Inflation and Welfare in Models with Trading Frictions*

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November 12, 2003

Abstract

We study the effects of inflation in models with various trading frictions. The framework is related to recent search-based monetary theory, in that trade takes place periodically in centralized and decentralized markets, but we consider three alternative mechanisms for price formation: bargaining, price taking, and posting. Both the value of money per transaction and market composition are endogenous, allowing us to characterize intensive and extensive margin effects. In the calibrated model, under posting the cost of inflation is similar to previous estimates, around 1% of consumption. Under bargaining, it is considerably bigger, between 3% and 5%. Under price taking, the cost of inflation depends on parameters, but tends to be between the bargaining and posting models. In some cases, moderate inflation may increase output or welfare.

*We are grateful to the Central Bank Institute at the Federal Reserve Bank of Cleveland for research support, as well as Ricardo Lagos, S. Boragan Aruoba, Paul Chen and Miquel Faig for their input. Wright thanks the NSF and Rocheteau thanks the Faculty of Economics and Commerce of the ANU for their support.

1 Introduction

We study the effects of inflation in models with various trading frictions. Our economic environment is based on recent search-theoretic models of monetary exchange following Lagos and Wright (2002), in that trade takes place periodically in both centralized and decentralized markets. However, following Rocheteau and Wright (2003), we extend previous analyses of that framework in two ways. First, by endogenizing the composition of agents in the market we analyze the extensive margin (the frequency of trade) as well as the intensive margin (the quantity exchanged per trade). Second, we study several alternative trading or pricing mechanisms, including bargaining as in previous studies, but also competitive price taking and price posting. The main contribution here is as follows. In Lagos and Wright (2002) the welfare costs of inflation were found to be considerably higher than previous estimates. But in Rocheteau and Wright (2003) we show qualitatively that this conclusion can depend critically on the assumed mechanism. Here ask, *how much*? That is, we study quantitatively the effects of inflation under the different mechanisms.

To do this, we present a version of the framework that is simple enough to take to the data, yet general enough to capture some of the key ideas discussed in the relevant literature. One such idea is that the frequency of trade should be endogenous. We use something like the standard matching function from equilibrium search theory to capture the time-consuming nature of trade and how it depends on the endogenous composition of agents in the market.[1] Modeling the extensive margin explicitly is important because inflation may well affect it differently from the way it affects output along

[1] In this sense our framework is similar to much of the search-based labour literature, which relies heavily on composition effects (so-called "market tightness") and an aggregate matching function. See Pissarides (2000) for a textbook treatment. Some related work in monetary theory is discussed below.

the intensive margin. As we will see, endogenizing the frequency of trade not only affects the magnitude of the cost of inflation, it can even change its sign – depending on parameters, and on the assumed trading mechanism, inflation may actually increase output or welfare, at least over some range.

The three trading/pricing mechanisms we consider have been used before in various contexts. Bilateral bargaining is the assumption most often used in the microfoundations of money in the search tradition since Shi (1995) and Trejos and Wright (1995), and we refer to equilibrium in the model with bargaining as *search equilibrium.* Price taking is the standard Walrasian assumption used in monetary theory in, say, overlapping generations models by Wallace (1980) e.g. and turnpike models by Townsend (1980) e.g., and we refer to it as *competitive equilibrium.* By the final mechanism, we mean more than simply price posting, which has been used in monetary models by several authors; we mean the combination of posting and directed search. This combination had not been studied in monetary theory before Rocheteau and Wright (2003), although it had been used in search models of the labour market since Shimer (1996) and Moen (1997), and following that literature we refer to this model as *competitive search equilibrium.*[2]

We calibrate the model to match standard real and monetary observations, and ask how the welfare cost of inflation differs across mechanisms. Our findings are as follows. In competitive search equilibrium, with price posting, our estimated welfare cost is similar to previous estimates such as Lucas (2000) or Cooley and Hansen (1989, 1991): going from 10% to 0% inflation is worth between 0.67% and 1.1% of consumption, depending the calibration. In search equilibrium, with bargaining, the estimated cost can

[2]The essential feature of competitive search equilibrium is that agents get to direct their search to locations posting attractive prices, which induces competition among price setters. Price setting with undirected (purely random) search is a very different equilibrium concept. See Curtis and Wright (2003) for a discussion and citations.

be between 3% and 5% – considerably bigger than what is found in most of the literature, although consistent with Lagos and Wright (2003). Further, we find something here that cannot happen in Lagos and Wright (2003) or the extensions in Rocheteau and Wright (2003): in those models, with bargaining the Friedman Rule is always the optimal policy, while this is *not* necessarily the case here. In competitive equilibrium, the cost of inflation is sensitive to parameter values, but for a benchmark case it tends to be in between the estimates from the posting and bargaining models. Also, with price taking, the optimal inflation rate may again exceed the Friedman Rule. The result that some inflation with either bargaining or competitive pricing may improve welfare is sensitive to the calibration, but nevertheless interesting. For example, for some parameters a (perfectly-anticipated, long-run) inflation can have a positive effect on output.[3]

The intuition for our results is as follows. To trade in the decentralized market buyers must invest by acquiring cash. If the terms of trade are bargained ex post, buyers do not get the full return on their investment – a standard *holdup problem.*[4] This holdup problem reduces the value of money, and this makes trade inefficient along the intensive margin in search equilibrium. By contrast, in competitive equilibrium or competitive search equilibrium, there is no holdup problem, and inflation is less costly on the intensive margin. Along the extensive margin search equilibrium is also inefficient. In principle inflation can amplify or mitigate this problem, but for our calibration it usually amplifies it. In competitive equilibrium the Friedman Rule achieves the efficient outcome along the intensive but not the extensive

[3] There is some evidence that this may be true at least for moderate inflation rates; see Bullard and Keating (1995).

[4] It is well known in some applications of search theory the holdup problem vanishes if bargaining power is just right – i.e. if the Hosios (1990) condition holds. However, in our model this condition requires buyers to have all the bargaining power, and this means that there will be no sellers in equilibrium so the market shuts down. Thus, it is *not* possible in general to achieve efficiency on both the intensive and extensive margins here.

margin. For some parameters inflation makes things better on the extensive and worse on the intensive margin, but the latter effect is second order near the Friedman Rule, the net effect is positive. In competitive search equilibrium, the Friedman Rule is efficient on both margins, so inflation is always bad but the effect is second order near the Friedman Rule.

It is interesting to compare these results with Shi (1997). In his model a household is composed of a large number of members, and chooses the fractions to be buyers and sellers each period.[5] Buyers and sellers from different households meet and bargain. In equilibrium the number of sellers may be too high, in which case a deviation from the Friedman Rule improves welfare by reducing the seller-buyer ratio. It is not true here that when the number of sellers is too high a deviation from the Friedman Rule necessarily improves welfare. However, we show by example that inflation may improve welfare when the number of sellers is inefficiently low. A difference in the models is that Shi imposes a bargaining procedure that avoids the holdup problem, which means the intensive margin is efficient close to the Friedman Rule.[6] In fact, our results are closer to those of Shi when we impose competitive price taking. Nevertheless, it is the case that for some parameters inflation may actually increase output and/or welfare.

The rest of the paper is organized as follows. Section 1 describes the basic environment. Section 2 presents the different trading/pricing mechanisms. Section 3 analyzes the welfare cost of inflation through a series of calibration experiments. Section 4 concludes.

[5] The large household assumption allows family members to share money at the end of each trading round, which means that in equilibrium all buyers hold the same amount of money at the start of the next period. One does not need this assumption in the Lagos-Wright framework because individuals have periodic access to centralized markets where they can adjust their cash balances, so we get all buyers to hold the same amount of money in equilibrium without invoking large families.

[6] For a comparison of the bargaining solutions in these models, and the implications for the optimality of the Friedman Rule, see Berentsen and Rocheteau (2003).

