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This paper develops a model of currency circulation under asymmetric information. Agents are heterogeneous and trade in bilateral matches. Coins are intrinsically valuable and are available in two weights, light and heavy. We characterize the equilibrium under complete information and under imperfect information about the quality of coins. We determine a set of conditions under which the two currencies circulate and are traded according to di¤erent terms of trade. We study how output, welfare, and the velocity of currency are a¤ected by the recognizability of coins. We show that society.s welfare increases as coins become more easily recognizable.

Keywords: Commodity money, Gresham's law, Search, Informational asymmetries.
JEL Classification: D80, E40.

Richard Dutu is at the University of Waikato. Ed Nosal is at the Federal Reserve Bank of Cleveland and may be reached at Ed.Nosal@clev.frb.org (216) 579-2021. Guillaume Rocheteau is at the University of Pennsylvania.
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"*He who had more Goods that he had used for, would choose to barter them for Silver, though he had no use for it; Because, Silver was certain in its Quality*".

John Law, *Money and Trade Considered with a Proposal for Supplying the Nation with Money,* 1705, p.7.

It has long been argued that recognizability is a desirable property for a medium of exchange. In the above quotation, John Law makes precisely this point: Silver was used as a medium of exchange because its value was readily known or recognizable. More than a century later, Jevons (1875) reiterated the importance of recognizability, or what he calls *cognizability,* as one property that money should possess:

"*As a medium of exchange, money has to be continually handed about, and it will occasion great trouble if every person receiving currency has to scrutinize, weigh, and test it. If it requires any skill to discriminate good money from bad, poor ignorant people are sure to be imposed upon*" (Stanley Jevons, *Money and the Mechanism of Exchange*, 1875, p. 40).[1]

More recently, Brunner and Meltzer (1971) and Alchian (1977) have argued that a widely recognizable object—money—may emerge in trade as an attempt to reduce the transaction costs associated with asymmetries of information. In particular, when trading with a recognizable medium of exchange, individuals do not have to spend real resources in order to assess the quality of the object that they receive in exchange for the goods they sell.[2]

In practice, however, assessing the intrinsic value of money was, and still may be, problematic. The quantity and quality of precious metal contained in coins are always subject to uncertainty. First, the technology for making coins was little changed from ancient Greece until the 16th century. Coins were struck by hand, using a hammer, trussel, and pile, which produced

[1] Jevons (1875) defined *cognizability* as the property of a substance for being easily recognized and distinguished from all other substances. He identified six other important properties of money: utility and value, portability, indestructibility, homogeneity, divisibility, and stability of value.

[2] Williamson and Wright (1994) formalized the idea that money may be an efficient response to asymmetric information regarding the quality of goods. See King and Plosser (1986), Bernhardt and Engineer (1991), and Berentsen and Rocheteau (2004) for other models in this vein.

imperfect coins of varying size, weight, and fineness. To make matters worse, institutions associated with metallic monetary systems could amplify the uncertainty associated with the true value of coins. There are a number of examples from history. The way in which quality was measured in medieval France encouraged mint masters to produce lighter coins than that the authorities prescribed and to pocket the difference.[3] It was also in the sovereign's interest to (secretly) lower the intrinsic content of coins as a way to increase seigniorage revenue. Although professionals—such as moneychangers and bullion dealers—would not be fooled for very long, lowering the intrinsic value of some coins would increase the uncertainty associated with the medium of exchange among nonexperts. Clipping was another source of discrepancies between two coins bearing the same imprint. Before the invention of the milled coin at the end of the 17th century, agents could make a profit by clipping the edges of coins and minting the clippings into new coins. This kind of operation could be very lucrative and was difficult for authorities to control or prevent. The consequences of clipping before the 1696 Great Recoinage in Britain have been well documented by British historian Thomas Macaulay, who reported that, because of the bad state of the coins, "*nothing could be purchased without a dispute. Over every counter there was wrangling from morning to night*" (Macaulay, 1855, p. 187).[4] All these examples suggest that uncertainty about the quality of coins may have a negative impact on trade and, consequently, welfare.

In this paper we investigate how the recognizability of the object that serves as a medium

[3] Weight could be accurately determined using precise scales. Fineness was harder to verify and required proof by using fire or a touchstone. The touchstone test consisted in rubbing a coin on a special stone and comparing the color of the trace left with that of coins of known fineness. Assay by fire entailed melting down the coins in order to separate gold or silver from less precious metals like copper, and to weight the resulting quantity. All these operations were costly because of the expertise and tools required. See Gandall and Sussman (1997) for a detailed description of the monitoring scheme employed by the French crown to supervise mint masters. For the period 1385–1410, Gandall and Sussman (1997) calculate that a mint master's expected gain from fraud on weight amounted to 11 percent of his net income when striking silver and 66 percent when striking gold. The expected gain from fraud on fineness increased to 100 percent for silver and 600 percent for gold.

[4] Macaulay (1855, p.186) recounted the following story about clipping: "*There was, indeed, some northern districts into which the clipped money had only begun to find its way. An honest Quaker, who lived in one of these districts, recorded (. . .) the amazement with which, when he travelled southward, shopkeepers and innkeepers stared at the broad and heavy halfcrowns with which he paid his way. They asked whence he came, and where such money was to be found. The guinea which he purchased for twenty-two shillings at Lancaster bore a different value at every stage of his journey. When he reached London it was worth thirty shillings.*"

of exchange matters for output, welfare and the functioning of markets. To do so, we develop a model of commodity money where a standard double-coincidence-of wants problem generates a need for a medium of exchange. Consumption goods are perfectly recognizable, but coins can be produced in different qualities or weights, and assessing their quality or weight may be difficult. Owing to heterogeneity among agents, coins of different weights—light and heavy—will be minted to promote efficient exchanges. As in Sargent and Wallace (1983) and Sargent and Smith (1997), agents have access to a minting and melting technology that enables them to turn goods into coins and vice versa. Before going out shopping, agents must decide which coin to mint. After their minting decision, buyers and sellers meet randomly and trade coins for output.

We begin by studying a version of the model in which agents have full information about the quality of the coins used in payment. We characterize the optimal denomination structure—i.e., the optimal weights of coins—and the composition and velocity of the money supply for this economy. Taking as given this denomination structure, we then introduce imperfect information about the medium of exchange. The nature of the information imperfection is that the seller only learns with a probability of less than one the true value of the coin that is being offered for trade. The terms of trade in a bilateral match are set by the buyer, who knows the true value of the coin, and lotteries are allowed to trade indivisible coins.

We show the existence of various types of equilibria. We distinguish single-currency equilibria, where only one kind of coin circulates, from dual-currency equilibria, where two kinds circulate. Within the class of dual-currency equilibria, an equilibrium may be separating or pooling. In a pooling or *by-tale* equilibrium, two different coins circulate but they trade at the same terms. In a separating or *by-weight* equilibrium, two different coins circulate and trade at different terms according to their intrinsic content. It may seem surprising that buyers with coins whose differences are indistinguishable to the seller, can fully separate themselves in the by-weight equilibrium. This separation can be explained as follows: Since buyers are making take-it-or-leave-it offers, the bargaining game has the structure of a signaling game, i.e., buyers make offers that signal the quality of their coins. In some equilibria, buyers with heavy coins can separate themselves from buyers with light coins by offering to trade their coins with a lower probability for less output. In such equilibria, the velocity of heavy coins and the quantity they buy are lower than light coins'. This velocity result hints at Gresham's law: In particular, the velocity of the more valuable coin is lower than that of the less valuable one.

We evaluate the effects of coins' recognizability on output and welfare. When coins circulate by weight, aggregate output and welfare increase with recognizability. When coins circulate by tale, welfare increases with recognizability but aggregate output can either increase or decrease. When the recognizability problem becomes severe, the dual-currency equilibria will cease to exist. In that case, all buyers will mint light coins irrespective of their transaction needs. This phenomenon also hints at Gresham's law: In particular, bad (or less valuable) money drives out good (or more valuable) money. We show that a decrease in the recognizability of coins that triggers a transition from a dual-currency equilibrium to a single-currency equilibrium necessarily reduces welfare.

There is a small literature on commodity money with imperfect information. Using the Kiyotaki–Wright (1989) framework, Cuadras-Morato (1994) and Li (1995) study the endogenous emergence of a medium of exchange when goods are subject to quality uncertainty. Haegler (1997) shows that fiat money can circulate alongside commodity money that is subject to uncertainty and that it is welfare improving provided that the quantity of money is sufficiently small. These paper mainly concern the emergence of a medium of exchange, while our paper focuses on the determination of terms of trade in bilateral matches with asymmetric information. Burdett, Trejos, and Wright (2001) and Velde, Weber, and Wright (1999) consider a related environments where the terms of trade are endogenous. Here, our discussion highlights the paper by Velde, Weber, and Wright (1999), which is the closest to our analysis. Velde, Weber, and Wright (1999) consider an economy with two coins of different weights and study the typology of equilibria in terms of circulation-by-weight or circulation-by-tale. Their model differs from what we do in several dimensions. In their environment, coins cannot be minted or melted freely and agents cannot choose which coin to hold when going into the market: their only decision is whether or not to spend a coin. Since we let buyers choose their portfolios, our model can be viewed as a commodity-money version of the model by Lagos and Wright (2005). The fact that buyers choose which coin they hold increases the competition between currencies. Also, we introduce heterogenous buyers so that it is optimal to have coins of different weights. Last, we allow agents to use lotteries to determine terms of trade. Such randomization devices allow us to eliminate some equilibria and to uncover new ones. For instance, in our equilibrium with circulation by weight, both coins are traded in uninformed matches for some output. In contrast, in the by-weight equilibrium of Velde, Weber, and Wright, only light coins are traded in uninformed matches, whereas heavy coins are hoarded. Also, in the standard specification for the utility function adopted in search models, the equilibrium with circulation by tale would not exist. Moreover, the availability of lotteries introduces a notion of divisibility which may convey some information about the quality of coins. In particular, buyers can use lotteries to send signals to sellers about the type of coin that they hold. Our by-weight equilibrium is separating in that buyers' offers fully reveal the coins they hold. Without lotteries, such separation is not possible.

The rest of the paper is organized as follows: Section 1 presents the physical environment of our model. Section 2 characterizes the benchmark economy with full information. Section 3 studies how imperfect information affects agents' minting decisions. Section 4 focusses on the consequences for output, welfare and velocity. Section 5 concludes.

