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On November 14–15, 2008, the Federal Reserve Bank of Cleveland hosted a conference on “Liquidity in Frictional Asset Markets.” In this paper we review the literature on asset markets with trading frictions in both finance and monetary theory using a simple search-theoretic model, and we discuss the papers presented at the conference in the context of this literature. We will show the diversity of topics covered in this literature, e.g., the dynamics of housing and credit markets, the functioning of payment systems, optimal monetary policy and the cost of inflation, the role of banks, and the effect of informational frictions on asset trading.

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“Liquidity is the object of bilateral search. In a bilateral search, buyers search for sellers, and sellers search for buyers. When a buyer finds a seller who will trade at mutually acceptable terms, the buyer has found liquidity. Likewise, when a seller finds a buyer who will trade at mutually acceptable terms, the seller has found liquidity.”

Larry Harris, *Trading and Exchanges: Market Microstructure for Practitioners*, 2003.

On November 14-15, 2008, the Federal Reserve Bank of Cleveland hosted a conference on “Liquidity in frictional asset markets.” The objective of this conference was to highlight a line of research which has analyzed trading frictions in asset markets using search-and-matching models, thereby providing an explicit and rigorous meaning to different notions of liquidity.

Traditionally, asset market modeling has been the realm of the competitive frictionless paradigm, where the matching of buyers to sellers is regarded as an instantaneous and costless process. A new body of research, pioneered by Kiyotaki and Wright (1991, 1993) in monetary theory and Duffie, Gârleanu, and Pedersen (2005, 2007) in finance, adopts the view that trading frictions and the mechanics of trade are important for understanding the functioning of asset markets. These trading frictions are obvious in the housing market (e.g., Caplin and Leahy, 2010, this issue): It takes weeks or months to buy or sell a house. Search frictions are also relevant in secondary markets for physical goods (Gavazza, 2010), and in the markets for financial securities, e.g., fixed-income securities, which are typically traded in a decentralized manner in over-the-counter (OTC) markets (Harris and Piowar, 2006; Edwards, Harris and Piowar, 2007). Similarly, credit relationships are often formed in markets where borrowers and lenders search for each other and negotiate contracts bilaterally. Examples are the market for overnight loans of federal funds (Ashcraft and Duffie, 2007; Ashcraft, McAndrews, and Skeie, 2010, this issue) and markets for credit derivative instruments. Finally, a commonly held view is that during financial disruptions, even very liquid asset markets resemble search markets, as investors face significant delays in obtaining acceptable quotes and trading.

Search-and-matching models provide new insights for the different dimensions of liquidity. They deliver a role for trading volume — which does not arise in pure pricing models — they offer an explicit notion of trading delays, and they generate bid-ask spreads even in the absence of informational asymmetries or dealers’ inventories. The search-theoretic approach emphasizes how the structure of the market — such as how agents meet and are matched and what the pricing mechanism is — matter for liquidity and efficiency. The search-and-matching model can be used to discuss the advantages of OTC versus exchange trading,

and to understand frictions that are at the center of the policy debate. It can also be used for a normative analysis of the provision of liquidity by market participants, financial intermediaries, and policymakers.

An explicit description of the frictions that plague some markets is also crucial to explain the role that assets play to facilitate exchange by serving as means of payment or collateral. A case in point is fiat money, an intrinsically useless object, that has a positive value because of its role as a medium of exchange. The same approach that is used in monetary theory to explain the liquidity value of fiat money can be applied to other assets, and it can help us to understand the liquidity structure of asset yields, how monetary policy affect asset prices, and the optimal provision of liquidity.

1 A simple framework

In order to present an overview of this literature, and the papers contained in this volume, we construct a simple framework to analyze frictions in asset markets. The model specification is a variation of Vayanos and Wang (2007), which itself builds upon Diamond (1982) and the follow-up literature.¹ We consider a continuous-time economy with a fixed supply, S , of trees that yield a constant flow of fruits, d . At each point in time, there is a constant flow, F , of potential buyers entering the economy. Buyers would like to hold one tree in order to consume its fruits. For tractability, trees are indivisible and agents cannot hold more than one tree. After a period of time of random length, exponentially distributed with mean $1/\kappa$, an agent no longer wants to hold a tree: If she holds one, she incurs a flow cost, γ . One common interpretation of this feature is that the agent receives a liquidity shock that makes her want to sell her asset. We will be more explicit about this liquidity shock in Section 5 when we formalize the role of assets as means of payment. Alternatively, one could think of a change in agents' hedging needs requiring some portfolio rebalancing, as in Duffie, Gârleanu, and Pedersen (2007) and Vayanos and Weill (2008). What matters for our purpose is that this shock generates a flow of agents who want to sell their assets. Once the asset has been sold, the agent leaves the economy permanently.² Trading frictions arise because finding a trading partner takes time. With some Poisson arrival rate, λ_b , buyers find a seller and acquire the asset. Similarly, sellers find buyers

¹The methodology to formalize markets with bilateral matching and bargaining is reviewed in Osborne and Rubinstein (1990). Our model is related to the monetary models of Kiyotaki and Wright (1991, 1993), Shi (1995), and Trejos and Wright (1995), the labor market model of Pissarides (2000), the housing model of Wheaton (1990), and the models of asset markets of Vayanos and Weill (2008), Afonso (2008), and Kim (2008, 2009).

²Instead of having agents exiting permanently, we could assume that agents who exit the market can become buyers again in the future. For such a formulation see, e.g., Duffie, Gârleanu, and Pedersen (2005).

with Poisson arrival rate, λ_s . We will assume that agents are risk-neutral and discount future utility at rate $r > 0$. The flows of agents in the economy are represented in Figure 1.