2 The Environment

Time is discrete and continues forever. Each period is divided into two subperiods, called *day* and *night*. During the day there will be a centralized and frictionless market, while at night trade occurs in more or less decentralized markets, subject to frictions as described in detail below.[7] In the centralized day market all agents can produce consumption goods from labor using a linear technology. At night each agent can do one of two things: either he can produce intermediate goods, or he can use these intermediate goods to produce a consumption good at home after the night market has closed. This generates a simple double coincidence problem in the decentralized market: some agents can make intermediate goods at night but they do not have the home technology to use them, while others do have the home technology but cannot produce intermediate goods or anything else to trade for them in the night market.[8]

Assuming a $[0, 1]$ continuum of agents, let n be the measure who have the technology for intermediate good production but not home production, and $1 - n$ the measure who have the technology for home production but not intermediate good production. Because of the way they will interact in the decentralized market, we call the former sellers and the latter buyers. By letting an agent choose whether to be a seller or a buyer we endogenize the composition of the decentralized market – i.e. the buyer-seller ratio –

[7] The day-night story introduced in Lagos and Wright (2002) is convenient but not at all necessary in this class of models. In Williamson and Wright (2003) e.g. there are both centralized and decentralized market running simultaneously each period, with agents subject to random location shocks; that setup is basically the same for most purposes.

[8] The only reason for invoking home production, as opposed to generating a double coincidence problem by having one type with a direct preference for the goods produced by another type, say, is that we want to allow agents to choose their type, and some people seem to find the choice of preference less palatable than the choice of technology. Calling it home production matters for little else, except maybe the way we do national income accounting when we calibrate the model.

and therefore the number of trades.[9] We assume that goods are nonstorable. We also assume that buyers in the decentralized market are *anonymous*; hence, they cannot get credit in the night market because they could default without fear of punishment, and this makes money essential (Kocherlakota 1998; Wallace 2001). Let the quantity of money at t be $M_t > 0$ and assume $M_{t+1} = \gamma M_t$, where γ is constant and new money is injected by lump-sum transfers to all agents.

The utility function of an agent within a full day-night period is

$$\mathcal{U} = U(x) - C(y) + \beta[u(q) - c(l)], \tag{1}$$

where x is consumption and y is production (equals labor supply) of the day good, while q is consumption of the home produced good and l is intermediate good production (equals labor supply) at night. Agents discount between day and night, but not between night and the next day; this is without loss in generality since as in Rocheteau and Wright (2003) all that matters is total discounting between one day-night period and the next. In any case, since buyers consume home produced goods but do not produce intermediate goods while the opposite is true for sellers, and since by a change of notation we can always make home output equal to the input, $q = l$, we may as well write buyers' and sellers' utility functions as

$$\mathcal{U}^b = U(x) - C(y) + \beta u(q) \tag{2}$$

$$\mathcal{U}^s = U(x) - C(y) - \beta c(q). \tag{3}$$

An assumption that is crucial in terms of tractability, although not crucial in principle if one adopts more sophisticated computational methods, is that

[9] Again, this is similar to the model in Shi (1997) where households choose the fraction of members to be buyers and sellers. Other methods of introducing extensive margin effects include Li (1995, 1997), Berentsen, Rocheteau and Shi (2001), and Lagos and Rocheteau (2003), who assume endogenous search intensities, and Rocheteau and Wright (2003), who allow entry on one side of the market. The method used here is slightly easier for calibration purposes.

utility is linear in day labor: $C(y) = y$. This assumption is what makes the Lagos-Wright framework easy to study analytically, because it implies that all agents of a given type (e.g. all buyers) will choose to carry the same amount of money out of the centralized market, independent of their histories. That is, conditional on type, there will be a degenerate distribution of money holdings in the decentralized market. Note also that this assumption makes our model similar to some previous well-known inflation studies, like Cooley and Hansen (1989), as well as many other macro models following Rogerson (1998) that also assume utility is linear in labor. In terms of the other functions, we assume $U'(x) > 0$, $U''(x) < 0$, $u'(q) > 0$, $u''(q) < 0$, $u(0) = c(0) = c'(0) = 0$, $c'(q) > 0$, $c''(q) > 0$, and $c(\bar{q}) = u(\bar{q})$ for some $\bar{q} > 0$. Let q^* denote the solution to $u'(q^*) = c'(q^*)$ and x^* the solution to $U'(x^*) = C'(x^*) = 1$; $q^* \in (0, \bar{q})$ exists by the previous assumptions, and we assume such an $x^* > 0$ also exists.

The final important element of the model is that we assume there trading frictions in the decentralized market: at night, a buyer gets an opportunity to trade with probability $\alpha_b = \alpha(n)$ and a seller gets an opportunity with probability $\alpha_s = (1-n)\alpha(n)/n$. One can interpret the buyer trading probability $\alpha(n)$ as being derived from an underlying constant returns to scale matching technology, although other interpretations are possible, and notice that $n\alpha_s = (1-n)\alpha_b$ so that one can think of trade as bilateral if so desired. We assume $\alpha'(n) > 0$, $\alpha(n) > \alpha'(n)n(1-n)$, $\alpha(n) \leq \min\{1, \frac{n}{1-n}\}$, $\alpha(0) = 0$, and $\alpha(1) = 1$. Let

$$\eta(n) = \frac{\alpha'(n)n(1-n)}{\alpha(n)} \tag{4}$$

be the contribution of sellers to the trading process. For example, if $\alpha(n) = n$, as in models like Kiyotaki and Wright (1991,1993), then $\eta(n) = 1 - n$.[10]

[10] More generally, if $\alpha(n)$ is derived from an underlying matching technology η is the elasticity of this matching function with respect to the measure of sellers. To see this,

This completes the description of the physical environment, and we now begin to describe what happens. Let $V^b(m)$ and $W^b(m)$ be the value functions of a buyer with m dollars in the night and day market, respectively. Similarly, let $V^s(m)$ and $W^s(m)$ be the value functions for sellers. We omit the time subscript t and shorten $t+1$ to $+1$, etc. in what follows. As we said earlier, agents get to choose to be buyers or sellers of intermediate goods at the beginning of each period (by stationarity, they effectively could choose this once and for all). Therefore the payoff to an agent with m at the start of the day is

$$W(m) = \max\left[W^b(m), W^s(m)\right]. \tag{5}$$

Bellman's equation for a buyer in the decentralized night market is

$$V^b(m) = \alpha_b(n)\left\{u\left[q(m)\right] + W_{+1}\left[m - d\left(m\right)\right]\right\} + \left[1 - \alpha_b(n)\right] W_{+1}(m), \tag{6}$$

where, in general, the quantity of intermediate good he buys q and the dollars he spends d may depend on his money holdings. Given $V^b(m)$, in the centralized market the problem for a buyer is

$$W^b(m) = \max_{\hat{m},x,y}\left\{U(x) - y + \beta V^b(\hat{m})\right\} \tag{7}$$

$$\text{s.t.} \quad x + \phi\hat{m} = y + \phi(m+T), \tag{8}$$

where ϕ is the price of money in terms of goods, T the lump-sum transfer, and $\hat{m}$ the money taken into the night market. Substituting y from (8) into (7) we obtain[11]

$$W^b(m) = \max_{\hat{m},x}\left\{U(x) - x - \phi\left(\hat{m} - T - m\right) + \beta V^b(\hat{m})\right\}. \tag{9}$$

write the matching function as $\mathcal{M}(b,s) = b\alpha\left(\frac{s}{b+s}\right)$ where b is the measure of buyers and s the measure of sellers. Then $\eta = \mathcal{M}_s(b,s)s/\mathcal{M}(b,s)$.

[11] We do not impose nonnegativity on y, but after finding an equilibrium one can easily adopt conditions to guarantee $y \geq 0$ (see Lagos and Wright 2002).