1 The model

Time is discrete and goes on for two periods, $t = 1, 2$.[5] The economy is composed of a continuum of agents of measure two, divided evenly between buyers and sellers. The set of buyers is denoted $\mathcal{B}$ and the set of sellers is denoted $\mathcal{S}$. Buyers are divided into two subgroups: a set $\mathcal{B}_H$ of buyers with measure λ have a high marginal utility of consumption; a set $\mathcal{B}_L$ of buyers with measure $1 - \lambda$ have a low marginal utility of consumption. There are three kinds of goods: a general

[5] Our two-period model is equivalent to an infinite-period horizon version: Since there are no state variables that link the different periods, there is no loss in generality by considering only two periods. Note also that our model is essentially a commodity-money version of the Lagos and Wright (2005) framework.

good that can be produced and consumed by all agents, a special good that is only consumed by buyers and only produced by sellers, and coins. Both general and special goods are perfectly divisible and perishable. In contrast, coins are perfectly durable and indivisible.

General goods can be turned into coins according to a perfectly reversible technology. For simplicity, we suppose that there is no transaction cost associated with the minting and melting processes.[6] The minting technology allows for two types of coins: *heavy coins* that are made of z_h units of general goods and *light coins* that are made of $z_\ell < z_h$ units of general goods. We will sometimes refer to z as the weight of the coin. For tractability purposes, we will assume that agents cannot carry more than one coin across periods.[7]

The sequence of events is as follows: At $t = 1$, agents can produce the general good and have access to the minting technology. At $t = 2$, buyers and sellers are matched pairwise and at random, where the special goods are produced in bilateral matches by the seller. Since there is the same number of buyers and sellers, we assume that each buyer is randomly assigned to a seller. At the end of the second period, bilateral matches are dissolved, agents melt any coin that they possess, and consumption takes place.

The utility function of a buyer, $\mathcal{U}^b(\varepsilon)$, is given by

$$\mathcal{U}^b(\varepsilon) = c_1 + \beta\left[\varepsilon u(q) + c_2\right], \tag{1}$$

where c_t is the net consumption of general goods in period t (if $c_t < 0$ agents consume less than they produce), q is the quantity of special goods consumed, $\beta \in (0, 1)$ is the discount factor across periods, and ε is a preference parameter where $\varepsilon = \varepsilon_L$ for buyers in $\mathcal{B}_L$ and $\varepsilon = \varepsilon_H > \varepsilon_L$ for buyers in $\mathcal{B}_H$. Note that general goods enter linearly in the utility function. We denote $r = \beta^{-1} - 1$ as the agents' rate of time preference. The function $u(q)$ is continuously differentiable, strictly increasing, and concave, and it satisfies $u(0) = 0$ and $u'(0) = \infty$.

[6] For a model where minting and melting are costly processes, see Sargent and Wallace (1983).

[7] This assumption is made to simplify the bargaining problem when there is imperfect information about the quality of coins. In several parts of the analysis, we will choose the sizes of the coins to make this constraint nonbinding.

The utility function of a seller, $\mathcal{U}^s$, is given by

$$\mathcal{U}^s = c_1 + \beta\left[-\psi(q) + c_2\right]. \tag{2}$$

The "cost" function, $\psi(q)$, is continuously differentiable, strictly increasing, and convex. Furthermore, $\psi(0) = 0$, $\psi'(0) = 0$, and there exists a $q_\varepsilon^* < \infty$ such that $\varepsilon u'(q_\varepsilon^*) = \psi'(q_\varepsilon^*)$. Output q_ε^* corresponds to the level of production that maximizes the total surplus in a match between a seller and a buyer of type ε.

The terms of trade in pairwise matches are determined by bargaining. For simplicity, we assume that buyers make take-it-or-leave-it offers.

2 Equilibrium with complete information

In this section, we describe the equilibrium of the model when there is no information problem about the weight of coins. We first describe the determination of the terms of trade in bilateral meetings. We assume that agents in a match have access to a randomization device that allows them to bargain over lotteries. We allow lotteries because they lead to a Pareto improvement in bilateral matches when coins are indivisible.[8] We will also see that lotteries play an important role in the presence of incomplete information. Since goods are divisible, agents only randomize over the transfer of the coin.[9] Denote (q, p) as the terms of trade, where $q \in \mathbb{R}_+$ is the quantity of the special good produced by the seller and consumed by the buyer, and $p \in [0, 1]$ is the probability that the buyer gives his coin to the seller. Suppose that the buyer holds a coin of weight z, whereas the seller holds a coin of weight z'. Since buyers make take-it-or-leave-it offers, the terms of trade, (q, p), are given by the solution to the following problem,

$$\max_{q\geq 0, p\in[0,1]} \varepsilon u(q) + (1-p)\, z \tag{3}$$

$$\text{s.t.} \quad -\psi(q) + p(z + z') + (1-p)z' \geq z', \tag{4}$$

$$p \leq 1.$$

[8] Lotteries in search models of money were introduced by Berentsen, Molico, and Wright (2002). Berentsen and Rocheteau (2002) discuss inefficiencies associated with indivisible money and compare models with lotteries and models with divisible money.

[9] This result was established in Berentsen, Molico, and Wright (2002).

The buyer chooses (q,p) in order to maximize his expected utility (3) subject to the seller's participation constraint (4). From (3) the expected utility of the buyer is the utility of consuming the special good, $\varepsilon u(q)$, plus the utility of consuming the general good embodied in the coin, z, if it is not handed over to the seller, an event which occurs with probability $1-p$. The left-hand side of (4) has a similar interpretation. The right-hand side of (4) is the utility of the seller from consuming the general good embodied in his coin if the buyer's offer is rejected. Problem (3)–(4) can be simplified to read

$$\max_{q,p\leq 1} \varepsilon u(q) - pz \quad \text{s.t.} \quad -\psi(q) + pz \geq 0. \tag{5}$$

It is clear from (5) that the terms of trade (q,p) do not depend on the coin held by the seller, z'. The solution to (5) is

$$q(\varepsilon,z) = \begin{cases} q_\varepsilon^* & \text{if } \psi(q_\varepsilon^*) \leq z \\ \psi^{-1}(z) & \text{otherwise} \end{cases}, \tag{6}$$

$$p(\varepsilon,z) = \begin{cases} \psi(q_\varepsilon^*)/z & \text{if } \psi(q_\varepsilon^*) \leq z \\ 1 & \text{otherwise} \end{cases}. \tag{7}$$

If the buyer's coin has insufficient value to purchase the efficient level of output, i.e., if $z < z_\varepsilon^* \equiv \psi\left(q_\varepsilon^*\right)$, then he gives his coin to the seller with probability one in exchange for as much output as his coin can purchase, which is equal to $\psi^{-1}(z)$; if, on the other hand, the buyer's coin has a value that exceeds the efficient level of output, i.e., $z \geq z_\varepsilon^* \equiv \psi\left(q_\varepsilon^*\right)$, then he will give the coin to the seller with probability $\psi(q_\varepsilon^*)/z$ in exchange for q_ε^* units of output.

From (1), (6) and (7), the expected utility of a buyer in period 1 satisfies

$$\max_{z\in\{0,z_\ell,z_h\}} -z + \beta\left\{u\left[q(\varepsilon,z)\right] + \left[1-p(\varepsilon,z)\right]z\right\}. \tag{8}$$

According to (8), a buyer chooses which coin to mint, if any. The disutility of minting a coin of weight z is z, where $z \in \{0, z_\ell, z_h\}$. In period 2, the buyer consumes q units of special goods, where q depends on the weight z of his coin and the buyer hands over his coin to the seller with probability p, where again p depends on z. Using the constraint in (5) with an equality, (8) can be simplified as

$$\max_{z\in\{0,z_\ell,z_h\}} \left\{-rz + \varepsilon u[q(\varepsilon,z)] - \psi[q(\varepsilon,z)]\right\}. \tag{9}$$

Problem (9) has a simple interpretation. The buyer chooses a coin that maximizes his surplus in a bilateral match, $\varepsilon u - \psi$, net of the (opportunity) cost of holding a coin, rz.

The problem of a seller is

$$\max_{z \in \{0, z_\ell, z_h\}} -z + \beta \int \left\{ -\psi[q(\varepsilon, z^b)] + p(\varepsilon, z^b) z^b + z \right\} dF(\varepsilon, z^b), \tag{10}$$

where $F(z^b, \varepsilon)$ is the distribution of buyers indexed by their type, ε, and the coin that they hold, z^b, where $(z^b, \varepsilon) \in \{0, z_\ell, z_h\} \times \{\varepsilon_L, \varepsilon_H\}$. The interpretation of (10) is analogous to that of (8). Problem (10) can be simplified to

$$\max_{z \in \{0, z_\ell, z_h\}} \left\{ -rz + \int \left\{ -\psi[q(\varepsilon, z^b)] + p(\varepsilon, z^b) z^b \right\} dF(\varepsilon, z^b) \right\}. \tag{11}$$

Since the integral in (11) does not depend on the seller's coin, the seller's optimal choice of the coin is $z = 0$. The intuition for this result is straightforward: The utility of consuming general goods is linear, so there is no smoothing motive for holding coins; and the bargaining outcome is independent of the seller's coin holdings, so there is no strategic motive for holding coins.

We are now in a position to define and characterize the equilibrium to the economy under complete information.

Definition 1 *An equilibrium is a list* $\left\{ (z^i)_{i \in \mathcal{B}_\ell}, (z^j)_{j \in \mathcal{B}_h} \right\}$ *such that* $z^i \in \{0, z_\ell, z_h\}$ *is solution to (9) with* $\varepsilon = \varepsilon_L$ *for all* $i \in \mathcal{B}_L$*, and* $z^j \in \{0, z_\ell, z_h\}$ *is solution to (9) with* $\varepsilon = \varepsilon_H$ *for all* $j \in \mathcal{B}_H$*.*

Proposition 2 *An equilibrium exists and it is generically unique.*

Proof. The function in (9) is maximized over a finite set. Therefore, a solution exists. Denote

$$\tilde{z}_\varepsilon = \max_{z \in \mathbb{R}_+} \left\{ -rz + \varepsilon u[q(\varepsilon, z)] - \psi[q(\varepsilon, z)] \right\}.$$

The function in (9) is strictly increasing for all $z \in (0, \tilde{z}_\varepsilon)$ and is strictly decreasing for all $z > \tilde{z}_\varepsilon$. Therefore, the function in (9) cannot take the same value for more than two distinct values for z. So the solution to (9) is unique except for a set of parameter values of measure 0, in which case the problem in (9) admits two solutions. ■

Figure 1 shows the payoff for an ε-type buyer, $-rz + \varepsilon u\left[q\left(\varepsilon, z\right)\right] - \psi\left[q\left(\varepsilon, z\right)\right]$, as a function of the weight z of the coin that he is holding. The variable $\tilde{z}_\varepsilon$ denotes an ε-buyer's optimal weight of coin, or his *ideal coin*, assuming there is no restriction on the weight of the coin that can be minted. The buyer's payoff is maximized at $z = \tilde{z}_\varepsilon$. The weight that maximizes the surplus of a match is given by z^*_ε, and the maximum weight that an ε-type buyer is willing to hold is given by $\bar{z}_\varepsilon$. This means that $\bar{z}_\varepsilon$ is the critical value for the weight of a coin for an ε buyer, above which the buyer has no incentive to hold a coin, i.e., the coin is "too heavy" This critical value is given implicitly by $r\bar{z}_\varepsilon = \varepsilon u[q(\bar{z}_\varepsilon)] - \psi[q(\bar{z}_\varepsilon)]$. Figure 1 depicts the (unlikely) situation in which $z_\ell < \tilde{z}_\varepsilon < z_h$ *and* the buyer is indifferent between holding the heavy and light coin.

