Debt Overhang and Credit Risk in a Business Cycle Model

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Abstract

We study the macroeconomic implications of the debt overhang distortion. The probability that a firm will default acts like a tax that discourages its current investment. This is because the marginal return of the firm's investment will be seized by its creditors in the event of default, so the higher the firm's probability of default, the lower its expected marginal return of investment. The dynamics of this distortion, which moves counter-cyclically, amplify and propagate the effects of productivity, volatility, wealth redistribution and government spending shocks. Both the size and the persistence of these effects are quantitatively important, and the fiscal multiplier is large and hump-shaped. The model replicates important features of the joint dynamics of macro variables and credit risk variables, like default rates, recovery rates and credit spreads.

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1 Introduction

We investigate the macroeconomic effects of a financial distortion that arises when firms are so levered that the probability that they will default on their liabilities becomes strictly positive. In this case, the burden of the debt—the *debt overhang*—creates a disconnect between the socially optimal level of investment and the firms' privately optimal one, reducing the firms' incentive to invest.

The firms' probability of default plays a key role: Because the marginal return of a firm's investment will be seized by its creditors in the event of default, the higher its probability of default, the lower the marginal return that the firm expects to receive from its investment, the smaller its incentive to invest. The probability of default acts like a tax that discourages the firm's investment. The sub-optimality of the investment choice stems from the fact that the firm does not internalize the positive effect of its investment choice on its creditors' payoff in the event of default[1].

We incorporate this debt overhang distortion in a business cycle framework and we find that it can dramatically amplify and propagate the effects of productivity, volatility and wealth redistribution shocks. There are two positive feedback loop mechanisms at work, both acting through the probability of default. First, shocks that increase the probability of default, exacerbate the debt overhang distortion, and decrease investment; in turn, a lower level of investment further increases the probability of default, in a static feedback amplification mechanism. Also, shocks that increase the probability of default and decrease investment, have a persistent negative effect on the firm's capital, thereby increasing the probability of default persistently over time, in a dynamic feedback propagation mechanism.

Through these mechanisms, productivity shocks have ampler and more persistent effects than in the standard real business cycle model. In addition, shocks that increase the volatility of productivity and wealth redistribution shocks from debtors to creditors, which do not have any effect in the standard real business cycle model, increase the probability of default, exacerbate the debt overhang distortion, and have ample and persistent negative effects on investment.

Recent empirical work in corporate finance has stressed the quantitative importance of the debt overhang effect. Hennessy (2004) shows that debt overhang distorts both the level and composition of investment, with under-investment being more severe for long-lived assets. He finds a statistically significant debt overhang effect regardless of firms' ability to issue additional secured debt. Using firm level data and studying a large variety of credit frictions, Hennessy, Levy and Whited (2007) document that the magnitude of the debt overhang drag on investment is substantial, especially for distressed (high probability of default) firms. Moyen (2007) measures a large overhang cost both with long-term debt and with short-term debt. She finds that the debt overhang problem is larger when investment in the capital stock is reversible.

[1] Myers (1977) is the early reference paper that focus on how the issuance of corporate debt leads to sub-optimal investment decisions. Lamont (1995) clarifies that "debt overhang occurs when existing debt deters new investment because the benefits from new investment will go to the existing creditors, not to the new investors". The following paragraph by Stein (2003) best summarizes the debt overhang distortion: "[A] large debt burden on a firm's balance sheet discourages further new investment ... This is because if the existing debt is trading at less than face value, it acts as a tax on the proceeds of the new investment: part of any increase in value generated by the new investment goes to make the existing lenders whole, and is therefore unavailable to repay those claimants who put up the new money. "

While the corporate finance and international finance literature[2] have long acknowledged the debt overhang effect, there is no recent macroeconomic model that explicitly considers the overhang effects of corporate debt. On one hand, a strand of the literature, following the seminal contribution of Kiyotaky and Moore (1997), assumes that there is no enforceability for unsecured lending, and studies equilibria where loans are fully collateralized and no default occurs. On the other hand, most of the financial frictions literature in macroeconomics has focused on the role of agency costs, arising from the asymmetric information associated with debt contracts, in generating frictions that affect the cost of credit and the level of investment. In the works of Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), and Bernanke, Gertler and Gilchrist (1999), and in the subsequent agency costs literature, monitoring real resources are used whenever defaults occur. Ex-ante, this generates an external finance premium that contributes to amplify fluctuations.

Although the qualitative predictions of our framework are close to the ones of the agency costs literature, the amplification mechanism generated by the debt overhang distortion is quantitatively much more important. This is because the debt overhang distortion depends directly on the probability of default, whose volatility is large both in the model and in the data. In contrast, the agency costs friction depends crucially on the level of the monitoring costs and on the dynamics of the external finance premium: When the monitoring costs tend to zero, although leverage and defaults are still present, they become irrelevant for the evolution of the aggregate variables. As we document in Section 3 with plausible values for the monitoring costs, the resulting amplification mechanism is quantitatively small.

There are qualitative differences between the two frameworks as well. The agency costs financial friction entails an increase in the marginal *cost* of investing, whereas the debt overhang distortion decreases the marginal *benefit* of investing. Also, only with debt overhang, shocks that increase the volatility of aggregate productivity end up increasing the probability of default and having first-order macroeconomic effects. Finally, in the debt overhang framework, default probabilities, credit spreads and default rates all unambiguously decrease after an expansionary productivity shock. The predictions of the agency costs framework are, in contrast, less clear-cut, as first pointed out by Carlstrom and Fuerst (1997)[3]. In their model, after a positive productivity shock, entrepreneurs need to borrow more in order to expand investment, which leads to counter-factually higher risk premia and bankruptcy rates. In other models with a financial friction based on agency monitoring costs, the sign of the responses of credit spreads and default rates to productivity shocks varies depending on the specific parametrization adopted.

The paper is organized as follows: Section 2 describes the economy, and defines the equilibrium; the system describing the equilibrium and its log-linear approximation is contained in Appendix B; Section 3 studies the amplification and propagation mechanisms,

[2]Because foreign debt effectively generates a tax on domestic investment, debt overhang effects have also been studied in the international finance literature. Examples are Krugman (1988) and Bulow and Rogoff (1991). See Obstfeld and Rogoff (1996), Sections 6.2.3 and 6.2.4 for a review.

[3]The following is the relevant excerpt from Carlstrom and Fuerst (1997): "The foremost problem is the cyclical behavior of bankruptcy rates and the risk premia. Because of our linearity assumptions, these variables are functions solely of the aggregate price of capital. Hence, the increase in the price of capital that occurs with a positive technology shock also leads to an increase in bankruptcy rates and risk premia. From a theoretical perspective this behavior is not surprising: The supply curve for capital is upward sloped because of agency costs, so that a demand-induced movement up this curve must imply an increase in risk premia."

documents the model's quantitative predictions, and evaluates the model empirically; and Section 4 concludes.

2 The model

The model economy has a population of mass 1 divided into λ households and $1-\lambda$ entrepreneurs. There is a continuum of firms of mass 1 owned by entrepreneurs. Firms operate a constant returns to scale technology that produces a final output using physical capital and labor as factors of production. Both factors are homogenous and can be freely reallocated across firms, and the relative price of investment to output is constant and normalized to 1. Labor contracts are signed and wages are paid one-period in advance.

Firms have access to capital markets where they can borrow to fund their investment in capital and labor and to smooth their dividends over time. A financial intermediary, owned by households, channels households' savings into the productive sector: It collects deposits from households and purchases risky corporate bonds. Firms can default on their liabilities whenever the value of their output is lower than the face value of their bonds; in this case, the output is seized by the creditors, with no further additional penalty or exclusion from financial markets. We rule out any strategic dynamic behavior involving the dynamics of reputation over time by assuming anonymity, i.e. creditors do not have information on the firms' history of default.

2.1 Households

The utility function is $[u(c)-v(l)]$, with $u'(c)\equiv c^{-\gamma}$, $\gamma>0$, and $v'(l)=\psi l^{\varphi}$, $\psi>0, \varphi>0$. Household h chooses consumption demand c_t^h, labor supply l_{t+1}^h, and a risk-free asset d_{t+1}^h to solve the following problem:

$$\max_{\{c_t^h, l_{t+1}^h, d_{t+1}^h\}_{t=0}^{\infty}} E_0\left\{\sum_{t=0}^{\infty}\beta^t\left[u(c_t^h)-v(l_t^h)\right]\right\}$$
$$\text{subject to: } c_t^h+d_{t+1}^h/R_t=w_{t+1}l_{t+1}^h+d_t^h+\Pi_t^h/\lambda$$

given the initial values of the state, the contingent sequences of risk-free prices $\{1/R_t\}_{t=0}^{\infty}$, wage rates $\{w_{t+1}\}_{t=0}^{\infty}$, profits $\{\Pi_t^h\}_{t=0}^{\infty}$ from the capital fund, and a no-Ponzi-game constraint. Notice that both labor is determined and wages are paid one-period in advance.

The households' necessary conditions are

$$1/R_t = E_t\{\beta u'(c_{t+1}^h)/u'(c_t^h)\}$$
$$u'(c_t^h)w_{t+1} = \beta v'(l_{t+1}^h)$$

The first equation governs the optimal consumption path depending on the risk-free rate R, while the second equation determines the consumption-labor choice in response of the wage rate w.

All households face the same problem, so we focus on equilibria where individual variables are the same across all households, and we denote the aggregate variables with the corresponding upper case letters: $C_t^h=\lambda c_t^h$, $L_t^h=\lambda l_t^h$, and $D_t^h=\lambda d_t^h$.

2.2 Entrepreneurs

Entrepreneur e chooses consumption demand c_t^e and labor supply l_{t+1}^e to solve the following problem:

$$\max_{\{c_t^e, l_{t+1}^e\}_{t=0}^{\infty}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[u(c_t^e) - v(l_t^e) \right] \right\}$$
$$\text{subject to: } c_t^e = w_{t+1} l_{t+1}^e + \Pi_t^e/(1-\lambda)$$

given the initial values of the state, the wage rates w_t, dividends $\Pi_t^e \equiv \int_{i=0}^1 \Pi_t^e(i) di$ from the continuum of firms, and a no-Ponzi-game constraint.

The entrepreneurs' necessary conditions are

$$u'(c_t^e) w_{t+1} = \beta v'(l_{t+1}^e)$$

As in the household case, all entrepreneurs face the same problem, so we focus on equilibria where individual variables are the same across all entrepreneurs, and we denote the aggregate variables with the corresponding upper case letters: $C_t^e = (1-\lambda) c_t^e$ and $L_t^e = (1-\lambda) l_t^e$.

2.3 Firms

There is a continuum of mass 1 of perfectly competitive firms. The i-th firm accumulates capital $k_{t+1}(i)$ and hires labor $l_{t+1}(i)$, to produce a homogenous output $y_t(i)$ with a constant return technology

$$y_t(i) \equiv \omega_t(i) \theta_t f(k_t(i), l_t(i))$$

where $f(k,l) \equiv A k^{\alpha} l^{1-\alpha}$, $A > 0$, $\alpha \in (0,1)$, is a production function common across all firms, θ_t is the aggregate productivity shock, and $\omega_t(i)$ is an idiosyncratic productivity shock, i.i.d. across all the firms indexed by $i \in [0,1]$.

The aggregate productivity θ follows the law of motion:

$$\ln(\theta_{t+1}) = \rho_\theta \ln(\theta_t) + \sigma_{\theta,t} \epsilon_{\theta,t+1}$$
$$\ln(\sigma_{\theta,t+1}/\sigma_\theta) = \rho_{\theta,\sigma} \ln(\sigma_{\theta,t}/\sigma_\theta) + \sigma_{\theta,\sigma} \eta_{\theta,t+1}$$

where $\epsilon_{\theta,t+1}$ and $\eta_{\theta,t+1}$ are two i.i.d. standard normal shocks.

The idiosyncratic productivity $\omega(i)$ follows the law of motion:

$$\ln(\omega_{t+1}(i)) = \sigma_{\omega,t} \epsilon_{\omega,t+1}(i)$$
$$\ln(\sigma_{\omega,t+1}/\sigma_\omega) = \rho_{\omega,\sigma} \ln(\sigma_{\omega,t}/\sigma_\omega) + \sigma_{\omega,\sigma} \eta_{\omega,t+1}$$

where $\epsilon_{\omega,t+1}(i)$, all i, and $\eta_{\omega,t+1}$ are i.i.d. standard normal shocks.

Let σ be the volatility of each firm's total productivity $\omega_t(i)\theta_t$, that is the product of the idiosyncratic productivity $\omega(i)$ and the aggregate productivity θ. The volatility σ is determined as follows:

$$\sigma_t^2 \equiv \sigma_{\omega,t}^2 + \sigma_{\theta,t}^2$$

Firms are owned by entrepreneurs and pay them dividends $\Pi_t^e(i)$. They discount future dividends using the same stochastic discount factor as the entrepreneurs:

$$\Lambda_{t,t+j}^e \equiv \beta^j u'(C_{t+j}^e)/u'(C_t^e)$$

where $C_t^e \equiv (1-\lambda)c_t^e$ is the entrepreneurs' aggregate consumption. For our choice of class of utility functions, this stochastic discount factor coincides with the one of any individual entrepreneur $\Lambda_{t,t+j}^e = \beta^j u'(c_{t+j}^e)/u'(c_t^e)$.

To finance investment and to smooth dividends over time, a firm, in each period, can access the corporate bond market and issue $b_{t+1}(i)$ one-period risky bonds, at the price $q_t(i)$. Institutional arrangements are such that the firm's debt obligation is limited by the value of its output $y_{t+1}(i)$. If the value of its output is lower than the bond face value, the firm can default on its debt without any penalty. In this case, however, the creditors seize the firm's output[4]. The level of output is perfectly observable, so there is no asymmetric information agency problem. The timing of events within one period can be thought as follows: first, the shocks are realized and production takes place; then, either debt is fully repaid or default occurs and the firm's output is seized; after that, new debt is issued, dividends are paid, and the labor and investment decisions are taken.

