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OPTIMAL FISCAL POLICY WHEN PUBLIC CAPITAL IS PRODUCTIVE: A BUSINESS CYCLE PERSPECTIVE

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ABSTRACT

This paper develops a dynamic general-equilibrium model with productive public capital to help account for differences in the business cycle characteristics of public- versus private-sector expenditures in postwar U.S. data. A specification that allows for multiple stochastic shocks (to technology and depreciation rates) can reproduce a number of features describing the cyclical behavior of U.S. public investment and public consumption as well as other fiscal variables, such as average marginal tax rates and the government debt-to-output ratio. The model also delivers reasonable predictions for the behavior of private-sector aggregates. It is less successful, however, in capturing the large variability of public consumption expenditures in U.S. data, and it overpredicts the variability of the capital tax relative to the labor tax.

1. Introduction

In postwar U.S. data, the cyclical behavior of public investment and public consumption is very different from the behavior of their private-sector counterparts. In comparison, public investment is much less variable than private investment, while the reverse holds true for consumption, i.e., public consumption is about twice as variable as private consumption. Moreover, the correlation between public expenditures and output is very weak. This contrasts sharply with the highly procyclical nature of private-sector expenditures. In this paper, I develop a dynamic general-equilibrium model with productive public capital to help account for these types of differences, as well as other features describing the cyclical behavior of U.S. fiscal policy. The approach represents an effort to carefully model the behavior of an entity--namely, the government--that directly accounts for about one-quarter of U.S. GNP and whose policies greatly impact the behavior of agents that generate the remaining three-fourths. This approach can be viewed as a natural extension of recent efforts to integrate fiscal policy into equilibrium models of the business cycle.

This paper builds on the recent work of Chari, Christiano, and Kehoe (1994a,b), who develop a competitive real business cycle (RBC) model in which a government policymaker chooses an optimal sequence of distortionary taxes in a dynamic version of the Ramsey (1927) optimal tax problem. The model developed here differs from theirs in three important respects. First, public capital is introduced as an input to the economy's production technology. This feature motivates the public investment process and is consistent with an expanding body of theoretical and empirical research which suggests that public capital may play an important role in the dynamics of growth and output.[1] Second, due to the specification of constant returns to scale in all inputs, competitive firms realize positive economic profits in equilibrium. This implies that the optimal steady-state tax on capital is positive, consistent with U.S. observations. Furthermore, I show that when profits are taxed at the same rate as capital rental income, the stochastic version of the model pins down a unique decentralization that is consistent with the optimal allocations. In a model without profits, the optimal steady-state tax on capital is zero, and

[1]Theoretical models with productive public expenditures include Arrow and Kurz (1970), Barro (1990), Barro and Sala-i-Martin (1992), Jones, Manuelli, and Rossi (1993a), Baxter and King (1993), and Glomm and Ravikumar (1994). Some recent empirical applications include Finn (1993) and Kocherlakota and Yi (1995). Aschauer (1993) provides a review of the empirical evidence regarding the productive effects of public capital.

there exist many decentralizations that can support the optimal allocations.[2] Third, public consumption expenditures are endogenized by introducing a separable term in the household utility function. In the Chari, Christiano, and Kehoe model, government spending follows an exogenous stochastic process.

I compare simulation results from four versions of the model that differ according to the tax regime (distortionary versus lump-sum taxes) and the process governing stochastic shocks (single versus multiple shocks). The cyclical behavior of the allocations turns out to be similar under the two tax regimes, regardless of the shock specification. However, the distortionary tax model encompasses a larger set of endogenous fiscal variables. The additional variables are the distortionary tax rates on labor and capital income and the government debt-to-output ratio.

The model with distortionary taxes and multiple shocks (to technology and depreciation rates) can reproduce the following stylized facts describing the behavior of U.S. fiscal policy: Public consumption and public investment are much less procyclical than their private-sector counterparts; private investment is more variable than public investment; the capital tax is more variable than the labor tax; the correlation between tax rates and output is relatively weak; and the government debt ratio is about twice as variable as output. The model also delivers reasonable predictions for the cyclical behavior of private-sector aggregates.

The model's principal shortcomings are its inability to capture the high variability of public consumption expenditures observed in the data and its overprediction of the variability of the capital tax relative to the labor tax. Other deficiencies are that the debt-to-output ratio is too countercyclical, and the correlation between public expenditures and output is still higher than that observed in the data. I find that the introduction of additional features, such as shocks to the demand for public goods and higher profit levels, can help remedy some of these shortcomings. I also experiment with an alternative tax structure in which labor and capital incomes are taxed at the same rate. However, I find that this specification does not have much impact on the cyclical behavior of the optimal allocations.

Some related research includes Rojas (1993), who examines the optimal behavior of public

[2] The zero tax result is discussed by Arrow and Kurz (1970), pp. 195-203, and has been further elaborated on by Judd (1985) and Chamley (1986). The non-uniqueness result is shown by Zhu (1992) and Chari, Christiano, and Kehoe (1994a).

investment in an RBC model with a capital tax and a continuously balanced budget. An alternative modeling approach treats fiscal policy as a series of exogenous shocks. For example, Christiano and Eichenbaum (1992) include stochastic government spending in the household utility function to shift the labor supply curve and thereby help explain the low observed correlation between postwar U.S. labor hours and real wages (as measured by average labor productivity). Braun (1994), McGrattan (1994), and Dotsey and Mao (1994) show that similar results can be obtained by introducing stochastic distortionary taxes to shift the labor supply curve. Ambler and Paquet (1994a) develop an RBC model in which some of the government's fiscal variables (such as public investment and nonmilitary public consumption) are endogenous, while others (such as the income tax and military spending) are exogenous.

The remainder of this paper is organized as follows: Sections 2 and 3 describe the model and define a competitive equilibrium under each tax regime. The computation procedure and the choice of parameter values is discussed in section 4. Section 5 examines the quantitative implications of the model and presents the simulation results. Section 6 concludes.

2. The Model

The model economy consists of households, firms, and the government. All goods are produced using a privately owned technology that exhibits constant returns to scale in the three productive inputs: labor, private capital, and public capital. This specification implies that private firms earn an economic profit equal to the difference between the value of output and payments made to the private inputs. The existence of profits yields a positive optimal tax rate on capital in the steady state for the model with distortionary taxes.[3] As owners of the firms, households receive net profits in the form of dividends. Various options regarding the taxation of these dividends are considered.

[3]Jones, Manuelli, and Rossi (1993b) show that when profits derive from productive public goods, the absence of a separate profits tax yields a positive optimal tax rate on capital in the steady state. However, if profits derive from the monopoly power of firms, Guo and Lansing (1995a) show that the optimal steady-state tax on capital can be positive, negative, or zero. In a perfectly competitive environment with no profits, the optimal steady-state tax on capital is zero.

2.1 The Household's Problem

There is a large number of identical, infinitely lived households, each of which maximizes a stream of discounted utilities over sequences of consumption and leisure:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln c_t - A h_t + B \ln g_t \right) \qquad 0<\beta<1, \quad A, B>0. \tag{1}$$

In this utility function, β is the household discount factor and c_t represents private consumption goods. The symbol E_t is the expectation operator conditional on information available at time t. Each household is endowed with one unit of time each period and works h_t hours during period t. The fact that utility is linear in hours worked draws on the formulation of indivisible labor described by Rogerson (1988) and Hansen (1985). This means that all fluctuations in total labor hours are due to changes in the number of workers employed, as opposed to variations in hours per worker.[4] Household preferences also include a term representing the utility provided by public consumption goods g_t. For simplicity, I interpret g_t as representing a *per capita* quantity of public goods that is free from congestion effects or specific user charges. The separability in c_t and g_t implies that public consumption does not affect the marginal utility of private consumption, a specification supported by parameter estimates in McGrattan, Rogerson, and Wright (1993). Households view g_t as outside their control.

The representative household faces the following within-period budget constraint:

$$c_t + x_t + b_{t+1} \leq (1-\tau_{ht}) w_t h_t + (1-\tau_{kt}) (r_t k_t + \hat{\pi}_t + r_{bt} b_t) + \tau_{kt} \delta_t k_t + b_t - T_t, \quad k_0,\ b_0 \text{ given}, \tag{2}$$

where x_t is private investment, k_t is private capital, and b_{t+1} represents one-period, real government bonds carried into period t+1 by the household. Households derive income by supplying labor and capital services to firms at rental rates w_t and r_t. Two additional sources of household income are the firm's net profits $\hat{\pi}_t$ (which are distributed to households as dividends) and the interest earned on government bonds $r_{bt} b_t$.

[4]In postwar U.S. data, about two-thirds of the variance in total hours is due to changes in the number of workers. See Kydland and Prescott (1990).