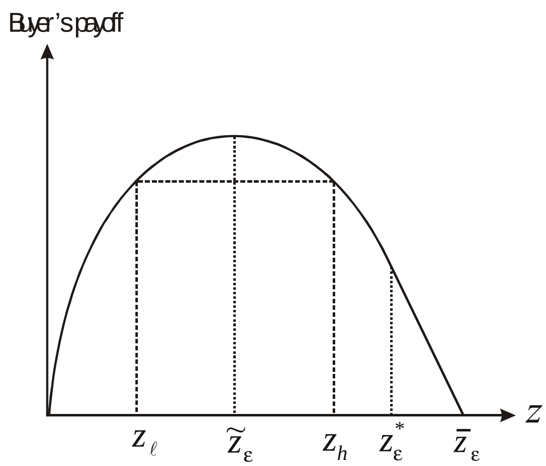

Figure 1: Buyer's payoff.

For arbitrary coins z_ℓ and z_h, one can show that there are two thresholds $\varepsilon_0 > 0$ and $\varepsilon_1 > \varepsilon_0$ such that: if $\varepsilon < \varepsilon_0$, then an $\varepsilon-$buyer holds no coin; if $\varepsilon \in (\varepsilon_0, \varepsilon_1)$, then an $\varepsilon-$buyer holds a light coin; and if $\varepsilon > \varepsilon_1$, then an $\varepsilon-$buyer holds a heavy coin. The next lemma characterizes the ideal coin of a buyer.

Lemma 3 *The weight of the ideal coin of an ε buyer is $\tilde{z}_\varepsilon$ that satisfies*

$$\frac{\varepsilon u'[\psi^{-1}(\tilde{z}_\varepsilon)]}{\psi'[\psi^{-1}(\tilde{z}_\varepsilon)]} = 1 + r. \tag{12}$$

Proof. $\tilde{z}_\varepsilon$ is solution to (9) when $z \in \mathbb{R}_+$. The first-order condition to problem (9) is $-r + \varepsilon u'q' - \psi'q' = 0$, where $q' = \partial q/\partial z$. For all $z > z^*_\varepsilon$, $q'(z, \varepsilon) = 0$. Therefore, $z \leq z^*_\varepsilon$ and $p = 1$. From (6), $q'(\varepsilon, z) = 1/\psi'[q(\varepsilon, z)]$. Substituting this into the above first-order condition and rearranging, we obtain equation (12). ■

Note from (12) that the weight of the ideal coin decreases with the rate of time preference, r, which is the opportunity cost of holding a coin.[10]

We will assume throughout the remainder of the paper that the minting technology is such that the weight of the coins is the ideal weights for each type of buyer; that is, we will assume that $z_h \equiv \tilde{z}_h$, which is the ideal coin for the high marginal-utility buyer, and $z_\ell \equiv \tilde{z}_\ell$, which is the ideal coin for the low marginal-utility buyer, where $\tilde{z}_\varepsilon$ is solution to (12). In the absence of asymmetries of information, the equilibrium outcome is characterized by all low (marginal-utility) buyers minting light coins and all high (marginal-utility) buyers minting heavy coins at date 1; sellers do not mint any coins. At date 2, high buyers make the take-it-or-leave it offer $\left(\psi^{-1}(z_h), 1\right)$ to the seller, which the seller accepts; low buyers make the offer $\left(\psi^{-1}(z_\ell), 1\right)$, which the seller accepts. At the end of period 2, the seller consumes either z_h or z_ℓ, depending upon whether he produced for the high or low buyer. For convenience, we will denote $\psi^{-1}(z_h) \equiv q_h$ and $\psi^{-1}(z_\ell) \equiv q_\ell$.

3 Equilibrium with asymmetric information

We now consider how equilibrium outcomes are affected when information about the weight (or quality) of coins is imperfect. We capture the notion of imperfect information by appealing to the information structure used in Williamson and Wright (1994): In any match, the seller receives a common-knowledge signal regarding the weight of the coin held by the buyer. With

[10]Note that the equation for $\tilde{z}$ is the same as the equation for the choice of real balances in the Lagos–Wright model. Also, it is equivalent to the choice of capital goods under the assumption of a linear storage technology in the model by Lagos and Rocheteau (2004).

probability $\theta \in (0,1)$, the signal is informative and the weight of the coin is revealed to the seller; with probability $1-\theta$, the signal is uninformative. The parameter θ captures the extent of the informational asymmetries. Also, the buyer's preference parameter, ε, is private information.

In matches where the seller is uninformed, the take-it-or-leave-it bargaining game has the structure of a signaling game.[11] The buyer makes an offer $(q,p) \in \mathbb{R}_+ \times [0,1]$, and the seller uses this offer to update his prior belief about the weight of the coin held by the buyer.[12] Let $\lambda(q,p) \in [0,1]$ represent the updated belief of a seller that the coin held by the buyer is a heavy coin conditional on the offer (q,p). If (q,p) corresponds to an equilibrium offer, then $\lambda(q,p)$ is derived from the seller's prior belief, according to Bayes' rule. If (q,p) is an out-of-equilibrium offer, then Bayes' rule cannot be applied and the seller's belief is arbitrary.

In an *uninformed* match, the buyer who holds a coin of weight z makes the offer (q,p) that solves the problem

$$\max_{q\geq 0, p\in[0,1]} \varepsilon u(q) - pz \tag{13}$$

$$\text{s.t.} \quad -\psi(q) + p\left\{\lambda(q,p)z_h + [1-\lambda(q,p)]z_\ell\right\} \geq 0. \tag{14}$$

The buyer chooses an offer (q,p) that maximizes his surplus from the trade (13), subject to the seller's participation constraint (14). From (14), the buyer takes into account that his offer will affect the seller's belief regarding the weight of his coin. We will restrict our attention to equilibria such that whenever (14) holds with equality, the buyer's offer will be accepted with probability one.

Let $[q^u(\varepsilon,z), p^u(\varepsilon,z)]$ denote the offer made by an ε–type buyer holding a coin of weight z in an uninformed match. We will restrict our attention to equilibria where all buyers of a given type (ε,z) make the same offer. We will call an equilibrium where $q^u(\varepsilon_L,z_\ell) = q^u(\varepsilon_H,z_h)$ a *by-tale* (or pooling) equilibrium because in all uninformed trades a light coin buys as much as a heavy coin, and an equilibrium where $q^u(\varepsilon_L,z_\ell) \neq q^u(\varepsilon_H,z_h)$ a *by-weight* (or separating) equilibrium because coins will always trade according to their weight.

[11] For a formal description of a signaling game, see Cho and Kreps (1987).

[12] In contrast to a standard signaling game, the type of the buyer in our bargaining game is really (ε,z), which is endogenous because buyers choose the weight of the coin they hold.

The buyer's choice of a coin, which modifies (9) in the obvious way, is now given by

$$\max_{z \in \{0, z_\ell, z_h\}} \left\{ -rz + \theta \left\{ \varepsilon u[q(\varepsilon, z)] - \psi[q(\varepsilon, z)] \right\} + (1-\theta) \left[\varepsilon u \left[q^u(\varepsilon, z) \right] - p^u(\varepsilon, z) z \right] \right\}, \tag{15}$$

where $\varepsilon = \varepsilon_H$ for high buyers and $\varepsilon = \varepsilon_L$ for low buyers. Because information may be imperfect in the bargaining games, the previous definition of an equilibrium must be modified.

Definition 4 *An equilibrium is a list* $\{(z^i)_{i \in \mathcal{B}_L},\ (z^j)_{j \in \mathcal{B}_H},\ [q(\varepsilon, z), p(\varepsilon, z), q^u(\varepsilon, z)],\ p^u(\varepsilon, z);\ (\varepsilon, z) \in \{\varepsilon_H, \varepsilon_L\} \times \{z_h, z_\ell\}),\ \lambda(q, p)\}$ *such that:*

1. z^i *is solution to (15) with* $\varepsilon = \varepsilon_L$ *for all* $i \in \mathcal{B}_L$*, and* z^j *is solution to (15) with* $\varepsilon = \varepsilon_H$ *for all* $j \in \mathcal{B}_H$*.*
2. $[q(\varepsilon, z), p(\varepsilon, z)]$ *is given by (6)–(7) for all* $(\varepsilon, z) \in \{\varepsilon_H, \varepsilon_L\} \times \{z_h, z_\ell\}$*.*
3. $[q^u(\varepsilon, z), p^u(\varepsilon, z)]$ *is solution to (13)–(14) for all* $(\varepsilon, z) \in \{\varepsilon_H, \varepsilon_L\} \times \{z_h, z_\ell\}$*.*
4. *The belief system* $\lambda(q, p)$ *is deduced from Bayes' rule whenever possible.*

A crucial element of the above definition is the belief system $\lambda(q, p)$. Below, we will put more structure on these beliefs by adopting a particular refinement. Before turning to the refinement, we can establish that the existence of imperfect information does not affect the strategy of a low buyer. In particular,

Lemma 5 *In any equilibrium, a low buyer always mints a light coin.*

Proof. See Appendix. ■

Intuitively, if imperfect information causes the value of a coin to deviate from its intrinsic value, then the light coin will tend to be overvalued and the heavy coin will tend to be undervalued. As well, the light coin is the ideal weight for low buyers in informed matches. These observations imply that a low buyer will never have an incentive to mint a heavy coin. Since in all equilibria the low buyer mints a light coin, a characterization of an equilibrium requires, among other things, that we determine whether a high buyer mints a heavy coin or a light one.

