Physical Review D **84**, 026001 (2011)
S. Sachdev, to appear

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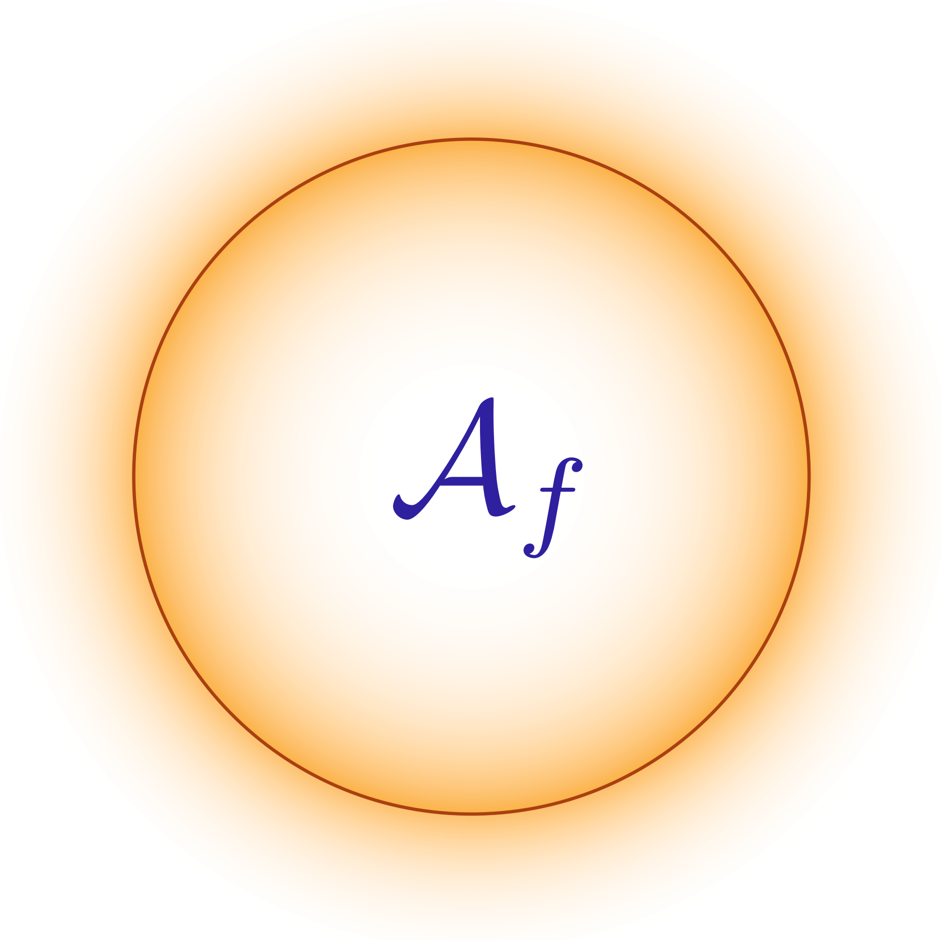


$$b \to f_1 f_2$$

Gauge invariance:

$$f_1(x) \to f_1(x) e^{i\theta(x)},$$
$$f_2(x) \to f_2(x) e^{-i\theta(x)}$$

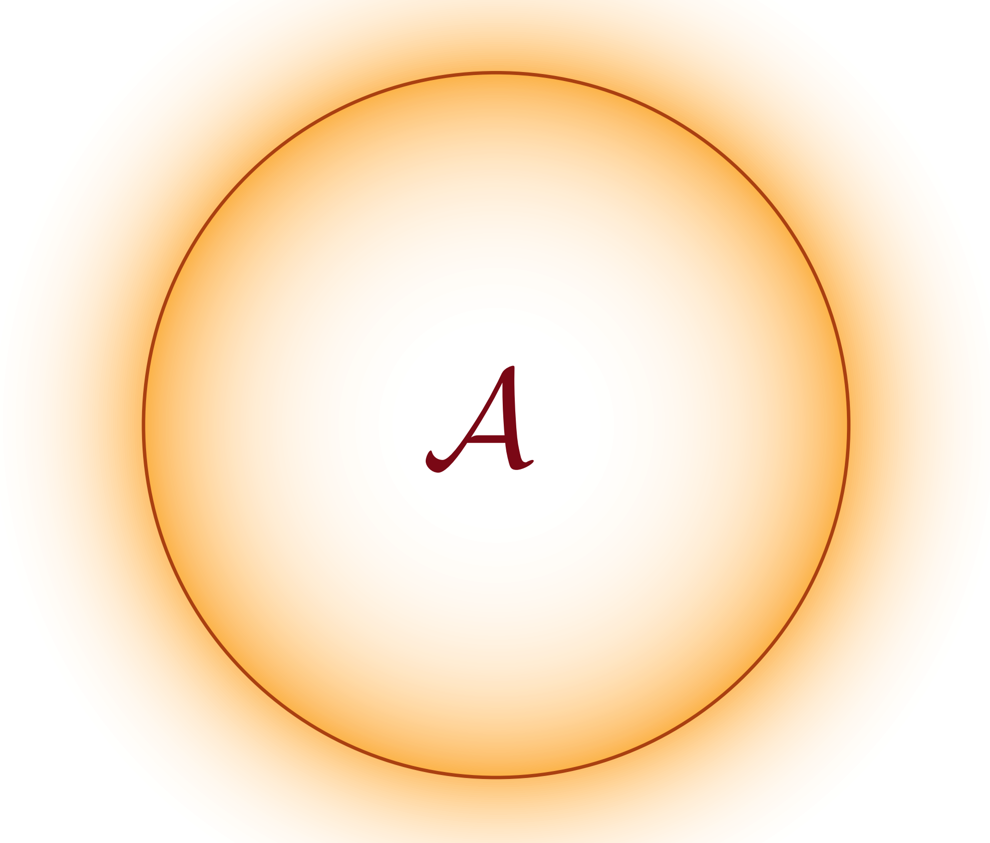
O. I. Motrunich and M. P. A. Fisher, *Physical Review* B **75**, 235116 (2007)
L. Huijse and S. Sachdev, *Physical Review* D **84**, 026001 (2011)
S. Sachdev, to appear