I impose a restriction that prevents the government from taxing away profits. In particular, I assume that the tax authority does not distinguish between profits and other types of capital income. As a result, the tax on capital income also functions as a tax on profits, but one with an endogenous upper bound. This scenario is reflected in equation (2), where net profits $\hat{\pi}_t$, capital rental income $r_t k_t$, and bond interest $r_{bt} b_t$ are all taxed at the same rate τ_{kt}. Labor income is taxed at the rate τ_{ht}. The term $\tau_{kt} \delta_t k_t$ represents the depreciation allowance built into the U.S. tax code, where δ_t is the depreciation rate. The depreciation rate for tax purposes is assumed to coincide with the rate of economic depreciation. Finally, T_t represents a lump-sum tax.

The following equation describes the law of motion for the private capital stock:

$$k_{t+1} = (1-\delta_t)k_t + x_t . \tag{3}$$

In equations (2) and (3), the depreciation rate can vary over time according to an exogenous stochastic process that will be specified shortly. The depreciation rate for public capital (δ_{Gt}) is also allowed to vary. Variable depreciation rates can be thought of as capturing the impact of external forces (for example, energy prices) on the utilization level or on the degree of obsolescence of the existing capital stock.[5] Households view tax rates, wages, interest rates, dividends, and depreciation as determined outside their control.

[5]Several recent examples where shocks of this sort have been successfully used in business cycle applications are Greenwood, Hercowitz, and Huffman (1988), Hercowitz and Sampson (1991), and Ambler and Paquet (1994b).

The household first-order conditions with respect to the indicated variables and the associated transversality conditions (TVC) are

$$c_t: \qquad \lambda_t = \frac{1}{c_t} \tag{4a}$$

$$h_t: \qquad \lambda_t (1-\tau_{ht}) w_t = A \tag{4b}$$

$$k_{t+1}: \qquad \lambda_t = \beta\, E_t \lambda_{t+1}\left[(1-\tau_{kt+1})(r_{t+1} - \delta_{t+1}) + 1\right] \tag{4c}$$

$$b_{t+1}: \qquad \lambda_t = \beta\, E_t \lambda_{t+1}\left[(1-\tau_{kt+1})\, r_{bt+1} + 1\right] \tag{4d}$$

$$\text{TVC:} \qquad \lim_{t\to\infty} E_0\, \beta^t \lambda_t k_{t+1} = 0, \qquad \lim_{t\to\infty} E_0\, \beta^t \lambda_t b_{t+1} = 0, \tag{4e}$$

where λ_t is the Lagrange multiplier associated with the budget constraint (2). The transversality conditions ensure that (2) can be transformed into an infinite-horizon, present-value budget constraint.

2.2 The Firm's Problem

Output (y_t) is produced by identical private firms that seek to maximize after-tax profits, subject to a technology that exhibits constant returns to scale in the three productive inputs, h_t, k_t, and k_{Gt}, where k_{Gt} is the *per capita* stock of public capital. As with g_t, I assume that public capital is free from congestion effects and specific user charges.[6] The production technology is subjected to serially correlated exogenous shocks z_t that are revealed to agents at the beginning of period t. These shocks generate equilibrium business cycle fluctuations in the model. Since the focus here is on the (detrended) business cycle movements of variables, the model abstracts from exogenous technical progress. The firm's decision problem can be summarized as

[6] Specifying k_{Gt} as a per capita quantity ensures that there are no scale effects associated with the number of firms (see Barro and Sala-i-Martin. [1992]). This setup can be viewed as incorporating an implicit congestion effect associated with the number of firms (which is equal to the number of households here). For a model with an explicit congestion effect that is linked to the size of the private capital stock, see Glomm and Ravikumar (1994).

$$\max_{k_t,\, h_t} (1-\tau_{kt})^{\gamma-1}(y_t - r_t k_t - w_t h_t), \qquad \gamma \geq 0, \tag{5}$$

subject to

$$y_t = \exp(z_t)\, k_t^{\theta_1} h_t^{\theta_2} k_{Gt}^{\theta_3} \qquad 0<\theta_i<1, \qquad \theta_1+\theta_2+\theta_3 = 1. \tag{6}$$

The above expressions allow for the possibility that the firm's profits may be taxed directly. The government's use of the tax rate τ_{kt} for this purpose implies that the tax authority does not distinguish between households and firms when assessing taxes on capital income. As before, this ensures that profits will not be completely taxed away. When $\gamma=2$, profits are initially taxed at the firm level and then taxed again as dividends at the household level. When $\gamma=1$, dividends are taxed only at household level. When $\gamma=0$, the effective tax rate on profits is zero. In reduced form, the $\gamma=0$ case captures the possibility that the tax authority cannot observe pure profits. The first-order conditions from (5) are

$$r_t = \theta_1 y_t / k_t, \tag{7a}$$

$$w_t = \theta_2 y_t / h_t. \tag{7b}$$

Each household is the owner of a firm and receives the firm's after-tax profits in the form of dividends. The expression for dividends is

$$\hat{\pi}_t = (1-\tau_{kt})^{\gamma-1}(1-\theta_1-\theta_2)\, y_t. \tag{8}$$

The stochastic process governing the evolution of the technology shock and the capital depreciation rates is summarized below:

$$\begin{bmatrix} z_{t+1} \\ \delta_{t+1} \\ \delta_{Gt+1} \end{bmatrix} = \Phi \begin{bmatrix} z_t \\ \delta_t \\ \delta_{Gt} \end{bmatrix} + \Phi_0 + \varepsilon_{t+1}, \qquad \varepsilon_t \sim N(0, \Sigma_\varepsilon), \quad z_0,\ \delta_0,\ \delta_{G0} \ \text{given}. \tag{9}$$

In (9), Φ is a 3×3 transition matrix, Φ_0 is a 3×1 matrix of constants, and ε_{t+1} is a 3×1 matrix of serially uncorrelated residuals. The residuals are drawn from a normal distribution with mean 0 and covariance matrix Σ_ε. When the off-diagonal elements of Φ are non-zero, innovations in technology can have a direct impact on the capital depreciation rates, and vice versa.

2.3 The Government's Problem

The government chooses an optimal program of taxes, borrowing, and public expenditures to maximize the discounted utility of the household. Two versions of the government's problem are considered, depending on the type of tax instrument available to the policymaker. The first-best solution is obtained with lump-sum taxes. In this case, $\tau_{ht} = \tau_{kt} = 0$ for all t. The more realistic second-best solution is obtained when all taxes are distortionary, namely, when $T_t = 0$ for all t. To avoid time inconsistency problems, I assume that the government can commit to a sequence of policies announced at $t = 0$. Following the approach of Chari, Christiano, and Kehoe (1994a), I further assume that τ_{k0} and r_{b0} are specified exogenously such that tax revenue collected at $t = 0$ cannot finance all future expenditures. If the initial levy on private-sector assets is sufficiently large, then the government chooses $\tau_{ht} = \tau_{kt} = 0$ for some $t > \hat{t}$. This case is not very interesting because after period $\hat{t}$, the distortionary tax model looks identical to the model with lump-sum taxes. In per capita terms, the government's budget constraint and the law of motion for public capital are as follows:

$$\begin{aligned} & g_t + x_{Gt}\left[1 + \varphi(x_{Gt}/k_{Gt})\right] + b_t(1 + r_{bt}) = \\ & b_{t+1} + \tau_{ht} w_t h_t + \tau_{kt}\left[(r_t - \delta_t)k_t + r_{bt} b_t\right] + \left[1 - (1 - \tau_{kt})^{\gamma}\right](1 - \theta_1 - \theta_2) y_t + T_t, \end{aligned} \tag{10}$$

$$k_{Gt+1} = (1 - \delta_{Gt}) k_{Gt} + x_{Gt}. \tag{11}$$

Government expenditures on the left-hand side of (10) include public consumption g_t, public investment x_{Gt}, and an adjustment cost for public investment equal to $x_{Gt}\,\varphi(x_{Gt} / k_{Gt})$. Following Abel and Blanchard (1983), adjustment costs are formulated as a premium $\varphi(\cdot)$ paid for each unit of investment goods relative to consumption goods. The properties of $\varphi(\cdot)$ are $\varphi(0) = 0$, $\varphi'(\cdot) > 0$, and $\varphi''(\cdot) > 0$. In the computations, a simple quadratic version of $\varphi(\cdot)$ is employed: $\varphi(\cdot) = \frac{1}{2}\alpha(x_{Gt}/k_{Gt})^2$, where $\alpha > 0$. The presence of a small adjustment cost ensures that public investment always remains positive in the stochastic simulations. This cost can be viewed as reflecting differences in the way that investment decisions are made in the public versus the private sector. For example, the decision to undertake an infrastructure project may require a public debate or a voter referendum to settle issues about financing or environmental impact.

The summation of the household budget constraint (2) and the government budget constraint (10) yields the following per capita resource constraint for the economy:

$$y_t = c_t + g_t + x_t + x_{Gt}\left[1 + \varphi(x_{Gt}/k_{Gt})\right]. \tag{12}$$

Because the resource constraint and the government budget constraint are not independent equations, equation (12) can be used in place of (10) in formulating the government's problem.