From (9) several things are clear: the maximizing choice of x is x^* where $U'(x^*) = 1$; the maximizing choice of $\hat{m}$ is independent of m; W^b is linear in m with $W^b_m = \phi$; and if the solution is interior then $\hat{m}$ satisfies

$$\phi = \beta V^b_m(\hat{m}). \tag{10}$$

Condition (10) sets the marginal cost of taking money out of the centralized market equal to the marginal benefit, in terms of what it will do for you in the decentralized market. As long as V^b is strictly concave, $\hat{m}$ is unique. For the alternative specifications of the model discussed below, strict concavity holds under fairly weak conditions, and hence all buyers will choose the same $\hat{m}$.[12] This is due to the quasi-linearity assumption $C(y) = y$, which, heuristically speaking, eliminates wealth effects on money demand and hence implies that all agents of a given type choose the same $\hat{m}$ regardless of the m with which they come into the centralized market.

Because sellers do not want to purchase anything in the decentralized night market they all choose $\hat{m} = 0$; see Rocheteau and Wright (2003) for details. Hence, we ignore the argument of V^s in what follows. Also, given sellers carry no money at night, each buyer carries $M^b = M/(1-n)$, and Bellman's equation for a seller in the decentralized market becomes

$$V^s = \alpha_s(n)\left\{-c\left[q\left(M^b\right)\right] + W_{+1}\left[d\left(M^b\right)\right]\right\} + \left[1-\alpha_s(n)\right]W_{+1}(0), \tag{11}$$

where $d = d(M^b)$ and $q = q(M^b)$ are the equilibrium terms of trade. The seller's problem in the centralized market is similar to a buyer's, and after substituting the budget equation can be written

$$W^s(m) = \max_x \left\{U(x) - x + \phi(m+T) + \beta V^s\right\}. \tag{12}$$

[12] As we will see below, under price taking the strict concavity of V^b is a direct consequence of $u'' < 0$. See Lagos and Wright (2002) for details under bargaining, and Rocheteau and Wright (2003) under price posting.

As in the buyer's problem, we again have $x = x^*$, and W^s is again linear in m with $W^s_m = \phi$.

Next, to discuss the choice of each agent about whether to be a buyer or a seller of intermediate goods, notice that linearity implies $W^b(m) = \phi m + W^b(0)$ and $W^s(m) = \phi m + W^s(0)$. Therefore, from (5), $W(m) = \phi m + \max\left[W^b(0), W^s(0)\right]$. Consequently the decision to be a buyer or a seller is independent of one's money holdings, and n is determined simply by $W^b(0) = W^s(0)$. Substituting (6) into (9) and (12) into (11), this condition can be reduced to

$$-\phi_{+1}M^b\left(\frac{\phi}{\beta\phi_{+1}} - 1\right) + \alpha_b(n)\left[u(q) - \phi_{+1}d\right] = \alpha_s(n)\left[\phi_{+1}d - c(q)\right]. \quad (13)$$

This has a simple interpretation: the left side is the expected payoff of being a buyer and the right side is the expected payoff of being a seller in the decentralized night market. Notice that the first term on the left is the cost for buyers of carrying M^b dollars into this market.[13]

To close this section, define welfare as the utility of a representative agent within a period composed of a night and the following day,

$$\mathcal{W} = (1-n)\alpha(n)\left[u(q) - c(q)\right] + U(x) - x. \quad (14)$$

On the intensive margin, the first-best allocation requires $x = x^*$ and $q = q^*$, where $U'(x^*) = C'(x^*) = 1$ and $u'(q^*) = c'(q^*)$. On the extensive margin, it requires that we maximize the number of trades, $(1-n)\alpha(n)$, which means

$$(1-n)\alpha'(n) = \alpha(n). \quad (15)$$

If $\alpha(n) = n$, e.g., (15) implies $n = 1/2$.[14] In any case, using the definition of

[13] This is easiest to see if we use the fact that in steady state $\phi_{+1} = \phi/\gamma$ and the inflation rate is $\pi = \gamma - 1$. Then defining the nominal interest rate by $1 + i = (1+r)(1+\pi)$, the first term is simply $-\phi_{+1}M^b i$, or the real cost of forgone nominal interest.

[14] This is reminiscent of a result in search-based monetary models like Kiyotaki and Wright (1993), generalized in Rocheteau (2000) and Berentsen (2002), that says efficiency dictates equal numbers of buyers and sellers.

$\eta(n)$ in (4), (15) can be expressed generally as

$$n = \eta(n). \tag{16}$$

Hence, efficiency implies the fraction of sellers must equal their contribution in the matching process.

3 Equilibrium

As discussed in above, here we consider the following three mechanisms: bargaining, price taking, and price posting. The first we refer to as *search equilibrium*, the second *competitive equilibrium*, and the third *competitive search equilibrium*. We present each in turn.

3.1 Search Equilibrium (Bargaining)

We assume here that in the decentralized market, as in most search models, agents are matched bilaterally and the terms of trade (q, d) are determined by the generalized Nash bargaining solution

$$\max_{(q,d)} \left[u(q) - \phi_{+1}d\right]^{\theta} \left[-c(q) + \phi_{+1}d\right]^{1-\theta} \text{ s.t. } d \leq M^b, \tag{17}$$

where θ is the bargaining power of a buyer.[15] In any monetary equilibrium the constraint $d \leq M^b$ binds in this model; intuitively, it should be clear that you would not bring to the decentralized market money that you do not want to spend, but see Lagos and Wright (2002) for details. Hence the seller receives $d = M^b$ and produces the quantity that solves the first order condition for q, which we write as $\phi_{+1}M^b = g(q)$ where

$$g(q) = \frac{\theta u'(q)c(q) + (1-\theta)c'(q)u(q)}{\theta u'(q) + (1-\theta)c'(q)}. \tag{18}$$

[15] To derive this, observe that the surplus (payoff minus threat point) of a buyer is $u(q) + W_{+1}(m-d) - W_{+1}(m) = u(q) - \phi_{+1}d$, using the linearity of W. The surplus of a seller is similar.

From this we have $q'(m) = \phi_{+1}/g'(q)$.

Now consider the centralized market price ϕ. From (6), we have $V_m^b(m) = \alpha_b(n)u'(q)q'(m) + [1 - \alpha_b(n)]\phi_{+1}$, and therefore[16]

$$V_m^b(m) = \left[\alpha_b(n)\frac{u'(q)}{g'(q)} + 1 - \alpha_b(n)\right]\phi_{+1}. \tag{19}$$

Inserting this into the first order condition $\phi = \beta V_m^b(M)$ and using the fact that $\phi = \gamma\phi_{+1}$ in steady state, after minor simplification we get

$$\frac{i}{\alpha_b(n)} + 1 = \frac{u'(q)}{g'(q)} \tag{20}$$

where $i = \frac{\gamma-\beta}{\beta}$ is the nominal interest rate defined by $1 + i = (1 + r)(1 + \pi)$ with $r = \beta^{-1} - 1$ and $\pi = \gamma - 1$ (as is standard, if we open a market for bonds here they will not trade in equilibrium, but we can still price them). For future reference denote by $\tilde{q}$ the solution to (20) when $i = 0$, and note that $\tilde{q} < q^*$ unless $\theta = 1$.

Also, the condition (13) determining the composition of buyers and sellers can be simplified using $\phi_{+1}M^b = g(q)$ to

$$-ig(q) + \alpha_b(n)\left[u(q) - g(q)\right] = \alpha_s(n)\left[g(q) - c(q)\right]. \tag{21}$$

We can now define an equilibrium for this model; in what follows, when we say an equilibrium we mean a steady state monetary equilibrium.

Definition 1 *A search equilibrium is a pair (q, n) that satisfies (20) and (21).*

The existence and uniqueness or multiplicity of equilibrium can be analyzed using methods similar to those used for a different but closely related model in Rocheteau and Wright (2003). Our goal here is instead to describe

[16] Notice from this that for V^b to be strictly concave we need $u'(q)/g'(q)$ strictly decreasing in q. Lagos and Wright (2002) show that this will be satisfied if either: $c(q)$ is linear and $u'(q)$ is log-concave; or θ is close to 1.

things quantitatively, which we do in the next section. As a benchmark, however, consider equilibrium at the Friedman Rule $i = 0$.[17] From (20) and (21), this implies

$$q_F = \tilde{q} \quad (22)$$

$$n_F = \frac{(1-\theta)c'(\tilde{q})}{(1-\theta)c'(\tilde{q}) + \theta u'(\tilde{q})}. \quad (23)$$

Since (20) implies $q < \tilde{q}$ for all $i > 0$, the Friedman Rule maximizes q. If $\theta < 1$ we have $\tilde{q} < q^*$; if $\theta = 1$ we have $n_F = 0$. This reflects a tension between the intensive and extensive margins: $q = q^*$ requires giving all the bargaining power to buyers, but then no one chooses to become a seller and the night market shuts down.