In particle physics: Quarks and gauge fields are "fundamental", and two quarks can bind to form a bosonic meson.

In particle physics: Quarks and gauge fields are "fundamental", and two quarks can bind to form a bosonic meson.

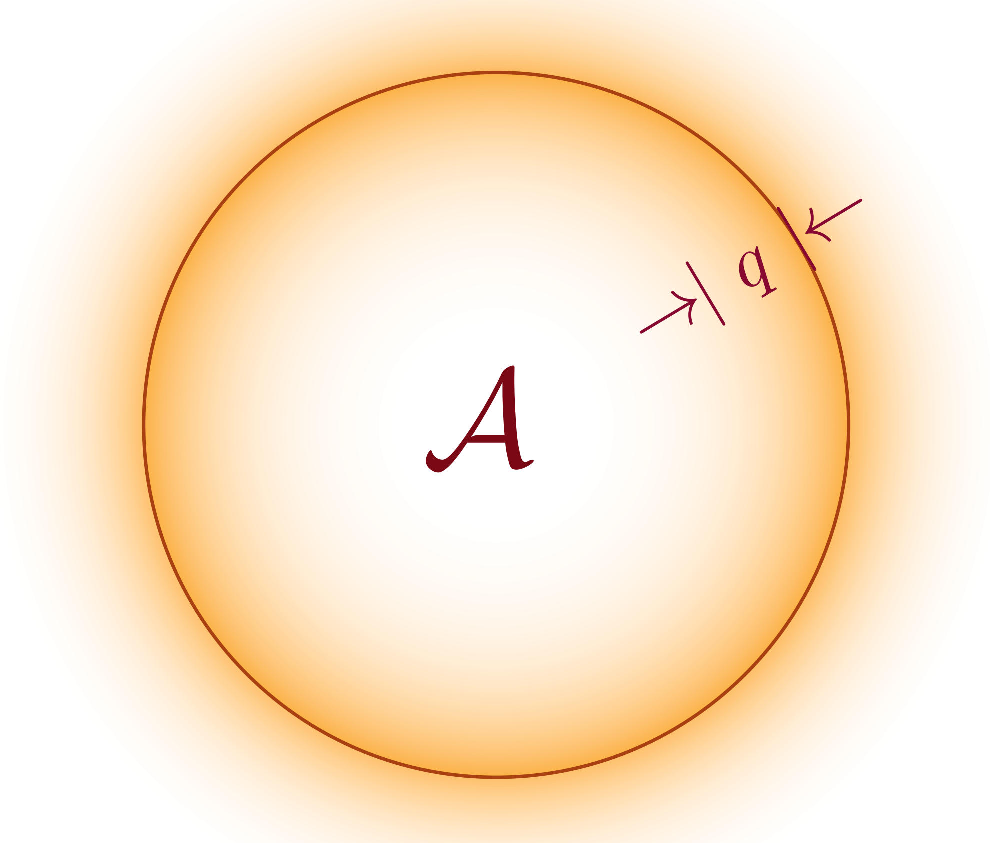
In condensed matter: The lattice boson is "fundamental", but it can *fractionalize* into fermionic quarks and *emergent* gauge fields.