As a condition for equilibrium, government policy must take into account the rational responses of the private sector, as summarized by (2), (3), (4), (7), and (8). For the distortionary tax case, these equations can be conveniently summarized by the following "implementability constraint":

$$\begin{aligned} E_0 \sum_{t=1}^{\infty} \beta^t \left[1 - A h_t - \lambda_t (1-\tau_{kt})^\gamma (1-\theta_1-\theta_2) y_t\right] + 1 - A h_0 \, - \\ \lambda_0 \left\{(1-\tau_{k0})^\gamma (1-\theta_1-\theta_2) y_0 + \left[(1-\tau_{k0})(r_0-\delta_0)+1\right] k_0 + \left[(1-\tau_{k0}) r_{b0} + 1\right] b_0\right\} = 0. \end{aligned} \tag{13}$$

Equation (13) is obtained by substituting the first-order conditions of the household and the firm into the present-value household budget constraint, with $T_t = 0$ for all t.[7] When $\gamma = 0$, the capital tax for $t \geq 1$ does not appear in the implementability constraint. Since τ_{k0} and r_{b0} are specified exogenously, the government's problem in this case amounts to choosing a set of allocations λ_t (= $1/c_t$), h_t, g_t, k_{t+1}, and k_{Gt+1} for all t to maximize household utility (1) subject to (12) and (13). Given the optimal allocations, the appropriate set of prices r_t and w_t, and policy variables τ_{ht}, τ_{kt}, and r_{bt} that decentralize them, can be computed using the profit-maximization conditions (7), the household first-order conditions (4), and the household budget constraint (2). For example, the optimal allocations uniquely determine w_t through equation (7b). Given λ_t and w_t, equation (4b) uniquely determines the government's optimal choice for

[7] More specifically, equation (13) is obtained as follows: Multiply both sides of the household budget constraint (2) by λ_t, take expectations at $t = 0$, substitute in (4a)-(4d) and (8), iterate the resulting expression forward and sum over time, and finally, apply the transversality conditions (4e).

τ_{ht}. The government has much more flexibility, however, in choosing the optimal capital tax and the optimal interest rate on government debt. The expectation operators in (4c) and (4d) imply that the after-tax returns on capital and bonds (weighted by marginal utility) must be the same "on average." In response to a series of shocks, the government can satisfy this ex ante arbitrage condition and implement the optimal allocations using many different decentralizations involving τ_{kt} and r_{bt}. Consequently, when $\gamma = 0$ (or when $1 - \theta_1 - \theta_2 = 0$), the model does not uniquely pin down the time-series behavior of these policy variables. (See Zhu [1992] and Chari, Christiano, and Kehoe [1994a] for a more complete description.)

When $\gamma > 0$, the capital tax for $t \geq 1$ appears in (13). In this case, (4c) must be imposed as a separate constraint on the government's problem, and τ_{kt} for $t \geq 1$ is an additional decision variable. In contrast to the earlier indeterminate case, the model now implies a unique decision rule for τ_{kt} that is consistent with the optimal allocations. Given this decision rule, equation (2) and the optimal allocations uniquely determine r_{bt}. Intuitively, the restriction that profits and dividends must be taxed at the same rate as capital rental income pins down a unique decentralization to support the allocations.[8]

Under lump-sum taxes, the optimal allocations can be obtained by maximizing (1) subject only to the resource constraint (12). In this case, government expenditures can be financed by any arbitrary sequence of lump-sum taxes and government debt, provided that the government budget constraint is satisfied in present-value terms (see Sargent [1987], chapter 3).

[8]Bohn (1994) provides a related example where a restriction on the menu of available policy instruments pins down a unique decentralization. In his model, capital incomes derived from different technologies must all be taxed at the same (state-contingent) tax rate.

The general version of the government's problem is

$$\max_{\substack{\lambda_t,\, h_t,\, g_t,\, \tau_{kt},\\ \mu_t,\, k_{t+1},\, k_{Gt+1}}} E_0 \sum_{t=1}^{\infty} \beta^t \Big\{ \ln(1/\lambda_t) - A\,h_t + B \ln g_t + \Lambda\Big[1 - A\,h_t - \lambda_t (1-\tau_{kt})^{\gamma}(1-\theta_1-\theta_2)\,y_t\Big] +$$

$$\mu_{t-1}\lambda_t\Big[(1-\tau_{kt})(\theta_1 y_t/k_t - \delta_t) + 1\Big] - \mu_t\lambda_t \Big\} + \ln(1/\lambda_0) - A\,h_0 + B\ln g_0 - \mu_0\lambda_0 +$$

$$\Lambda\Big\{1 - A\,h_0 - \lambda_0(1-\tau_{k0})^{\gamma}(1-\theta_1-\theta_2)\,y_0 - \lambda_0\Big[(1-\tau_{k0})(r_0-\delta_0)+1\Big]k_0 - \lambda_0\Big[(1-\tau_{k0})r_{b0}+1\Big]b_0\Big\},$$

subject to

$$\begin{aligned} g_t &= y_t - 1/\lambda_t - k_{t+1} + k_t(1-\delta_t) - \Big[k_{Gt+1} - k_{Gt}(1-\delta_{Gt})\Big](1+\varphi_t),\\ y_t &= \exp(z_t)\,k_t^{\theta_1}\,h_t^{\theta_2}\,k_{Gt}^{\theta_3},\\ \varphi_t &= \tfrac{\alpha}{2}\big(k_{Gt+1}/k_{Gt} - 1 + \delta_{Gt}\big)^2, \end{aligned} \tag{14}$$

$$\begin{bmatrix} z_{t+1}\\ \delta_{t+1}\\ \delta_{Gt+1}\end{bmatrix} = \Phi \begin{bmatrix} z_t\\ \delta_t\\ \delta_{Gt}\end{bmatrix} + \Phi_0 + \varepsilon_{t+1},$$

with z_0, δ_0, δ_{G0}, k_0, k_{G0}, b_0, τ_{k0}, and r_{b0} given. The Lagrange multiplier Λ associated with (13) is determined endogenously at $t=0$ and is constant over time. To impose (4c) as a constraint, I apply the law of iterated expectations and group terms such that the return function at time t involves only variables dated t or earlier. This is done to facilitate a recursive solution algorithm. Notice that the first-order condition with respect to the Lagrange multiplier μ_t recovers (4c). The lagged multiplier μ_{t-1} is treated as an additional state variable for $t \geq 1$ in the recursive version of (14).[9] When $\gamma = 0$, the first-order condition with respect to τ_{kt} implies $\mu_t = 0$ for all t. This reflects the fact that (4c) does not need to be imposed as a separate constraint when $\gamma = 0$.

With distortionary taxes, the solution to (14) for $t \geq 1$ can be characterized by a set of stationary

[9] Including μ_{t-1} in the state vector at time t is the mechanism by which the commitment assumption is maintained when choosing τ_{kt} each period in the recursive version of (14). This solution method was developed by Kydland and Prescott (1980) in a deterministic setting. Rojas (1993) shows how the method can be applied in a stochastic environment.

decision rules $\lambda_t(s_t, \Lambda)$, $h_t(s_t, \Lambda)$, $g_t(s_t, \Lambda)$, $k_{t+1}(s_t, \Lambda)$, $k_{Gt+1}(s_t, \Lambda)$, $\tau_{kt}(s_t, \Lambda)$, and $\mu_t(s_t, \Lambda)$, where $s_t = \{z_t, \delta_t, \delta_{Gt}, k_t, k_{Gt}, \mu_{t-1}\}$. Given these decision rules, a stationary decision rule for the government bond allocation $b_{t+1}(s_t, \Lambda)$ can be computed as the solution to the following recursive equation:

$$\lambda_t(k_{t+1} + b_{t+1}) = \beta E_t\left[1 - A h_{t+1} - \lambda_{t+1}(1-\tau_{kt+1})^{\gamma}(1-\theta_1-\theta_2)y_{t+1} + \lambda_{t+1}(k_{t+2} + b_{t+2})\right]. \tag{15}$$

Equation (15) is the household budget constraint at $t+1$ (with $T_{t+1} = 0$), after taking expectations and substituting in the first-order conditions of the private sector. At $t = 0$, the government chooses λ_0, h_0, g_0, k_1, k_{G1}, and μ_0. The $t = 0$ allocations, together with the decision rules for $t \geq 1$, determine Λ for a given set of initial conditions.

With lump-sum taxes, $\Lambda = \mu_t = 0$ for all t, and (14) collapses to a standard social planning problem. The solution to the planner's problem for $t \geq 0$ can be characterized by a set of stationary decision rules $\lambda_t(s_t)$, $h_t(s_t)$, $g_t(s_t)$, $k_{t+1}(s_t)$, and $k_{Gt+1}(s_t)$, where $s_t = \{z_t, \delta_t, \delta_{Gt}, k_t, k_{Gt}\}$.