Given $q = \tilde{q}$, comparing (16) and (23) we see that n_F coincides with the efficient n^* iff

$$\eta(n_F) = \frac{(1-\theta)c'(\tilde{q})}{(1-\theta)c'(\tilde{q}) + \theta u'(\tilde{q})}. \quad (24)$$

This is the familiar Hosios (1990) condition: the measures of buyers and sellers are efficient iff the seller's share of the surplus from matching equals their contribution to the trading process. Given that η is independent of θ, u, and c, this condition will not hold in general, and it is possible for n to be either too high or too low in equilibrium. Therefore, in theory, having the composition of buyers and sellers endogenous may either exacerbate or mitigate the welfare cost of inflation, and it is even possible that some inflation could improve welfare.

3.2 Competitive Equilibrium (Price Taking)

Consider next imposing a standard Walrasian mechanism at night: agents in the intermediate goods market now trade in large groups taking the price as

[17] The condition $i = 0$ is equivalent to $\gamma = \beta$; in this model there is no difference between nominal interest targeting, money supply targeting, or inflation targeting. Also, as is standard, it is impossible to set $i < 0$ ($\gamma < \beta$) here because monetary equilibrium exists only if $\gamma \geq \beta$.

given, and the price adjusts to clear the market. In order to have trading frictions in this setting we assume that agents need to spend a stochastic amount of time before being able to trade. This idea is clearly related to the Lucas and Prescott (1974) search model, where agents incur a cost to move from one competitive market to another. More precisely, each period here a buyer gets into the competitive market with probability $\alpha_b(n)$ whereas a seller gets in with probability $\alpha_s(n)$. Therefore, in each period only a measure $(1-n)\alpha(n)$ of buyers and sellers trade each night.[18] We still call the night market decentralized, even though it has competitive pricing. Also, note that as long as agents are anonymous in this market, money will still be essential.

A buyer who gets into the market at night maximizes $u(q^b) - \phi_{+1}pq^b$ subject to $q^b \leq \frac{M^b}{p}$ where p is the nominal price of the intermediate good. A seller who gets in maximizes $-c(q^s) + \phi_{+1}pq^s$. The price clears the market, which with equal numbers on each side requires $q^s = q^b = q$. Therefore,

$$c'(q) = p\phi_{+1} \tag{25}$$

$$q = \frac{M^b}{p}, \tag{26}$$

where we have used the fact that $q^b \leq \frac{M^b}{p}$ is binding in equilibrium; see Rocheteau and Wright (2003) for details. In this model,[19]

$$V_m^b(M^b) = \alpha_b(n)u'\left(\frac{M^b}{p}\right)\frac{1}{p} + [1-\alpha_b(n)]\phi_{+1}. \tag{27}$$

Inserting this into $\phi = \beta V_m^b(M^b)$, using (25) and rearranging, we get

$$\frac{i}{\alpha_b(n)} + 1 = \frac{u'(q)}{c'(q)}. \tag{28}$$

[18] We assume equal measures of buyers and sellers get into the night market, but this of course does not mean $n = 1/2$; n is the total measure of sellers, not all of whom get in. In any case, the assumption that the measures of buyers and sellers who get in are equal is used to make the different trading mechanisms more comparable because if so desired, one can still think of trade as bilateral even if pricing is Walrasian. As shown in Rocheteau and Wright (2003) this can easily be relaxed to allow a different measure of buyers and sellers to get in, although then of course trade cannot in general be bilateral.

[19] Notice that the strict concavity of V^b requires only $u'' < 0$ here.

Also, (25) and (26) imply $\phi_{+1}M^b = c'(q)q$, and so (13) reduces to

$$-iqc'(q) + \alpha_b(n)\left[u(q) - qc'(q)\right] = \alpha_s(n)\left[qc'(q) - c(q)\right]. \tag{29}$$

Definition 2 *A competitive equilibrium is a pair* (q, n) *that satisfies (28) and (29).*

The equilibrium conditions here are generally different from those in search equilibrium given above. Now the Friedman Rule implies

$$q_F = q^* \tag{30}$$

$$n_F = \frac{q^*c'(q^*) - c(q^*)}{u(q^*) - c(q^*)}. \tag{31}$$

From (30), q is always efficient at the Friedman Rule in competitive equilibrium. This is because, when agents are price takers, there is no holdup problem in money demand. From (31), $n_F = 0$ if $c(q)$ is linear because profit is zero so no one would want to be a seller. Hence we need c to be nonlinear for this model to be interesting.

Finally, (16) and (31) coincide and $n = n^*$ iff

$$\eta(n_F) = \frac{q^*c'(q^*) - c(q^*)}{u(q^*) - c(q^*)}. \tag{32}$$

This is again a Hosios condition, but different from the one in search equilibrium. It is again not likely to hold, as it relates the properties of η with those of u and c. Since the Friedman Rule gives $q = q^*$, it is possible that inflation in excess of the Friedman Rule could generate a welfare improvement if it moves n in the right direction – which is possible since n could be too big or too small. It is true that inflation also reduces q, which is bad along the intensive margin, but the effect on welfare of a change in q is second order near $i = 0$. Hence it is again possible that some inflation could improve welfare.

3.3 Competitive Search Equilibrium (Posting)

We now consider a price-posting mechanism where the terms of trade are publicly announced and then agents can direct their search. There are still trading frictions because agents may or may not get to trade at that price. This corresponds to the notion of competitive search equilibrium in Moen (1997) and Shimer (1996). Several interpretations of the mechanism have been proposed, and here we adopt the one used in Moen (1997) and Mortensen and Wright (2002). This story is that there are competing *market makers* who can open *submarkets*, where a given submarket is characterized by the terms (q, d) at which agents commit to trade and by the fraction of sellers n. Obviously this assumes a certain amount of commitment; this is the essence of posting and competitive search. One could argue about whether this type of commitment is reasonable, but we emphasize logically it does not make money inessential: committing to the terms of decentralized trade within the period is not the same as committing to repayment of credit.

The different submarkets are announced at the beginning of each period and agents can choose to go to any open submarket at night. In each submarket, buyers and sellers are matched bilaterally and at random, and hence get to trade with probabilities $\alpha_b(n)$ and $\alpha_s(n)$, respectively. The sequence of events is as follows. At the beginning of every period, each agent chooses to be a buyer or a seller. Then market makers announce terms of trade; we find it convenient to write these terms as (q, z) here, where $z = \phi_{+1}d$. Market makers compete to attract buyers and sellers to their submarkets because they charge entry fees, although in equilibrium this fee is 0 since there is free entry into market making.[20]

[20] We assume here that market makers must charge the same fee to buyers and sellers, say because they cannot identify types when the enter; see Faig and Huangfu (2003) for an analysis of the case when the fees can differ. In any case, market makers are not crucial for the competitive search equilibrium concept; one can alternatively let buyers or sellers

In designing submarkets market makers effectively maximize the expected utility of buyers subject to the constraint that they can attract some sellers to their submarket. Let $\mathcal{S}$ be the set of active submarkets described by (q, z, n), let s denote an element of $\mathcal{S}$, and let $\mathcal{V} \equiv \max_{\text{s}\in\mathcal{S}} \{\alpha_s(n)[-c(q) + z]\}$ be the expected utility of sellers in equilibrium. Then for any active submarket, the problem can be formulated as

$$\max_{(q,z,n)} \{\alpha_b(n)[u(q) - z] - iz\} \tag{33}$$

$$\text{s.t.} \quad \alpha_s(n)[-c(q) + z] = \mathcal{V}. \tag{34}$$

It is shown in Rocheteau and Wright (2003) that, except for at most a countable number of values for $\mathcal{V}$, the solution to this problem is unique, and so all submarkets are the same in equilibrium.[21] Hence, we may as well assume there is only one active submarket.