Bose metals



- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the fermion density

Bose metals



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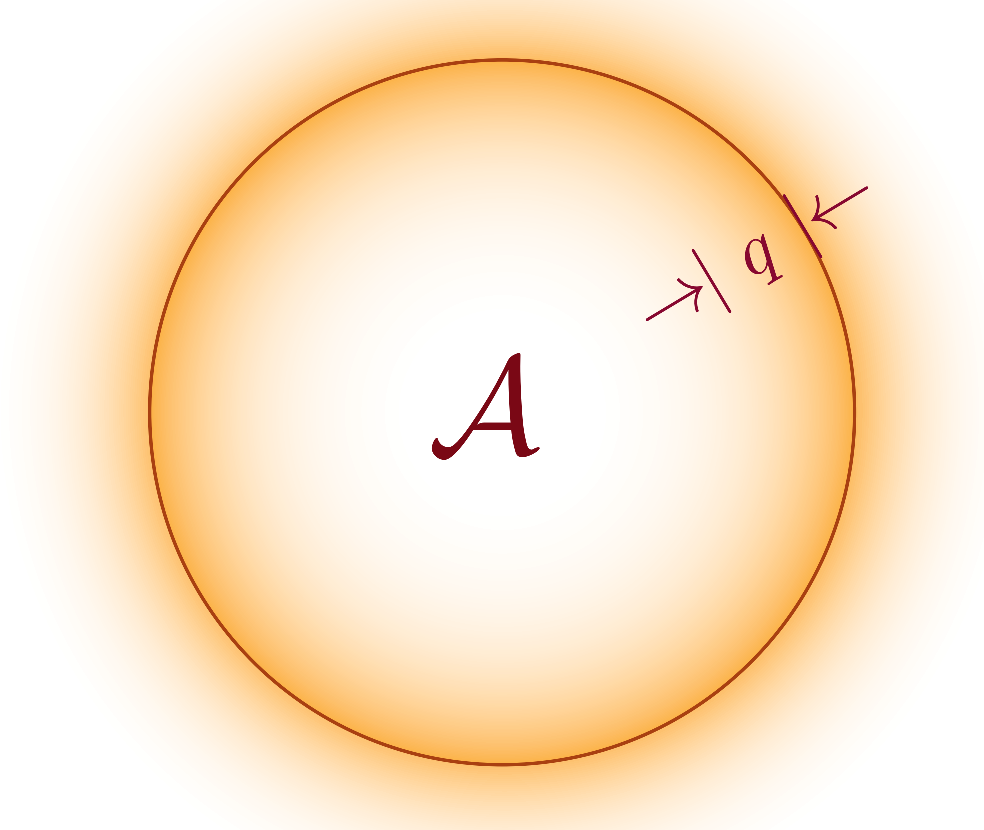
- Particle and hole of excitations near the Fermi surface with energy $\omega \sim |q|^z$; three-loop computation shows $z = 3/2$.

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Bose metals

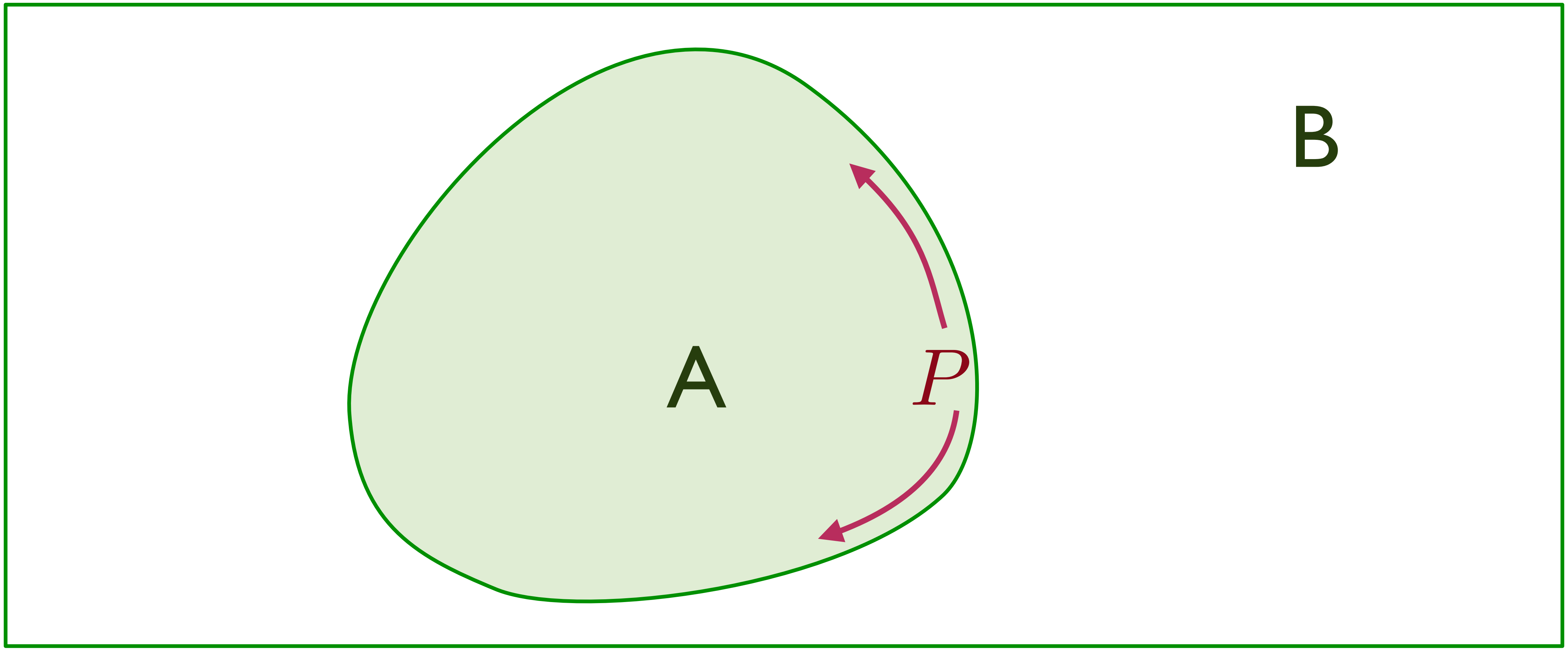


- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the fermion density

- Particle and hole of excitations near the Fermi surface with energy $\omega \sim |q|^z$; three-loop computation shows $z = 3/2$.

- The phase space density of fermions is effectively one-dimensional, so the entropy density $S \sim T^{(d-\theta)/z}$ with $\theta = d - 1$.