3. Defining an Equilibrium

This section defines a competitive equilibrium under each tax regime. In what follows, I assume that $\gamma > 0$ so that the solution to (14) pins down a unique decentralization under distortionary taxes. In this case, a competitive (Ramsey) equilibrium is defined as

(*i*) A set of stationary decision rules $\lambda_t(s_t, \Lambda)$, $h_t(s_t, \Lambda)$, $g_t(s_t, \Lambda)$, $k_{t+1}(s_t, \Lambda)$, $k_{Gt+1}(s_t, \Lambda)$, $\tau_{kt}(s_t, \Lambda)$, and $\mu_t(s_t, \Lambda)$ that satisfy (14) for $t \geq 1$, where $s_t = \{z_t, \delta_t, \delta_{Gt}, k_t, k_{Gt}, \mu_{t-1}\}$.

(*ii*) A stationary decision rule for the bond allocation $b_{t+1}(s_t, \Lambda)$ that satisfies (15) for $t \geq 1$.

(*iii*) A set of allocations λ_0, h_0, g_0, k_1, and k_{G1}, and Lagrange multipliers μ_0 and Λ, that satisfy (14) and (2) given the decision rules for $t \geq 1$ and the initial conditions z_0, δ_0, δ_{G0}, k_0, k_{G0}, b_0, τ_{k0} and r_{b0}.

(*iv*) A sequence of factor prices r_t and w_t for $t \geq 0$ defined by (7a) and (7b).

(*v*) A sequence of labor tax rates τ_{ht} for $t \geq 0$ defined by (4b).

(*vi*) A sequence of bond interest rates r_{bt} for $t \geq 1$ defined by (2).

Under lump-sum taxes, a competitive equilibrium is defined as

(*i*) A set of stationary decision rules $\lambda(s_t)$, $h_t(s_t)$, $g_t(s_t)$, $k_{t+1}(s_t)$, and $k_{Gt+1}(s_t)$ that satisfy (14) with $\Lambda = \mu_t = 0$ for $t \geq 0$, where $s_t = \{z_t, \delta_t, \delta_{Gt}, k_t, k_{Gt}\}$, given the initial conditions z_0, δ_0, δ_{G0}, k_0, and k_{G0}.

(*ii*) A sequence of factor prices r_t and w_t for $t \geq 0$ defined by (7a) and (7b).

(*iii*) An arbitrary sequence of lump-sum taxes T_t and government debt b_t that satisfies:

$$g_0 + x_{G0}\left[1 + \varphi(x_{G0}/k_{G0})\right] - T_0 + \sum_{t=1}^{\infty} \frac{g_t + x_{Gt}\left[1 + \varphi(x_{Gt}/k_{Gt})\right] - T_t}{\prod_{i=1}^{t} (1 + r_{bi})} = 0, \tag{16}$$

$$\lim_{t \to \infty} \frac{b_{t+1}}{\prod_{i=1}^{t} (1 + r_{bi})} = 0. \tag{17}$$

(*iv*) A sequence of bond interest rates r_{bt} for $t \geq 0$ defined by (2).

4. Computation Procedure and Calibration

The stationary decision rules governing the optimal allocations in the model are obtained by solving a linear-quadratic approximation of (14). The algorithm makes use of the fact that (14) is recursive for $t \geq 1$ under distortionary taxes and for $t \geq 0$ under lump-sum taxes. The resulting decision rules, which are log-linear functions of the state variables, are valid in the neighborhood of the deterministic steady state.[10] Under distortionary taxes, the stationary decision rules depend on the Lagrange multiplier Λ, which is computed as follows. First, given an initial guess for Λ, I compute the deterministic steady state from the first-order conditions of (14) with respect to λ_t, h_t, g_t, k_{t+1}, k_{Gt+1}, μ_t, and τ_{kt}. The steady-state version of (15) is then used to compute the steady-state level of government debt $\bar{b}$. I repeat this procedure, adjusting Λ until a desired level of steady-state debt is obtained. Next, I use the first-order conditions of (14) at $t = 0$, together with the log-linear decision rules for $t \geq 1$, the household budget constraint (2) evaluated at $t = 0$ and $t = 1$, and the initial conditions z_0, δ_0, δ_{G0}, k_0,

[10] The computation procedure is based on the algorithm described by Hansen and Prescott (1994). Chari, Christiano, and Kehoe (1994b) assess the accuracy of the log-linear approximation method in an optimal policy problem similar to the one presented here. They conclude that the approximation yields very accurate results for the allocations, but less accurate results for the tax rates, in comparison to a minimum-weighted residual method.

k_{G0}, τ_{k0}, and r_{b0}, to compute an initial level of debt b_0 that is consistent with Λ and $\bar{b}$. The economy is assumed to be in stationary equilibrium. All simulations represent small fluctuations around the deterministic steady state.

Parameters are assigned values based on empirically observed features of postwar U.S. data. The sample period begins in 1954 to avoid the influence of the Korean War. The time period in the model is taken to be one year, which is consistent with both the time frame of most government fiscal decisions and the frequency of available data on average marginal tax rates. The discount factor β $(=0.962)$ implies a real rate of interest of 4 percent. The parameter A in the household utility function is chosen such that the fraction of time spent working is equal to 0.3 in the steady state. This coincides with time-use studies, such as Juster and Stafford (1991), which indicate that households spend approximately one-third of their discretionary time in market work. The value of B is chosen to yield a steady-state ratio $\bar{g}/\bar{y}$ equal to 0.17, the average value for the U.S. economy from 1954 to 1992. In computing this average, public consumption was estimated by subtracting total public investment (including military investment) from an annualized series for government purchases of goods and services (GGEQ from Citibase). This was done to reduce double counting, since the GGEQ series does not distinguish between government consumption and investment goods.[11]

The exponents in the Cobb-Douglas production function are chosen on the basis of two criteria. First, the selected values of θ_1 $(=0.32)$ and θ_2 $(=0.60)$ are in the range of the estimated shares of GNP received by private capital and labor in the U.S. economy (see Christiano [1988]). Second, given the steady-state depreciation rate $\bar{\delta}_G$ (described below), the output elasticity of public capital θ_3 $(=0.08)$ is chosen to yield a steady-state ratio of public investment to output equal to 0.03. This coincides with the average ratio of nonmilitary public investment to GNP in the U.S. economy from 1954 to 1992.[12]

With constant returns to scale in all inputs, the value of θ_3 $(=1-\theta_1-\theta_2)$ also determines the

[11]The specific parameter values used in the computations are $A=2.480$, $B=0.267$ for the model with distortionary taxes, and $A=3.330$, $B=0.285$ for the model with lump-sum taxes.

[12]The range of direct empirical estimates for θ_3 at the aggregate national level is quite large. Aschauer (1989) and Munnell (1990) estimate values of 0.39 and 0.34, respectively. Finn (1993) estimates a value of 0.16 for highway public capital. Aaron (1990) and Tatom (1991) argue that removing the effects of trends and taking account of possible missing explanatory variables (such as oil price shocks) can yield point estimates for θ_3 that are not statistically different from zero.

steady-state level of firm profits. Since profits do not affect household decisions at the margin, the government would like to tax profits as much as possible to obtain non-distortionary revenue. Choosing $\tau_{kt} > 0$ accomplishes this objective in varying degrees, depending on the value of γ. As γ increases, the capital tax collects a larger fraction of revenue from profits. This motivates the government to choose a higher capital tax and lower labor tax for a given level of profits. Since dividends are subject to double taxation under the U.S. tax code, I choose $\gamma = 2$ as the baseline, which implies that the optimal steady-state tax on capital in the model is $\overline{\tau}_k = 0.29$. This value coincides with the average effective corporate tax rate in the United States from 1954 to 1980, as estimated by Jorgenson and Sullivan (1981). The steady-state tax on labor ($\overline{\tau}_h$) turns out to be 0.22. This is close to the average marginal tax rate on labor income estimated by Barro and Sahasakul (1986) from 1954 to 1983. When $\gamma = 0$, the steady-state tax rates are $\overline{\tau}_k = 0.10$ and $\overline{\tau}_h = 0.33$.[13]

The steady-state private capital depreciation rate $\overline{\delta}$ ($= 0.067$) is estimated by a least squares regression of $x_t - (k_{t+1} - k_t)$ on k_t. An analogous regression yields the steady-state public capital depreciation rate $\overline{\delta}_G$ ($= 0.022$).[14] The steady-state value of the technology shock is set equal to zero. The transition matrix Φ governing the evolution of the shocks is estimated by a vector autoregression using detrended annual data on z_t, δ_t, and δ_{Gt} from 1954 to 1992.[15] The constant vector Φ_0 is chosen to agree

[13]When profits are zero ($\theta_1 + \theta_2 = 1$), the optimal steady-state tax on capital is zero. If a separate profits tax were available, the government would tax profits at 100 percent and other capital income at 0 percent in the steady state (see footnote 2).