Firm i solves the following problem:

$$\begin{aligned} &\max_{\{\Pi_t^e, k_{t+1}, l_{t+1}, b_{t+1}\}_{t=0}^{\infty}} E_0 \left\{ \sum_{t=0}^{\infty} \Lambda_{0,t}^e \Pi_t^e(i) \right\} \\ &\text{subject to: } \Pi_t^e(i) + k_{t+1}(i) - (1-\delta)k_t(i) + \min\{y_t(i), b_t(i)\} \\ &\qquad = y_t(i) - w_{t+1} l_{t+1}(i) + q_t(i) b_{t+1}(i) \\ &\text{where } y_t(i) \equiv \omega_t(i)\theta_t f(k_t(i), l_t(i)) \end{aligned}$$

given the initial values of the state, the sequence of stochastic discount factors $\{\Lambda_{0,t}^e\}_{t=0}^{\infty}$, bond prices $q_t(i)$, wage rates w_t, and a no-Ponzi-game constraint. Again, notice that labor is determined and wages are paid one-period in advance. When labor is introduced this way, it becomes an investment choice for the firm, with the decision taken and the cost sustained in the current period, while the benefit is received in the next period.

Dropping the index i to ease notation, the firms' necessary conditions are

$$\begin{aligned} q_t &= \frac{\partial E_t\{\Lambda_{t,t+1}^e \min\{y_{t+1}, b_{t+1}\}\}}{\partial b_{t+1}} \\ 1 &= \frac{\partial E_t\{\Lambda_{t,t+1}^e[(1-\delta)k_{t+1} + y_{t+1} - \min\{y_{t+1}, b_{t+1}\}]\}}{\partial k_{t+1}} \\ w_{t+1} &= \frac{\partial E_t\{\Lambda_{t,t+1}^e[y_{t+1} - \min\{y_{t+1}, b_{t+1}\}]\}}{\partial l_{t+1}} \end{aligned}$$

To gain intuition on these necessary conditions, notice that k_{t+1}, l_{t+1} and b_{t+1} are all known in period t+1, and that $\ln(\omega_{t+1}\theta_{t+1})$ is normally distributed with mean equal to

[4]This is the same modeling choice of default as in the agency costs literature. If the recovery value included part of the aggregate capital stock (or any other aggregate variables), besides the individual's firm output, then one could subtract this extra-collateral from both arguments of the min operator, redefine risky corporate debt as the difference between debt and this extra-collateral, and show that only the agents' budget constraints would be modified, with no effect on the other equilibrium conditions.

$\rho_\theta \ln(\theta_t)$ and standard deviation equal to σ_t. Then, well-known analytical results holding for log-normally distributed random variables yield[5]:

$$
\begin{aligned}
\frac{\partial E_t\{\min\{y_{t+1}, b_{t+1}\}\}}{\partial b_{t+1}} &= \Phi(d_{2,t}) \\
\frac{\partial E_t\{\min\{y_{t+1}, b_{t+1}\}\}}{\partial k_{t+1}} &= E_t\{\omega_{t+1}\theta_{t+1}\} f_k(k_{t+1}, l_{t+1})[1 - \Phi(d_{1,t})] \\
\frac{\partial E_t\{\min\{y_{t+1}, b_{t+1}\}\}}{\partial l_{t+1}} &= E_t\{\omega_{t+1}\theta_{t+1}\} f_l(k_{t+1}, l_{t+1})[1 - \Phi(d_{1,t})] \\
\text{where } d_{2,t} \equiv & \frac{\rho_\theta \ln(\theta_t) + \ln(f(k_{t+1}, l_{t+1})) - \ln(b_{t+1})}{\sigma_t} \qquad \text{and } d_{1,t} \equiv d_{2,t} + \sigma_t
\end{aligned}
$$

f_k and f_l denote the derivatives of f with respect to its two arguments, and $\Phi(\cdot)$ is the cumulative distribution function of the standard normal random variable.

Using these results and the fact that the expectation of a product is equal to the product of the expectations plus a covariance term, $E(xz) = E(x)E(z) + Cov(x, z)$, we can express the firm's necessary conditions as follows:

$$
\begin{aligned}
q_t &= E_t\{\Lambda^e_{t,t+1}\}\Phi(d_{2,t}) + \chi^e_{b,t} && (1) \\
1 &= E_t\{\Lambda^e_{t,t+1}\}[1 - \delta + E_t\{\omega_{t+1}\theta_{t+1}\} f_k(k_{t+1}, l_{t+1})\Phi(d_{1,t})] + \chi^e_{k,t} && (2) \\
w_{t+1} &= E_t\{\Lambda^e_{t,t+1}\}E_t\{\omega_{t+1}\theta_{t+1}\} f_l(k_{t+1}, l_{t+1})\Phi(d_{1,t}) + \chi^e_{l,t} && (3)
\end{aligned}
$$

where we have defined $\chi^e_{j,t} \equiv \partial Cov_t(\Lambda^e_{t,t+1}, \min\{\omega_{t+1}\theta_{t+1} f(k, l), b\})/\partial j$, for $j = b, k, l$[6].

To interpret these conditions notice that $\Phi(d_{2,t})$ is the probability that the debt will be fully repaid, so $1 - \Phi(d_{2,t})$ is the default probability. $\Phi(d_{1,t})$ can be similarly interpreted as an (adjusted) repayment probability. The difference between $\Phi(d_{1,t})$ and $\Phi(d_{2,t})$ is quantitatively negligible and does not play any role in our model. With regard to $d_{1,t}$ and $d_{2,t}$, they both can be interpreted as distances to default.

Recall that firms can use borrowing as a way to optimally smooth dividends over the cycle, paying higher dividends in periods when the entrepreneurs' consumption is relatively low. Equation (1) equates the marginal benefit of paying more dividends in the current period to the marginal cost of increasing borrowing and paying less dividends in the next period.

The second and third equations are the ones crucially affected by the debt overhang distortion. The two equations are similar to the corresponding ones of a standard real business cycle model with labor-in-advance, except for the presence of the (adjusted) probability of repayment $\Phi(d_{1,t})$. When the value of production exceeds the face value of bonds, an event that occurs with probability $\Phi(d_{1,t})$, the firm repays its liabilities and receives the full marginal return from its investment, as in the standard case. However, when production falls short of the face value of bonds, the firm defaults, the creditors seize its output, and the firm does not receive the marginal return from its investment.

[5]These results are routinely used in option pricing to compute the price of options and its derivatives (the *greeks*). Appendix A details the computation of the derivatives, which involves two terms canceling each other out.

[6]The terms χ^e can be loosely interpreted as risk premia associated with the co-movement between the bond risky payoff and the stochastic discount factor. In fact, the terms are identically zero both in the absence of aggregate uncertainty and in the absence of bond risk (when the default probability is zero). Their contribution to the cycle is of second-order importance when the economy is hit by relatively small shocks, so it will not appear in our analysis based on a first-order approximation method.

Hence, the lower the repayment probability $\Phi(d_{1,t})$, the lower the firm's expected marginal return on investment, the lower its incentive to invest. The default probability $1-\Phi(d_1)$ appears as a wedge in both the investment and labor equations, discouraging investment and labor demand.

Equation (2) shows how the debt overhang distortion affects the investment decision: a high debt-recovery ratio, b/y, induces a relatively short distance to default, d_1, and a relatively low repayment probability, $\Phi(d_1)$, that, in turn, reduces the firm's expected marginal return on investment, $f_k\Phi(d_1) < f_k$. An analogous argument applies to the labor hiring decision, as shown in equation (3).

Over the cycle, this default probability acts like a *counter-cyclical tax*, strengthening the firm's incentive to reduce the capital stock in periods when output is below trend, the debt-recovery ratio is high, and the default probability is high.

It is worth noting that, as the debt-recovery ratio, b/y, tends to zero, the default probability tends to zero as well, and the debt overhang distortion becomes unimportant[7]. This suggests that the debt overhang effect may play a quantitatively more important role over the cycle in periods when the corporate sector has already accumulated substantial debt.

2.4 The capital fund

The role of the capital fund in the model is to collect household savings and channel them to the productive sector of the economy. The capital fund is a financial intermediary that borrows risk-free and purchases the firms' risky debt in the corporate bond market. It takes bond prices as given, and perfectly observes the total stock of debt of each firm.

The capital fund is owned by households, pays them profits Π_t^h, and discounts future profits using the same stochastic discount factor as the households:

$$\Lambda_{t,t+j}^h \equiv \beta^j u'(C_{t+j}^h)/u'(C_t^h)$$

where $C_t^h \equiv \lambda c_t^e$ is the households' aggregate consumption. For our choice of utility function, this stochastic discount factor coincides with the one of any individual household $\Lambda_{t,t+j}^h = \beta^j u'(c_{t+j}^h)/u'(c_t^h)$.

The capital fund problem is

$$\max_{\{\Pi_t^h, D_{t+1}, b_{t+1}^h(i)\}_{t=0}^{\infty}} E_0 \left\{ \sum_{t=0}^{\infty} \Lambda_{0,t}^h \Pi_t^h \right\}$$

$$\text{subject to: } \Pi_t^h + D_t + \int_{i=0}^{1} q_t(i) b_{t+1}^h(i) di = D_{t+1}/R_t + \int_{i=0}^{1} \min\{y_t(i), b_t^h(i)\} di$$

given the initial values of the state, the sequence of stochastic discount factors $\{\Lambda_{0,t}^h\}_{t=0}^{\infty}$, risk-free prices $\{1/R_t\}_{t=0}^{\infty}$, bond prices $\{q_t(i)\}_{t=0}^{\infty}$, for all i, and a no-Ponzi-game constraint.

The integral is taken with respect to the unitary distribution of firms. The portfolio of loans $\{b_{t+1}^h(i)\}_{i\in[0,1]}$ is risky and sells for a price vector $\{q_t(i)\}_{i\in[0,1]}$, with future firms' output $\{y_{t+1}(i)\}_{i\in[0,1]}$ acting as recovery value. The payoff of each loan is equal to the minimum between the face value $b_t^h(i)$ and the recovery value $y_t(i)$.

[7] Under our assumptions of log-normality, the default probability becomes zero only when debt tends to zero; however, for other probability distributions, the default probability may become zero for strictly positive values of debt.

The necessary conditions for optimality are

$$1/R_t = E_t\{\Lambda^h_{t,t+1}\} \tag{4}$$
$$q_t(i) = \frac{\partial E_t\{\Lambda^h_{t,t+1}\min\{y_{t+1}(i), b^h_{t+1}(i)\}\}}{\partial b^h_{t+1}(i)} \text{ for all } i.$$

Using the same strategy as in the previous section we rewrite the optimality condition as

$$q_t(i) = E_t\{\Lambda^h_{t,t+1}\}\Phi(d_{2,t}(i)) + \chi^h_{b,t}(i) \text{ for all } i, \tag{5}$$

where $\chi^h_{b,t} \equiv \partial Cov_t(\Lambda^h_{t,t+1}, \min\{\omega_{t+1}\theta_{t+1}f(k_{t+1}, l_{t+1}), b_{t+1}\})/\partial b_{t+1}$. Equation (5) describes the optimal lending decision to each individual firm, taking into account both the firm's bond price and its risk of default, captured by $\Phi(d_{2,t}(i))$. At the aggregate level, equation (5) equates the marginal cost of paying less profits in the current period to the marginal benefit of purchasing more corporate bonds and paying more profits in the next period[8].

From equations (4) and (5), disregarding the χ^h term, it follows that

$$q_t(i) = \Phi(d_{2,t}(i))/R_t \text{ for all } i.$$

For each firm i, the bond price is equal to the inverse of the gross risk-free rate times the firm's repayment probability. Equivalently, the bond yield $1/q$ is equal to the risk-free rate R plus the credit spread, which in turn is approximately equal (up to the first-order) to the firm's probability of default $1 - \Phi(d_2)$.

2.5 Equilibrium conditions

The following equations describe the equilibrium conditions for the labor, good, bond, and deposit markets

$$L^h_{t+1} + L^e_{t+1} = \int_{i=0}^1 l_{t+1}(i)di$$
$$C^h_t + C^e_t + \int_{i=0}^1 k_{t+1}(i) = (1-\delta)\int_{i=0}^1 k_t(i)di + \int_{i=0}^1 \omega_t(i)\theta_t f(k_t(i), l_t(i))di$$
$$b^h_{t+1}(i) = b_{t+1}(i) \text{ all } i$$
$$D^h_{t+1} = D_{t+1}$$

These conditions complete the set of equilibrium conditions needed to characterize the equilibrium.

Because of the presence of the idiosyncratic shock $\omega_t(i)$, firms do not face the same problem. However, one can easily verify that a solution where bond prices are the same for all firms, the capital fund demands the same amount of bonds from all firms, and all firms choose the same values of capital, labor and bonds, satisfy all the budget constraints, the necessary conditions and the markets equilibrium conditions. We therefore focus on this equilibrium where, for all i, $q_t(i) = q_t$, $b^h_{t+1}(i) = B^h_{t+1}$, $k_{t+1}(i) = K_{t+1}$, $l_{t+1}(i) = L_{t+1}$,

[8]The term $\chi^h_{b,t}(i)$ has a similar interpretation to the corresponding one in equation (1).

and $b_{t+1}(i) = B_{t+1}$. Of course, firms pay different levels of dividends depending on the realization of their idiosyncratic productivity shock[9].

The system describing the equilibrium and its log-linear approximation is spelled out in Appendix B. Once the equilibrium has been determined, one can compute several variables related with credit risk. Appendix C defines the expected default frequency, the default rate, the loss rate, the loss given default, and the recovery rate.

3 Results

In this section, we document the model's quantitative predictions and compares them with data.