Entanglement entropy of a Bose metal

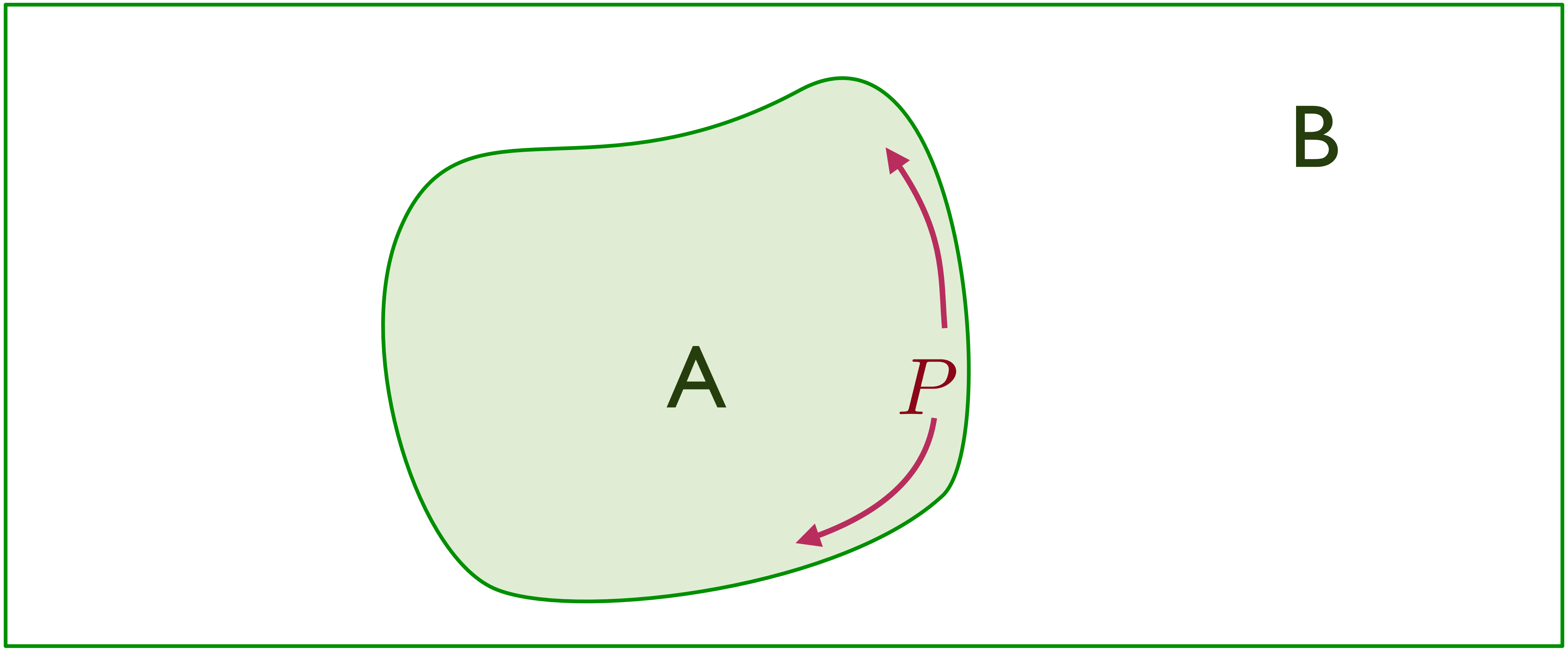


Logarithmic violation of "area law": $S_E \propto (k_F P)\ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)
B. Swingle, *Physical Review Letters* **105**, 050502 (2010)
Y. Zhang, T. Grover, and A. Vishwanath, *Physical Review Letters* **107**, 067202 (2011)

Entanglement entropy of a Bose metal



Logarithmic violation of "area law": $S_E \propto (k_F P) \ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A.
The coefficient is *independent* of the shape of A.

