# SIGNIFICANT FIGURES

NUMERICAL DATA that are used to record observations or solve problems are seldom exact. The numbers are generally rounded off and, consequently, are estimates of some true value, and the mathematical operations or assumptions involved in the calculations commonly are approximations. In numerical computations, no more than the necessary number of digits should be used; to report results with too many or too few digits may be misleading. To avoid surplus digits, numbers should be rounded off at the point where the figures cease to have real meaning. Conversely, the number of significant figures may be unnecessarily reduced by choosing the less meaningful of several possible methods of calculation. Careful consideration, therefore, should be given to the significant digits and arithmetic involved in each measurement.

The number of significant figures resulting from any calculation involving simple arithmetic operations on measured quantities should not exceed the number of significant figures of the least precise number entering into the calculation. In the calculation itself, one more significant figure may be retained in the more precise numbers than exist in the least precise number.

The digits 1 through 9 are always significant, regardless of their position in a number. The digit 0 is significant when it is between other significant digits but not when it is at the right or left of the number that locates the decimal point, because that location may be changed by changing dimensions—for example, grams to milligrams. At the right of a number, 0 is significant if it indicates actual precision, but not if it is used only to complete a rounded number. For example, the number 0.0046 has only two significant figures, but 4,103 has four significant figures. In a number such as 53,200 we do not know the number of significant figures unless we know whether the zeros at the end were actually determined experimentally. To remove this ambiguity the number may be written as $5.3200 \times 10^4$ if the zeros are significant, and $5.32 \times 10^4$ if they are not. Use of five significant figures indicates that the author knows that the two zeros have real meaning. Nonsignificant zeros should never be used at the right of the decimal part of the number. In tabulating data, an alternative is to list only the significant figures and absorb the superfluous zeros in the general heading, as follows:

| Specimen | Temperature (°C × $10^3$) |
|---|---|
| A | 1.4 |
| B | 2.0 |
| C | 1.8 |
| D | 1.2 |

## ROUNDING OFF NUMBERS

A consistent procedure should be followed in rounding off numbers to *n* significant figures. All digits to the right of the *n*th digit should be discarded, as illustrated in the following six examples of rounded numbers, each of which has only three significant figures:

| Example | Original number | Rounded number |
|---|---|---|
| 1 | 0.32891 | 0.329 |
| 2 | 47,543 | 47,500 |
| 3 | 11.65 | 11.6 |
| 4 | 22.75 | 22.8 |
| 5 | 18.05 | 18.0 |
| 6 | 18.051 | 18.1 |

If the first of the discarded digits is greater than 5, add 1 to the *n*th digit (example 1). If the first of the discarded digits is less than 5, leave the *n*th digit unchanged (example 2). If the first of the discarded digits is 5 and all the following digits are zero, round off to the nearest even number (examples 3–5). If the 5 is followed by any of the digits 1 through 9, add 1 to the *n*th digit (example 6).

If the difference between successive numbers is more important than the total or average, it may be desirable to round consistently in one direction all numbers in which the first dropped digit is followed by zeros only, instead of rounding to the nearest even number.

In presenting numerical data, give only those digits that convey actual information. The last digit should represent the uncertainty in the data. Unless stated otherwise, it is generally assumed that the last significant figure is uncertain by one unit. To illustrate, if the length of a drill core is given as 3.12 cm, true length is implied to be $3.12 \pm 0.01$ and is thus somewhere between 3.11 and 3.13 cm. If the uncertainty in the last figure is appreciably different from one unit, attention can be called to the uncertainty by expressing the measurement at $3.12 \pm 0.03$ cm.

Special problems arise in converting English-to-metric or metric-to-English units. These problems can be avoided if (1) the precision of the original measurement is stated and (2) the author adheres strictly to the concept of significant figures. Most readers will assume that the first-listed number represents the system used for the actual measurement; hence they should not be confused by reconverting the second-listed number. Thus, the measurement "500 ft (152 m)" implies a precision of "500 ± 1 ft," not the "500 ± 3 ft" that would result from converting the 152 m back to feet. (The Survey no longer encourages dual measurements in its formal reports.)

## Absolute and Relative Errors

The absolute error of a number or measurement generally is defined as the numerical difference between the true value and the approximate value as given by the number or measurement. The relative error can be defined as the absolute error divided by the true value of the quantity. The true index of a measurement is expressed by the relative error, which in turn is indicated by the number of significant figures required to express the measurement. For this reason, the number of significant figures is vitally important in reporting measured or computed quantities.

The following example[1] illustrates the difference between absolute and relative errors: Assume that the length of a carefully prepared core of rock 2 inches long has been measured to the nearest thousandth of an inch and that a mile of railroad track has been measured to the nearest foot. The absolute errors are 0.0005 inches for the core and 0.5 ft for the track, whereas the relative errors are, respectively,

$$\frac{0.0005}{2} = \frac{1}{4{,}000} \text{ and } \frac{0.5}{5{,}280} = \frac{1}{10{,}560}$$

The track measurement is relatively better.