3.1 Data and calibration

Data are quarterly for the period 1981:I—2008:IV. We use output and hours (both Nonfarm Business Sector) from the Bureau of Labor Statistics, consumption (Nondurable Goods and Services), capital and investment (both Private Fixed Nonresidential) from the Bureau of Economic Analysis, debt (Nonfarm Nonfinancial Corporate Business Liabilities) from the Flow of Funds, default rates (All Rated) and recovery rates (All Bonds) from Moody's, and credit spreads (difference of Seasoned Baa Corporate Bond Yield and 10-Year Treasury Note Yield at Constant Maturity) from Moody's and the Treasury Department. The quarterly capital series has been obtained by interpolating the annual data.

Table 1 lists our benchmark parametrization. The values of all preferences and production parameters are standard. The parameters of the technology process θ_t are estimated from the HP-filtered Solow residual. First, the autocorrelation and the volatility of the technology process are estimated. Then, the first-order autocorrelation of the log-volatility process is set equal to one following Justiniano and Primiceri (2008). Finally, the volatility of the log-volatility process is estimated via quasi-maximum-likelihood following Harvey, Ruiz and Shephard (1994).

To calibrate the process of the idiosyncratic productivity ω_t, we set its average volatility σ_ω so that the model (approximately) matches the empirical correlations of the growth rates of output and debt. Also, we set the volatility of the log-volatility process equal to zero, so $\sigma_{\omega,t}$ is actually constant and equal to σ_ω. However, we find instructive to show in the next subsection the impulse response function to a shock to the log-volatility process. To this end, we set the autocorrelation of the log-volatility process equal to one, again following Justiniano and Primiceri (2008).

To calibrate the fraction of households, λ, we notice that the crucial difference between households and entrepreneurs in the model is that only the latter own the firms, so firms use the entrepreneurs' stochastic discount factor to make their optimal choices. According to the 1995 (the mid-period year) Survey of Consumer Finances, the percentage of families holding stocks was 15.3; holding mutual funds was 12.0; holding retirement accounts was 43.0. As a benchmark, we then set $\lambda = 2/3$, so the fraction of entrepreneurs in the model is equal to 33%.

[9] In particular, the firms' budget constraints imply that $\Pi^e_t(i) + \min\{y_t(i), b_t(i)\} - y_t(i)$ is the same for all i, so all firms that default pay the same dividends, whereas firms that do not default pay dividends directly related with their realized production.

Table 1: Benchmark parameter values

Parameter	Value	Description
A	1	Production function scale parameter
α	0.33	Production function capital share
δ	0.025	Capital depreciation rate
β	0.99	Preferences discount factor
γ	2	Relative risk aversion
φ	1	Inverse of labor supply elasticity
ρ_θ	0.7	Autocorrelation of technology
σ_θ	0.006	Average technology volatility
$\rho_{\sigma,\theta}$	1	Autocorrelation of technology log-volatility
$\sigma_{\sigma,\theta}$	0.0758	Volatility of technology log-volatility
σ_ω	0.0115	Average idiosyncratic-productivity volatility
$\rho_{\sigma,\omega}$	1	Autocorrelation of idiosyncratic-productivity log-volatility
$\sigma_{\sigma,\omega}$	0	Volatility of idiosyncratic-productivity log-volatility
λ	2/3	Fraction of households
$1-\Phi(d_2)$	0.005	Probability of default

The steady state level of debt b is set so that the steady state repayment probability $\Phi(d_2)$ is equal to 0.995, so the model steady state default rate is equal to the 0.5% quarterly average default rate for All Corporates from Moody's.

3.2 The effect of financial shocks: VAR evidence

Before considering the model's impulse response function, we show some suggestive evidence about the response of investment to financial shocks. Figure 1 shows the impulse response function of a 4-lags Vector Auto-Regression of technology, the default rate, and investment. Consistently with our model, the shocks are identified with a Cholesky decomposition with the variables ordered as listed above, so that technology does not respond simultaneously to a default rate shock, whereas investment does. The first column refers to the response to a technology shock, and agrees with intuition, with technology and investment responding positively, and the default rate negatively (although its response is significant only marginally). Focusing on the second column, we note that a default rate shock has an important delayed negative effect on investment: A shock increasing the default rate by 25 basis points decreases investment by more than 2 percent after 2 years. In addition, a F-test on the joint significance of the coefficients of the default rate in the investment equation strongly rejects the null of no significance, so lags of the default rate help linearly predict future investment, even after including in the regression lags of technology and investment themselves.

Figure 2 shows the impulse response function for a similar experiment with the credit spread in place of the default rate. The observations for the previous case apply to this case as well. The effect of a credit spread shock on investment is negative and significant for ten quarters. A shock increasing the credit spread by 8 basis points decreases investment by more than 4 percent after 10 quarters. The coefficients of the credit spread in the investment equation are also jointly statistically significant.

Overall the previous evidence is suggestive of an influence of credit risk variables on investment and real activity. In particular, the impulse response functions show that the negative response of investment to credit risk shocks is significant, both statistically and from an economic point of view. We now turn to the model's impulse response function to technology and volatility shocks, as well as the response to a one-time increase in the endogenous state variable debt.

3.3 Impulse responses

The crucial effect of the debt overhang distortion is on the equilibrium conditions determining investment and labor. From equations (1) through (5), disregarding the χ terms and evaluating the equations at equilibrium, the following two conditions can be derived:

$$\begin{aligned} R_t &= 1 - \delta + E_t\{\omega_{t+1}\theta_{t+1}\} f_k(K_{t+1}, L_{t+1})\Phi(d_{1,t}) \\ R_t w_{t+1} &= E_t\{\omega_{t+1}\theta_{t+1}\} f_l(K_{t+1}, L_{t+1})\Phi(d_{1,t}) \\ \text{where } d_{2,t} &\equiv \frac{\rho_\theta \ln(\theta_t) + \ln(f(K_{t+1}, L_{t+1})/B_{t+1})}{\sigma_t} \text{ and } d_{1,t} \equiv d_{2,t} + \sigma_t \end{aligned}$$

These two equations are similar to the corresponding ones of a standard real business cycle model with labor-in-advance, except for the presence of the probability of repayment $\Phi(d_{1,t})$. As already noted, the default probability $1-\Phi(d_{1,t})$ acts like a wedge discouraging investment and labor demand.

As a result, in the model with debt overhang, shocks affect the real economy through an additional channel, by affecting the distance to default $d_{1,t}$ and the default probability $1 - \Phi(d_{1,t})$. The effect of shocks on the distance to default is both direct and indirect through their effect on the endogenous state variables, capital and debt. As mentioned in the introduction, there are two positive feedback loop mechanisms at work. A static one, by which shocks that increase the current probability of default decrease current investment and future capital, decrease the distance to default $d_{1,t}$ and further increase the current probability of default $1 - \Phi(d_{1,t})$. And a dynamic one, by which shocks that increase the current probability of default decrease investment and future capital, and increase the future probability of default.

Technology shocks

Figure 3 shows the impulse response to an expansionary technology shock. The thick solid and thin lines respectively refer to our debt overhang model, and a corresponding standard model without any financial friction, calibrated with the same parameter values. We will comment on the dashed line in the next subsection on the financial friction based on agency monitoring costs.

The standard effect of an expansionary productivity shock consists in increasing the expected marginal product of capital, thereby encouraging investment. The debt overhang distortion adds an additional effect: The expansionary productivity shock increases the distance to default $d_{1,t}$, thereby increasing the repayment probability and further encouraging investment. Notice the static feedback loop mechanism: an increase in investment increases future capital, which, in turn, increases the repayment probability and leads to a further increase in investment. Moreover, the debt overhang correction adds persistence to the propagation mechanism, because the higher capital level tends to increase the repayment probability for several periods, even as productivity dies out.

In line with the VAR evidence, the probability of default decreases substantially, implying a smaller investment wedge, a higher expected marginal return of the firms' investment, and a higher investment and future production. Labor responds similarly to investment. The qualitative response of all variables agrees with intuition: Bond credit spreads decrease, recovery rates increase, loss rates decrease, and default rates decrease[10].

Under our baseline parametrization, debt increases after an expansionary productivity shock. When debt increases, it tends to increase the probability of default and to weaken the effect of an expansionary productivity shock, so debt contributes negatively to the dynamic feedback loop mechanism. In numerical experiments, we find that the dynamics of debt do not fully offset the dynamics of capital, so that the effects of productivity shocks are always stronger and more persistent in the economy with default relatively to the model without debt overhang.

Volatility and wealth redistribution shocks

Figures 4 and 5 respectively show the impulse response functions to a shock to the volatility of technology θ and the idiosyncratic productivity ω. Recall that both types of shocks do not have any effect in the log-linearized version of the standard model without debt overhang; equivalently, they do not have any first-order effect there[11]. In contrast, they have sizeable effects in the economy with default. Both shocks have very similar effects, the main difference being that the quantitative effect of the second shock is larger because the standard deviation of the idiosyncratic productivity process is calibrated to be larger than the one of the technology process. An unanticipated increase in volatility both directly and indirectly (through capital and debt) decreases the distances to default. As a result, default probabilities increase, and the expected marginal return from the firms' investment decrease. As the debt overhang distortion gets larger, investment and future production decrease. Bond credit spreads increase, recovery rates decrease, loss rates increase, and default rates increase. Notice that a shock that increases the volatility of the idiosyncratic productivity, by thickening the tail of firms that default, has an especially strong effect on the recovery rate and on the default rate.

It is also instructive to consider the response to a one-time unanticipated increase in the endogenous state variable debt, shown in Figure 6. This can be interpreted as the response to a shock redistributing wealth from firms (the debtors) to households (the creditors). The default-free economy (not shown) responds quite intuitively, debt increases permanently, households' consumption increases permanently, entrepreneurs' consumption decreases permanently, and there is no change in the other variables. In our framework, instead, a wealth redistribution shock increases the probability of default, exacerbates the financial friction, and decreases the firms' expected marginal return from investment, investment and future production. Credit spreads, loss rates and default

[10] Notice that the recovery rate refers to the subset of firms that default. Hence, the effect on the recovery rate is the result of the effect on the recovery value per given firm and the effect on the selection of firms that default. The positive effect on the recovery value is then attenuated by the decrease in the default rate, which leaves firms with relatively lower idiosyncratic productivity in the pool of firms that default.

[11] To be precise, the shock to the volatility of the idiosyncratic productivity does have a small effect through its effect on the factor $e^{\sigma^2_{\omega,t-1}/2}$ that multiplies the production function. However, the corresponding log-linear term is quantitatively negligible and of no theoretical interest.

rates increase, whereas recovery rates decrease.

Finally, notice that, in the debt overhang model, volatility shocks and debt shocks only affect the aggregate economy through the probability of default, which is closely related to the default rate and the credit spread. Hence, within the context of the model, they can be interpreted as credit risk shocks, and the model impulse response function to volatility shocks and debt shocks can be meaningfully compared with the empirical impulse response function to default rate shocks and credit spread shocks, documented in the previous subsection. The model correctly predicts the qualitative response to credit risk shocks, although, quantitatively, it tends to under-predict the size of the response of investment relative to either the default rate or the credit spread.

3.4 Comparison with the agency monitoring costs financial friction

We now contrast our debt overhang financial friction with the financial friction based on agency monitoring costs, although we notice that the two frictions are not alternative to each other. We add to the standard model with labor-in-advance and without debt overhang the monitoring costs financial friction described in Christiano, Motto and Rostagno (2003)[12].

The parameters specific to the monitoring costs friction are calibrated in the plausible way suggested by Bernanke, Gertler, and Gilchrist (1999). The calibration is as follows: the monitoring costs parameter is $\mu = 0.12$; the average and autocorrelation of the entrepreneurs' survival probability are respectively $z^* = 0.9845$ and $\rho_z = 0.95$; the standard deviation of the idiosyncratic shock is $\sigma^* = 0.28$. The other parameters are calibrated as in our model, as described in Table 1.

The dashed line in Figures 3, 5 and 6 refers to the impulse responses of the monitoring costs model. We do not plot the response to shocks to the volatility of technology, because they do not have any first-order effect. The first observation is that the qualitative response of most variables to shocks is similar in the two models, highlighting some common elements between the two frictions. Notice in particular the similarity of the response to a debt shock in the model with debt overhang with the response to a net worth shock in the model with monitoring costs, plotted in Figure 6. The qualitative response to a shock to the volatility of the idiosyncratic productivity is also similar in the two models (except for the recovery rate).

The monitoring costs model, however, does not have clear-cut predictions as to the sign of the response of credit spreads and default rates to technology shocks. In the model with monitoring costs, entrepreneurs finance their investment through their net worth or bank loans. After an expansionary technology shock, since net worth is pre-determined, they have to increase their debt in order to increase their investment. Depending on the choice of parameters, this may lead to an increase in credit spreads and default rates after an expansionary technology shock. In contrast, the response of credit spreads and default rates is always negative in our debt overhang framework. Turning to the data, Figures 7 and 8 plot the default rate and the credit spread against the growth rates of production, labor and investment and show their correlations. The correlations of both

[12] Appendix D briefly describes the friction. For a detailed description of the friction, see Bernanke, Gertler, and Gilchrist (1999) and Christiano, Motto, and Rostagno (2003). Appendix E clarifies why the debt overhang correction is not present in standard agency costs models.

the default rate and the credit spread with the three macro variables are negative in all cases.

Furthermore, the amplification mechanism of technology shock is quantitatively small in the model with monitoring costs, unless the monitoring costs are set equal to an unplausibly high level. The reason behind the quantitative difference between the two models is quite instructive. In the model with monitoring costs, the credit spread is the sum of the default probability and the external finance premium. The financial friction, however, is related to the external finance premium only, which in turn is linked with the monitoring costs. When the monitoring costs tend to zero, although leverage and defaults are still present, they become irrelevant for the evolution of the aggregate variables. A plausible calibration of the monitoring costs leads to a very small variability of the external finance premium, and to a very small amplification mechanism. In contrast, in the model with debt overhang, the distortion is related to the default probability (there is no external finance premium), whose response is sizeable for plausible parameter values.