It is also shown in Rocheteau and Wright (2003) that the correspondence $n(\mathcal{V})$ emerging from this program is non-empty and upper hemi-continuous, and any selection from $n(\mathcal{V})$ is decreasing in $\mathcal{V}$. Furthermore, the maximum expected utility of the buyer defined by (33) is continuous and decreasing in $\mathcal{V}$. This means that there is a unique $\mathcal{V}$ such that the expected utility of a buyer is equal to the expected utility of a seller and this determines the equilibrium. Substituting z from (34) into (33) and taking the first order conditions for q and n, we get

$$\frac{u'(q)}{c'(q)} = 1 + \frac{i}{\alpha(n)} \tag{35}$$

$$\eta(n)[u(q) - c(q)] = \left\{1 + \frac{i}{\alpha(n)}[1 - \eta(n)]\right\} \frac{n\mathcal{V}}{\alpha(n)(1-n)}, \tag{36}$$

post prices in order to attract potential trading partners (see e.g. Acemoglu and Shimer (1999)).

[21] When the solution is not unique, buyers and sellers obtain the same expected payoff no matter the solution chosen by the market maker. In such cases, on can assume without loss of generality that all market makers choose the same solution.

where $\eta(n)$ is as defined above. Notice that (35), which determines q for any given n, is the same as the equilibrium condition (28) from competitive equilibrium. To derive the equilibrium condition for n, substitute $\mathcal{V}$ from (34) and $i/\alpha(n)$ from (35) into (36) to obtain

$$-c(q) + z = \frac{c'(q)\eta(n)}{c'(q)\eta(n) + u'(q)\left[1 - \eta(n)\right]} \left[u(q) - c(q)\right]. \tag{37}$$

From this we see that the terms of trade in competitive search equilibrium coincide with those in search equilibrium when the seller's bargaining power is given by $1 - \theta = \eta(n)$. Equivalently, the seller's effective bargaining power (i.e. the trading surplus) adjusts in order to reflect his contribution in the matching process. Hence z satisfies a condition analogous to (18) where θ is replaced by $1 - \eta$,

$$z = f(q, n) = \frac{\left[1 - \eta(n)\right] u'(q)c(q) + \eta(n)c'(q)u(q)}{\left[1 - \eta(n)\right] u'(q) + \eta(n)c'(q)}. \tag{38}$$

Finally, (13) implies

$$-if(q, n) + \alpha_b(n)\left[u(q) - f(q, n)\right] = \alpha_s(n)\left[f(q, n) - c(q)\right]. \tag{39}$$

Definition 3 *A competitive search equilibrium is a pair (q, n) that satisfies (35) and (39).*

From (35), the intensive margin is efficient and $q = q^*$ in competitive search equilibrium iff $i = 0$, as in competitive equilibrium, but generally not search equilibrium. Posting again eliminates the holdup problem in money demand. Furthermore, at $i = 0$ and $q = q^*$ (39) becomes

$$nu(q^*) + (1 - n)c(q^*) = \left[1 - \eta(n)\right] c(q^*) + \eta(n)u(q^*),$$

which reduces to $n = \eta(n)$. Hence, posting generates endogenously the Hosios condition, and therefore it is also efficient along the extensive margin when $i = 0$. To summarize, the Friedman Rule in competitive search equilibrium generates the first best allocation, $q = q^*$ and $n = n^*$. A corollary is that any deviation from the Friedman Rule must reduce welfare, although for a small deviation the effect is second order.

4 Quantitative Analysis

We now move to the quantitative experiments. While the period length in this model can be anything, and while it may seem that a shorter period makes more sense in terms of the story, for now we set it to a year mainly because we want to use the same methods as, and compare our results to, Lucas (2000). The results are actually quite robust to period length, however, as we will discuss briefly below. Thus, for now we set $\beta^{-1} = 1.03$, as in Lucas. The utility function for goods traded in the centralized market is $U(x) - y$, and we use $U(x) = A \ln x$; except for notation, $A \ln x - y$ is exactly what Cooley and Hansen (1989) use. With $U(x) = A \ln x$, notice that $x^* = A$.

The utility function over home produced goods is

$$u(q) = \frac{(q+b)^{1-a} - b^{1-a}}{1-a},$$

where $a > 0$ and $b \in (0, 1)$. This generalizes the typical CRRA utility function to guarantee $u(0) = 0$ for any a, which is a maintained assumption in the model; for calibration we actually set $b \approx 0$ so that $u(q)$ is close to the standard CRRA specification. Regarding the disutility of production for sellers of the intermediate input, we take $c(q) = q^\delta/\delta$ with $\delta \geq 1$. We set $\alpha(n) = n$, which as we said earlier is a common specification in search-theoretic models of money.

We now choose the vector of parameters $\Omega = (a, A, \delta, \theta)$. Regarding the bargaining power parameter θ, which is only relevant in search equilibrium, we start with the symmetric case $\theta = 0.5$ and then check to see how varying θ affects the results. For the other parameters, we follow Lucas (2000) and choose Ω to match the money demand data. Thus, define $L = M/PY = L(i)$ where P is the nominal price level and Y real output. One can think of this as money demand in the sense that desired real balances M/P are proportional to real spending Y, with a factor of proportionality $L(i)$ that depends on the nominal interest rate. We measure i by the short term commercial paper rate, Y by GDP, P by the GDP deflator, and M by $M1$, as in Lucas; as he points out, the choice of $M1$ is somewhat arbitrary, but we use it here to make the analyses comparable. We consider the period 1900-2000, which is just slightly longer than Lucas' sample; for the sake of comparison, we will also consider the shorter period 1959-2000.

In the model L is constructed as follows. In the decentralized market we measure output by the production of intermediate goods (since home produced goods are not traded). Nominal output in this market is therefore $(1-n)\alpha(n)M^b$. Nominal output in the centralized market is x^*/ϕ_{+1}. Hence, $PY = (1-n)\alpha(n)M^b + x^*/\phi_{+1}$. Using the fact that $M = (1-n)M^b$, $z = \phi_{+1}M^b$ and $x^* = A$, we have

$$L = \frac{(1-n)\,z}{A + (1-n)\,\alpha(n)\,z}. \tag{40}$$

Since the endogenous variables z and n depend on nominal interest rate through the equilibrium conditions of the model, (40) defines a relation $L = L(i)$, where $L(i)$ depends on the underlaying parameter vector.

We first tried to choose (a, A, δ) to minimize the squared residuals between L in the data and L in the model. However, numerically we were not able to pin down the parameters precisely; roughly speaking the routine picks A to adjust the level of L and (a, δ) to adjust the curvature, and there is more than one combination of (a, δ) that can generate basically the same curvature. We therefore let the data identify (A, a) and set δ to an arbitrary value. We choose $\delta = 1.1$ so that $c(q)$ is close to be linear and therefore close to the specification for $C(y)$; we cannot take $\delta = 1$ because this implies sellers earn zero profit, and hence there are no sellers, in competitive equilibrium (this is not an issue in search equilibrium or competitive search equilibrium). We will discuss below how the value of δ matters.