When bargaining with a seller, a buyer can attempt to signal the weight of his coin. But signaling raises the thorny issue of how a seller should interpret an offer that is not supposed to occur in equilibrium. In this regard, we will restrict sellers' out-of-equilibrium beliefs to be consistent with the Cho–Kreps equilibrium refinement. The intuition behind this refinement is as follows: Suppose that a seller receives the out-of-equilibrium offer $(\hat{q}, \hat{p})$. If offer $(\hat{q}, \hat{p})$ reduces the utility of an ε buyer who holds a coin that weighs z compared to his equilibrium payoff, then, according to the Cho–Kreps criterion, the seller should assign a probability equal to zero that this offer came from an ε buyer holding a coin that weighs z. The criterion allows the seller to place a positive probability weight only on buyers who would benefit from having the out-of-equilibrium offer accepted.

The following proposition specifies the necessary conditions for a by-weight equilibrium and a by-tale equilibrium to exist.

Proposition 6 *Consider an equilibrium where high buyers hold heavy coins. If $\varepsilon_L/z_\ell > \varepsilon_H/z_h$, then the outcome in uninformed matches is separating. If $\varepsilon_L/z_\ell < \varepsilon_H/z_h$, then the outcome in uninformed matches cannot be separating.*

Proof. See Appendix. ■

The condition $\varepsilon_L/z_\ell > \varepsilon_H/z_h$ says something about relative marginal rates of substitution between q and p for a high buyer with a heavy coin and a low buyer with a light coin. It indicates that in order to obtain the same increase in consumption, a low buyer with a light coin is willing to give up his coin with a higher probability than a high buyer with a heavy coin. When this condition is satisfied, it is *not* possible to have a by-tale equilibrium. If a by-tale (or pooling) allocation is proposed as an equilibrium, then a high buyer with a heavy coin could instead offer to trade his coin for a lower quantity of output and with a lower probability in a way that makes him better off compared to the proposed equilibrium and, at the same time, would make the low buyer with the light coin worse off. This point is illustrated in figure 2. We denote $U^b_{L\ell} = \varepsilon_L u(q) - pz_\ell$ as the surplus that a low buyer holding a light coin receives if the terms of trade are (q, p), and $U^b_{Hh} = \varepsilon_H u(q) - pz_h$ as the surplus that a high buyer holding a heavy coin receives. Consider a proposed pooling equilibrium where all buyers offer (q^u, p^u). Suppose that a high buyer defects from a proposed equilibrium play and instead offers $(\hat{q}^u, \hat{p}^u)$, which lies to the right of U^b_{Hh} and to the left of $U^b_{L\ell}$ in figure 2. According to the Cho–Kreps refinement, the seller should interpret this out-of-equilibrium offer as coming from a high buyer with a heavy coin. The seller will accept this offer because it provides him with a positive surplus, i.e., allocation $(\hat{q}^u, \hat{p}^u)$ lies above his reservation indifference curve U^s_h, defined by $-\psi(q) + pz_h = 0$. Therefore, if the two coins coexist, they will be traded at different terms of trade in uninformed matches.

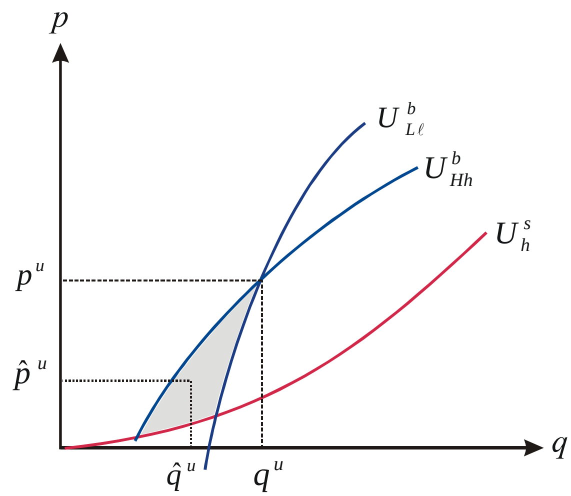

Figure 2: Ruling out pooling equilibria ($\varepsilon_L/z_\ell > \varepsilon_H/z_h$).

If, on the other hand, $\varepsilon_L/z_\ell < \varepsilon_H/z_h$, then it is not possible to have a by-weight (separating) equilibrium. If a separating allocation was proposed as an equilibrium, then the low buyer with a light coin would make the full information offer $(q_\ell, 1)$, since in a separating equilibrium the buyer is revealed as being a low type with a light coin. However, when $\varepsilon_L/z_\ell < \varepsilon_H/z_h$, the indifference curves for the buyers are "reversed" compared to figure 2, i.e., U^b_{Hh} is steeper than $U^b_{L\ell}$, which implies that the incentive-compatible offer for the high buyer holding the heavy coin that maximizes his surplus is given by $(q_\ell, 1)$. Hence, the equilibrium cannot be separating and, in an equilibrium where both heavy and light coins circulate, they will circulate by tale.

Because a by-tale equilibrium will turn out to be qualitatively equivalent to an equilibrium where lotteries are not allowed, which have already been described in various papers (Velde, Weber, and Wright (1999) and Burdett, Trejos, and Wright (2001)), we will focus most of our attention on equilibria that features circulation by weight. These equilibria are of particular interest because they allow for the possibility of signaling. Also, the required condition for circulation by weight, $\varepsilon_L/z_\ell > \varepsilon_H/z_h$, will be satisfied for standard specifications for utility and cost functions (e.g., $u(q) = q^a$, with $a \in (0,1)$ and $\psi(q) = q$).

3.1 By-weight circulation

In this section we will examine equilibria where circulation is by weight. We will first characterize the set of by-weight equilibrium offers assuming that the high buyer mints a heavy coin at date 1. In informed meetings, a buyer holding the heavy coin will make the offer $(q_h, 1)$, and a buyer holding the light coin will make the offer $(q_\ell, 1)$. The lemmas below describe what happens in uninformed meetings.

Lemma 7 *Assume $\varepsilon_L/z_\ell > \varepsilon_H/z_h$. In any equilibrium where high-type buyers hold heavy coins, the low buyer will mint a light coin and will make the full information offer,*

$$[q^u(\varepsilon_L, z_\ell), p^u(\varepsilon_L, z_\ell)] = (q_\ell, 1). \tag{16}$$

The high buyer's offer maximizes his surplus

$$[q^u(\varepsilon_H, z_h), p^u(\varepsilon_H, z_h)] = \arg\max_{q,p\leq 1} \varepsilon_H u(q) - pz_h \tag{17}$$

subject to the seller accepting and the low buyer not mimicking the offer, i.e.,

$$-\psi(q) + pz_h \geq 0 \tag{18}$$

$$\varepsilon_L u(q_\ell) - z_\ell \geq \varepsilon_L u(q) - pz_\ell, \tag{19}$$

respectively. Furthermore, if buyers defect from their equilibrium minting strategies, their date 2 offers are

$$\begin{aligned} [q^u(\varepsilon_H, z_\ell), p^u(\varepsilon_H, z_\ell)] &= (q_\ell, 1), \\ [q^u(\varepsilon_L, z_h), p^u(\varepsilon_L, z_h)] &= [q^u(\varepsilon_H, z_h), p^u(\varepsilon_H, z_h)]. \end{aligned}$$

Proof. See Appendix. ■

The equilibrium offer by a low buyer holding a light coin, $(q_\ell, 1)$, is represented in figure 3 at the intersection of the participation constraint of the seller who believes that the buyer is holding a light coin, $U^s_\ell = 0$, and $p = 1$. The indifference curve of a low buyer holding a light coin that goes through this point, $U^b_{L\ell} = \varepsilon_L u(q_\ell) - z_\ell$, represents the equilibrium surplus of the low buyer. Given that the equilibrium is separating, i.e., by their offers the buyers essentially reveal their type and the coin that they are holding, the low buyer holding the light coin can do no better than he could in an informed match.

In contrast, signaling is costly for the high buyer holding a heavy coin, and his payoff is lower than what it would be in an informed match. The best offer that a high buyer holding a heavy coin can propose, (17), must (i) satisfy the participation constraint of seller who believes that the buyer is holding a heavy coin, (18), and (ii) not be imitated by the low buyer holding a light coin, (19). The seller's participation constraint with an equality is depicted by U^s_h in figure 3, and the low buyer's incentive-compatibility constraint with an equality is given by $U^b_{L\ell}$. It can be seen from figure 3 that the solution to (17)–(19) is at the intersection of the seller's participation constraint, U^s_h, and the low buyer's incentive-compatibility constraint, $U^b_{L\ell}$. Notice that $q^u(\varepsilon_H, z_h) < q^u(\varepsilon_L, z_\ell) = q_\ell$ and $p^u(\varepsilon_H, z_h) < p^u(\varepsilon_L, z_\ell) = 1$. In order to signal the weight of his coin, the high buyer proposes an offer with lower output and a lower probability to deliver his coin, compared to the low buyer's offer.

The above lemma simply *assumes* that the high buyer will mint the heavy coin in date 1. Will a high buyer have an incentive to mint the heavy coin? From (15), the high buyer will mint a heavy coin if

$$-rz_h + \theta\Delta_{Hh} + (1-\theta)\,\Delta^u_{Hh} \geq -rz_\ell + \Delta_{H\ell}, \tag{20}$$

where $\Delta_{ji} = \varepsilon_j u[q(\varepsilon_j, z_i)] - p(\varepsilon_j, z_i) z_i$ is the surplus of a buyer of type (j,i) in an informed match with $i \in \{\ell, h\}$ and $j \in \{L, H\}$. Similarly, Δ^u_{ji} is the surplus of a buyer of type (j,i) in an uninformed match, $\Delta^u_{ji} = \varepsilon_j u[q^u(\varepsilon_j, z_i)] - p^u(\varepsilon_j, z_i) z_i$. Condition (20) can be re-expressed

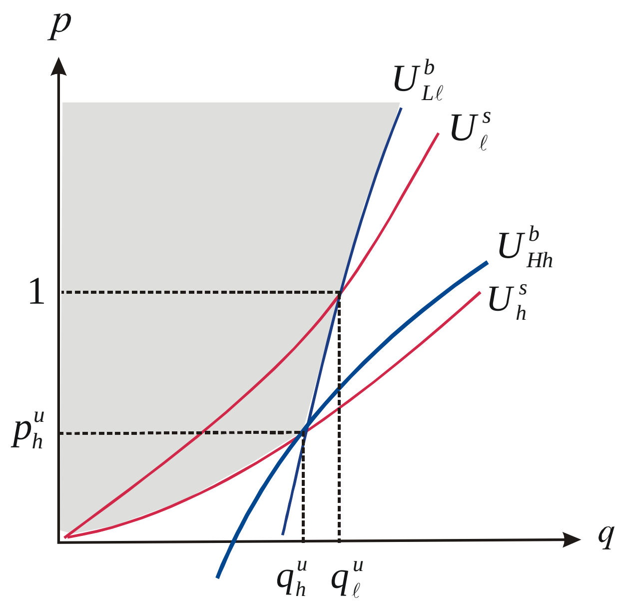

Figure 3: Separating offer.

as

$$\theta \geq \theta_c \equiv \frac{r\left(z_h - z_\ell\right) + \Delta_{H\ell} - \Delta^u_{Hh}}{\Delta_{Hh} - \Delta^u_{Hh}}. \tag{21}$$

It can be demonstrated that $0 < \theta_c < 1$.[13] As long as the information problem is not too severe, high buyers will mint heavy coins and low buyers will mint light ones. The benefits from trading with heavy coins in informed matches outweigh the costs associated with signaling for high buyers in uninformed matches.