The monitoring costs friction can lead to a larger amplification mechanism when it acts in combination with other features, as in the Bernanke, Gertler, and Gilchrist (1999) model. In numerical experiments, we found that the most important features of their model necessary for the friction to generate a large amplification mechanism are a variable price of capital, a very large (close to one) autocorrelation of the technology shock, and sticky prices together with a monetary policy rule implying a very small (about 1.1) monetary policy long-run response of the nominal interest rate to inflation. Given the monetary policy rule, the degree of persistence of the technology process has dramatic effects on the amplification mechanism. Indeed, when the autocorrelation of technology is small, the monitoring costs friction can lead to an attenuation mechanism. Notice however that, even under the parametrization of Bernanke, Gertler, and Gilchrist (1999), the amplification and propagation mechanism is substantially smaller than the one generated by the debt overhang distortion.

3.5 Correlations with credit spreads and default rates

Tables 2 and 3 provide some evidence in support of the debt overhang model, by comparing the second moments of several variables of interest in the model and in the data. The variables are the growth rates of debt, labor, investment, consumption and output, and the level of credit spread, recovery rate and default rate. The moments are correlations with credit spread, with default rate, and with the output growth rate, and autocorrelations. Recall that one parameter, the volatility σ_ω of the idiosyncratic productivity shocks, has been calibrated to approximately match the correlation between the growth rates of output and debt.

The signs of all moments match the ones in the data. Also, the correlations of the credit variables, namely the bond spread, the recovery rate and the default rate, with output and investment are all consistent with data. The autocorrelations of the credit risk variables are also consistent with data. Finally, a comparison between the autocorrelations in the models with and without debt overhang reveals how the debt overhang distortion significantly enhances the persistence of the macro variables growth rates, helping to better match their empirical counterparts.

Table 2: Correlations with credit spread, default rates and output

	Corr. with Credit Spread		Corr. with Default Rate		Corr. with Output	
	Model	Data	Model	Data	Model	Data
Debt	-0.5462	-0.2370	-0.7172	-0.2164	0.1821	0.1721
Investment	-0.4256	-0.5805	-0.2398	-0.5261	0.9990	0.3545
Consumption	-0.7523	-0.3162	-0.9083	-0.1900	0.3313	0.5141
Production	-0.4567	-0.4398	-0.2781	-0.2038	1.0000	1.0000
Credit Spread	1.0000	1.0000	0.8852	0.3952	-0.4567	-0.4398
Recovery Rate	-0.8852	-0.2137	-1.0000	-0.6514	0.2781	0.2251
Default Rate	0.8852	0.3952	1.0000	1.0000	-0.2781	-0.2038

Table 3: First-order autocorrelations

	Model without Debt Overhang	Model	Data
Debt	—	0.7718	0.4115
Investment	0.1129	0.3050	0.5464
Consumption	0.2193	0.7545	0.4664
Production	0.1042	0.3063	0.4213
Credit Spread	—	0.7147	0.8090
Recovery Rate	—	0.8439	0.9804
Default Rate	—	0.8439	0.7560

3.6 Fiscal multipliers

Among others, Blanchard and Perotti (2002) and Galí, López-Salido and Vallés (2007) document that the empirical fiscal multiplier[13] of a government spending shock on output is smaller than one at impact (in a neighborhood of 0.75), but eventually reaches values larger than one (estimates vary widely and often reach values larger than two). The real business cycle framework can hardly replicate a fiscal multiplier greater than one. Here, we show how the presence of the debt overhang distortion helps obtain a fiscal multiplier greater than one.

We consider a temporary fiscal shock with a half-life of about 4 years (equal to the estimates by Galí, López-Salido and Vallés (2007)), namely a shock to an exogenous government spending process with a first-order autoregressive coefficient $\rho_g = 0.90$. The share of government spending in aggregate production is set equal to 20%. Figure 9 shows the fiscal multipliers on output, investment and consumption as well as the responses of some key rates to a unitary shock to government spending. In the debt overhang model, the shock generates a boost to the expected corporate revenues that, at impact, induces expectations of a reduction of the corporate sector leverage and, thus, of the expected default rate. This effect is described by the evolution of the credit spread that falls immediately even if output and the default rate are predetermined at the time of the shock. In the next periods, the positive dynamic feedback induced by the debt overhang more than offsets the crowding out effects of the real rate increase on consumption and investment. Under our baseline calibration, the fiscal multiplier is 0.95 after one quarter, 1.22 after one year, and reaches a peak at 1.7 after 7 years, so the fiscal multiplier is hump-shaped, and the government spending shock contributes positively to output growth for several years. These values are remarkably larger than in the case of the standard real business cycle model, where the multiplier never reaches values above 0.4[14].

The dashed line shows the multiplier of the Bernanke, Gertler, and Gilchrist (1999) model with agency monitoring costs. The weak monetary policy reaction to inflation increases the multiplier relative to the case (not shown) without monitoring cost, but the magnification of the financial accelerator mechanism is relatively small: The difference between the responses of the models with and without monitoring costs never exceeds 0.1. This is also evident from the fact that the external finance premium barely moves. In fact, in the case (not shown) of a permanent fiscal shock ($\rho_g = 1$), entrepreneurs increase their leverage so much that the external finance premium increases and the agency costs friction leads to an *attenuation* effect.

4 Conclusions

In this paper, we have studied the business cycle implications of the debt overhang distortion described by Myers (1977). The dynamics of this distortion, which moves countercyclically, substantially amplify and propagate the response of a standard business cycle model to technology, volatility, wealth redistribution and government spending shocks.

The model is able to capture the effects of credit risk variables, such as credit spreads and default rates, on macroeconomic variables. The predictions of the model as to the co-

[13]The fiscal multiplier at time $t+j$ on the variable X is defined as $\Delta X_{t+j}/\Delta G_t$ where G is government spending.

[14]Excluding the first period, the multiplier of the real business cycle model with labor-in-advance is almost identical to its version without labor-in-advance.

movement of investment, output and credit risk variables are consistent with data. The debt overhang distortion adds persistence to the macro variables processes, and generates a fiscal multiplier close to the VAR literature estimates.

Our model is easy to describe and adopt, and its mechanisms can be easily incorporated in other macroeconomic models. The solution method developed in this paper may also be useful to analyze other problems involving non-linearities.

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A Analytical results for log-normals

This appendix applies some well-known analytical results holding for the expectation of the minimum of log-normal random variables, and for its derivatives, in order to derive equations (1), (3), and (2).

Notice that k_{t+1}, l_{t+1} and b_{t+1} are all known in period $t+1$, and that $\ln(\omega_{t+1}\theta_{t+1})$ is normally distributed with mean equal to $\rho_\theta \ln(\theta_t)$ and standard deviation equal to σ_t. Then, a well-known analytical result holding for log-normally distributed random variables yields

$$E_t\{\min\{\omega_{t+1}\theta_{t+1}f(k_{t+1},l_{t+1}),b_{t+1}\}\} = E_t\{\omega_{t+1}\theta_{t+1}\}f(k_{t+1},l_{t+1})[1-\Phi(d_{1,t})] + b_{t+1}\Phi(d_{2,t})$$
$$\text{where } d_{2,t} \equiv \frac{\rho_\theta \ln(\theta_t) + \ln(f(k_{t+1},l_{t+1})/b_{t+1})}{\sigma_t} \text{ and } d_{1,t} \equiv d_{2,t} + \sigma_t$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal random variable.

Turning to the derivatives, and dropping the time subscripts to ease notation,

$$\begin{aligned}
\frac{\partial E\{\min\{\omega\theta f(k,l),b\}\}}{\partial b} &= \frac{\partial[E\{\omega\theta\}f(k,l)[1-\Phi(d_1)] + b\Phi(d_2)]}{\partial b} \\
&= -E\{\omega\theta\}f(k,l)\Phi'(d_1)\frac{\partial d_1}{\partial b} + b\Phi'(d_2)\frac{\partial d_2}{\partial b} + \Phi(d_2) \\
&= \Phi(d_2)
\end{aligned}$$

where the last step follows from $\frac{\partial d_1}{\partial b} = \frac{\partial d_2}{\partial b}$ and from

$$\begin{aligned}
-E\{\omega\theta\}f(k,l)\Phi'(d_1) + b\Phi'(d_2) &= -e^{\ln(E\{\omega\theta\}f(k,l))}\Phi'(d_1) + e^{\ln(b)}\Phi'(d_2) \\
&= -e^{\ln(E\{\omega\theta\}f(k,l))}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_1^2} + e^{\ln(b)}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_2^2} \\
&= -e^{\ln(E\{\omega\theta\}f(k,l))}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_1^2} + e^{\ln(b)}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_1^2+\ln(E\{\omega\theta\}f(k,l)/b)} \\
&= -e^{\ln(E\{\omega\theta\}f(k,l))}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_1^2} + \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_1^2+\ln(E\{\omega\theta\}f(k,l))} \\
&= 0
\end{aligned}$$

Similarly,

$$\begin{aligned}
\frac{\partial E\{\min\{\omega\theta f(k,l),b\}\}}{\partial k} &= \frac{\partial[E\{\omega\theta\}f(k,l)[1-\Phi(d_1)] + b\Phi(d_2)]}{\partial k} \\
&= E\{\omega\theta\}f_k(k,l)[1-\Phi(d_1)] - E\{\omega\theta\}f(k,l)\Phi'(d_1)\frac{\partial d_1}{\partial k} + b\Phi'(d_2)\frac{\partial d_2}{\partial k} \\
&= E\{\omega\theta\}f_k(k,l)[1-\Phi(d_1)]
\end{aligned}$$

where the last step follows from $\frac{\partial d_1}{\partial k} = \frac{\partial d_2}{\partial k}$ and from the previous result

$$-E\{\omega\theta\}f(k,l)\Phi'(d_1) + b\Phi'(d_2) = 0$$

Similarly,

$$\frac{\partial E\{\min\{\omega\theta f(k,l),b\}\}}{\partial l} = E\{\omega\theta\}f_l(k,l)[1-\Phi(d_1)]$$

B Solution

This appendix spells out the system describing the equilibrium and its log-linear approximation.

The following convenient consolidated constraint follows from the budget constraints of the households and the capital fund, together with the bank deposits equilibrium condition:

$$C_t^h + \int_{i=0}^{1} q_t(i) b_{t+1}^h(i) di = w_{t+1} L_{t+1}^h + \int_{i=0}^{1} \min\{\omega_t(i)\theta_t f(k_t(i), l_t(i)), b_t^h(i)\} di$$

$$C_t^h + q_t B_{t+1}^h = w_{t+1} L_{t+1}^h + \int_{i=0}^{1} \min\{\omega_t(i)\theta_t f(K_t, L_t), B_t^h\} di$$

Notice that, because $\omega_t(i)$ is i.i.d. across a continuum of firms indexed by $i \in [0,1]$, $\int_{i=0}^{1} h(\omega_t(i))di = E_t h(\omega_t)$ for a generic function h, where ω_t is a random variable (unknown in period t) distributed as $\omega_t(i)$. In other words, the average of a variable across firms is equal to the expectation of the same variable for one firm, prior to the realization of ω_t. Hence,

$$\int_{i=0}^{1} \omega_t(i) di = E_t\{\omega_t\}$$

$$\int_{i=0}^{1} \min\{\omega_t(i)\theta_t f(K_t, L_t), B_t^h\} di = E_t\{\min\{\omega_t \theta_t f(K_t, L_t), B_t^h\}\}$$

where E_t is the average with respect to ω_t.

Notice that θ_t, K_t and B_t are all known in period t, and $\ln(\omega_t)$ is normally distributed with mean equal to zero and standard deviation equal to $\sigma_{\omega,t-1}$. Then, a well-known analytical result holding for log-normally distributed random variables yields

$$E_t\{\min\{\omega_t \theta_t f(K_t, L_t), B_t\}\} = E_t\{\omega_t\}\theta_t f(K_t, L_t)[1 - \Phi(d_{1,t}^\omega)] + B_t \Phi(d_{2,t}^\omega)$$

$$\text{where } d_{2,t}^\omega \equiv \frac{\ln(\theta_t f(K_t, L_t)/B_t)}{\sigma_{\omega,t-1}} \text{ and } d_{1,t}^\omega \equiv d_{2,t}^\omega + \sigma_{\omega,t-1}$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal random variable. The budget constraint then becomes:

$$C_t^h + q_t B_{t+1}^h = w_{t+1} L_{t+1}^h + E_t\{\omega_t\}\theta_t f(K_t, L_t)[1 - \Phi(d_{1,t}^\omega)] + B_t^h \Phi(d_{2,t}^\omega)$$