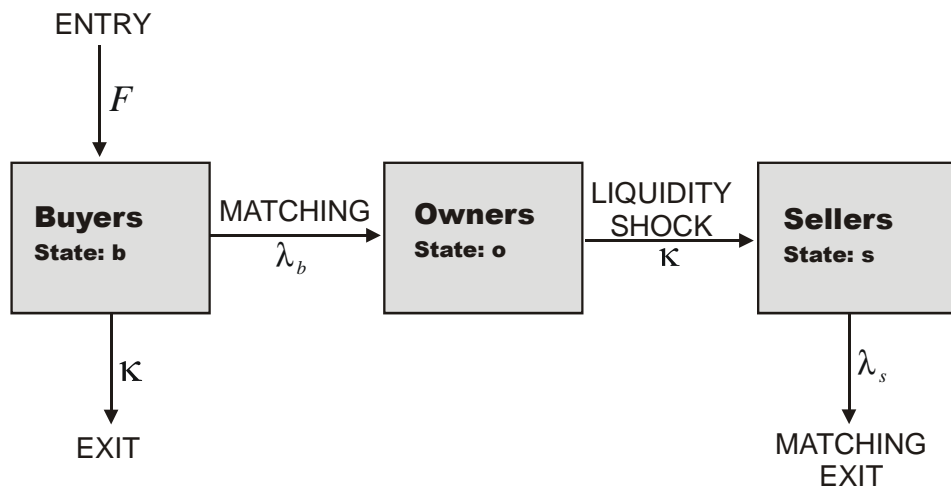


Figure 1: Flows

2 Matching frictions

The first building block of the model is the description of the process through which buyers and sellers are brought together. In Harris’s (2003, p.398) words:

“We can characterize the expected outcome of a search problem as a production function that explains how the inputs to the search are related to the expected products of the search.”

This approach is similar to the one used by Pissarides (2000) to formalize search-matching activities in the labor market.

2.1 The matching function

Consider a market composed of μ_b buyers and μ_s sellers. Taking the search efforts of buyers and sellers as given, the inputs to the search problem are the numbers of buyers and sellers. The output is the number of matches that are formed. There are multiple reasons for why the matching process is not instantaneous. Buyers and sellers might be heterogenous in terms of their portfolio needs; it takes time for an agent to locate someone on the other side of the market; buyers and sellers might be asymmetrically informed about

the characteristics of the asset that is traded; the arrivals of buyers and sellers in a marketplace are not synchronized; there are technological constraints that prevent all orders from being matched instantly.

It is common to summarize these trading frictions using a reduced-form matching function, $\lambda M(\mu_b, \mu_s)$, which specifies the number of trading opportunities per unit of time. The parameter λ is a scaling variable, which determines the efficiency of the matching process. The matching function is continuous and increasing with respect to each of its arguments, and equal to 0 when one side of the market has no agent, i.e., when $\mu_b = 0$ or $\mu_s = 0$. In labor-market applications, empirical evidence has convinced many researchers that $M(\mu_b, \mu_s)$ has constant returns to scale (see Petrongolo and Pissarides, 2001). For housing and financial markets, however, as of now there is no compelling empirical evidence that the constant-returns assumption should (or should not) apply.³

In some markets, observable trading mechanics are naturally represented by more specific matching technologies.⁴ For instance, in the housing market, search frictions are well described by a stock-flow matching process (Taylor, 1995; Coles and Smith, 1998; Ebrahimi and Shimer, 2006). In the context of the housing market, a stock-flow matching process assumes that a new buyer first inspects all the houses that are on sale. If he finds a house he likes, he is instantly matched. Otherwise, he has to enter the stock of unmatched buyers and wait until new vacant houses are put on the market. A similar matching process arises in limit order books (Rosu, 2009; Biais, Hombert, and Weill, 2010). For instance, a new limit order to buy can be immediately executed if its limit price is high enough to be matched with a limit-sell order stocked in the book. Otherwise, the new limit order is stocked in the book until some other trader places a low enough limit-sell order.

Trading frictions can also be explained by informational asymmetries between buyers and sellers regarding the quality of assets.⁵ There can be assets of different qualities, and the holder of the asset might be better informed about the characteristics of his assets (e.g., its future cash flows). For instance, according to Plantin (2009), securitized pools of loans are sold to institutional investors who receive a flow of future privileged information about their cash flows. Hopenhayn and Werner (1996) and Lester, Postlewaite, and Wright

³To the contrary, Weill (2008) shows that, in financial markets, the sign of the cross-sectional relationship between various liquidity proxies and asset tradeable shares is consistent with *increasing* returns in matching, and inconsistent with constant or decreasing returns.

⁴For a survey of the microfoundations of the matching function in the context of the labor market, see Petrongolo and Pissarides (2001).

⁵Similarly, a large part of the market microstructure literature resorts to asymmetries of information to endogenize transaction costs in financial markets. See, e.g., Kyle (1985) and Glosten and Milgrom (1985).

(2010, this issue) introduce private information problems about the quality of assets into search models and show that there are circumstances under which a buyer will refuse to trade with a seller who holds an asset of unknown quality.⁶ In Rocheteau (2008) and Li and Rocheteau (2009) private information problems affect the sizes of the trades.

Matching frictions are also generated by the heterogeneity of buyers and sellers. Agents might value the asset differently depending on their liquidity and hedging needs, or they can have different rates of time preference or different costs of participating in the market. While in our simple model we do not take this heterogeneity explicitly into account, it is certainly an important aspect of the search-and-matching frictions that characterize some asset markets and a key determinant of the supply and demand for liquidity. Such two-sided heterogeneity has been shown to matter for the extent to which the market is segmented (e.g., Burdett and Coles, 1997; Shimer and Smith, 2000; Jacquet and Tan, 2007).