Now let's turn to single-currency equilibria in which high buyers mint light coins. In that case, the following lemma describes the equilibrium offers that buyers make:

Lemma 8 *Assume $\varepsilon_L/z_\ell > \varepsilon_H/z_h$. In any equilibrium where high-type buyers mint light coins,*

$$[q^u(\varepsilon_H, z_\ell), p^u(\varepsilon_H, z_\ell)] = [q^u(\varepsilon_L, z_\ell), p^u(\varepsilon_L, z_\ell)] = (q_\ell, 1)\,.$$

[13] From (19), it can be checked that

$$\Delta^u_{Hh} = \Delta_{H\ell} - \left(\frac{\varepsilon_H}{\varepsilon_L} - 1\right) z_\ell - p^u(\varepsilon_H, z_h)\left(z_h - \frac{\varepsilon_H}{\varepsilon_L} z_\ell\right),$$

i.e., the surplus of a high-type buyer with a heavy coin in an uninformed meeting is lower than that of a high-type buyer with a light coin. This guarantees that $\theta_c > 0$.

Furthermore, if buyers defect from equilibrium play and mint heavy coins, their (out-of-equilibrium) offers satisfy $[q^u(\varepsilon_H, z_h), p^u(\varepsilon_H, z_h)] = [q^u(\varepsilon_L, z_h), p^u(\varepsilon_L, z_h)]$*, where* $[q^u(\varepsilon_H, z_h), p^u(\varepsilon_H, z_h)]$ *is given by the solution to (17)–(19).*

Proof. See Appendix. ■

Intuitively, the above equilibrium offer extracts all of the surplus from the seller, given that the buyer—who is either high or low—is holding a light coin. A single-currency equilibrium will exist only if the high-type buyer has an incentive to hold the light coin, and this will happen only when θ is smaller than the threshold θ_c. One can also show that there is no single-currency equilibrium for $\theta > \theta_c$.[14] The following proposition summarizes the above discussion.

Proposition 9 *Assume* $\varepsilon_L/z_\ell > \varepsilon_H/z_h$*. There exists a threshold* θ_c *such that: (i) If* $\theta > \theta_c$*, then there can only exist a circulation by-weight equilibrium, in which both heavy and light coins circulate; (ii) If* $\theta \leq \theta_c$*, then there can only exist a single currency equilibrium where all buyers hold light coins.*

Proposition 9 shows that even though high buyers can separate themselves from low buyers, there is a threshold for θ below which heavy coins are driven out of circulation. The reason a high buyer would choose to mint a light coin is that when the information problem is severe, the holder of a heavy coin incurs a large signaling cost by reducing his average consumption in the second period. It is better for the high buyer to avoid these signaling costs by holding a light coin.

3.2 By-tale circulation

In this section, we examine equilibria in which heavy and light coins buy exactly the same amount of goods in uninformed matches. Since a pooling equilibrium is rather straightforward

[14] To see this, suppose that there exists an offer $[q^u(\varepsilon_H, z_h), p^u(\varepsilon_H, z_h)]$ for which a single-currency equilibrium exists for $\theta > \theta_c$. In this case, $[q^u(\varepsilon_H, z_h), p^u(\varepsilon_H, z_h)]$ does not satisfy (17)–(19); therefore, the surplus of a high buyer with a heavy coin in an uninformed match is less than he would get if he made the offer in (17)–(19). But then a high buyer would have a profitable deviation by holding a heavy coin and making the offer in (17)–(19). Since $\theta > \theta_c$, the payoff associated with this deviation dominates the equilibrium payoff of a high buyer holding a light coin; therefore, it should be attributed to a buyer with a heavy coin, from the Cho–Kreps refinement.

to characterize, our treatment can be brief.

Circulation by tale requires, from proposition 6, that $\varepsilon_L/z_\ell < \varepsilon_H/z_h$ and that the high buyer has an incentive to mint a heavy coin at date 1. We will restrict our attention to what we think is the natural by-tale equilibrium: one where all coins are traded with probability one, i.e., $p^u(\varepsilon_H, z_h) = p^u(\varepsilon_L, z_\ell) = 1$.[15] The quantity traded in uninformed matches, q^u, is not uniquely determined, but it must satisfy the seller's participation constraint,

$$\psi(q^u) \leq \lambda z_h + (1-\lambda) z_\ell, \tag{22}$$

and it must be at least as good as the offer of a low buyer in an informed match, $q^u \geq q_\ell$. A high buyer will have an incentive to mint a heavy coin if

$$-rz_h + \theta\Delta_{Hh} + (1-\theta)\left[\varepsilon_H u(q^u) - z_h\right] > -rz_\ell + \theta\Delta_{H\ell} + (1-\theta)\left[\varepsilon_H u(q^u) - z_\ell\right] \tag{23}$$

or if

$$\theta > \bar{\theta} = \frac{(1+r)\,(z_h - z_\ell)}{\varepsilon_H u\left[q\left(\varepsilon_H, z_h\right)\right] - \varepsilon_H u\left[q\left(\varepsilon_H, z_\ell\right)\right]}. \tag{24}$$

As in the previous section, both heavy and light coins can co-circulate provided that the information problem is not too severe; but unlike the previous section, heavy and light coins buy exactly the same amount of the specific good.

When the information problem does become severe, then—as in the previous section—high buyers will choose to mint light coins. High buyers will, in fact, choose to mint light coins if $\theta \leq \bar{\theta}$.[16] The results of this section are conveniently summarized in the following proposition,

Proposition 10 *Assume $\varepsilon_L/z_\ell < \varepsilon_H/z_h$. If $\theta > \bar{\theta}$ then there is an equilibrium with by-tale circulation. A by-tale equilibrium is not unique; although both heavy and light coins will be*

[15] If high buyers strictly prefer holding heavy coins to holding light ones, then the Cho–Kreps refinement implies that $p^u(\varepsilon_H, z_h) = p^u(\varepsilon_L, z_\ell) = 1$. We view this as being the "natural" equilibrium outcome because $p^u(\varepsilon_H, z_h) = p^u(\varepsilon_L, z_\ell) < 1$ is a knife-edge case in which the high buyer is just indifferent between minting a light and heavy coin.

[16] If a high buyer deviates from the single-currency equilibrium strategy of minting a light coin by minting a heavy coin, his best offer is $(q_\ell, 1)$. Indeed, from the assumption $\varepsilon_L/z_\ell < \varepsilon_H/z_h$, U^b_{Hh} is steeper than $U^b_{L\ell}$ in the (q,p) space. Therefore, the best offer that a high-type buyer with a heavy coin can make is $(q^u(\varepsilon_L, z_\ell), p^u(\varepsilon_L, z_\ell)) = (q_\ell, 1)$. Hence, the high buyer's equilibrium payoff is given by the right-hand side of (23), while the defection payoff is given by the left-hand side of (23).

traded with probability one, the output level is not pinned down. A single-currency equilibrium exists if $\theta \leq \bar{\theta}$.

Proposition 10 describes a Gresham's-law type of phenomenon. The heavy and light coins are traded at par in some matches because of asymmetries of information. As a consequence, if the fraction of uninformed matches gets sufficiently large, then Gresham's law is activated: All buyers choose to mint light coins and heavy coins disappear from circulation.

4 Recognizability and welfare

We now consider various positive and normative aspects associated with the recognizability of coins. We examine how recognizability affects output, welfare and the velocity of currency. We define the velocity of coin $i \in \{\ell, h\}$, v_i, as the average probability that the coin changes hands in a bilateral match, i.e.,

$$v_i = \theta p_i + (1-\theta) p_i^u, \tag{25}$$

where p_i and p_i^u are the probabilities that the coin of weight i changes hands in informed and uninformed matches, respectively. We measure aggregate output, Y, as the sum of the quantities traded in bilateral matches, i.e.,

$$Y = \int \left[\theta q(\varepsilon, z) + (1-\theta) q^u(\varepsilon, z)\right] dF(\varepsilon, z), \tag{26}$$

where $F(\varepsilon, z)$ is the distribution of buyers' types $(\varepsilon, z) \in \{\varepsilon_L, \varepsilon_H\} \times \{z_\ell, z_h\}$. Finally, social welfare, W, is the sum of the utilities of all agents in the economy, i.e.,

$$W = \int \mathcal{U}^b(\varepsilon, z) dF(\varepsilon, z) + \mathcal{U}^s, \tag{27}$$

where $\mathcal{U}^s$ is the expected utility of a seller and $\mathcal{U}^b(\varepsilon, z)$ is the expected utility of an ε buyer who mints a coin of weight z, and $\mathcal{U}^b(\varepsilon, z)$, satisfies

$$\beta^{-1}\mathcal{U}^b(\varepsilon, z) = -rz + \left\{\theta\left[\varepsilon u\left[q(\varepsilon, z)\right] - p(\varepsilon, z)z\right] + (1-\theta)\left[\varepsilon u\left[q^u(\varepsilon, z)\right] - p^u(\varepsilon, z)z\right]\right\}.$$

Note that $\mathcal{U}^s = 0$ for all single-currency equilibria and for the dual-currency equilibria where circulation is by weight. However, $\mathcal{U}^s$ can differ from zero in some equilibria with circulation by tale.

We first describe the effects of a change in recognizability on the different types of equilibria previously studied (assuming this change does not put the economy in a different type of equilibrium). We then investigate the effects of a change in recognizability that triggers a transition from a dual-currency equilibrium to a single-currency equilibrium on output, welfare, and velocity. Finally, we discuss how our model might also be viewed as a model of counterfeiting.