We measure the welfare cost of a $\pi = \gamma - 1$ percent inflation by asking how much agents would be willing to give up in terms of total consumption to reduce γ to 1. Expected utility for an agent given γ is measured by $\mathcal{W}_\gamma$ as defined in (14). Suppose we reduce γ to 1 but also reduce consumption of all goods by a factor Δ. Expected utility becomes

$$\mathcal{W}_1(\Delta) = (1 - n_1)\,\alpha(n_1)[u(q_1\Delta) - c(q_1)] + U(x^*\Delta) - x^*,$$

where q_γ and n_γ are the equilibrium values for n and q given γ. The welfare cost of inflation is the value of Δ_1 that solves $\mathcal{W}_1(\Delta) = \mathcal{W}_\gamma(1)$. We also report how much consumption agents would be willing to give up to reduce γ to β (the Friedman Rule). The measure Δ_F is interesting because the Friedman Rule is the optimal monetary policy under some of the mechanisms we consider. In the following, we let $\bar{\Delta}_1 = 100(1 - \Delta_1)$ and $\bar{\Delta}_F = 100(1 - \Delta_F)$, and take as a benchmark $\gamma = 1.1$; i.e. $\bar{\Delta}_1$ is the percentage of total consumption agents would give up to have 0% instead of 10% inflation and $\bar{\Delta}_F$ is the percentage they would give up to have the Friedman Rule instead of 10% inflation.

4.1 Competitive Search Equilibrium

We present our quantitative results in a different order from the way we presented the theory, beginning with competitive search equilibrium, because as we saw in the previous section this mechanism delivers the first best allocation at the Friedman Rule, and hence offers a natural benchmark. When we fit the model to the data we find $(a, A) = (0.0976, 0.9562)$. As it can be seen in the upper panels of the following figure, this simple procedure generates a very good fit, where the left panel is for the whole sample 1900-2000 and the right panel is for the shorter sample 1959-2000. The lower diagrams show the equilibrium values of q and n as functions of i implied by the fitted parameter values in each case.

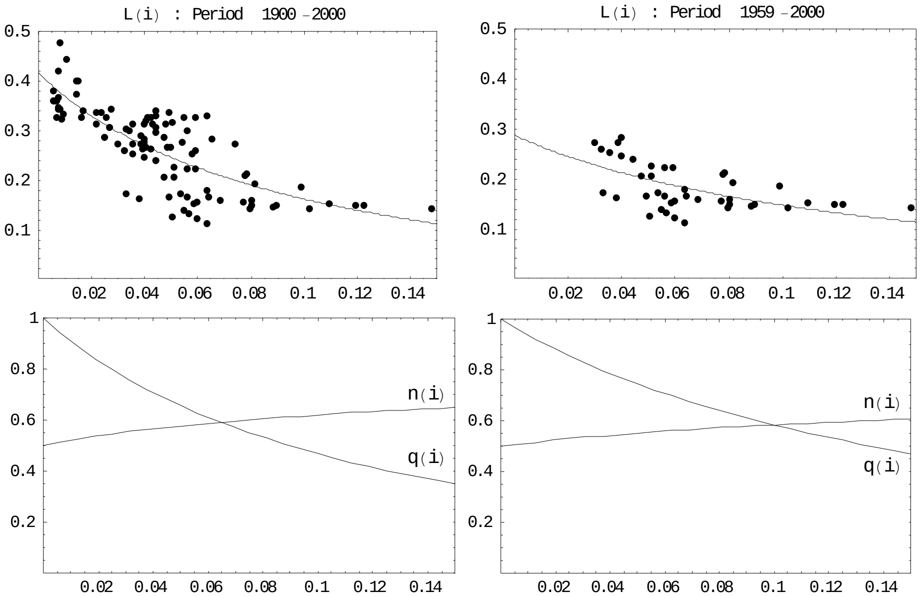

Figure 1. Quality of fit and equilibrium values of q and n in Competitive Search Equilibrium.

At the Friedman Rule, in competitive search equilibrium we have $q = 1$ and $n = 0.5$, corresponding to the first-best allocation. We find that a 10% inflation here is worth just over 1% of consumption: $\bar{\Delta}_1 = 1.11$ and $\bar{\Delta}_F = 1.22$. This is a little bigger than most previous estimates – e.g. Cooley and Hansen (1989) or Lucas (2000) – but it is in the same ballpark (Lucas reports slightly less than 1%). The results are fairly robust to changes in δ. For example, if we assume $\delta = 1.2$, we obtain $(a, A) = (0.0156, 0.8766)$, $\bar{\Delta}_1 = 1.09$ and $\bar{\Delta}_F = 1.20$. An upper bound is obtained at $\delta = 1$, which yields $(a, A) = (0.1797, 1.0519)$, $\bar{\Delta}_1 = 1.13$ and $\bar{\Delta}_F = 1.25$. Note that the cost shrinks when we consider more recent data: going back to the case $\delta = 1.1$, if we fit the model to the 1959-2000 data, we find $(a, A) = (0.1946, 1.5987)$, $\bar{\Delta}_1 = 0.67$ and $\bar{\Delta}_F = 0.74$.

As seen in the figure, the implied parameters from either sample indicate that an increase in inflation reduces q and increases n. To see how this works, we represent the equilibrium conditions (35) and (39) by the curves Q and N in the following figure. As i increases, Q shifts to the left and N shifts upward: the black curves correspond to $i = 0.03$ and the grey curves to $i = 0.13$. In any case, to summarize we have seen that competitive search equilibrium generates a welfare cost of inflation that is very much in line with estimates found in the previous literature, including Lucas (2000). This is interesting, we believe, because it shows that introducing frictions in the trading process does not necessarily raise the cost of inflation if one is willing to adopt a particular mechanism.

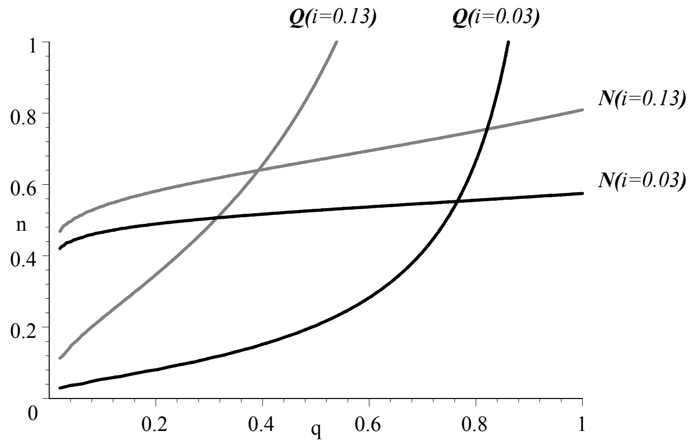

Figure 2. Competitive search equilibrium

4.2 Search Equilibrium

We now move to search equilibrium. As a benchmark, consider symmetric bargaining, $\theta = 1/2$. Now when we fit the model $(a, A) = (0.2450, 0.8942)$. For these parameters a 10% inflation implies $\bar{\Delta}_1 = 3.10$ and $\bar{\Delta}_F = 3.77$. These measures are bigger than those in most of the literature and what we found under competitive search, but similar to what is reported in Lagos and Wright (2002) (which is also a bargaining model, but does not have extensive margin effects). If we recalibrate to the 1959-2000 data, the results do not change very much in this case: we find $(a, A) = (0.4064, 1.4671)$, $\bar{\Delta}_1 = 3.02$ and $\bar{\Delta}_F = 3.82$. In Figure 3, we represent the fit and the equilibrium values of q and n as a function of i. Again the left panel is for the whole sample 1900-2000, and the right panel is for the shorter sample 1959-2000.