## Arithmetic Operations

A simple arithmetic operation, such as addition or multiplication, may affect the number of significant figures in the result. In addition and subtraction, the placement of the decimal point is important in the retention of significant figures. The general rule can be illustrated thus: Suppose you want to add the numbers 120.632, 8.14, 980.3, and 1,401.0023, each number being correct to its last figure. Inasmuch as the third number listed is correct only to the first decimal place, it is meaningless to retain more than two decimal places in the other numbers. Consequently,

$$\begin{array}{r} 120.63 \\ 8.14 \\ 980.3 \\ \underline{1{,}401.00} \\ 2{,}510.07 \end{array}$$

and the result is rounded to 2,510.1, or to five significant figures. Note that only one decimal place is retained in the sum and that the number of significant figures in the sum is less than the number of significant figures in two of the original numbers. The procedure of rounding off applies to measurements but not to whole numbers that are correct to the last digit. If the whole numbers in the example given above applied to individual persons or digits and represented counts that were correct to the last digit, they would be shown as:

$$\begin{array}{r} 120 \\ 8 \\ 980 \\ \underline{1{,}401} \\ 2{,}509 \end{array}$$

and the total would not be rounded off.

If small numbers are added to (or subtracted from) large numbers of limited accuracy, the total should retain no more significant figures than are justified by the accuracy of the larger numbers. For example, in adding 356,000 (good to only three figures) and 1,420 (good also to three figures), the sum is 357,000, not 357,420. The figures that are dropped are within the limits of error of the larger number and are meaningless in the sum. By the same reasoning, the addition of a very large group of numbers of limited accuracy cannot produce a total more accurate than the respective items. Therefore, if several hundred objects have been weighed individually with an accuracy of three figures, the total weight of all the objects should be rounded off to three significant figures.

In subtraction, the number of significant figures in the difference may be considerably reduced if the numbers are close to each other in numerical value. Suppose 0.1189 is subtracted from 0.1204. The difference is 0.0015, which contains only two significant figures.

In the multiplication or division of two or more approximate numbers of different accuracies, the more accurate numbers should be rounded off so as to contain one more significant figure than the least accurate number. In this procedure, the error of the product is due almost entirely to the error of the least accurate number. Therefore, the final result should be given to as many significant figures as are contained in the least accurate number, and no more. As illustrations, two calculations may be given:

$$103.24 \times 0.0081 = 103 \times 0.0081 = 0.83$$

and

$$\frac{56.3}{2.23612} = \frac{56.3}{2.236} = 25.2.$$

In computing with logarithms, no more decimals need be retained in the mantissa of the logarithm than the number of significant figures in the numerical factors that enter the computation. Thus, log 352.3 = 2.5469. It is sometimes easier to use logarithms directly from the tables without rounding off, but the results of computation should never be presented as being more accurate than the original data.

## Misusing Significant Figures

A result cannot be more accurate than the data used to obtain it. Thus the number of significant figures of the result cannot be greater than is justified by the least accurate number entering into the calculation. Despite this rule, many published data contain incorrect significant figures.

Many estimates of ore reserves are carried to as many as six significant figures—for example, 123,415 tons. Such a number gives a spurious impression of accuracy, if not a suspicion that the estimator is incompetent. To see the fallacy, just consider how reserve tonnages are calculated. The estimated volume, which is usually determined from drill-hole information, is multiplied by the density of the ore. At best, the volume can be determined accurately to only three significant figures, and probably to no more than two. The density of the ore may be correct to two significant figures. Consequently, the calculation of the estimated tonnage can produce no more than two significant figures. The figure in the foregoing example should be given as 120,000 tons.

Again, the depth to a geologic structure, as computed from gravity determinations, might be given as 13,016 ft. If, as is usual, this figure was calculated on the assumption of a density contrast for the ore body good to only two significant figures, the figure should be reported as "about 13,000 ft."

Reports on results of chemical analyses provide yet another illustration. Typically the results might be reported as 1,061.39 for $SO_4$, 880.90 for Na, and 205.62 for Cl, all in milligrams per liter (mg/L). Each of these numbers contains five or six significant figures, whereas the analytical procedures used justified only two or three. Moreover, concentrations of more than 1,000 mg/L are customarily reported to only three significant figures; for concentrations between 10 and 1,000 mg/L only whole numbers are reported. It follows that the above results should be listed as 1,060, 881, and 206 mg/L.

Certain other field measurements, some of them crude, are improperly reported to a greater number of significant figures than would be justified by even the most refined laboratory methods. In these, as well as in laboratory measurements, care should be taken to use only as many significant figures as are justified.

Some published stratigraphic measurements[1] indicate unrealistic accuracies. The calculated thickness of a sedimentary formation of Tertiary age might be given as 14,633 ft, but if the top and bottom are as ill defined as most Tertiary units, a more acceptable figure would be "about 15,000 ft." Calculations of the thickness of such rock units based on measurements of strike and dip along a measured traverse inevitably contain many uncertainties (exact amount and direction of dip, magnetic declination, nature of exposure, and others), which are almost impossible to evaluate and which limit the acceptable value to a few significant figures. Mining geologists have been known to pace the length of an adit but to use a steel tape to measure the last few feet and to record the total distance in fractions of a foot. So too have stratigraphers been known to measure the poorly exposed parts of a section by hand leveling but to measure cliff-forming beds by tape and then to construct a columnar section in which some units, and the total thickness, are reported in inches or even in fractions of an inch.

[1]STA 7 was in press before the Survey adopted the policy of using metric units in all its formal reports.