Finally, trading frictions can result from technological constraints on the ability of intermediaries to process large volume of orders. For instance, electronic trading platforms have capacity constraints that can generate bottlenecks and trading delays. Weill (2009) formalizes this view in a competitive financial market, where marketmakers face a capacity constraint on their number of trades per unit of time with outside investors.⁷

2.2 Time in the market

Let us go back to our model with the reduced-form matching function, $M(\mu_b, \mu_s)$. It can be used to construct the expected times that buyers and sellers spend in the market. Time is taken to be continuous, and the time spent in the market for buyers and sellers is characterized by Poisson processes with respective arrival rates:

$$\begin{aligned}\lambda_b &= \frac{\lambda M(\mu_b, \mu_s)}{\mu_b} \\ \lambda_s &= \frac{\lambda M(\mu_b, \mu_s)}{\mu_s}.\end{aligned}$$

Assuming for simplicity that the matching function has constant returns to scale, the matching rates depend only on the ratio of buyers to sellers $\theta = \mu_b/\mu_s$: $\lambda_s = \lambda M(\theta, 1) \equiv \lambda\alpha(\theta)$ and $\lambda_b = \lambda M(1, 1/\theta) = \lambda M(\theta, 1)/\theta = \lambda\alpha(\theta)/\theta$. Note that the assumption that $M(0, \mu_s) = M(\mu_b, 0) = 0$ implies that $\alpha(0) = 0$ and $\alpha(\theta)/\theta \rightarrow 0$

⁶Williamson and Wright (1994) were the first to introduce a private information problem in a monetary search model. See Section 5.

⁷See also Sattinger (2002) and Stevens (2007) for an application of queueing theory to describe search frictions.

as $\theta \rightarrow \infty$. Likewise, the assumption that $M(\mu_b, \mu_s)$ is increasing in both arguments means that $\alpha(\theta)$ is increasing, while $\alpha(\theta)/\theta$ is decreasing.⁸ If the matching technology exhibits increasing returns to scale then the matching rates depend not only on the composition of the market but also on its size.

The measures of buyers and sellers are endogenous. Let μ_o denote the measure of asset owners who are not seeking to sell their asset. The flows in and out of the buyer, owner, and seller states obey the ordinary differential equations:

$$\dot{\mu}_b = F - (\lambda_b + \kappa)\mu_b \quad (1)$$

$$\dot{\mu}_o = \lambda_b\mu_b - \kappa\mu_o \quad (2)$$

$$\dot{\mu}_s = \kappa\mu_o - \lambda_s\mu_s \quad (3)$$

According to the right side of (1), the measure of buyers increases with the flow of new agents entering the market, F , and decreases with the flow of buyers who are matched with sellers, $\lambda_b\mu_b$, and with the flow of buyers who receive a preference shock and exit the market, $\kappa\mu_b$. Equations (2) and (3) have a similar interpretation. In a steady state, $\mu_b = F/(\lambda_b + \kappa)$, $\mu_o = \frac{\lambda_b}{\lambda_b + \kappa} \frac{F}{\kappa}$, and $\mu_s = \frac{\lambda_b}{\lambda_b + \kappa} \frac{F}{\lambda_s}$. In equilibrium, the measure of agents holding an asset, $\mu_o + \mu_s$, must equal the fixed stock of the asset, S . If the matching function has constant returns to scale, this market-clearing condition can be rewritten as:

$$\left(\frac{F}{\kappa} - S\right) \lambda \frac{\alpha(\theta)}{\theta} + \frac{F}{\theta} = \kappa S, \quad (4)$$

and it determines a unique ratio of buyers per seller, θ . It can be checked that θ decreases with S and κ , and it increases with F . In the case where $\frac{F}{\kappa} > S$, θ also increases with λ . The volume of trades is $\lambda\alpha(\theta)\mu_s$, and the turnover of the asset is $\lambda\alpha(\theta)\mu_s/S$.

It is useful to consider the frictionless limiting economy as the efficiency of the matching process becomes infinite, $\lambda \rightarrow \infty$. Consider the case when $\frac{F}{\kappa} > S$: i.e., the total measure of traders who would be happy to own, F/κ , is greater than the asset supply, S . From (4), one easily verifies that as the matching efficiency goes to infinity, $\lambda \rightarrow \infty$, the number of buyers per seller goes to infinity, $\theta \rightarrow \infty$, and the matching rate of sellers goes to infinity as well, $\lambda_s \rightarrow \infty$. Therefore, in the frictionless limit, sellers can sell their asset instantly. Buyers, on the other hand, must wait until asset owners receive a preference shock and sell their

⁸ A popular matching function in both the monetary and finance literatures, the so-called Kiyotaki-Wright matching function, is one where the contact rate of an agent is proportional to the fraction of market participants that are on the other side of the market. In this case, $M(\mu_b, \mu_s) = \frac{\mu_b\mu_s}{\mu_b + \mu_s}$. Duffie and Sun (2007) and Stevens (2007) provide explicit microfoundations for this matching function.

assets. Precisely, λ_b tends to the solution to $(\frac{F}{\kappa} - S) \lambda_b = \kappa S$. Note that in the frictionless limit $F/\kappa - S$ is the measure of buyers, i.e., the measure, F/κ , of agents who would be happy to own an asset, minus the measure, S , of agents who actually own one. Thus, buyers' search times adjusts so that the flow of buyers who buy the asset is exactly equal to the flow of assets, κS , put on the market by owners who receive a preference shock.

3 Asset prices

The second building block of the model is the collection of values (lifetime maximum expected utility) of buyers, owners, and sellers at a steady state. These solve the following Hamilton–Jacobi–Bellman equations:

$$rV_b = \lambda_b(-p + V_o - V_b) - \kappa V_b \quad (5)$$

$$rV_o = d + \kappa(V_s - V_o) \quad (6)$$

$$rV_s = d - \gamma + \lambda_s(p - V_s) \quad (7)$$

According to the right side of (5), a buyer finds a seller with Poisson arrival rate λ_b , in which case she pays the price p to purchase the asset and makes a transition to the owner state. On the other hand, with Poisson arrival rate, κ , the buyer no longer wants to hold the asset and leaves the market. The other Bellman equations have similar interpretations.