System describing the equilibrium

After using the bond market equilibrium condition, the system describing the equilibrium is:

$$
\begin{aligned}
\ln(\theta_{t+1}) &= \rho_\theta \ln(\theta_t) + \sigma_{\theta,t}\epsilon_{\theta,t+1} \\
\ln(\sigma_{\theta,t+1}/\sigma_\theta) &= \rho_{\theta,\sigma} \ln(\sigma_{\theta,t}/\sigma_\theta) + \sigma_{\theta,\sigma}\eta_{\theta,t+1} \\
\ln(\sigma_{\omega,t+1}/\sigma_\omega) &= \rho_{\omega,\sigma} \ln(\sigma_{\omega,t}/\sigma_\omega) + \sigma_{\omega,\sigma}\eta_{\omega,t+1} \\
\sigma_t^2 &\equiv \sigma_{\omega,t}^2 + \sigma_{\theta,t}^2 \\
u'(C_t^h)q_t &= \beta E_t\{u'(C_{t+1}^h)\}\Phi(d_{2,t}) + \partial Cov_t^h/\partial B_{t+1} \\
u'(C_t^h)w_{t+1} &= \beta v'(L_{t+1}^h) \\
C_t^h + q_t B_{t+1} &= w_{t+1}L_{t+1}^h + Y_t[1-\Phi(d_{1,t}^\omega)] + B_t\Phi(d_{2,t}^\omega) \\
u'(C_t^e)q_t &= \beta E_t\{u'(C_{t+1}^e)\}\Phi(d_{2,t}) + \partial Cov_t^e/\partial B_{t+1} \\
u'(C_t^e)w_{t+1} &= \beta v'(L_{t+1}^e) \\
u'(C_t^e) &= \beta E_t\{u'(C_{t+1}^e)\}[(1-\delta) + E_t\{\alpha Y_{t+1}/K_{t+1}\}\Phi(d_{1,t})] + \partial Cov_t^e/\partial K_{t+1} \\
u'(C_t^e)w_{t+1} &= \beta E_t\{u'(C_{t+1}^e)\}E_t\{(1-\alpha)Y_{t+1}/L_{t+1}\}\Phi(d_{1,t}) + \partial Cov_t^e/\partial L_{t+1} \\
L_{t+1}^h + L_{t+1}^e &= L_{t+1} \\
C_t^h + C_t^e + K_{t+1} &= (1-\delta)K_t + Y_t \\
\text{where } d_{2,t}^\omega &\equiv \frac{\ln(\theta_t f(K_t,L_t)/B_t)}{\sigma_{\omega,t-1}} \text{ and } d_{1,t}^\omega \equiv d_{2,t}^\omega + \sigma_{\omega,t-1} \\
d_{2,t} &\equiv \frac{\rho_\theta \ln(\theta_t) + \ln(f(K_{t+1},L_{t+1})/B_{t+1})}{\sigma_t} \text{ and } d_{1,t} \equiv d_{2,t} + \sigma_t \\
Y_t &\equiv E_t\{\omega_t\}\theta_t f(K_t,L_t) = e^{\sigma_{\omega,t-1}^2/2}\theta_t f(K_t,L_t)
\end{aligned}
$$

where $Cov_t^h \equiv Cov_t(\beta u'(C_{t+1}^h), \min\{\omega_{t+1}\theta_{t+1}f(K_{t+1},L_{t+1}), B_{t+1}\})$ and similarly $Cov_t^e \equiv Cov_t(\beta u'(C_{t+1}^e), \min\{\omega_{t+1}\theta_{t+1}f(K_{t+1},L_{t+1}), B_{t+1}\})$.

Log-linear approximation

This system can be solved with standard methods, log-linearizing it around its non-stochastic steady state. Notice that the derivatives of the covariance terms disappear from the log-linearized approximation because, in the non-stochastic steady state, the covariances are identically equal to zero, so their derivatives are equal to zero as well.

The following is the log-linear approximation of our equilibrium system:

$$
\begin{aligned}
\hat{\theta}_{t+1} &= \rho_\theta \hat{\theta}_t + \sigma_\theta \epsilon_{\theta,t+1} \\
\hat{\sigma}_{\theta,t+1} &= \rho_{\theta,\sigma} \hat{\sigma}_{\theta,t} + \sigma_\theta \sigma_{\theta,\sigma} \eta_{\theta,t+1} \\
\hat{\sigma}_{\omega,t+1} &= \rho_{\omega,\sigma} \hat{\sigma}_{\omega,t} + \sigma_\omega \sigma_{\omega,\sigma} \eta_{\omega,t+1} \\
\sigma \hat{\sigma}_t &= \sigma_\omega \hat{\sigma}_{\omega,t} + \sigma_\theta \hat{\sigma}_{\theta,t} \\
-\gamma \hat{c}^h_t + \hat{q}_t &= -\gamma \hat{c}^h_{t+1} + \frac{\Phi'(d_2)}{\Phi(d_2)} \hat{d}_{2,t} \\
-\gamma \hat{c}^h_t + \hat{w}_{t+1} &= \varphi \hat{l}^h_{t+1} \\
c^h \hat{c}^h_t + qb(\hat{q}_t + \hat{b}_{t+1}) &= w l^h (\hat{w}_{t+1} + \hat{l}^h_{t+1}) + \dots \\
&\dots + y[1 - \Phi(d^\omega_1)] \left(\hat{y}_t - \frac{\Phi'(d^\omega_1)}{1 - \Phi(d^\omega_1)} \hat{d}^\omega_{1,t} \right) + b \Phi(d^\omega_2) \left(\hat{b}_t + \frac{\Phi'(d^\omega_2)}{\Phi(d^\omega_2)} \hat{d}^\omega_{2,t} \right) \\
-\gamma \hat{c}^e_t + \hat{q}_t &= -\gamma \hat{c}^e_{t+1} + \frac{\Phi'(d_2)}{\Phi(d_2)} \hat{d}_{2,t} \\
-\gamma \hat{c}^e_t + \hat{w}_{t+1} &= \varphi \hat{l}^e_{t+1} \\
-\gamma \hat{c}^e_t &= -\gamma \hat{c}^e_{t+1} + [1 - \beta(1-\delta)] \left\{ \hat{y}_{t+1} - \hat{k}_{t+1} + \frac{\Phi'(d_1)}{\Phi(d_1)} \hat{d}_{1,t} \right\} \\
-\gamma \hat{c}^e_t + \hat{w}_{t+1} &= -\gamma \hat{c}^e_{t+1} + \hat{y}_{t+1} - \hat{l}_{t+1} + \frac{\Phi'(d_1)}{\Phi(d_1)} \hat{d}_{1,t} \\
l^h \hat{l}^h_{t+1} + l^e \hat{l}^e_{t+1} &= l \hat{l}_{t+1} \\
c^h \hat{c}^h_t + c^e \hat{c}^e_t + k \hat{k}_{t+1} &= (1-\delta) k \hat{k}_t + y \hat{y}_t \\
\text{where } \hat{d}^\omega_{2,t} &\equiv \frac{\hat{\theta}_t + \alpha \hat{k}_t + (1-\alpha)\hat{l}_t - \hat{b}_t}{\sigma_\omega} - \frac{d^\omega_2}{\sigma_\omega} \hat{\sigma}_{\omega,t-1} \text{ and } \hat{d}^\omega_{1,t} \equiv \hat{d}^\omega_{2,t} + \hat{\sigma}_{\omega,t-1} \\
\hat{d}_{2,t} &\equiv \frac{\rho_\theta \hat{\theta}_t + \alpha \hat{k}_{t+1} + (1-\alpha)\hat{l}_{t+1} - \hat{b}_{t+1}}{\sigma} - \frac{d_2}{\sigma} \hat{\sigma}_t \text{ and } \hat{d}_{1,t} \equiv \hat{d}_{2,t} + \hat{\sigma}_t \\
\hat{y}_t &\equiv \sigma_\omega \hat{\sigma}_{\omega,t-1} + \hat{\theta}_t + \alpha \hat{k}_t + (1-\alpha)\hat{l}_t
\end{aligned}
$$

where hatted variables represent deviations (in the case of σ, σ_θ, σ_ω, d^ω_1, d^ω_2, d_1 and d_2) or log-deviations (in the case of all other variables) from steady state, whereas non-hatted variables represent steady state values.

C Credit risk variables

This appendix defines the credit risk variables that we study.

Expected Default Frequency:

$$\begin{aligned}
EDF_t &\equiv E_t \int_{i=0}^{1} I\{\omega_{t+1}(i)\theta_{t+1}f(k_{t+1}(i), l_{t+1}(i)) \leq b_{t+1}(i)\}di \\
&= Prob_t\{\omega_{t+1}\theta_{t+1}f(K_{t+1}, L_{t+1}) \leq B_{t+1}\} \\
&= Prob_t\left\{\frac{\ln(\omega_{t+1}) + \ln(\theta_{t+1}) - \rho_\theta \ln(\theta_t)}{\sigma_t} \leq \frac{\ln(B_{t+1}/f(K_{t+1}, L_{t+1})) - \rho_\theta \ln(\theta_t)}{\sigma_t}\right\} \\
&= \Phi\left(-d_{2,t}\right) \\
&= 1 - \Phi\left(d_{2,t}\right)
\end{aligned}$$

where $d_{2,t} \equiv \dfrac{\rho_\theta \ln(\theta_t) + \ln(f(K_{t+1}, L_{t+1})/B_{t+1})}{\sigma_t}$ and $d_{1,t} \equiv d_{2,t} + \sigma_t$

where $I(\cdot)$ is the indicator function, and $\Phi(\cdot)$ is the cumulative distribution function of the standard normal random variable.

Default Rate:

$$\begin{aligned}
DR_t &\equiv \int_{i=0}^{1} I\{\omega_t(i)\theta_t f(k_t(i), l_t(i)) \leq b_t(i)\}di \\
&= Prob_t\{\omega_t\theta_t f(K_t, L_t) \leq B_t\} \\
&= Prob_t\left\{\frac{\ln(\omega_t)}{\sigma_{\omega,t-1}} \leq \frac{\ln(B_t/\theta_t f(K_t, L_t))}{\sigma_{\omega,t-1}}\right\} \\
&= \Phi\left(-d^\omega_{2,t}\right) \\
&= 1 - \Phi\left(d^\omega_{2,t}\right)
\end{aligned}$$

where $d^\omega_{2,t} \equiv \dfrac{\ln(\theta_t f(K_t, L_t)/B_t)}{\sigma_{\omega,t-1}}$ and $d^\omega_{1,t} \equiv d^\omega_{2,t} + \sigma_{\omega,t-1}$

Loss rate:

$$\begin{aligned}
LR_t &\equiv \frac{\int_{i=0}^{1} \max\left\{b_t(i) - \omega_t(i)\theta_t f(k_t(i), l_t(i)), 0\right\} di}{\int_{i=0}^{1} b_t(i)di} \\
&= \frac{E_t \max\left\{B_t - \omega_t\theta_t f(K_t, L_t), 0\right\}}{B_t} \\
&= \frac{(1 - \Phi(d^\omega_{2,t}))B_t - (1 - \Phi(d^\omega_{1,t}))\theta_t f(K_t, L_t)}{B_t} \\
&= (1 - \Phi(d^\omega_{2,t})) - (1 - \Phi(d^\omega_{1,t}))\frac{\theta_t f(K_t, L_t)}{B_t}
\end{aligned}$$

where E_t is the average with respect to ω_t.

Loss given default:

$$
\begin{aligned}
LGD_t &\equiv \frac{\int_{i=0}^{1} \max\{b_t(i) - \omega_t(i)\theta_t f(k_t(i), l_t(i)), 0\}\, di}{\int_{i=0}^{1} b_t(i) I\{\omega_t(i)\theta_t f(k_t(i), l_t(i)) \leq b_t(i)\} di} \\
&= E_t \left\{ \frac{B_t - \omega_t\theta_t f(K_t, L_t)}{B_t} \,|\, \omega_t\theta_t f(K_t, L_t) \leq B_t \right\} \\
&= \frac{1}{B_t} E_t \{B_t - \omega_t\theta_t f(K_t, L_t) \,|\, \omega_t\theta_t f(K_t, L_t) \leq B_t\} \\
&= \frac{1}{B_t} \frac{E_t \max\{B_t - \omega_t\theta_t f(K_t, L_t), 0\}}{DR_t} \\
&= \frac{1}{B_t} \frac{(1 - \Phi(d_{2,t}^{\omega}))B_t - (1 - \Phi(d_{1,t}^{\omega}))\theta_t f(K_t, L_t)}{1 - \Phi(d_{2,t}^{\omega})} \\
&= 1 - \frac{1 - \Phi(d_{1,t}^{\omega})}{1 - \Phi(d_{2,t}^{\omega})} \frac{\theta_t f(K_t, L_t)}{B_t}
\end{aligned}
$$

Recovery rate:

$$
RR_t \equiv 1 - LGD_t = \frac{1 - \Phi(d_{1,t}^{\omega})}{1 - \Phi(d_{2,t}^{\omega})} \frac{\theta_t f(K_t, L_t)}{B_t}
$$

D A brief description of the monitoring costs financial friction

In this appendix, we briefly describe the monitoring costs financial friction, based on Bernanke, Gertler and Gilchrist (1999), that we add to a standard model with labor-in-advance for the purpose of comparing its effect with the debt overhang distortion.

Households supply funds to a perfectly competitive banking sector at the risk free rate R_t. In turn, banks lend those funds to risk-neutral entrepreneurs at the risky rate R_{t+1}^e. Entrepreneurs combine their own funds N_{t+1} with the bank loans B_{t+1}, and purchase capital K_{t+1} at a price Q_t:

$$
Q_t K_{t+1} = N_{t+1} + B_{t+1}
$$

We neglect investment adjustment costs, so output is freely transformable into capital and consumption, and the price of capital relative to consumption, Q_t, is one.

After capital is purchased, each entrepreneur is subject to an idiosyncratic shock, ω_{t+1}, with distribution $F(\omega_{t+1}; \sigma_{\omega,t})$, that changes the level of capital from K_{t+1} to $\omega_{t+1}K_{t+1}$. The next period, entrepreneurs rent their capital to firms at the rental rate R_{t+1}^k and firms produce output from capital and labor. Finally, after production occurs, the entrepreneur receives back the depreciated capital, $(1-\delta)\omega_{t+1}K_{t+1}$, and pays his debt to the banks. The loan, however, is risky, because the entrepreneur's liability is limited to the rent that he receives, so the entrepreneur effectively repays

$$
\min\{\omega_{t+1} R_{t+1}^k Q_t K_{t+1}, R_{t+1}^e B_{t+1}\}
$$

It is useful to define a threshold $\bar{\omega}_{t+1}$ such that all entrepreneurs for whom $\omega_{t+1} < \bar{\omega}_{t+1}$ have not enough resources to repay the debt, so $F(\bar{\omega}_{t+1})$ is the default rate:

$$
\bar{\omega}_{t+1} R_{t+1}^k Q_t K_{t+1} = R_{t+1}^e B_{t+1}
$$

Credit market frictions arise because the realization of ω_{t+1} is observable to the lender only after paying a monitoring cost $\mu\omega_{t+1}R^k_{t+1}Q_tK_{t+1}$, with $0 \leq \mu < 1$. In equilibrium, banks pay the costs to monitor all the entrepreneurs that default.