[14]Data sources are as follows. The capital and investment series are in 1987 dollars from *Fixed Reproducible Tangible Wealth in the United States*, U.S. Department of Commerce (1993). The series for k_{Gt} and x_{Gt} include nonmilitary government-owned equipment, structures, and residential components. The series for k_t and x_t include business equipment and structures, consumer durables, and residential components. The "capital input" measure of the net stock was used for all capital data. Annualized series for the following variables were constructed using the indicated quarterly series from Citibase: y_t = GNPQ, c_t = GCNQ + GCSQ (nondurables and services), h_t = LHOURS (household survey), y_t / h_t = GNPQ/LHOURS, g_t = GGEQ - x_{Gt} - military investment. The series for b_t / y_t is federal debt held by the public as a fraction of GNP, where the debt series is from *Federal Debt and Interest Costs*, U.S. Congressional Budget Office (1993), table A-2. All variables were normalized by the total population series PAN from Citibase to obtain per capita quantities analogous to those in the model. Average marginal tax rates on labor income (τ_{ht}) are from Barro and Sahasakul (1986) for 1954-83, McGrattan, Rogerson, and Wright (1993) for 1954-87, and Mendoza, Razin, and Tesar (1994) for 1965-88. Average marginal tax rates on capital income (τ_{kt}) are from Jorgenson and Sullivan (1981, table 11) for 1954-80, McGrattan, Rogerson, and Wright (1993) for 1954-87, and Mendoza, Razin, and Tesar (1994) for 1965-88.

[15]The technology shock was measured as $z_t = \ln y_t - 0.32 \ln k_t - 0.60 \ln h_t - 0.08 \ln k_{Gt}$. The depreciation rates were measured as $\delta_t = [x_t - (k_{t+1} - k_t)] / k_t$ and $\delta_{Gt} = [x_{Gt} - (k_{Gt+1} - k_{Gt})] / k_{Gt}$. Trends were removed by regressing each series on a constant and a time trend and subtracting the trend. The specification with δ_t and δ_{Gt} in levels yielded a much better fit of the data than did a logarithmic specification.

with the steady-state values of the shock variables. The covariance matrix Σ_ε is estimated from the data and imposed in the simulations. The stationary stochastic process used in the simulations is

$$\begin{bmatrix} z_{t+1} \\ \delta_{t+1} \\ \delta_{Gt+1} \end{bmatrix} = \begin{bmatrix} 0.854 & -1.714 & -1.278 \\ -0.030 & 0.280 & -0.019 \\ 0.014 & 0.027 & 0.258 \end{bmatrix} \begin{bmatrix} z_t \\ \delta_t \\ \delta_{Gt} \end{bmatrix} + \begin{bmatrix} 0.143 \\ 0.048 \\ 0.015 \end{bmatrix} + \varepsilon_{t+1}, \tag{18}$$

$$\Sigma_\varepsilon = S' \cdot S, \text{ where } S = \begin{bmatrix} 0.00642 & -0.00126 & -0.00019 \\ 0 & 0.00531 & 6.224e{-5} \\ 0 & 0 & 0.00192 \end{bmatrix}. \tag{19}$$

The following matrix shows the correlation coefficients among the residuals:

$$\rho_\varepsilon = \begin{bmatrix} 1.00 & -0.23 & -0.10 \\ & 1.00 & 0.05 \\ & & 1.00 \end{bmatrix}. \tag{20}$$

An interesting aspect of ρ_ε is that innovations in the cyclical component of the technology variable display a weak negative correlation with innovations in the cyclical components of the depreciation rates. Also, the correlation between innovations in the two depreciation rates is weak ($=0.05$).[16] The model is also simulated with technology shocks alone. This provides a benchmark for comparison with the standard real business cycle specification. In this case, the depreciation rates are constant over time and the evolution of z_t is described by

$$z_{t+1} = 0.941\, z_t + \varepsilon_{t+1}, \qquad \varepsilon_t \sim N(0,\ 0.0124^2). \tag{21}$$

The parameter α ($=40$) is chosen such that $\varphi(\cdot) = 0.01$ in steady state. This implies that adjustment costs are equal to 1 percent of public investment, or 0.03 percent of output, in steady state. Costs of such small magnitude seem reasonable and are sufficient to ensure that x_{Gt} always remains

[16] This specification differs from that employed by Ambler and Paquet (1994b). They use Euler equations from their model together with quarterly investment data to construct an artificial time series of capital stocks and corresponding (quarterly) depreciation rates. In the resulting specification, depreciation shocks are strongly correlated with one another, but have no persistence and are not correlated with technology shocks. Here, annual depreciation rates are computed from actual capital stock data.

positive during the simulations. Under distortionary taxes, the value of Λ $(=0.313)$ implies a steady-state ratio $\bar{b}/\bar{y}$ equal to 0.37. This is the average level of U.S. federal debt held by the public as a fraction of GNP from 1954 to 1992. The initial conditions z_0, δ_0, δ_{G0}, k_0, k_{G0}, and τ_{k0} are set to their corresponding steady-state levels, and $r_{b0} = r_0 - \delta_0$. The initial debt ratio b_0/y_0 that is consistent with these values is 0.362.

Table 1 summarizes steady-state values for the model versus the corresponding U.S. averages from 1954 to 1992. For the data, three values are shown for the average marginal tax rates $\bar{\tau}_h$ and $\bar{\tau}_k$. These correspond to the sample means from different studies (see footnote 14 for sources).

5. Quantitative Implications of the Model

5.1 Optimal Decision Rules

Table 2 shows the optimal decision rules for selected variables in the case of distortionary taxes and multiple shocks.[17] The optimal behavior of public versus private expenditures over the business cycle can be inferred by examining the coefficients on the shock variables in the decision rules. For x_{Gt} and g_t, the coefficients on z_t are both positive, implying procyclical behavior. The private-sector counterparts, x_t and c_t, are also procyclical, as evidenced by their positive coefficients on z_t. Notice that public and private investment exhibit a sign difference regarding the coefficient on δ_t. This feature of the model, together with the fact that innovations in the depreciation rates tend to be negatively correlated with innovations in z_t, tends to reduce the contemporaneous correlation between public and private expenditures relative to the single-shock version of the model. This is a desirable outcome, given the weak correlation between public and private expenditures in postwar U.S. data.

The decision rules also show that the capital tax responds much more strongly to exogenous shocks than does the labor tax. This reflects the government's use of the capital tax as a tool for absorbing shocks to its budget (which are caused by changes in the size of the tax base over the business cycle). An increase in z_t expands the tax base, allowing government spending requirements to be met

[17]For the chosen parameter values, it can be shown that the linearized dynamic system implied by (14) possesses a unique set of stable, stationary decision rules for $t \geq 1$. (See Guo and Lansing [1995b] for more details.) Where possible, the nonlinear versions of the equilibrium conditions are used to compute the period-by-period values of endogenous variables during the simulations.

with a lower value of τ_{kt}. Due to the depreciation allowance, an increase in δ_t tends to reduce the tax base, which calls for an increase in τ_{kt}. Absorbing shocks mainly by changes in τ_{kt}, as opposed to changes in τ_{ht}, is efficient because household assets (capital and government bonds) cannot be quickly adjusted in response to a change in the capital tax. In contrast, the household can instantaneously adjust labor supply in response to a change in the labor tax.[18] Given the behavior of τ_{kt}, the optimal response of government borrowing to shocks is relatively small. With constant depreciation rates, the procyclical nature of the tax base, especially the level of profits, implies strong countercyclical behavior for τ_{kt} and τ_{ht}. With variable depreciation rates, however, the correlation between output and tax rates is reduced. Again, this improves comparison with the data.

5.2 Simulation Results

The model's quantitative predictions for the cyclical behavior of fiscal policy and private-sector aggregates are shown in tables 3 and 4. Figures 1-8 provide a visual comparison for some selected variables.

The cyclical behavior of the allocations turns out to be similar under the two tax regimes, regardless of the shock specification. However, the distortionary tax model includes a larger set of endogenous fiscal variables. The additional variables are the distortionary tax rates on labor and capital income and the government debt-to-output ratio.

Under both tax regimes, the single-shock specification underpredicts the variability of output and, consequently, most of the other variables in table 3. Actually, this is a desirable feature, since evidence suggests that the contribution of technology shocks to fluctuations in U.S. output is less than 100 percent (see Aiyagari [1994]). Both tax regimes capture the fact that public investment is only about half as variable as private investment in postwar U.S. data. Moreover, in the model with distortionary taxes, the capital tax is more variable than the labor tax, a feature which also tends to characterize U.S. tax rates.

The single-shock specification suffers from a number of serious shortcomings. First, public

[18] The optimality of using a state-contingent capital tax to absorb budget shocks has been shown previously by Judd (1989), Zhu (1992), and Chari, Christiano, and Kehoe (1994a).

expenditures in the model (g_t and x_{Gt}) are strongly procyclical in contrast to the relatively weak correlation with output observed in the data. The predicted correlations between the components of public expenditures and output are both close to 1.0. In the data, however, the correlation between public consumption and output is only 0.05, while the correlation between public investment and output is 0.38. This is further reflected in table 4, where the predicted correlations between public expenditures and their private-sector counterparts are much higher than those in the data. Second, the standard deviation of g_t in the model is too low, both in absolute terms and relative to the standard deviation of c_t. Third, under distortionary taxes, the predicted standard deviation of the labor tax is much lower than any of the values based on U.S. tax rates. Fourth, the model predicts strong countercyclical behavior for tax rates and the government debt ratio, in contrast to the weak correlations observed in the data. Finally, the correlation between hours worked and average labor productivity is close to 1.0 in the model versus a near-zero correlation in the data. This last observation is a well-known deficiency of single-shock RBC models (see Christiano and Eichenbaum [1992]).

