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This paper investigates how market structure affects efficiency and several dimensions of liquidity in an asset market. To this end, we generalize the search-theoretic model of financial intermediation of Darrell Duffie et al. (2005) to allow for entry of dealers and unrestricted asset holdings.

Keywords: bid-ask spread, execution delay, liquidity, search, trade volume

JEL Classification: G11, G12, G21

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1 Introduction

In many markets, trade is facilitated by intermediaries, e.g., dealers, market-makers, specialists. The degree of market power that these intermediaries have is commonly viewed as a key determinant of the liquidity of the market in which they operate. Recently, several regulations have been introduced to foster competition in financial markets.[1] The available evidence suggests that these reforms have had an impact on trading costs and have also affected the incentives of financial intermediaries to make markets.

In order to understand precisely how the organization of the market in which an asset is traded affects the standard financial measures of liquidity, we generalize the search-theoretic model of financial intermediation of Darrell Duffie et al. (2005) (DGP hereafter) by introducing entry of dealers and a nontrivial choice of asset holdings. (Asset holdings are restricted to lie in $\{0, 1\}$ in DGP.) These extensions allow us to discuss several dimensions of market liquidity, such as trading costs, trade volume and execution delays. Motivated by the recent regulatory changes, we center our analysis around the effects of changes in the dealers' market power.

We find that a reform which reduces the dealers' market power can quite naturally lead to lower trading costs, higher trade volume, and a net entry of dealers, in line with the evidence provided by James Weston (2000).[2] Our model can also generate multiple steady-state equilibria, suggesting that markets with similar structures may

[1] See Michael Barclay et al. (1999) and James Weston (2000) for accounts of these regulatory reforms.

[2] In Table 6, Weston (2000) documents that the 1997 reform in the NASDAQ was accompanied by a net entry of dealers, higher turnover of stocks and lower spreads.

differ considerably in terms of their liquidity outcomes.

2 The environment

Time is continuous and goes on forever. There are two types of infinitely-lived agents: a unit measure of investors and a large measure of dealers. There is one asset, one perishable good called *special good*, and a general consumption good defined as numéraire. The asset is durable, perfectly divisible and in fixed supply $A \in \mathbb{R}_+$. Each unit of the asset produces a unit flow of special good. There is no market for the special good. The numéraire good is produced and consumed by all agents. The instantaneous utility function of an investor is $\varepsilon_i u(a) + c$, where $a \in \mathbb{R}_+$ is the consumption of special goods (which coincides with the investor's asset holdings), $c \in \mathbb{R}$ is the net consumption of the numéraire good ($c < 0$ if the investor produces more of the numéraire good than he consumes), and $i \in \{L, H\}$ indexes an idiosyncratic component, with $\varepsilon_L < \varepsilon_H$. The function $u(a)$ is continuous and twice differentiable, with $u' > 0$ and $u'' < 0$. Each investor receives an idiosyncratic preference shock with Poisson arrival rate δ. Conditional on the preference shock, the probability the investor draws preference type $i \in \{L, H\}$ is π_i, with $\pi_L + \pi_H = 1$. Dealers' instantaneous utility is simply c, their consumption of the numéraire good. Dealers who choose to participate in the market also incur a flow cost $\gamma > 0$ which represents the ongoing costs of running the dealership. All agents discount at rate $r > 0$.

Participating dealers have continuous access to a competitive asset market. An investor can adjust his asset holdings only through a dealer whom he contacts at random and bilaterally with Poisson rate α. Once they have made contact, the dealer and the investor negotiate over the quantity of assets that the dealer will acquire on behalf of the investor and an intermediation fee; they execute the transaction and part ways.

The rate at which investors contact dealers, α, is a continuously differentiable function of the measure of active dealers in the market, υ. Investors contact a dealer faster when the measure of active dealers is larger , i.e., $\alpha'(\cdot) > 0$. Furthermore, $\alpha(0) = 0$, $\alpha(\infty) = \infty$, $\alpha(\infty)/\infty = 0$ and $\alpha'(0) = \infty$. We capture the notion of competition for order flow by assuming that the rate at which dealers contact investors, $\alpha(\upsilon)/\upsilon$, is decreasing in υ.