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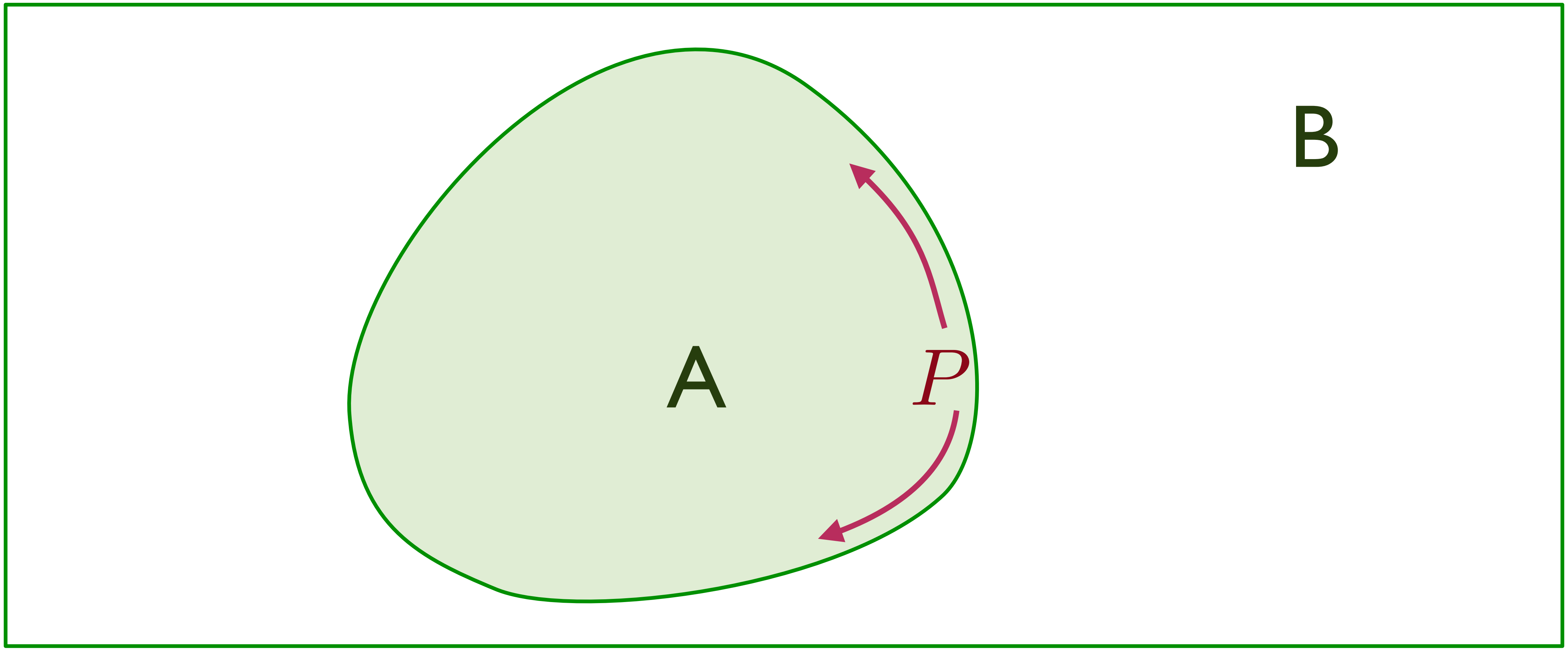

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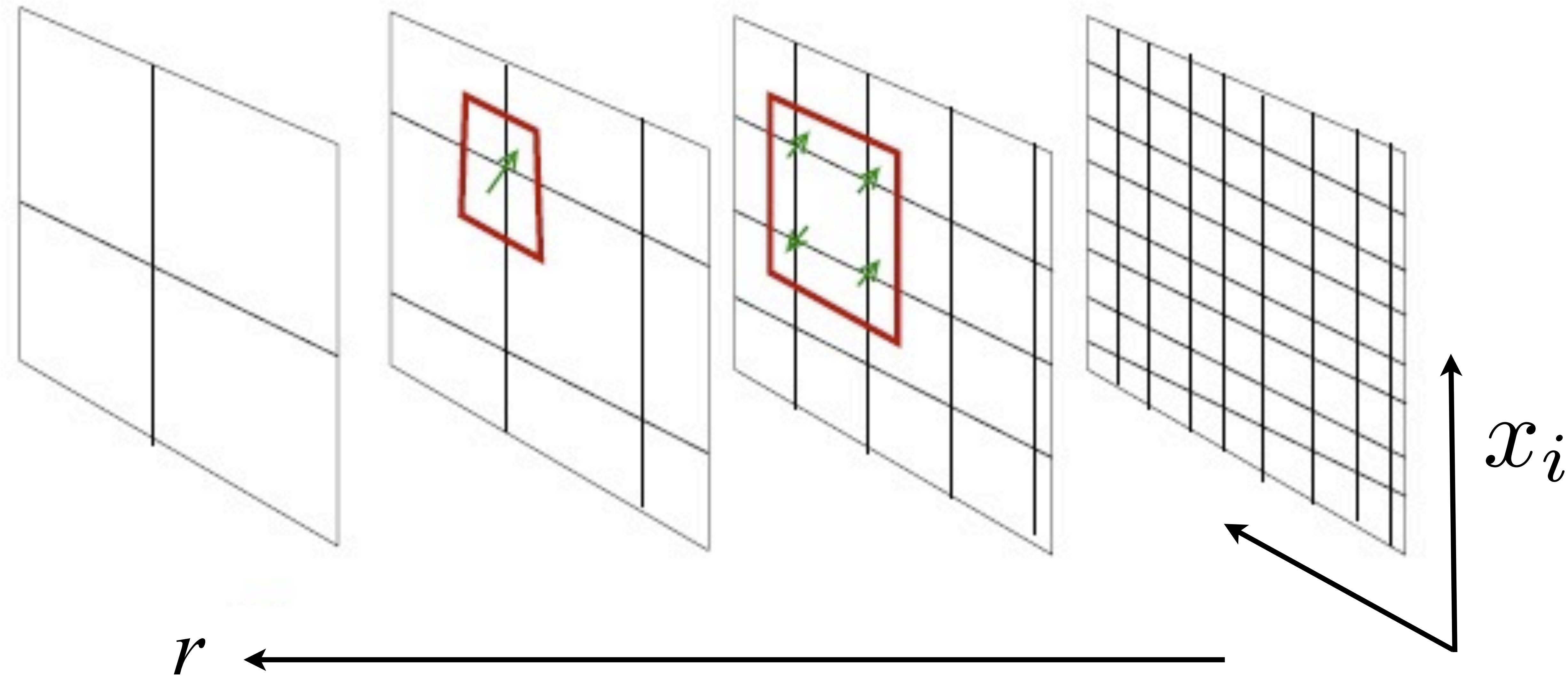
Holography