4.1 By-weight circulation

Consider first the case where equilibrium in the economy is characterized as by-weight circulation. In such an equilibrium, light coins are traded with probability one in all matches, i.e., $v_\ell = 1$, and a light coin buys q_ℓ units of output. In contrast, from (18) and (19), heavy coins are traded with probability less than one in uninformed matches, $p^u(\varepsilon_H, z_h) < 1$, and the velocity of heavy coins is given by

$$v_h = \theta + (1-\theta)p^u(\varepsilon_H, z_h). \tag{28}$$

From (18) and (19), the terms of trade are determined by the incentive-compatibility condition for low buyers and the individual-rationality condition for sellers. From this, $p^u(\varepsilon_H, z_h)$ is independent of the fraction of informed matches, θ. Therefore, the velocity of money increases with the level of recognizability, θ, because heavy coins have a higher velocity in informed matches.

The higher velocity associated with greater recognizability translates into higher aggregate output and higher welfare. To see this, note from (26) that aggregate output is

$$Y = \lambda\left[\theta q_h + (1-\theta)q^u(\varepsilon_H, z_h)\right] + (1-\lambda)q_\ell. \tag{29}$$

According to (29), high buyers, who represent a fraction λ of all buyers, consume q_h in informed matches and $q^u(\varepsilon_H, z_h)$ in uninformed matches; low buyers consume q_ℓ in all matches. Because

$q^u(\varepsilon_H, z_h) < q_h$, aggregate output will increase as coins become more recognizable, i.e., as θ increases.

From (27), society's welfare is given by

$$W = \lambda \left\{ -z_h + \beta\theta\varepsilon_H u(q_h) + \beta(1-\theta)\left[\varepsilon_H u(q_h^u) + (1-p_h^u)z_h\right]\right\} \\ +(1-\lambda)\left[-z_\ell + \beta\varepsilon_L u(q_\ell)\right]. \tag{30}$$

Equation (30) has the following interpretation: A high buyer produces z_h units of output in the first period in order to mint the heavy coin. In the second period, he consumes q_h and trades his coin with probability one in informed matches; he consumes q_h^u and trades his coin with probability p_h^u in uninformed matches. A low buyer produces z_ℓ units of output in the first period in order to mint a light coin and always consumes q_ℓ in the second period. Whereas the expected utility of a low buyer is independent of θ, the expected utility of a high buyer increases with θ because $\varepsilon_H u(q_h) - z_h > \varepsilon_H u(q_h^u) - p_h^u z_h$. Hence, social welfare increases with the recognizability of coins because, in order to separate themselves from buyers holding light coins, buyers with heavy coins trade with a lower probability and buy less output in uninformed matches. Consequently, as the recognizability of coins improves, both output and welfare increase.

These results are summarized in

Proposition 11 *Consider an equilibrium with circulation by weight. Output, welfare, and the velocity of heavy coins all increase with θ.*

4.2 By-tale circulation

Consider now the case where the equilibrium is characterized as by-tale circulation. Recall that there are multiple equilibria with by-tale circulation that differ in the amount of output produced in uninformed matches. Even when we restrict our attention to equilibria where $p^u = 1$, the output traded in an uninformed match can be anywhere between q_ℓ and

$\psi^{-1}[\lambda z_h + (1-\lambda)z_\ell]$.[17] Here, we will focus on the equilibrium where coins are traded with probability one in uninformed matches, $p^u = 1$, and the participation constraint for the seller binds, $\psi(q^u) = \lambda z_h + (1-\lambda)z_\ell$.[18] Because all coins are traded with probability one, the velocity of coins is not affected by a change in the recognizability; hence, $v_\ell = v_h = 1$ for all θ. Aggregate output is given by

$$Y = \lambda\theta q_h + (1-\lambda)\theta q_\ell + (1-\theta)q^u. \tag{31}$$

From the convexity of $\psi(q)$, it is easy to check that $\lambda q_h + (1-\lambda)q_\ell \leq q^u$, where the inequality is strict if ψ is strictly convex.[19] Hence, as coins become more recognizable, aggregate output actually *falls*. The reason is that the output traded in an uninformed match is larger than the expected output in an informed match. So in contrast with what was obtained in the equilibrium with circulation by weight, aggregate output does not increase as the recognizability of the currency improves. This result, however, is not robust across all by-tale equilibria. For example, a by-tale equilibrium with $q^u = q_\ell$ can be sustained by the belief that any out-of-equilibrium offer in an uninformed match comes from a buyer holding a light coin. In such an equilibrium, output, Y, increases with θ.

When $p^u = 1$ and $\psi(q^u) = \lambda z_h + (1-\lambda)z_\ell$, social welfare is measured by

$$\begin{aligned}\beta^{-1}W = \lambda\left\{-rz_h + \theta\left[\varepsilon_H u(q_h) - \psi(q_h)\right] + (1-\theta)\left[\varepsilon_H u(q_u) - \psi(q_u)\right]\right\} + \\ +(1-\lambda)\left\{-rz_\ell + \theta\left[\varepsilon_L u(q_\ell) - \psi(q_\ell)\right] + (1-\theta)\left[\varepsilon_L u(q_u) - \psi(q_u)\right]\right\}.\end{aligned} \tag{32}$$

It can be shown that society's welfare increases with θ (see the appendix). Even though aggregate output can increase as the currency becomes less recognizable, social welfare will fall because of misallocation of output between high and low buyers. In other words, the coins'

[17] Recall that any q between q_ℓ and $\psi^{-1}[\lambda z_h + (1-\lambda)z_\ell]$ can be sustained by the following belief: If a buyer makes an offer asking for more output than q, the seller believes that this offer comes from a buyer holding a light coin and will reject it.

[18] This equilibrium corresponds to the no-lottery equilibrium studied in Velde et al. (1999) and Burdett et al. (2001).

[19] Since $\psi^{-1}(\cdot)$ is concave,

$$\psi^{-1}[\lambda z_h + (1-\lambda)z_\ell] \geq \lambda\psi^{-1}(z_h) + (1-\lambda)\psi^{-1}(z_\ell).$$

lack of recognizability implies that high and low buyers consume the same quantity of output in uninformed matches, although efficiency would dictate that high buyers consume more than low buyers.

The above results are summarized in

Proposition 12 *Consider an equilibrium with circulation by weight. Although an increase in θ increases welfare, it does not affect the velocity of coins and it can either increase or decrease output.*

Propositions 11 and 12 both capture the idea that there is a welfare gain associated with making a currency more recognizable. The gains in welfare, however, arise for different reasons. In a by-weight equilibrium, social welfare increases with recognizability because the velocity of the heavy coin increases. The low velocity of the heavy coin originates in high buyers' need to signal the quality of their coin by offering to consume less. In a by-tale equilibrium, there are no signaling costs, because the equilibrium is pooling. The welfare gain from having a currency that is more recognizable arises here from a better allocation of output among low and high buyers: Buyers with a high marginal utility of consumption consume more, whereas buyers with a low marginal utility of consumption consume less.

4.3 Gresham's law

We know from propositions 9 and 10 that a reduction in the recognizability of coins triggers a Gresham's-law type of phenomenon. If $\varepsilon_L/z_\ell > \varepsilon_H/z_h$ and $\theta < \theta_c$ or if $\varepsilon_L/z_\ell < \varepsilon_H/z_h$ and $\theta < \bar{\theta}$, then all buyers mint light coins, and heavy coins are driven out circulation. We now want to assess the welfare consequences of a transition from a dual-currency to a single-currency equilibrium.

Consider first the case where $\varepsilon_L/z_\ell > \varepsilon_H/z_h$, so that the equilibrium corresponds to circulation by weight, provided that θ is sufficiently large. Low buyers trade z_ℓ for q_ℓ in both the single- and dual-currency equilibria. Hence, the welfare of low buyers does not depend upon θ. The welfare of high buyers is, however, minimized when $\theta < \theta_c$. To see this, recall that high buyers always have the option of minting light coins in the first period of their lives. If they choose to mint heavy coins, as they do when $\theta > \theta_c$, their welfare must be higher than what they would obtain by minting light coins.

Consider next the case where $\varepsilon_L/z_\ell < \varepsilon_H/z_h$, so that the equilibrium corresponds to circulation by tale, provided that θ is sufficiently large. If $\theta > \bar{\theta}$, then there is an equilibrium with circulation by tale such that $q^u \geq q_\ell$. Thus low buyers are as well-off or worse-off at the single-currency equilibrium compared to the dual-currency, by-tale equilibrium, because the circulation of heterogenous coins allows low buyers to consume more than the intrinsic value of their light coins in uninformed matches. In contrast, the welfare of high buyers is lower in the single-currency equilibrium. To see this, consider an equilibrium where both high and light coins are minted. A high buyer always has the option to mint light coins, in which case he consumes q_ℓ in informed matches and $q^u \geq q_\ell$ in uninformed matches. In a single-currency equilibrium, they would mint a light coin and consume q_ℓ in all matches. Consequently, high buyers have a higher expected utility in the dual-currency equilibrium than in the single-currency equilibrium.

Proposition 13 *A decrease in θ that triggers a transition from a dual-currency equilibrium to a single-currency equilibrium is welfare-worsening in a Pareto sense.*

A decrease in coins' recognizability reduces welfare when it drives heavy coins out of circulation. Heavy coins, which are useful to high buyers, may no longer be used if the asymmetries of information are sufficiently severe.[20]

4.4 Counterfeiting

Propositions 11, 12, and 13 have all shown that coins' imperfect recognizability imposes welfare costs on society. One can make this point in a rather dramatic way by focusing on a limiting case where the weight of the light coin approaches zero. One can interpret the situation where

[20] It should be noted that the above proposition does not imply that a dual coin arrangement is necessarily better than an arrangement with a single coin. Indeed, the experiment that we have considered consists in taking the denomination structure $\{z_\ell, z_h\}$ that is ideal in the absence of an information problem and seeing how, *given this denomination structure*, the recognizability of coins affects welfare. Designing an *optimal* denomination structure in the presence of asymmetric information is left for future investigation.

$z_\ell \to 0$ as one of counterfeiting, where the heavy coin is the "genuine currency" and the light coin is the counterfeit. The assumption that the weight of the light coin is almost zero captures the idea that the marginal cost of producing counterfeit currency is close to zero.[21]

Assume $\varepsilon_L/z_\ell > \varepsilon_H/z_h$ so that the equilibrium outcome has the potential to be separating. The following proposition characterizes the terms of trade in uninformed matches:

Proposition 14 *Assume $\varepsilon_L/z_\ell > \varepsilon_H/z_h$. As $z_\ell \to 0$, both $q^u(\varepsilon_H, z_h)$ and $q^u(\varepsilon_H, z_\ell)$ approach 0.*

Proof. From (19), we have $q^u(\varepsilon_L, z_\ell) \geq q^u(\varepsilon_H, z_h)$. But $q^u(\varepsilon_L, z_\ell) = q_\ell \to 0$ as z_ℓ approaches 0. ■

According to Proposition 14, the quantities produced in uninformed matches tends to zero as the intrinsic value of light coins tend to zero. In other words, if the light coin is almost costless to produce, trade in uninformed meetings shuts down. The intuition for this result is simple. The buyer with a genuine coin who wishes to separate himself from a buyer with a counterfeit coin looks for an offer that the buyer with a counterfeit coin does not want to imitate. However, in a separating equilibrium, the utility of a buyer with a counterfeit coin is almost zero. Therefore, the only offer that a buyer with genuine coin can make is such that q^u is close to 0.