The equilibrium rates and external finance premium are determined by two key conditions. The first one is the zero-profit condition for banks, which is assumed to hold state-by-state[15]:

$$[1 - F(\bar{\omega}_{t+1})]R^e_{t+1}B_{t+1} + \int_0^{\bar{\omega}_{t+1}} (1-\mu)\omega R^k_{t+1}Q_tK_{t+1}dF(\omega) = R_tB_{t+1}$$

The ex-post revenues from banking activity—the interest payments plus the recovered values net of monitoring costs—must equal the banks' cost of funds.

The second condition follows from the solution of an optimal loan contract between banks and entrepreneurs. The optimal contract maximizes the entrepreneurs' expected wealth at the end of the contract

$$E_t\left\{\int_{\bar{\omega}_{t+1}}^{\infty} \omega R^k_{t+1}Q_tK_{t+1}dF(\omega) - [1 - F(\bar{\omega}_{t+1})]R^e_{t+1}B_{t+1}\right\}$$

given the banks' zero-profit condition.

The solution of the contracting problem gives, after some manipulations, a relation between the external finance premium, $E_t\{R^k_{t+1}\}/R_t - 1$, and the default threshold, $\bar{\omega}_{t+1}$, that can be expressed as

$$E_t\{g(\bar{\omega}_{t+1})[R^k_{t+1}/R_t - \tilde{g}(\bar{\omega}_{t+1})]\} = 0$$

where g and $\tilde{g}$ are functions of $\bar{\omega}_{t+1}$ (and $\sigma_{\omega,t}$). When the monitoring costs are zero, $\mu = 0$, both g and $\tilde{g}$ are identically equal to one, that is $g(\bar{\omega}_{t+1}) \equiv 1$ and $\tilde{g}(\bar{\omega}_{t+1}) \equiv 1$, for all $\bar{\omega}_{t+1}$. This implies that the expected return on capital must equal the risk-free rate, $E_tR^k_{t+1} = R_t$, as in the linearized standard business cycle model: even though defaults may occur, $F(\bar{\omega}) > 0$, the external finance premium is zero and no financial accelerator arises.

E Debt overhang correction and agency costs models

In this appendix, we introduce a stylized example designed to identify the essential elements needed for the debt overhang distortion to arise, and to clarify why the debt overhang correction is not present in standard agency costs models.

In the example, firms sell banks an exogenous amount of $B > 0$ bonds at the bond price q. The bonds are risky, and their payoff is the minimum between their face value and the value of output produced by firms. Production is given by $\omega f(K)$, where ω is a productivity shock, $f(\cdot)$ is a production function, and K is the amount of capital invested by firms.

The timing of events is as follows: First the debt contract is signed, and then the investment decision is taken, so firms know the bond price q before choosing the capital level K.

[15] A consequence of this assumption is that R^e_{t+1}, the lending rate between t and $t+1$, will be a function of the $t+1$ aggregate shocks, which rules out both banks' default and positive profits. The state-contingent nature of the debt contract makes more difficult the mapping of the lending rate R^e to a data counterpart.

Firms choose capital K to solve the following problem:

$$\max_{K} E\{\omega f(K) - K + qB - \min\{\omega f(K), B\}\}$$

where E is the expectation operator, given the bond price q and the distribution of the productivity shock ω.

The necessary condition is

$$E\{\omega\}f'(K) = 1 + \frac{\partial E\{\min\{\omega f(K), B\}\}}{\partial K}$$

The debt overhang correction term is the last one on the right hand side. The presence of this term, whose sign is positive, implies that the level of K is less than the socially optimal one, which is determined by the condition $E\{\omega\}f'(K) = 1$. The debt overhang correction is present because part of the benefits of the firms' investment choice accrues to the banks, and firms do not internalize this positive externality on the banks' profits.

Nothing else is needed to study the debt overhang distortion. However, for the purpose of comparing this example with the following one, we can close the model by imposing that a zero-profit condition holds for banks in equilibrium:

$$E\{\min\{\omega f(K), B\}\} - qB = 0$$

The system describing the equilibrium is characterized by two equations, the necessary condition and the zero-profit condition, that jointly determine K and q. Clearly, the debt overhang correction is still present even when the banks' zero-profit condition is imposed. What is crucial for the debt overhang distortion to arise is that, when firms choose investment, they have a positive externality on the banks' profits that they do not internalize.

A key modeling assumption

The reason why the debt overhang correction does not appear in standard agency costs models lies in the following key modeling assumption. In the agency costs literature, entrepreneurs choose investment to maximize their objective function *subject to a banks' zero-profit condition*. They take into account that the lending rate varies in such a way that the banks' profits are identically equal to zero. Hence, when entrepreneurs choose their level of investment, they do not have any externality effect on the banks' profits, and they fully internalize all the benefits of their investment choice. To clearly see this interesting point, we now modify the previous example adding this modeling assumption.

The timing of events is now reversed: first the investment decision is taken, and then the debt contract is signed. Moreover, the bond price q, that is the inverse of the lending rate, varies with the capital level K in such a way that the banks' profits are equal to zero for all possible levels of K.

Firms solve the following modified problem:

$$\max_{K,q} E\{\omega f(K) - K + qB - \min\{\omega f(K), B\}\}$$
$$\text{subject to } E\{\min\{\omega f(K), B\}\} - qB = 0$$

When choosing K, now firms take into account that q will adjust so that the banks' zero-profit condition will hold. In other words, they take into account that the terms of the debt contract will vary exactly to offset any impact of the firms' investment decision on the banks' profits, so firms do not have an externality effect on the banks' profits any more.

After substituting the banks' zero-profit condition into the firms' objective function, the problem can be stated as follows:

$$\max_{K} E\{\omega f(K) - K\}$$

which clearly shows that firms fully internalize all the benefits of their investment choice. Notice how adding the banks' zero-profit condition as a constraint greatly simplifies the firms' optimization problem.

The necessary condition is now

$$E\{\omega\} f'(K) = 1$$

so the capital level K is equal to the socially optimal one.

The system describing the equilibrium is characterized by two equations, the new necessary condition and the zero-profit condition, that jointly determine K and q. The system is the same as in the original version of the example, except that the debt overhang correction is not present any more. The reason for this is that the terms of the debt contract vary depending on the investment level in such a way that the banks' profits are identically equal to zero, so firms cease to have a positive externality on the banks' profits, and fully internalize all the benefits of their investment choice.

VAR impulse response to a default rate shock

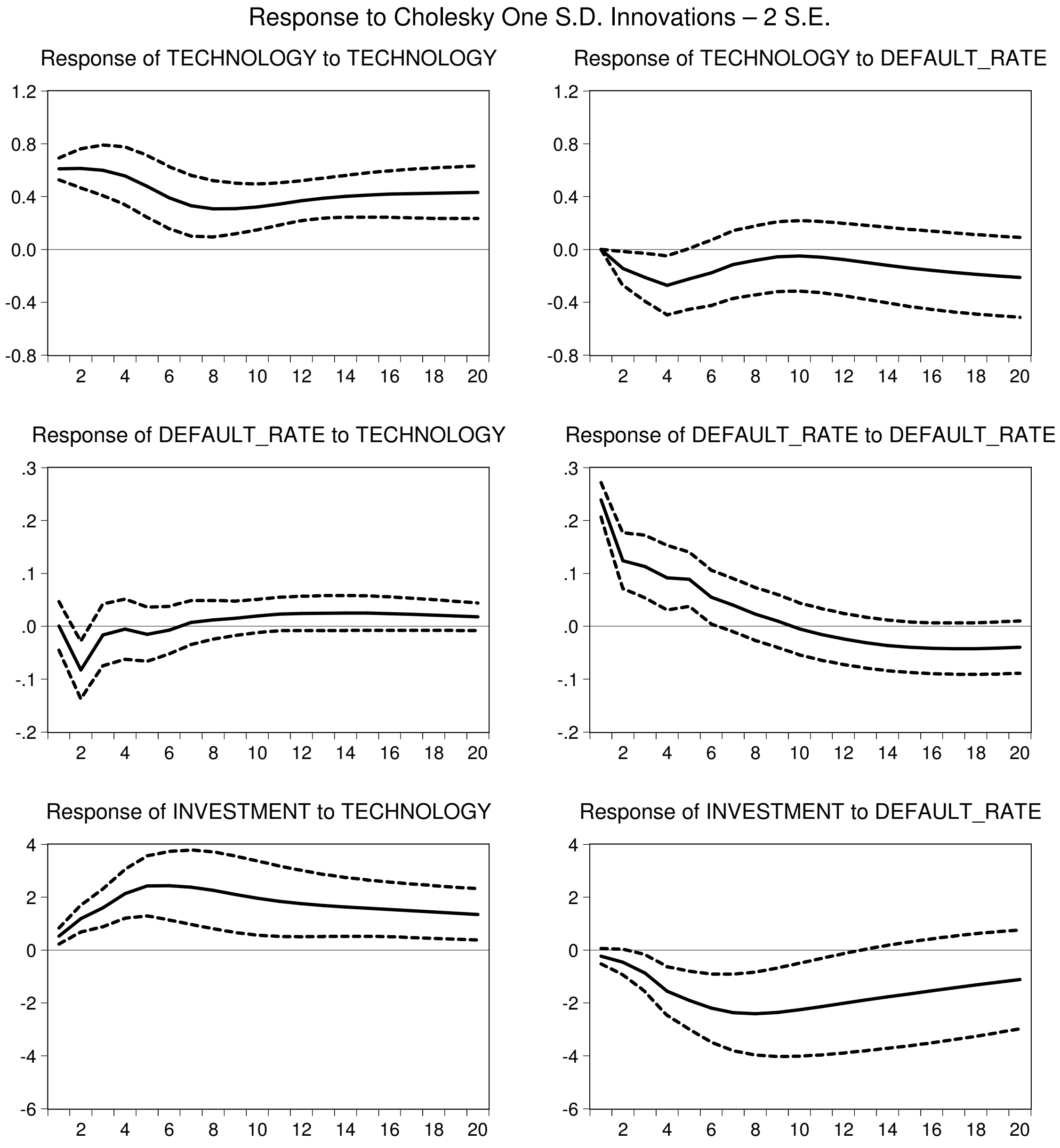

Figure 1: VAR impulse response function to shocks identified with Cholesky decomposition. Technology and investment are in log-levels. All variables are multiplied by 100.

VAR impulse response to a credit spread shock

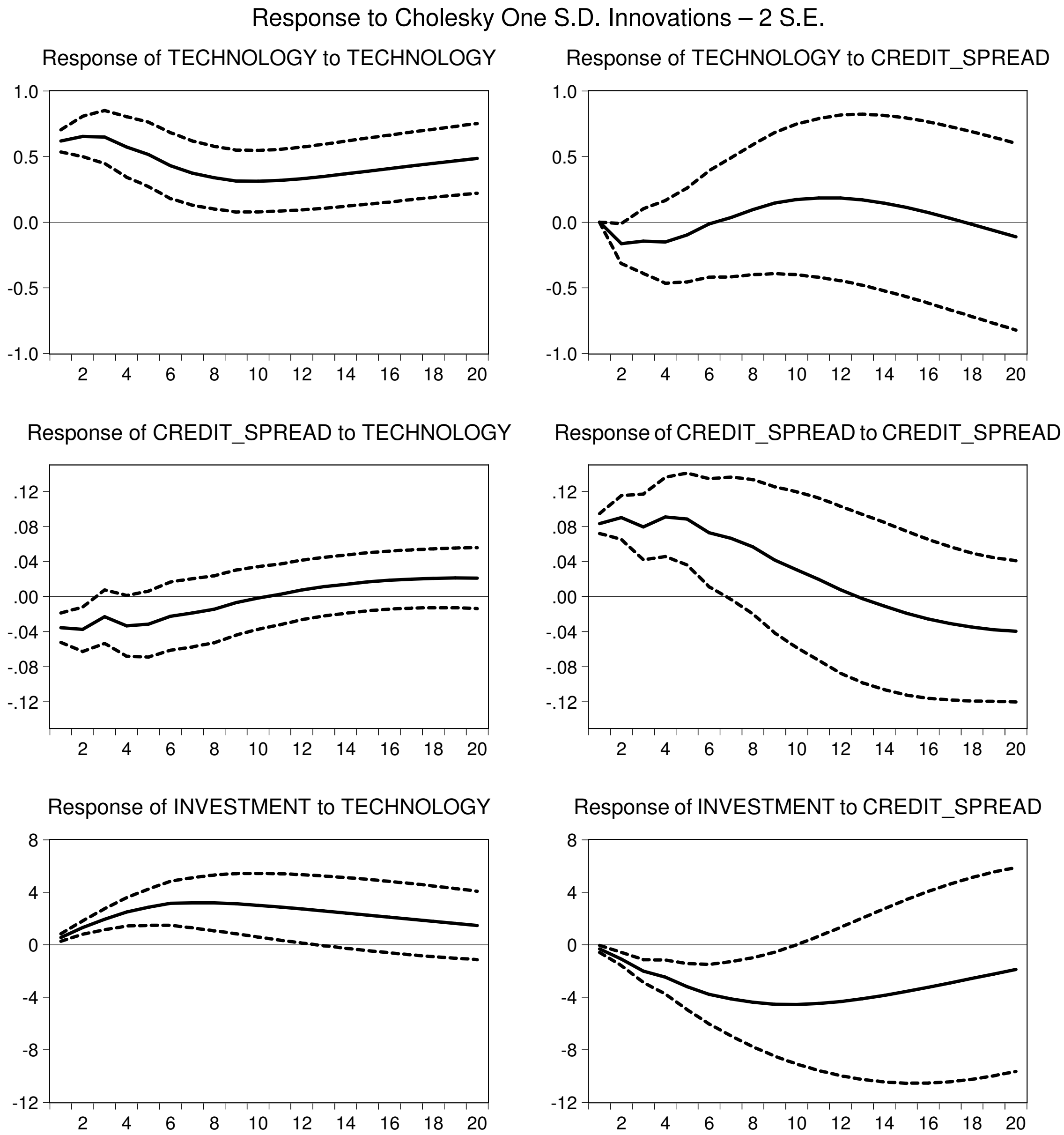

Figure 2: VAR impulse response function to shocks identified with Cholesky decomposition. Technology and investment are in log-levels. All variables are multiplied by 100.

