Calculating Standard Errors and Confidence Intervals for the AHRQ Quality Indicators

All of the AHRQ Quality Indicator modules begin with estimating a logit model of a 0/1 outcome variable and a set of patient[1]-level covariates as dependent variables, and using the results to form the expected outcome for each patient (e.g. P=pr(outcome=1)).

I. Notation:

Y_{ij} = 0 or 1, outcome for patient j in hospital i.
X_{ij} = covariates (e.g., gender, age, DRG, comorbidity)
P_{ij} = predicted probability from logit of Y on X
= $\exp(X_{ij}\beta)/[1+ \exp(X_{ij}\beta)]$
where β is estimated from logit on entire sample.
e_{ij} = $Y_{ij} - P_{ij}$ = logit residual (difference between actual and expected).
n_i = number of patients in sample at hospital i.
α = average outcome in the entire sample[2] (e.g. Ybar).

II. Estimating the Risk Adjusted Rate (RAR) and SE using the *Ratio* Method[3] of Indirect Standardization for each Hospital:

1. Estimating RAR:

let $O_i = (1/n_i)\sum(Y_{ij})$ be the observed rate at hospital i
let $E_i = (1/n_i)\sum(P_{ij})$ be the expected rate at hospital i

RAR_i
$= \alpha(O_i/E_i) = \alpha\,[(1/n_i)\sum(Y_{ij})]/\,[(1/n_i)\sum(P_{ij})]$ (where sum is for $j = 1$ to $j = n_i$)
= reference population rate * observed/expected at hospital i.

2. Estimating Variance of RAR (SE is the square root)[4]:

$Var(RAR_i)$
$= Var[\alpha(O_i/E_i)]$
$= (\alpha/E_i)^2 Var[O_i]$ (since $var(aX) = a^2 var(X)$ for any constant a)
$= (\alpha/E_i)^2 Var[(1/n_i)\sum(Y_{ij})]$ (by the definition of O_i)
$= (\alpha/E_i)^2(1/n_i)^2 Var[\sum(Y_{ij})]$ (since $var(aX) = a^2 var(X)$ for any constant a)
$= (\alpha/E_i)^2(1/n_i)^2[\sum Var(Y_{ij})]$ (since $var(\sum X_i)=\sum var(X_i)$ if Xi are independent)
$= (\alpha/E_i)^2(1/n_i)^2\sum [P_{ij}(1-P_{ij})]$ (since Y is 0/1, so $var(Y) = P(1-P)$)

[1] The same methods apply to the area-level indicators, but for clarity the methods refer only to the provider-level indicators.
[2] For the AHRQ QI, the sample is the entire reference population consisting of the discharges in the SID for the participating states. Therefore, the "average outcome for the entire sample" is the reference population rate.
[3] Risk-adjusted rate = (Observed rate / Expected Rate) * Reference Population Rate
[4] For a discussion of this variance estimation method, see Hosmer D and Lemeshow S. Confidence Interval Estimates of an Index of an Index of Quality Performance Based on Logistic Regression Models. Statistics in Medicine, Volume 14, 2161-2172 (1995).

III. Computing the 95 percent Confidence Interval around RAR

To compute the 95 percent confidence interval around the RAR, multiple the SE (the square root of the variance computed in Section II) by 1.96 and add (subtract) from the RAR:

Lower Bound of the CI = RAR – (1.96 * SE)
Upper Bound of the CI = RAR + (1.96 * SE)

The RAR of Hospital i (RAR_i) is *significantly lower* than some external point estimate of performance (e.g., the state or national average rate) if the <u>upper bound</u> of the CI is *lower than* the point estimate. The RAR of Hospital i (RAR_i) is *significantly higher* than some external point estimate of performance (e.g., the state or national average rate) if the <u>lower bound</u> of the CI is *higher than* the point estimate.

[Note: the same method may be used to compute confidence intervals around the observed rates by using the reference population rate in place of the expected rate]