From (5), the buyer's surplus from a trade is $-p + V_o - V_b$, while from (7), the seller's surplus is $p - V_s$. So the total match surplus is $\Sigma \equiv V_o - V_b - V_s$. It is the value of an owner minus the values of a buyer and a seller. Subtracting (5) and (7) from (6) and rearranging, we find that the total surplus of a match is

$$\Sigma = \frac{\gamma}{r + \kappa + \lambda_b \phi + \lambda_s(1 - \phi)}, \quad (8)$$

where ϕ and $1 - \phi$ are the respective fractions of the match surplus appropriated by the buyer and the seller, usually interpreted as their respective bargaining powers.⁹ From (8) the match surplus is proportional to the holding cost, γ . More precisely, it can be interpreted as the present value of the holding cost that would be incurred by the seller in case trade does not occur until either: the buyer loses her desire to

⁹Because of the bilateral monopoly problem between the buyer and the seller, there are many ways one could split the match surplus. In over-the-counter markets, prices are usually determined through a bargaining process between the buyer and the seller. (See, e.g., Shi, 1995; Trejos and Wright, 1995; Duffie, Gârleanu, and Pedersen, 2005.) A solution to this bargaining problem is provided by the Nash solution, where ϕ is the exogenous bargaining power of the buyer. Explicit alternating-offer bargaining games predict similar outcomes. See Osborne and Rubinstein (1990).

trade with intensity κ , in which case the potential match surplus vanishes; the buyer (seller) finds another counterparty with intensity λ_b (λ_s) and recoups a fraction ϕ ($1 - \phi$) of the surplus. In this model the surplus is strictly positive because trading takes time. If either the buyer or the seller could trade without delays and $\lambda_b\phi + \lambda_s(1 - \phi) = \infty$, then the match surplus would shrink to zero.

A convenient expression for the asset price is found by subtracting (5) and the identity $rp = rp$ from (6):

$$r(V_o - V_b - p) = d - rp - \kappa(V_o - V_b - V_s) - \lambda_b(V_o - V_b - p),$$

which implies

$$rp = d - \kappa\Sigma - (r + \lambda_b)\phi\Sigma. \tag{9}$$

The first term is standard: It is the flow dividend payment, which is capitalized in the asset price. The second and the third terms arise because of search-and-matching frictions. The second term is a liquidity discount because, with intensity κ , a buyer who receives a preference shock no longer wants to hold the asset and, as a consequence, the match surplus, Σ , is lost. The third term is a bargaining discount, because the buyer is able to extract a fraction, ϕ , of the surplus from the seller. So this simple model delivers a rich theory of asset prices that depend on the structure of the market, which includes the matching technology and agents' bargaining powers. In the context of the housing market Caplin and Leahy (2010, this issue) show that such a model of trade with matching frictions can generate the large price changes and the positive correlation between prices and sales that we see in the data.

Let us consider the limit of the asset price when trading frictions vanish.¹⁰ As before, suppose that $\frac{E}{\kappa} > S$; i.e., the market is a seller's market. Then, $\lambda_s \rightarrow \infty$, and λ_b stays bounded. From (8), as long as $\phi < 1$, $\Sigma \rightarrow 0$. From (9), $p \rightarrow 1/r$, the discounted sum of dividends.

Our model can be used to analyze traditional measures of transaction costs, such as bid-ask spreads. For this, consider the payoff-equivalent economy in which the buyer's bargaining power is a random variable with support $[0, 1]$ and mean ϕ , independently distributed across encounters. Then, the maximum buying price (the ask) is $p^a = V_o - V_b$ and the minimum selling price (the bid) is $p^b = V_s$. The bid-ask spread is then equal to the total match surplus:

$$p^a - p^b = \Sigma = \frac{\gamma}{r + \kappa + \lambda_b\phi + \lambda_s(1 - \phi)}.$$

¹⁰For related limiting results, see Duffie, Gârleanu, and Pedersen (2005) and Miao (2005). Earlier results were derived in different contexts by Gale (1987) and Spulber (1996).

The bid-ask spread is proportional to the seller's cost of holding the asset, γ . But if the matching process becomes infinitely efficient and $\lambda_b\phi + \lambda_s(1 - \phi) = \infty$, then the bid-ask spread vanishes.

Because it assumes that only one asset is traded, the above model analyzes the impact of liquidity on asset prices only by comparative statics, and thus cannot explain why assets may differ in their liquidity in the first place. Vayanos and Wang (2007), Vayanos and Weill (2008), and Weill (2008) provide multi-asset models starting from the assumption that all asset markets share the same matching technology. Liquidity and price differences arise endogenously in equilibrium as a consequence of investors' optimal search behavior, and can be related to asset fundamental characteristics. We will revisit this theme in Section 5 when we endogeneize the role that assets play to facilitate exchange.

So far, the price of the asset is determined by investors themselves. In many markets, prices are set by some intermediaries (e.g., brokers or dealers). Such intermediaries can be described, as in Rubinstein and Wolinsky (1987), Spulber (2006), Duffie, Gârleanu, and Pedersen (2005), or Weill (2007), as agents with a higher meeting rate. For instance, in Duffie, Gârleanu, and Pedersen (2005) dealers can contact each other instantly in a competitive interdealer market. Alternatively, middlemen might have a better technology to recognize the quality of assets, as in Li (1998). The model can then provide a natural definition of the bid-ask spread as the difference between the price at which intermediaries sell the asset and the price at which they buy it.

We have assumed that the asset price was determined through a negotiation. Alternatively, prices can be posted by some market makers. If investors differ in terms of their information about the prices in the market—some investors are sophisticated while others are unsophisticated—then, following the logic in Burdett and Judd (1983), the model is able to predict a distribution of posted prices. This is consistent with the finding of Green, Hollifield, and Schurhoff (2007) for the market of municipal bonds.