In general, the addition of multiple shocks improves the model's performance in a number of ways. The standard deviation of output, hours, and the government debt ratio all increase to values that are very close to those in the data. Public expenditures become less procyclical, and as a result, the contemporaneous correlations between public and private expenditures decline. These correlations are still stronger than those observed in the data, however, especially in the case of g_t (see table 4 and figures 1-4). With multiple shocks, the correlations between the tax rates and output become much weaker, and the correlation between hours worked and average labor productivity substantially declines. The last observation confirms earlier results obtained by Hercowitz and Sampson (1991) and Ambler and Paquet (1994b) using RBC models with a shock to the capital accumulation equation.

The multiple-shock specification does not help much to increase the standard deviations of g_t and τ_{ht}, and the government debt ratio remains strongly countercyclical. The correlation between the debt ratio and output is -0.85 in the model versus -0.18 in the data. A notable drawback to the multiple-shock specification is that the standard deviations of x_t and τ_{kt} are now much higher than the corresponding U.S. values.

To summarize, the model with multiple shocks and distortionary taxes is able to capture the following stylized facts describing the behavior of U.S. fiscal policy: Public expenditures (g_t and x_{Gt}) are much less procyclical than private expenditures (c_t and x_t); private investment is more variable than public investment; the capital tax is more variable than the labor tax; the correlation between tax rates and output is relatively weak; and the government debt ratio is about twice as variable as output. The model's principal shortcomings are its inability to capture the high variability of g_t relative to c_t, and its overprediction of the variability of τ_{kt} relative to τ_{ht}. Some other deficiencies are that b_t / y_t is too strongly countercyclical and that g_t and x_{Gt} remain more strongly correlated with output than in the data.

Figure 3 reveals that the high variability of public consumption relative to private consumption in U.S. data is strongly influenced by the sizable expansion of government spending that occurred in the late 1960s, coinciding with the Vietnam war. To capture this sort of phenomenon, I experimented with a modified version of the model that included an independent stochastic shock applied to the parameter B in the household utility function. This shock, which can be viewed as representing changes in the demand for public goods (perhaps due to wars, riots, or disasters), improved the model's performance in two ways: first, by increasing the variability of g_t relative to c_t, and second, by reducing the contemporaneous correlation between g_t and y_t. The countercyclical nature of b_t / y_t was also reduced, but only to a small degree.[19]

Figures 5-8 display the U.S. and model tax rates *before* detrending. For quantitative comparisons (tables 3 and 4), detrending is necessary because U.S. labor tax estimates all display a distinct upward trend, while two out of the three capital tax estimates display a downward trend. These trends have no counterpart in the model. The fact that the model overpredicts the variability of the capital tax relative to the labor tax is consistent with the results of Chari, Christiano, and Kehoe (1994a). In their model, with log utility and technology shocks alone, the standard deviation of the capital tax is 17.67 percent (for the decentralization with uncontingent debt) versus 0.08 percent for the labor tax. In comparison, table 3 indicates that the standard deviation of τ_{kt} in the single-shock specification is 4.39 percent versus

[19] Ambler and Paquet (1994b) adopt a different approach by introducing an exogenous stochastic component of government expenditures in their model. These expenditures are calibrated to U.S. data on military spending. The income tax in their model is also taken to be exogenous (and constant over time). In contrast, I have chosen to endogenize all of the government's fiscal variables.

0.38 percent for τ_{ht}. The fact that τ_{kt} is less variable here is due primarily to the existence of profits.[20]

I experimented with varying both the size of firm profits (by changing θ_3 relative to θ_1) and the level of dividend taxation (by changing γ). As either θ_3 or γ increased, the percentage standard deviation of τ_{kt} declined. The intuition for this result is straightforward. Recall that dividends (equal to after-tax profits) do not distort household decisions because profits are determined outside households' control. Higher profits or a higher γ implies a larger and less elastic tax base for the capital tax. Consequently, smaller changes in the tax rate can produce the same revenue effect when responding to shocks. Although higher levels of θ_3 reduce the standard deviation of τ_{kt}, the resulting steady-state ratio $\bar{x}_G/\bar{y}$ exceeds the U.S. average, which is undesirable from a calibration standpoint. Similarly, values of γ greater than two become difficult to justify in comparison to the U.S. tax structure.

The model's tax structure imposes a sharp distinction between the taxation of labor and capital incomes. In reality, however, the personal income tax encompasses income from many different sources, such as wage earnings and investment income. To investigate the effects of an alternative tax structure, I introduced the following additional constraint in (14) to require that labor and capital incomes be taxed at the same rate: $(1 - \tau_{kt}) = A/(\lambda_t w_t)$, where $w_t = \theta_2 y_t / h_t$. Table 5 shows the results of this experiment. The standard deviation of the optimal flat-rate income tax τ_t turns out to be very low (0.80 percent with multiple shocks). Given that labor's share of income is 0.60 in the model, it is not surprising that τ_t behaves similarly to τ_{ht} under the original tax structure. In steady state, $\bar{\tau} = 0.243$, which exceeds $\bar{\tau}_h = 0.221$. Ceteris paribus, a higher labor tax exerts more of a dampening effect on hours worked (and output) in response to shocks. This effect, and the fact that the tax base for τ_t is higher than for τ_{ht}, helps compensate for the absence of the capital tax as a separate shock absorber. This is evidenced by the standard deviations of the allocations in table 5, which are mostly very close to those in table 3.

[20]There are a number of other differences with the Chari, Christiano, and Kehoe model that are likely to affect the quantitative comparisons. First, their solution technique involves a minimum-weighted residual method applied to a discrete state space. Second, simulated tax rates in their model are not subjected to the Hodrick-Prescott filter before computing the statistics.

6. Concluding Remarks

This paper develops a model that combines elements from the theory of optimal public finance with an equilibrium view of aggregate fluctuations. Interestingly, the model predicts that if the government responds systematically to external forces and takes into account the rational responses of the private sector, then fiscal variables can fluctuate substantially over the business cycle. Moreover, these policy variations might even be in the best interests of households. More specifically, this paper demonstrates that an equilibrium business cycle model with optimal fiscal policy can reproduce a number of features describing the cyclical behavior of public expenditures, tax rates, and the government debt-to-output ratio in the postwar U.S. economy. Although the model's quantitative predictions do not coincide with some key aspects of the data, the results provide encouragement for an approach that seeks to explicitly model government behavior in a general equilibrium framework. The consideration of additional features, such as voting behavior, monetary policy, or institutions that affect the commitment mechanism (or lack thereof), are likely to improve our understanding of the links between government policy and the business cycle.

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Table 1: Model Steady States versus U.S. Averages

Variable	Model: Distortionary Taxes	Model: Lump-Sum Taxes	U.S. Economy[a]
$\bar{x}/\bar{y}$	0.17	0.20	0.22
$\bar{x}_G/\bar{y}$	0.03	0.03	0.03
$\bar{c}/\bar{y}$	0.63	0.60	0.56
$\bar{g}/\bar{y}$	0.17	0.17	0.17
$\bar{k}/\bar{y}$	2.61	3.01	2.61
$\bar{k}_G/\bar{k}$	0.50	0.42	0.23
$\bar{b}/\bar{y}$	0.37	--	0.37
$\bar{\tau}_h$	0.22	0	0.28, 0.24, 0.25
$\bar{\tau}_k$	0.29	0	0.29, 0.56, 0.43

[a]For the U.S. economy, investment, consumption, and capital averages are for the period 1954 to 1992. Tax rate averages correspond to the sources and periods listed in footnote 14.

Table 2: Optimal Decision Rules (Distortionary Taxes with Multiple Shocks)

Variable	Constant	z_t	δ_t	δ_{Gt}	$\ln(k_t)$	$\ln(k_{Gt})$	$\ln(\mu_{t-1})$
x_t =	0.071	0.654	0.773	0.626	-0.162	0.047	-0.020
x_{Gt} =	0.005	0.033	-0.035	0.023	0.035	-0.065	0.021
c_t =	0.256	0.198	-0.248	-0.148	0.179	0.039	-0.045
g_t =	0.104	0.047	-0.091	-0.049	0.027	0.009	0.012
τ_{ht} =	0.383	-0.105	0.197	-0.096	-0.091	-0.018	0.138
τ_{kt} =	-1.684	-1.519	12.078	-0.735	1.428	-0.046	-0.658
b_{t+1} =	-0.128	-0.080	1.002	-0.049	0.320	0.344	-0.258

Source: Author's calculations.