3 Equilibrium

We restrict our attention to steady-state equilibria where the price of the asset in terms of the numéraire good, $p \in \mathbb{R}_+$, is constant over time. The value function of an investor with a preference type $i \in \{L, H\}$ who holds a quantity of assets a, $V_i(a)$, satisfies

$$rV_i(a) = \varepsilon_i u(a) + \delta\pi_j \left[V_j(a) - V_i(a)\right] + \alpha(\upsilon)[V_i(a_i) - V_i(a) - p(a_i - a) - \phi_i\,(a)],$$

where $\{j\} = \{L, H\}\backslash\{i\}$. The investor enjoys a utility flow $\varepsilon_i u(a)$ from holding portfolio a. He receives a new preference type with instantaneous probability $\delta\pi_j$ and enjoys a capital gain $V_j(a) - V_i(a)$. Upon contacting a dealer, with instantaneous probability $\alpha(\upsilon)$, the investor buys $a_i - a$ (sells if negative) and pays the dealer an intermediation fee, $\phi_i\,(a) \in \mathbb{R}_+$. The quantity traded, a_i, and the fee, $\phi_i\,(a)$, correspond to the Nash solution of the bargaining problem,

$$(a_i, \phi_i) = \arg\max_{(a',\phi)} [V_i(a') - V_i(a) - p(a' - a) - \phi]^{1-\eta} \phi^{\eta},$$

where $\eta \in [0, 1]$ is the dealer's bargaining power. After some calculations,

$$\bar{\varepsilon}_i u'(a_i) \leq rp, \qquad \text{“}=\text{”} \quad \text{if } a_i > 0, \tag{1}$$

$$\phi_i(a) = \frac{\eta \left\{ \bar{\varepsilon}_i \left[u(a_i) - u(a) \right] - rp(a_i - a) \right\}}{r + \alpha(\upsilon)(1 - \eta)}, \tag{2}$$

where $\bar{\varepsilon}_i = \frac{[r+\alpha(\upsilon)(1-\eta)]\varepsilon_i + \delta\bar{\varepsilon}}{r+\delta+\alpha(\upsilon)(1-\eta)}$ and $\bar{\varepsilon} = \pi_L \varepsilon_L + \pi_H \varepsilon_H$. Note that $\bar{\varepsilon}_i u'(a)$ is a weighted average of current and future expected marginal utilities. The weight on the current marginal utility decreases with trading delays, $1/\alpha$, and with dealers' bargaining power, η.

The free entry of dealers implies

$$\Gamma(\upsilon, \eta) \equiv \frac{\alpha(\upsilon)}{\upsilon} \left\{ \phi_L \left[a_H(\upsilon, \eta) \right] n_{HL} + \phi_H \left[a_L(\upsilon, \eta) \right] n_{LH} \right\} - \gamma = 0, \tag{3}$$

where n_{ji} denotes the measure of investors with preference type i who hold the quantity of assets a_j, given by

$$n_{HL} = n_{LH} = \frac{\delta \pi_L \pi_H}{\alpha(\upsilon) + \delta}, \tag{4}$$

$$n_{ii} = \frac{\alpha(\upsilon) + \delta \pi_i}{\alpha(\upsilon) + \delta} \pi_i, \tag{5}$$

where $a_H(\upsilon, \eta)$ and $a_L(\upsilon, \eta)$ are implicitly defined by (1). According to (3) the expected net profit of dealers, $\Gamma(\upsilon, \eta)$, must be 0 in equilibrium. The expected flow revenue of a dealer equals the expected intermediation fee he earns when he trades for a random investor, an event which occurs with Poisson rate $\alpha(\upsilon)/\upsilon$.

Finally, p is determined by the market-clearing condition $\sum_{i,j} n_{ij} a_i = A$, which using (4), can be written as

$$\pi_H a_H + \pi_L a_L = A. \tag{6}$$

Condition (6) equates the aggregate demand for the asset by investors (the left-hand side) to the asset supply.

A steady-state equilibrium is a list $\{(n_{ij}), (a_i, \phi_i(\cdot)), \upsilon, p\}$ satisfying (1)–(6). It is easy to show that $\lim_{\upsilon \to 0} \Gamma(\upsilon, \eta) = \infty$ and $\lim_{\upsilon \to \infty} \Gamma(\upsilon, \eta) = -\gamma$ for all $\eta > 0$. So an equilibrium with $\upsilon > 0$ exists provided that $\eta > 0$.

4 Liquidity and welfare

In order to isolate the direct effects of changes in the bargaining power, η, on the dealers' incentives to participate in the market from the general equilibrium effects that operate through the implied changes in the investors' asset holdings, we first assume $u(a) = a^{1-\sigma}/(1-\sigma)$ with $\sigma \to 0$. In this limiting case $a_i = A\bar{\varepsilon}_i^{1/\sigma} / \sum_j \pi_j \bar{\varepsilon}_j^{1/\sigma}$ is independent of η, specifically, $a_L \to 0$ and $a_H \to A/\pi_H$. From (6), the asset price is $p = \left(\pi_H \bar{\varepsilon}_H^{1/\sigma} + \pi_L \bar{\varepsilon}_L^{1/\sigma}\right)^\sigma / rA^\sigma \to \bar{\varepsilon}_H / r$. Thus, from (2), $\phi_{LH} \equiv \phi_H(a_L) \to 0$ (dealers do not charge a fee when they sell) and

$$\phi_{HL} \equiv \phi_L(a_H) \to \frac{\eta\left(\varepsilon_H - \varepsilon_L\right)A}{\left[r + \delta + \alpha(\upsilon)(1-\eta)\right]\pi_H}.$$