For simplicity, we have restricted agents' asset holdings to $\{0, 1\}$. This assumption is made for tractability and is common in the literature, dating back to Diamond (1982) and more recently Duffie, Gârleanu, and Pedersen (2005). The model has been extended in Gârleanu (2007) and Lagos and Rocheteau (2007, 2009) to allow for unrestricted asset holdings. They show that if agents can adjust their asset holdings in a continuous fashion, then trading frictions have significant impact on the distribution of asset holdings, trade volume, and trading costs, but not necessarily on asset prices.

4 Market participation

Since every buyer enters the market seeking to purchase one share of the asset, the flow of buyers, F , is naturally interpreted as the amount of capital flowing in the market, what Brunnermeier and Pedersen (2009) refer as "funding liquidity". This is a crucial parameter of our model because it ultimately determines the ease of transacting in the market, represented by search times, what Brunnermeier and Pedersen refer as "market liquidity." In what follows, by endogenizing F , we provide a simple model of the two-way interaction between "funding liquidity" and "market liquidity," and analyze its welfare implications. Related models are studied by Huang and Wang (2009, 2010), and our current formalization is similar to the one in Pissarides (2000, ch.7) in the context of the labor market, and Vayanos and Wang (2007) and Afonso (2008) in the context of asset markets.

Suppose that agents differ in terms of their participation costs in the market. These costs represent the opportunity costs of giving up investment opportunities in other markets. Let $F(c)$ denote the flow of agents with an entry cost less than c and assume, to simplify the analysis, that the entry cost is paid once and for all upon entry. An agent is willing to enter the market if her entry cost is smaller than some threshold c_R . Hence, the flow of participating agents is $F(c_R)$, and from (4),

$$\left(\frac{F(c_R)}{\kappa} - S\right) \lambda \frac{\alpha(\theta)}{\theta} + \frac{F(c_R)}{\theta} = \kappa S, \quad (10)$$

which gives an increasing relationship between the ratio of buyers to sellers, θ , and the threshold for the entry cost below which agents enter, c_R . Intuitively, if c_R increases, then there is larger flow of buyers entering the market, which must be balanced by a larger outflow of sellers in a steady state; i.e., θ increases.

Buyers at the threshold entry cost must be indifferent between entering or not; i.e., $V_b = c_R$. From (5) and (8), this indifference condition can be written as

$$(r + \kappa) c_R = \gamma \frac{\lambda \frac{\alpha(\theta)}{\theta} \phi}{r + \kappa + \lambda \frac{\alpha(\theta)}{\theta} \phi + \lambda \alpha(\theta)(1 - \phi)}. \quad (11)$$

This defines a decreasing relationship between c_R and θ . Intuitively, if the ratio θ of buyers to sellers increases, then buyers have a harder time finding sellers and so they are willing to incur smaller entry costs. The two relationships (10) and (11), labelled "steady state" and "entry," respectively, in Figure 2, uniquely define the equilibrium values of c_R and θ .

This simple model can be used to study how changes in fundamentals affect participation decisions. Suppose, for instance, that sellers' cost of holding the asset, γ , increases. Graphically, the entry curve in

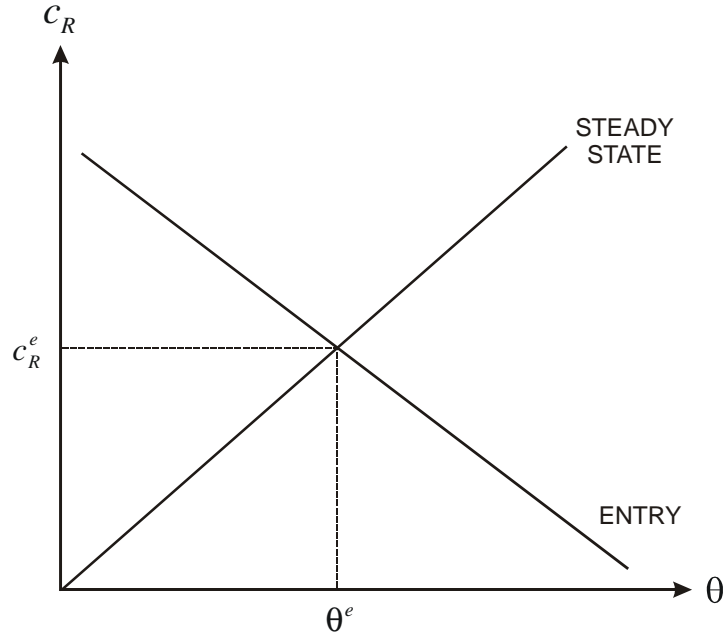


Figure 2: Equilibrium with endogenous participation

Figure 2 shifts upward. More buyers enter the market (c_R increases) in order to respond to sellers' higher demand for liquidity. As a result θ increases, i.e. sellers can trade faster on average.

So far, we have only considered a participation decision on the buyer's side. We could make the buyer's decision to sell her assets endogenous as well. One could assume, for instance, that the cost of holding the asset, γ , follows a stochastic process. When this cost is sufficiently high, sellers would choose to participate in order to sell their assets.

A natural question is whether agents' participation decisions (and hence "funding" liquidity) are socially efficient. By participating in the market, a buyer reduces the search times of potential sellers; this is a thick market externality. But the new entrant reduces the search times of other buyers; this is a congestion externality (Afonso, 2008). As it has been well-known since Hosios (1990), in environments where prices are set through bargaining, these externalities in general do not cancel out and participation is socially inefficient. To see this point, suppose that the matching function exhibits constant returns to scale (a necessary condition for social efficiency). By Euler Homegenous Function Theorem, $M = M_{\mu_b}\mu_b + M_{\mu_s}\mu_s$, where M_{μ_b} and M_{μ_s} denote the partial derivative of the matching function with respect to μ_b and μ_s , respectively. According to

this formula, buyers collectively create a fraction

$$\frac{M_{\mu_b} \mu_b}{M} = \frac{\alpha'(\theta)\theta}{\alpha(\theta)}$$

of the matches. Since the buyer's share in the match surplus is ϕ , the social and private gains from buyers' participation coincide if $\phi = \alpha'(\theta)\theta/\alpha(\theta)$; i.e., the buyer's surplus share must reflect her contribution to the matching process.¹¹ If this condition is not satisfied, then participation is socially inefficient, and Pigouvian taxes or subsidies are welfare improving.