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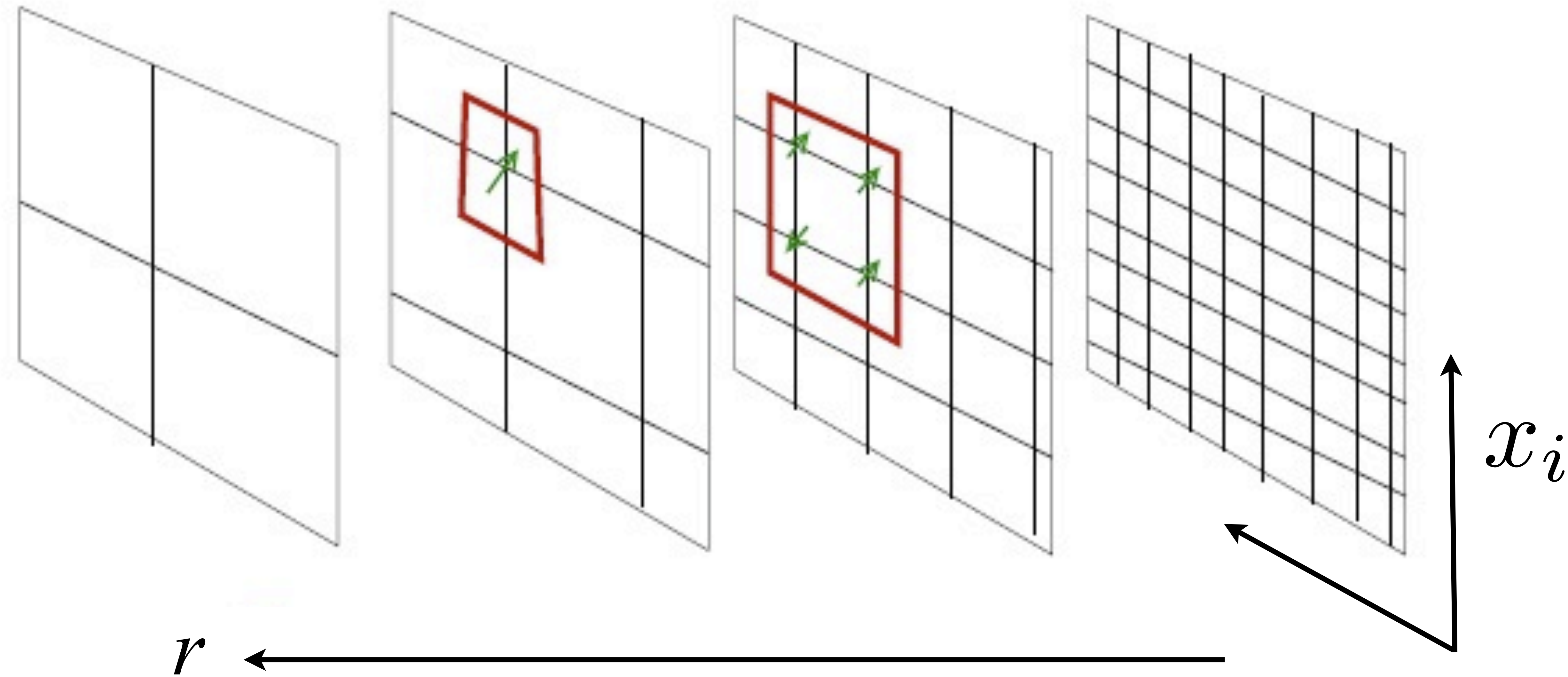
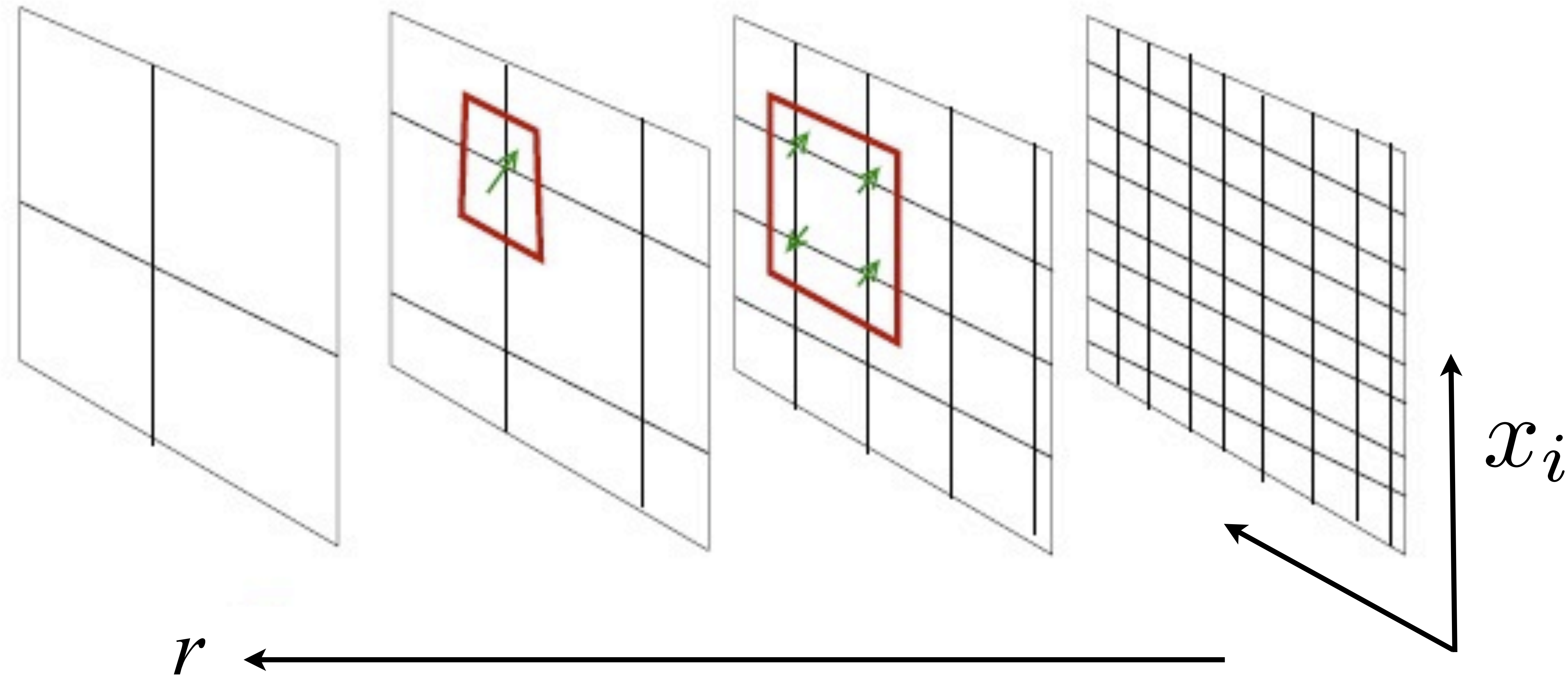