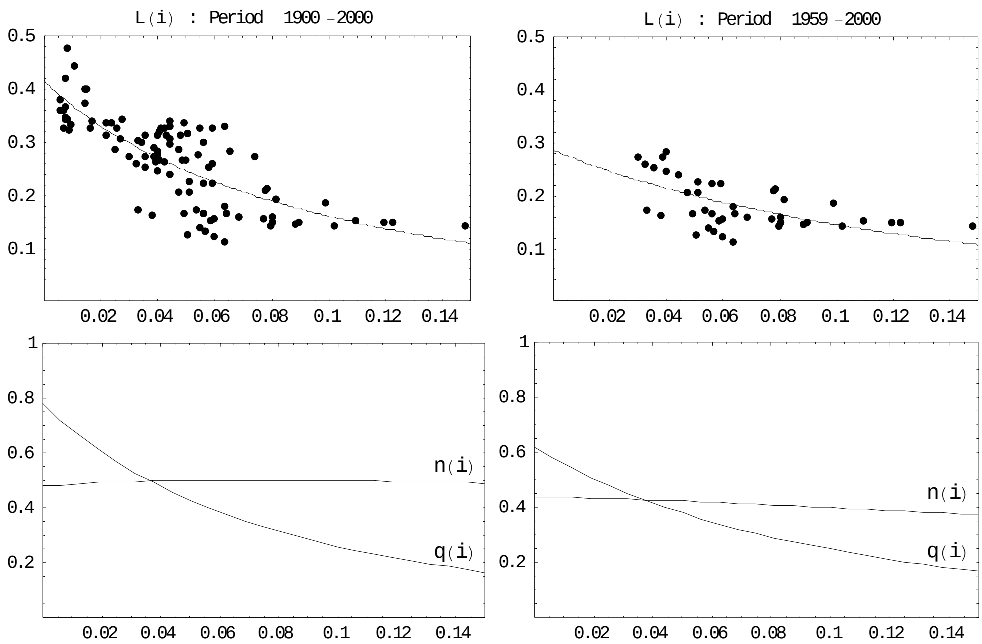

Figure 3. Quality of fit and equilibrium values of q and n in Search Equilibrium.

To explain the difference across models, first note that under bargaining there is a holdup problem in money demand. Second, when agents decide to become sellers they do not internalize the effect of their decisions on the composition of the market and the frequency of trade. These two frictions raise the cost of inflation. Interestingly, n is now nonmonotonic in i. This reflects the fact that inflation has two effects on agents' incentive to become a seller. First, it raises the opportunity cost of holding money, which hurts buyers. Second, it reduces q which affects buyers' and sellers' shares of the match surplus. The buyer's share in equilibrium is $\frac{\theta u'(q)}{\theta u'(q)+(1-\theta)c'(q)}$, which is decreasing in q. Therefore as i increases, q decreases and buyers extract a larger fraction of the gains from trade. This first effect dominates for low values of i while the second dominates for larger values. To illustrate these two effects, we represent (20) and (21) by the curves Q and N in the following figure.

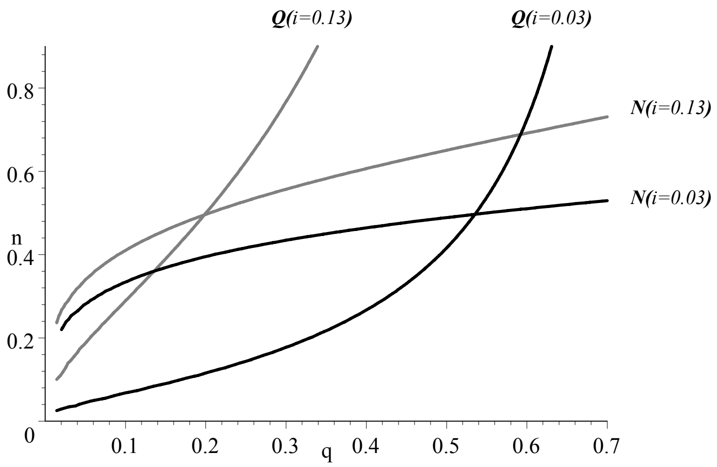

Figure 4. Equilibrium under bargaining.

The cost of inflation depends on θ, and a change in bargaining power can mitigate or exacerbate the effects described above. As θ gets bigger the holdup problem should be less severe, but the effect on the extensive margin is less obvious. To investigate this, we first vary θ while keeping (a, A) constant. As reported in Table 1, in this case $\bar{\Delta}_F$ decreases with θ whereas $\bar{\Delta}_1$ is actually nonmonotonic and, in particular, is smaller at $\theta = 0.2$ than $\theta = 0.5$. This is because at $\theta = 0.2$ inflation has a positive effect on the number of trades. However, this positive effect on the extensive margin is outweighed by the negative effect on the intensive margin, and so any inflation is still bad for welfare at $\theta = 0.2$. At $\theta = 0.8$, on the other hand, the positive effect on the extensive margin outweighs the effect on the intensive margin, and a small deviation from the Friedman Rule is good for welfare, as seen in Figure 5. The reason this happens is that when θ is big the holdup problem is not too severe. We think that it is always interesting to see a

θ	0.2	0.4	0.5	0.6	0.8
$\bar{\Delta}_1$	2.78	3.21	3.10	2.95	2.83
$\bar{\Delta}_F$	4.14	4.11	3.77	3.40	2.95
$q_{1.1}$	0.07	0.17	0.20	0.21	0.16
q_F	0.63	0.74	0.78	0.83	0.92
$n_{1.1}$	0.75	0.58	0.49	0.40	0.20
n_F	0.77	0.57	0.48	0.38	0.19

Table 1: Equilibrium and welfare.

model where inflation may be beneficial, simply because the Friedman Rule is so robust in monetary economics.[22]

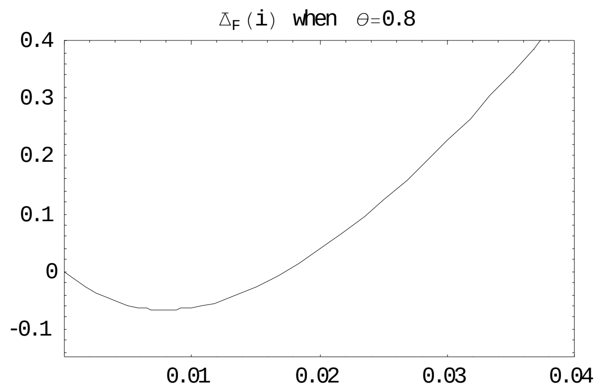

Figure 5. Welfare cost of inflation when $\theta = 0.8$.

We repeat that in the above calculations we vary θ but keep the same (a, A). We can also do the exercise where we refit (a, A) for each value of θ. As shown in Table 2, the cost of inflation is now a non-monotonic function of θ, and tends to be bigger when θ is further from 1/2. In all cases shown in this table, the Friedman Rule is the optimal monetary policy. Also, we can show how the extensive margin matters by computing Δ_F when n is

[22] As discussed in the Introduction, some inflation may also be good in the model of Shi (1997), for different but not unrelated reasons. This contrasts sharply with the model in Rocheteau and Wright (2003), where the extensive margin is captured using free entry by sellers, and we can prove that the Friedman rule is optimal in search equilibrium for any bargaining power.

θ	0.2	0.4	0.5	0.6	0.8
$\bar{\Delta}_1$	7.41	4.01	3.10	2.56	4.48
$\bar{\Delta}_F$	10.14	5.06	3.77	2.99	5.44
$q_{1.1}$	0.08	0.17	0.20	0.22	0.16
q_F	0.55	0.73	0.78	0.81	0.78
$n_{1.1}$	0.71	0.58	0.49	0.39	0.10
n_F	0.76	0.57	0.48	0.38	0.18

Table 2: Equilibrium and welfare.

θ	0.2	0.4	0.5	0.6	0.8
$\bar{\Delta}_F$	10.97	5.03	3.78	3.03	3.36

Table 3: Welfare cost of inflation when n is exogenous.

exogenous and equal to its value at the Friedman Rule, as shown in Table 3. Comparison of Tables 2 and 3 shows that having n endogenous mitigates the cost of inflation when θ is small and exacerbates it when θ is high. The reasoning is that for low values of θ, inflation has a positive effect on the extensive margin while for high values of θ it has a negative effect.

Although symmetric bargaining may be a natural benchmark, another way to pick θ is to choose it to generate a markup μ (price over marginal cost) consistent with the data. We target $\mu = 1.1$, which is standard following Basu and Fernald (1997). In the model, real marginal cost is $c'(q)$ and nominal marginal cost is $c'(q)/\phi_{+1}$. The price in the decentralized market is M^b/q. Therefore, the markup in the decentralized market is $\phi_{+1}M^b/\left[c'(q)q\right] = z(q)/\left[c'(q)q\right]$. The markup in the centralized market is one. We aggregate markups using the shares of output produced in each sector. A markup of $\mu = 1.1$ implies $\theta = 0.3$, $(a, A) = (0.2615, 0.4964)$, and this yields $\bar{\Delta}_1 = 5.36$ and $\bar{\Delta}_F = 7.03$. One has to interpret this somewhat cautiously, however. If there are other reasons for a positive markup, such as elements of monopolistic competition in the centralized market, say, one may not want to attribute $\mu = 1.1$ entirely to bargaining in the decentralized market.