Alternatively, these externalities can be internalized by requiring that prices be set in advance and be made publicly available. This notion of equilibrium, first formalized by Moen (1997), is called *competitive search equilibrium*.¹² Suppose, for instance, that sellers can post the price at which they commit to sell their asset. Buyers can observe all the posted prices and direct their search toward the price of their choosing. We can then define a submarket as a subset of sellers posting a price and a subset of buyers looking for that price. On each submarket there are search frictions as described earlier. When setting their prices sellers take into account that the buyer-to-sellers ratio in the submarket, $\theta(p)$, will adjust so that buyers are indifferent between searching in that submarket and searching in a different one. The seller's problem can then be written as:

$$rV_s = d - \gamma + \max_{p, \theta} \lambda \alpha(\theta) (p - V_s) \tag{12}$$

$$\text{s.t. } \lambda \frac{\alpha(\theta)}{\theta} (-p + V_o - V_b) = (r + \kappa)V_b, \tag{13}$$

where the seller takes as given the buyer's values V_b and V_o of searching in other submarkets and of becoming an owner. Equation (13), which gives a decreasing relationship between θ and p , captures the key trade-off faced by the seller: If she increases her price, p , then θ decreases; i.e., her market is less attractive to buyers, and she has to accept longer search times.

One can use (13) to obtain an expression for $\alpha(\theta)p$ as a function of θ . Plugging this expression into the seller's objective, (12), to eliminate p , and taking derivatives with respect to θ leads to the first-order condition:

$$(r + \kappa)V_b = \lambda \alpha'(\theta) (V_o - V_b - V_s),$$

¹¹This line of reasoning was developed in Mortensen (1982).

¹²See also Mortensen and Wright (2002) for a presentation of this equilibrium concept and some extensions.

or, equivalently,

$$(r + \kappa)V_b = \lambda \frac{\alpha(\theta)}{\theta} \left[\frac{\alpha'(\theta)\theta}{\alpha(\theta)} \right] (V_0 - V_b - V_s).$$

The price is the one that would prevail in an economy with ex-post bargaining, where the buyer's bargaining power corresponds to the contribution that a marginal buyer makes to the matching process, as measured by $\alpha'(\theta)\theta/\alpha(\theta)$. As argued above, in this case, participation decisions are socially efficient.

The competitive search equilibrium is also convenient for discussing how investors' heterogeneity affects participation. Suppose, for instance, that there are two types of buyers with different participation costs. The market will then be endogenously segmented, with one submarket for buyers with a low participation cost and a different submarket for buyers with a high participation cost. In the submarket where buyers have a high participation cost, the price will be high but θ will be low to reduce buyers' search times.

Another approach to endogenize the ease of trading is to explicitly model financial intermediation. For instance, dealers could choose the intensity with which they match buyers and sellers (Duffie, Gârleanu, and Pedersen, 2005), whether or not to participate in the market at some cost (Lagos and Rocheteau, 2007, 2009), or the size of their asset inventories (Shevshenko, 2004; Weill, 2007; Lagos, Rocheteau, and Weill, 2008). The model can then be used to investigate time-varying liquidity, i.e., how the ease of trading can vary over time as a function of market conditions.

5 Liquidity and payments

So far we have captured a motive for trading the asset by introducing idiosyncratic shocks that affect agents' utility flow from holding the asset. In the following we will be more explicit and we will assume the asset can be used to finance random spending opportunities, as in the monetary models of Shi (1995), Trejos and Wright (1995), and Wallace (1996, 2000). In contrast with the previous section, this model gives rise to a liquidity premium (instead of a liquidity discount), which capitalizes the transaction services provided by the asset in the presence of search frictions.

Consider the same environment as before with two consumption goods: the fruits produced by the asset, and a consumption good traded in a bilateral match between a consumer and a producer, call it a search good. There is flow F of agents who enter the economy with a single opportunity to produce search goods. They will search for a consumer and trade their output for an asset that they can spend later in their lives when it is their turn to consume. The search good is perfectly divisible and perishable, and consumption and

production take place on the spot. After having produced, an agent has the desire to consume after a period of random length exponentially distributed with mean $1/\kappa$. In order to produce $q \in \mathbb{R}_+$ units of the search good, the producer incurs a disutility cost q . The consumption of q units generates $u(q)$ in terms of utility, where $u(\cdot)$ has the usual properties. After having consumed, the agent leaves the economy permanently. We assume that agents cannot commit and that there is no record-keeping of individual trades. As shown by Kocherlakota (1998), in such an economy there is an essential role for an asset as a medium of exchange.

The distribution of agents across states (μ_b, μ_o, μ_s) and the matching rates are determined as before. Let us turn to the determination of the output traded in bilateral meetings, q . The flow Bellman equations are

$$rV_b = \lambda_b(-q + V_o - V_b) \tag{14}$$

$$rV_o = d + \kappa(V_s - V_o) \tag{15}$$

$$rV_s = d + \lambda_s[u(q) - V_s]. \tag{16}$$

According to (14) a buyer of the asset, who is also a producer of the search good, meets a seller at rate λ_b . The buyer produces q units of the consumption good in exchange for the asset. According to (15), an agent with no production opportunity and no desire to consume the search good enjoys only the consumption of the fruits generated by his asset. With Poisson arrival rate κ , he receives a preference shock and wishes to consume the search good. According to (16) the seller of the asset enjoys the dividend flow, d , and meets a buyer at rate λ_s . He receives q units of the consumption good, valued according to the utility function $u(q)$, in exchange for his asset, and he leaves the market permanently.