Consider a metric which transforms under rescaling as

$$x_i \to \zeta\, x_i, \quad t \to \zeta^z\, t, \quad ds \to \zeta^{\theta/d}\, ds.$$

Recall: conformal matter has $\theta = 0$, $z = 1$, and the metric is anti-de Sitter

Holography



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The value $\theta = d-1$ reproduces *all* the essential characteristics of the entropy and entanglement entropy of a Bose metal.

N. Ogawa, T. Takayanagi, and T. Ugajin, JHEP **1201**, 125 (2012).
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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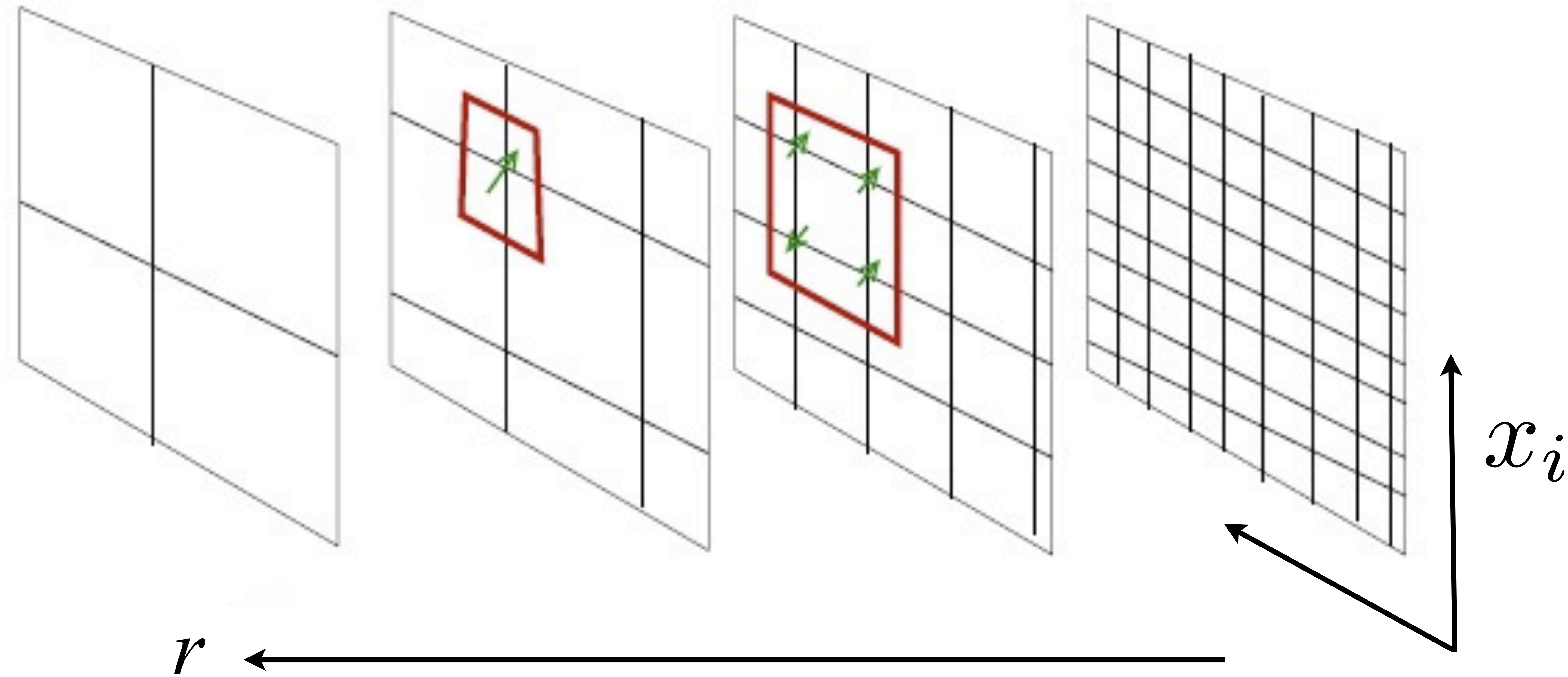