Table 3: Business Cycle Statistics for Models and U.S. Economy

Standard Deviation (percent)

Variable	Model with Multiple Shocks[a]: Distortionary Taxes	Model with Multiple Shocks[a]: Lump-Sum Taxes	Model with Technology Shocks Only[a]: Distortionary Taxes	Model with Technology Shocks Only[a]: Lump-Sum Taxes	U.S. Economy 1954-92[b]
y_t	1.61	1.66	1.32	1.27	1.60
c_t	0.49	0.43	0.56	0.48	0.82
g_t	0.57	0.54	0.52	0.51	1.63
x_t	7.96	7.37	4.84	4.28	5.06
x_{Gt}	2.14	2.96	2.96	2.69	2.36
k_t	0.52	0.46	0.35	0.31	0.49
k_{Gt}	0.15	0.15	0.08	0.08	0.21
h_t	1.35	1.48	0.90	0.81	1.46
y_t/h_t	0.39	0.43	0.45	0.48	0.59
b_t/y_t	3.74	--	1.62	--	3.32
τ_{ht}	0.45	--	0.38	--	3.37, 2.25, 2.06
τ_{kt}	18.12	--	4.39	--	12.9, 2.74, 4.87

Contemporaneous Correlation with Output

Variable	Model with Multiple Shocks: Distortionary Taxes	Model with Multiple Shocks: Lump-Sum Taxes	Model with Technology Shocks Only: Distortionary Taxes	Model with Technology Shocks Only: Lump-Sum Taxes	U.S. Economy 1954-92
c_t	0.69	0.52	0.96	0.96	0.88
g_t	0.38	0.40	0.91	0.90	0.05
x_t	0.96	0.98	0.98	0.99	0.90
x_{Gt}	0.68	0.75	0.97	0.92	0.31
k_t	0.16	0.21	-0.26	-0.28	0.70
k_{Gt}	0.14	-0.07	-0.50	-0.53	0.02
h_t	0.98	0.97	0.99	0.99	0.93
y_t/h_t	0.73	0.52	0.95	0.96	0.41
b_t/y_t	-0.85	--	-0.96	--	-0.18
τ_{ht}	-0.44	--	-0.99	--	0.15, -0.21, -0.30
τ_{kt}	0.29	--	-0.94	--	-0.17, 0.01, -0.31

[a]Model statistics are means over 250 simulations, each 39 periods long, after dropping the first 50 periods. Before computing the statistics, all series were logged (except for the tax rates) and detrended using the Hodrick-Prescott filter (see Prescott [1986]). The smoothing parameter for the filter was set equal to 10, which is appropriate for annual data (see Baxter and King [1995]).

[b]For the U.S. economy, data sources are described in footnote 14. Data on average marginal tax rates do not extend over the full sample. All series were logged and/or detrended as in the model.

Table 4: Comparison of Selected Statistics

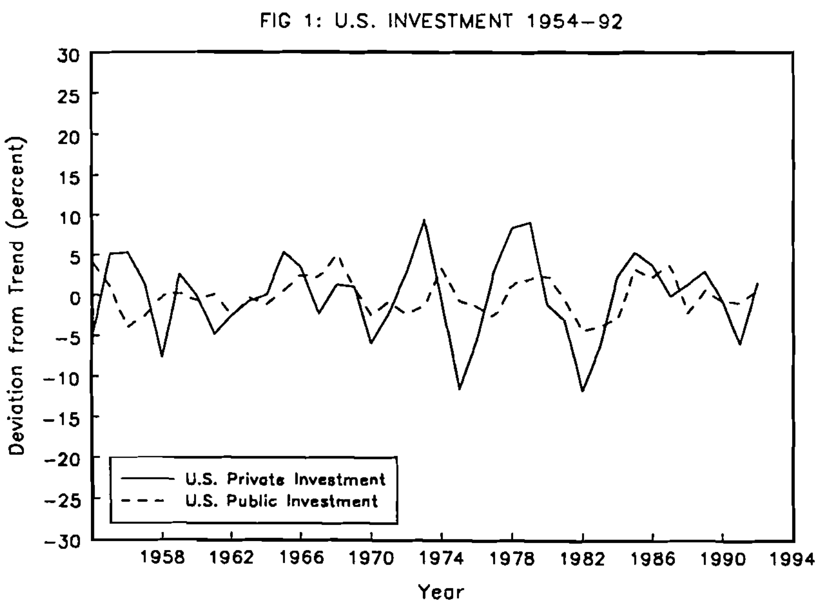
	Model with Multiple Shocks[a]		Model with Technology Shocks Only[a]		
Statistic	Distortionary Taxes	Lump-Sum Taxes	Distortionary Taxes	Lump-Sum Taxes	U.S. Economy 1947-92[b]
σ_{xG}/σ_x	0.27	0.40	0.61	0.63	0.47
corr (x_G, x)	0.51	0.71	0.92	0.87	0.26
σ_g/σ_c	1.15	1.26	0.93	1.06	1.98
corr (g, c)	0.78	0.81	0.94	0.94	0.10
$\sigma_h/\sigma_{y/h}$	3.43	3.45	2.02	1.69	2.47
corr $(h, y/h)$	0.58	0.29	0.89	0.91	0.04
$\sigma_{\tau k}/\sigma_{\tau h}$	39.9	--	11.44	--	3.82, 1.22, 2.37
corr (τ_k, τ_h)	0.67	--	0.89	--	0.29, 0.47, 0.89

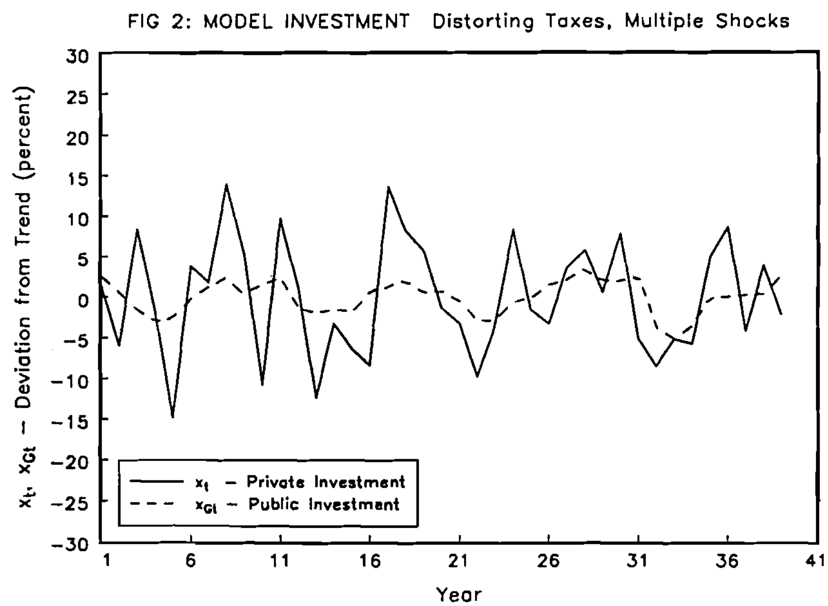
Table 5: Model with Flat-Rate Income Tax

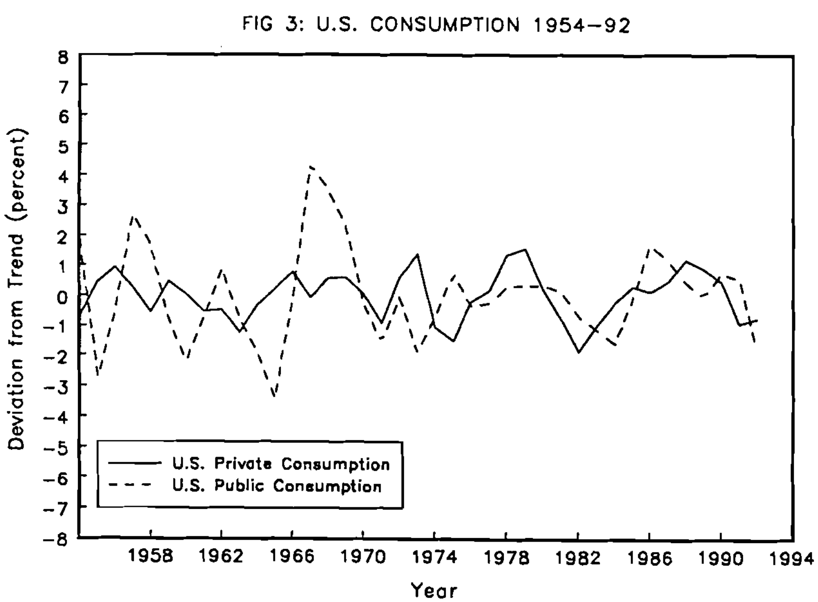
	Multiple Shocks		Technology Shocks Only	
Variable	Standard Deviation (%)	Correlation with Output	Standard Deviation (%)	Correlation with Output
y_t	1.61	1.00	1.31	1.00
c_t	0.54	0.46	0.55	0.98
g_t	0.54	0.44	0.50	0.89
x_t	7.92	0.96	4.70	0.98
x_{Gt}	2.94	0.71	2.66	0.92
k_t	0.50	0.14	0.34	-0.28
k_{Gt}	0.15	-0.08	0.08	-0.53
h_t	1.36	0.98	0.89	0.99
y_t/h_t	0.39	0.70	0.46	0.95
b_t/y_t	1.31	-0.89	1.30	-0.95
τ_t	0.80	0.09	0.36	-0.97