Response to technology shocks

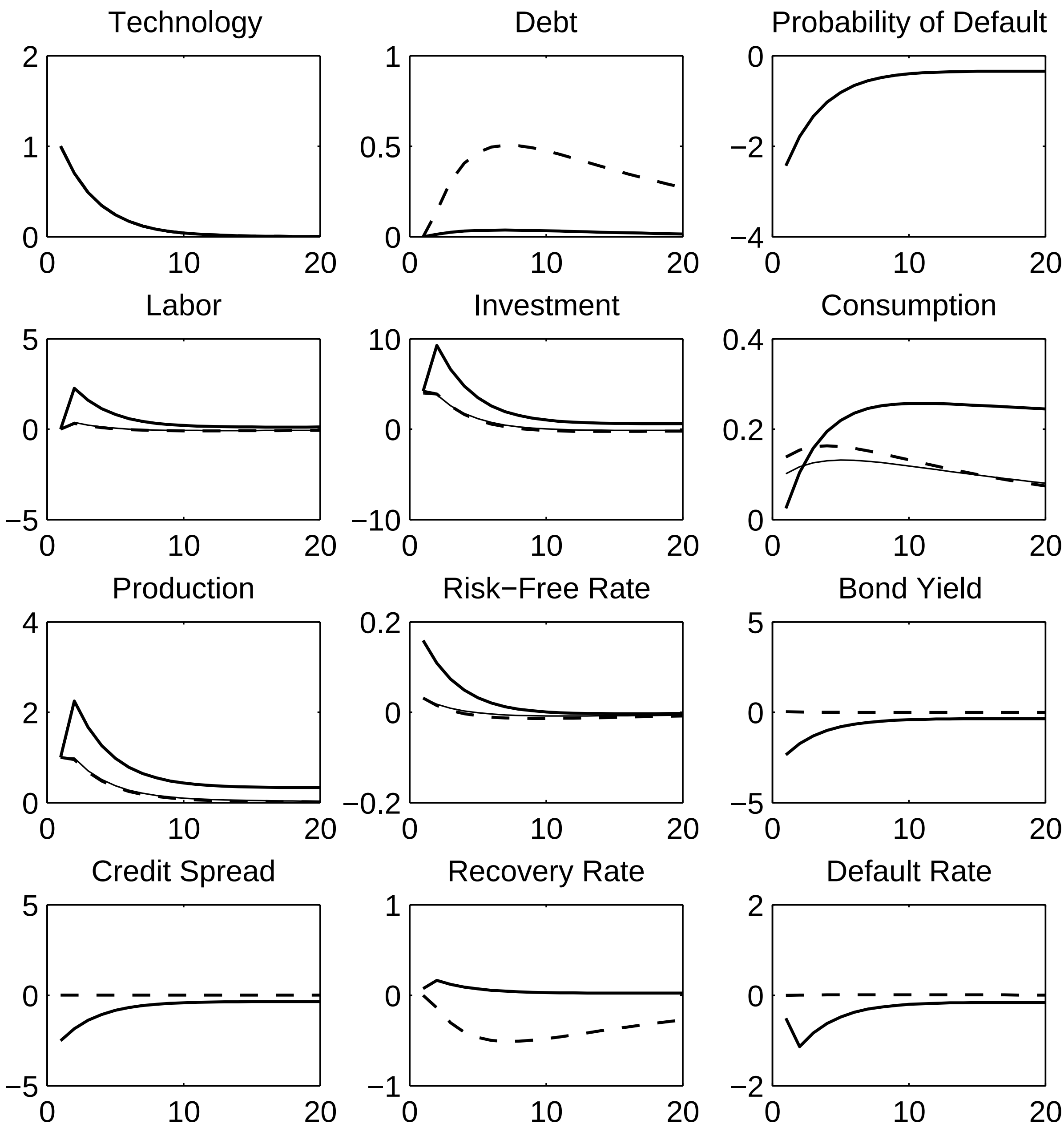

Figure 3: Model response to an expansionary technology shock. The thick solid, thin solid and dashed lines respectively refer to our debt overhang model, a corresponding model without any financial friction, and a model with a monitoring costs financial friction.

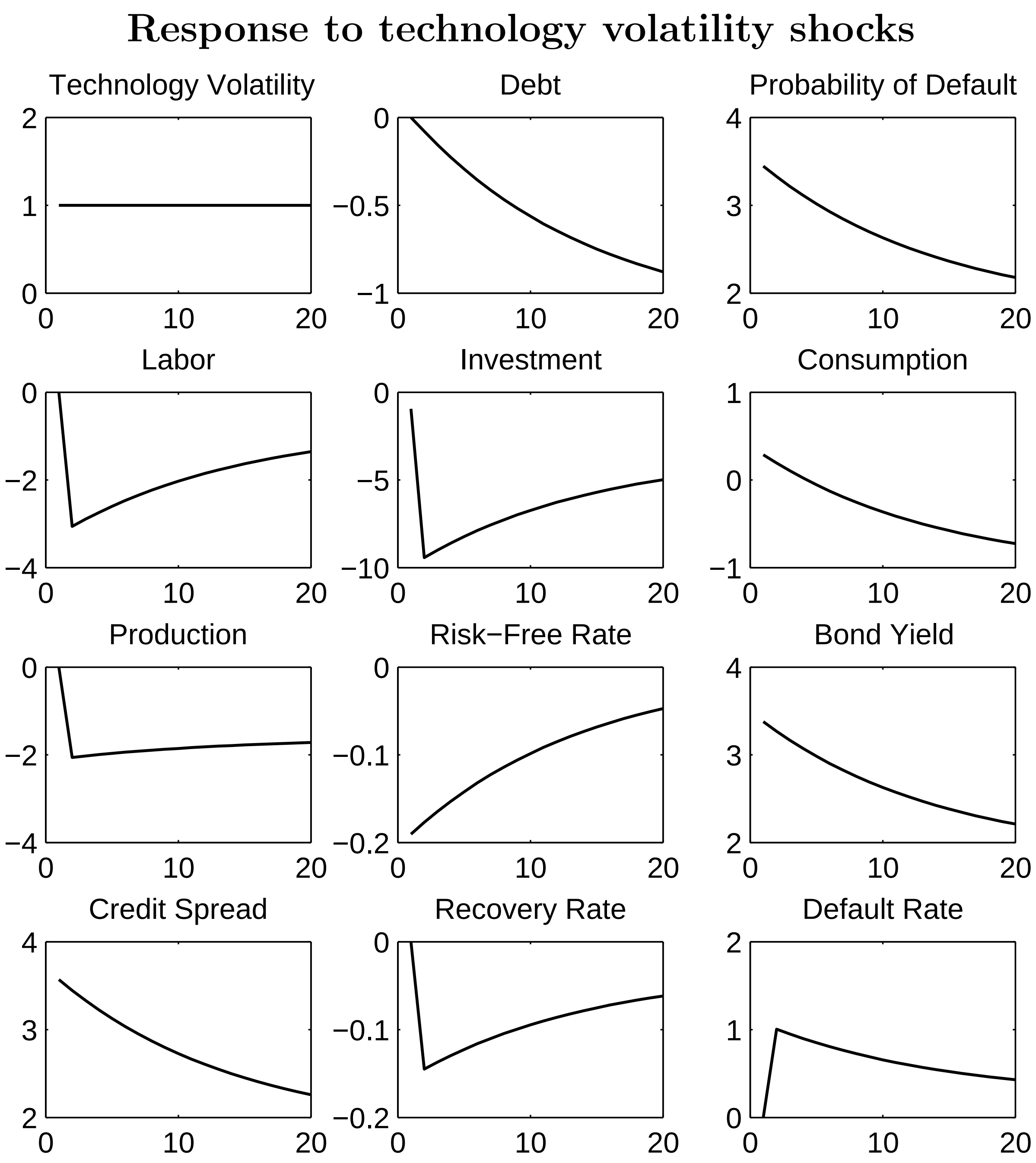

Figure 4: Model response to a positive shock to the volatility of technology. The thick solid line refers to our debt overhang model.

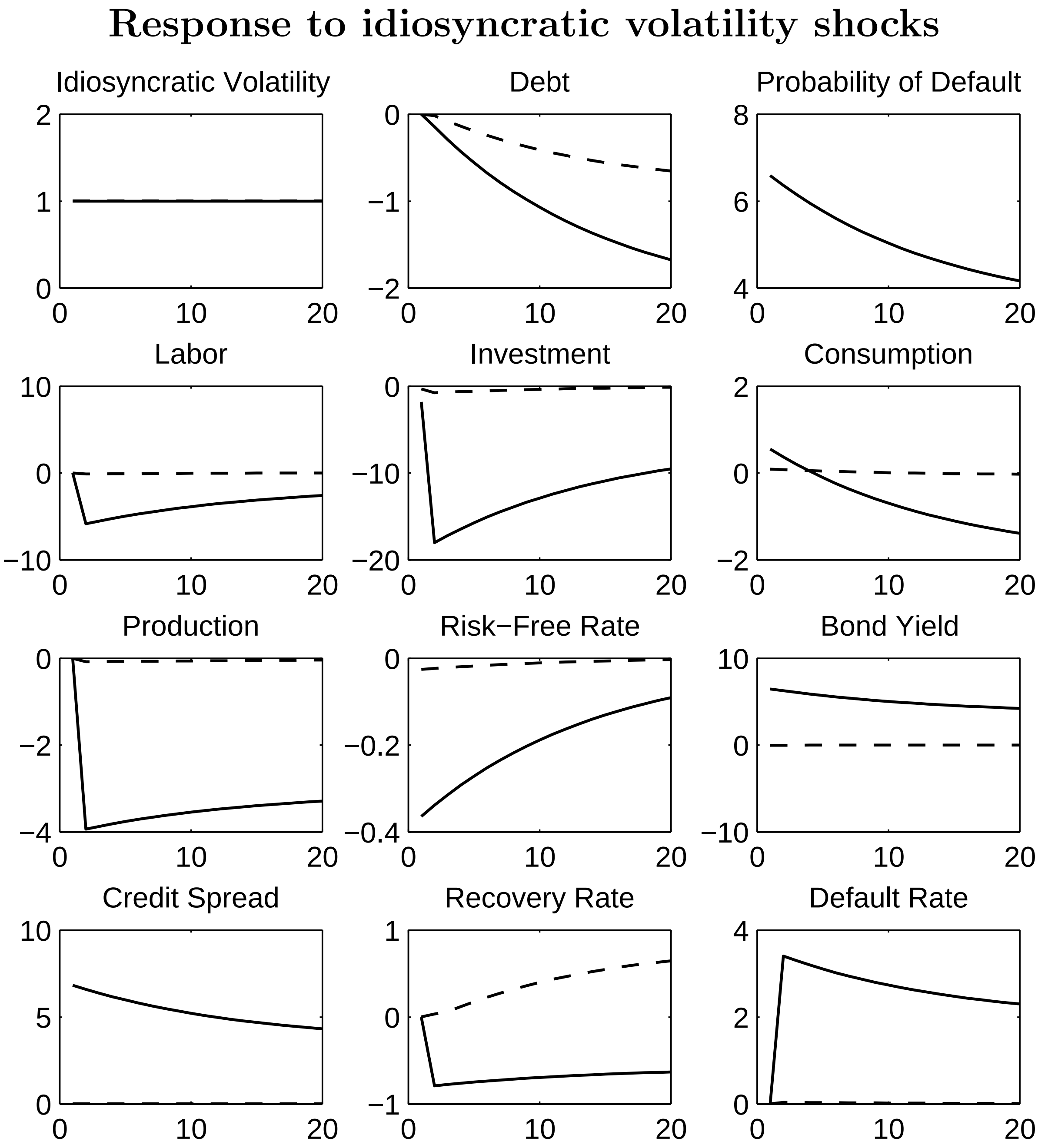

Figure 5: Model response to a positive shock to the volatility of idiosyncratic productivity. The thick solid and dashed lines respectively refer to our debt overhang model, and a model with a monitoring costs financial friction.