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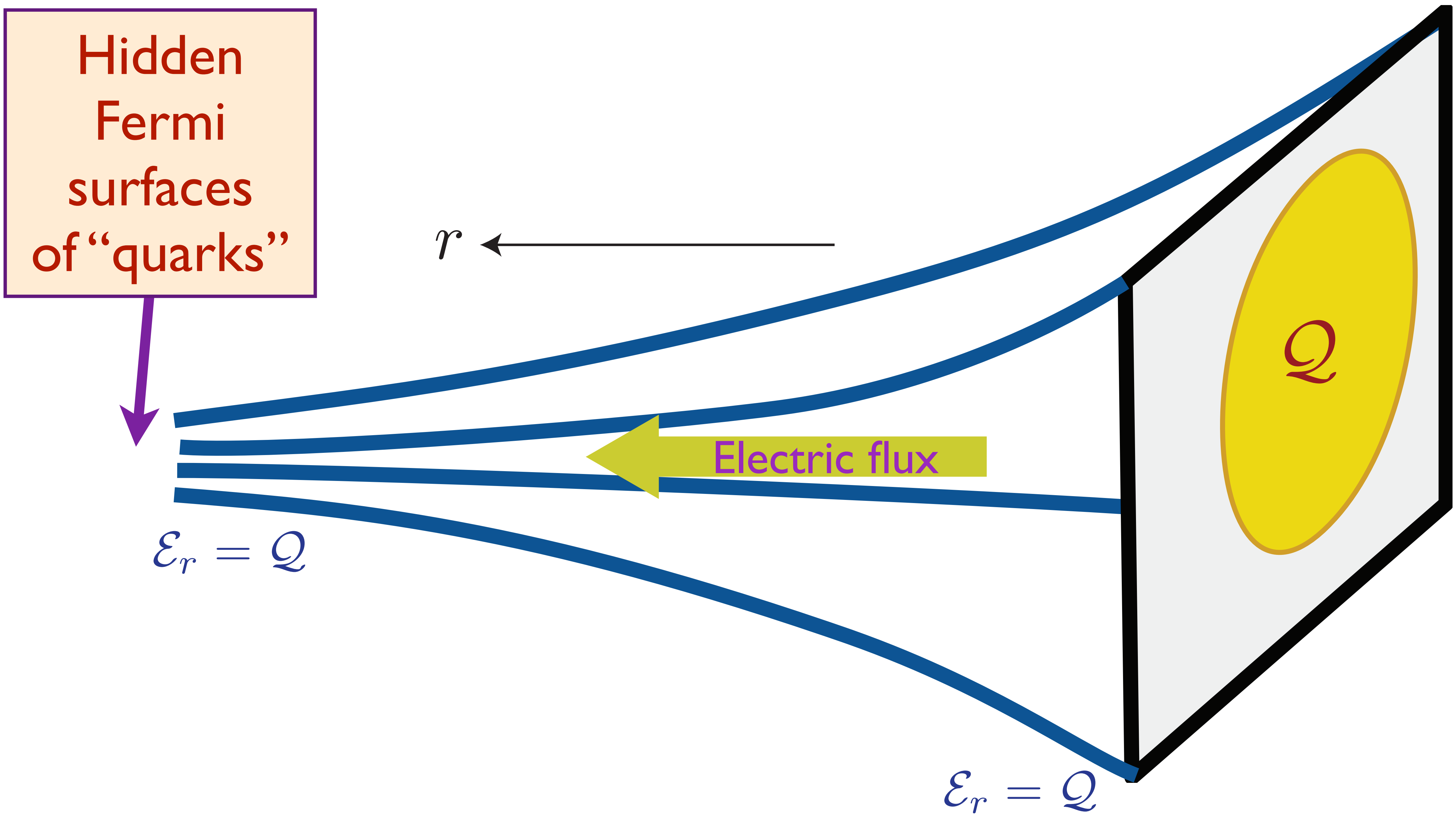
$$x_i \to \zeta\, x_i, \quad t \to \zeta^z\, t, \quad ds \to \zeta^{\theta/d}\, ds.$$

The null-energy condition of gravity yields $z \geq 1 + \theta/d$. In $d = 2$, this corresponds to $z \geq 3/2$ (recall: field theory yields $z = 3/2$!)

N. Ogawa, T. Takayanagi, and T. Ugajin, JHEP **1201**, 125 (2012).
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holographic theory of a Bose metal



Fully fractionalized state has all the electric flux exiting to the horizon at $r = \infty$

Conclusions

Realizations of many-particle entanglement:
Z_2 spin liquids and
conformal quantum critical points

Conclusions

More complex examples in metallic states are experimentally ubiquitous, but pose difficult strong-coupling problems to conventional methods of field theory

Conclusions

String theory and gravity in
emergent dimensions
offer a remarkable new approach
to describing states with many-particle quantum entanglement.

Much recent progress offers hope of a holographic description of "strange metals"