So for given υ the trading cost ϕ_{HL} increases with η. From (3) the net expected profit of a dealer is

$$\Gamma(\upsilon, \eta) = \frac{\alpha(\upsilon)}{\upsilon} \frac{\eta A \delta \pi_L \left(\varepsilon_H - \varepsilon_L\right)}{\left[r + \delta + \alpha(\upsilon)(1-\eta)\right]\left[\alpha(\upsilon) + \delta\right]} - \gamma.$$

The function $\Gamma(\upsilon, \eta)$ is strictly decreasing in υ, so the equilibrium is unique; Γ is strictly increasing in η so that $d\upsilon/d\eta > 0$. This last result implies that ϕ_{HL} increases with η. Finally, the total volume of trade, $\mathcal{V} \equiv \alpha(\upsilon)n_{HL}(a_H - a_L) = A\frac{\alpha(\upsilon)\delta\pi_L}{\alpha(\upsilon)+\delta}$, increases with υ. The following proposition summarizes the effects of η on the different dimensions of liquidity.

Proposition 1 *Assume $\sigma \to 0$. An increase in dealers' bargaining power raises trading costs (ϕ_{HL}), reduces trading delays ($1/\alpha(\upsilon)$) and increases trade volume ($\mathcal{V}$).*

Assuming $r \approx 0$, steady-state welfare is $\sum_{j\in\{L,H\}} n_{Hj}\varepsilon_j a_H - \upsilon\gamma$. If a planner could choose the measure of dealers in the market, he would pick the unique υ^* that solves $\alpha'(\upsilon)\frac{\delta\pi_L(\varepsilon_H-\varepsilon_L)}{[\alpha(\upsilon)+\delta]^2}A = \gamma$. (It is easy to check that the planner would choose the same portfolio allocations implied by the equilibrium.) By using $\Gamma(\upsilon^*, \eta) = 0$ to solve for η, the following proposition shows that the optimal allocation can be implemented provided that dealers enjoy a degree of market power that gives them enough incentives to participate in market-making.

Proposition 2 *Assume* $\sigma \to 0$ *and* $r \approx 0$. *The constrained-efficient allocation is achieved if and only if*

$$\eta = \left[\frac{\alpha\left(\upsilon^{*}\right)+\delta}{\alpha(\upsilon^{*})+\delta+\upsilon\alpha'(\upsilon^{*})}\right]\frac{\upsilon^{*}\alpha'(\upsilon^{*})}{\alpha(\upsilon^{*})} \in (0,1)\,.$$

5 Portfolio effects

Proposition 1 predicts a positive correlation between trading costs and the number of dealers. This result, however, hinges on the fact that a_H and a_L become independent of η as $\sigma \to 0$. For more general preferences, however, increases in the dealers' market power distorts individuals' asset holdings, which in turn affects the volume of trade and the profitability of dealers. The following lemma, immediate from (1), formalizes these portfolio effects.

Lemma 1 *(i)* $\partial a_H(\upsilon,\eta)/\partial\eta < 0$ *and* $\partial a_L(\upsilon,\eta)/\partial\eta > 0$. *(ii) If* $\eta < 1$ *then* $\partial a_H(\upsilon,\eta)/\partial\upsilon > 0$ *and* $\partial a_L(\upsilon,\eta)/\partial\upsilon < 0$.

According to part (i) of Lemma 1 a reduction in η (for given υ) raises the asset demand from high-marginal-utility investors and reduces the asset demand from low-marginal-utility investors. Intuitively, investors put more weight on their current utility relative to their future expected utility when dealers have less market power. Thus, all else equal, a reduction in η makes the distribution of portfolios more disperse. But changes in η also induce changes in υ. According to part (ii) of Lemma 1 the effects of an increase in υ on a_H and a_L are similar to those of a reduction in η. As shown in the following propositions, these portfolio effects can have important positive and normative implications.