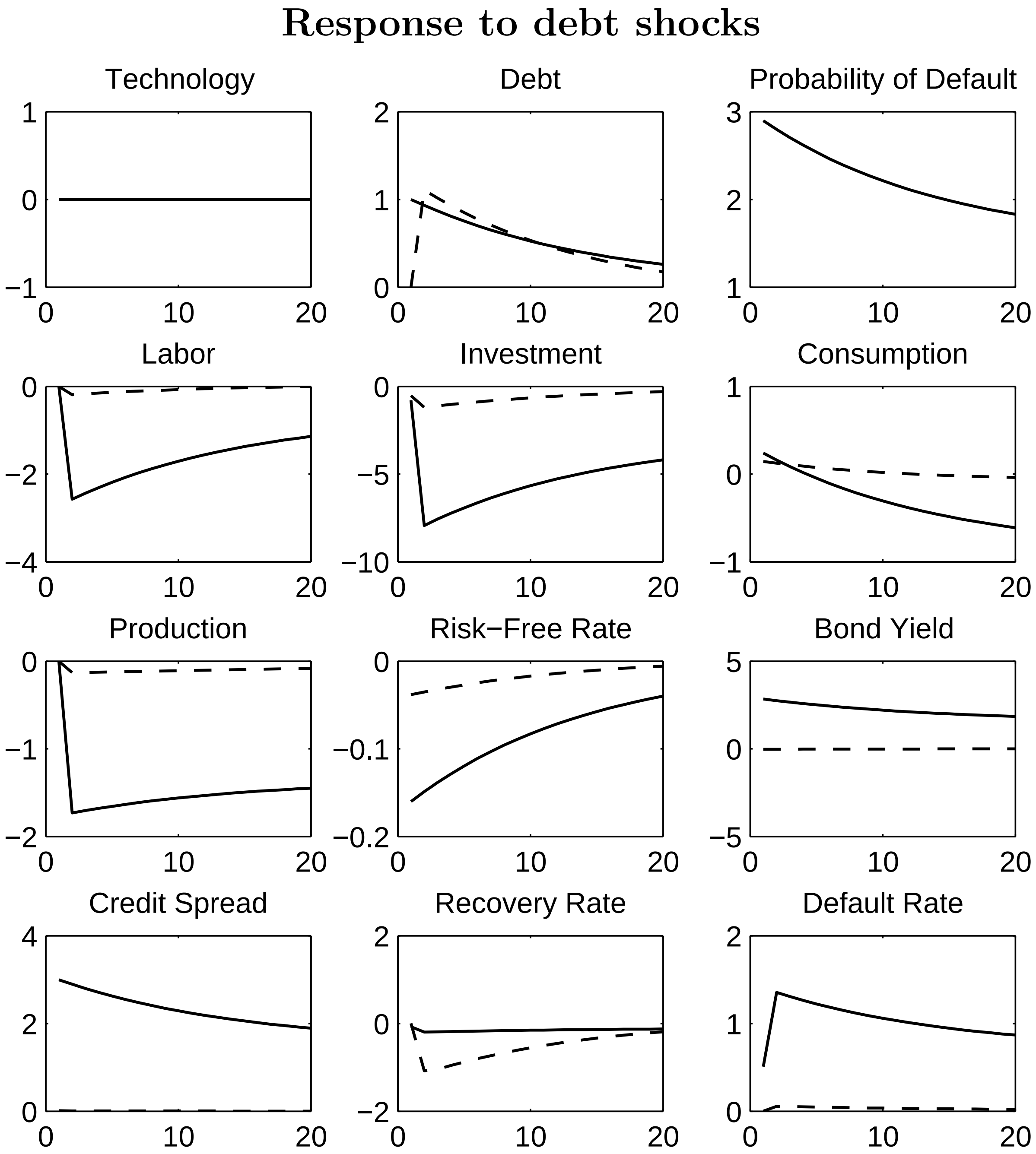

Figure 6: Model response to a positive debt shock. The thick solid and dashed lines respectively refer to our debt overhang model, and a model with a monitoring costs financial friction.

Default rate and macro variables

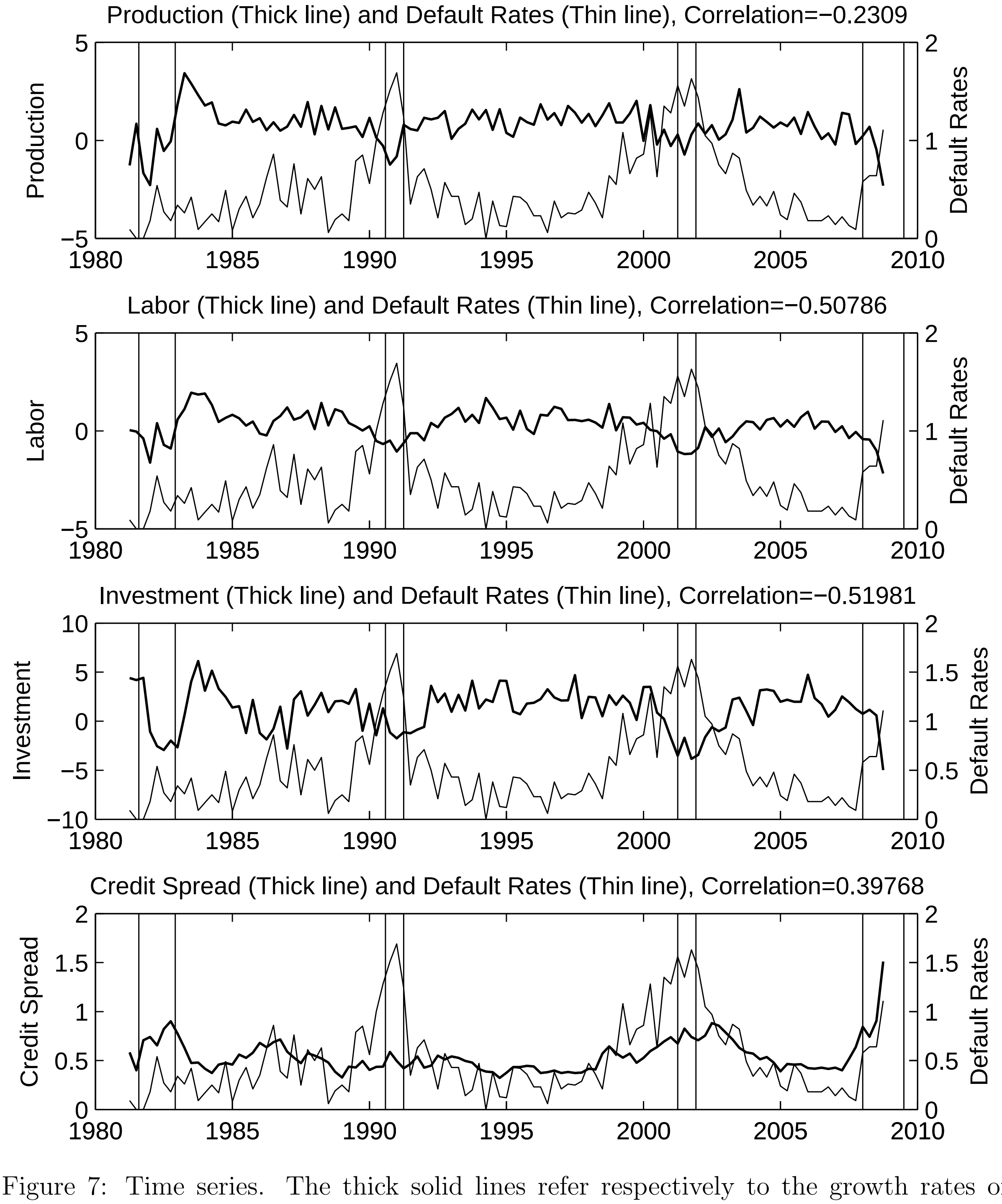

Figure 7: Time series. The thick solid lines refer respectively to the growth rates of production, labor, and investment, and to the level of the credit spread. The thin solid line refers to the default rate. The vertical bars indicate the beginning and end of NBER recessions.

Credit spread and macro variables

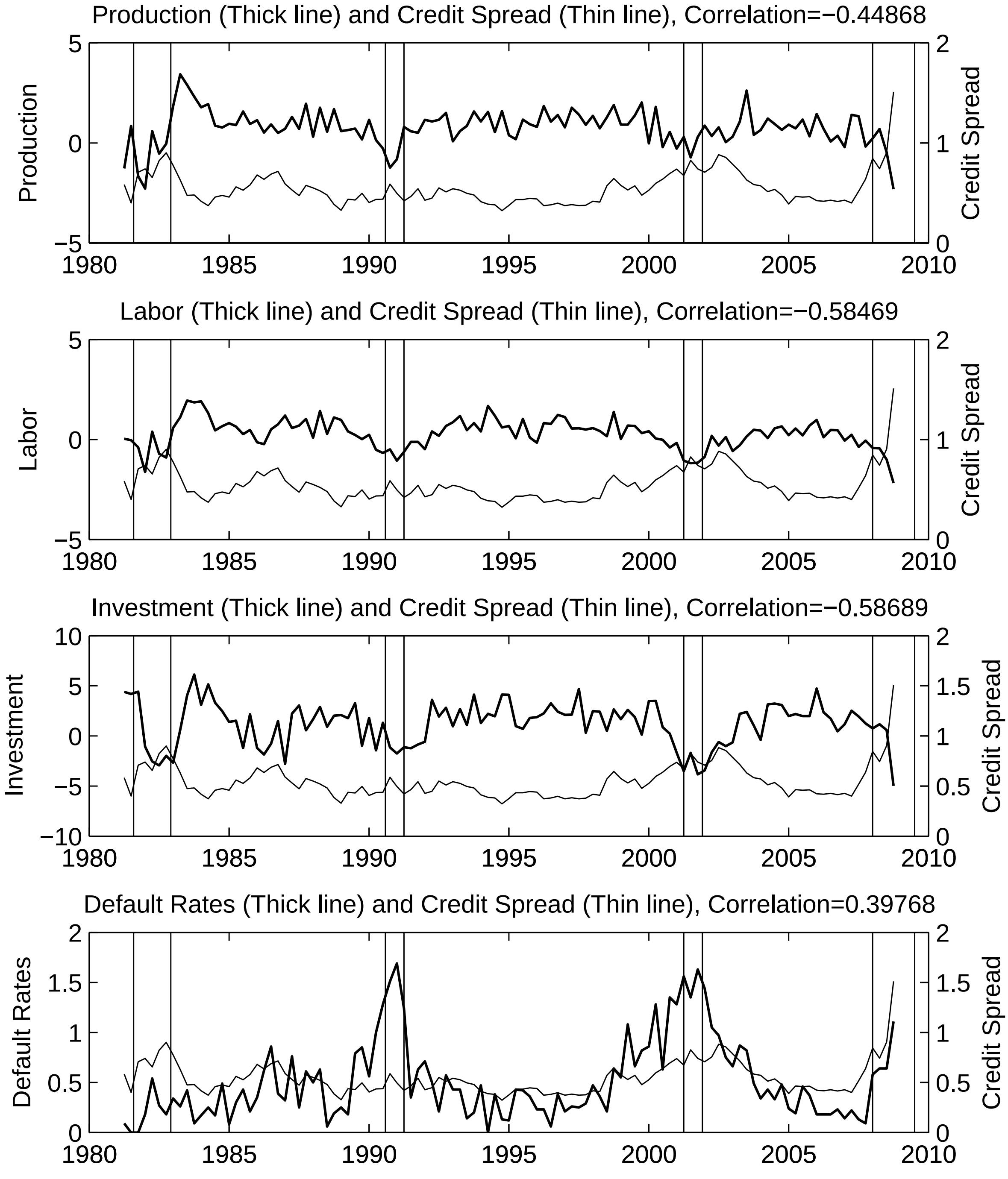

Figure 8: Time series. The thick solid lines refer respectively to the growth rates of production, labor, and investment, and to the level of the default rate. The thin solid line refers to the credit spread. The vertical bars indicate the beginning and end of NBER recessions.

Fiscal multipliers

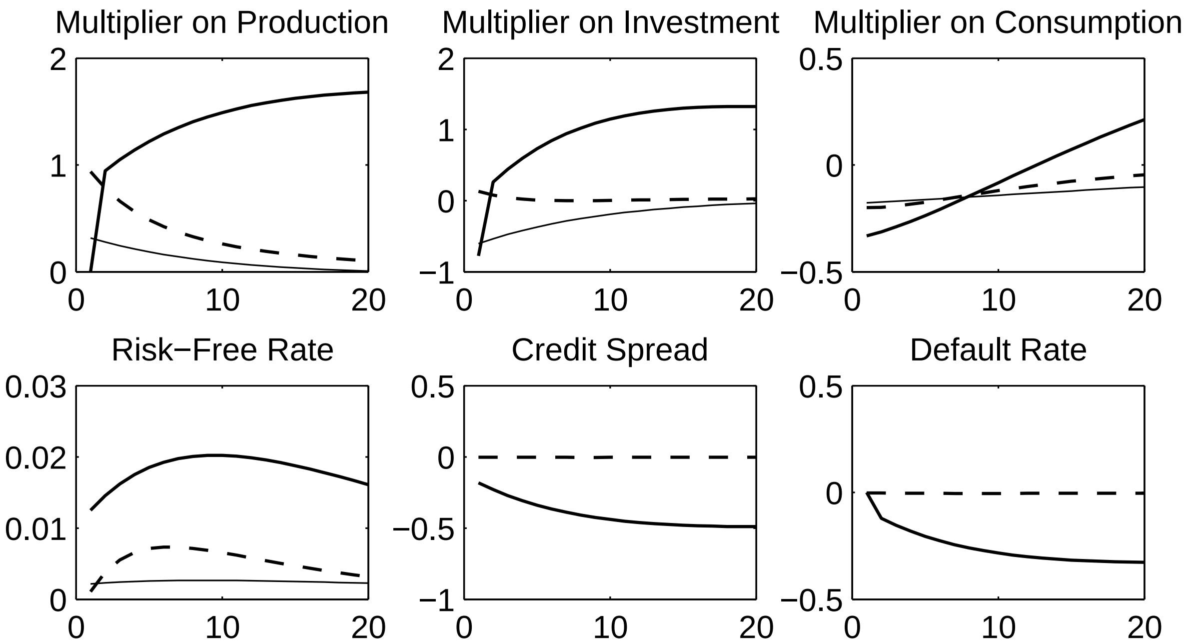

Figure 9: Model fiscal multipliers of a shock to a government purchases process with a first-order autoregressive coefficient $\rho_g = 0.90$. The fiscal multiplier at time $t + j$ on the variable X is defined as $\Delta X_{t+j}/\Delta G_t$ where G is government spending. The second row shows impulse responses to a unitary shock. The thick solid, thin solid and dashed lines respectively refer to our debt overhang model, a standard real business cycle model, and the Bernanke, Gertler and Gilchrist (1999) model with the monitoring costs financial friction.