To summarize, in the presence of bargaining the welfare cost of inflation is bigger than what is usually found in studies adhering to the competitive paradigm. Although the exact numbers depend on some details, for the most reasonable calibrations $\bar{\Delta}_1$ is in a range of approximately 3% to 5%. The key feature of the model is the holdup problems that are common in environments with bargaining. We found that extensive margin effects tends to mitigate the cost of inflation when the bargaining power of buyers is low, and to exacerbate the cost when it is high. Usually Friedman Rule is the optimal policy, although we found examples where it is not.

4.3 Competitive Equilibrium

In this model, the data yield $(a, A) = (0.0983, 1.1144)$.[23] A 10% inflation now implies $\bar{\Delta}_1 = 1.54$ and $\bar{\Delta}_F = 1.65$, which is smaller than the measure we obtained under bargaining but still a bit bigger than typical measures in the literature. In competitive equilibrium, the monetary holdup problem vanishes ($q_F = 1$), which reduces the cost of inflation as compared to search equilibrium. However, the market-clearing price does not internalize the effects on the extensive margin, since $n_F = 0.45 < 0.5 = n^*$. This inefficiency explains the relatively higher cost of inflation.

In this case, if we refit the model to the period 1959-2000, the best fit is obtained for $a \approx 0$. When we restrict a not to be smaller than 0.01 we get $(a, A) = (0.01, 0.2478)$ and $\bar{\Delta}_1 = 0.82$. Again, the estimated cost of inflation is lower in the more recent sample. For these parameter values, a deviation from the Friedman Rule is optimal, and welfare is maximized for $i \approx 0.01$. To explain this result, note that for these parameters $n_F = 0.9$. An increase in i above the Friedman Rule reduces n and therefore raises the number of

[23] This model was somewhat harder to fit to the money demand data. It seems to work better when we omit one observation, the year 1981, which is a bit of an outlier.

trades and welfare. While this result is sensitive to the calibration, we think it is at least interesting that a case where the optimal policy is $i > 0$ can be derived for parameters that are not implausible.

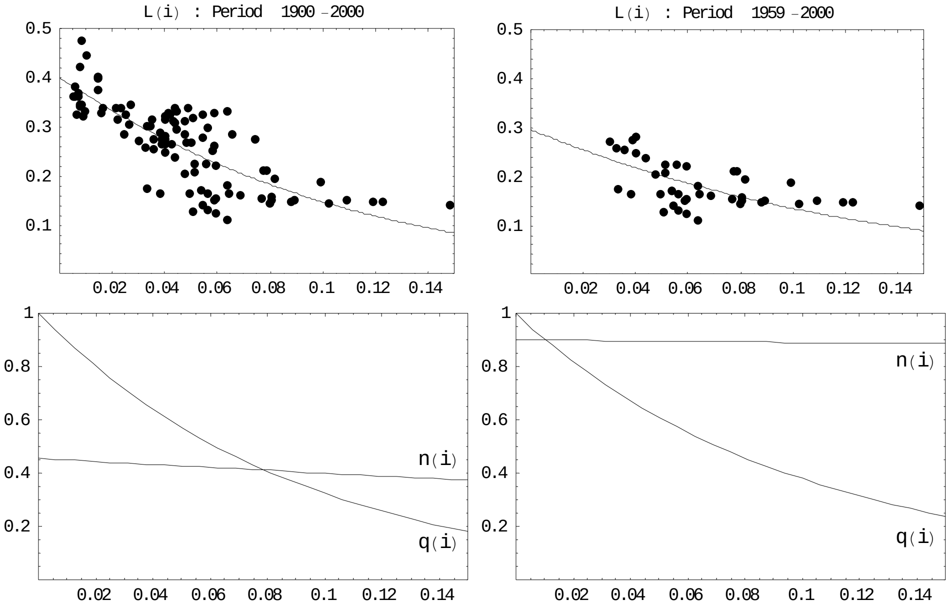

Figure 6. Quality of fit and equilibrium values of q and n in Competitive Equilibrium

Generally, the results for competitive equilibrium are sensitive to the choice of parameter values.[24] If we keep (A, a) constant but increase δ we find that a deviation from the Friedman Rule is welfare improving for all values of δ larger than 1.13. For these parameters, $n_F > 0.5$ and a deviation from the Friedman Rule brings n closer to n^*. Because $q = q^*$ at the Friedman Rule, a small change in q has only a second order effect and the positive welfare effect on n dominates. Furthermore, if δ is large enough (larger than 1.3) a deviation from the Friedman Rule also has a positive effect on output. This is interesting as there is some empirical evidence of a positive output effect of inflation for low inflation economies, such as Bullard and Keating (1995).

[24]For example, if $\delta = 1.2$ we get $(a, A) = (0.3915, 1.8643)$ and $\bar{\Delta}_F = 3.96$, and set $\delta = 1.5$ we get $(a, A) = (0.6496, 2.0866)$ and $\bar{\Delta}_F = 9.12$. The point is simply that results are very sensitive to parameter values with this mechanism.

To summarize, in competitive equilibrium the welfare cost of inflation is sensitive to parameter values, but under our benchmark calibration it is bigger than usually found in the literature because of the endogenous composition of buyers and sellers. It is smaller than under bargaining, however, because there is no holdup problem on money demand. In some cases, a deviation from the Friedman Rule can improve welfare if it happens to raise the number of trades, and can also increase output, at least for moderate inflation rates.[25]

5 Conclusion

In this paper we have analyzed inflation in some models with trading frictions. We did this under three alternative trading mechanisms: bargaining, price taking, and price posting. The quantity of output one gets in exchange for money as well as the frequency of trade are endogenous in the model, which allowed us to distinguish the effects of inflation on the intensive and extensive margins. We calibrated parameters to match some simple observations, and calculated the welfare cost of inflation under various scenarios.

The main conclusions are as follows. First, the cost of inflation is big under bargaining: assuming symmetry, eliminating a 10% inflation is worth about 3% of consumption. This is due to the holdup problem in money demand emphasized in Lagos and Wright (2002). That problem is absent under price taking or posting. Under price taking the cost of inflation can still be big, but for a different reason: the frequency of trade is generally

[25] We checked the robustness of our results to the period length by taking the period to be a month. That is, we did not use monthly data, but transformed the data and model into monthly equivalents. Given this, we also fit a more general function $\alpha(n) = \mu n$ to capture the trading frictions, since when the period is shorter it makes more sense to allow $\alpha(n) < 1$ even at $n = 1$. In any case, the results did not change substantially. For example, the competitive search equilibrium model now implies $\bar{\Delta}_F = 1.23$, almost exactly the same as the yearly model, while the search equilibrium model with $\theta = 0.5$ now implies $\bar{\Delta}_F = 2.93$, only slightly smaller than the yearly model.

inefficient, and inflation can make this worse. Depending on parameter values inflation can also raise the frequency of trade, in which case a deviation from the Friedman Rule may be optimal. Under price posting the cost of inflation is close to previous estimates, around 1%.

Several extensions seem worth exploring. First, we endogenized the frequency of trades by allowing agents to choose to be either buyers or sellers. This modeling choice was mainly to make calibration easier. More work remains to be done to see how the results compare to models that capture the extensive margin in other ways, including endogenous search intensity. It would be interesting to introduce other distortions to see how they interact with the effects in our model. Certainly there is a lot more to be done in terms of fitting the model to the data; we used the simple approach in Lucas (2000) mainly to facilitate comparison, but this can be considered a preliminary step. Finally, we only studied economies where the distribution of money holdings across buyers is degenerate in equilibrium. It is known that the cost of inflation changes when one considers models that do not have this property, such as Molico (1999). Exploring these extensions is left to future work.

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