Proposition 3 *For some parameter values, there are multiple equilibria.*

Consider the following parameter values and functional forms: $r = 0.1$, $\delta = 1$, $\varepsilon_H = 1$, $\varepsilon_L = 0$, $\pi_L = \pi_H = 0.5$, $\sigma = 1.7$, $A = 1$, $\gamma = 0.12$ and $\alpha(\upsilon) = 5\upsilon^{0.9}$. The first panel of Figure 1 plots $\Gamma(\upsilon, \eta)$ as a function of υ. It shows that there are multiple (three) steady-state equilibria if $\eta = 0.51$. This multiplicity is removed when η is raised to 0.55 —only the equilibrium with a high measure of dealers remains—or when η is reduced to 0.44—only the low equilibrium survives. Thus, a small change in dealers' market power can have a dramatic change on market outcomes. The possibility of multiple equilibria also suggests that markets with identical fundamental structure can exhibit very different equilibrium liquidity properties: an equilibrium with slow execution, wide spreads and low volume of trade can coexist with another with fast execution, narrow spreads and a large volume of trade.

Proposition 4 *A reduction in dealers' market power can generate: a reduction of trading costs, a net entry of dealers, an increase in trade volume and an increase in welfare.*

Keeping the investors' contact rate constant, an increase in η tends to reduce the dispersion of portfolios. But there is also a general equilibrium effect according to which an increase in η can raise υ (and the investors' contact rate), which in turn can increase the dispersion of portfolios. As shown in the second panel of Figure 1 the direct effect dominates for large values of η.[3] As a result, the volume of trade (third panel) and the dealers' revenue decline, and execution delays increase (fourth panel) with the dealers' bargaining power for sufficiently high values of η. The fifth panel shows that intermediation costs per unit of asset traded (expressed as a proportion of the asset price) increase with η. Thus, provided that η is high enough, a reduction in dealers' market power improves all dimensions of liquidity: the volume of trade is larger and trades are executed faster and at a lower cost. Furthermore, a reduction in η can raise welfare according to the last panel.[4]

[3] In the last five panels of Figure 1 we plot the equilibrium with the highest measure of dealers whenever there are multiple equilibria.

[4] In contrast to the linear case of Proposition 2, there is no value of η that implements the constrained-efficient allocation since dealers' market power distorts investors' asset holdings. See Ricardo Lagos and Guillaume Rocheteau (2006) for further details.

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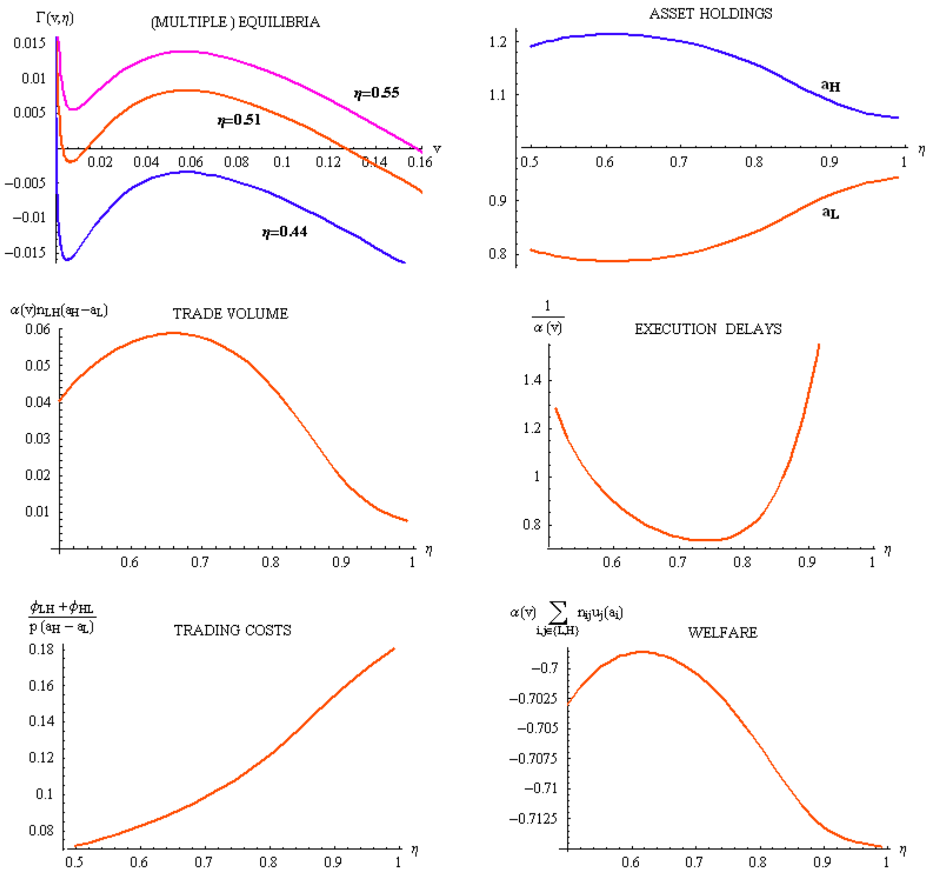

Figure 1: Effects of a change in dealers' market power