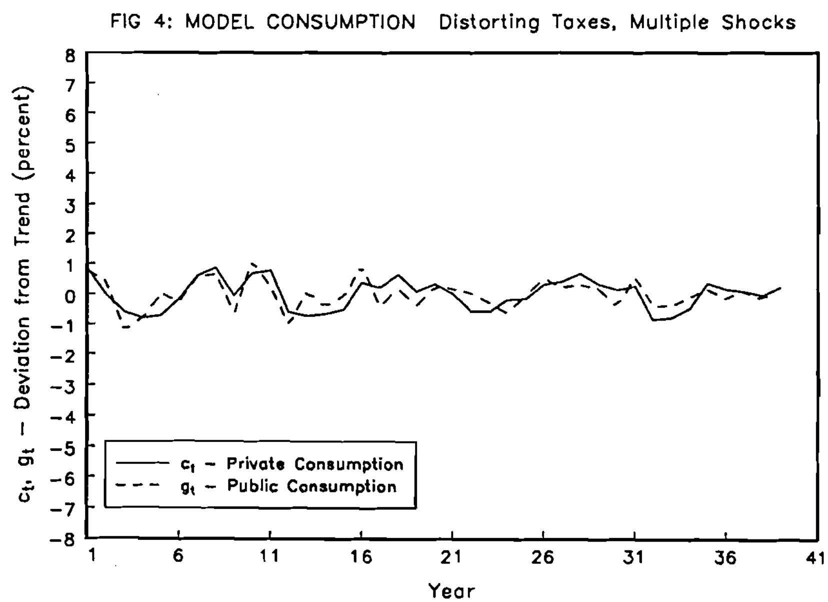
[a]Model statistics are means over 250 simulations, each 39 periods long, after dropping the first 50 periods. Before computing the statistics, all series were logged (except for the tax rates) and detrended using the Hodrick-Prescott filter (see Prescott [1986]). The smoothing parameter for the filter was set equal to 10, which is appropriate for annual data (see Baxter and King [1995]).

[b]For the U.S. economy, data sources are described in footnote 14. Data on average marginal tax rates do not extend over the full sample. All series were logged and/or detrended as in the model.









Sources: Author's calculations and Citibase.

FIG 5: U.S. LABOR TAX

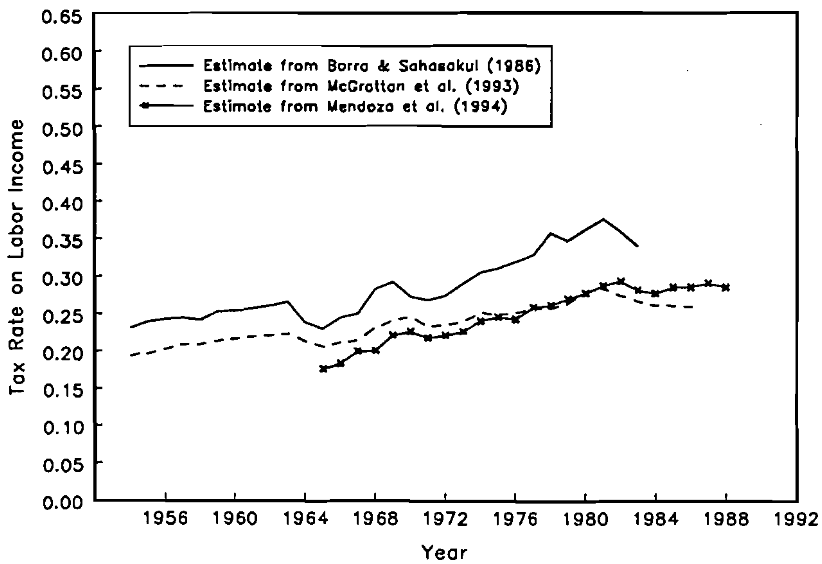


FIG 6: MODEL LABOR TAX

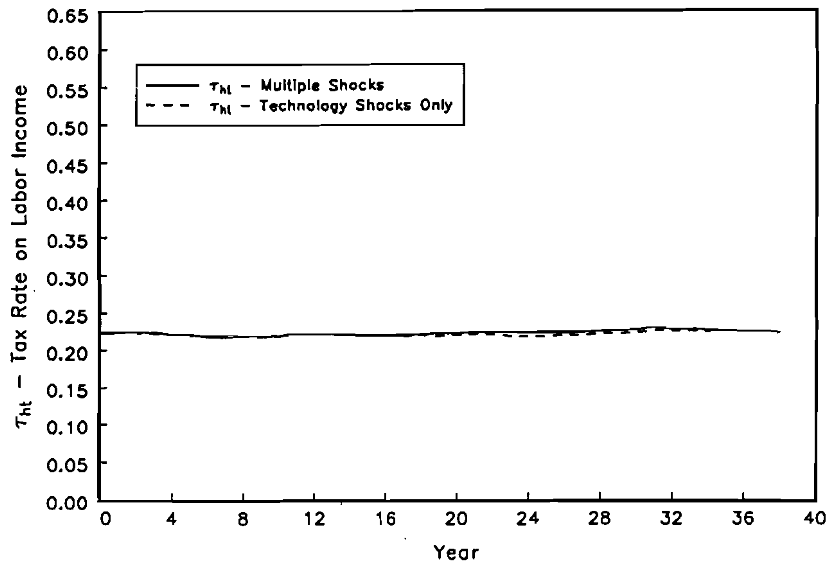


FIG 7: U.S. CAPITAL TAX

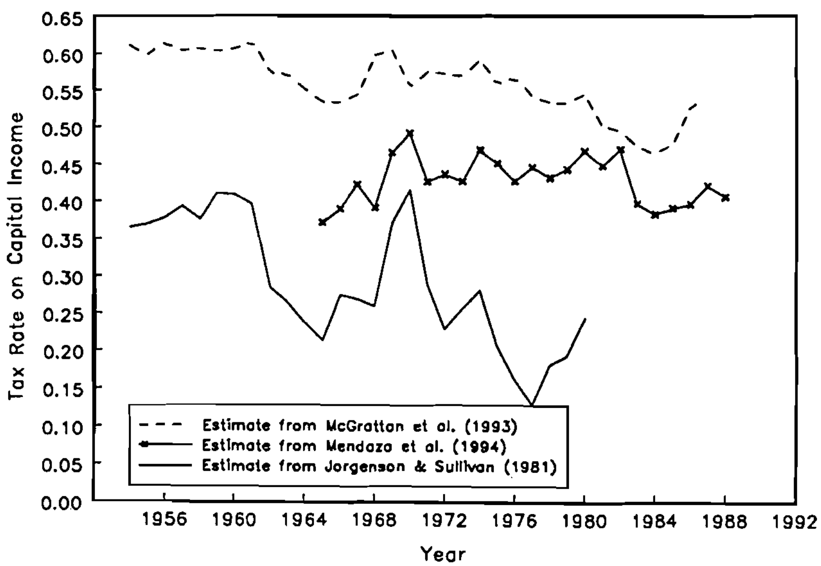
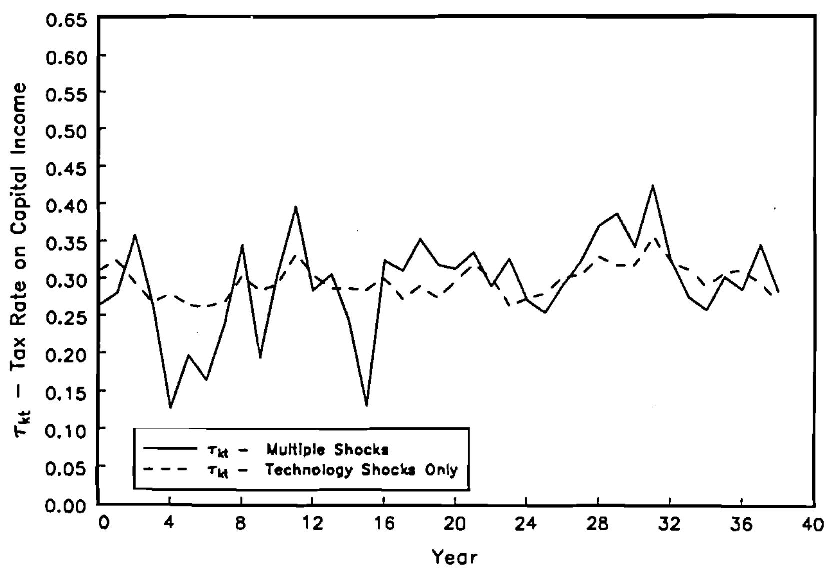


FIG 8: MODEL CAPITAL TAX



Sources: As noted, and author's calculations.