Proposition 14 has dramatic implications for the way the economy works. If $\theta > \theta_c$, then buyers with high marginal utility of consumption mint heavy coins and trade only if they are in informed meetings. As θ decreases, the number of meetings in which trades take place falls. As θ falls below the threshold θ_c, high buyers have no incentive to mint heavy coins because the probability that they can use them in a bilateral match is too small. As a consequence, the entire economy shuts down.

[21] For a models of counterfeiting with fiat currencies, see Kulti (1996), Green and Weber (1996), and Nosal and Wallace (2004).

5 Conclusion

In this paper we have studied the effects of the imperfect recognizability of coins on output, welfare and the velocity of money. We have developed a simple model in which heterogenous buyers can trade with two different coins, a light coin and a heavy one. The weights of the coins correspond to their optimal weights in a world with complete information. The terms of trade in bilateral matches are determined by take-it-or-leave-it offers by buyers, and we have allowed the use of lotteries to overcome the indivisibility of coins. We have characterized the different types of equilibria that can emerge in the presence of asymmetric information. If is it very difficult to distinguish between light and heavy coins, then the equilibrium will be characterized by a single coin, the light one. This outcome has a Gresham's-law flavor to it. If the recognizability problem is not too severe, then both heavy and light coins will circulate. We have established conditions under which equilibrium is separating in the sense that heavy and light coins are always traded according to different terms. In such equilibria, velocity, output, and welfare increase with the recognizability of coins. Furthermore, as the weight of the light coin tends to zero, so does the quantity traded in uninformed matches. Therefore, economic activity shuts down when agents can counterfeit good coins at a negligible cost. In pooling or by-tale equilibria, velocity and output do not necessarily increase with the recognizability of coins, but welfare does.

Our model could be extended to characterize the optimal denomination structure of the economy in the presence of incomplete information. In particular, one may wonder whether a uniform currency is better than two currencies for overcoming the recognizability problem. Our model could also be used to discuss issues related to counterfeiting and how different policies can prevent it. We leave these extensions to future research.

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Appendix

A1. Proof of Lemma 5

Step 1: From (14), a low buyer with a light coin weakly prefers to trade with uninformed sellers than with informed sellers. This means that

$$\varepsilon_L u\left[q^u(\varepsilon_L, z_\ell)\right] - p^u(\varepsilon_L, z_\ell) z_\ell \geq \varepsilon_L u\left(q_\ell\right) - z_\ell.$$

Consequently,

$$-r z_\ell + \theta \left\{\varepsilon_L u\left(q_\ell\right) - z_\ell\right\} + (1-\theta)\left\{\varepsilon_L u\left[q^u(\varepsilon_L, z_\ell)\right] - p^u(\varepsilon_L, z_\ell) z_\ell\right\} \geq$$
$$-r z_\ell + \varepsilon_L u\left(q_\ell\right) - z_\ell. \tag{33}$$

Step 2: If information is complete, a low buyer prefers to hold a light coin than a heavy coin. This means that

$$-r z_\ell + \varepsilon_L u\left(q_\ell\right) - z_\ell > -r z_h + \varepsilon_L u\left(q_h\right) - z_h. \tag{34}$$

Step 3: From (14), a low buyer holding a heavy coin weakly prefers to trade with an informed seller than with an uninformed one. It implies

$$-r z_h + \varepsilon_L u\left(q_h\right) - z_h \geq$$
$$-r z_h + \theta\left\{\varepsilon_L u\left(q_h\right) - z_h\right\} + (1-\theta)\left\{\varepsilon_L u\left[q^u(\varepsilon_L, z_h)\right] - p^u(\varepsilon_L, z_h) z_h\right\}. \tag{35}$$

From (33)–(35), we deduce that low-type buyers strictly prefer to hold light coins.

A2. Proof of Proposition 6

We first show that if $\varepsilon_L/z_\ell > \varepsilon_H/z_h$, then in any equilibrium where high buyers hold heavy coins, $[q^u(\varepsilon_H, z_h), p^u(\varepsilon_H, z_h)] \neq [q^u(\varepsilon_L, z_\ell), p^u(\varepsilon_L, z_\ell)]$. This implies that there exists no equilibrium with circulation by tale. The proof is diagrammatic. In figure A.1, we denote U^b_{ij} as the locus of points in (q,p)-space that generates the same surplus from the bargaining game for an ε_i buyer, where $i \in \{H, L\}$, holding a coin of weight z_j, where $j \in \{h, \ell\}$. The equation for this indifference curve is

$$U^b_{ij} = \varepsilon_i u(q) - p z_j, \qquad \forall (i,j) \in \{H, L\} \times \{h, \ell\}.$$

Similarly, U_j^s denotes the indifference curve for a seller who believes that the buyer he is matched with holds a coin of weight z_j, where $j \in \{h, \ell\}$. The equation for this indifference curve is

$$U_j^s = -\psi(q) + pz_j, \qquad \forall j \in \{h, \ell\}\,.$$

Consider an equilibrium where high buyers hold heavy coins and suppose that, contrary to the claim made above, $[q^u(\varepsilon_H, z_h), p^u(\varepsilon_H, z_h)] = [q^u(\varepsilon_L, z_\ell), p^u(\varepsilon_L, z_\ell)] = (q^u, p^u)$ in an uninformed meeting. We represent the equilibrium utility levels of a low buyer holding light coins by $U_{L\ell}^b$ and a high buyer holding heavy coin by U_{Hh}^b, when both buyers make the offer (q^u, p^u) in figure A.1. (We also depict the utility that a high buyer can expect to receive if he deviates from the proposed equilibrium of holding a heavy coin and, instead, chooses to hold a light coin and offers (q^u, p^u); this is indicated by the the dotted indifference curve labeled $U_{H\ell}^b$.)

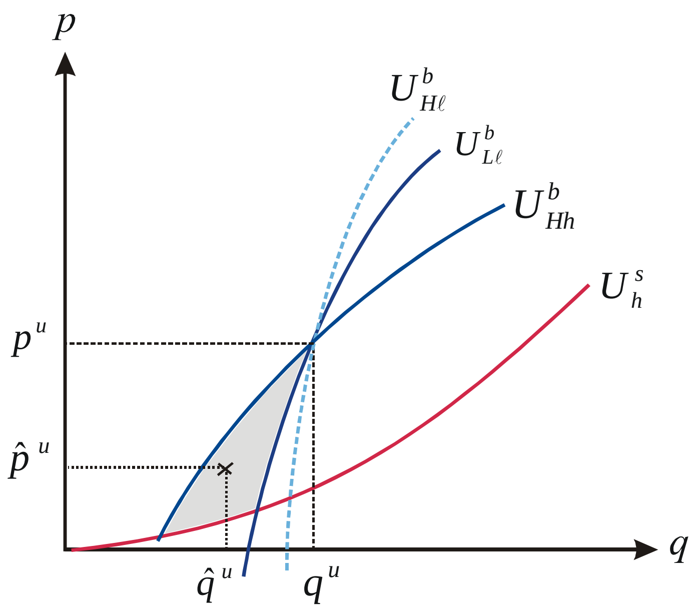

Figure A.1. No pooling equilibrium.

Because $\varepsilon_L/z_\ell > \varepsilon_H/z_h$, $U_{L\ell}^b$ is steeper than U_{Hh}^b. Consider now the out-of-equilibrium offer $(\hat{q}^u, \hat{p}^u)$ made by some buyer (see figure A.1). If such an offer were accepted, it would reduce the utility of *any* buyer holding a light coin compared to the utility associated with offer

(q^u, p^u); i.e., offer $(\hat{q}^u, \hat{p}^u)$ is located to the left of the indifference curves $U^b_{L\ell}$ and $U^b_{H\ell}$. However, offer $(\hat{q}^u, \hat{p}^u)$ would increase the utility of a high buyer holding a heavy coin compared to the utility associated with the proposed equilibrium offer (q^u, p^u); i.e., offer $(\hat{q}^u, \hat{p}^u)$ is located to the right of the indifference curve U^b_{Hh}. Therefore, according to the Cho–Kreps criterion, the seller should believe that offer $(\hat{q}^u, \hat{p}^u)$ comes from a high buyer holding a heavy coin. Finally, the offer $(\hat{q}^u, \hat{p}^u)$ provides the seller with a payoff that is greater than zero. To see this, note first that (q^u, p^u) is an acceptable offer given the seller's initial belief, λ, that the high buyer is holding a heavy coin. Therefore, (q^u, p^u) is located above the zero payoff indifference curve of the seller who believes that the buyer is holding a heavy coin, denoted by U^s_h in figure A.1. The deviating offer $(\hat{q}^u, \hat{p}^u)$ is also chosen to be located above the indifference curve of the seller who believes that the buyer is holding a heavy coin, so that the seller will accept the offer. Hence, it is not possible to have an equilibrium in which the high buyer with a heavy coin and the low buyer with a light coin make the same offer.

We next show that if $\varepsilon_L/z_\ell < \varepsilon_H/z_h$, then in any equilibrium where high buyers hold heavy coins,

$$[q^u(\varepsilon_H, z_h), p^u(\varepsilon_H, z_h)] = [q^u(\varepsilon_L, z_\ell), p^u(\varepsilon_L, z_\ell)] = (q^u, p^u).$$

Assume that $[q^u(\varepsilon_H, z_h), p^u(\varepsilon_H, z_h)] \neq [q^u(\varepsilon_L, z_\ell), p^u(\varepsilon_L, z_\ell)]$. Since low buyers are identified by their offers, they propose the same terms of trade as those that prevail in informed matches, i.e., $[q^u(\varepsilon_L, z_\ell), p^u(\varepsilon_L, z_\ell)] = (q_\ell, 1)$. The surplus of the low buyer is represented in figure A.2 by the indifference curve $U^b_{L\ell}$.