How is q determined? As before, we will assume that the output a unit of asset buys is determined through bargaining. Suppose first that the buyer of the asset has all the bargaining power. The seller of the asset is indifferent between selling or not selling so that $u(q) = V_s$ and $V_s = V_o = d/r$. The value from holding the asset corresponds to its fundamental value. In particular, if the dividend goes to 0, then the value of the asset is 0. The asset has no purchasing power, $q = 0$, and agents obtain no more than their autarky payoff, $V_b = V_o = V_s = 0$.

Suppose next that the seller of the asset has all the bargaining power. Then, the buyer of the asset is indifferent between buying or not buying so that $q = V_o$ and $V_b = 0$. From (15) and (16) the purchasing

power of the asset is determined by

$$(r + \kappa) q = \left(\frac{r + \lambda_s + \kappa}{r + \lambda_s} \right) d + \frac{\kappa \lambda_s}{r + \lambda_s} u(q). \quad (17)$$

The solution to (17) is represented in Figure 3 where “RHS” corresponds to the right side of (17) and “LHS” corresponds to the left side. If agents receive no spending shock, $\kappa = 0$, then the asset is priced at its fundamental value, $q = d/r$. In contrast, if the asset is useful as a means of payment, then its price rises above the fundamental value $q = d/r$. Because of its moneyness, the asset acquires a "liquidity premium" or a "convenience yield" (Cochrane, 2005). To see this, consider the limit case when d tends to 0 and the real asset approaches a fiat money. Although the asset pays no dividend, it still has a positive value in exchange when $\kappa > 0$, i.e., to the extent it helps finance agents' needs to consume the search good. Graphically, the right side of (17) is represented by the dashed curve in Figure 3. There are two equilibria: a monetary one with $q > 0$ and a nonmonetary one at the origin. One can use this simple model to study the effects of a change in the money stock on the value of money. As the stock of money increases, the number of sellers per buyer increases (λ_b increases and λ_s decreases). From (17) the purchasing power of money decreases.

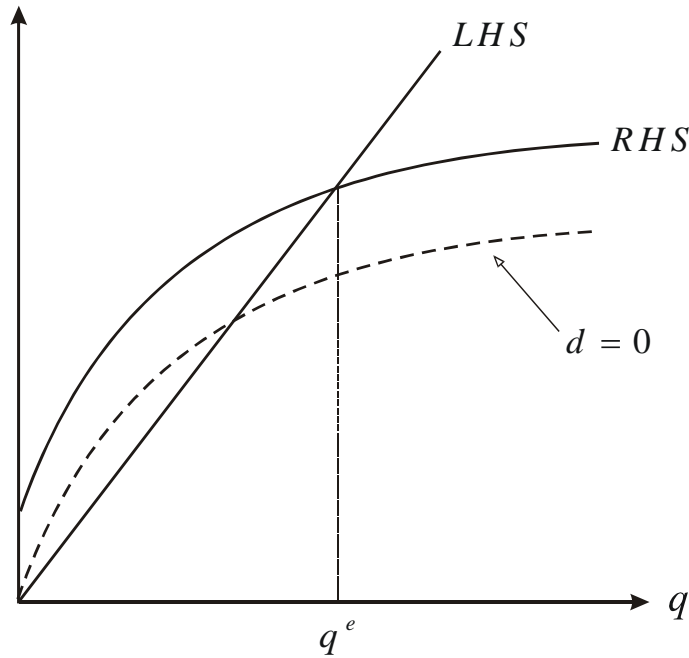


Figure 3: Value of the asset when used as a means of payment

Because of the unit upper bound on asset holdings and the indivisibility of assets, the model is limited

in its ability to discuss monetary policy. Shi (1997) and Lagos and Wright (2005) have extended the model (in the case where the asset is a fiat money, $d = 0$) to allow for divisible money and unrestricted asset holdings.¹³ The model can then be used to study conventional monetary policy, such as the optimal rate of growth of the money supply. For instance, Rocheteau and Wright (2005) investigate the optimality of the Friedman rule — one version of which requires the money supply to contract at the rate of time preference — under different trading mechanisms and with and without search externalities, and show that it is sometimes optimal to increase inflation above the Friedman rule to mitigate the congestion externalities in the goods market. Several papers in this issue adopt a similar framework to discuss monetary policy. Berentsen and Waller (2010, this issue) show that if the monetary authority pursues a price-level target, it can control inflation expectations and improve welfare by stabilizing short-run shocks to the economy. Jacquet and Tan (2010, this issue) study a version of the model in which money has a dual role as a self-insurance device and as a means of payment and show that the optimal monetary policy is such that the inflation rate is state contingent.

Another central question in monetary economics is the welfare loss for society associated with inflationary finance. Lucas (2000) provides a survey of this literature. Using a reduced-form money demand function he shows that the welfare cost of 10 percent inflation is about 1 percent of GDP every year. One insight from the microfounded model of monetary exchange of Lagos and Wright is to show that the welfare cost of inflation can be much larger than the traditional estimate, by a factor of three, once one spells out carefully the frictions in the environment that give rise to a positive money demand. In this line of work, Chiu and Molico (2010, this issue) quantify the welfare costs and the redistributive effects of inflation in the presence of idiosyncratic liquidity risk. They show that inflation induces important redistributive effects across households.

Recent evidence about the "moneyness" of real assets is provided by Krishnamurthy and Vissing-Jorgensen (2008) who document a negative relationship between the yield spread between corporate bonds and Treasury securities and the U.S. government debt-to-GDP ratio, based on annual observations from 1925 to 2005. Their notion of an aggregate demand for Treasury debt is directly borrowed from the traditional notion of aggregate money demand, such as the one used by Lucas (2000) to assess the welfare cost of inflation. Applying a similar calculation to Treasuries, Krishnamurthy and Vissing-Jorgensen found that "the value

¹³For a survey of this new generation of monetary models, see Williamson and Wright (2010).

of the liquidity provided by the current level of Treasuries is around 0.95 per cent of GDP per year.” Just as microfounded models of monetary exchange offer new estimates for the cost of inflation, microfounded models of liquidity offer a new avenue to think about the social benefits of the liquidity provided by Treasury securities.