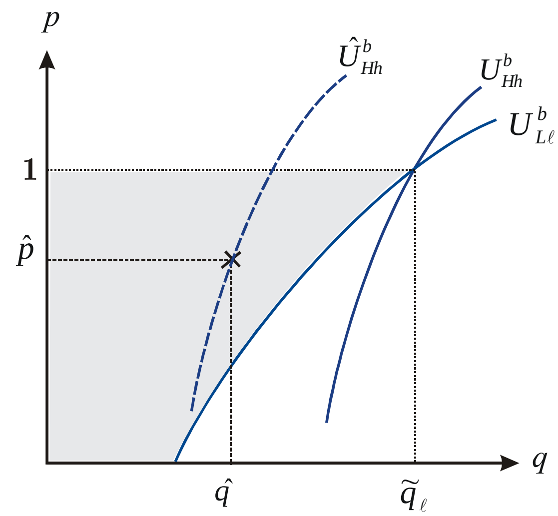

Figure A.2. No separating equilibrium.

Incentive compatibility requires that the offer made by a high buyer be located in the shaded area in figure A.2, where U^b_{Hh} is the indifference curve of a high buyer holding a heavy coin who makes the offer $(q_\ell, 1)$. It is easy to see that any offer [not equal to $(q_\ell, 1)$] in the shaded area—e.g., $(\hat{q}, \hat{p})$—provides a lower payoff to the high buyer than the payoff associated with offer $(q_\ell, 1)$. In figure A.2, $\hat{U}^b_{Hh} < U^b_{Hh}$; hence, there cannot exist a by-weight (separating) equilibrium.

A3. Proof of Lemma 7

Consider first the equilibrium offer in an uninformed match of a low buyer holding a light coins, $[q^u(\varepsilon_L, z_\ell), p^u(\varepsilon_L, z_\ell)]$. Since from proposition 6 a buyer reveals his type through his offer, $\lambda\,[q^u(\varepsilon_L, z_\ell), p^u(\varepsilon_L, z_\ell)] = 0$. Therefore, a low buyer holding a light coin can do no better than making an offer that assumes that the seller can observe the coin he is holding, i.e., $[q^u(\varepsilon_L, z_\ell), p^u(\varepsilon_L, z_\ell)] = (q_\ell, 1)$.

Let us turn to the equilibrium offer in an uninformed match of a high buyer holding a heavy coin, $[q^u(\varepsilon_H, z_h), p^u(\varepsilon_H, z_h)]$. Since equilibrium is separating, $\lambda[q^u(\varepsilon_H, z_h),\ p^u(\varepsilon_H, z_h)] = 1$. An offer cannot violate (18), otherwise, it would be rejected by a seller; nor can it violate (19);

otherwise, low buyers would have an incentive to deviate from their equilibrium offer. If the equilibrium offer did not maximize the utility of a high buyer holding a heavy coin in (17) subject to (18) and (19), then one could construct a profitable deviation, as in the proof of proposition 6.

Consider next the offer of a high buyer who deviates in the first period by minting a light coin, $[q^u(\varepsilon_H, z_\ell), p^u(\varepsilon_H, z_\ell)]$. Any acceptable offer must satisfy (19); otherwise, the low buyer holding the light coin would have a profitable deviation. Since the indifference curve $U^b_{H\ell}$ is steeper than indifference curve $U^b_{L\ell}$, a high buyer holding the light coin can do no better than offering $[q^u(\varepsilon_L, z_\ell), 1]$, as in figure 3. Hence, a high buyer has no incentive to mint a light coin.

Finally, consider next the offer of a low buyer who deviates in the first period by minting a heavy coin, $[q^u(\varepsilon_L, z_h), p^u(\varepsilon_L, z_h)]$. Since the indifference curve U^b_{Hh} is steeper than indifference curve U^b_{Lh}, a low buyer holding a heavy coin cannot do better than offering $[q^u(\varepsilon_H, z_h), p^u(\varepsilon_H, z_h)]$ because otherwise the participation constraint of the seller would be violated, see figure 3. Hence, a low buyer has no incentive to mint a heavy coin.

A4. Proof of Lemma 8

In equilibrium, all buyers hold light coins. Consequently, buyers cannot do better than the offer they would make in an informed match, namely, $(q_\ell, 1)$.

Consider next the offer of a high buyer who deviates and mints a heavy coin. This offer, $[q^u(\varepsilon_H, z_h), p^u(\varepsilon_H, z_h)]$, must satisfy (19), so that a low buyer with a light coin has no incentive to deviate from his equilibrium offer, and it must also satisfy the seller's participation constraint (18). In figure A.3, the offer $[q^u(\varepsilon_H, z_h), p^u(\varepsilon_H, z_h)]$ must be located on or to the left of $U^b_{L\ell}$. As well, high buyer with a light coin must not have an incentive to deviate from his equilibrium offer by proposing $[q^u(\varepsilon_H, z_h), p^u(\varepsilon_H, z_h)]$. In figure A.3, the offer must be located on or to the left of $U^b_{H\ell}$. Also, any such offer must satisfy the participation constraint of the seller under the belief that he faces a buyer with a heavy coin. In figure A.3, the offer must be located above the curve U^s_h.

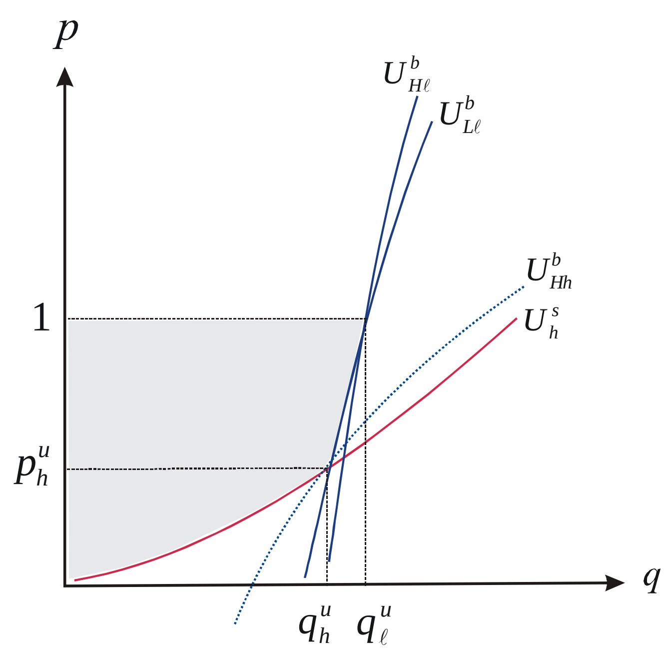

Figure A.3. Single currency equilibrium.

One can use the same reasoning as in lemma 7 to show that the offer $[q^u(\varepsilon_L, z_h), p^u(\varepsilon_L, z_h)]$ satisfies (18)–(19), and the best deviating offer is given by (q^u_h, p^u_h) in figure A.3. Hence, the high buyer will have no incentive to mint the heavy coin. The same reasoning applies to a low buyer who deviates and mints a heavy coin, i.e., the best deviating offer is given by (q^u_h, p^u_h).

A5. Proof of Proposition 12

We show that social welfare increases with θ. Here we will denote arbitrary weights for light and heavy coins as z_ℓ and z_h, respectively, and the ideal weights as $\tilde{z}_\ell$ and $\tilde{z}_h$. Social welfare is measured by

$$\beta^{-1} W^{\text{tale}} = \lambda \left\{ -r\tilde{z}_h + \theta \left[\varepsilon_H u(\psi^{-1}(\tilde{z}_h)) - \tilde{z}_h \right] + (1-\theta) \left[\varepsilon_H u(q^u) - \psi(q^u) \right] \right\} +$$
$$+ (1-\lambda) \left\{ -r\tilde{z}_\ell + \theta \left[\varepsilon_L u(\psi^{-1}(\tilde{z}_\ell)) - \tilde{z}_\ell \right] + (1-\theta) \left[\varepsilon_L u(q^u) - \psi(q^u) \right] \right\}.$$

Grouping all the terms in θ and $(1-\theta)$, we obtain

$$\beta^{-1}W^{\text{tale}} = \theta\left\{-(1+r)\left[\lambda\tilde{z}_h + (1-\lambda)\tilde{z}_\ell\right] + \lambda\varepsilon_H u\left[\psi^{-1}(\tilde{z}_h)\right] + (1-\lambda)\varepsilon_L u\left[\psi^{-1}(\tilde{z}_\ell)\right]\right\}$$
$$+(1-\theta)\left\{-(1+r)\left[\lambda\tilde{z}_h + (1-\lambda)\tilde{z}_\ell\right] + \left[\lambda\varepsilon_H + (1-\lambda)\varepsilon_L\right] u\left[\psi^{-1}\left[\lambda\tilde{z}_h + (1-\lambda)\tilde{z}_\ell\right]\right]\right\}. \quad (36)$$

We want to establish that

$$-(1+r)\left[\lambda\tilde{z}_h + (1-\lambda)\tilde{z}_\ell\right] + \lambda\varepsilon_H u\left[\psi^{-1}(\tilde{z}_h)\right] + (1-\lambda)\varepsilon_L u\left[\psi^{-1}(\tilde{z}_\ell)\right] >$$
$$-(1+r)\left[\lambda\tilde{z}_h + (1-\lambda)\tilde{z}_\ell\right] + \left[\lambda\varepsilon_H + (1-\lambda)\varepsilon_L\right] u\left[\psi^{-1}\left[\lambda\tilde{z}_h + (1-\lambda)\tilde{z}_\ell\right]\right]. \quad (37)$$

To do that, we consider the following maximization problem:

$$\max_{z_h, z_\ell}\left\{-(1+r)\left[\lambda z_h + (1-\lambda)z_\ell\right] + \lambda\varepsilon_H u\left[\psi^{-1}(z_h)\right] + (1-\lambda)\varepsilon_L u\left[\psi^{-1}(z_\ell)\right]\right\} \quad (38)$$
$$\text{s.t.} \quad \lambda z_h + (1-\lambda)z_\ell = \lambda\tilde{z}_h + (1-\lambda)\tilde{z}_\ell. \quad (39)$$

Note that the value of the objective function corresponds to the left-hand side of (37) when $(z_h, z_\ell) = (\tilde{z}_h, \tilde{z}_\ell)$ and it corresponds to the right-hand side of (37) when $z_h = z_\ell = \lambda\tilde{z}_h + (1-\lambda)\tilde{z}_\ell$. The solution to (38)–(39) is given implicitly by

$$\varepsilon_H \frac{u'\left[\psi^{-1}(z_h)\right]}{\psi'\left[\psi^{-1}(z_h)\right]} = \varepsilon_L \frac{u'\left[\psi^{-1}(z_\ell)\right]}{\psi'\left[\psi^{-1}(z_\ell)\right]}.$$

Therefore, $(z_h, z_\ell) = (\tilde{z}_h, \tilde{z}_\ell)$ maximizes (38) and, as a consequence, (37) holds. Therefore, W^{tale} is an increasing function of θ.

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