The simple model we described above has a single asset that can serve as medium of exchange. A central objective of monetary theory, however, is to explain which asset among multiple assets will serve as money to facilitate trades. From Kiyotaki and Wright (1989), we learned that fundamental features of supply and demand, social conventions, and the properties of assets matter for the moneyness of an asset. The relationship between the physical properties of an asset (cash flow, divisibility, storage cost) and its moneyness has been investigated further by Wallace (2000), who develops a theory of the liquidity structure of asset yields based on indivisibility. Financial economists often relate asset liquidity to the extent of asymmetric information about cash flows. Two papers in this issue embed this idea into search models and investigate the manner in which an asset’s “recognizability” affects its liquidity.¹⁴ Lester, Postlewaite, and Wright (2010) show that assets that exist in different qualities and that lack recognizability might not be acceptable in decentralized trades. Cavalcanti and Nosal (2010, this issue) use a version of the model above to study the counterfeiting of currency when agents are heterogenous in terms of their ability to produce counterfeits and show, using a mechanism design approach, that it is not efficient to eliminate counterfeiting activity completely.

If different assets have different liquidity properties, they should also have different rates of return. To the extent that monetary theory can help explain liquidity differences across assets, it can also help explain seemingly anomalous rate-of-return differences, such as the rate-of-return-dominance puzzle, or the equity-premium and risk-free-rate puzzles. The rate-of-return-dominance puzzle is about the observation that individuals hold money instead of interest-bearing assets. This puzzle has been studied in the context of search-theoretic models by Aiyagari, Wallace, and Wright (1997) and Zhu and Wallace (2007), among others. Lagos (2007) uses a version of the model with divisible bonds and equity to address the equity-premium and risk-free-rate puzzles. Geromichalos, Licari, and Suarez-Lledo (2007) replace bonds with money and study the effects of monetary policy on asset prices. An increase in the rate of growth of the money supply reduces the rate of return of fiat money, which induces agents to use more stocks for their liquidity needs.

¹⁴Williamson and Wright (1994) were the first to formalize the view that fiat money provides superior transaction services because it is more easily recognizable than other means of payments.

Because stocks are in fixed supply, their price must go up. Lagos (2010, this issue) considers the case in which stocks are risky and shows that the optimal monetary policies implement Friedman's prescription of zero nominal interest rates. Under an optimal policy, equity prices and returns are independent of monetary considerations.

Just like intermediaries are useful to make markets, they have a role in providing liquid assets. Banks, which supply circulating liabilities (inside money) are a case in point. Cavalcanti and Wallace (1999) propose a model similar to the one above in which banks are described as agents whose trading histories are public, so that they can be induced to redeem their liabilities. Bencivenga and Camera (2010, this issue) introduce banks in a model with divisible money and capital. Banks offer demand deposit contracts and hold primary assets to maximize depositors' utility. If banks' operating costs are small, banks reallocate liquidity, eliminating idle balances and improving the allocation.

The search paradigm offers a natural framework for thinking about the trading frictions prevailing in credit markets. Diamond (1987, 1990) and Shi (1996) used a model similar to the one presented in this section to describe how agents trade with IOUs. Chamley and Rochon (2010, this issue) develop a model of lending where banks and borrowers meet according to a search process. They show that there can be multiple equilibria that differ according to whether there is rollover of loans or not, and rollover is socially inefficient. The global dynamics display a continuum of equilibria, some of them with sudden crises in which the volume of outstanding loans is reduced. Ashcraft, McAndrews, and Skeie (2010, this issue) develop a model with credit and liquidity frictions in the interbank market consistent with the liquidity hoarding by banks and the extreme volatility of the fed funds rate that were observed during the 2007-08 financial crisis. Banks rationally hold excess reserves intraday and overnight as a precautionary measure to self-insure against liquidity shocks.

Finally, models of monetary exchange can be amended to analyze large payment systems and issues related to settlement.¹⁵ A case in point is Koepl, Monnet, and Temzelides (2008) who develop a dynamic general equilibrium model of payments that incorporates private information frictions. Infinitely lived agents trade in bilateral matches and are subject to a double-coincidence-of-wants problem. There is no currency, but there is a payments system that can record individual transactions and assign balances to its participants. The model is used to determine the optimal settlement frequency and the trade-off between trade sizes and

¹⁵The canonical model of settlement is due to Freeman (1996), who considers an overlapping-generations economy with heterogeneous agents, some trading with debt, others with money.

settlement frequency. Afonso and Shin (2010, this issue) construct a model of a payment system calibrated to reproduce features of the US Fedwire system and study its ability to withstand severe payment disruptions. They show that individually cautious behavior can accumulate into a significant and detrimental impact on the overall functioning of the payment system.

6 Conclusion

We provided an overview of the literature on asset markets with trading frictions in both finance and monetary theory using a simple search-theoretic model. By taking explicitly into account trade mechanics, search-theoretic models provide a natural platform to investigate various notions of liquidity. From the matching technology, one can explain trading delays and the volume of trade or, in a monetary context, the velocity of the asset. Using various pricing mechanisms (e.g., price posting, bargaining) and alternative assumptions about traders' information, one can obtain rich predictions for bid and ask prices. The search approach also has novel predictions for asset prices, which depend not only on the streams of dividends but also on trading frictions and buyers' and sellers' bargaining powers. In monetary environments asset prices can also exhibit liquidity premia, which can help explain various asset-pricing anomalies. Finally, the model makes it possible to discuss positive and normative aspects of liquidity provision, including participation in the market, market-making and intermediation, and monetary policy.

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